0/02/24

$\forall \rightarrow$ For All
$\exists \rightarrow$ There exists

closed under addition
closed under multiplication

$$\overset{odd}{3} + \overset{odd}{3} = \overset{even}{6}$$

$$\underset{odd}{3} * \underset{odd}{3} = \underset{odd}{9}$$

V: is a non empty set
V is called a vector space (V.S)
if it satisfies the following conditions,

A_1: $v_1, v_2 \in V \Rightarrow v_1 + v_2 \in V$ (closed under addition)

A_2: $v_1 + v_2 = v_2 + v_1$, $\forall v_1, v_2 \in V$ (commutative law)

A_3: $(v_1 + v_2) + v_3 = v_1 + (v_2 + v_3)$ $\forall v_1, v_2, v_3 \in V$ (Assosiative Law)

A_4: For each $v \in V$, $\exists\, 0 \in V$ s.t. $v + 0 = 0 + v = v$ (Identity

A_5: For each $v \in V$, $\exists\, (-v) \in V$ s.t $v + (-v) = (-v) + v = 0$ (Additive inverse)

M_1: (Close under scalar Multiplication) ~~$\forall v_1, v_2 \in V, kv_2 \in V$~~

$\forall v \in V$ and $K \in F$, $kv \in V$

M_2: $K(u+v) = Ku + Kv$

M_3: $(K+m)v = Kv + mv$

M_4: $K(mv) = (Km)v$

M_5: $1.v = v$ $(1 \in V)$

Vector space need to contain $\vec{0}$ $\begin{vmatrix} a & 0 \\ 0 & b \end{vmatrix}$ ✓

not a v.space X $\begin{vmatrix} a & 1 \\ 1 & b \end{vmatrix}$ X fixed

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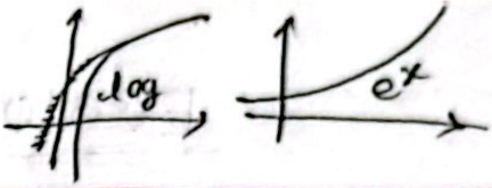
$\mathbb{F} \to$ scalar Quantity



- matrix addition is closed under addition.
- $\vec{0}$ is a vector space, $V = \{0\}$

Subspace: (sub space হতে হলে vector space হতে হবে)

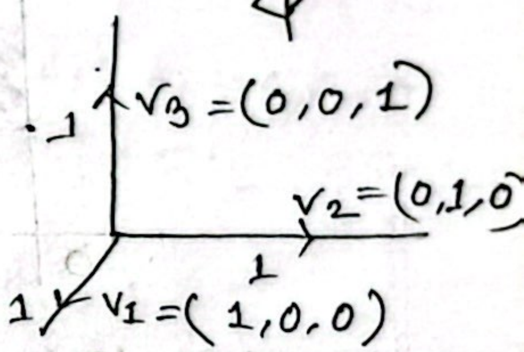
$$w = k_1v_1 + k_2v_2 + \dots + k_rv_r$$

$$w = (a,b,c)$$

$$= av_1 + bv_2 + cv_3)$$

$$= (a,0,0) + (0,b,0) + (0,0,c)$$

$$= (a,b,c)$$



Tribial Linear combination.

Example-9: $(9,2,7) = c_1(1,2,-1) + c_2(6,4,2)$

$$(9,2,7) = (c_1, 2c_1, -c_1) + (6c_2, 4c_2, 2c_2)$$

$$(9,2,7) = (c_1+6c_2, 2c_1+4c_2, -c_1+2c_2)$$

$$c_1 + 6c_2 = 9$$
$$2c_1 + 4c_2 = 2$$
$$-c_1 + 2c_2 = 7$$

$$\Rightarrow \left(\begin{array}{cc|c} 1 & 6 & 9 \\ 2 & 4 & 2 \\ -1 & 2 & 7 \end{array}\right)$$

$$\Rightarrow \left(\begin{array}{cc|c} 1 & 6 & 9 \\ 0 & -8 & -16 \\ 0 & 8 & 16 \end{array}\right) \Rightarrow \left(\begin{array}{cc|c} 1 & 6 & 9 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{array}\right)$$

$$\Rightarrow \left[\begin{array}{cc|c} 1 & 6 & 9 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right]$$

$c_2 = 2$;

$c_1 + 6 \times 2 = 9$

$\Rightarrow c_1 = -3$

$\therefore (9,2,7) = -3u+2$

MAT209 3-3-24

$(4,-1,8) = c_1(1,2,-1) + c_2(6,4,2)$

$(4,-1,8) = (c_1 + 6c_2,\ 2c_1 + 4c_2,\ -c_1 + 2c_2)$

$$\left(\begin{array}{cc|c} 1 & 6 & 4 \\ 2 & 4 & -1 \\ -1 & 2 & 8 \end{array}\right)$$

linear system:

$c_1 + 6c_2 = 4$

$2c_1 + 4c_2 = -1$

$-c_1 + 2c_2 = 8$

$$\Rightarrow \left(\begin{array}{cc|c} 1 & 6 & 4 \\ 0 & -8 & -9 \\ 0 & 8 & 12 \end{array}\right) \quad \begin{array}{l} (-2R_1 + R_2) \\ (R_1 + R_3) \end{array}$$

$$\Rightarrow \left(\begin{array}{cc|c} 1 & 6 & 4 \\ 0 & 2 & \frac{9}{4} \\ 0 & 2 & 3 \end{array}\right)$$

$$\Rightarrow \left(\begin{array}{cc|c} 1 & 6 & 4 \\ 0 & 1 & 9/8 \\ 0 & 0 & 3/8 \end{array}\right)$$

$\cancel{c_2 = 9/8}$

$\cancel{c_1 + 6c_2 = 4}$

$\cancel{\Rightarrow c_1 = 4 - \left(\overset{3}{\cancel{6}} \cdot \frac{9}{\underset{4}{\cancel{8}}}\right)}$

$\cancel{\Rightarrow c_1 = 4 - \frac{27}{4}}$

$0c_1 + 0c_2 = 3.$

There is no solution.

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MAT 209

A linear dependent set

$\{v_1, v_2, v_3 \ldots\ldots, v_k\}$

$c_1v_1 + c_2v_2 + \ldots\ldots + c_kv_k = 0$

$c_1 = c_2 = c_3 = \ldots = c_k = 0.$

Example-1 : $c_1 = 3,\ c_2 = 1,\ c_3 = -1$

$$c_1v_1 + c_2v_2 + c_3v_3 = 0.$$

$\Rightarrow c_1(2, -1, 0, 3) + c_2(1, 2, 5, -1) + c_3(7, -1, 5, 8) = 0$

$(0, 0, 0, 0)$

$\Rightarrow (2c_1 + c_2 + 7c_3,\ -c_1 + 2c_2 - c_3,\ 0c_1 + 5c_2 + 5c_3,\ 3c_1 - c_2 + 8c_3) = 0$

$$\Rightarrow \left.\begin{aligned} 2c_1 + c_2 + 7c_3 &= 0 \\ -c_1 + 2c_2 - c_3 &= 0 \\ 5c_2 + 5c_3 &= 0 \\ 3c_1 - c_2 + 8c_3 &= 0 \end{aligned}\right\}$$

$$\begin{aligned} 2c_1 + c_2 + 7c_3 &= 0 \\ 5c_2 + 5c_3 &= 0 \\ 5c_2 + 5c_3 &= 0 \\ 5c_2 + 5c_3 &= 0 \end{aligned}$$

$$\begin{cases} 2c_1 + c_2 + 7c_3 = 0 \\ 5c_2 + 5c_3 = 0 \end{cases}$$

$c_3 = s$ (Assume)

$\therefore c_2 = -s$

$\therefore c_1 = -\left(\dfrac{-s + 7s}{2}\right) = \dfrac{-6s}{2} = -3s.$ Linearly dependent

Example-1

linearly independent

□ $c_1v_1 + c_2v_2 + c_3v_3 = c_1(1, -2, 3) + c_2(5, 6, -1) + c_3(3, 2, 1)$

$\Rightarrow (c_1 + 5c_2 + 3c_3, -2c_1 + 6c_2 + 2c_3, 3c_1 - c_2 + c_3)$

$c_1 + 5c_2 + 3c_3 = 0$

$-2c_1 + 6c_2 + 2c_3 = 0$

$3c_1 - c_2 + c_3 = 0$

$$\begin{bmatrix} 1 & 5 & 3 \\ -2 & 6 & 2 \\ 3 & -1 & 1 \end{bmatrix}$$

$\cancel{c_1 = -(5c_2 + 3c_3)}$

$\cancel{-2(-5c_2 - 3c_3) + 6c_2 + 2c_3 = 0}$

$\cancel{\Rightarrow 10c_2 + 6c_3 + 6c_2 + 2c_3 = 0}$

Gaussian Elimination

$c_1 + 5c_2 + 3c_3 = 0$

$16c_2 + 8c_3 = 0$

$\Rightarrow c_3 = -2$

$c_2 = 1$

$c_1 = 1$

5.4

□ Basis and Dimension

□ $v_1 + v_2 - 2v_3 = 0$

* Theorem 5.3.3.

একটি Basis এর মধ্যে 8টা Linear

Basis → linearly independent,

vectors space কে span করে যাতে (x, y, z).

Basis of R_3

$(1, 0, 0)$

$(0, 1, 0)$

$(0, 0, 1)$

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Ex-3 : Demonstrating that a set of vectors is a Basis.

$$\left.\begin{aligned} c_1+2c_2+3c_3&=0\\ 2c_1+9c_2+3c_3&=0\\ c_1+0c_2+4c_3&=0 \end{aligned}\right\} \xrightarrow[\text{Elimination}]{\text{Gausisian}} \Rightarrow \boxed{\begin{aligned} c_1+2c_2+3c_3&=0\\ 5c_2-3c_3&=0\\ \cancel{2}c_2+c_3&=0 \end{aligned}}$$

$$\Rightarrow \boxed{\begin{aligned} c_1+2c_2+3c_3&=0\\ 5c_2-3c_3&=0\\ 2c_3&=0 \end{aligned}}$$

$$c_1=c_2=c_3=0$$

10/03/2024

MID – 31st March
(Linear Algebra)

Basis and Dimension

Ex-5.3 — 1, 2, 3 (Page 378) ***

Example - 3 ~~Example~~ Demonstating a set of vectors is a Basis.

$$c_1 + 2c_2 + 3c_3 = a$$
$$2c_1 + 9c_2 + 3c_3 = b$$
$$c_1 \quad + 4c_3 = c$$

$$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 9 & 3 \\ 1 & 0 & 4 \end{vmatrix} = 1(36-0) - 2(8-3) + 3(0-9) = 36 - 10 - 27 = -1 \neq 0$$

If it's determinant $= 0$ then it doesn't span.

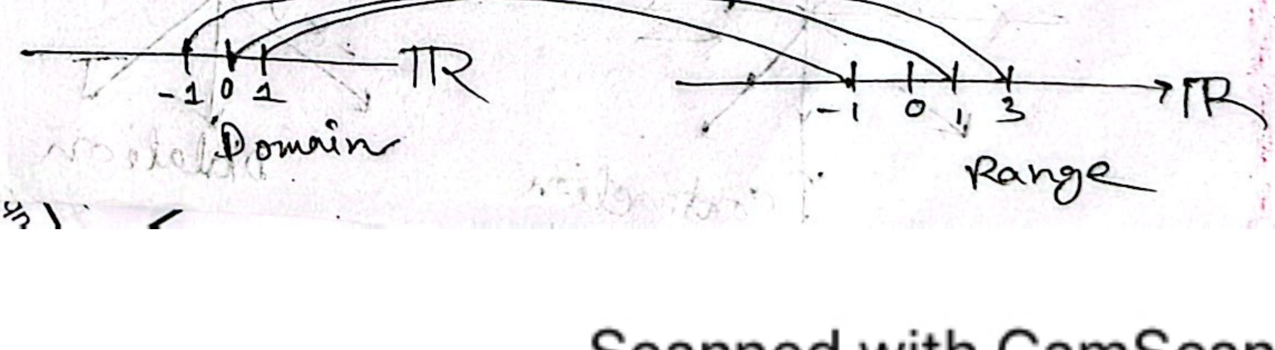
Example-10 $x_1 = -s-t$, $x_2 = s$, $x_3 = -t$.
$x_4 = 0$, $x_5 = t$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} \Rightarrow \begin{pmatrix} -s-t \\ s \\ -t \\ 0 \\ t \end{pmatrix} \Rightarrow \begin{pmatrix} -s \\ s \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -t \\ 0 \\ -t \\ 0 \\ t \end{pmatrix} \Rightarrow s\begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} -1 \\ 0 \\ -1 \\ 0 \\ 1 \end{pmatrix}$$

Basis এর মধ্যে যতটি vector থাকবে তাকে dimension of space বলা হয়।

2, 12-17

$f(x) = 2x + 1$



V.S = Vector space

$T : V \longrightarrow W$

T is a transformation from V.S V to V.S W

$T(u+v) = Tu + Tv$

$T(\alpha u) = \alpha Tu$

Zero transformation $T : V \longrightarrow \{0\}$

$T(u) = 0 \; \forall \; u \in V$

linear proved

$T(u+v) = 0 = 0 + 0 = Tu + Tv$

$T(\alpha u) = \alpha \cdot 0 = \alpha \cdot Tu$

Identity operator: same input, output

linear proved

$T : V \longrightarrow V$

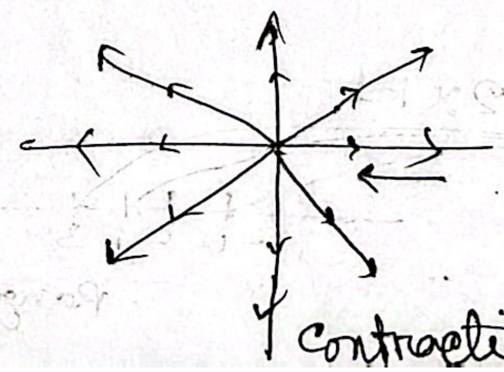
$Tu = u$

$T(v+w) = v + w = Tv + Tw$

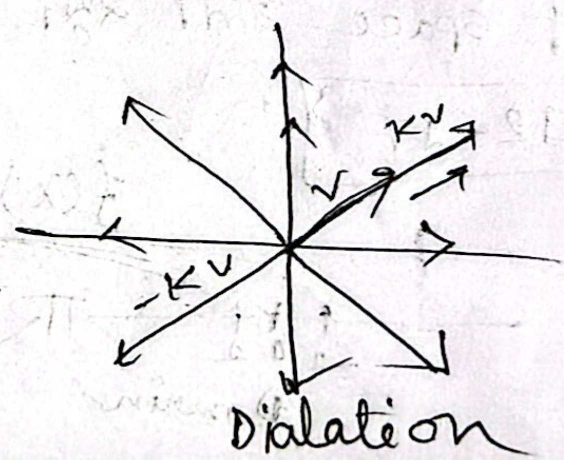
$T(\alpha v) = \alpha v = \alpha \cdot Tv$

Dilation and Contraction Operator:

$T(v) = kv$



contraction



Dialation

04-02-24

MAT205

Page 14-17

Augmanted Matrix

Book: Page 17. (1.1)

$2x+3y+5z=10$

$\Rightarrow 4x+6y+10z=20$

augmented matrix

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 3 & 4 & 5 \end{bmatrix} \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 6 & 7 & 8 \\ 0 & 0 & 4 & 5 \end{array}\right]$$

$$x_1+2x_2+3x_3=4$$
$$6x_2+7x_3=8$$
$$4x_3=5$$

Back ward substitution

Elementary Row operation

The augmanted matrix of our system A is,

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 2 & 4 & -3 & 1 \\ 3 & 6 & -5 & 0 \end{array}\right] \quad \begin{cases} R_2' = -2R_1+R_2 = 0 \\ R_3' = -3R_1+R_3 = 0 \end{cases}$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 2 & -7 & -17 \\ 0 & 3 & -11 & -27 \end{array}\right] \quad \{R_3'' = -3R_2' + 2R_3'\}$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 2 & -7 & -17 \\ 0 & 0 & -1 & -3 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 2 & -7 & -17 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad R_3''' = (-1)R$$

$x+y+2z=9$
$2y-7z=-17$
$z=3$

Backward substitution :-

$2y=-17+21$
$\Rightarrow 2y=4$
$\therefore y=2$

$x=9-y-2z$
$=9-2-2.3$
$=9-2-6=1$

$\therefore (x,y,z)=(1,2,3)$

This is the solution of unique ~~solutio~~ case.

⊞ (11)
$x-y=3$
$2x-2y=K$
$\left.\right\} \left[\begin{array}{cc|c} 1 & -1 & 3 \\ 2 & -2 & K \end{array}\right]$

$\Rightarrow \left[\begin{array}{cc|c} 1 & -1 & 3 \\ 0 & 0 & -6+K \end{array}\right]$ $R_2' = -2R_1+R_2$

If $K\neq 6$, ~~there will be~~ then the system does not have any solution.

If $K=6$, than $\left[\begin{array}{cc|c} 1 & -1 & 3 \\ 0 & 0 & 0 \end{array}\right] \rightarrow x-y=3$

then, we have $(2-1)=(\text{variable}-\text{eqn})=1=$ free variable

let it be, $y\in\mathbb{R}$ then $x=3+y$ $[y=t\in\mathbb{R}]$

$\therefore x=3+t$

$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3+t \\ t \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} + \begin{pmatrix} t \\ t \end{pmatrix}$

$(x,y) \Rightarrow (3+t, t)$

$= \begin{pmatrix} 3 \\ 0 \end{pmatrix} + t\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$

As, there is only one equation for 2 variables thus, there is no unique solution.

From Above, We ~~canu~~ can say, the system has only two solutions → no solution, infinite solution.

3-02-2

Reduced Row echelon form

Leading ~~tip~~ উপাদান গুলো 1 হতে হবে, Leading উপাদান এর উপরের সারির উপাদানটি 0 হবে,

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 2 & -7 & -17 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad \begin{bmatrix} R_1' = R_1 + (-2) R_3 \\ R_2' = R_2 + 7R_3 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & 2 & 0 & 4 \\ 0 & 0 & 1 & 3 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad \left[R_2'' = \frac{R_2'}{2}\right]$$

$$\left[R_1''' = R_1'' + (-1) R_2''\right]$$

$$\hookrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad Ex. \rightarrow 4, 6$$

$$x_1 = 1 - 2x_2 + x_4 - x_5$$

$$= 1 - 2.\frac{1}{3}(1 + 7x + t) + s - t$$

$$= 1 - 2/3 - \frac{1}{3} - \frac{2t}{3} + s - t$$

$$= 1/3 - \frac{11}{3}s - \frac{5}{3}t$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 1/3 & -11/3\,s & -5/3\,t \\ 1/3 & 7/3\,s & +1/3\,t \\ 1 & -7s & 0 \\ 0 & s & 0 \\ 0 & & 1 \end{pmatrix}$$

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$$= \begin{pmatrix} 1/3 \\ 1/3 \\ 1 \\ 0 \\ 0 \end{pmatrix} + s \begin{pmatrix} -11/3 \\ 7/3 \\ 7 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} -5/3 \\ 1/3 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$x_1 + 2x_2$ $\qquad$ $x_4 + x_5 = 1$

$\Rightarrow 3x_2 + x_3$ $\qquad$ $-x = 2$

$x_3 + 7x_4 = 1$

$$\left[\begin{array}{ccccc|c} 1 & 2 & 0 & -1 & 1 & 1 \\ 0 & 3 & 1 & 0 & -1 & 2 \\ 0 & 0 & 1 & 7 & 0 & 1 \end{array}\right]$$

$(5 - 3) = 2$ Free Variables)

Unknown – eqns

~~(5~~ Let $x_4 = s$

$x_5 = t \quad s, t \in \mathbb{R}$

$x_3 = 1 - 7x_4 = 1 - 7s \quad s \in \mathbb{R}$

we have to take x_4 and x_5 as free variables

As x_1, x_2 and x_3 are in leadin variable.

$3x_x = 2 - x_3 + x_5 = 2(1 - 7s) + t$

$= 1 + 7s + t$

$\therefore x_2 = 1/3(1 + 7s + t)$

13/02/24

MAT205

AB
~~BA~~

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 3 & 5 \\ 7 & 9 & 1 \\ 3 & 5 & 2 \end{bmatrix}$$

$$\underset{3\times3}{AB} = \begin{bmatrix} 1+14+9 & 3+18+15 & 5+2+6 \\ 4+35+18 & 12+45+30 & 20+5+12 \\ 7+56+27 & 21+72+45 & 35+8+18 \end{bmatrix} = \begin{bmatrix} 24 & 36 & 13 \\ 57 & 87 & 37 \\ 90 & 138 & 61 \end{bmatrix}$$

□ $A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 6 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & 1 & 4 & 3 \\ 0 & -1 & 3 & 1 \\ 2 & 7 & 5 & 2 \end{bmatrix}$

$$AB = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 6 & 0 \end{bmatrix} \begin{bmatrix} 4 & 1 & 4 & 3 \\ 0 & -1 & 3 & 1 \\ 2 & 7 & 5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+0+8 & 1+(-2)+28 & 4+6+20 & 3+2+8 \\ 8+0+0 & \cancel{8+0+0} \;\; 2-6+0 & 8+18+0 & 6+6+0 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & \cancel{26}\;27 & 30 & 13 \\ 8 & -4 & 26 & 12 \end{bmatrix} = 2\times4$$

Additive Identity = 0.
Multiplicative Identity = 1

linear combination

P-40-43 47

$$[c_1 \quad c_2 \quad c_3][c] = [B]$$

$$\Rightarrow 2[c_1] + (-1)c_2 + 3c_3 = [B]$$

$$a_1c_1 + a_2c_2 + \ldots + a_nc_n = b$$

□ Transpose Matrix: P-50

row → colume

$$\begin{bmatrix} 1 & 2 & 4 \\ 2 & 6 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 \\ 2 & 6 \\ 4 & 0 \end{bmatrix}$$

horizontal → vertical

matrix এর trace = তার Transpose এর trace

□ $AB = C$
$\Rightarrow B = A^{-1}C$

$AA^{-1} = I$ (Identity Matrix)

□ Inverse Matrix
- Square matrix হতে হবে
- Determinant $\neq 0$

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$$

$$[A|I] = \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 1 & 3 & 0 & 1 \end{array}\right] = \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & -1 & 1 \end{array}\right]$$

$$= \left[\begin{array}{cc|cc} 1 & 0 & 3 & -2 \\ 0 & 1 & -1 & 1 \end{array}\right]$$

$$\therefore A^{-1} = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$$

Prove :

$$AA^{-1} = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3-2 & -2+2 \\ 3-3 & -2+3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$|I_3| = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1(0-0) - 0(1-0) + 0(0-1)$$
$$= 1$$

(i) যদি Determinant $=0$ হয় তাহলে তা singular matrix

(ii) Inverse ২ প্রকার,

ⓐ Additive

② Multiplicative

$$[A|I] \longrightarrow [I|A^{-1}]$$

$$\therefore AA^{-1} = I = A^{-1}A$$

$$(A^{-1})^T = (A^T)^{-1}$$

$$\square\ A^{-1} = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} ;\ (A^T)^{-1} = \begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix} \qquad (A^{-1})^T = \begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix}$$

$$\cancel{(A^T)^{-1} \cdot A^T = I}$$
$$\cancel{\Rightarrow A^{-1} =}$$

$\boxed{\text{Example 1.}}$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$$

The matrix must be non-singular. Let us check

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{vmatrix} = 1(40-0) - 2(16-3) + 3(0-5)$$
$$= 40 - 26 - 15 = -1 \neq 0$$

$\therefore A$ has an inverse.

$$[A|I] = \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 3 & 0 & 1 & 0 \\ 1 & 0 & 8 & 0 & 0 & 1 \end{array}\right]$$

$$\left.\begin{array}{l} R_2' = -2R_1 + R_2 \\ R_3' = (-1)R_1 + R_3 \end{array}\right\} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & -2 & 5 & -1 & 0 & 1 \end{array}\right]$$

$2c_1 + 4c_2 = 0$

$4c_1 + 8c_2 = 0$

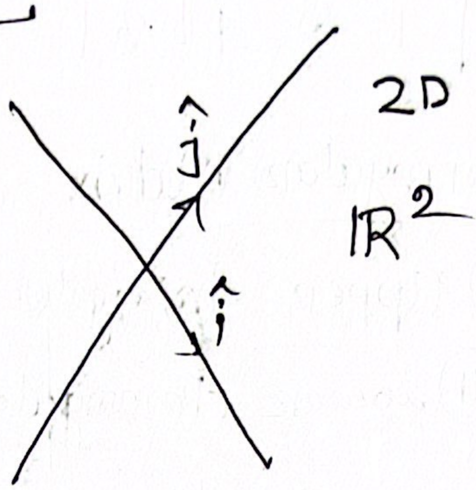
$\Rightarrow c_1 = -2c_2$

$\therefore c_1 = -2$ Not linearly Independent

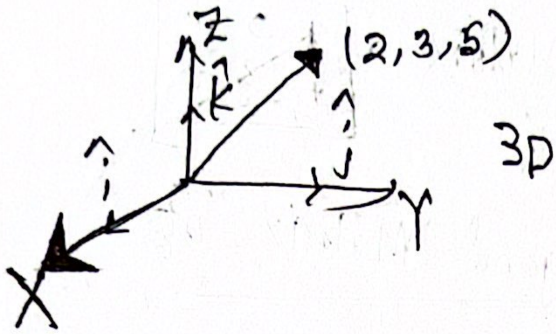
$c_2 = 1$

Vector

$A = 2\hat{i} + 3\hat{j} + 5\hat{k}$, $B = 4\hat{i} + 5\hat{j} + 7\hat{k}$



Vector space:



$A = (2,3,5)$, $B(4,5,7)$

$A + B = (6, 8, 12)$

A_1, A_2, A_3 যদি হয়,

$c_1A_1 + c_2A_2 + c_3A_3 = 0$

যদি $c_1 = c_2 = c_3 = 0$ তাহলে linearly Independent.

$A = (2,4)$

$B = (4,8)$

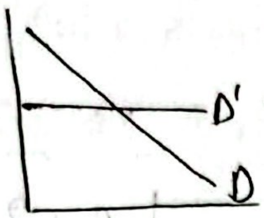
$B = 2A$

$c_1A + c_2B = 0$

$\Rightarrow c_1(2,4) + c_2(4,8) = 0$

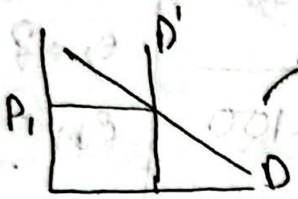
$\Rightarrow (2c_1, 4c_1) + (4c_2, 8c_2) = 0$

$(2c_1 + 4c_2, 4c_1 + 8c_2) = 0$



→ Perfectly elastic ($e = 0$)

$$e = \frac{\%\text{ change in } Q_D}{\%\text{ change in } P}$$



→ Perfectly inelastic ($e = 0$)

↳ means, at ~~any~~ P price, demand infinity

But at any other price, there is no demand

Elasticity and revenue:

$R = p \times q$ [price, quantity]

$|e| > 1 \Rightarrow$ % change in Q_D > % change in price

$|e| < 1 \Rightarrow$ % " " " < % " " "

$|e| = 1 \Rightarrow$ % " " " = % " " "

lowering the price ~~m~~ demand increases

$R_0 = P_0 \times q_0$

$R_N = P_N \times q_N$

$\Rightarrow \cancel{R_0 > R_N}\ R_0 < R_N$

Decision making by individuals and firms

Master degree সম্পূর্ণ. এর জন্য যত খরচ: stationary

Explicit cost -

Implicit cost - Master degree নিতে job off করা. salary sacrifice করা

Accounting profit = Revenue - explicit cost

Economic profit = Revenue - (Explicit + implicit)

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$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & -1 & -5 & 2 & 1 \end{array}\right] \quad [R_3'' = 2R_2' + R_3']$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right] \quad R_3''' = (-1)R_3''$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & -14 & 6 & 3 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right] \quad \begin{array}{l} R_1^{iv} = (-3)R_3''' + R_1''' \\ R_2^{iv} = 3R_3''' + R_2''' \end{array}$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \overset{-40}{\cancel{-14}} & \overset{16}{\cancel{6}} & \overset{9}{\cancel{3}} \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right] \quad R_1^{v} = (-2)R_2^{iv} + R_1^{iv}$$

$$A^{-1} = \begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix}$$

$$A \cdot A^{-1} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix} \begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix}$$

Raya

$$\Rightarrow \begin{bmatrix} \cancel{16}\ -40+26+15 & 16-10-6 & 9-6-3 \\ \cancel{16-10-6} & & \\ \cancel{9-6-3} & & \\ -80+65+15 & 32-25-6 & 18-15-3 \\ -40+0+40 & 16+0+(-16) & 9+0+(-8) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

□ $Ax = b$

$x = A^{-1}b$ ~~***~~

$\downarrow$

left side of b.

$$\begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix} \begin{pmatrix} 5 \\ 3 \\ 17 \end{pmatrix}$$

$$\boxed{\begin{aligned} x_1 + 2x_2 + 3x_3 &= 5 \\ 2x_1 + 5x_2 + 3x_3 &= 3 \\ x_1 + + 8x_3 &= 17 \end{aligned}}$$

$$\Rightarrow x = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

Example 2 (P-98)

Solve the system a, b

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

যদি leading element non-zero থাকে

$$\left[\begin{array}{ccc|c} 0 & 1 & 2 & 3 \\ 4 & 5 & 6 & 2 \\ 1 & 2 & 4 & 3 \end{array}\right] \quad R_1 \sim R_3 \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 4 & 3 \\ 4 & 5 & 6 & 2 \\ 0 & 1 & 2 & 3 \end{array}\right]$$

↳ Interchange

Triangular Matrix

(i) Upper triangular Matrix → $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$

(ii) lower triangular Matrix → $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{bmatrix}$

~~Transt~~ Inverse of diagonal Matrix of

$$\begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix}^T \rightarrow \begin{bmatrix} 1/d_1 & 0 & 0 \\ 0 & 1/d_2 & 0 \\ 0 & 0 & 1/d_3 \end{bmatrix}$$

(All diagonal element must be non-zero)

$A = (a_{ij})_{i,j} = 1, 3$

$a_{ij} \begin{cases} 0 \text{ if } i > j \\ \text{non zero otherwise} \end{cases}$ $A = \begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix}$

$A^2 = \begin{bmatrix} d_1^2 & 0 & 0 \\ 0 & d_2^2 & 0 \\ 0 & 0 & d_3^2 \end{bmatrix}$

Identity matrix এর Inverse matrix = Identity Matrix

symmetrix Matrix = $\begin{bmatrix} 1 & \boxed{2} & \boxed{3} \\ \boxed{2} & 4 & \boxed{5} \\ \boxed{3} & \boxed{5} & 6 \end{bmatrix}$

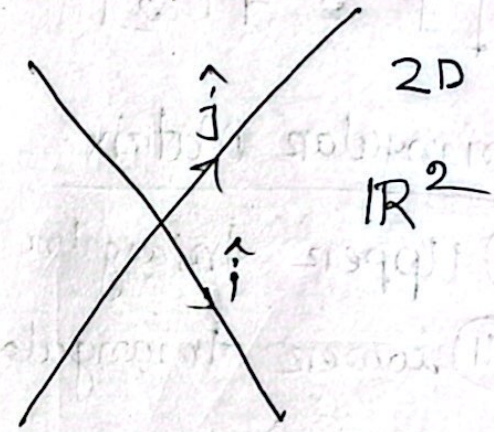
Anti symmetric matrix = $\begin{bmatrix} 1 & 2 & 3 \\ -2 & 4 & 5 \\ -3 & -5 & 6 \end{bmatrix}$

22/02/

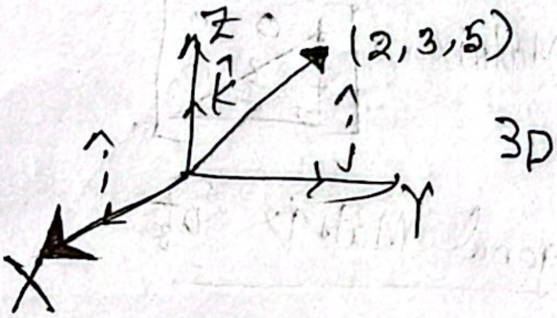
MAT205 03 March 2024 → Quiz 1 → (I) System of linear Eqns
(II) Gaussian Elimination
(III) Reduced Row echelon form

Vector

$A = 2\hat{i} + 3\hat{j} + 5\hat{k}$, $B = 4\hat{i} + 5\hat{j} + 7\hat{k}$



Vector space:



$A = (2, 3, 5)$, $B(4, 5, 7)$

$A + B = (6, 8, 12)$

A_1, A_2, A_3 যদি হয়,

$c_1A_1 + c_2A_2 + c_3A_3 = 0$

যদি $c_1 = c_2 = c_3 = 0$ তাহলে linearly Independent.

$A = (2, 4)$ | $c_1 A + c_2 B = 0$

$B = (4, 8)$ | $\Rightarrow c_1(2,4) + c_2(4,8) = 0$

$B = 2A$ | $\Rightarrow (2c_1, 4c_1) + (4c_2, 8c_2) = 0$

| $\Rightarrow (2c_1 + 4c_2, 4c_1 + 8c_2) = 0$

$$2c_1 + 4c_2 = 0$$

$$4c_1 + 8c_2 = 0$$

$$\Rightarrow c_1 = -2c_2$$

$$\therefore c_1 = -2$$ Not linearly Independent

$$c_2 = 1$$

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$\forall \rightarrow$ For All
$\exists \rightarrow$ There exists

closed under addition
closed under multiplication

$$\underset{odd}{3} + \underset{odd}{3} = \underset{even}{6}$$

$$\underset{odd}{3} * \underset{odd}{3} = \underset{odd}{9}$$

v: is among non empty set
v is called a vector space (V.S)
if it satisfies the following conditions,

A_1: $v_1, v_2 \in V \Rightarrow v_1 + v_2 \in V$ (closed under addition)

A_2: $v_1 + v_2 = v_2 + v_1$, $\forall\, v_1, v_2 \in V$ (commutative law)

A_3: $(v_1 + v_2) + v_3 = v_1 + (v_2 + v_3)$ $\forall\, v_1, v_2, v_3 \in V$ (Assosiative Law)

A_4: For each $v \in V$, $\exists\ 0 \in V$ s.t. $v + 0 = 0 + v = v$ (Identity)

A_5: For each $v \in V$, $\exists\ (-v) \in V$ s.t $v + (-v) = (-v) + v = 0$ (Additive inverse)

M_1: (Close under scalar Multiplication) ~~$\forall v_1, v_2 \in V, kv_2 \in V$~~

$\forall\, v \in V$ and $K \in F$, $kv \in V$

M_2: $K(u+v) = Ku + Kv$

M_3: $(K+m)v = Kv + mv$

M_4: $K(mv) = (Km)v$

M_5: $1.v = v$ $(1 \in V)$

Vector space need to contain $\vec{0}$

$$\begin{vmatrix} a & 0 \\ 0 & b \end{vmatrix} \checkmark$$

not a v.space $\times$

$$\begin{vmatrix} a & 1 \\ 1 & b \end{vmatrix} \times$$

$\rightarrow$ fixed

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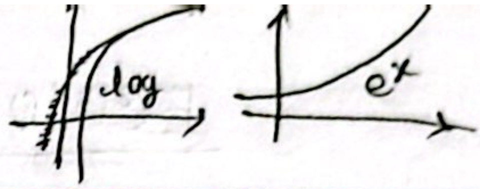
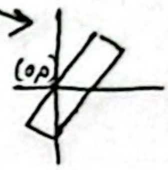
$\mathbb{F} \rightarrow$ scalar Quantity



- matrix addition is closed under addition.
- $\vec{0}$ is a vector space, $V = \{0\}$

Subspace: (sub space হতে হলে vector space হতে হবে



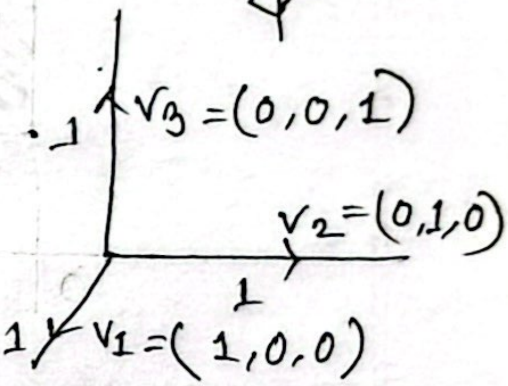
$w = k_1v_1 + k_2v_2 + \ldots\ldots + k_rv_r$

$w = (a, b, c)$

$= av_1 + bv_2 + cv_3)$

$= (a,0,0) + (0,b,0) + (0,0,c)$

$= (a, b, c)$



Tribial Linear combination.

Example-9: $(9,2,7) = c_1(1,2,-1) + c_2(6,4,2)$

Page 358

$(9,2,7) = (c_1, 2c_1, -c_1) + (6c_2, 4c_2, 2c_2)$

$(9,2,7) = (c_1 + 6c_2, 2c_1 + 4c_2, -c_1 + 2c_2)$

$c_1 + 6c_2 = 9$
$2c_1 + 4c_2 = 2$
$-c_1 + 2c_2 = 7$

$\Rightarrow \left(\begin{array}{cc|c} 1 & 6 & 9 \\ 2 & 4 & 2 \\ -1 & 2 & 7 \end{array}\right)$

$\Rightarrow \left(\begin{array}{cc|c} 1 & 6 & 9 \\ 0 & -8 & -16 \\ 0 & 8 & 16 \end{array}\right) \Rightarrow \left(\begin{array}{cc|c} 1 & 6 & 9 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{array}\right)$

$c_2 = 2;$

$c_1 + 6 \times 2 = 9$

$\Rightarrow c_1 = -3$

$\Rightarrow \left[\begin{array}{cc|c} 1 & 6 & 9 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right]$

$\therefore (9,2,7) = -3u + 2$

P-358

MAT209 3-3-24

$(4,-1,8) = c_1(1,2,-1) + c_2(6,4,2)$

$(4,-1,8) = (c_1 + 6c_2,\ 2c_1 + 4c_2,\ -c_1 + 2c_2)$

$$\left(\begin{array}{cc|c} 1 & 6 & 4 \\ 2 & 4 & -1 \\ -1 & 2 & 8 \end{array}\right)$$

linear system:

$c_1 + 6c_2 = 4$

$2c_1 + 4c_2 = -1$

$-c_1 + 2c_2 = 8$

$$\Rightarrow \left(\begin{array}{cc|c} 1 & 6 & 4 \\ 0 & -8 & -9 \\ 0 & 8 & 12 \end{array}\right) \quad \begin{array}{l} (-2R_1 + R_2) \\ (R_1 + R_3) \end{array}$$

$$\Rightarrow \left(\begin{array}{cc|c} 1 & 6 & 4 \\ 0 & 2 & \frac{9}{4}\ [illegible] \\ 0 & 2 & 3\ [illegible] \end{array}\right)$$

$$\Rightarrow \left(\begin{array}{cc|c} 1 & 6 & 4 \\ 0 & 1 & 9/8 \\ 0 & 0 & 3/8 \end{array}\right)$$

$0c_1 + 0c_2 = 3.$

There is no solution.

$c_2 = 9/8$

$c_1 + 6c_2 = 4.$

$\Rightarrow c_1 = 4 - \left(\cancel{6}^{3} \cdot \frac{9}{\cancel{8}_{4}}\right)$

$\Rightarrow c_1 = 4 - \frac{27}{4}$

p-361

Example 12 : Three Vectors that do not span R^3

$c_1v_1 + c_2v_2 + c_3v_3 = (x, y, z)$

$c_1 + c_2 + 2c_3 = x$

$c_1 + c_3 = y$

$2c_1 + c_2 + 3c_3 = z$

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07/03/20

MAT 209

A linear dependent set

$\Rightarrow \{v_1, v_2, v_3, \ldots, v_k\}$

$c_1v_1 + c_2v_2 + \ldots + c_kv_k = 0$

$c_1 = c_2 = c_3 = \ldots = c_k = 0$.

p. 370

Example-1 : $c_1 = 3,\ c_2 = 1,\ c_3 = -1$

$c_1v_1 + c_2v_2 + c_3v_3 = 0.$

$\Rightarrow c_1(2,-1,0,3) + c_2(1,2,5,-1) + c_3(7,-1,5,8) = (0,0,0,0)$

$\Rightarrow (2c_1 + c_2 + 7c_3,\ -c_1 + 2c_2 - c_3,\ 0c_1 + 5c_2 + 5c_3,\ 3c_1 - c_2 + 8c_3) = 0$

$$\left.\begin{aligned} 2c_1 + c_2 + 7c_3 &= 0 \\ -c_1 + 2c_2 - c_3 &= 0 \\ 5c_2 + 5c_3 &= 0 \\ 3c_1 - c_2 + 8c_3 &= 0 \end{aligned}\right\}$$

$$\begin{aligned} 2c_1 + c_2 + 7c_3 &= 0 \\ 5c_2 + 5c_3 &= 0 \\ 5c_2 + 5c_3 &= 0 \\ 5c_2 + 5c_3 &= 0 \end{aligned}$$

$$\left\{\begin{aligned} 2c_1 + c_2 + 7c_3 &= 0 \\ 5c_2 + 5c_3 &= 0 \end{aligned}\right.$$

$c_3 = s$ (Assume)

$\therefore c_2 = -s$

$\therefore c_1 = -\left(\frac{-s + 7s}{2}\right) = \frac{-6s}{2} = -3s.$ Linearly depend

Example-4

linearly dependent

P-371

□ $c_1v_1+c_2v_2+c_3v_3 = c_1(1,-2,3)+c_2(5,6,-1), c_3(3,2,1)$

$\Rightarrow (c_1+5c_2+3c_3, -2c_1+6c_2+2c_3, 3c_1-c_2+c_3)$

$$c_1+5c_2+3c_3=0$$
$$-2c_1+6c_2+2c_3=0$$
$$3c_1-c_2+c_3=0$$

$$\begin{bmatrix} 1 & 5 & 3 \\ -2 & 6 & 2 \\ 3 & -1 & 1 \end{bmatrix}$$

$\cancel{c_1 = -(5c_2+3c_3)}$

$\cancel{-2(-5c_2-3c_3)+6c_2+2c_3=0}$

$\cancel{\Rightarrow 10c_2+6c_3+6c_2+2c_3=0}$

Gaussian Elimination

$$c_1+5c_2+3c_3=0$$
$$16c_2+8c_3=0$$

$\Rightarrow c_3=-2$

$c_2=1$

$c_1=1$

□ $V_1+V_2-2V_3=0$

5.4
□ Basis and Dimension

** Theorem 5.3.3. P-375

একটার Basis এর মধ্যে ৪টা Linear

P-383

Basis → linearly independent, vector space এর span করে যায় x,y,z.

Basis of R_3

(1,0,0)

(0,1,0)

(0,0,1)

P-388

Ex-3 : Demonstrating that a set of vectors is a Basis.

$$\left.\begin{aligned} c_1 + 2c_2 + 3c_3 &= 0 \\ 2c_1 + 9c_2 + 3c_3 &= 0 \\ c_1 + 0c_2 + 4c_3 &= 0 \end{aligned}\right\} \xrightarrow[\text{Elimination}]{\text{Gausisian}} \boxed{\begin{aligned} c_1 + 2c_2 + 3c_3 &= 0 \\ 5c_2 - 3c_3 &= 0 \\ \cancel{2}c_2 + c_3 &= 0 \end{aligned}}$$

$$\Rightarrow \boxed{\begin{aligned} c_1 + 2c_2 + 3c_3 &= \\ 5c_2 - 3c_3 &= \\ 2c_3 &= \end{aligned}}$$

$$c_1 = c_2 = c_3 = 0$$

10/03/2024

MID – 31st March
(Linear Algebra)

Basis and Dimension

Ex-5.3 1, 2, 3 (Page 378) ***

$\boxed{\text{Example - 3}}$ ~~Example~~ Demonstrating a set of vectors is a Basis.

$$\begin{aligned} c_1 + 2c_2 + 3c_3 &= a \\ 2c_1 + 9c_2 + 3c_3 &= b \\ c_1 \quad + 4c_3 &= c \end{aligned}$$

$$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 9 & 3 \\ 1 & 0 & 4 \end{vmatrix} = 1(36-0)-2(8-3)+3(0-9) = 36-10-27 = -1 \neq 0$$

If it's determinant $= 0$ then it doesn't span.

$\boxed{\text{Example-10}}$ $x_1 = -s-t$, $x_2 = s$, $x_3 = -t$, $x_4 = 0$, $x_5 = t$

P-393

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} \Rightarrow \begin{pmatrix} -s-t \\ s \\ -t \\ 0 \\ t \end{pmatrix} \Rightarrow \begin{pmatrix} -s \\ s \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -t \\ 0 \\ -t \\ 0 \\ t \end{pmatrix} \Rightarrow s\begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} -1 \\ 0 \\ -1 \\ 0 \\ 1 \end{pmatrix}$$

Basis এর মধ্যে যতগুলি vector থাকবে তাকে dimension of space বলা হয়।

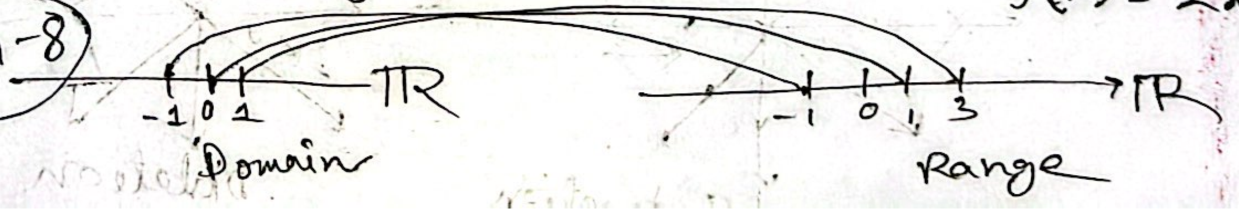
*** $\boxed{2, 12, 17}$ 5.4 P-397

Linear Transformation

$f(x) = 2x+1$

$f(x) = 2x+1$

Ch-8



V.S = Vector space

$T : V \longrightarrow W$

T is a transformation from V.S V to V.S W

$T(u+v) = Tu + Tv$

$T(\alpha u) = \alpha Tu$

Zero transformation $T : V \longrightarrow \{0\}$ (P-579)

$T(u) = 0 \; \forall \; u \in V$

linear proved

$T(u+v) = 0 = 0 + 0 = Tu + Tv$

$T(\alpha u) = \alpha . 0 = \alpha . Tu$

Identity Operator: Same input, output

linear proved

$T : V \rightarrow V$

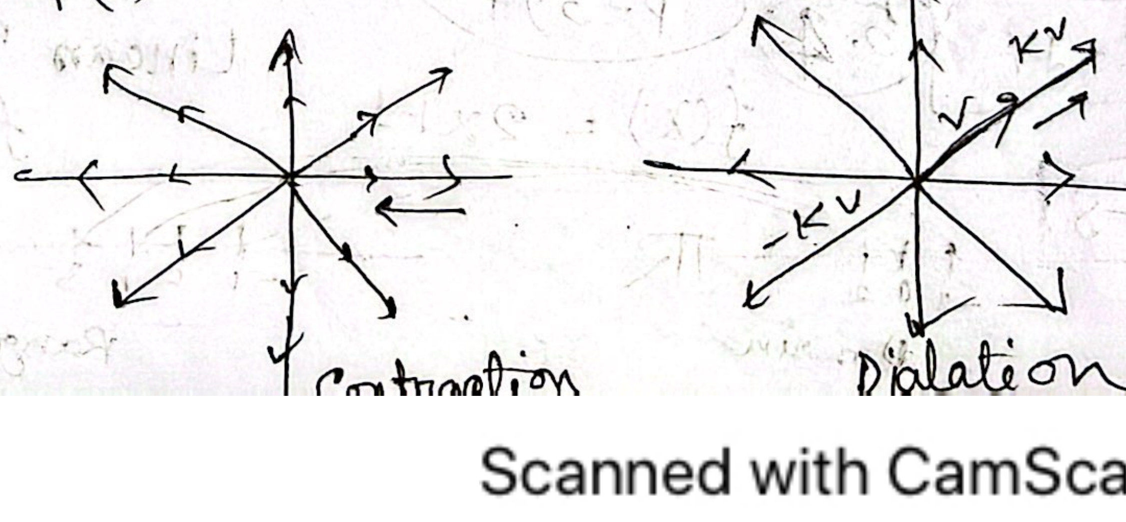
$Tu = u$

$T(v+w) = v + w = Tv + Tw$

$T(\alpha v) = \alpha v = \alpha . Tv$

Dilation and Contraction Operator:

$T(v) = kv$



$\alpha v = (\alpha v_1, \alpha v_2, \alpha v_3)$

Ex-8.1 (P-589)

(2) $T(x_1, x_2, x_3) = (2x_1 - x_2 + x_3, x_2 - 4x_3)$

$\therefore T(u+v)$ $u \in \mathbb{R}^3$

$v \in \mathbb{R}^3$

$u = (u_1, u_2, u_3)$, $v = (v_1, v_2, v_3)$

$\therefore u+v = (u_1+v_1, u_2+v_2, u_3+v_3)$

$\therefore T(u+v) = T(u_1+v_1, u_2+v_2, u_3+v_3)$

$= \{2(u_1+v_1) - (u_2+v_2) + (u_3+v_3), (u_2+v_2) - 4(u_3+v_3)\}$

$= (2u_1 + 2v_1 - u_2 - v_2 + u_3 + v_3, u_2 + v_2 - 4u_3 - 4$

$= (2u_1 - u_2 + u_3, u_2 - 4u_3) + (2v_1 - v_2 + v_3, v_2 - 4v_3$

$= T(u_1, u_2, u_3) + T(v_1, v_2, v_3)$

$= Tu + Tv$

$\therefore T(\alpha v) = T(\alpha v_1, \alpha v_2, \alpha v_3)$

$= (2\alpha v_1 - \alpha v_2 + \alpha v_3, \alpha v_2 - 4\alpha v_3)$

$= \alpha(2v_1 - v_2 + v_3, v_2 - 4v_3)$

$= \alpha T(v_1, v_2, v_3)$

$= \alpha Tv$

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Problem 3.4 (6), (7), (5), (8) p-589

(1) $T(x_1, x_2) = (x_1 + 2x_2, 3x_1 - x_2)$

$T(u+v) = T(u_1+v_1, u_2+v_2)$

$= (u_1+v_1+2(u_2+v_2), 3(u_1+v_1)-(u_2+v_2))$

$= (u_1+v_1+2u_2+2v_2, 3u_1+3v_1-u_2-v_2)$

$= (u_1+2u_2, 3u_1-u_2) + (v_1+2v_2, 3v_1-v_2)$

$= T(u_1, u_2) + T(v_1, v_2)$

$= Tu + Tv$

$T(\alpha v) = T(\alpha v_1 + \alpha v_2)$

$= (\alpha v_1 + 2\alpha v_2, 3\alpha v_1 - \alpha v_2)$

$= \alpha(v_1+2v_2, 3v_1 - v_2)$

$= \alpha T(v_1, v_2)$

$= \alpha Tv$

(6) $T(A) = tr(A)$

$T(A+B) = tr(A+B)$

$= tr(A) + tr(B)$

$= T(A) + T(B)$

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$$T(\alpha A) = tr(\alpha A)$$
$$= \alpha\ tr(A)$$
$$= \alpha T(A)$$

$$A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \qquad \alpha A \begin{bmatrix} \alpha & 0 & 0 \\ 0 & 2\alpha & 3\alpha \\ 4\alpha & 5\alpha & 6\alpha \end{bmatrix}$$

$$Tr(\alpha A) = \alpha\, Tr(A).$$

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1/10/23

MAT205

Eigen values & Eigen vectors

Page-538

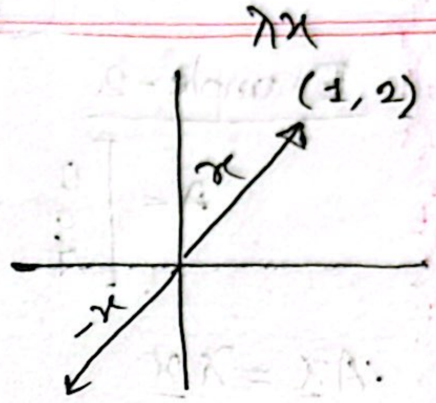

Figure 7.1.1

$Ax = \lambda x$

λ এর মানগুলোকে Eigen Value বলে

Eigen Value বসানোর ফলে যে মানগুলো আসবে তাকে Eigen vector বলে।

$Ax - \lambda x = 0$

$\Rightarrow Ax - \lambda I x = 0$

$\Rightarrow (A - \lambda I)x = 0$

$\Rightarrow |A - \lambda I| = 0$

Example-1: Eigen Vector of 2x2 Matrix

$A = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}, \; x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$Ax = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \end{bmatrix} = \underbrace{3}_{\text{Eigen Value}} \underbrace{\begin{bmatrix} 1 \\ 2 \end{bmatrix}}_{\text{Eigen vector}}$

$Ax = \lambda x$ $\quad \therefore \lambda = 3 =$ Eigen Value

- charasteristic polynomial $= \det(\lambda I - A)$
- charasteristic equation $= \det(\lambda I - A) = 0$

Example-2

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & -17 & 8 \end{bmatrix}, \quad \underline{\lambda} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$A\underline{x} = \lambda \underline{x}$$

$$\Rightarrow (A - \lambda I)x = 0 \quad \ldots\ldots (1)$$

$$P(\lambda) = |A - \lambda I| \qquad \lambda I = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

The characteristics of polynomial of A

$$\begin{vmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ 4 & -17 & 8-\lambda \end{vmatrix}$$

$$= -\lambda\left((8-\lambda)(-\lambda) + 17\right) - 1(0-4)$$

$$= -\lambda(-8\lambda + \lambda^2 + 17) + 4$$

$$= 8\lambda^2 - \lambda^3 - 17\lambda + 4$$

$= \cancel{-\lambda^3 + 8\lambda^2}$ Therefore, the characteristic equation of A is,

$$P(\lambda) = 0$$

$$\therefore -\lambda^3 + 8\lambda^2 - 17\lambda + 4 = 0$$

$$\Rightarrow \lambda^3 - 8\lambda^2 + 17\lambda - 4 = 0$$

$$\Rightarrow \cancel{\lambda^3}\ \lambda^2(\lambda - 4) - 4\lambda(\lambda - 4) + (\lambda - 4) = 0$$

$$\Rightarrow (\lambda - 4)(\lambda^2 - 4\lambda + 1) = 0$$

Trace of matrix = sum of eigen value

(541)

$\lambda^2+1=0$. $\therefore \lambda=-1$

P

~~$(\lambda-4)($~~

Either

$\lambda-4=0$

$\therefore \lambda=4$

or $\lambda^2-4\lambda+1=0$

~~$\Rightarrow \lambda^2-2\lambda-2\lambda+1=0$~~

~~$\Rightarrow \lambda(\lambda-2)-2(\lambda-$~~

$\lambda=\dfrac{4\pm\sqrt{16-4.1.1}}{2.1}$

$=\dfrac{4+2\sqrt{3}}{2}=2\pm\sqrt{3}$

$\lambda_1=4$

$\lambda_2=2+\sqrt{3}$

$\lambda_3=2-\sqrt{3}$

Ex-3 Upper Triangular / Lower Triangular Matrix এর diagonal eliment হবে eigen value.

□ Complex Eigenvalues :-

(541)

$\lambda^2+1=0$

$\lambda^2=-1$

$\lambda=\pm\sqrt{-1}$

$=\pm i$

$(A-\lambda I)x=0$ ...... (1)

$$\begin{bmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ 4 & -17 & 8-\lambda \end{bmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Put $\lambda=4$, the we have,

$$\begin{bmatrix} -4 & 1 & 0 \\ 0 & -4 & 1 \\ 4 & -17 & 4 \end{bmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$=\begin{bmatrix} -4 & 1 & 0 \\ 0 & 1 & -\frac{1}{4} \\ 0 & -16 & 4 \end{bmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$ (gaussian elemination

$$= \begin{bmatrix} -4 & 1 & 0 \\ 0 & 1 & -\frac{1}{4} \\ 0 & 1 & -\frac{1}{4} \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$= \begin{bmatrix} -4 & 1 & 0 \\ 0 & 1 & -1/4 \\ 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$-4x + y = 0$

$y - \frac{1}{4} z = 0$

Let, $z = s$

$y = \frac{1}{4} s$

$x = \frac{1}{16} s$

$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = s \begin{pmatrix} \frac{1}{16} \\ \frac{1}{4} \\ 1 \end{pmatrix}$

Let $s = 1$

$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1/16 \\ 1/4 \\ 1 \end{pmatrix}$ is the eigen vector corresponding to $\lambda = 4$.

$\boxed{\text{Example-5}}$ same

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$Ax = \lambda x$

$A(Ax) = A(\lambda x)$

$\Rightarrow A^2 x = \lambda (Ax)$

$\Rightarrow \lambda . \lambda x = \lambda^2 x$

$\Rightarrow A^n x = \lambda^n x$