Chapter 5 (Electricity)

▫ The Coulomb's Law:

$$\boxed{F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}} \quad (\varepsilon_0 = 8.854 \times 10^{-12}\ coul^2/nt\text{-}m^2)$$

There are two types electric charge: (I) positive charge(+)
(II) negative charge(−)

The coulomb's law states that if there two charges $q_1 q_2$ separated by a distance r, then the force between these two charges $\boxed{F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}}$

Coulomb: The quantity of electricity transported in one second by a current of one Ampere (A). The unit of charge (q) is coulomb.

▫ Charge is quantized:

charge is not continuous. It exists in discrete "packetes", so the property is called 'quantized'. It is made up of integral multiples of certain minimum electric charge ($e = 1.6 \times 10^{-19}$ coul)

$$\boxed{q = ne} \qquad (n = \text{integer '+' or '−'})$$

▫ **Electric Field:** An invisible field created by attraction and repulsion of electrical charges.

Electric field strength $\boxed{E = \frac{F}{q}}$ (N/coul)

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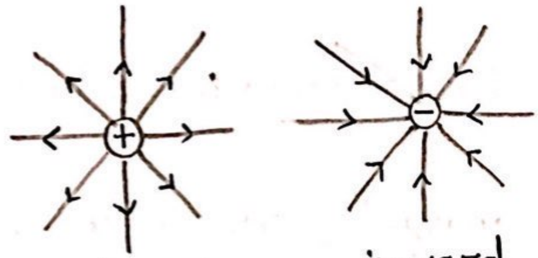
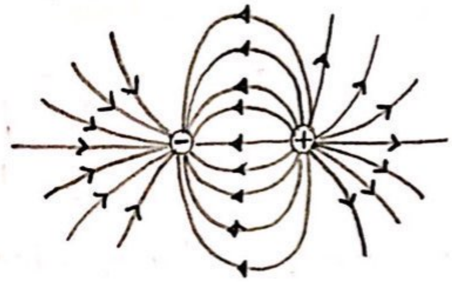
Lines of Force : Lines of force are path followed by an electric charge. (imaginary)

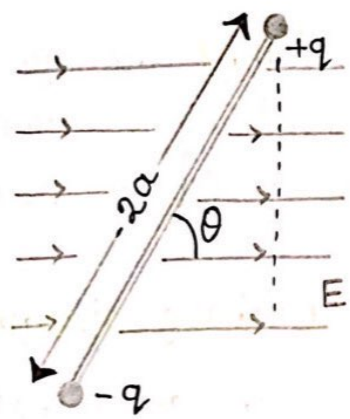




☐ A Dipole in an Electric Field:

two equal and opposite charges (+q and -q) ~~and~~ separated by a small distance ~~constit~~ (2a) constitute an electric dipole.

∴ dipole moment is, $\boxed{p = 2aq}$



The arrangement is placed in a uniform external electric field E. Two opposite and equal forces F and -F act. where, $\boxed{F = qE}$ The net force is clearly 0. but the net Torque is, $\tau = F(2a \sin\theta)$

$= 2aF\sin\theta$

$= 2aqE\sin\theta \quad [F = qE]$

$= pE\sin\theta \quad [p = 2aq]$

$\boxed{\tau = p \times E}$

Uniform electric field:
A field in which the lines of force are parallel.

The work done is, $\boxed{W = \int dW = \int_{\theta_0}^{\theta} \tau d\theta = U}$

$[W = Fd]$
$\downarrow \quad \downarrow$
$\tau \quad \theta$

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Here, Work done, $W = \int dW = \int_{\theta_0}^{\theta} T d\theta = U$ (potential energy)

$$\therefore U = \int_{\theta_0}^{\theta} \tau d\theta = \int_{\theta_0}^{\theta} pE \sin\theta d\theta \quad [\tau = pE\sin\theta]$$

$$= pE\int_{\theta_0}^{\theta} \sin\theta d\theta$$

$$= -pE[\cos\theta]_{\theta_0}^{\theta} = -pE[\cos\theta - \cos\theta_0]$$

Let, $\theta_0 = 90°$ $\quad \therefore U = -pE\cos\theta - 0$

$$= -pE\cos\theta \; \cancel{= p}\cdot$$

in vector form, $\boxed{U = -p.E}$

□ **Electric flux:** The measure of flow of the electric field through a given area (hypothetical / imaginary) which maybe closed or open.

$$\boxed{\phi_E \cong \Sigma E.\Delta S = \iint E.dS} \quad \boxed{= ES\cos\theta}$$

S = area of surface
ϕ_E = Electric flux
E = Electric field

Unit: $N\text{-}m^2/coul$

□ **Gauss's Law:** It is applied to any closed hypothetical surface (called Gaussian surface) which gives connection between electric flux (ϕ_E) for the surface and net charge (q) enclosed by the surface.

$$\boxed{\varepsilon_0 \phi_E = q}$$

$$\boxed{\varepsilon_0 \oint\!\!\oint E.ds = q}$$

It shows the relation / connection between ϕ_E and q

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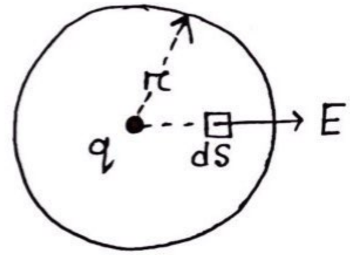
□ Prove Coulomb's Law from Gauss's Law

We know, the Gauss's law is,



A Gaussian surface

$$\varepsilon_0 \oiint E\,ds = q$$

$\Rightarrow \varepsilon_0 E \oiint dS = q$ [E will be same on all points]

$\Rightarrow \varepsilon_0 E (4\pi r^2) = q$ [Area of sphere $= 4\pi r^2$]

$\Rightarrow E = \dfrac{1}{4\pi r^2} \cdot \dfrac{q}{\varepsilon_0}$ $\qquad \therefore E = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q}{r^2}$

We know, $F = q_0 E$

$\therefore F = \dfrac{1}{4\pi\varepsilon_0} \dfrac{q_0 q}{r^2}$ [proved]

□ Electric Potential:

electric potential energy difference $\boxed{V_B - V_A = \dfrac{W_{AB}}{q_0}}$

W_{AB} → (a) positive(+) → B will be higher than A
(b) negative (−) → B will be lower " A
(c) zero (0) → B " " same as A

SI unit of potential is Volt (V).

1 Volt = 1 Joule/coul.

electric potential at a point, $\boxed{V = \dfrac{W}{q}}$

□ Work done in Electric Field is Path-independent.

Equipotential surface: The locus which have same electric potential at all points.

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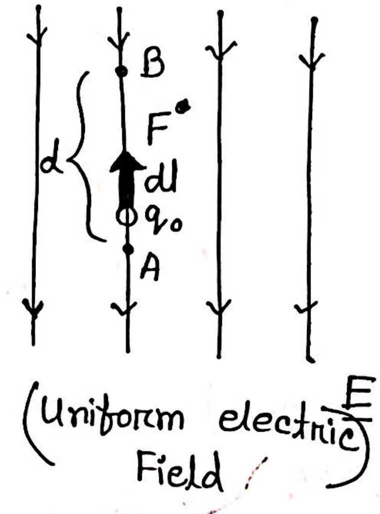
Relation between Electric potential (V) and Electric Field (E) (uniform)

Let, A and B are two points in a Uniform Electric Field E. The distance between A and B is d. Assuming that, a positive charge q_0 is moved by an external agent from A to B along straight line connecting them.

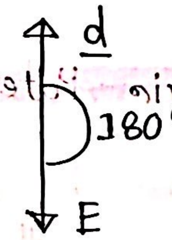


Since, the direction of the electric field is downward, There will be a Force,

$$(\underline{F}' = q_0 \underline{E}) \text{ on } q_0 \text{ charge}$$

The Force applied by the ~~aece~~ external agent,

$$\underline{F} = -q_0 \underline{E}$$



$$W_{AB} = \underline{F}.d = -q_0 E.d = -q_0 Ed \cos 180°$$

$$= -q_0 Ed(-1)$$

$$W_{AB} = q_0 Ed$$

$\therefore$ The potential difference, $V_B - V_A = \dfrac{W_{AB}}{q_0} = Ed$

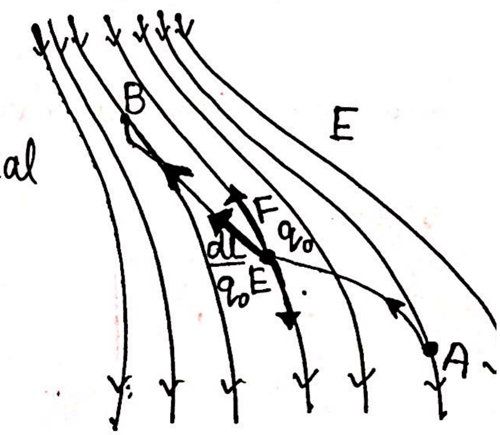
$$\therefore E = \frac{V_B - V_A}{d} \quad (\text{unit: } Vm^{-1})$$

When, the electric Field is not-uniform (non-uniform):

$$\underline{F} = -q_0 \underline{E}$$

~~dW~~ Element of work done by an external agent to move a displacement dl,

$$dW = F.dl = -q_0 E dl$$



The total work done by the external agent to move the charge particle q_0 from A to B,

$$W_{AB} = \int dW = \int_A^B (-q_0 \vec{E}\cdot\vec{dl})$$

$$W_{AB} = -q_0 \int_A^B \underline{E}\cdot\underline{dl}$$

Potential Energy, $V_B - V_A = \dfrac{W_{AB}}{q_0} = -\int_A^B E\cdot dl$

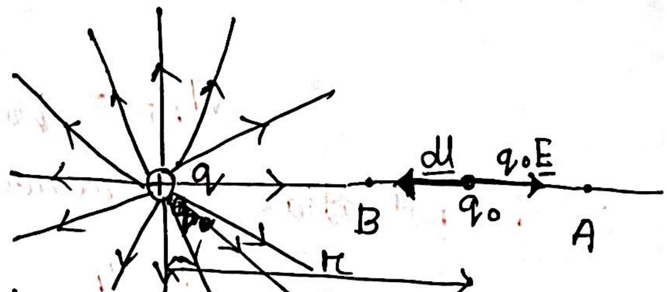
If we take point A to infinity distance and $V_A = 0$, let $V_B = V$; So, $\therefore V_B - V_A = -\int_A^B \underline{E}\cdot\underline{dl}$

$$\Rightarrow V - 0 = -\int_\infty^B \underline{E}\cdot\underline{dl} \qquad \boxed{\therefore V = -\int_\infty^B \underline{E}\cdot\underline{dl}}$$

potential energy at any point

□ Electric Potential due to a Point Charge:

Work done by an external force to have the distance dl. (In the figure)



$E\cdot dl = E\,dl\cos 180 = -E\,dl$

increasing l decreasing r

$\therefore dl = -dr$

$\underline{E}\cdot\underline{dl} = E\,dr$

a test charge q_0 is moved by an external agent from A to B in the field set up by a point charge q

$\therefore$ Potential Energy $V_B - V_A = -\int_A^B E\,dl = -\int_A^B E\,dr = -\int_A^B \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2}\cdot dr$

$$= -\frac{q}{4\pi\varepsilon_0}\int \frac{dr}{r^2}$$

We know,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

$$= -\frac{q}{\ldots}\left[\frac{-1}{r}\right]^{r_B}$$

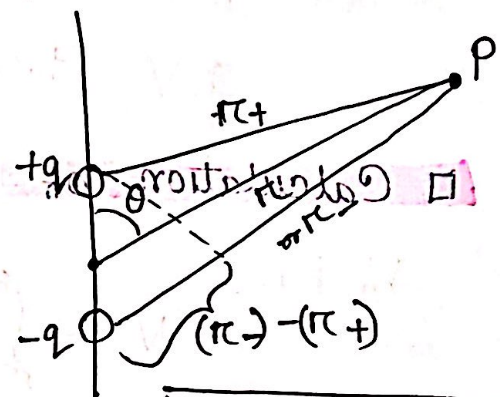
$$\therefore V_B - V_A = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

let, point A to be at infinity (letting $r_A \to \infty$) and $V_A = 0$ at this position, $V_B = V$,

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$$

So, equipotential surfaces for an isolated point charge are spheres concentric with point charge.

□ Potential Due to ~~Dipode~~ dipole:



Potential at P, $V = V_+ + V_-$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{+q}{r_+}\right) + \frac{1}{4\pi\varepsilon_0}\left(\frac{-q}{r_-}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_+} - \frac{1}{r_-}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{r_- - r_+}{r_+ r_-}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{2a\cos\theta}{r^2}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{2aq\cos\theta}{r^2}\right)$$

$$\boxed{\begin{array}{l}(r_-)-(r_+) \approx 2a\cos\theta \\ r_- r_+ \approx r^2\end{array}}$$

if point P is far away from the dipole

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{p\cos\theta}{r^2}\right) \quad [2aq = p]$$

Potential at point P due to an electric dipole.

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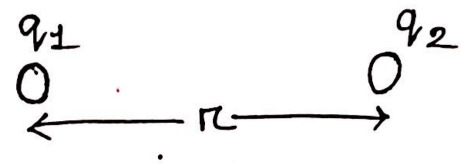
□ Electrostatic Potential Energy.

$$U = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{12}} = W$$



Electrostatic potential energy is ~~written~~ denoted by U

□ Electron volt: $V_A - V_B = 1V$

then, $1\ eV$ = work done by external agent to move an electron from higher potential to lower potential.

$$1eV = 1.6 \times 10^{-19} J$$

□ Calculation of $\underline{E}$ from V:

We have,

$$V_B = -\int_{\infty}^{B} \underline{E} \cdot \underline{dl}$$

electric field can be obtained by the gradient of the potential (V).

$$\underline{E} = -\underline{\nabla} V \quad [\underline{\nabla} = \text{partial differential operator}]$$

So, "The Electric Field Strength is the negative of the gradient of electric potential.

$$E_x = -\frac{dV}{dx}$$

$$\underline{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

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