Stefanie Shidoosh

CS 181: HW #2

2 (from HW#1). Let Σ= { *x, y, z*, 0, 1 }. Let X be the set {x, y, z} and let B be the set {0, 1}.

d. List the elements of the language concatenation: B • X

{0x, 0y, 0z, 1x, 1y, 1z}

e. What is the language concatenation B^+ • {}?

It is the empty set, {}.

Stefanie Shidoosh

1. Let Σ= {a, b}. Let $L_{reflection}$ = $\{ww^R$ | w in Σ* }. Let L_{PAL} = {w in Σ* | $w = w^R\}$.
Let L_1 = Σ*•((Σ*$)^R$).

a. Are $L_{reflection}$ and L_{PAL} equal?

No, $L_{reflection}$ and L_{PAL} are not equal.

$L_{reflection}$ takes w that is concatenated it with its reversal. This creates a form of a particular type of string known as a palindrome.

L_{PAL} takes only takes w that is equal to its reversal. This also exclusively produces palindromes.

However, L_{PAL} is a more exhaustive set of palindromes, as it includes an odd length of w, |w| % 2 = 1. For example, consider the string w = bab. $L_{reflection}$ would reject this string (but would accept babbab), whereas L_{PAL} would accept it.

b. Are L_{PAL} and L_1 equal?

No, L_{PAL} and L_1 are not equal. Consider the string w of length 1 in L_1, Σ* concatenated with a reversal in Σ* would accept the string ab (or bba), which is not in L_{PAL}.

Stefanie Shidoosh

2. Recall from lecture that a *directed rooted tree* is a connected, directed graph which is acyclic even when viewed as an undirected graph where one node is designated as the root and all edges are directed away from the root towards the leaves. The *height of a directed rooted tree* is defined as the length of a longest path from the root to a leaf. The height can be expressed in terms of the number of nodes on a longest path or in terms of the number of edges on a longest path. We define a *k-ary directed rooted tree* to be a directed rooted tree in which each node has *at most* k children.

a. What is the fewest number of nodes and edges a k-ary directed rooted tree of height n nodes (n-1 edges) can have? *Briefly justify* your answer.

The fewest number of nodes a k-ary directed rooted tree of height n nodes is n. Consider that the height can be expressed in terms of the number of nodes on a longest path. Therefore, consider a k-ary directed rooted tree in which there is one path from root to leaf, and that happens to contain all nodes in the tree. This path would use n nodes at the minimum.

The fewest number of edges a k-ary directed rooted tree of height n nodes is n-1. Consider that the height can be expressed in terms of the number of edges on a longest path. Therefore, consider a k-ary directed rooted tree in which there is one path from root to leaf, and that happens to contain all nodes in the tree. This path, having used n nodes, where n is the height of the tree expressed by the number of nodes on the longest path, must use n-1 edges at the minimum.

b. What is the largest number of nodes and edges a k-ary directed rooted tree of height n nodes (n-1 edges) can have? *Prove your answer by induction on n.*

$k^n - 1$ nodes, which requires $(k^n - 1) - 1$ edges to create the path.

For the root (base case), n = 1, and the number of nodes is $k^{n-1} = 1$

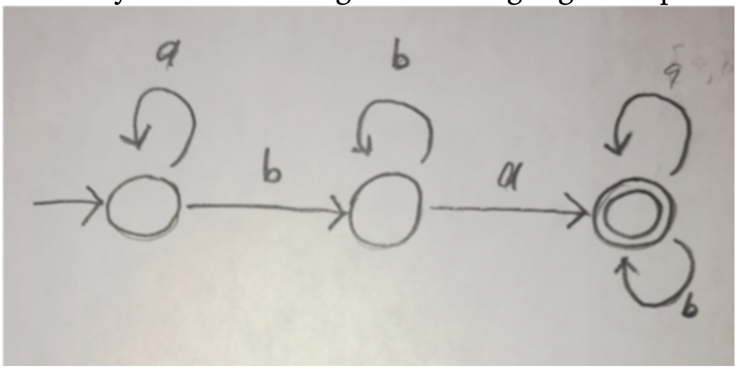
(k can be anything, as anything to the 0 is 1).

Assume that maximum number of nodes on height n is k^{n-1}

Since in a k-ary tree every node has at most k children, the next level would have k times as many nodes at most. A tree has maximum nodes if all levels have maximum nodes. Thus, by induction, the maximum number of nodes in a binary tree of height h is $k^0 + k^1 + k^2 + \ldots + k^{n-1}$, i.e. the simple geometric series with n terms and sum of this series is $k^n - 1$. Finally, for $k^n - 1$ nodes, $(k^n - 1) - 1$ edges are needed create the path.

Stefanie Shidoosh

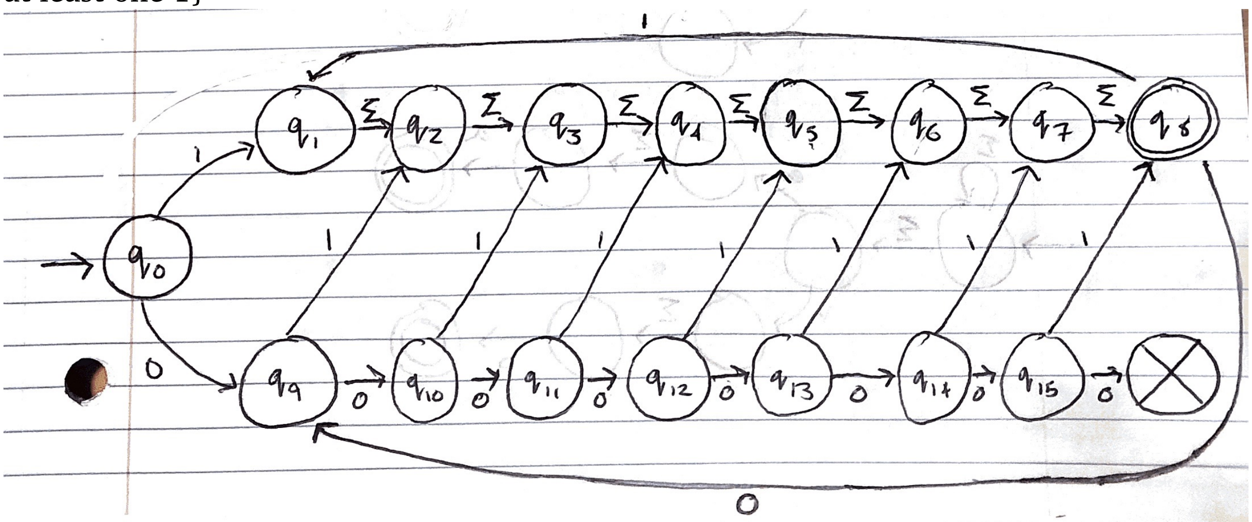
3. Briefly describe in English the language accepted by the following DFA over Σ={a, b}:



The language accepted by the this DFA over Σ={a, b}:
Strings that have at least one b and at least one a, where the first occurrence of b must be followed by at least one occurrence of a.

Stefanie Shidoosh

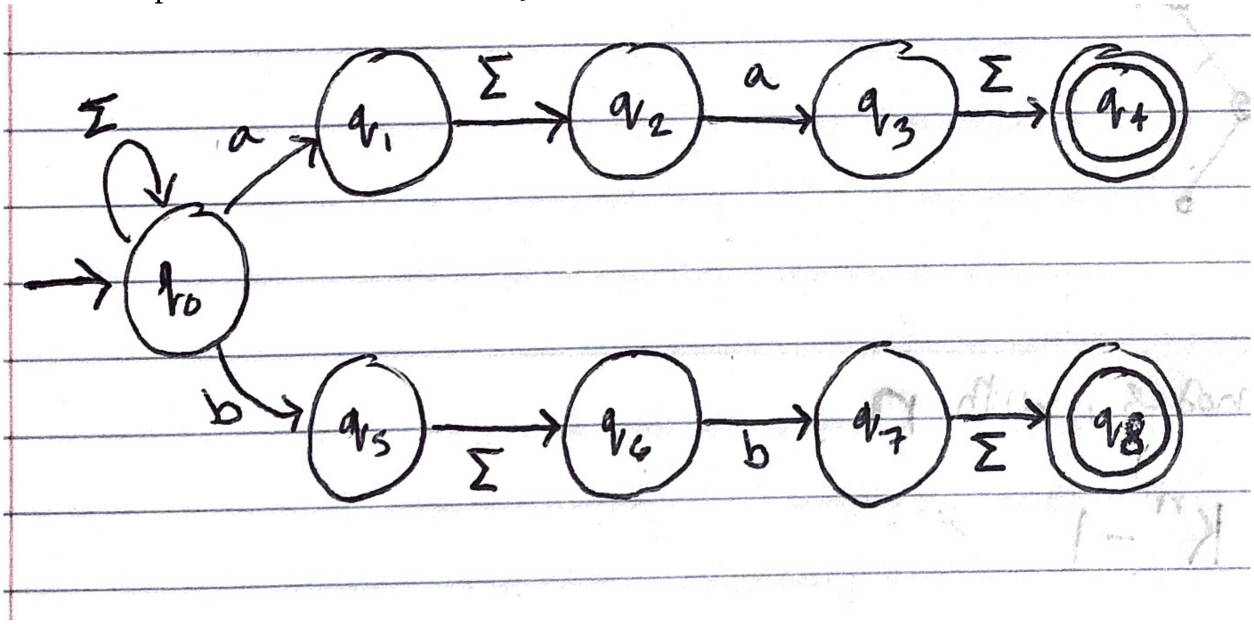
4. Let Σ = {0, 1}. Give a DFA for the following language:
L = {w in Σ+ | |w| is a multiple of 8 and every consecutive block of 8 symbols contains at least one 1}



The start state is q_0, and the accepting state is q_8. This DFA ensures that at least one 1 must be seen before accepting, as a 1 is required to reach to the route to the accepting state. It ensures that |w| is a multiple of 8 as a multiple of 8 traversals is required to end at the accepting state. If there any consecutive block of 8 zeroes, a dead state is reached.

Stefanie Shidoosh

5. Let Σ = {a, b}. Give a DFA or NFA for the following language:
L = {w in Σ+ | |w| > 3 and the same symbol appears in the position 4th from the end and in the position 2nd from the end}



The start state is q_0, and the accepting states are q_4 and q_8. This NFA allows for any symbols and any number of symbols to precede the 4th to end symbol.

If the 4th from the end symbol is an a:

It can be followed by exactly any one symbol,

The symbol after that, the symbol 2nd from the end, must be a

The symbol after that, the final symbol, can be any one symbol and is then accepted.

If the 4th from the end symbol is an b:

It can be followed by exactly any one symbol,

The symbol after that, the symbol 2nd from the end, must be b

The symbol after that, the final symbol, can be any one symbol and is then accepted.

Dead states are left out, as blocking can be assumed for nondeterminism, allowing for the leaving out transitions that will eventually lead to the rejecting of the input.

Stefanie Shidoosh

6. Let $\Sigma = \{a,b,c\}$. Give a regular expression for the following language:
$L = \{w \text{ in } \Sigma+ \mid w \text{ contains at least one b and at least one c}\}$

$$(\Sigma^*b\Sigma^*c\Sigma^*) \cup (\Sigma^*c\Sigma^*b\Sigma^*)$$

$(\Sigma^*b\Sigma^*c\Sigma^*)$ considers w in which the first occurrence of b occurs before the first occurrence of c. It allows for any number of characters of any kind in Σ to precede and proceed b until the first occurrence of c, which then can be followed by any number of characters of any kind in Σ.
$(\Sigma^*c\Sigma^*b\Sigma^*)$ considers w in which the first occurrence of c occurs before the first occurrence of b. It allows for any number of characters of any kind in Σ to precede and proceed c until the first occurrence of b, which then can be followed by any number of characters of any kind in Σ.
The union is to consider both w, in which the first occurrence of b occurs before the first occurrence of c or the first occurrence of c occurs before the first occurrence of b.