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<u>CS 181: HW #5</u>

Zero. One of the CFGs for the language of strings of balanced ()'s shown in lecture: S -> S S | (S) | ε

was shown to be ambiguous by pointing out that strings have more than one parse tree in the grammar. Besides showing multiple parse trees for the same string, there are two other ways to show that a CFG is ambiguous.

a. State briefly in English what the other two ways to show that a grammar is ambiguous are.

> 1. Determine if there exists a string generated by the grammar such that it has more than one corresponding leftmost derivation
>
> 2. Determine if there exists a string generated by the grammar such that it has more than one corresponding rightmost derivation

b. Choose one of those other two ways, and use it to show that this grammar is ambiguous. State which of the other two ways you have chosen to use.

> I will show (1) – there exists a string generated by the grammar such that it has more than one corresponding leftmost derivation
>
> S => SS => (S)S => ( )S => ( )SS=> ( )(S)S => ( )( )S=> ( )( )(S) => ( ) ( ) ( )
>
> OR
>
> S => SS => SSS => (S)SS=> ( )SS => ( )(S)S => ( )( )S => ( ) ( ) (S) => ( ) ( ) ( )

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One. Show an example of two languages, N and R, such that:

N is not a finite state language,

R is a finite state language,

and N $\subset$ R (proper subset)

Briefly justify your answer. You may want consult the lists of languages posted on CCLE for the Midterm, and you may choose to cite any of those languages in your answer.

Let R = $\{0^a 1^b \mid a,b \in \mathbb{N}\}$ which is a regular language.
Let N = $\{0^n 1^n \mid n \in \mathbb{N}\}$, which is a known non-finite state language.

N $\subset$ R, N, a known non-finite state language, is a proper subset of R, a finite state language.

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Two. Let L2 be a language over some alphabet, and suppose the complement of L2 is not a FSL. What does that tell us about L2?

> FSLs are closed under the complement operation. If L2's complement was a FSL, then we could conclude that L2 is also an FSL. Therefore, if L2's complement is not an FSL, then L2 cannot be an FSL.
>
> Note: If the language in question was a CFL, CFLs are not closed under the complement operation, and therefore the characterization of L2 would be unknown without additional information.

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Three. Show that the following language over alphabet Sigma = {a, b} is not a FSL using the Pumping Lemma:

$$\{a^i b^j \in \sum + \mid i \neq j\}$$

Proof: Let L = $\{a^i b^j \in \sum + \mid i \neq j\}$
Assume for the sake of contradiction that L is a FSL
By pumping lemma, there exists a pumping length p, p ≥ 1
Consider the case s = $a^p b^{p+\beta}, s \in$ L, i≠j, $\beta \geq 1$
By the pumping lemma, there exists x, y, z such that s = xyz and |xy| ≤ p
Since s starts with p a's, then x and y consists of a's
Let x = a^α, y = a^β, z = $a^{p-\alpha-\beta} b^{p+\beta}$
By the pumping lemma, |y| ≥ 1, $\beta \geq 1$
Then, by the pumping lemma, for all i ≥ 0, xy^iz ∈ L
In particular, consider i = 2

$$\begin{aligned} L \ni (xy^2z) &= a^\alpha (a^\beta)^2 a^{p-\alpha-\beta} b^{p+\beta} \\ &= a^\alpha (a^{2\beta})(a^{p-\alpha-\beta} b^{p+\beta}) \\ &= a^{\alpha+2\beta+p-\alpha-\beta} b^{p+\beta} \\ &= a^{p+\beta} b^{p+\beta} \end{aligned}$$

$a^{p+\beta} b^{p+\beta}$ is NOT in L, i = $p+\beta$, j = $p+\beta$, $i = j$, xy^2z is NOT in L
Therefore, by contradiction of the pumping lemma, L is a non-FSL.