Derivative

Q: 1

(a) $f(z) = 3z^2 - 2z + 4$

$$f'(z) = \frac{d}{dz}(3z^2 - 2z + 4)$$

$$= \frac{d}{dz}(3z^2) - \frac{d}{dz}(2z) + \frac{d}{dz}(4)$$

$$= 3(2z) - 2$$

$$\therefore f'(z) = 6z - 2$$

(b) $f(z) = (1 - 4z^2)^3$

$$\frac{d}{dz}(1-4z^2) = \frac{d}{dz}(1) - \frac{d}{dz}(4z^2)$$

$$= 0 - (4 \times 2)z$$

$$= -8z$$

$$f'(z) = \frac{d}{dz}(1-4z^2)^3 \cdot (-8z)$$

$$= 3(1-4z^2)^2(-8z)$$

$$= -24z(1-4z^2)^2$$

$$\therefore f'(z) = -24z(1-4z^2)^2$$

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(c) $f(z) = \dfrac{z-1}{2z+1} \quad (z = -1/2)$

We know, $\dfrac{d}{dz}\left[\dfrac{f(z)}{F(z)}\right] = \dfrac{F(z)f'(z) - f(z)F'(z)}{[F(z)]^2}$

$f(z) = z-1$

$F(z) = 2z+1$

$$f'(z) = \frac{d}{dz}\left(\frac{z-1}{2z+1}\right) = \frac{2z+1(1)-(z-1)(2)}{(2z+1)^2}$$

$$= \frac{2z+1-2z+2}{(2z+1)^2}$$

$$= \frac{3}{(2z+1)^2}$$

$$\therefore f'(z) = \frac{3}{(2z+1)^2}, \quad \left(z = -\frac{1}{2}\right)$$

(d) $f(z) = \dfrac{(1+z^2)^4}{z^2} \quad (z \neq 0)$

$$f'(z) = \frac{d}{dz}\left(\frac{(1+z^2)^4}{z^2}\right)$$

$$= \frac{\frac{d}{dz}(1+z^2)\cdot 4(z^2+1)^3\cdot z^2 - (z^2+1)^4\cdot 2z}{(z^2)^2}$$

$$= \frac{(2z)\,4(z^2+1)^3(z^2) - (z^2+1)^4\cdot 2z}{z^4}$$

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$$= \frac{8z^3(z^2+1)^3 - 2z(z^2+1)^4}{z^4}$$

$$= \frac{8(z^2+1)^3}{z} - \frac{2(z^2+1)^4}{z^3}$$

$$= \frac{2(z^2+1)^3(3z^2-1)}{z^3}$$

Q:2

(a) $P(z) = a_0 + a_1 z + a_2 z^2 + \dots + a_n z^n \quad (a_n \neq 0)$

$$P'(z) = \frac{d}{dz}(a_0) + \frac{d}{dz}(a_1 z) + \frac{d}{dz}(a_2 z^2) + \dots + \frac{d}{dz}(a_n z^n)$$

$$= 0 + a_1 + 2a_2 z + \dots \dots n a_n z^{n-1}.$$

Using formula $\frac{d}{dz} c = 0$, $\frac{d}{dz} z = 1$, $\frac{d}{dz} cf(z) = cf'(z)$

and $\frac{d}{dz} z^n = n z^{n-1}$ it is proved that,

$$P'(z) = a_1 + 2a_2 z + \dots + n a_n z^{n-1}$$

when $a_n \neq 0$ of degree $n\ (n \geq 1)$.

$\therefore$ $P(z)$ is differentiable everywhere with $P'(z)$

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(b) $$P(z) = a_0 + a_1 z + a_2 z^2 + \ldots + a_n z^n$$

$$(a_n \neq 0)$$

$$P(0) = a_0 + a_1 \cdot 0 + a_2 \cdot 0 + \ldots + a_n \cdot 0$$

$$\therefore P(0) = a_0$$

$$P'(z) = \frac{d}{dz}\left(a_0 + a_1 z + a_2 z^2 + \ldots + a_n z^n\right)$$

$$= 0 + (a_1 \cdot 1) + 2 z a_2 + \ldots + n a_n z^{n-1}$$

$$= a_1 + 0 + 0 \ldots + 0$$

$$\therefore P'(0) = a_1$$

$$P''(z) = \frac{d}{dz}\left(a_1 + 2 z a_2 + \ldots + \cancel{(n-1)}\, n a_n z^{n-1}\right)$$

$$= 0 + 2a_2 + \ldots + (n-1)(n) a_n{}^{z-2}$$

$$\therefore P''(0) = 2a_2$$

From the analysis, we can say $a_n = \frac{P^{(n)}(0)}{n!}$

$$a_0 = P(0)$$

$$a_1 = P'(0) = \frac{P'(0)}{1!}$$

$$a_2 = \frac{P''(0)}{2} = \frac{P''(0)}{2!}$$

Same as above, $a_n = a_n = \frac{P^n(0)}{n!}$ (Showed)

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Q:3 $\quad f(z) = \frac{1}{z} \quad (z \neq 0)$

Complex derivative of a function $f(z)$ at a point z is $\quad f'(z) = \lim_{\Delta z \to 0} \frac{f(z+\Delta z) - f(z)}{\Delta z}$

Given,

$$\therefore f(z) = \frac{1}{z}$$

$$f'(z) = \lim_{\Delta z \to 0} \frac{\frac{1}{z+\Delta z} - \frac{1}{z}}{\Delta z}$$

$$= \lim_{\Delta z \to 0} \frac{\frac{z - z - \Delta z}{z(z+\Delta z)}}{\Delta z}$$

$$= \lim_{\Delta z \to 0} \frac{-\Delta z}{z(z+\Delta z)\Delta z}$$

$$= \lim_{\Delta z \to 0} \frac{-1}{z(z+\Delta z)}$$

$$= \frac{\lim_{\Delta z \to 0} -1}{\lim_{\Delta z \to 0} z(z+\Delta z)}$$

$$= \frac{-1}{z(z+0)}$$

$$\therefore f'(z) = \frac{-1}{z^2}$$

(proved)

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