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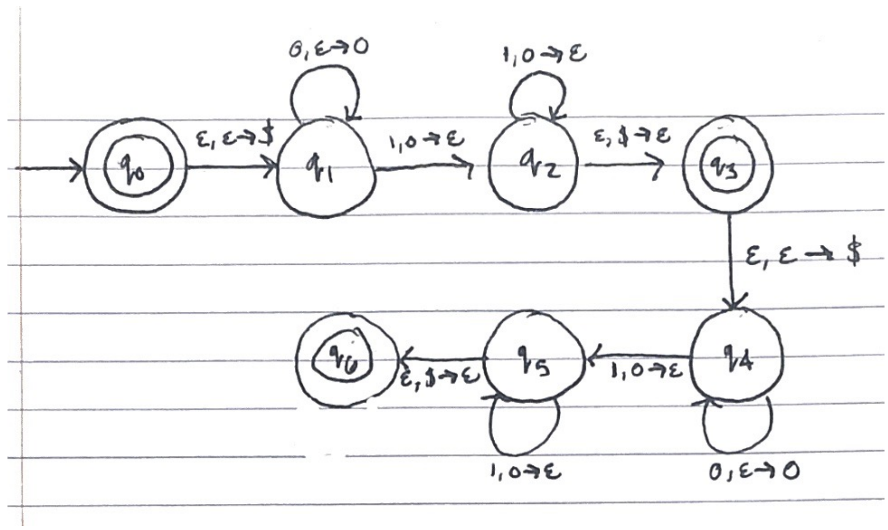
CS181: HW #6

1. Let $\Sigma = \{0,1\}$, and let

$$L = \{0^i 1^i 0^j 1^j \mid i, j \geq 0\}$$

(You may recognize the basic structure of this language from last week.) Show a PDA for this language, and *very briefly* describe how it correctly recognizes the language. There are no additional requirements for your PDA.



q_0 is the initial state. 0^i1^i is guaranteed if the input reaches q_3: i zeros are pushed, i ones need to be popped to reach the accepting state, q_3. In order to ensure 0^j1^j, the process repeats with a "new" stack (the stack is empty by q_3): j zeros are pushed, j ones need to be popped to reach the accepting state, q_6.

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2. Let Σ = {a, b, c}, and let

L = {w ∈ Σ* | w = rat, where r, t ∈ {b, c}*, |r| = |t|, and in substring t the number of b's and c's are equal}

Show that this language is not a CFL using the Pumping Lemma for CFLs. You should be able to do this directly using the lemma without invoking any other results.

Proof: Suppose for the sake of contradiction that L is a CFL
By the pumping lemma, there exists a pumping length, p ≥ 1.
Consider s = $b^p c^p \mathrm{a} b^p c^p$ ∈ L
By the pumping lemma, since |s| > p, there exists strings u, v, x, y, z such that s = uvxyz, |vxy| ≤ p, |vy| > 0, and for all i ≥ 0, $\mathrm{u}v^i\mathrm{x}y^i\mathrm{z}$ ∈ L

Case 1: Neither v nor y can contain a, as considering i = 0, $\mathrm{u}v^0\mathrm{x}y^0\mathrm{z}$ does not contain a and therefore is not in L. This is a contradiction.
Case 2: Both v and y are nonempty and occur on the left-hand side of the a.
Consider i =2, The string uv^2xy^2z is not in L because it is longer on the left-hand side of the a, |r| > |t|. This is a contradiction.
Case 3: Both strings occur on the right-hand side of the a.
Consider i = 0, the string $\mathrm{u}v^0\mathrm{x}y^0\mathrm{z}$ is not in L, because it is again longer on the left-hand side of the a, |r| > |t|. This is a contradiction.
Case 4: Only one of v and y is nonempty (both cannot be nonempty).
Treat them as if both occurred on the same side of the a as above. This is a contradiction.
Case 5: Both v and y are nonempty and straddle the a.
Then, v consists of c's and y consists of b's because of $|vxy|$ ≤ p.
Consider i = 2, uv^2xy^2z contains more c's on the left-hand side of the a then there is b's, so it is not in L. This is a contradiction.

Therefore, since we have a contradiction in all cases, L is not a CFL.