Exponential Distribution

1. Data collected at Toronto Pearson International Airport suggests that an exponential distribution with $\theta = 0.37$ is a good model for rainfall duration in hours.

   a) What proportion of rainfall durations at this location is at least 2 hours? At most, 3 hours? Between 2 and 3 hours?

   b) What must the rainfall duration be to place it among the longest 5% of all time?

2. Extensive experience with fans of a certain type used in diesel engines has suggested that the exponential distribution with $\theta = 0.00004$ provides a good model for a time until failure (hour).

   a) What proportion of fans will last at least 20,000 hours? At most 30,000 hours? Between 20,000 and 30,000 hours?

   b) What must the lifetime of a fan be to place it among the best 1% of all fans? Among the worst 1%?

3. The article "Probabilistic Fatigue Evaluation of Riveted Railway Bridges" (J. of Bridge Engr., 2008: 237–244) suggested the exponential distribution with $\theta = \frac{1}{6}$ as a model for the distribution of stress range (MPa) in certain bridge connections.

   a) What proportion of stress ranges are at least 2 MPa? At most, 7 MPa? Between 5 and 10 MPa?

   b) What value separates the highest 2% of the stress ranges from the remaining 98%?

Stochastic and Queueing Process

1. A radio repair shop in a shopping mall has one serviceman. Computers are brought in for repair and arrive according to a Poisson process with a constant arrival rate of two pairs per hour. The repair time distribution is exponential, with a mean of 20 minutes, and the repair and arrival process is independent. Consider a pair of radios to be the unit to be served, and do the following:

   a) Find the probability that the number of pairs of radios in the system exceeds 2.

   b) The mean number of pairs waiting for service.

   c) Find the mean turnaround time for a pair of radios (time to wait plus repair).

2. In a supermarket, the average arrival rate of customers is five every 30 minutes. The arrival time it takes to list and calculate the customers' purchase at the cash desk is 4.5min and this time is exponentially distributed

   a) How long will the customer expect to wait for service?

   b) What is the chance that the queue length will exceed 1?

   c) What is the probability that the cashier is working?

3. A communication satellite is launched via a booster system with a discrete-time guidance control system. Course correction signals form a sequence $\{X_n\}$ where the state space for X is as follows:

0: No correction required

1: Minor correction required

2: Major correction required

$$\begin{pmatrix} 1 & 0 & 0 \\ \frac{2}{3} & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & 1 \end{pmatrix}$$

If today has no required for correction in communication satellite launched, find the probability that it will be required to do major correction exactly 2 days hence.

4. A television repairman finds that the time spent on his job is exponentially distributed with a mean of 30 min. The average inter-arrival time of TV sets is 120 min, where the arrival follows Poisson distribution. What fraction of his time would he be busy repairing?

5. Weather data are analyzed for a particular locality, and a Markov chain is employed as a model for weather change as follows. The conditional probability of change from rain to clear weather in one day is 0.5. The conditional probability of transition from clear to rain in one day is 0.6. The model will be a discrete-time model, with the transition occurring only between days.

a) Determine the matrix P of the one-step transition probability.
b) Find the steady-state probability of a rainy day.
c) If today is clear, find the probability that it will be clear the day after tomorrow.
d) The initial probabilities are: $p_R = 0.3, p_c = 0.7$. Find the probability that starting Monday raining, it will also be raining on Thursday.

6. Question: Here is a matrix P of one-step transition probability. Now find the probability of the cure in the next three days if today is sick.

$$P=\begin{pmatrix} 0.3 & 0.1 & 0.6 \\ 0.2 & 0.7 & 0.1 \\ 0.5 & 0.3 & 0.2 \end{pmatrix}$$

Let, 0= sick; 1= Under treatment; 2= Cure

7. Customers arrive (at a supermarket) at a Poisson rate of 1 per every 12 minutes, and the service time is exponential at a rate of 1 per every 8 minutes. Then find

a) Find the proportion of time there is no customer in the supermarket.
b) Find the proportion of time the supermarket is busy.
c) Find the average number of customers in the supermarket.
d) The average time a customer spends in that queue to get into the supermarket.

Hypothesis Testing

1. From past experience a television manufacturer found that 10 percent or less of its sets needed any type of repair in the first two years of operation. In a sample of 50 sets manufactured two years ago, 9 needed repairs. At the .05 significance level, has the percent of sets needing repair increased?

2. The chamber of commerce of a Florida Gulf Coast community advertises that area residential property is available at a mean cost of $125,000 or less per lot. Suppose a sample of 32 properties provided a sample mean of $130,000 per lot with standard deviation of $12,500. Use a 0.01 level of significance to test the validity of the advertising claim.

3. A radio station in Myrtle Beach announced that at least 90% of the hotels and motels would be full for the Memorial Day weekend. The station advised listeners to make reservations in advance if they planned to be in the resort over the weekend. On Saturday night a sample of 58 hotels and motels showed 49 with a no-vacancy sign and 9 with vacancies. Can we conclude that the announcement was wrong? Use $\alpha = 0.05$ in making the statistical test.

4. The mean length of a small counterbalance bar is 43 millimeters. The production supervisor is concerned that the adjustments of the machine producing the bars have increased or not. He asks the Engineering Department to investigate. Engineering selects a random sample of 12 bars and measures each. The results are reported below in millimeters.

42	39	42	45	43	40	38	41	40	42	43	42

Is it reasonable to conclude that the mean length of the bars increased? Use the 0.01 significance level.