Chapter #4: Diodes

from **Microelectronic Circuits** Text

by Sedra and Smith

Oxford Publishing

Introduction

- **IN THIS CHAPTER WE WILL LEARN**
  - the characteristics of the ideal diode and how to analyze and design circuits containing multiple ideal diodes together with resistors and dc sources to realize useful and interesting nonlinear function
  - the details of the i-v characteristic of the junction diode (which was derived in Chapter 3) and how to use it to analyze diode circuits operating in the various bias regions: forward, reverse, and breakdown
  - a simple but effective model of the diode i-v characteristic in the forward direction: the constant-voltage-drop model

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Introduction

- a powerful technique for the application and modeling of the diode (and in later chapters, transistors): dc-biasing the diode and modeling its operation for small signals around the dc-operating point by means of the small-signal model
- the use of a string of forward-biased diodes and of diodes operating in the breakdown region (zener diodes), to provide constant dc voltages (voltage regulators)
- application of the diode in the design of rectifier circuits, which convert ac voltages to dc as needed for powering electronic equipment
- a number of other practical and important applications

4.1.1. **Current-Voltage Characteristic** of the Ideal Diode

- **ideal diode** – most fundament nonlinear circuit element
  - two terminal device
  - circuit symbol shown to right
  - operates in two modes
    - on and off

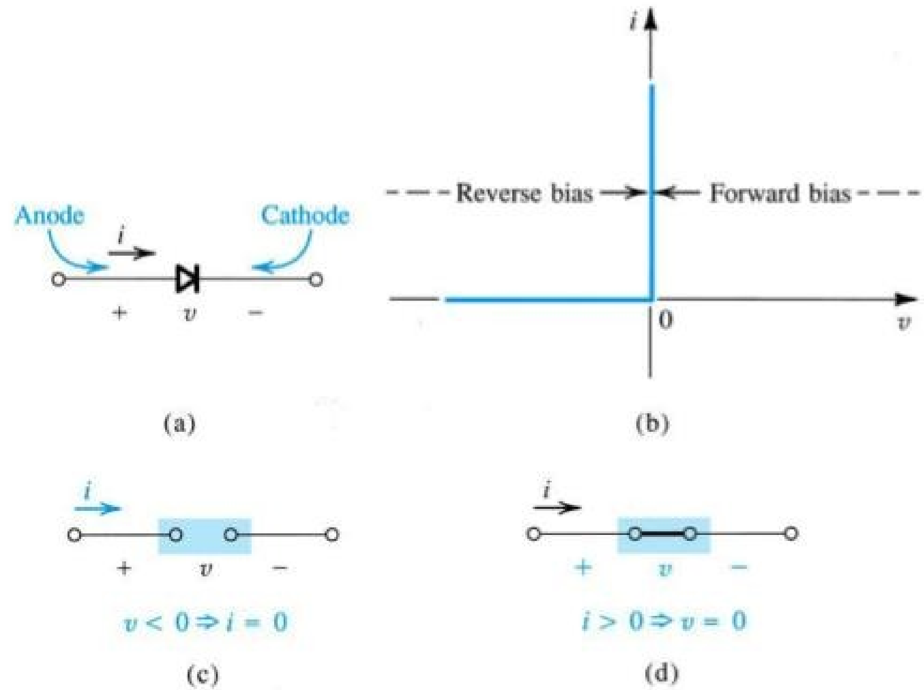

Figure 4.1: Diode characteristics

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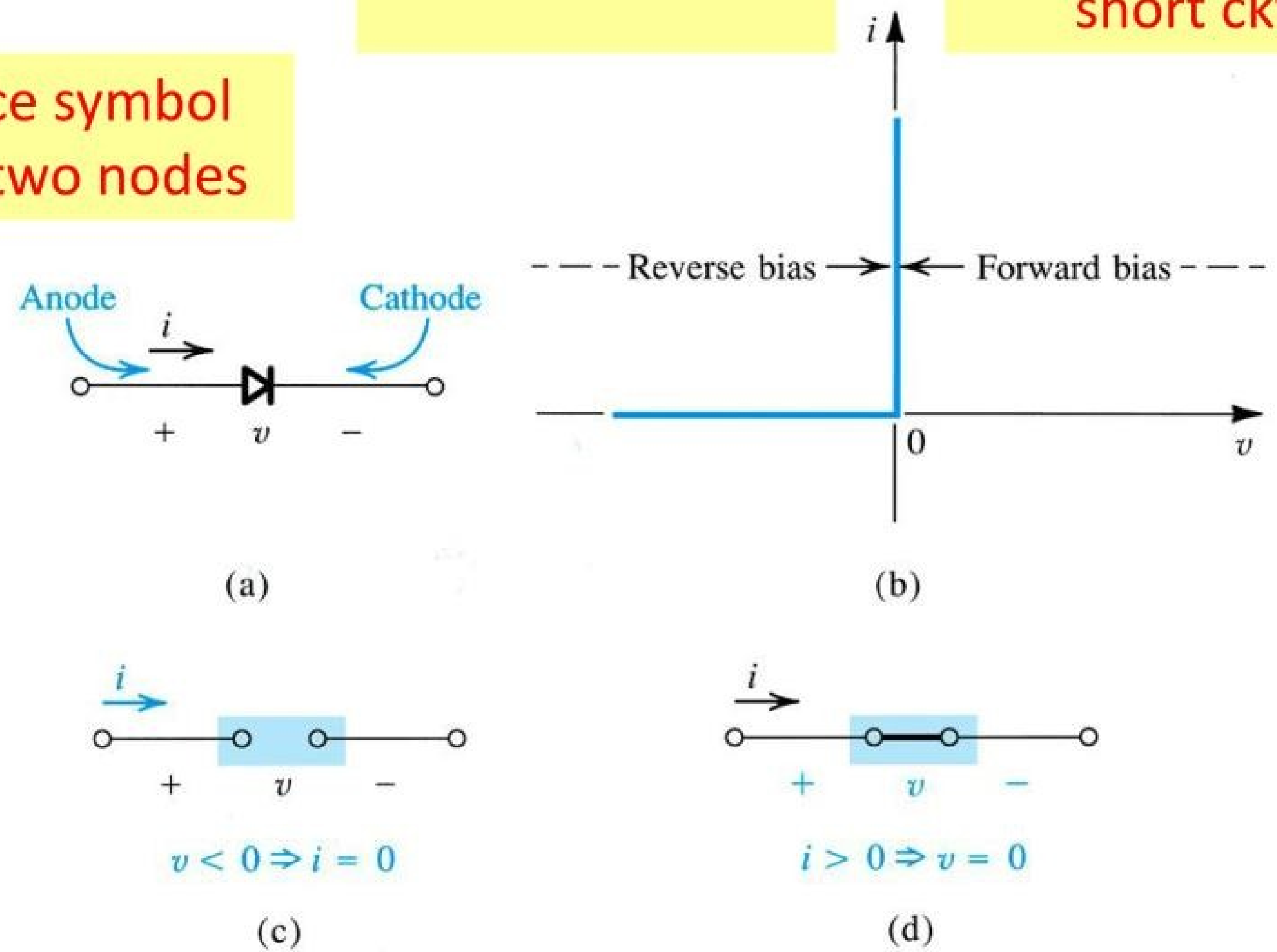
4.1.1. **Current-Voltage Characteristic**

- **cathode** – negative terminal, from which current flows
- **anode** – positive terminal of diode, into which current flows
- voltage-current (*VI*) behavior is:
  - piecewise linear for rated values
  - nonlinear beyond this range

mode #2: reverse bias = open ckt.

mode #1: forward bias = short ckt

device symbol with two nodes



4.1.1. Current-Voltage Characteristic

- External circuit should be designed to limit…
  - current flow across conducting diode
  - voltage across blocking diode
- Examples are shown to right…

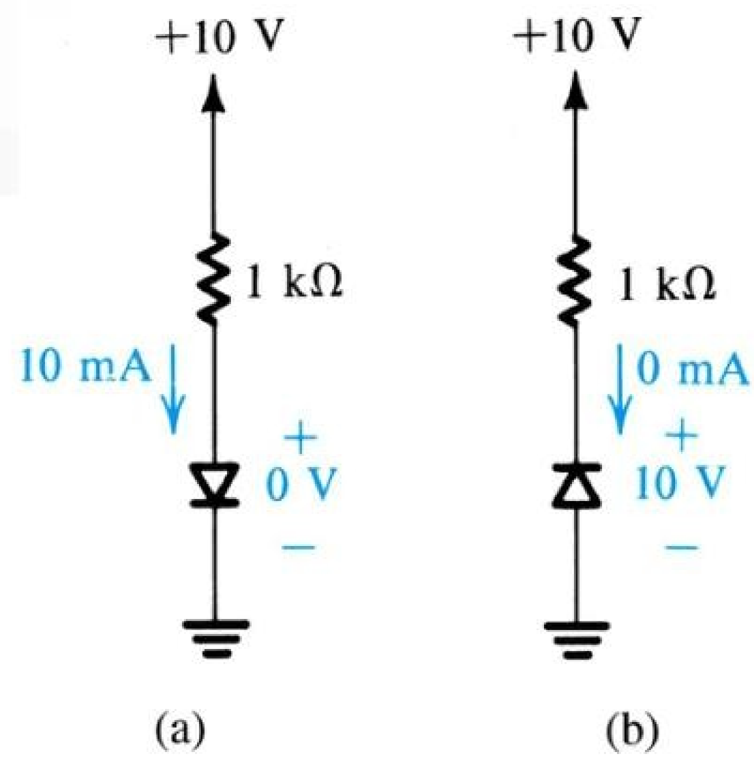


Figure 4.2: The two modes of operation of ideal diodes and the use of an external circuit to limit **(a)** the forward current and **(b)** the reverse voltage.

4.1.2: **A Simple Application – The Rectifier**

- One fundamental application of this piecewise linear behavior is the rectifier.
- **Q:** What is a **rectifier**?
  - **A:** Circuit which converts AC waves in to DC…ideally with no loss.

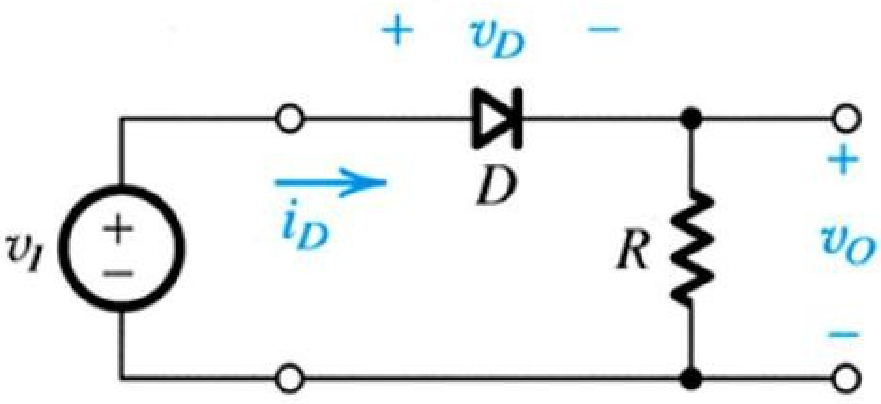


Figure 4.3(a): Rectifier Circuit

4.1.2: **A Simple Application – The Rectifier**

- This circuit is composed of diode and series resistor.
- **Q:** How does this circuit operate?
  - **A:** The diode blocks reverse current flow, preventing negative voltage across *R*.

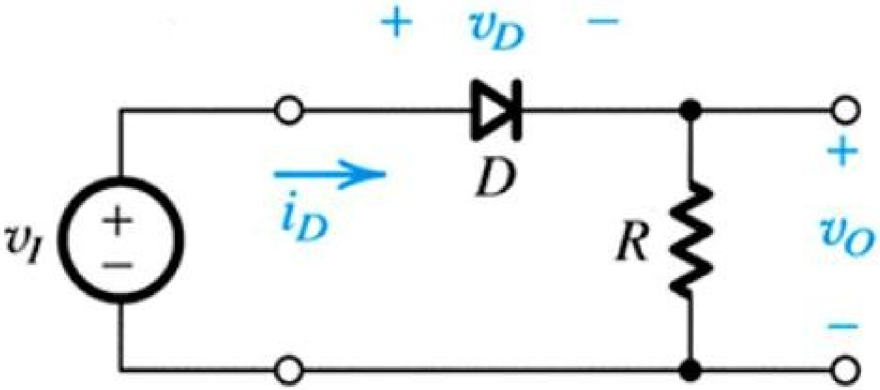


Figure 4.3(a): Rectifier Circuit

Example 4.1:
Diode Rectifier

- Consider the circuit of Figure 4.4. A source (v_S) with peak amplitude of 24V is employed to charge a 12V dc-battery.
  - **Q(a):** Find the fraction of each cycle during which the diode conducts.
  - **Q(b):** Find peak value of diode current and maximum reverse-bias voltage that appears across the diode.

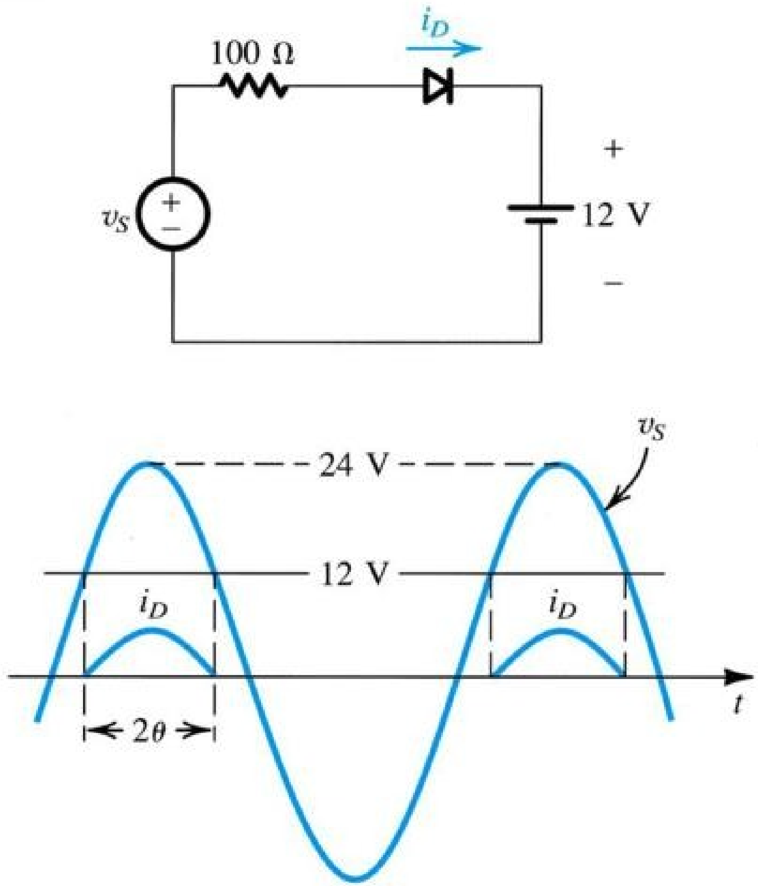


Figure 4.4: Circuit and Waveforms for Example 4.1.

4.1.3. Another Application, Diode Logic Gates

- **Q:** How may diodes be used to create logic gates?
  - **A:** Examples of AND / OR gates are shown right.
    - Refer to next slide.

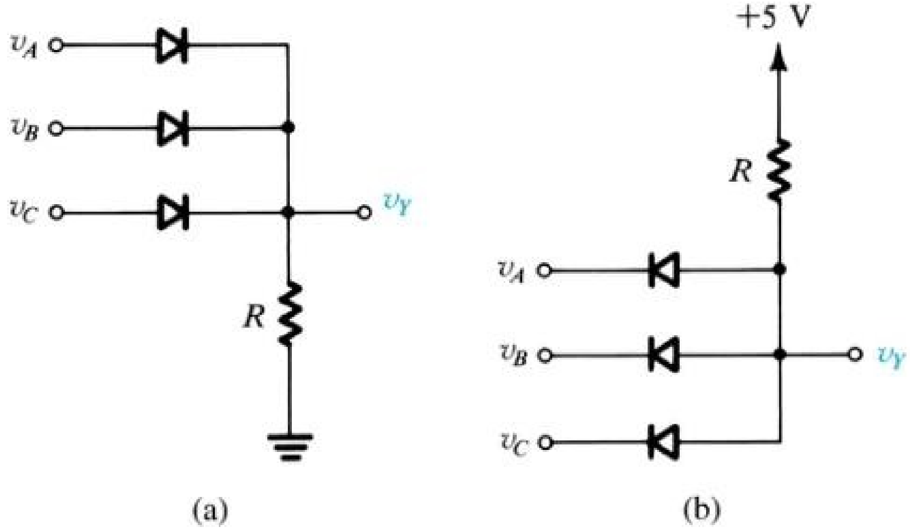
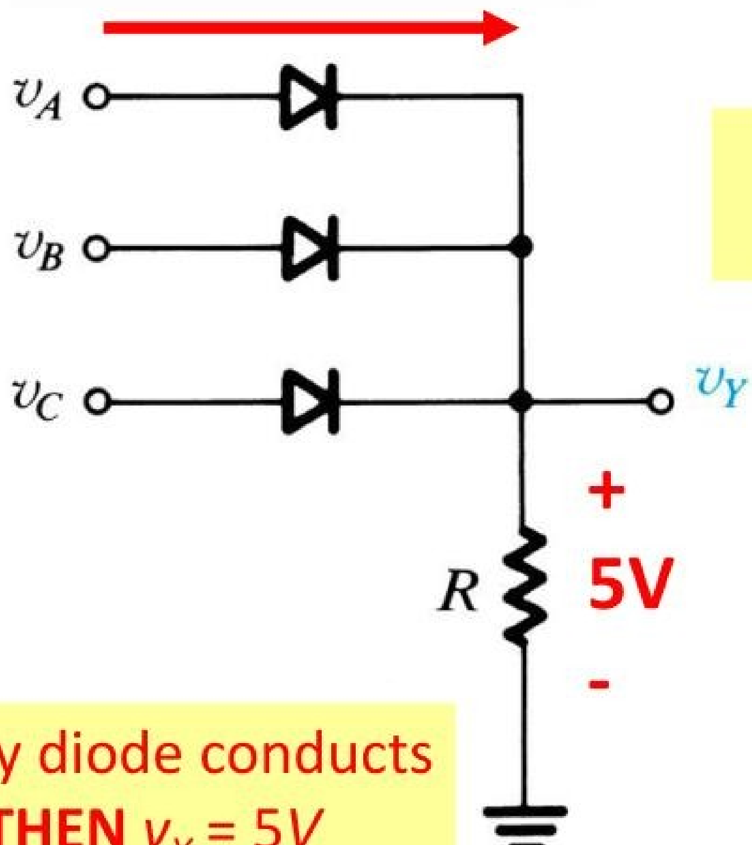


Figure 4.5: Diode logic gates: **(a)** OR gate; **(b)** AND gate (in a positive-logic system).

OR GATE

IF $v_A = 5V$ **THEN** diode_A will conduct **AND** $v_Y = v_A = 5V$

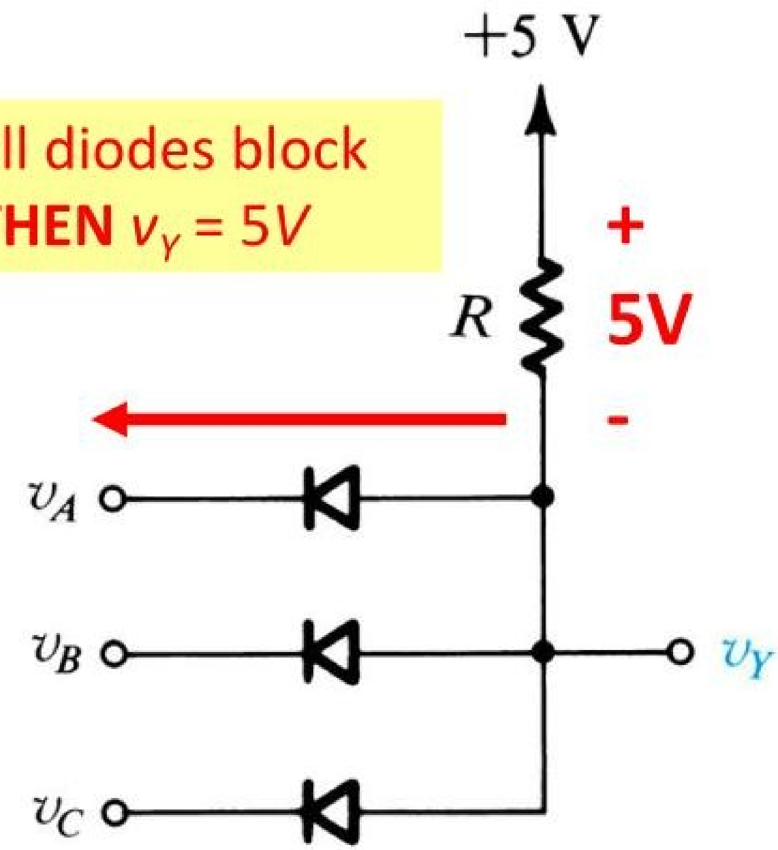


IF any diode conducts **THEN** $v_Y = 5V$

AND GATE

IF $v_A = 0V$ **THEN** diode_A will conduct **AND** $v_Y = v_A = 0V$

IF all diodes block **THEN** $v_Y = 5V$



Example 4.2: More Diodes

To apply nodal / mesh techniques, one must have knowledge of all component impedances.

- **Q:** What difficulties are associated with multi-diode circuits?
  - **A:** Circuit cannot be solved without knowledge of diodes' statuses. Yet, statuses are dependent on the solution.

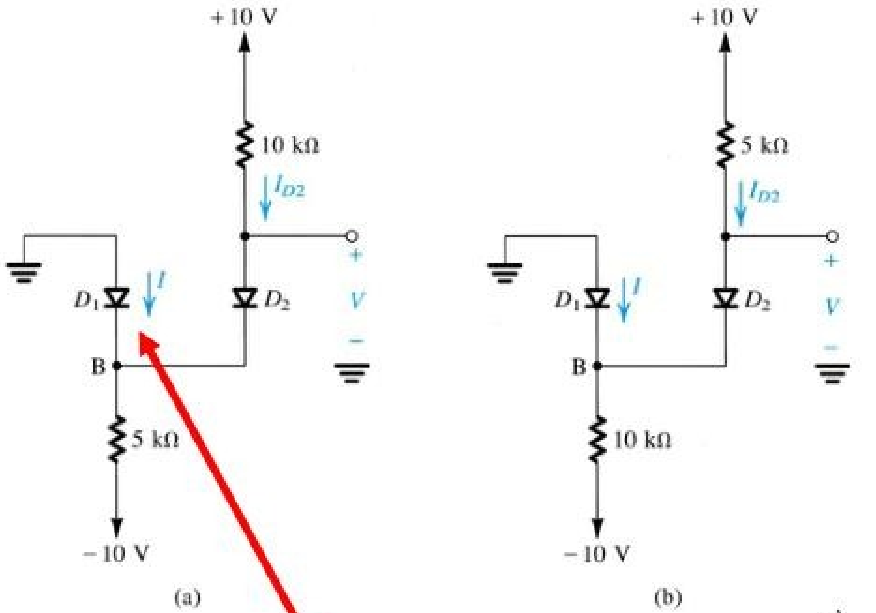


Figure 4.6: Circuits for Example 4.2.

IF $v_B < 0$ **THEN** $Z_{D1} = 0 ohms$
ELSE Z_{D1} = open circuit

Example 4.2:
More Diodes

- **Q:** How does one solve these circuits?
  - **A:** One must use the following steps…
    - **1)** assume the status of all diodes
    - **2)** solve via mesh / nodal analysis
    - **3)** check for coherence

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Example 4.2: More Diodes

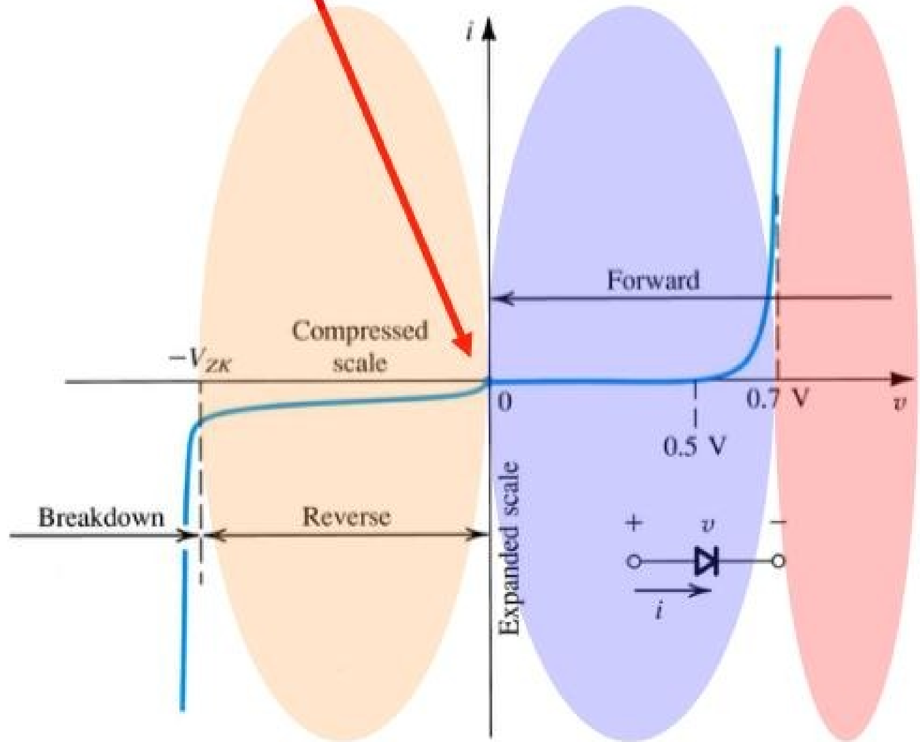
If answer to either of these is **no**, then the solution is **not physically realizable**.

- **Q:** How does one check for coherence?
  - **A:** One must ask the following questions…
    - **1)** Are calculated voltages across all "assumed conducting" diodes forward-biased?
    - **2)** Are the calculated currents through all "assumed blocking" diodes zero?
- **Q:** What does one do, if the solution is not coherent?
  - **A:** One must change one or more of these assumptions and solve as well as check for coherence again.

4.2. **Terminal Characteristics of Junction Diodes**

discontinuity caused by differences in scale

- Most common implementation of a diode utilizes pn junction.
- *I-V* curve consists of three characteristic regions
  - forward bias: $v > 0$
  - reverse bias: $v < 0$
  - breakdown: $v << 0$



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4.2.1. **The Forward-Bias Region**

- The **forward-bias region** of operation is entered when $v > 0$.
- ***I-V* relationship** is closely approximated by equations to right.

(4.3) is a simplification suitable for large *v*

$$\text{(eq4.1)}\ i = \underbrace{I_S}_{I_S = \text{constant for diode at given temperature (aka. saturation current)}}(e^{v/V_T} - 1)$$

V_T = thermal voltage

k= Boltzmann's constant (8.62**E**-5 eV/K)

q= magnitude of electron charge (1.6**E**-19 C)

$$\text{(eq4.2)}\ V_T = \frac{kT}{q} = \underbrace{25.8mV}_{\text{at room temperature}}$$

I_S = constant for diode at given temperature (aka. saturation current)

$$\text{(eq4.3)}\ i = I_S e^{v/V_T}$$

4.2.1. **The Forward-Bias Region**

- Equation (4.3) may be reversed to yield (4.4).
- This relationship applies over as many as seven decades of current.

$$\text{(eq4.4)}\ v = \overbrace{V_T \ln\left(\frac{i}{I_S}\right)}^{I_S = \text{ constant for diode at given temperature (aka. saturation current)}}$$

4.2.1. **The Forward-Bias Region**

- **Q:** What is the relative effect of current flow (i) on forward biasing voltage (v)?
- **A:** Very small.
  - 10x change in i, effects 60mV change in v.

step #1: consider two cases (#1 and #2)

$$I_1 = I_S e^{V_1/V_T} \text{ and } I_2 = I_S e^{V_2/V_T}$$

step #2: divide I_2 by I_1

$$\frac{I_2}{I_1} = \frac{\cancel{I_S} e^{V_2/V_T}}{\cancel{I_S} e^{V_1/V_T}}$$

step #3: combine two exponentials

$$\frac{I_2}{I_1} = e^{(V_2 - V_1)/V_T}$$

step #4: invert this expression

$$V_2 - V_1 = V_T \ln\left(I_2 / I_1\right)$$

step #5: convert to log base 10

$$V_2 - V_1 = \underbrace{2.3 V_T \log\left(I_2 / I_1\right)}_{60mV \approx 2.3 V_T \log(10/1)}$$

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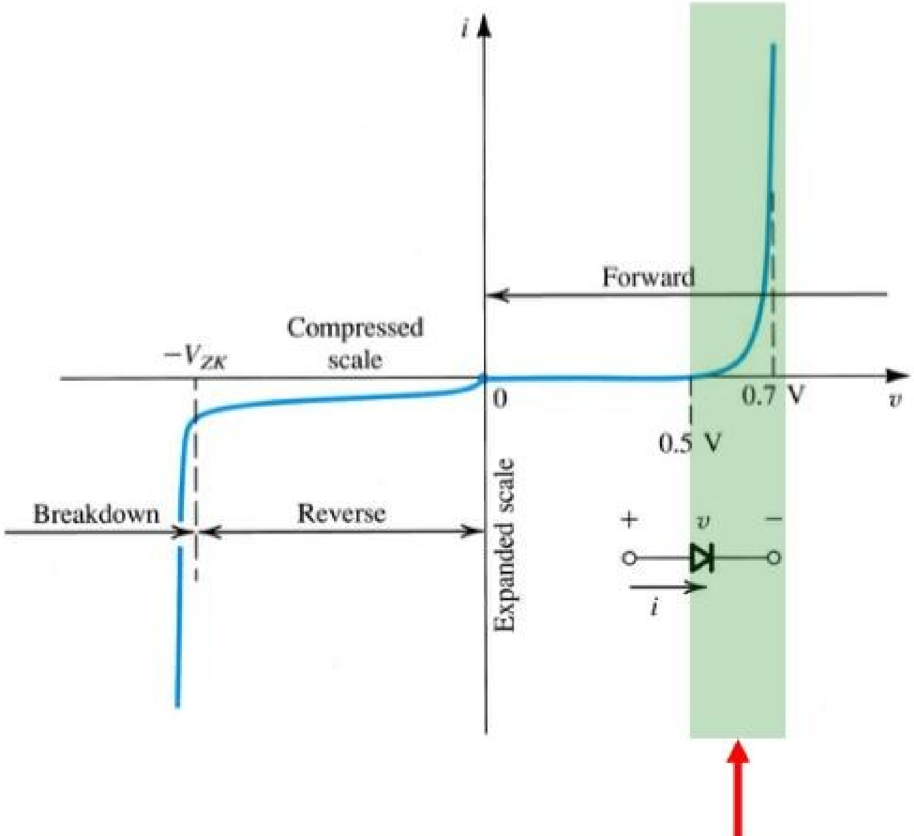
4.2.1: The Forward-Bias Region

- **cut-in voltage** – is voltage, below which, minimal current flows
  - approximately 0.5V
- **fully conducting region** – is region in which R_{diode} is approximately equal 0
  - between 0.6 and 0.8V



fully conducting region

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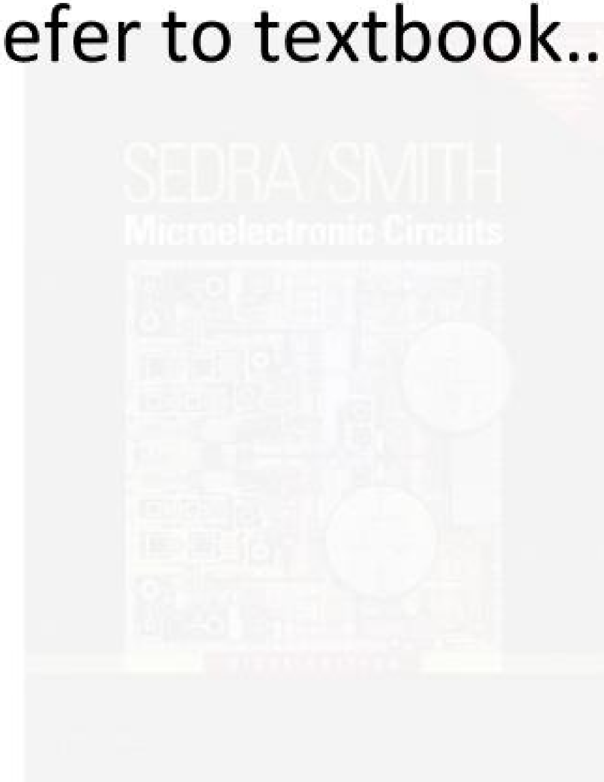
Example 4.3

- Refer to textbook…



4.2.2. **The Reverse-Bias Region**

- The reverse-bias region of operation is entered when $v < 0$.
- *I-V* relationship, for negative voltages with $|v| > V_T$ (25mV), is closely approximated by equations to right.

$$\overbrace{i = -I_S e^{-|v|/V_T}}^{\text{this expression applies for negative voltages}}$$

$$\overbrace{i = -I_S \underbrace{\left(\frac{1}{e^{|v|/V_T}}\right)}_{\approx 0 \text{ for larger voltage magnitudes}}}^{\textbf{action:}\text{ invert exponential}}$$

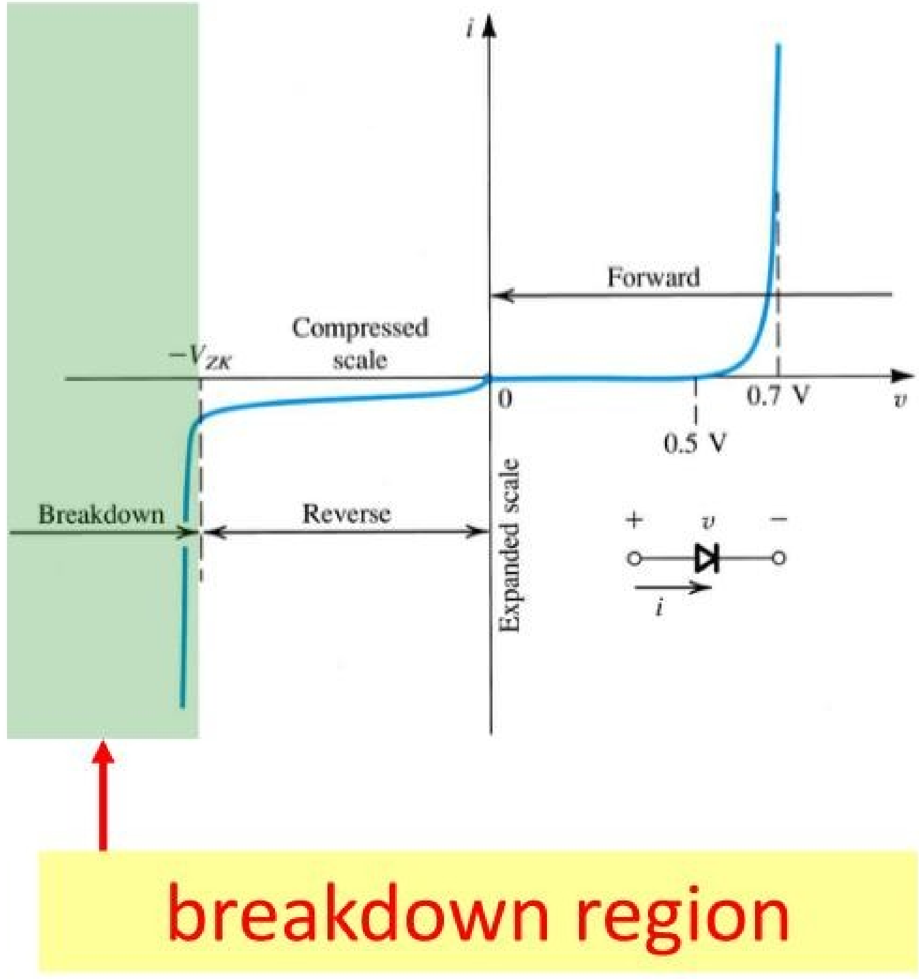
$$i = -I_S$$

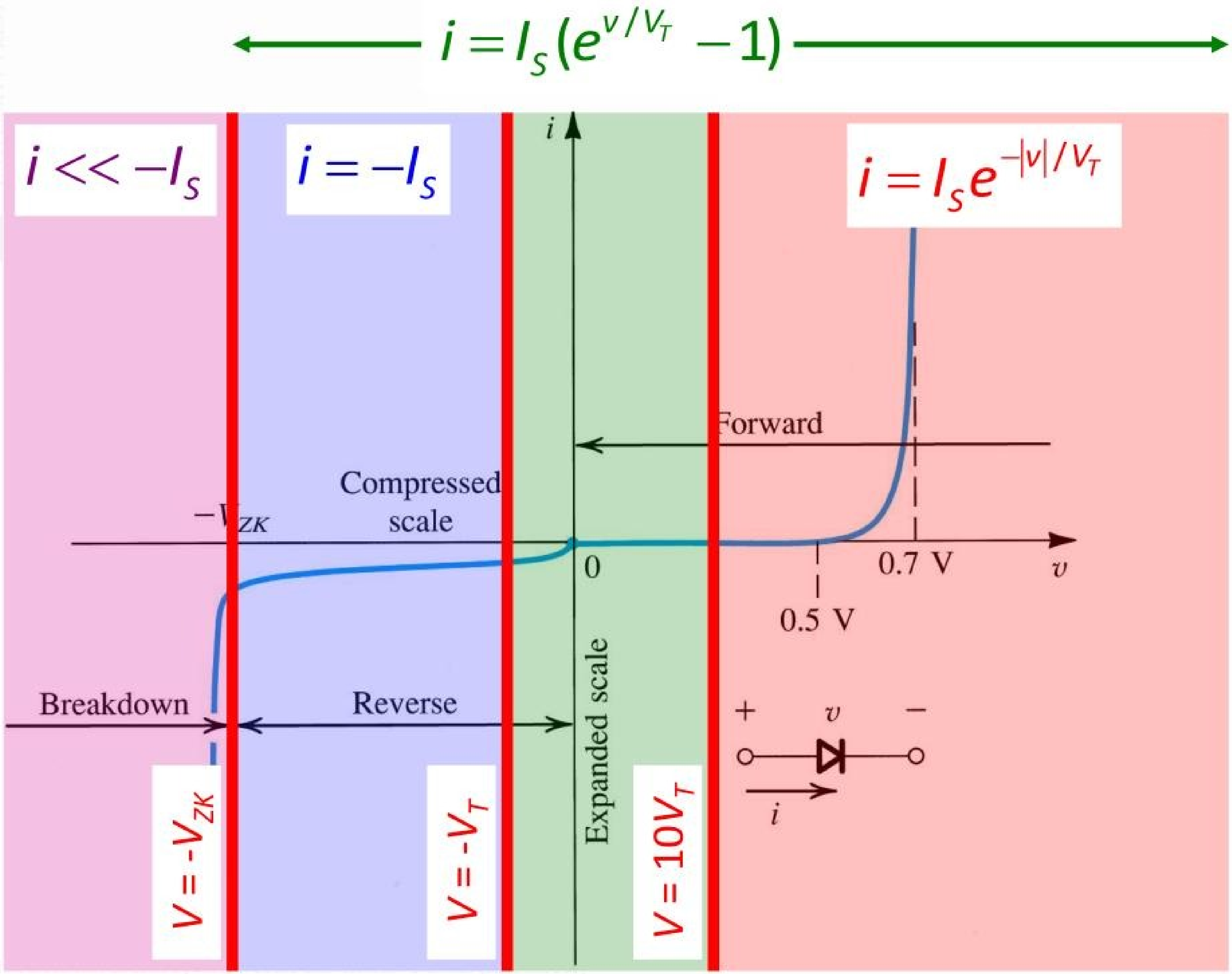
4.2.2. **The Reverse-Bias Region**

- A "real" diode exhibits reverse-bias current, although small, much larger than I_S.
  - 10^{-9} vs. 10^{-14} *Amps*
- A large part of this reverse current is attributed to leakage effects.

4.2.3. **The Breakdown Region**

- The breakdown region of operation is entered when $v < V_{ZK}$.
  - **Zener-Knee Voltage (V_{ZK})**
- This is normally non-destructive.





$i = I_S(e^{v/V_T} - 1)$
$i << -I_S$
$i = -I_S$
$i = I_S e^{-|v|/V_T}$
i
Forward
Compressed scale
$-V_{ZK}$
0
0.7 V
v
0.5 V
Expanded scale
Breakdown
Reverse
+
v
−
i
$V = -V_{ZK}$
$V = -V_T$
$V = 10V_T$

4.3. **Modeling the Diode Forward Characteristic**

- The previous slides define a robust set of diode models.
- Upcoming slides, however, discuss simplified diode models better suited for use in circuit analyses:
  - exponential model
  - constant voltage-drop model
  - ideal diode model
  - small-signal (linearization) model

4.3.1. The Exponential Model

- **exponential diode model**
  - most accurate
  - most difficult to employ in circuit analysis
    - due to nonlinear nature

$$\underbrace{\text{(eq4.6)}\quad I_D = I_S e^{V_D/V_T}}_{\substack{V_D = \text{voltage across diode} \\ I_D = \text{current through diode}}}$$

4.3.1. The Exponential Model

- **Q:** How does one solve for I_D in circuit to right?
  - $V_{DD} = 5V$
  - $R = 1kOhm$
  - $I_D = 1mA$ @ $0.7V$
- **A:** Two methods exist…
  - **graphical method**
  - **iterative method**

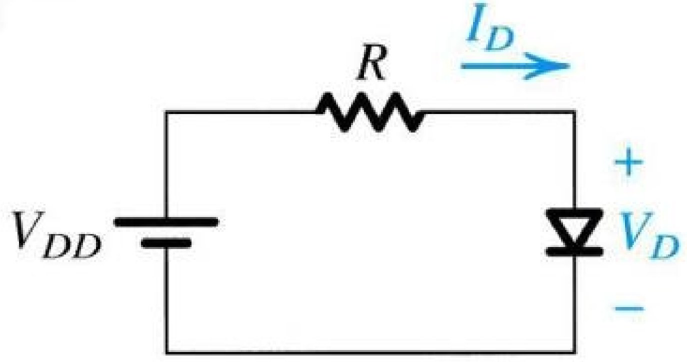


Figure 4.10: A simple circuit used to illustrate the analysis of circuits in which the diode is forward conducting.

$$\text{(eq4.7)}\quad I_D = \frac{V_{DD} - V_D}{R}$$

4.3.2. **Graphical Analysis Using Exponential Model**

- **step #1:** Plot the relationships of (4.6) and (4.7) on single graph
- **step #2:** Find intersection of the two…
  - **load line** and diode characteristic intersect at **operating point**

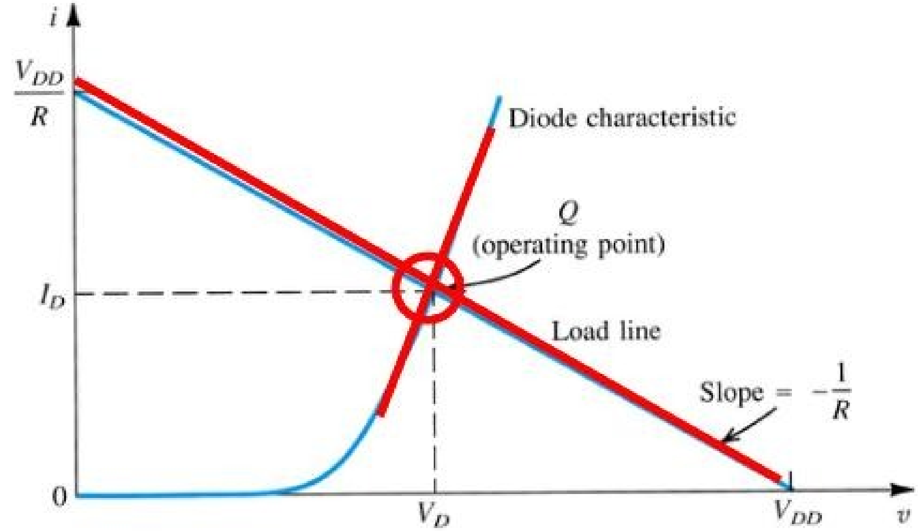


Figure 4.11: Graphical analysis of the circuit in Fig. 4.10 using the exponential diode model.

4.3.2. **Graphical Analysis Using Exponential Model**

- **Pro's**
  - Intuitive
    - b/c of visual nature
- **Con's**
  - Poor Precision
  - Not Practical for Complex Analyses
    - multiple lines required

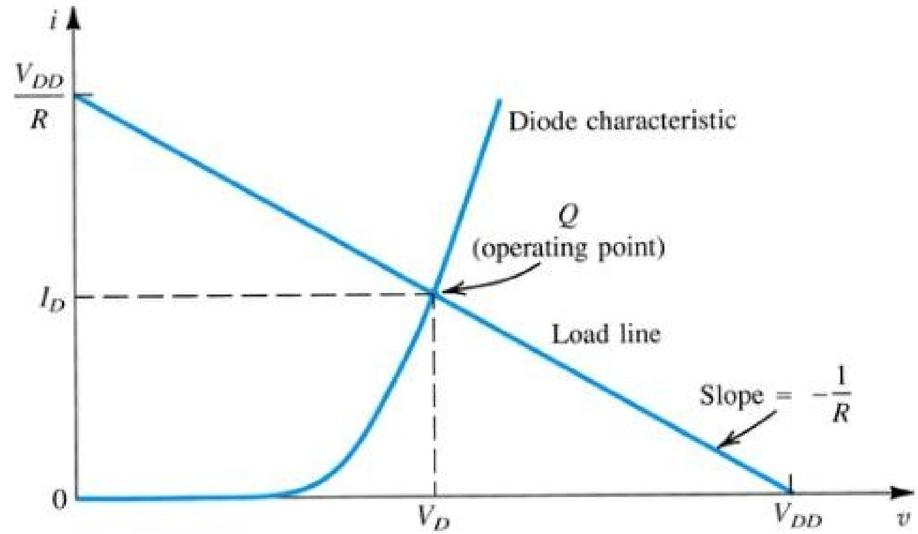


Figure 4.11: Graphical analysis of the circuit in Fig. 4.10 using the exponential diode model.

4.3.3. Iterative Analysis Using Exponential Method

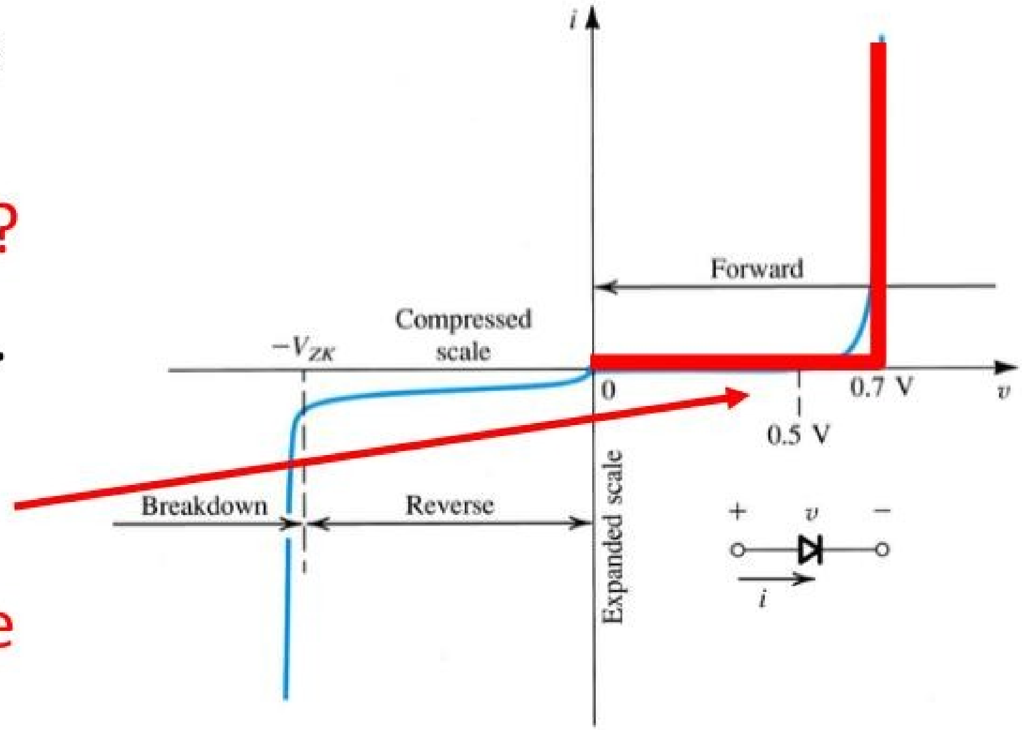
- **step #1:** Start with initial guess of V_D.
  - $V_D^{(0)}$
- **step #2:** Use nodal / mesh analysis to solve I_D.
- **step #3:** Use exponential model to update V_D.
  - $V_D^{(1)} = \mathbf{f}(V_D^{(0)})$
- **step #4:** Repeat these steps until $V_D^{(k+1)} = V_D^{(k)}$.
  - Upon convergence, the new and old values of V_D will match.

4.3.3. Iterative Analysis Using Exponential Method

- Pro's
  - High Precision
- Con's
  - Not Intuitive
  - Not Practical for Complex Analyses
    - 10+ iterations may be required

4.3. **Modeling the Diode Forward Characteristic**

- **Q:** How can one analyze these diode-based circuits more efficiently?
- **A:** Find a simpler model.
  - One example is assume that voltage drop across the diode is constant.



4.3.5. **The Constant Voltage-Drop Model**

- The **constant voltage-drop diode model** assumes that the slope of I_D vs. V_D is vertical *@ 0.7V*
- **Q:** How does example 4.4 solution change if CVDM is used?
  - **A:** 4.262*mA* to 4.3*mA*

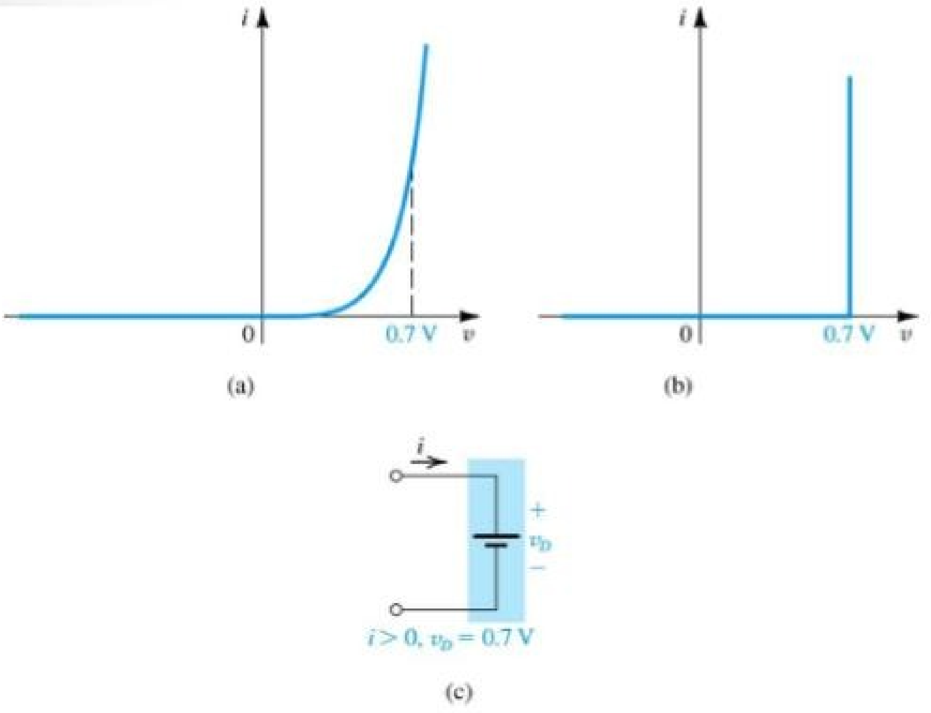
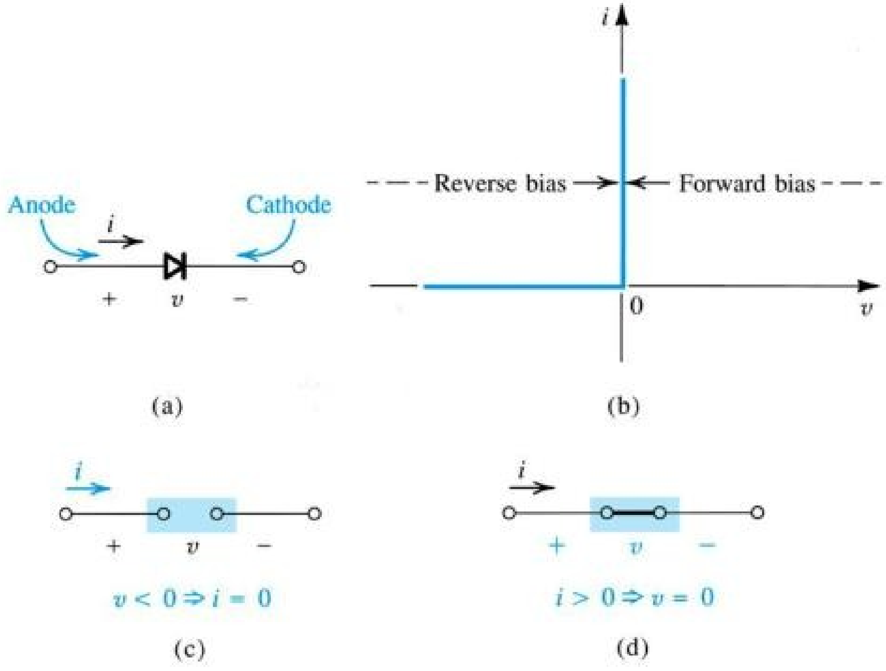


Figure 4.12: Development of the diode constant-voltage-drop model: **(a)** the…

4.3.6. **Ideal Diode Model**

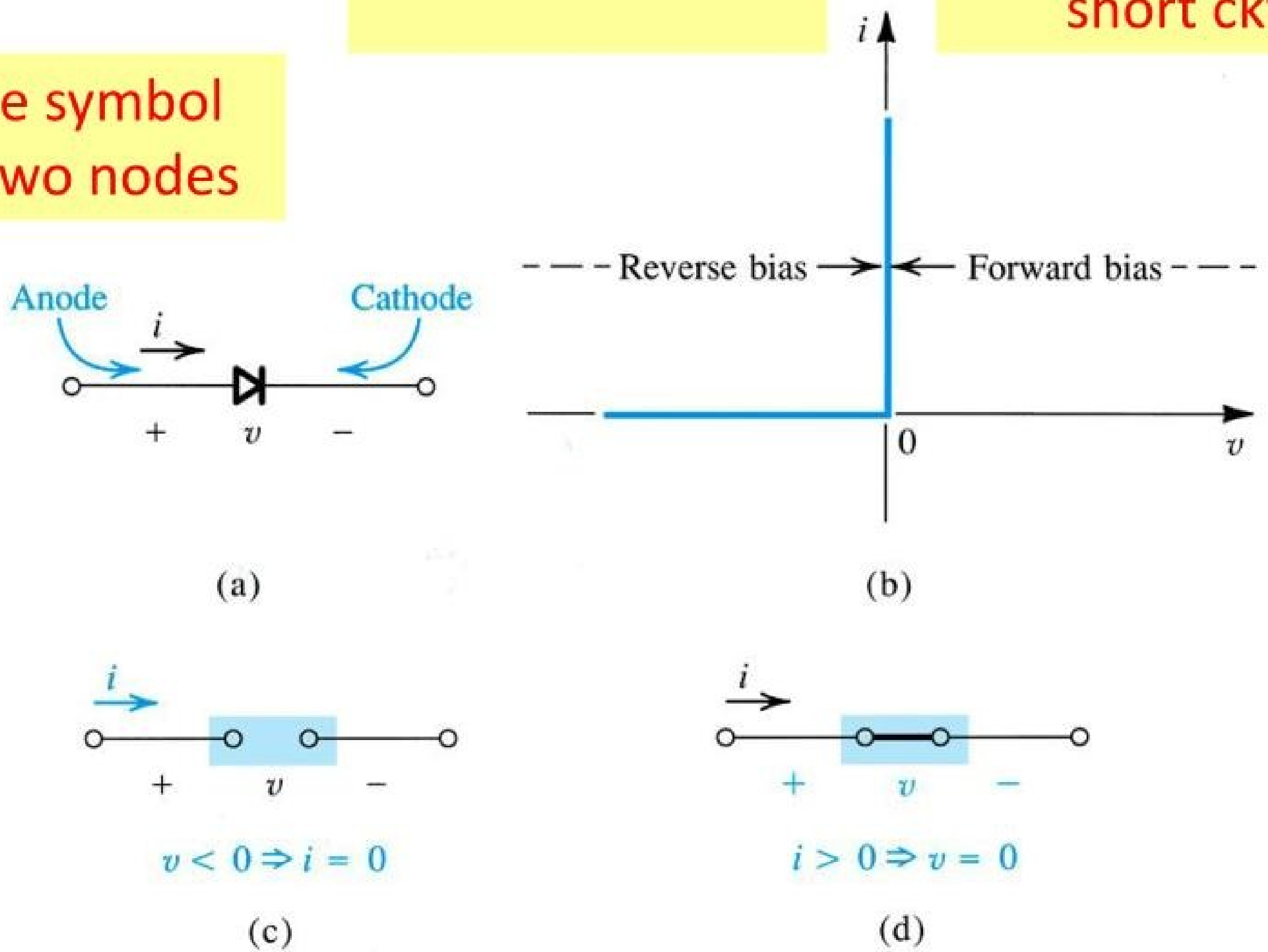
- The **ideal diode model** assumes that the slope of I_D vs. V_D is vertical @ $0V$
- **Q:** How does example 4.4 solution change if ideal model is used?
  - **A:** 4.262mA to 5mA



mode #2: reverse bias = open ckt.

mode #1: forward bias = short ckt

device symbol with two nodes



When to use these models?

- **exponential model**
  - low voltages
  - less complex circuits
  - emphasis on accuracy over practicality
- **constant voltage-drop mode**:
  - medium voltages = 0.7*V*
  - more complex circuits
  - emphasis on practicality over accuracy
- **ideal diode model**
  - high voltages >> 0.7*V*
  - very complex circuits
  - cases where a difference in voltage by 0.7*V* is negligible
- **small-signal model**
  - this is next…

4.3.7. **Small-Signal Model**

- **small-signal diode model**
  - Diode is modeled as variable resistor.
  - Whose value is defined via linearization of exponential model.
  - Around bias point defined by constant voltage drop model.
    - $V_D^{(0)} = 0.7V$

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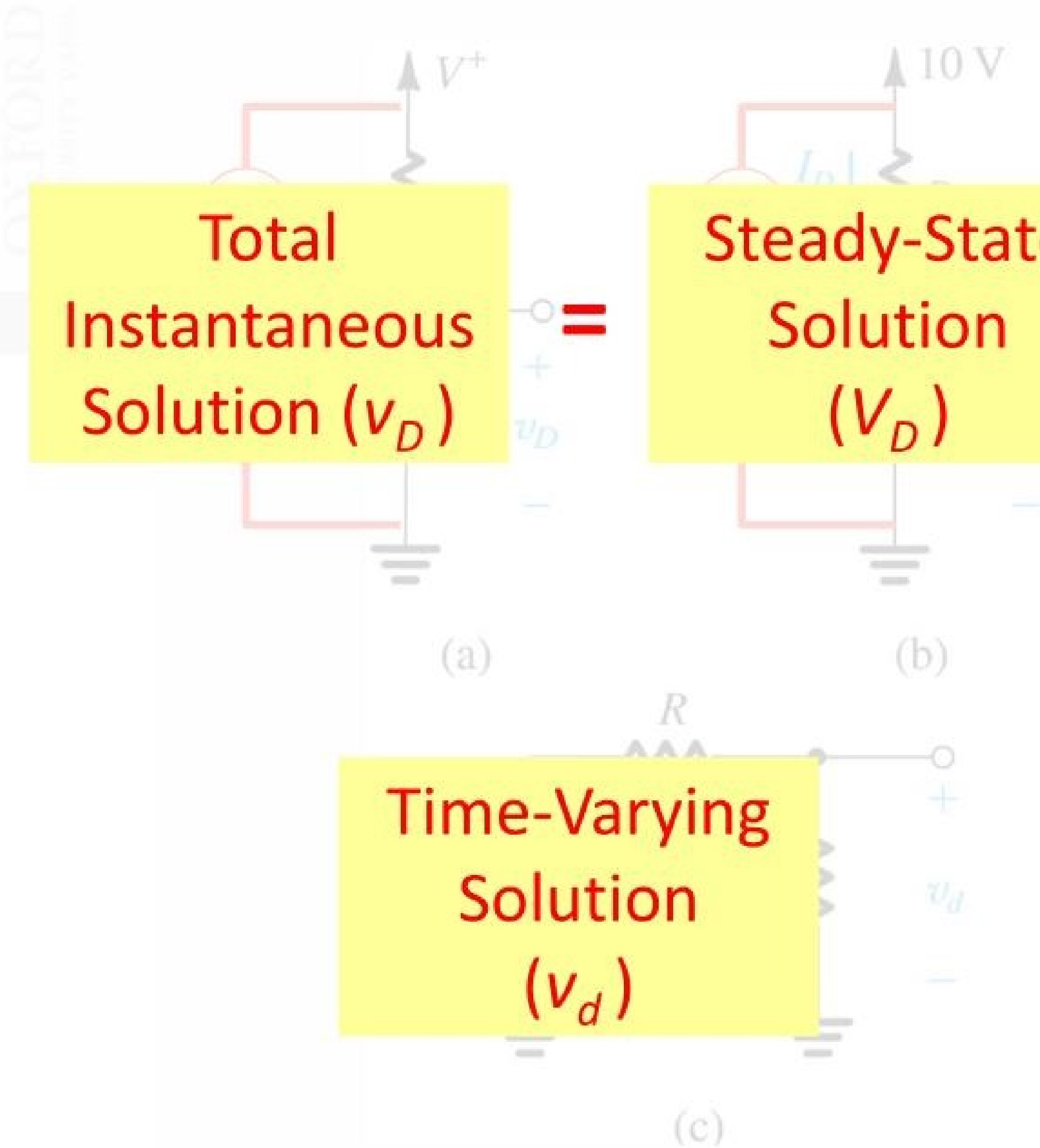
4.3.7. **Small-Signal Model**

Neither of these circuits employ the exponential model – simplifying the "solving" process.

- **Q:** How is the small-signal diode model defined?
  - **A:** The total instantaneous circuit is divided into steady-state and time varying components, which may be analyzed separately and solved via algebra.
    - In steady-state, diode represented as CVDM.
    - In time-varying, diode represented as resistor.

CVDM

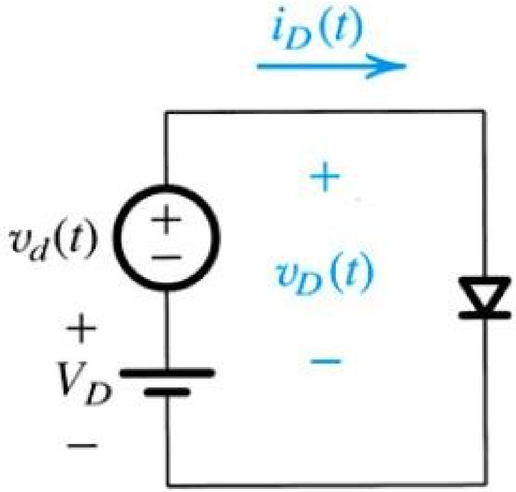


Total Instantaneous Solution (v_D) = Steady-State Solution (V_D) + Time-Varying Solution (v_d)

Figure 4.14: (a) Circuit for Example 4.5. **(b)** Circuit for calculating the dc operating point. **(c)** Small-signal equivalent circuit.

4.3.7. **Small-Signal Model**

- **Q:** How is the small-signal diode model defined?
  - **step #1:** Consider the conceptual circuit of Figure 4.13(a).
    - DC voltage (V_D) is applied to diode
    - Upon V_D, arbitrary time-varying signal v_d is super-imposed



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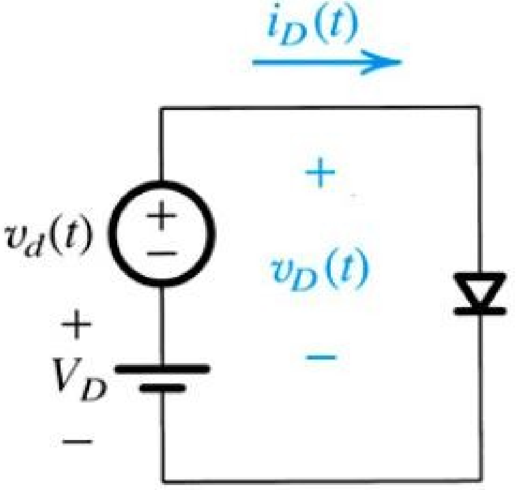
4.3.7. **Small-Signal Model**

- **DC only** – upper-case w/ upper-case subscript
- **time-varying only** – lower-case w/ lower-case subscript
- **total instantaneous** – lower-case w/ upper-case subscript
  - DC + time-varying



4.3.7. **Small-Signal Model**

- **step #2:** Define DC current as in (4.8).
- **step #3:** Define total instantaneous voltage (v_D) as composed of V_D and v_d.
- **step #4:** Define total instantaneous current (i_D) as function of v_D.

(eq4.8) $I_D = I_S e^{V_D / V_T}$

(eq4.9) $\underbrace{v_D(t) = V_D + v_d(t)}$

$v_D(t)$= total instantaneous voltage across diode
V_D= dc component of $v_D(t)$
$v_d(t)$= time varying component of $v_D(t)$

(eq4.10) $\underbrace{i_D(t) = I_S e^{v_D / V_T}}$

note that this is different from (4.8)

4.3.7. **Small-Signal Model**

- **step #5:** Redefine (4.10) as function of both V_D and v_d.
- **step #6:** Split this exponential in two.
- **step #7:** Redefine total instant current in terms of DC component (I_D) and time-varying voltage (v_d).

(eq4.11) $i_D(t) = I_S e^{(V_D + v_d)/V_T}$

action: split this exponential using appropriate laws

(eq4.11) $i_D(t) = \underbrace{I_S e^{V_D/V_T}}_{I_D} e^{v_d/V_T}$

(eq4.12) $i_D(t) = I_D e^{v_D/V_T}$

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4.3.7. **Small-Signal Model**

- **step #8:** Apply power series expansion to (4.12).
- **step #9:** Because $v_d/V_T << 1$, certain terms may be neglected.

example: $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots$

(eq4.12a) $$i_D(t) = I_D \overbrace{\underbrace{\left(1 + \frac{v_d}{V_T} + \overbrace{\left[\left(\frac{v_d}{V_T}\right)^2 \frac{1}{2!}\right] + \left[\left(\frac{v_d}{V_T}\right)^3 \frac{1}{3!}\right] + \ldots}^{\text{because } v_d/V_T<<1\text{, these terms are assumed to be negligible}}\right)}_{\text{power series expansion of } e^{v_d/V_T}}}^{\textbf{action: } \text{apply power series expansion to (4.12)}}$$

(eq4.14) $$i_D(t) = \overbrace{I_D \left(1 + \frac{v_d}{V_T}\right)}^{\textbf{action: } \text{eliminate negligible terms}}$$

4.3.7. Small-Signal Model

$$i_D(t) = I_D + \underbrace{\left(\frac{I_D}{V_T}\right)v_d}_{i_d}$$

- **small signal approximation**
  - Shown to right for exponential diode model.
    - total instant current (i_D)
    - small-signal current (i_d)
    - small-signal resistance (r_d)
  - Valid for for $v_d < 5mV$ amplitude (not peak to peak).

$$i_D(t) = I_D + i_d$$

$$i_d = \frac{1}{r_d} v_d$$

$$r_d = \frac{V_T}{I_D}$$

4.3.7. **Small-Signal Model**

- This method may be used to approximate any function $y = \mathbf{f}(x)$ around an operating point (x_0, y_0).

$$y(t) = y_0 + \left(\left. \frac{\partial y}{\partial x} \right|_{y=Y} \right)^{-1} \overbrace{\left(x(t) - x_0 \right)}^{\Delta x}$$

4.3.7: Small-Signal Model

- **Q:** How is **small-signal resistance** r_d defined?
  - **A:** From steady-state current (I_D) and thermal voltage (V_T) as below.
    - Note this approximation is only valid for small-signal voltages $v_d < 5mV$.

$$r_d = \frac{V_T}{I_D}$$

Example 4.5:
Small-Signal Model

- Consider the circuit shown in Figure 4.14(a) for the case in which $R = 10kOhm$.
- The power supply $V+$ has a dc value of $10V$ over which is super-imposed a $60Hz$ sinusoid of $1V$ peak amplitude (known as the **supply ripple**)
  - **Q:** Calculate both amplitude of the sine-wave signal observed across the diode.
    - **A:** v_d (peak) = $2.68mV$
- Assume diode to have $0.7V$ drop at $1mA$ current.

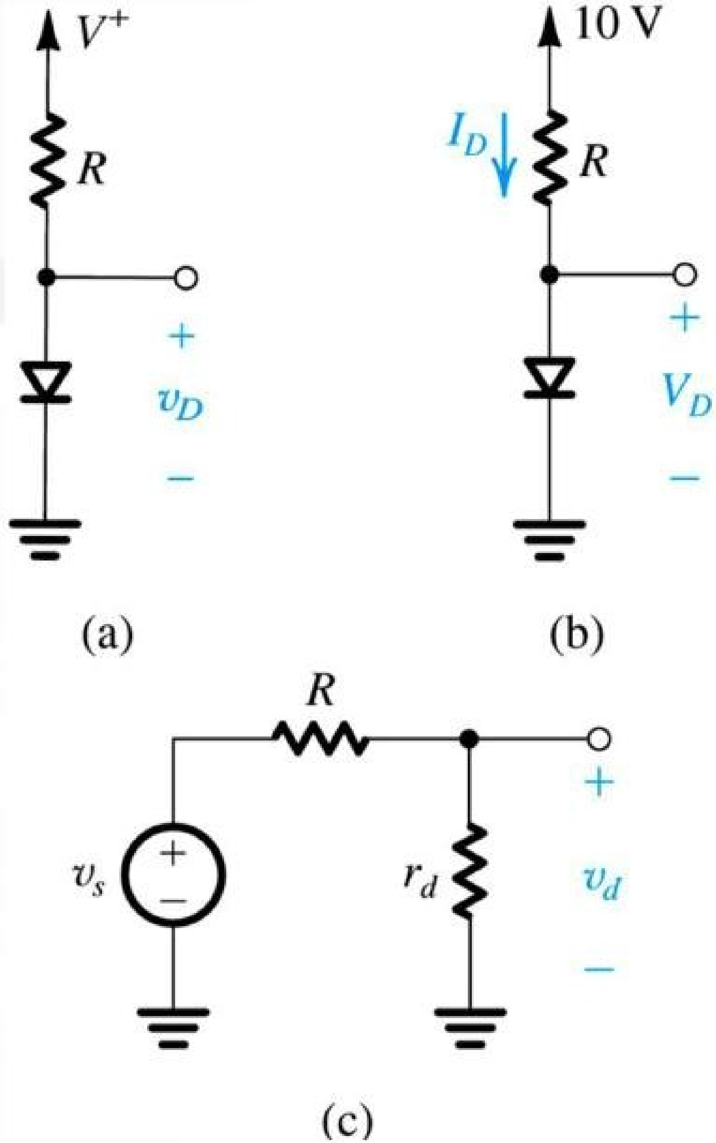


Figure 4.14: (a) circuit for Example 4.5. **(b)** circuit for calculating the dc operating point. **(c)** small-signal equivalent circuit.

4.3.8. Use of Diode Forward Drop in Voltage Regulation

- **Q:** What is a **voltage regulator**?
  - **A:** Circuit whose voltage output remains stable in spite of changes in supply and load.
- **Q:** What characteristic of the diode facilitates voltage regulation?
  - **A:** The approximately constant voltage drop across it (0.7*V*).

Example 4.6:
Diode-Based Voltage Regulator

- Consider circuit shown in Figure 4.15. A string of three diodes is used to provide a constant voltage of 2.1*V*.
  - **Q:** What is the change in this regulated voltage caused by **(a)** a +/- 10*%* change in supply voltage and **(b)** connection of 1*kOhm* load resistor.

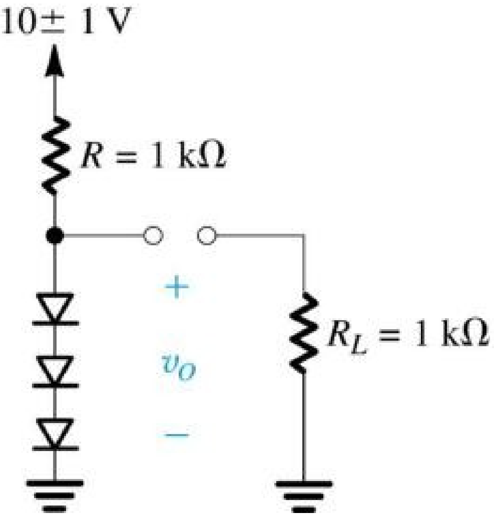


Figure 4.15: Circuit for Example 4.6.

4.4. Operation in the Reverse Breakdown Region – Zener Diodes

- Under certain circumstances, diodes may be intentionally used in the reverse breakdown region.
- These are referred to as **Zener Diodes**.

4.5. **Rectifier Circuits**

- One important application of diode is the **rectifier** –
  - Electrical device which converts alternating current (AC) to direct current (DC)
- One important application of rectifier is dc power supply.

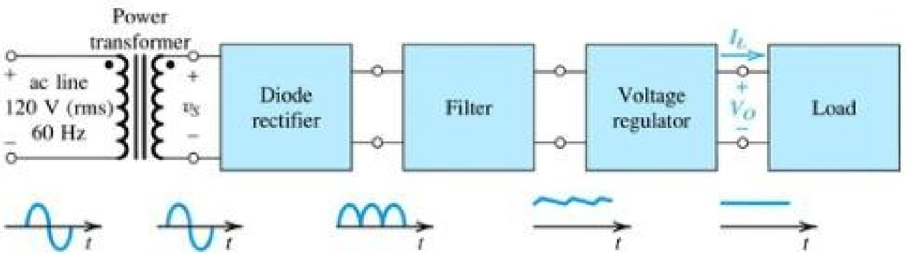


Figure 4.20: Block diagram of a dc power supply

step #1: increase / decrease rms magnitude of AC wave via power transformer

step #2: convert full-wave AC to half-wave DC (still time-varying and periodic)

step #3: employ low-pass filter to reduce wave amplitude by > 90%

step #4: employ voltage regulator to eliminate ripple

step #5: supply dc load

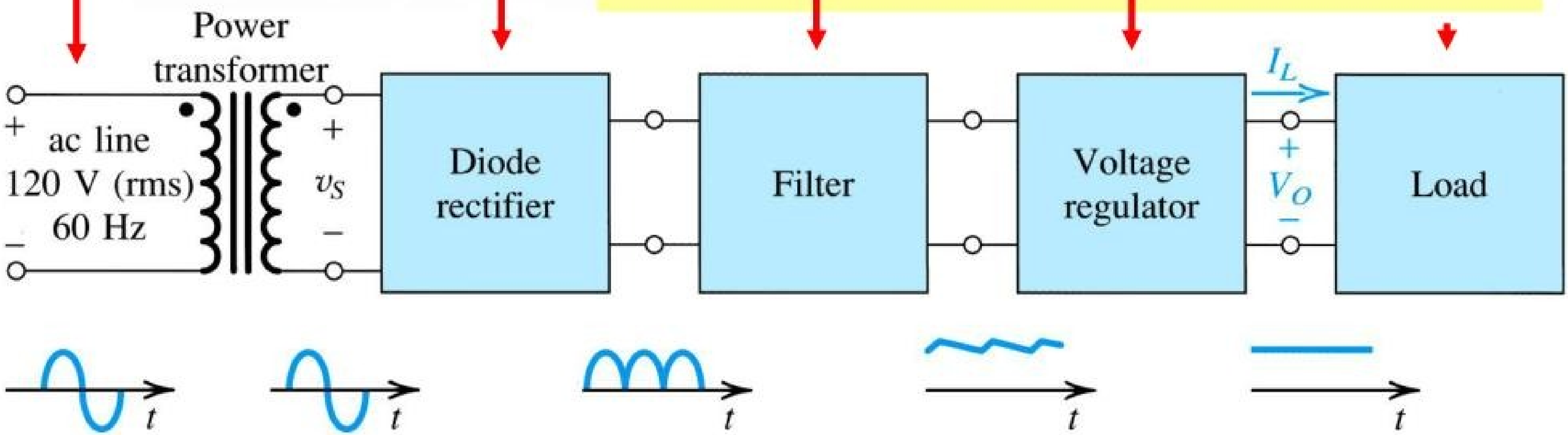


Figure 4.20: Block diagram of a dc power supply

4.5.1. **The Half-Wave Rectifier**

- **half-wave rectifier** – utilizes only alternate half-cycles of the input sinusoid
  - Constant voltage drop diode model is employed.

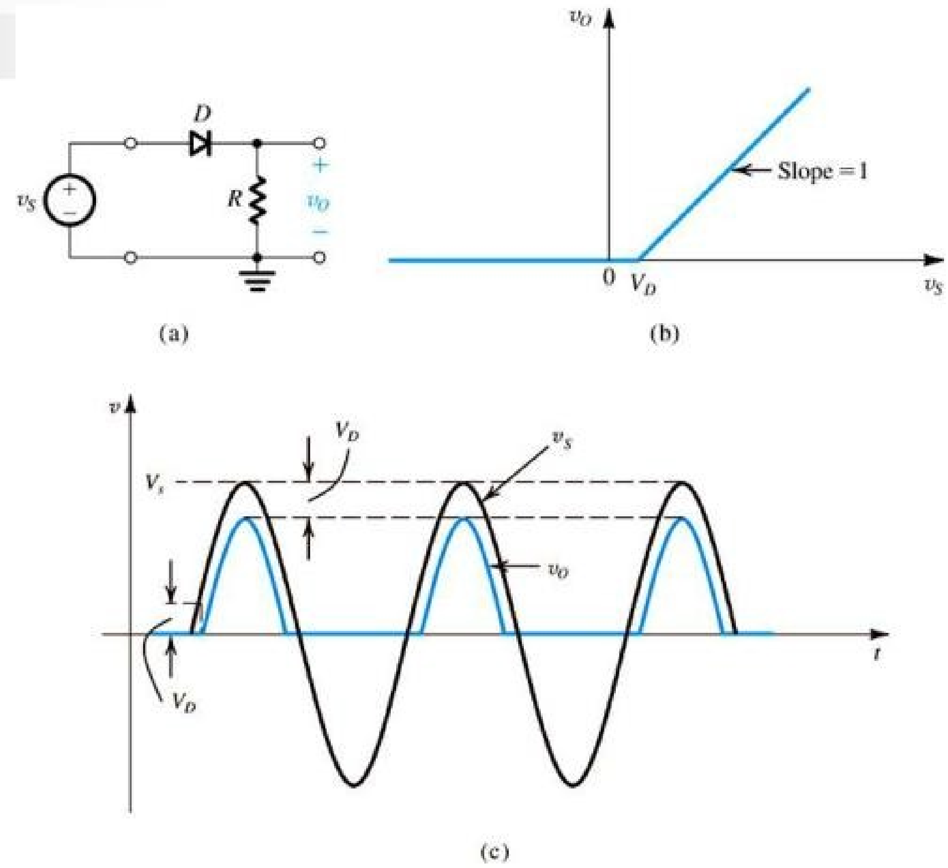


Figure 4.21: (a) Half-wave rectifier **(b)** Transfer characteristic of the rectifier circuit **(c)** Input and output waveforms

4.5.1. **The Half-Wave Rectifier**

- **current-handling capability** – what is maximum forward current diode is expected to conduct?
- **peak inverse voltage (PIV)** – what is maximum reverse voltage it is expected to block w/o breakdown?

4.5.1. **The Half-Wave Rectifier**

- **exponential model?** It is possible to use the diode exponential model in describing rectifier operation; however, this requires too much work.
- **small inputs?** Regardless of the model employed, one should note that the rectifier will not operate properly when input voltage is small ($< 1V$).
  - Those cases require a **precision rectifier.**

4.5.2. The Full-Wave Rectifier

- **Q:** How does full-wave rectifier differ from half-wave?
  - **A:** It utilizes both halves of the input
    - One potential is shown to right.

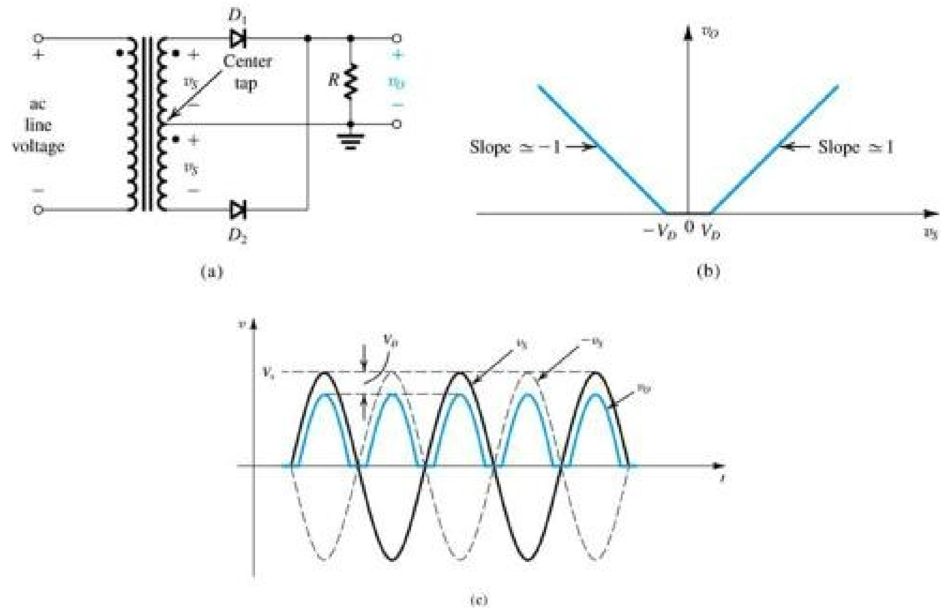


Figure 4.22: Full-wave rectifier utilizing a transformer with a center-tapped secondary winding.

The key here is center-tapping of the transformer, allowing "reversal" of certain currents…

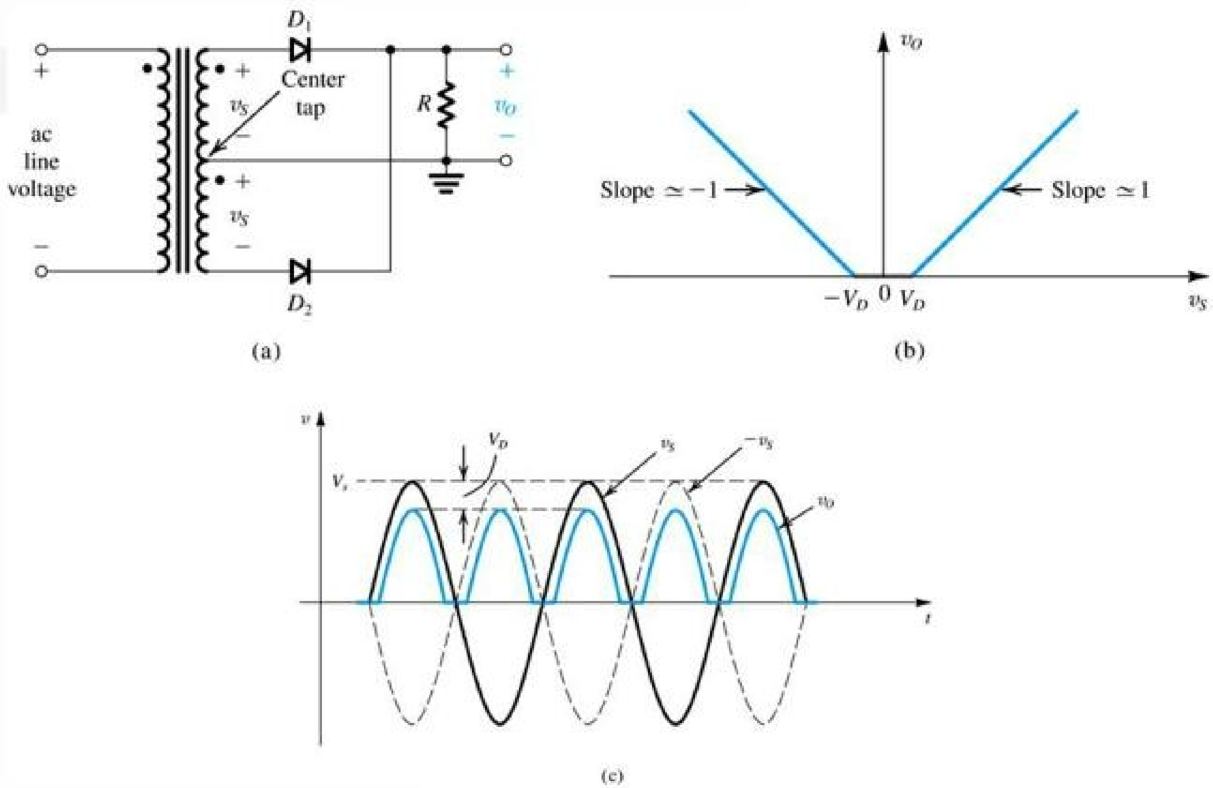
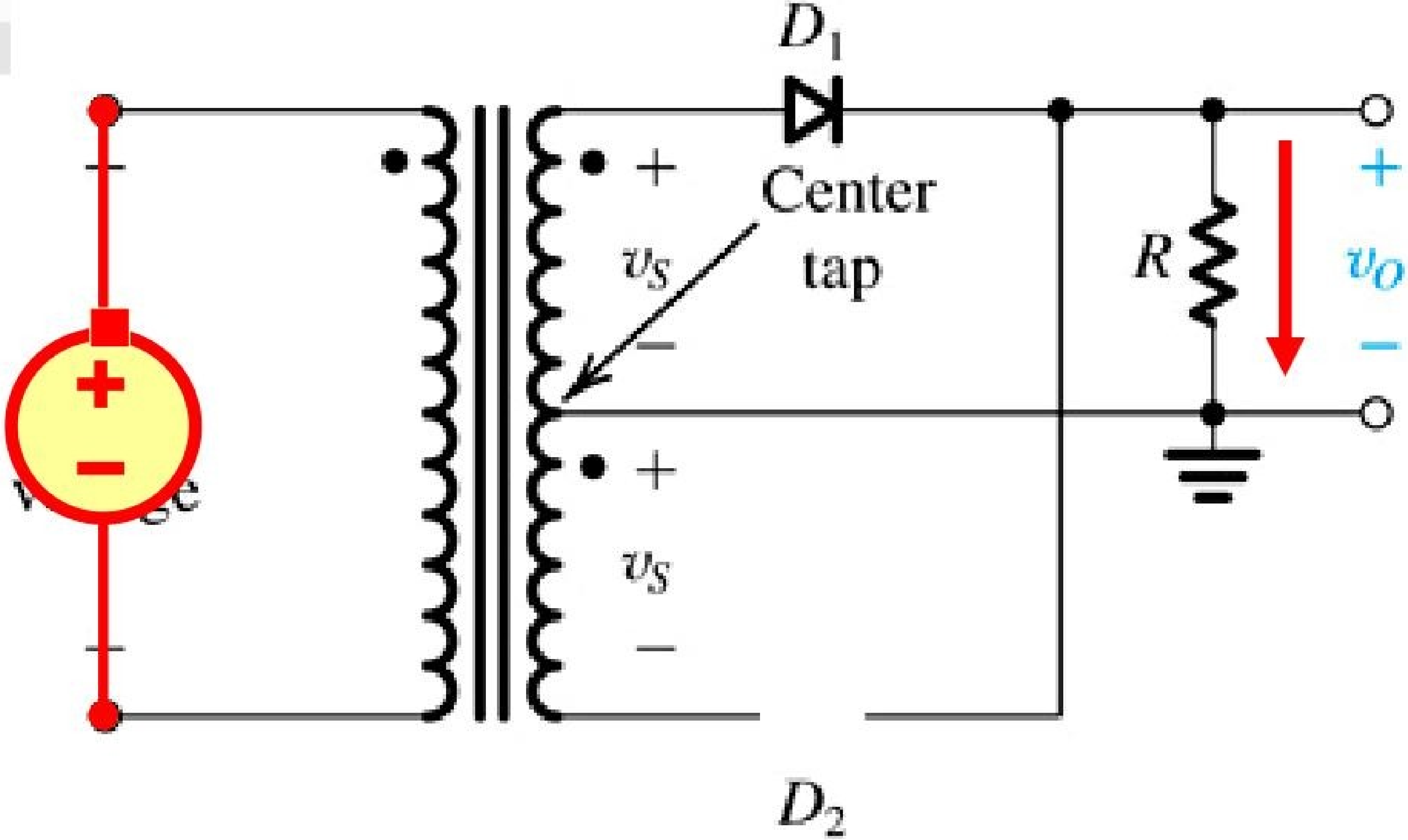
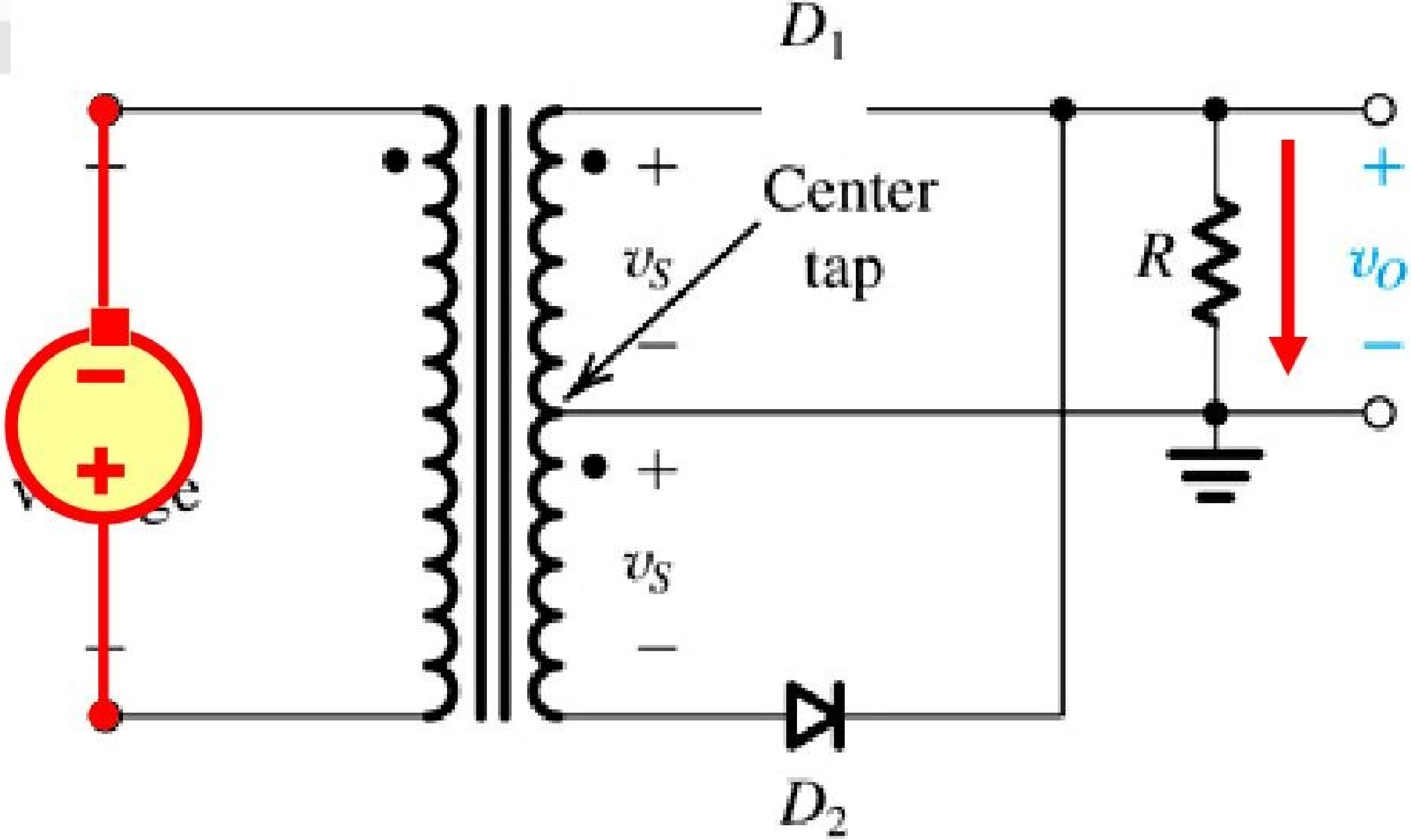


Figure 4.22: full-wave rectifier utilizing a transformer with a center-tapped secondary winding: **(a)** circuit; **(b)** transfer characteristic assuming a constant-voltage-drop model for the diodes; **(c)** input and output waveforms.

When instantaneous source voltage is **positive**, D_1 conducts while D_2 blocks…



when instantaneous source voltage is **negative**, D_2 conducts while D_1 blocks



4.5.2. **The Full-Wave Rectifier**

- **Q:** What are most important observation(s) from this operation?
  - **A:** The direction of current flowing across load never changes (both halves of AC wave are rectified). The full-wave rectifier produces a more "energetic" waveform than half-wave.
    - PIV for full-wave = $2V_S - V_D$

4.5.3. **The Bridge Rectifier**

- An alternative implementation of the full-wave rectifier is **bridge rectifier.**
  - Shown to right.

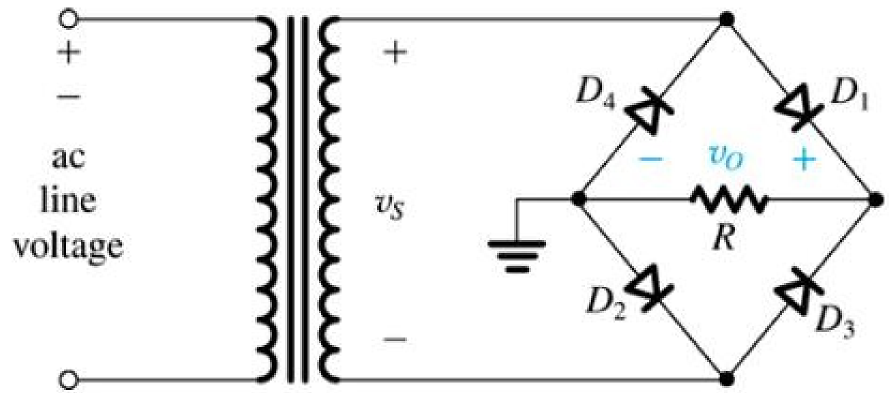


Figure 4.23: The bridge rectifier circuit.

when instantaneous source voltage is **positive**, D_1 and D_2 conduct while D_3 *and* D_4 block

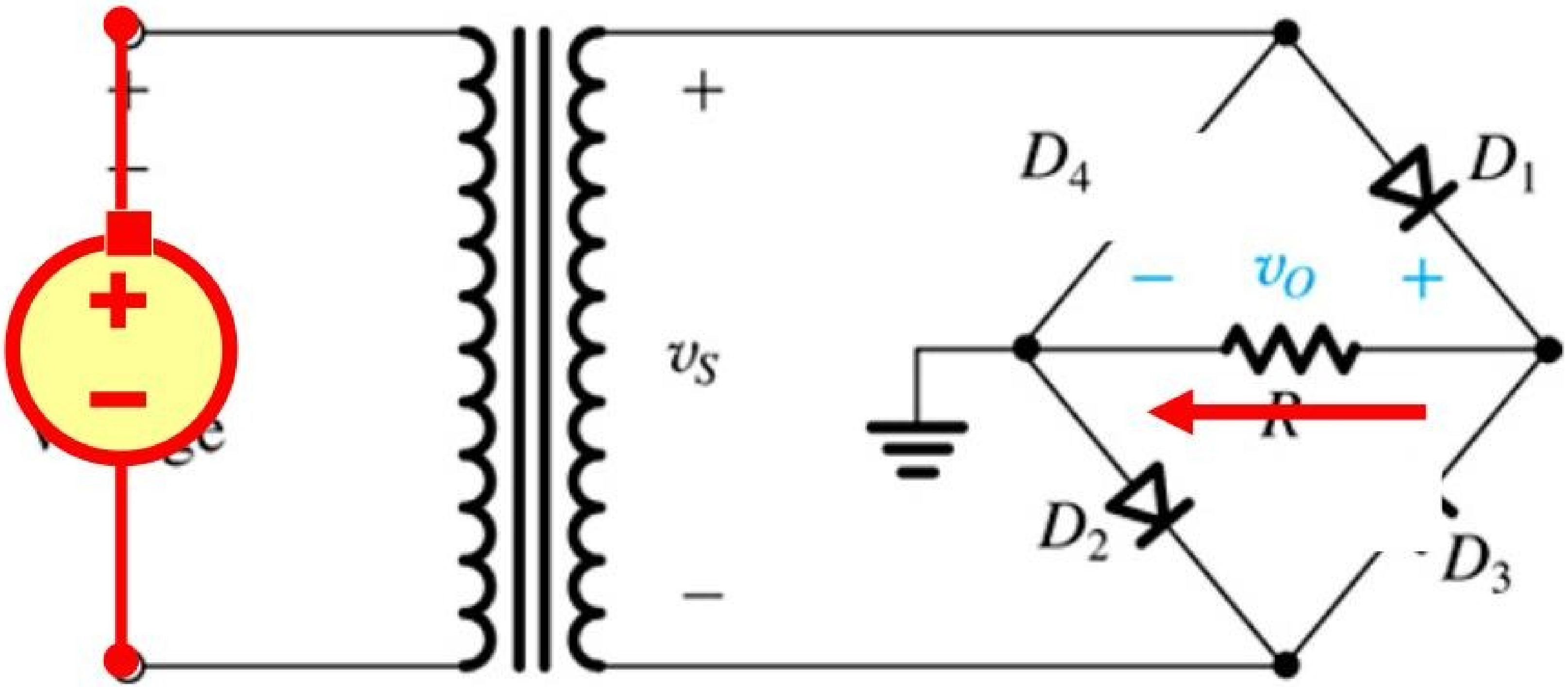


Figure 4.23: The bridge rectifier circuit.

when instantaneous source voltage is **positive**, D_1 and D_2 conduct while D_3 *and* D_4 block

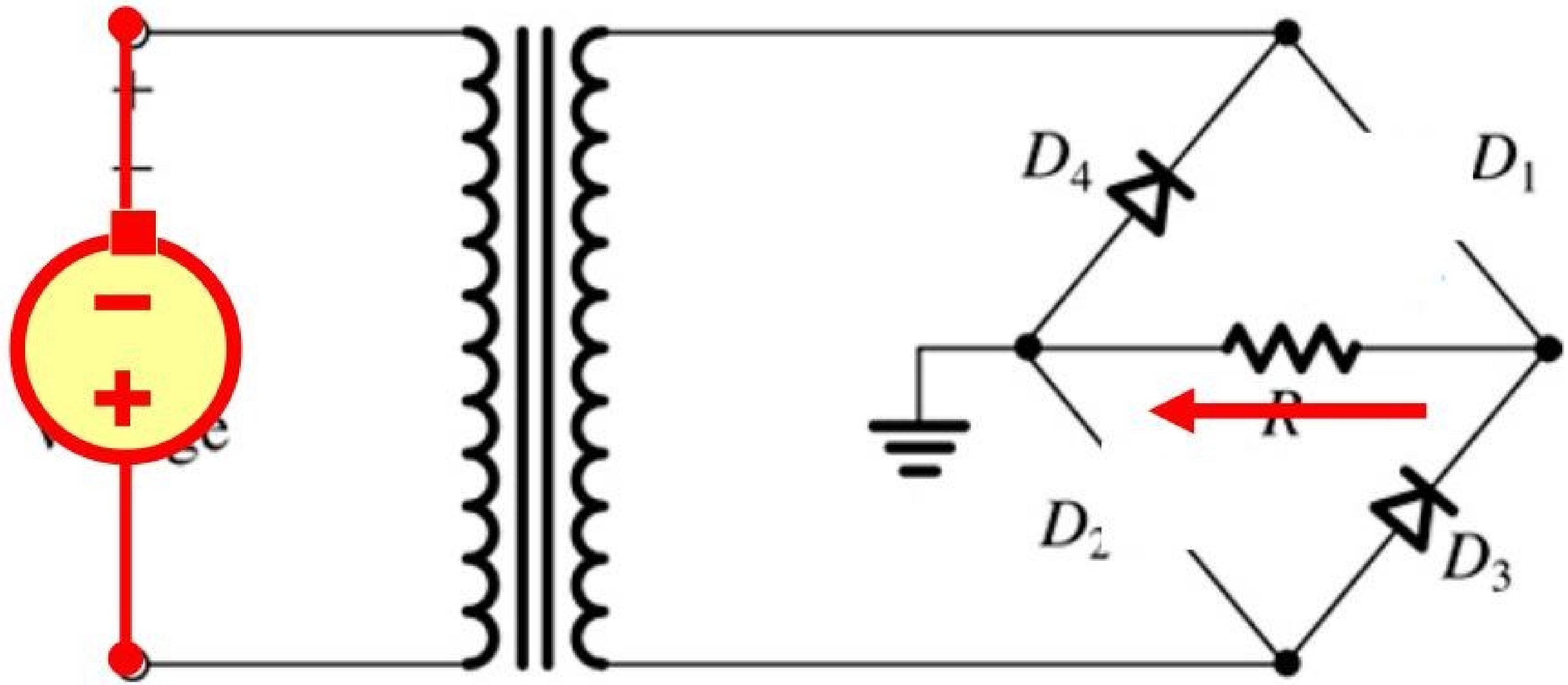


Figure 4.23: The bridge rectifier circuit.

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4.5.3: **The Bridge Rectifier (BR)**

- **Q:** What is the main advantage of BR?
  - **A:** No need for center-tapped transformer.
- **Q:** What is main disadvantage?
  - **A:** Series connection of TWO diodes will reduce output voltage.
- PIV = $V_S - V_D$

4.5.4. **The Rectifier with a Filter Capacitor**

- Pulsating nature of rectifier output makes unreliable dc supply.
- As such, a **filter capacitor** is employed to remove ripple.

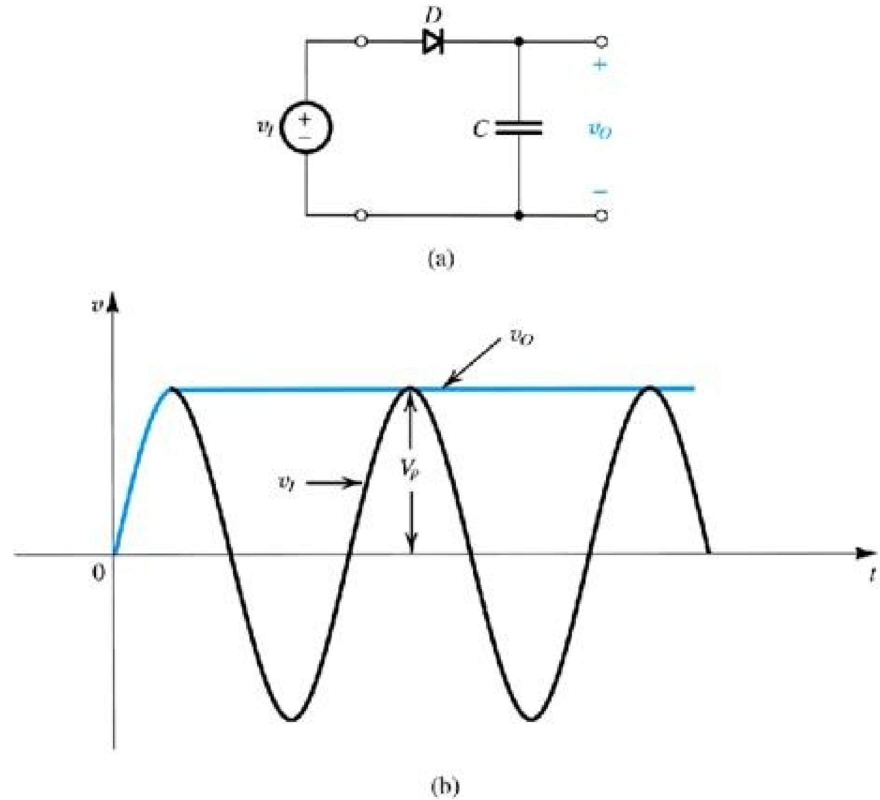

Figure 4.24: (a) A simple circuit used to illustrate the effect of a filter capacitor. **(b)** input and output waveforms assuming an ideal diode.

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4.5.4. **The Rectifier with a Filter Capacitor**

- **step #1:** source voltage is positive, diode is forward biased, capacitor charges.
- **step #2:** source voltage is reverse, diode is reverse-biased (blocking), capacitor cannot discharge.
- **step #3:** source voltage is positive, diode is forward biased, capacitor charges (maintains voltage).

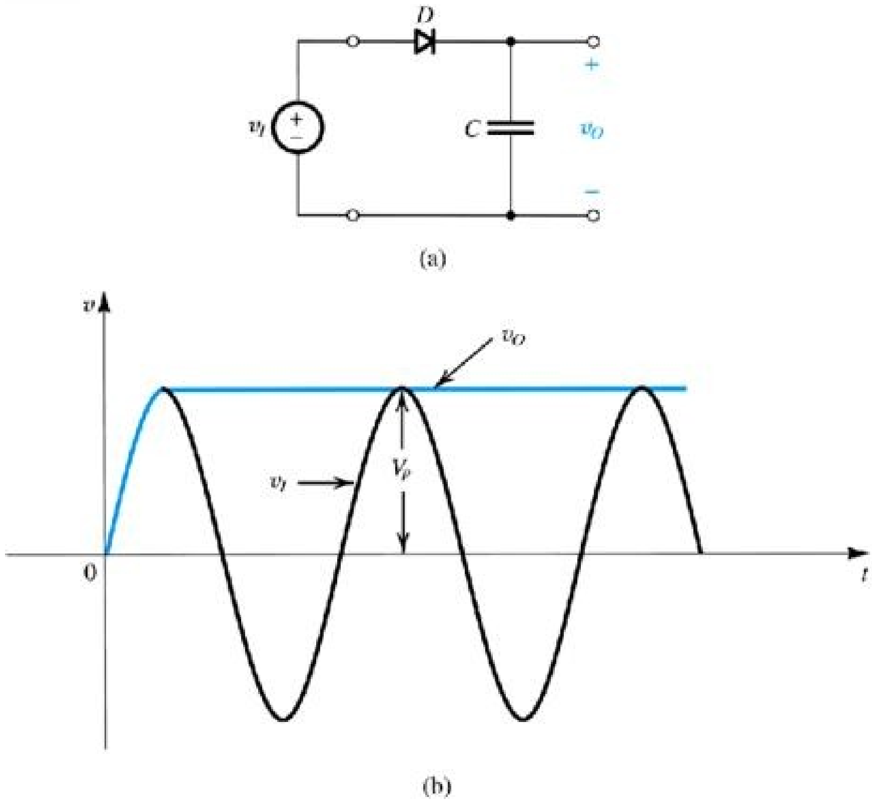


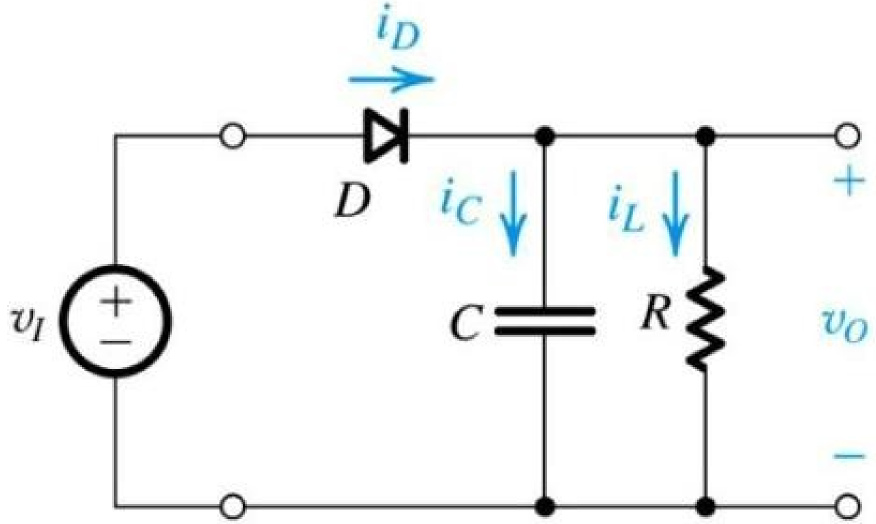
Figure 4.24 (a) A simple circuit used to illustrate the effect…

4.5.4. **The Rectifier with a Filter Capacitor**

- **Q:** Why is this example unrealistic?
  - **A:** Because for any practical application, the converter would supply a load (which in turn provides a path for capacitor discharging).

4.5.4. **The Rectifier with a Filter Capacitor**

- **Q:** What happens when **load resistor** is placed in series with capacitor?
  - **A:** One must now consider the discharging of capacitor across load.

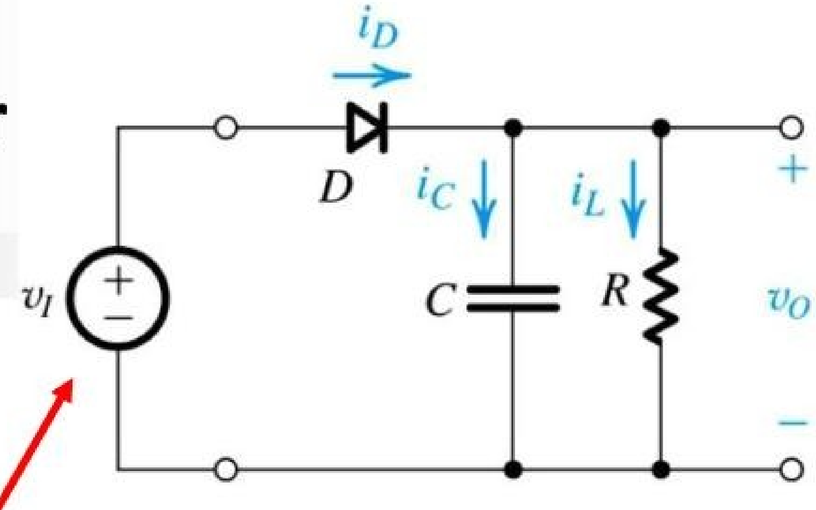


4.5.4. **The Rectifier with a Filter Capacitor**

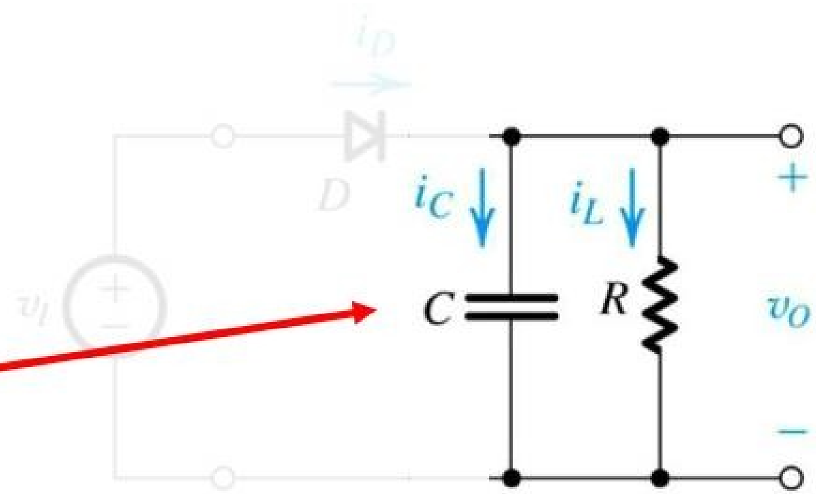
- The textbook outlines how **Laplace Transform** may be used to define behavior below.

$$\overbrace{v_O(t)=v_I(t)-v_D}^{\text{output voltage for state \#1}}$$

$$\underbrace{v_O(t)=V_{peak}e^{-\frac{t}{RC}}}_{\text{output voltage for state \#2}}$$



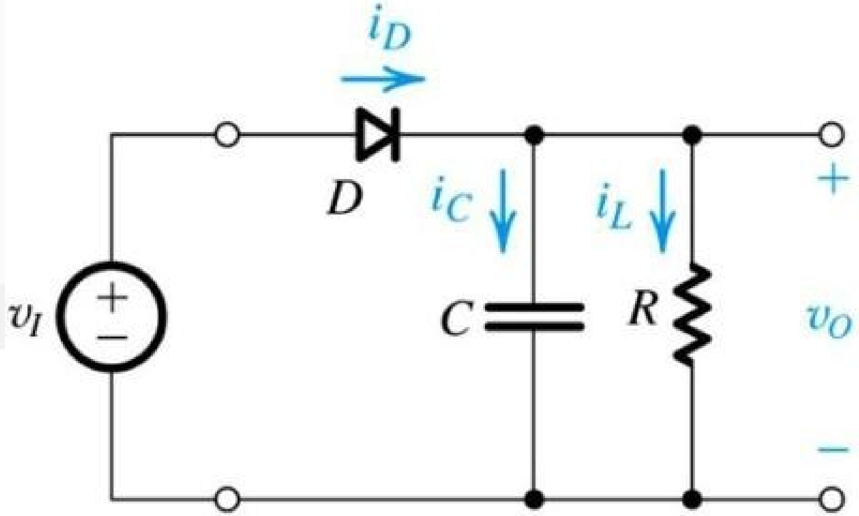
circuit state #1



circuit state #2

Q: What happens when load resistor is placed in series with capacitor?

- **step #1:** Analyze circuit state #1.
  - When diode is forward biased and conducting.
- **step #2:** Input voltage (v_I) will be applied to output (v_O), minus 0.7V drop across diode.



circuit state #1

$$i_L = \frac{v_O}{R}$$

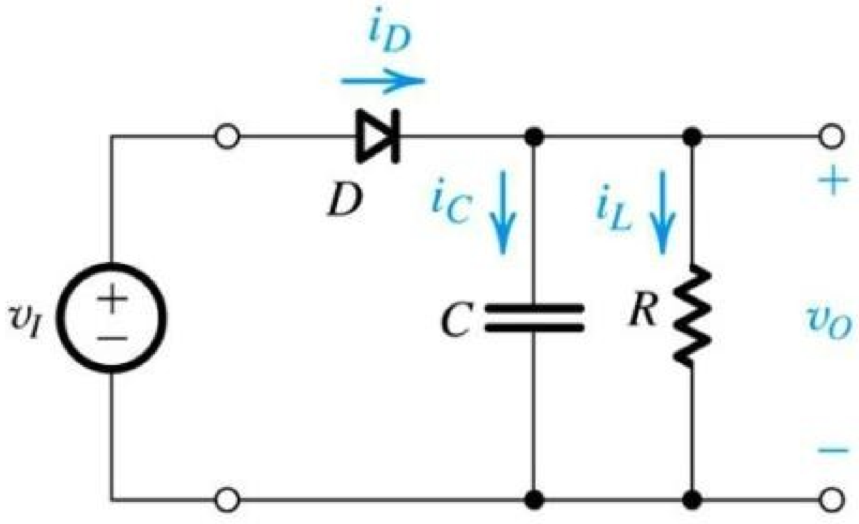
$$i_D = i_C + i_L$$

$$i_D = \underbrace{C\frac{dv_I}{dt}}_{\text{action: define capacitor current differentially}} + i_L$$

Q: What happens when load resistor is placed in series with capacitor?

- **step #3:** Define output voltage for state #1.

$$\underbrace{v_O = v_I - v_D}_{\text{output voltage for state \#1}}$$



circuit state #1

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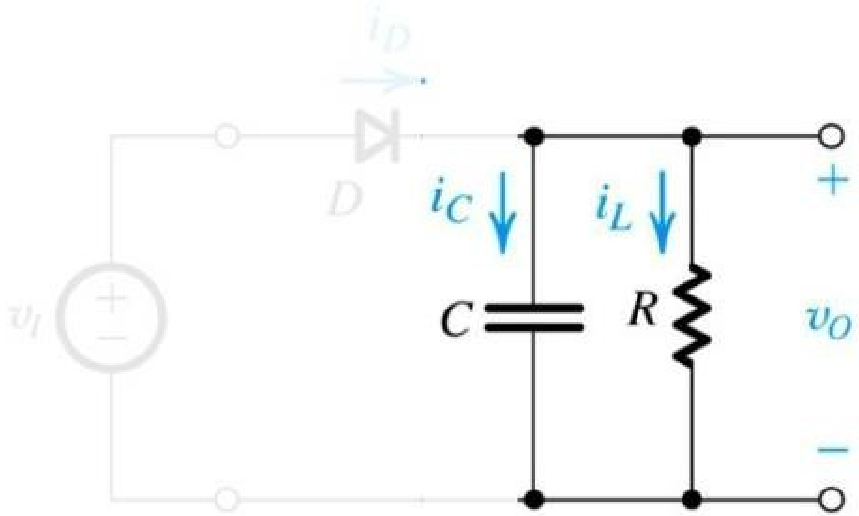
Q: What happens when load resistor is placed in series with capacitor?

- **step #4:** Analyze circuit state #2.
  - When diode is blocking and capacitor is discharging.
- **step #5:** Define KVL and KCL for this circuit.
  - $v_O = Ri_L$
  - $i_L = -i_C$



circuit state #2

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Q: What happens when load resistor is placed in series with capacitor?

- **step #6:** Use combination of circuit and Laplace Analysis to solve for $v_O(t)$ in terms of initial condition and time…

4.5.4. The Rectifier with a Filter Capacitor

$$v_O = Ri_L$$

action: replace i_L with $-i_C$

$$v_O = -Ri_C$$

action: define i_C differentially

$$v_O = -R\underbrace{\left(C\frac{dv_O}{dt}\right)}_{i_C}$$

action: change sides

$$v_O + RC\frac{dv_O}{dt} = 0$$

action: take Laplace transform

$$\mathbf{L}\left\{v_O + RC\frac{dv_O}{dt} = 0\right\}$$

action: take Laplace transform

$$V_O(s) + RC\underbrace{\left[sV_O(s) - V_O(0)\right]}_{\text{transform of } \frac{dv_O}{dt}} = 0$$

action: seperate disalike / collect alike terms

$$\underbrace{V_O(s) + RCsV_O(s)}_{(1+RCs)V_O(s)} = RC\underbrace{V_O(0)}_{\text{initial condition}}$$

action: pull out RC

$$\underbrace{(1+RCs)V_O(s)}_{RC\left(s+\frac{1}{RC}\right)V_O(s)} = RCV_O(0)$$

action: eliminate RC from both sides

$$RC\left(s+\frac{1}{RC}\right)V_O(s) = RCV_O(0)$$

action: solve for $V_O(s)$

$$V_O(s) = V_O(0)\frac{1}{s+\frac{1}{RC}}$$

action: take inverse Laplace

$$\mathbf{L}^{-1}\left\{V_O(s) = V_O(0)\frac{1}{s+1/(RC)}\right\}$$

action: solve

$$\boxed{v_O(t) = V_O(0)e^{-\frac{t}{RC}}}$$

4.5.4. **The Rectifier with a Filter Capacitor**

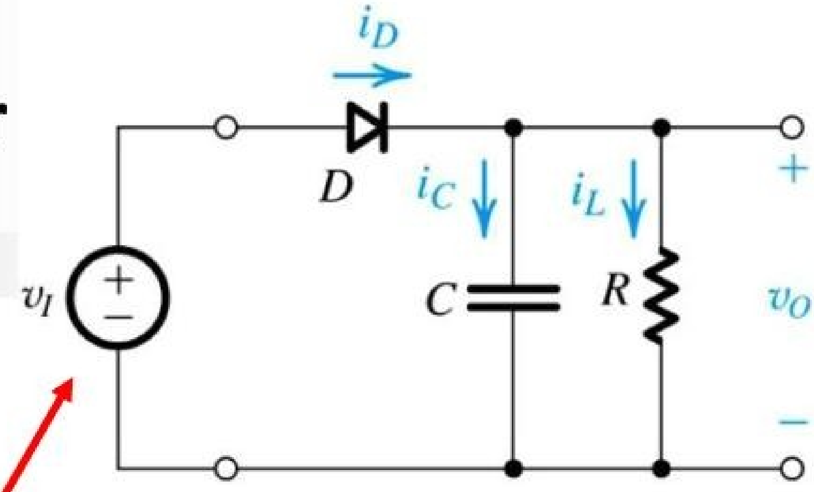
- **Q:** What is $V_O(0)$?
  - **A:** Peak of v_I, because the transition between state #1 and state #2 (aka. diode begins blocking) approximately as v_I drops below v_C.

4.5.4. **The Rectifier with a Filter Capacitor**

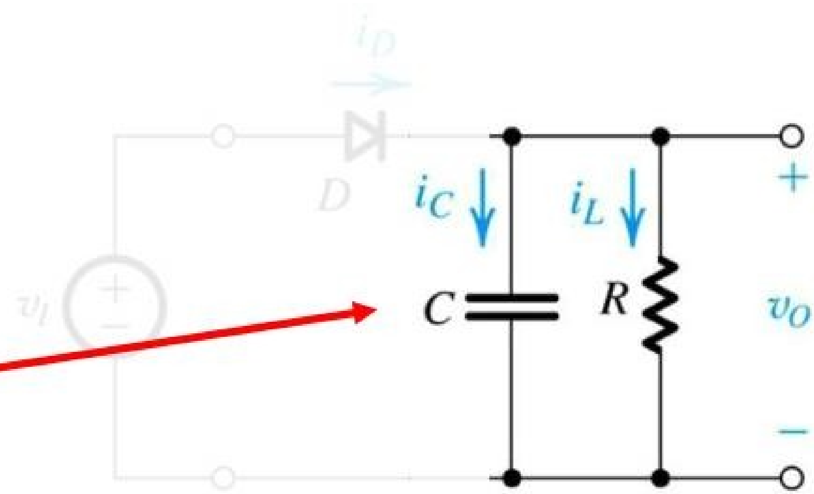
- **step #7:** Define output voltage for states #1 and #2.

$$\overbrace{v_O(t) = v_I(t) - v_D}^{\text{output voltage for state \#1}}$$

$$\underbrace{v_O(t) = V_{peak} e^{-\frac{t}{RC}}}_{\text{output voltage for state \#2}}$$



circuit state #1



circuit state #2

$$\overbrace{v_O(t) = v_I(t)}^{\text{output voltage for state \#1}}$$

$$\underbrace{v_O(t) = V_{peak} e^{-\frac{t}{RC}}}_{\text{output voltage for state \#2}}$$

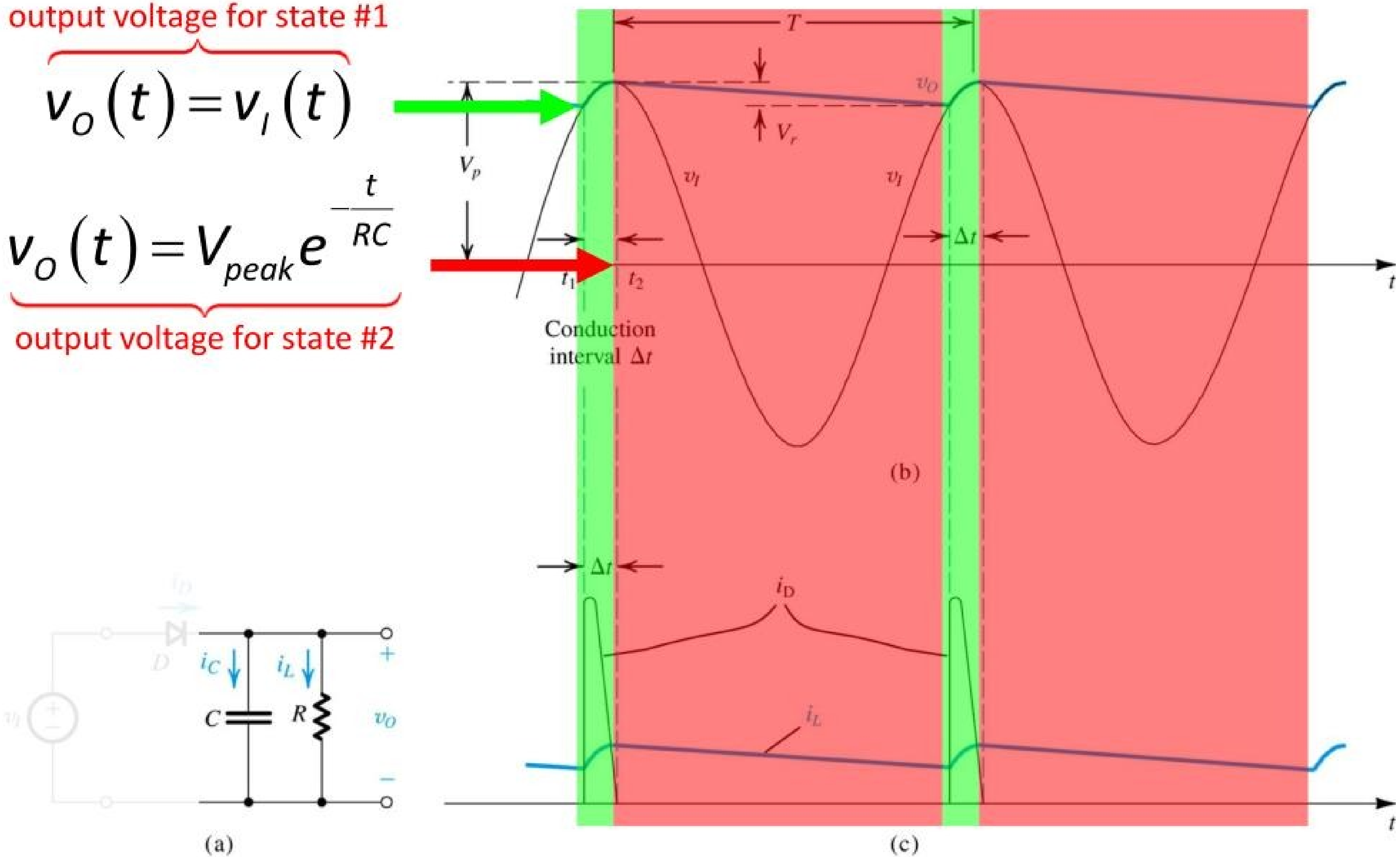


Figure 4.25: Voltage and Current Waveforms in the Peak Rectifier Circuit WITH *RC* >> *T*. The diode is assumed ideal.

A Couple of Observations

- The diode conducts for a brief interval (Δt) near the peak of the input sinusoid and supplies the capacitor with charge equal to that lost during the much longer discharge interval. The latter is approximately equal to T.
- Assuming an ideal diode, the diode conduction begins at time t_1 (at which the input v_I equals the exponentially decaying output v_O). Diode conduction stops at time t_2 shortly after the peak of v_I (the exact value of t_2 is determined by settling of I_D).

A Couple of Observations

- During the diode off-interval, the capacitor C discharges through R causing an exponential decay in the output voltage (v_O). At the end of the discharge interval, which lasts for almost the entire period T, voltage output is defined as follows – $v_O(T) = V_{peak} - V_r$.
- When the ripple voltage (V_r) is small, the output (v_O) is almost constant and equal to the peak of the input (v_I). the average output voltage may be defined as below…

$$\text{(eq4.27)}\ \mathbf{avg}\left(V_O\right) = V_{peak} - \frac{1}{2}V_r \quad \approx V_{peak} \text{ if } V_r \text{ is small}$$

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4.5.4. **The Rectifier with a Filter Capacitor**

- **Q:** How is **ripple voltage** (V_r) defined?
  - **step #1:** Begin with transient response of output during "off interval."
  - **step #2:** Note T is discharge interval.
  - **step #3:** Simplify using assumption that $RC >> T$.
  - **step #4:** Solve for ripple voltage V_r.

$$v_O(t) = V_{peak} e^{-\frac{t}{RC}}$$

$$\underbrace{V_{peak} - V_r = v_O(T)}_{T \text{ is discharge interval}}$$

$$V_{peak} - V_r \approx V_{peak} \times \underbrace{\left(e^{-\frac{T}{RC}} \right)}_{\text{because } RC >> T \text{, we can assume...}}$$

$$e^{-\frac{T}{RC}} \approx 1 - \frac{T}{RC}$$

action: solve for ripple voltage V_r

$$\text{(eq4.28)} \quad V_r \approx V_{peak} \left(\frac{T}{RC} \right)$$

$$1 - \frac{T}{RC} - 1$$

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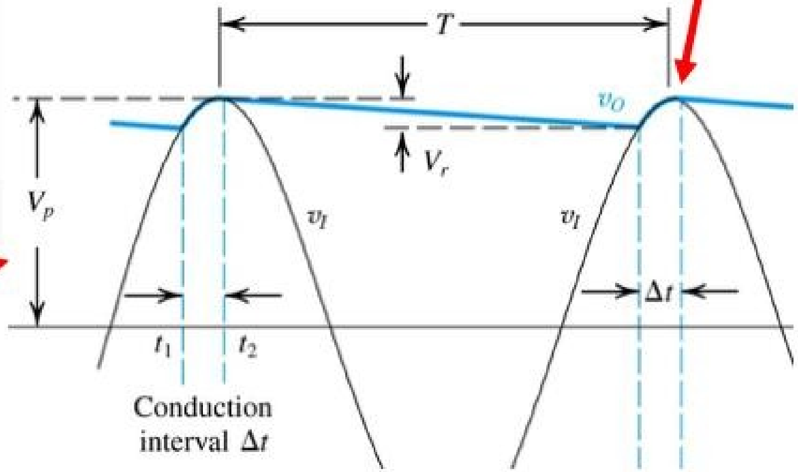
4.5.4. The Rectifier with a Filter Capacitor

- **step #5:** Put expression in terms of frequency ($f = 1/T$).
  - Observe that, as long as V_r << V_{peak}, the capacitor discharges as constant current source (I_L).
- **Q:** How is **conduction interval** (Δt) defined?
  - **A:** See following slides…

$$\text{(eq4.29)} \quad V_r \approx \frac{V_{peak}}{fRC} = \frac{\overbrace{I_L}^{\frac{V_{peak}}{R}}}{fC}$$

expression to define ripple voltage (V_r)

Q: How is **conduction interval** (Δt) defined?



- **step #1:** Assume that diode conduction stops (very close to when) v_I approaches its peak.
- **step #2:** With this assumption, one may define expression to the right.
- **step #3:** Solve for $\omega\Delta t$.

$$\underbrace{V_{peak}\cos(\omega\Delta t) = V_{peak} - V_r}_{}$$

note that peak of vI represents cos(0^O), therefore **cos**($\omega\Delta t$) represents variation around this value

(eq4.30) $\underbrace{\omega\Delta t = \sqrt{2V_r / V_{peak}}}_{}$

as assumed, conduction interval Δt will be small when $V_r << V_{peak}$

4.5.4. The Rectifier with a Filter Capacitor

- **Q:** How is peak-to-peak ripple (V_r) defined?
  - **A:** (4.29)
- **Q:** How is the conduction interval (Δt) defined?
  - **A:** (4.30)

(eq4.29) $$V_r \approx \frac{V_{peak}}{fRC} = \frac{I_L}{fC}$$

$I_L = \frac{V_{peak}}{R}$

(eq4.30) $$\underbrace{\omega \Delta t = \sqrt{2V_r / V_{peak}}}_{\text{as assumed, conduction interval } \Delta t \text{ will be small when } V_r << V_{peak}}$$

4.5.4. **The Rectifier with a Filter Capacitor**

- **precision rectifier** – is a device which facilitates rectification of low-voltage input waveforms.

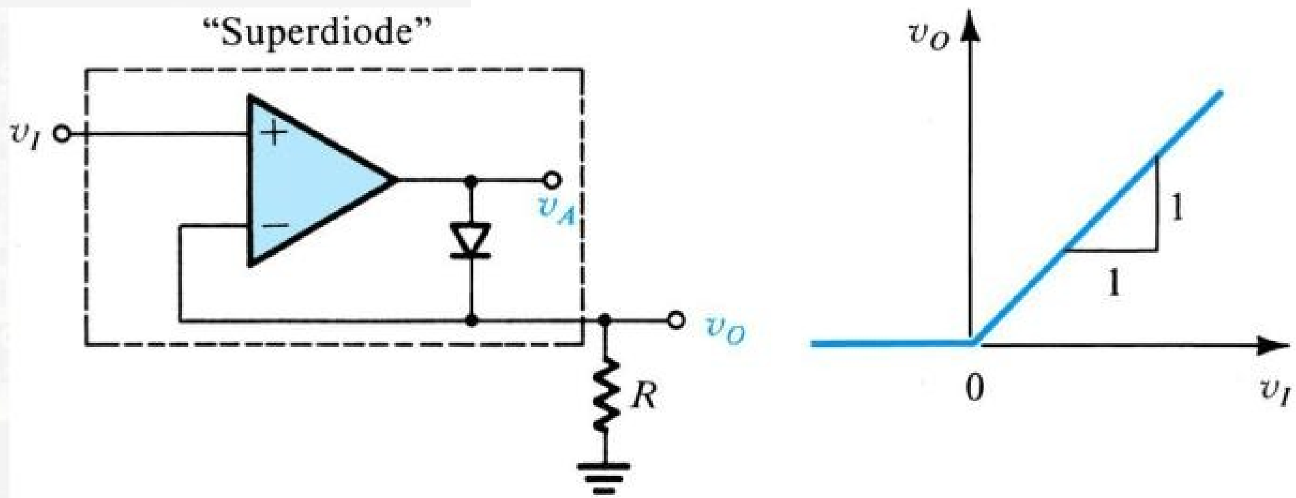


Figure 4.27: The "Superdiode" Precision Half-Wave Rectifier and its almost-ideal transfer characteristic.

4.6: **Limiting and Clamping Circuits**

- **Q:** What is a **limiter circuit**?
  - **A:** One which limits voltage output.

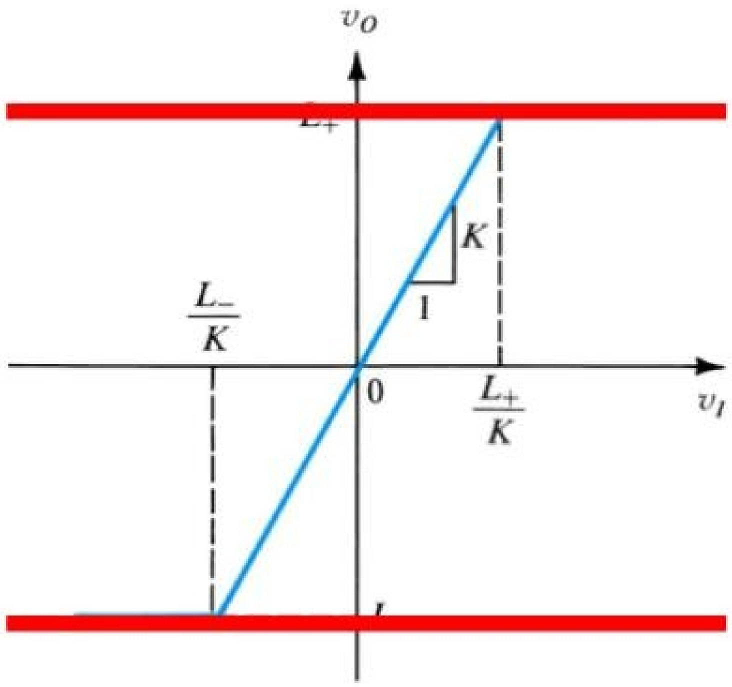


Figure 4.28: General transfer characteristic for a limiter circuit

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4.6. **Limiting and Clamping Circuits**

- **passive limiter circuit**
  - has linear range
  - has nonlinear range
  - $K < 1$
  - examples include
    - single limiter operate in uni-polar manner
    - double limiter operate in bi-polar manner

$$v_O = \begin{cases} \overbrace{Kv_I}^{\text{over linear range}} \\ \underbrace{\text{constant value(s)}}_{\text{outside linear range}} \end{cases}$$

$$v_O = \begin{cases} L_- & v_I \leq \frac{L_-}{K} \\ Kv_I & \frac{L_-}{K} < v_I < \frac{L_+}{K} \\ L_+ & v_I \geq \frac{L_+}{K} \end{cases}$$

4.6. **Limiting and Clamping Circuits**

- soft vs. hard limiter

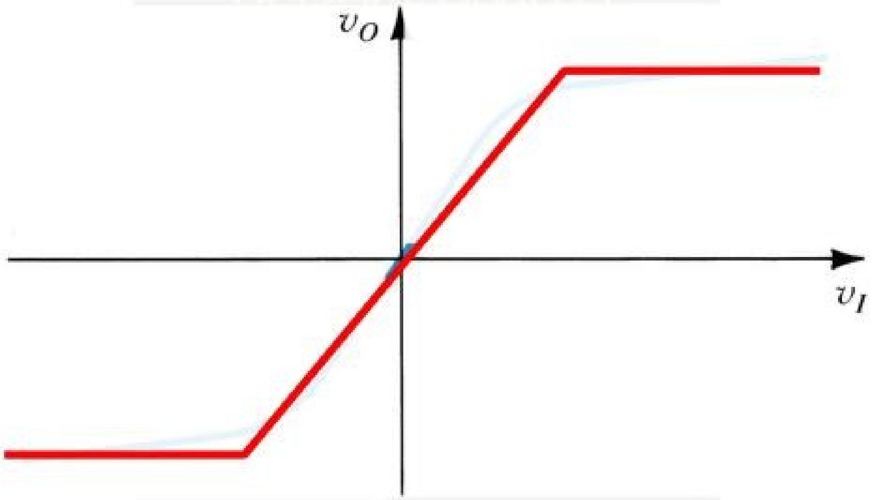

Figure 4.30: Hard vs. Soft Limiting.

- **Q:** How are limiter circuits applied?
  - **A:** Signal processing, used to prevent breakdown of transistors within various devices.

single limiters employ one diode

double limiters employ two diodes of opposite polarity

linear range may be controlled via string of diodes and dc sources

zener diodes may be used to implement soft limiting

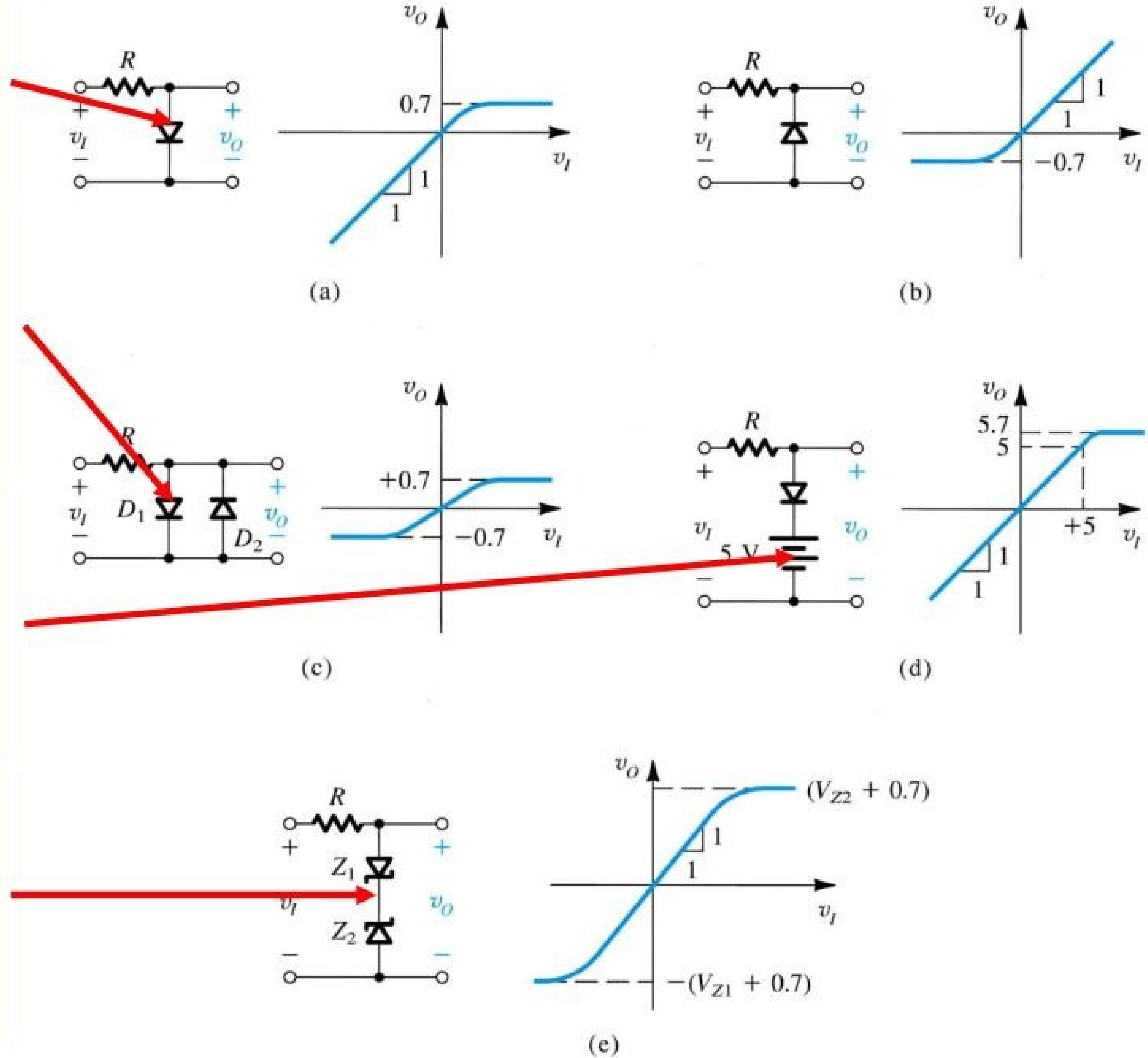

Figure 4.31: Variety of basic limiting circuits.

4.6.2. **The Clamped Capacitor or DC Restorer**

- **Q:** What is a **dc restorer**?
  - **A:** Circuit which removes the dc component of an AC wave.
- **Q:** Why is this ability important?
  - **A:** Average value of this output (w/ dc = 0) is effective way to measure duty cycle

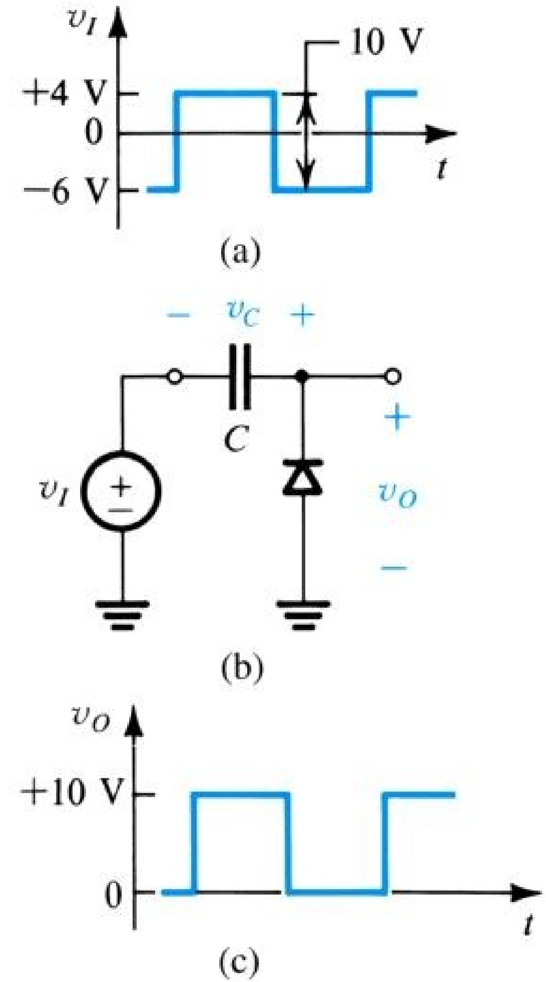

Figure 4.32: The clamped capacitor or dc restorer with a square-wave input and no load

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4.6.3: **The Voltage Doubler**

- **Q:** What is a **voltage doubler**?
  - **A:** One which multiplies the amplitude of a wave or signal by two.

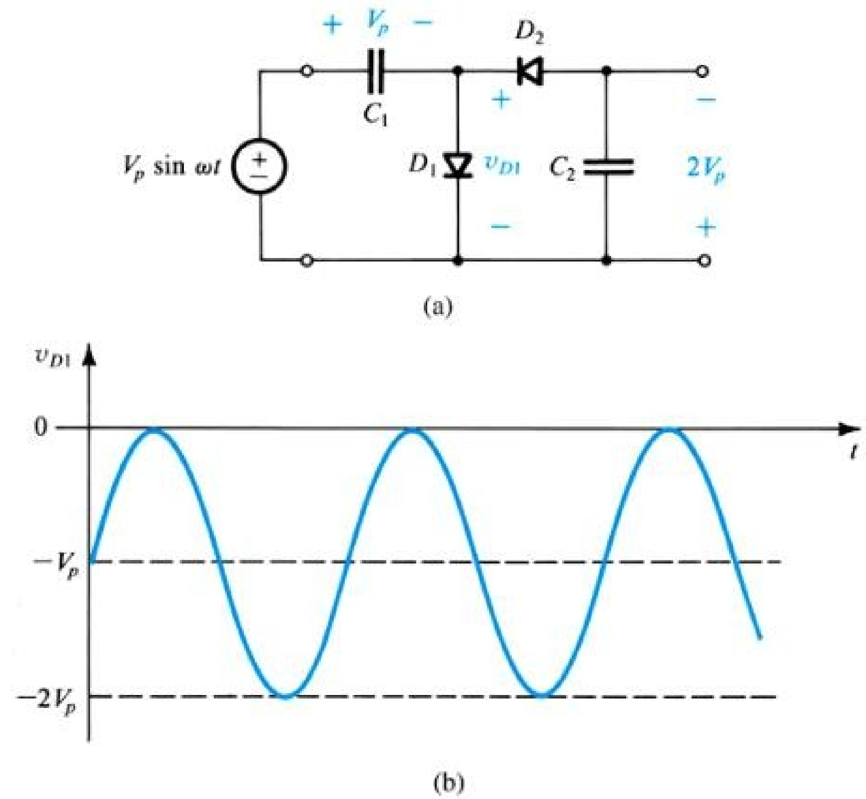

Figure 4.34: Voltage doubler: **(a)** circuit; **(b)** waveform of the voltage across D_1.

Summary (1)

- In the forward direction, the ideal diode conducts any current forced by the external circuit while displaying a zero-voltage drop. The ideal diode does not conduct in reverse direction; any applied voltage appears as reverse bias across the diode.
- The unidirectional current flow property makes the diode useful in the design of rectifier circuits.
- The forward conduction of practical silicon-junction diodes is accurately characterized by the relationship $i = I_S e^{V/VT}$.

Summary (2)

- A silicon diode conducts a negligible current until the forward voltage is at least 0.5*V*. Then, the current increases rapidly with the voltage drop increasing by 60mV for every decade of current change.
- In the reverse direction, a silicon diode conducts a current on the order of $10^{-9}A$. This current is much greater than I_S and increases with the magnitude of reverse voltage.

Summary (3)

- Beyond a certain value of reverse voltage (that depends on the diode itself), breakdown occurs and current increases rapidly with a small corresponding increase in voltage.
- Diodes designed to operate in the breakdown region are called zener diodes. They are employed in the design of voltage regulators whose function is to provide a constant dc voltage that varies little with variations in power supply voltage and / or load current.

Summary (4)

- In many applications, a conducting diode is modeled as having a constant voltage drop – usually with value of approximately 0.7V.
- A diode biased to operate at a dc current I_D has small signal resistance $r_d = V_T/I_D$.
- Rectifiers covert ac voltage into unipolar voltages. Half-wave rectifiers do this by passing the voltage in half of each cycle and blocking the opposite-polarity voltage in the other half of the cycle.

Summary (5)

- The bridge-rectifier circuit is the preferred full-wave rectifier configuration.
- The variation of the output waveform of the rectifier is reduced considerably by connecting a capacitor C across the output load resistance R. The resulting circuit is the peak rectifier. The output waveform then consists of a dc voltage almost equal to the peak of the input sine wave, V_p, on which is superimposed a ripple component of frequency $2f$ (in the full-wave case) and of peak-to-peak amplitude $V_r = V_p/2fRC$.

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Summary (6)

- Combination of diodes, resistors, and possible reference voltage can be used to design voltage limiters that prevent one or both extremities of the output waveform from going beyond predetermined values – the limiting levels.
- Applying a time-varying waveform to a circuit consisting of a capacitor in series with a diode and taking the output across the diode provides a clamping function.
- By cascading a clamping circuit with a peak-rectifier circuit, a voltage doubler is realized.

Summary (6)

- Beyond a certain value of reverse voltage (that depends on the diode itself), breakdown occurs and current increases rapidly with a small corresponding increase in voltage.
- Diodes designed to operate in the breakdown region are called zener diodes. They are employed in the design of voltage regulators whose function is to provide a constant dc voltage that varies little with variations in power supply voltage and / or load current.