Stefanie Shidoosh

CS 181: HW #1

0. Briefly explain the system used in the Sipser textbook to number the sections, subsections, exercises, problems, figures, examples, theorems, etc..

Chapters are numbered and have a title page with the corresponding heading.
Sections are titled in all capitals by its subject and numbered with respect to the chapter and the order of its appearance. As an example, the second section of the first chapter is 1.2 Nondeterminism.
Subsections are capitalized and titled by their subject, and not numbered.
Problems are listed after Exercises at the end of the chapter.
Figures, examples, theorems, corollaries, etc., are labeled by their respective category, and are numbered by the chapter and the order of its appearance.

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1. Let G and H be two undirected graphs, which are disjoint from each other. Assume each of G and H is connected, acyclic, and non-empty (at least one node). Prove the following two statements:

a. Adding one edge between a node in G and a node in H results in a single connected, acyclic graph.

Suppose there exists a cycle C in the graph, G', produced by adding an edge, e, between G and H.

For all v ∈ V(C), v ~∈ V(G) and v ~∈ V(H)

Thus, C ∩ G and C ∩ H have at least one node

For any v_0 ∈ C ∩ G and v_1 C ∩ H, the path between v_0 and v_1 must contain e

For C to exist, there must exist at least 2 distinct paths, such that they do not both utilize e, from v_0 and v_1

This is a contradiction as any such path from v_0 and v_1 must contain the added edge e

Therefore, no such C exists, and G' is noncyclic.

G' is connected as connecting the connected graphs G and H maintains connectedness.

b. Adding *two distinct* edges between G and H, as in 3.a, results in a single connected, *cyclic* graph.

Consider "For any v_0 ∈ C ∩ G and v_1 C ∩ H, the path between v_0 and v_1 must contain e. For C to exist, there must exist at least 2 distinct paths, such that they do not both utilize e, from v_0 and v_1" from 1.a.

However, adding 2 distinct edges, e_0, e_1, allow for at least 2 distinct paths from v_0 and v_1 as each path contains e_0 and e_1, respectively.

Therefore, G' is consequently cyclic.

G' is connected as connecting the connected graphs G and H maintains connectedness.

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2. Let Σ={a, b, c, 0, 1}. Let X be the set {x, y, z} and let B be the set {0, 1}. Answer the following questions.

a. List the elements of B x (B x X):
{(0, (0,x)), (0, (0,y)), (0, (0,z)),
(0, (1,x)), (0, (1,y)), (0, (1,z)),
(1, (0,x)), (1, (0,y)), (1, (0,z)),
(1, (1,x)), (1, (1,y)), (1,(1,z))}

b. List the elements of (B x B) x X:
{((0,0), x), ((0,0, y), ((0,0), z),
((0,1), x), ((0,1), y), ((0,1), z),
((1,0), x), ((1,0), y), ((1,0), z),
((1,1), x), ((1,1), y), ((1,1), z)}

c. List the elements of the power set $\mathcal{P}$(X) (all subsets of X)
{{}, {x}, {y}, {z}, {x,y}, {x,z}, {y,z}, {x, y z}}

~~d. List the elements of the language concatenation: B • X~~
~~{0x, 0y, 0z, 1x, 1y, 1z}~~

~~e. What is the language concatenation B+ • {}?~~
~~It is the empty set, {}.~~

f. What is the Cartesian Product {} x X*?
It is the empty set, {}.

g. What is the Cartesian Product {ε, 1} x X?
{(ε,x), (ε,y), (ε,z), (1,x), (1,y), (1,z)}

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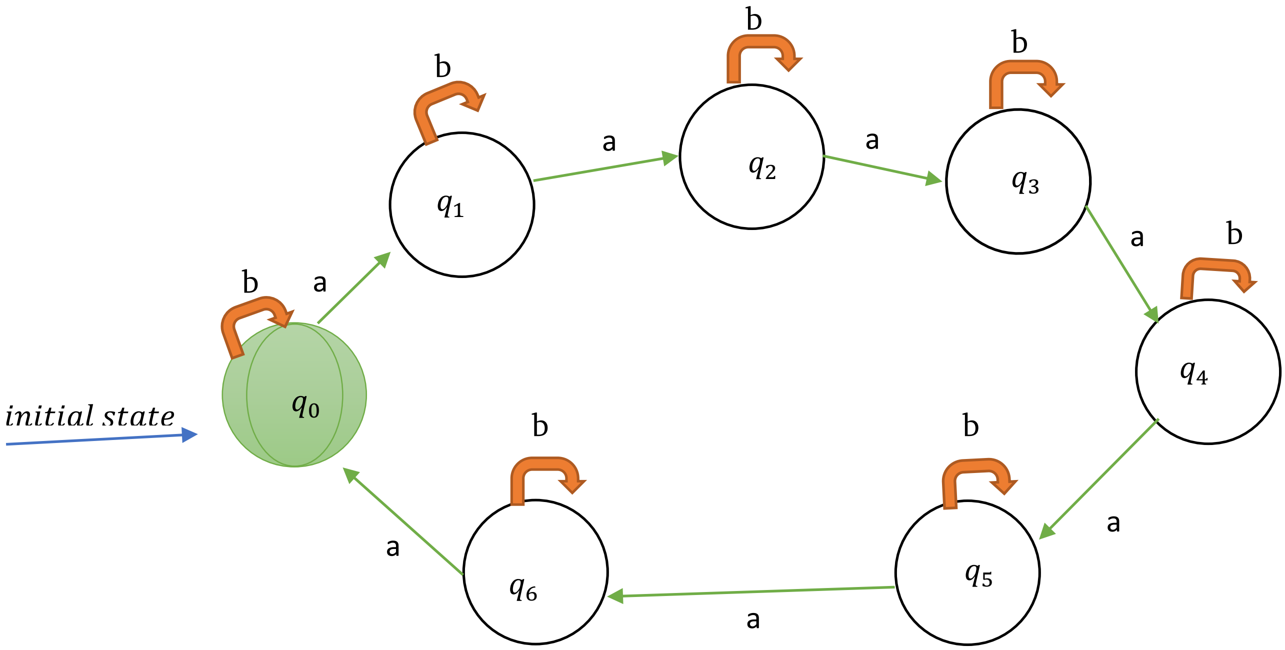
3. Let alphabet $\Sigma = \{a,b\}$. Show a DFA which recognizes the following language over Sigma. Show the DFA as a fully specified state diagram. Be sure to clearly indicate your initial state and accepting state(s).

L = {w ∈ Σ* | the number of a's in w is evenly divisible by 7}

The state q_0 is the initial state and only accepting state.

The design checks for multiple of 7 exact occurrences of "a" in the word. If "b" is the next symbol, the state is not changed, as "b" does not contribute to L's definition. If "a" is the next symbol, the next state is visited; every 7th traversal of this kind, the word is reaches the accept state.



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4. Let alphabet $\Sigma = \{0,1\}$. Show a DFA which recognizes the following language over Sigma. Show the DFA using the formal 5-tuple notation. Make sure you clearly defne all five components of the DFA, and ensure your transition function is full specified. You may define the transition function using a function table or using more succinct mathematical notation. Briefly describe how your design works.

$L = \{w \in \Sigma^* \mid$ w has an even parity, and no run of 0's is longer than 3$\}$

Let $M = (Q, \Sigma, \delta, q_0, F)$

$Q = \{q_{[1,3]}, q_{[1,4]}, q_{[1,5]}, q_{[1,6]}, q_{[1,7]}, q_{[1,F]}, q_{[2,3]}, q_{[2,4]}, q_{[2,5]}, q_{[2,6]}, q_{[2,F]}\}$

$\Sigma = \{0, 1\}$

$\delta =$

$\boldsymbol{\delta}$	**0**	**1**
$q_{[1,3]}$	$q_{[1,4]}$	$q_{[2,3]}$
$q_{[1,4]}$	$q_{[1,5]}$	$q_{[2,3]}$
$q_{[1,5]}$	$q_{[1,6]}$	$q_{[2,3]}$
$q_{[1,6]}$	$q_{[1,F]}$	$q_{[2,3]}$
$q_{[1,F]}$	$q_{[1,F]}$	$q_{[1,F]}$
$q_{[2,3]}$	$q_{[2,4]}$	$q_{[1,3]}$
$q_{[2,4]}$	$q_{[2,5]}$	$q_{[1,3]}$
$q_{[2,5]}$	$q_{[2,6]}$	$q_{[1,3]}$
$q_{[2,6]}$	$q_{[2,F]}$	$q_{[1,3]}$
$q_{[2,F]}$	$q_{[2,F]}$	$q_{[2,F]}$

$q_0 = q_{[1,3]}$ ($q_{[1,3]}$ is the start state)

$F = \{ q_{[1,3]}, q_{[1,4]}, q_{[1,5]}, q_{[1,6]}\}$, the set of accept states.

The dead states are $\{q_{[1,F]}, q_{[2,F]}\}$.

This design has 4 accept states, all only reachable if w has an even parity and no run longer than three zeroes. If there is a run of zeroes longer than three zeroes, w reaches a dead state.

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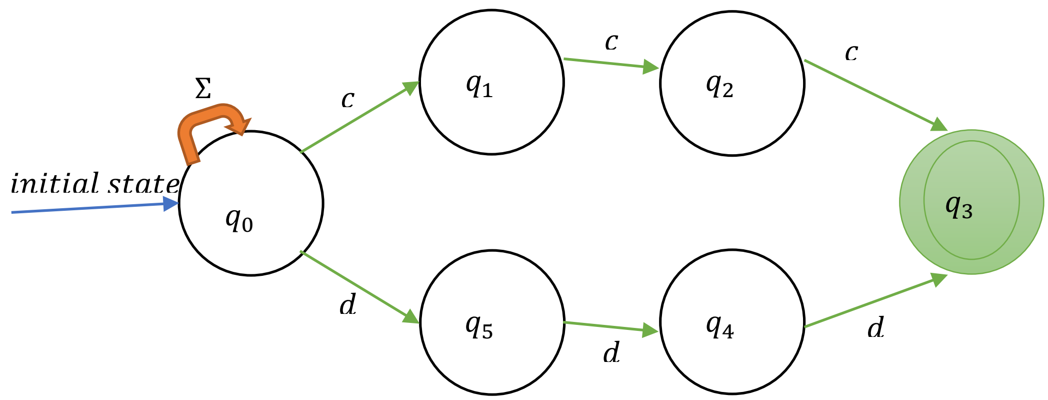
5. Let alphabet $\Sigma = \{c,d,e\}$. Show an NFA which recognizes the following language over Σ. Specify the NFA as a fully specified state diagram. Be sure to clearly indicate your initial state and accepting state. Briefly describe how your design works.

L = { w $\in$ Σ+ | w contains substring ccc or ddd}.

The initial state is q_0, and the accept state is q_3.



This design utilizes an NFA, assuming that it will guess on L's definition and guess right to reach the accept state. When w has substring ccc or ddd, the respective traversal will reach the accept state, q_3. The wrong guesses are contained in the self-loop defined by Σ.

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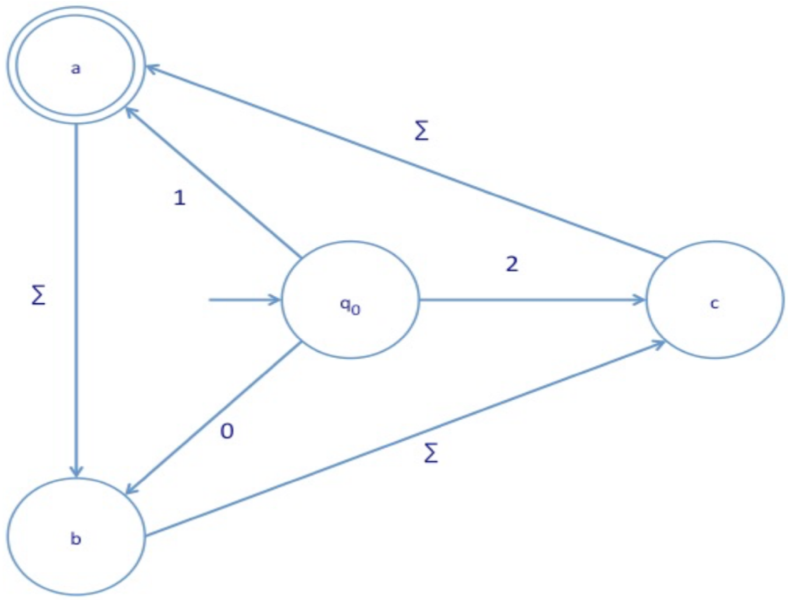
6. Briefly describe in English the language over Σ = {0,1,2} accepted by this DFA



The language accepted by this DFA is:
Words that begin with 1 and followed by multiple of three characters;
Words that begin with 0 and followed by two characters + a multiple of three characters;
Words that begin with 2 and followed by one character + a multiple of three characters.