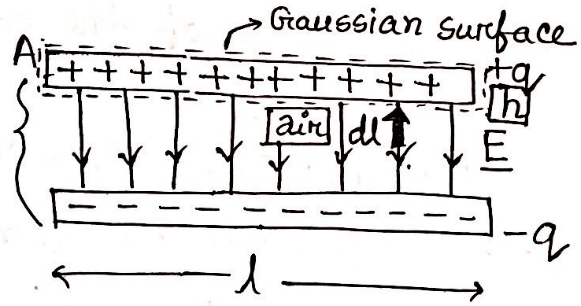
Capacitor: If we have an arrangement of two ~~equal and opposite charges, we pl~~ such arra nearby conductors carrying equal and opposite charges, such arrangement is called capacitor. the conductors are called plates.

$$C = \frac{q}{V}$$

q = magnitude of charge of one plates
V = potential difference between the two plates.

Simplest capacitor is the parallel plate capacitor.

A parallel-plate Capacitor



~~Go~~ Parallel-plate capacitor formed with two parallel conducting plates of area (A) separated by distance (d) in air. The electric field strength (E) between the plates is uniform. If we connect each plate to a battery, a charge $+q$ and other charge $-q$ will appear on the plates.

Gauss's law: $\epsilon_0 \oint\!\!\oint E.ds = q$

$$\Rightarrow \epsilon_0 \iint E\, ds \cos 0^\circ = q$$

$$\Rightarrow \epsilon_0 E \iint ds = q$$

$$\Rightarrow \epsilon_0 E A = q$$

We know, potential $(V) = -\int E.dl$

$$= -\int E\,dl \cos 180^\circ = -E\int dl(-1)$$

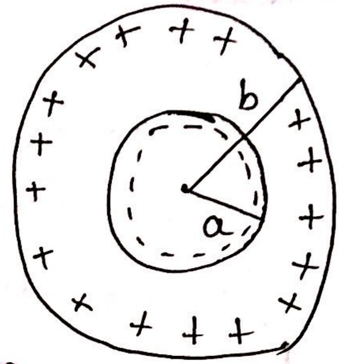
$dl \rightarrow$ element of displacement from (−) plate toward the (+) plate

$$\boxed{V = Ed}$$

$\therefore$ The capacitance of parallel-plate $C = \frac{q}{V} = \frac{\epsilon_0 E A}{Ed} = \boxed{\frac{\epsilon_0 A}{d} = C}$

A cylindrical capacitor

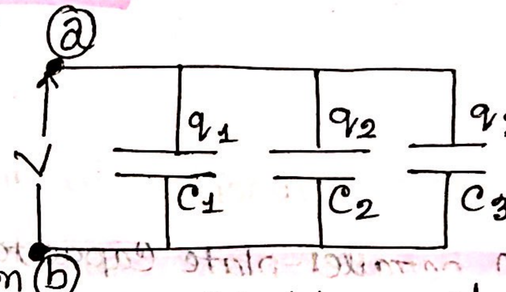
$$\text{Capacitance, } C = \frac{2\pi\epsilon_0 l}{\ln(b/a)}$$



l = length of the cylinder
(prove in slide 31-32)

Parallel combination of Capacitors

We know, $C = \frac{q}{V}$



parallel plate capacitors

$q_1 = C_1 V$, $q_2 = C_2 V$, $q_3 = C_3 V$

The total charge of the combination

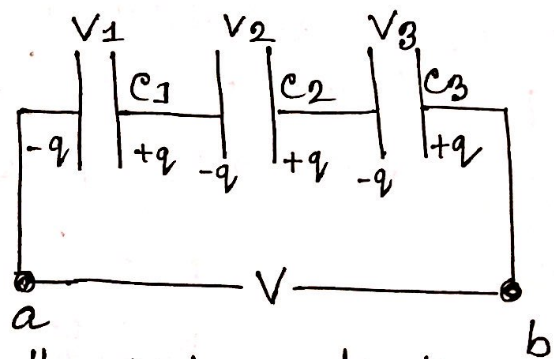
$$q = q_1 + q_2 + q_3$$

$$q = C_1 V + C_2 V + C_3 V = V(C_1 + C_2 + C_3)$$

If the equivalent Capacitance is C, then,

$$C = \frac{q}{V} = C_1 + C_2 + C_3$$

Series Combination of Capacitor



We know, $C = \frac{q}{V}$

$$V_1 = \frac{q}{C_1}, \quad V_2 = \frac{q}{C_2}, \quad V_3 = \frac{q}{C_3}$$

The potential difference between the series combination,

$$V = V_1 + V_2 + V_3 = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

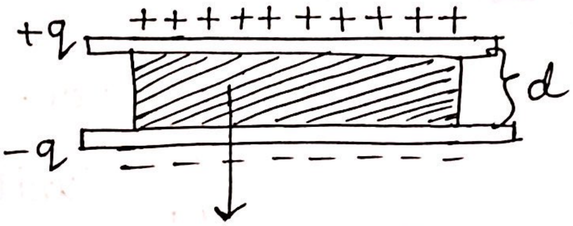
If the equivalent Capacitance is C, then,

$$C = \frac{q}{V} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}} \qquad \therefore \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

A Parallel-plate capacitor with a Dielectric

The capacitance

$$C = \frac{K \varepsilon_0 A}{d}$$



dielectric medium with " constant K

Energy Storage in an $\underline{E}$

let us suppose, at a time (t) a charge $q'(t)$ has been transferred from one plate to the other. The potential difference between the plates is, $V(t) = \frac{q'(t)}{C}$

if dq' is transferred, work will be, $dW = V dq' = \frac{q'}{C} dq'$

∴ total work will be,

$$W = \int dW = \int_0^q \frac{q'}{C} dq' = \frac{1}{2}\frac{q^2}{C} = \frac{q^2}{2C}$$

Since, $q = CV$

$$W = \frac{(CV)^2}{2C} = \frac{1}{2}CV^2 = U$$

→ potential energy stored in the capacitor resides in the electric field $\underline{E}$

The energy density (u)

↓

(stored energy per unit volume)

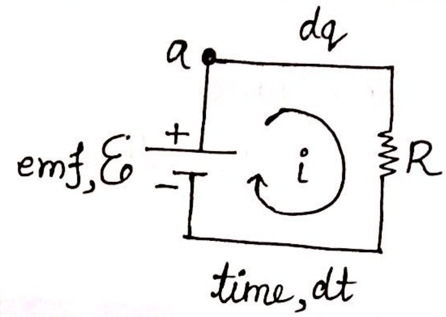
$$\text{energy density}(u) = \frac{\text{Energy}}{\text{Volume}} = \frac{U}{V} = \frac{U}{Ad} = \frac{CV^2/2}{Ad} \quad [\text{Volume} = Ad]$$

$$u = \frac{CV^2}{2Ad} = \frac{K\varepsilon_0 A/d}{2Ad}(Ed)^2$$

$$\therefore \text{energy density}, (u) = \frac{1}{2} K \varepsilon_0 E^2$$

Electromotive force and circuits

A simple Electric Circuit



electromotive force,

$$\mathcal{E} = \frac{dW}{dq} \quad \text{(unit : Volt)}$$

dW = small amount of work done to bring the charge dq from (−)ve to (+)ve terminal of the source

Now, let us calculate the current in the simple circuit, We know, current $i = \frac{dq}{dt}$

Joule Heat $\cdot i^2Rdt$ (appear in the resistare)

Work done, $dW = \mathcal{E}dq = \mathcal{E}idt$

$\therefore$ From, energy principle,

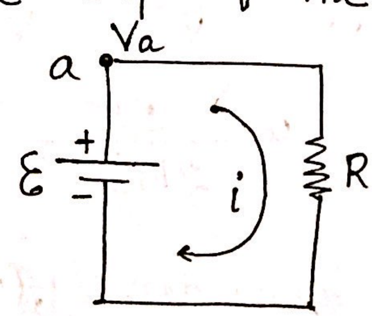
$$\mathcal{E}idt = i^2Rdt$$

$$\mathcal{E} = iR \qquad \therefore \boxed{i = \frac{\mathcal{E}}{R}}$$

Laws of solving Circuit Equations:

Kirchhof's Voltage Law: (Loop theorem)

"The algebric sum of the changes in potential encountered in making a complete loop of the circuit must be zero."



$$V_a - iR + \mathcal{E} = V_a$$

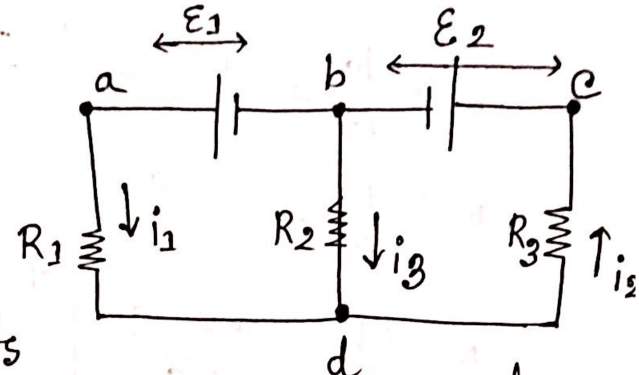
$$\therefore -iR + \mathcal{E} = 0$$

independent of the Value V_a. (Rule : Slide-38)

□ Kirchhof's current Law:

(slide 39 (2nd Para))

$$i_1 + i_3 - i_2 = 0$$

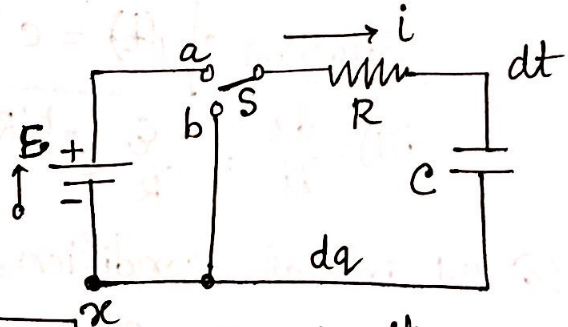


multiloop circuit

"At any junction the algebric sum of the currents must be zero."

~~Itd~~ It is also known as junction theorem.

□ A simple RC circuit:



RC circuit

dt → small time

dq → small amount of charge

$$dq = i\,dt$$

Work done, $\boxed{dW = \varepsilon dq = i^2 R dt + dU}$

$$\therefore \; \varepsilon dq = i^2 R dt + d\left(\frac{q^2}{2C}\right) \qquad \left[dU = d\frac{q^2}{2C}\right]$$

$$\Rightarrow \varepsilon dq = i^2 R dt + \frac{1}{2C} 2q\,dq$$

$$\Rightarrow \varepsilon dq = i^2 R dt + \frac{q}{C} dq$$

charging capacitor

dividing both sides by dt, we get,

$$\varepsilon \frac{dq}{dt} = i^2 R + \frac{q}{C}\frac{dq}{dt}$$

$$\Rightarrow \varepsilon i = i^2 R + \frac{q}{C} i \qquad \left[i = \frac{dq}{dt}\right]$$

$$\Rightarrow \varepsilon = iR + \frac{q}{C}$$, can be derived by loop theorem:

$$\Rightarrow \varepsilon = R \cdot \frac{dq}{dt} + \frac{q}{C} \qquad \left[i = \frac{dq}{dt}\right]$$

$$\Rightarrow R\frac{dq}{dt} + \frac{q}{c} = \mathcal{E}$$

$$\Rightarrow R\frac{dq}{dt} = \mathcal{E} - \frac{q}{c}$$

$$\Rightarrow \frac{dq}{dt} = \frac{c\mathcal{E} - q}{RC}$$

$$\Rightarrow \frac{dq}{c\mathcal{E}-q} = \frac{1}{RC}\,dt$$

Integrating both sides we get,

$$\int_0^q \frac{1}{c\mathcal{E}-q}\,dq = \int_0^t \frac{1}{RC}\,dt$$

$$\Rightarrow \left[\frac{\ln(c\mathcal{E}-q)}{-1}\right]_0^q = \frac{1}{RC}\left[t\right]_0^t$$

$$\Rightarrow \frac{\ln(c\mathcal{E}-q)}{\quad} =$$

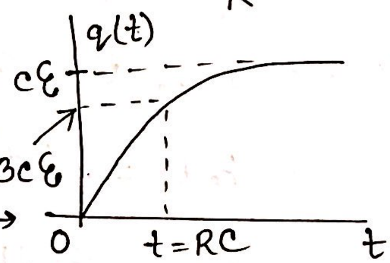
Solution : $\boxed{q(t) = c\mathcal{E}\,(1 - e^{-t/RC})}$

$$i(t) = \frac{dq}{dt} = \frac{\mathcal{E}}{R}e^{-t/RC}$$

(a) At initial condition, $t=0$, $q=0$ and $i = \frac{\mathcal{E}}{R}$

(b) as $t \to \infty$, $q = c\mathcal{E}$ and $i \to 0$

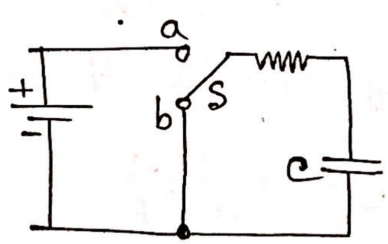
charging the capacitor w.r.to time



RC → RC time constant

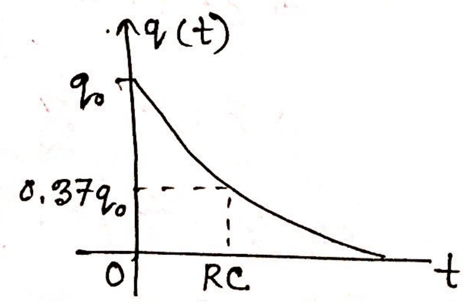
☐ Discharging the capacitor

$$R\frac{dq}{dt} + \frac{1}{c}q = 0 \text{ (No emf)}$$



Solution: $q(t) = q_0 e^{-t/RC}$

$q_0 \rightarrow$ initial charge on the capacitor



Discharging capacitor w.r.to time