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CS 181: HW #4

Zero. The grammar for strings of balanced ( )'s shown in lecture:

$$S \rightarrow SS \mid (S) \mid ()$$

was shown to be ambiguous by pointing out that strings like ( )( )( ) have more than one parse tree in the grammar. What does this tell us about whether that language is an inherently ambiguous CFL? Briefly explain.

> A language is an inherently ambiguous CFL if all the context-free grammars of that language are ambiguous. The grammar for strings of balanced ( )'s is ambiguous, considering that some strings such as ( ) ( ) ( ) have more than one parse tree in the grammar. However, this does not necessarily show that all context-free grammars of this language are ambiguous. For example, this can be shown by the following grammar, as it is deterministic for this language, and thus this language cannot be inherently ambiguous: $S \rightarrow \varepsilon \mid (S)S$

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Consider the following CFL:
Let $\sum$ = {a, b, c}. Show a Context Free Grammar with terminal symbols $\sum$ for the following language:

$$\{a^i b^j c^k \in \textstyle\sum^* \mid i=j \text{ or } i=k\}$$

One. Show that this is not a FSL using the Pumping Lemma.

Proof: Let L = $\{a^i b^j c^k \in \sum^* \mid i=j \text{ or } i=k\}$
Assume for the sake of contradiction that L is a FSL
By pumping lemma, there exists a pumping length p, p ≥ 1
Consider the case s = $a^p b^p c$ ∈ L, i=j
By the pumping lemma, there exists x, y, z such that s = xyz and |xy| ≤ p
Since s starts with p a's, then x and y consists of a's
Let x = a^α, y = a^β, z = $a^{p-\alpha-\beta} b^p$c
By the pumping lemma, since |y| ≥ 1, $\beta \geq 1$
Then, by the pumping lemma, for all i ≥ 0, xy^iz ∈ L
In particular, consider i = 2

$$\begin{aligned} L \ni (xy^2z) &= a^\alpha (a^\beta)^2 a^{p-\alpha-\beta} b^p c \\ &= a^\alpha (a^{2\beta})(a^{p-\alpha-\beta} b^p c) \\ &= a^{\alpha+2\beta+p-\alpha-\beta} b^p c \\ &= a^{\beta+p} b^p c \end{aligned}$$

$a^{\beta+p}b^p$c is NOT in L, i = $\beta + p$, j = p, k = 1, $i \neq j, i \neq k$; xy^2z is NOT in L
Now, consider the case s = $a^p b c^p$ ∈ L, i=k
By the pumping lemma, there exists x, y, z such that s = xyz and |xy| ≤ p
Since s starts with p a's, then x and y consists of a's
Let x = a^α, y = a^β, z = $a^{p-\alpha-\beta} b c^p$
By the pumping lemma, since |y| ≥ 1, $\beta \geq 1$
Then, by the pumping lemma, for all i ≥ 0, xy^iz ∈ L
In particular, consider i = 2

$$\begin{aligned} L \ni (xy^2z) &= a^\alpha (a^\beta)^2 a^{p-\alpha-\beta} b c^p \\ &= a^\alpha (a^{2\beta})(a^{p-\alpha-\beta} b c^p) \\ &= a^{\alpha+2\beta+p-\alpha-\beta} b c^p \\ &= a^{\beta+p} b c^p \end{aligned}$$

$a^{\beta+p}bc^p$is NOT in L, i = $\beta + p$, j =1, k = p, $i \neq j, i \neq k$; xy^2z is NOT in L
Therefore, by contradiction, L is not a FSL.

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Two. Give a CFG for this language.

G = (V, Σ, R, S), V = $\{S_0, S_1, S_2, X_1, X_2, Y_1, Y_2\}$, Σ = {a, b, c}, R, the set of rules, is:

$S_0 \rightarrow S_1 \mid S_2$

$S_1 \rightarrow \mathrm{a}X_1 bY_1 \mid \varepsilon$

$X_1 \rightarrow \mathrm{a}X_1\mathrm{b} \mid \varepsilon$

$Y_1 \rightarrow \mathrm{c}Y_1 \mid \varepsilon$

$S_2 \rightarrow \mathrm{a}X_2 c \mid \varepsilon$

$X_2 \rightarrow \mathrm{a}X_2 Y_2\mathrm{c} \mid \varepsilon$

$Y_2 \rightarrow \mathrm{b}Y_2 \mid \varepsilon$

S_0 is the start variable.