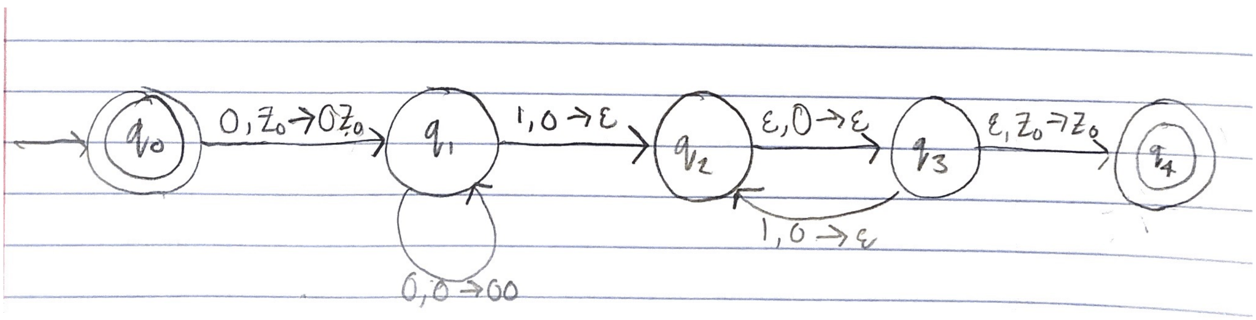
CS 181 HW #7

1. Let $\Sigma = \{0, 1\}$ and let $L = \{0^{2n}1^{n} \mid n \geq 0\}$

Show a DPDA for this language, and *very briefly* describe how it correctly recognizes the language deterministically. There are no additional requirements for your DPDA. In particular, you may find it easier if you do *not* try to make it fully specified; but that is up to you.



q_0 is the initial state. Z_0 denotes the top of the stack. q_0 accepts when $n = 0$. q_4 is an accepting state that accepts input such that $0^{2n}1^{n}$, as q_2 and q_3 require that 2 zeroes are popped for every 1 input.

2. Let $\sum$ = {a, b, c} and let L = $\{a^i b^{2i} c^i \mid i \geq 0\}$
Show that this language is not a CFL using the Pumping Lemma for CFLs. Do this directly using the lemma without employing any other results.

Proof: Suppose for the sake of contradiction that L is a CFL
By the pumping lemma, there exists a pumping length, p ≥ 1.
Consider s = $a^p b^{2p} c^p$, |s| = 4p > p, s is in L
By the pumping lemma, since |s| > p, there exists strings u, v, x, y, z such that s = uvxyz, |vxy| ≤ p, |vy| > 0, and for all i ≥ 0, uv^ixy^iz ∈ L

Case 1: vxy consists of only a's
Consider i = 0, uv^0xy^0z
The number of a's is equal to p - |vy| which is < p since |vy| > 0 by the pumping lemma. The number of c's remains unchanged, and the number of a's does not equal the number of c's. s not in L when i = 0, and thus is a contradiction.

Case 2: vxy consists of only b's
Consider i = 0, uv^0xy^0z
This will have less than 2p b's, as the number of b's is equal to 2p - |vy|, which is < 2p since |vy| > 0 by the pumping lemma. The number of a's and c's remain unchanged, and the number of b's cannot be twice the number of a's and c's respectively. s is not in L when i=0, and thus is a contradiction.

Case 3: vxy consists of only c's
Consider i = 0, uv^0xy^0z
The number of c's is equal to p - |vy| which is < p since |vy| > 0 by the pumping lemma. The number of a's remains unchanged, and the number of c's does not equal the number of a's. s not in L when i = 0, and thus is a contradiction.

Case 4: vxy consists of a's and b's
Since |vy| > 0, vy must contains at least one a or one b. If vy contains only a's or only b's, then this is the same as cases 1 and 2, respectively. Else, vy must contain both a's and b's, then vy has k a's for some k ≥ 1; then, consider i = 0, uv^0xy^0z; this will have p - k a's and n c's, in which the number of a's and the number of c's are not equal. s in not in L when i = 0, and this is a contradiction.

Case 5: vxy consists of b's and c's
Since |vy| > 0, vy must contains at least one c or one b. If vy contains only c's or only b's, then this is the same as cases 3 and 2, respectively. Else, vy must contain both c's and b's, then vy has k c's for some k ≥ 1; then, consider i = 0, uv^0xy^0z; this will have p-k c's and n a's, in which the number of a's and the number of c's are not equal. s in not in L when i = 0, and this is a contradiction.

Therefore, since we have a contradiction in all cases, L is not a CFL.