

Part I

1. let $x = 4 + 5$ in $(3 * x)$
 let $x = 9$ in $(3 * x)$ arithmetic
 $(3 * 9)$ let reduction
 27 arithmetic
2. $(\lambda x \rightarrow 3 * x) (4 + 5)$
 $(\lambda x \rightarrow 3 * x) 9$ arithmetic
 $3 * 9$ lambda reduction
 27 arithmetic
3. $((\lambda x \rightarrow (\lambda y \rightarrow x + (3 * y))) 4) 1$
 $(\lambda y \rightarrow 4 + (3 * y)) 1$ lambda reduction
 $4 + (3 * 1)$ lambda reduction
 $4 + 3$ arithmetic
 7 arithmetic
4. let $x = 4$ in (let $y = 1$ in $(x + (3 * y)))$
 (let $y = 1$ in $(4 + (3 * y)))$ let reduction
 $(4 + (3 * 1))$ let reduction
 $4 + 3$ arithmetic
 7 arithmetic
5. let $x = 4$ in (let $y = 1 + x$ in $(x + (3 * y)))$
 (let $y = 1 + 4$ in $(4 + (3 * y)))$ let reduction
 (let $y = 5$ in $(4 + (3 * y)))$ arithmetic
 $(4 + (3 * 5))$ let reduction
 $(4 + 15)$ arithmetic
 19 arithmetic
6. $((\lambda x \rightarrow (\lambda y \rightarrow x + (3 * x))) 4) 1$
 $(\lambda y \rightarrow 4 + (3 * 4)) 1$ lambda reduction
 $4 + (3 * 4)$ lambda reduction
 $4 + 12$ arithmetic
 16 arithmetic
7. $((\lambda x \rightarrow (\lambda y \rightarrow y + (3 * y))) 4) 1$
 $(\lambda y \rightarrow y + (3 * y)) 1$ lambda reduction
 $1 + (3 * 1)$ lambda reduction
 $1 + 3$ arithmetic
 4 arithmetic
8. $(\lambda y \rightarrow y + ((\lambda y \rightarrow 3 * y) 4)) 5$
 $(\lambda y \rightarrow y + (3 * 4)) 5$ lambda reduction
 $5 + (3 * 4)$ lambda reduction
 $5 + 12$ arithmetic
 17 arithmetic
9. $(\lambda y \rightarrow ((\lambda y \rightarrow 3 * y) 4) + y) 5$
 $(\lambda y \rightarrow (3 * 4) + y) 5$ lambda reduction
 $(3 * 4) + 5$ lambda reduction
 $12 + 5$ arithmetic
 17 arithmetic

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10. $(\lambda x \rightarrow x * (\text{let } x = 3 * 2 \text{ in } (x + 7)) + x) 4$

$(\lambda x \rightarrow x * (\text{let } x = 6 \text{ in } (x + 7)) + x) 4$

$(\lambda x \rightarrow x * ((6 + 7) + x) 4$

$4 * (6 + 7) + 4$

$4 * 13 + 4$

$52 + 4$

56

arithmetic

let reduction

lambda reduction

arithmetic

arithmetic

arithmetic

11. $g ((\text{let } x = 4 \text{ in } (\lambda y \rightarrow x + y)) 2)$

$g ((\lambda y \rightarrow 4 + y) 2)$

$g (4 + 2)$

$g (6)$

$(\lambda z \rightarrow z + 4) 6$

$6 + 4$

10

let reduction

lambda reduction

arithmetic

substitution from file

lambda reduction

arithmetic

12. $\text{let } x = 5 \text{ in } (\lambda z \rightarrow x * z)$

$(\lambda z \rightarrow 5 * z)$

let reduction

13. $(\lambda x \rightarrow (\lambda z \rightarrow x * z)) 5$

$(\lambda z \rightarrow 5 * z)$

lambda reduction

14. $f ((\lambda n \rightarrow \text{fn Rock}) (\lambda x \rightarrow \text{whatItBeats } x))$

$f ((\lambda x \rightarrow \text{whatItBeats } x) \text{ Rock})$

$f (\text{whatItBeats } \text{Rock})$

$f ((\lambda s \rightarrow \text{case } s \text{ of } \{\text{Rock} \rightarrow \text{Scissors}; \text{Paper} \rightarrow \text{Rock}; \text{Scissors} \rightarrow \text{Paper}\}) \text{Rock})$

$f (\text{case } \text{Rock} \text{ of } \{\text{Rock} \rightarrow \text{Scissors}; \text{Paper} \rightarrow \text{Rock}; \text{Scissors} \rightarrow \text{Paper}\})$

$f (\text{Scissors})$

$(\lambda s \rightarrow \text{case } s \text{ of } \{\text{Rock} \rightarrow 334; \text{Paper} \rightarrow 138; \text{Scissors} \rightarrow 99\}) \text{Scissors}$

$\text{case } \text{Scissors} \text{ of } \{\text{Rock} \rightarrow 334; \text{Paper} \rightarrow 138; \text{Scissors} \rightarrow 99\}$

99

lambda reduction

lambda reduction

file substitution

lambda reduction

case reduction

file substitution

lambda substitution

case reduction

15. $((\lambda f \rightarrow (\lambda x \rightarrow f (f x))) \text{whatItBeats}) \text{Paper}$

$(\lambda x \rightarrow \text{whatItBeats } (\text{whatItBeats } x)) \text{Paper}$

$(\text{whatItBeats } (\text{whatItBeats } \text{Paper}))$

$(\text{whatItBeats } ((\lambda s \rightarrow \text{case } s \text{ of } \{\text{Rock} \rightarrow \text{Scissors}; \text{Paper} \rightarrow \text{Rock}; \text{Scissors} \rightarrow \text{Paper}\}) \text{Paper}))$

$(\text{whatItBeats } (\text{case } \text{Paper} \text{ of } \{\text{Rock} \rightarrow \text{Scissors}; \text{Paper} \rightarrow \text{Rock}; \text{Scissors} \rightarrow \text{Paper}\}))$

$(\text{whatItBeats } (\text{Rock}))$

$((\lambda s \rightarrow \text{case } s \text{ of } \{\text{Rock} \rightarrow \text{Scissors}; \text{Paper} \rightarrow \text{Rock}; \text{Scissors} \rightarrow \text{Paper}\}) \text{Rock})$

$\text{case } \text{Rock} \text{ of } \{\text{Rock} \rightarrow \text{Scissors}; \text{Paper} \rightarrow \text{Rock}; \text{Scissors} \rightarrow \text{Paper}\}$

Scissors

lambda reduction

lambda reduction

file substitution

lambda reduction

case reduction

file substitution

lambda reduction

case reduction

16. $\text{whatItBeats } (\text{case } \text{Paper} \text{ of } \{\text{Rock} \rightarrow \text{Paper}; \text{Paper} \rightarrow \text{Rock}; \text{Scissors} \rightarrow \text{Scissors}\})$

$\text{whatItBeats } (\text{Rock})$

$((\lambda s \rightarrow \text{case } s \text{ of } \{\text{Rock} \rightarrow \text{Scissors}; \text{Paper} \rightarrow \text{Rock}; \text{Scissors} \rightarrow \text{Paper}\}) \text{Rock})$

$\text{case } \text{Rock} \text{ of } \{\text{Rock} \rightarrow \text{Scissors}; \text{Paper} \rightarrow \text{Rock}; \text{Scissors} \rightarrow \text{Paper}\}$

Scissors

case reduction

file substitution

lambda reduction

case reduction

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17. (case (Win Rock) of {Draw -> whatItBeats; Win z -> (\s -> Scissors)}) Paper
(\s -> Scissors) Paper case reduction
Scissors lambda reduction

18. case (Win (whatItBeats Rock)) of {Draw -> n; Win x -> (n + f x)}

case (Win ((\s -> case s of {Rock -> Scissors; Paper -> Rock; Scissors -> Paper}) Rock)) of {Draw -> n; Win x -> (n + f x)} file substitution
case (Win (case Rock of {Rock -> Scissors; Paper -> Rock; Scissors -> Paper}) of {Draw -> n; Win x -> (n + f x)}) lambda reduction
case (Win Scissors) of {Draw -> n; Win x -> (n + f x)} case reduction
(n + f Scissors) case reduction
1 + f Scissors file substitution
1 + (\s -> case s of {Rock -> 334; Paper -> 138; Scissors -> 99}) Scissors file substitution
1 + (case Scissors of {Rock -> 334; Paper -> 138; Scissors -> 99}) lambda reduction
1 + 99 case reduction
100 arithmetic

19. let y = 2 in (case (Win (whatItBeats Rock)) of {Draw -> n; Win y -> (n + f y)} + y)

let y = 2 in (case (Win ((\s -> case s of {Rock -> Scissors; Paper -> Rock; Scissors -> Paper}) Rock)) of {Draw -> n; Win x -> (n + f y)+y}) file substitution
let y = 2 in (case (Win (case Rock of {Rock -> Scissors; Paper -> Rock; Scissors -> Paper}) of {Draw -> n; Win x -> (n + f y)+y}) lambda reduction
let y = 2 in case (Win Scissors) of {Draw -> n; Win y -> (n + f y)} + y case reduction
let y = 2 in n + f Scissors + y case reduction
n + f Scissors + 2 let reduction
1 + f Scissors + 2 file substitution
1 + (\s -> case s of {Rock -> 334; Paper -> 138; Scissors -> 99}) Scissors + 2 file substitution
1 + (case Scissors of {Rock -> 334; Paper -> 138; Scissors -> 99}) + 2 lambda reduction
1 + 99 + 2 case reduction
100 + 2 arithmetic
102 arithmetic

Part II.

Construct an expression e, whose value depends on a shape represented by the variable s and a number represented by the variable x, such that: if the shape s is Rock, then e evaluates to the square of the number x; if the shape s is Paper, then e evaluates to the cube of the number x; and if the shape s is Scissors, then e evaluates to the number x.

$$e \text{ s } x = \text{case } s \text{ of } \{\text{Rock} \rightarrow x * x; \text{Paper} \rightarrow x * x * x; \text{Scissors} \rightarrow x\}$$

$$[\text{Rock}/s][4/x]e \Rightarrow 16 \quad = \quad e \text{ Rock } 4$$

$$e \text{ Rock } 4 = \text{case Rock of } \{\text{Rock} \rightarrow 4 * 4; \text{Paper} \rightarrow 4 * 4 * 4; \text{Scissors} \rightarrow 4\}$$

$$4 * 4 \quad \text{case reduction}$$

$$16 \quad \text{arithmetic}$$

$$[\text{Paper}/s][4/x]e \Rightarrow 64 \quad = \quad e \text{ Paper } 4$$

$$e \text{ Paper } 4 = \text{case Paper of } \{\text{Rock} \rightarrow 4 * 4; \text{Paper} \rightarrow 4 * 4 * 4; \text{Scissors} \rightarrow 4\}$$

$$4 * 4 * 4 \quad \text{case reduction}$$

$$64 \quad \text{arithmetic}$$

$$[\text{Scissors}/s][4/x]e \Rightarrow 4 \quad = \quad e \text{ Scissors } 4$$

$$e \text{ Paper } 4 = \text{case Paper of } \{\text{Rock} \rightarrow 4 * 4; \text{Paper} \rightarrow 4 * 4 * 4; \text{Scissors} \rightarrow 4\}$$

$$4 \quad \text{case reduction}$$