MAT205 01-02-2024

MAT205

Linear Algebra & Complex Variables

Book: Elementary Linear Algebra with Applications
By Howard Anton

System of Linear Equation:-

An eqn is of the form $a_1x+a_2x_2+a_3x_3+\dots+a_nx_n=b$ is called a linear eqn over F of n variables $x_1, x_2, x_3 \dots x_n$ where $a_1, a_2, a_3 \dots a_n, b \in F$

Two or more than two of these type of linear equations are called system of linear equations.

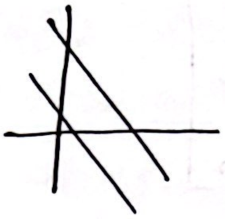
Homogeneous:	Non-Homogeneous:
$2x+3y=0$	$x+5y-3z=5$
$4x+7y=0$	$2x-3y+z=1$

$$\begin{matrix} a_{11}x_1+a_{12}x_2+\dots+a_nx_n=b_1 \\ a_{21}x_1+a_{22}x_2+\dots+a_{2n}x_n=b_2 \\ \vdots \\ a_{m1}x_1+a_{m2}x_2+\dots+a_{mn}x_n=b_m \end{matrix}$$

may have unique solution or may have no solution or may have infinite solution

~~No solution =~~ Zero Solution = Trivial Solution

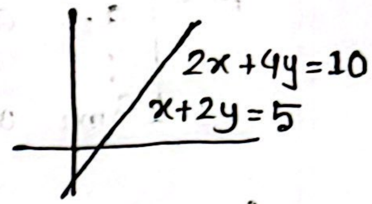
☐ Parallel = No Intersecting point



= No Solution

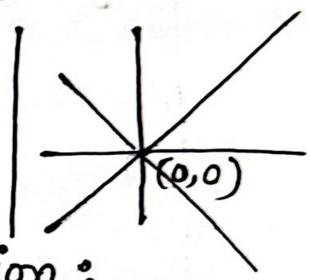
☐ Same line ≐ Infinite Solution

$$\left.\begin{array}{l} 2x+4y=10 \\ x+2y=5 \end{array}\right\}$$



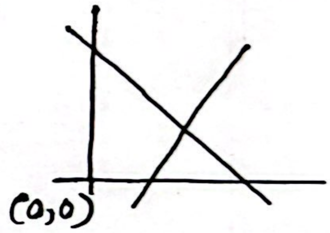
☐ For Homogeneous system of linear Equation:

(R.H.S = 0) → সবসময় unique solution or infinite solution হবে। কখনও No solution হবে না।
~~Gipto~~ Graph (0,0) দিয়ে যাবে Always.



☐ Non-Homogeneous System of linear Equation:

(R.H.S ≠ 0) → Graph (0,0) দিয়ে যাবে এমন কোনো কথা নেই।



☐ Gaussian Elimination:-

To solve a system of linear Equation with great number of variables, we need to use a method known as Gaussian Elimination.

$$\underbrace{\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}}_{b}$$

$$A\underline{x} = \underline{b}$$

Column Vector (1 Column)

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}, \quad \underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad \underline{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

$A\underline{x} = \underline{b} \rightarrow$ system of non-homogeneous linear Equation. If $\underline{b} = 0$, than it will be a system of homogeneous linear Equation.

Augmanted Matrix

Book: Page 17. (1.1)

$2x+3y+5z=10$

$\Rightarrow 4x+6y+10z=20$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 3 & 4 & 5 \end{bmatrix} \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 6 & 7 & 8 \\ 0 & 0 & 4 & 5 \end{array}\right]$$

augmented matrix

$x_1+2x_2+3x_3=4$

$6x_2+7x_3=8$

$4x_3=5$

Backward substitution

Elementary Row operation

The augmanted matrix of our system A is,

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 2 & 4 & -3 & 1 \\ 3 & 6 & -5 & 0 \end{array}\right] \quad \left\{\begin{array}{l} R_2' = -2R_1+R_2 = 0 \\ R_3' = -3R_1+R_3 = 0 \end{array}\right.$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 2 & -7 & -17 \\ 0 & 3 & -11 & -27 \end{array}\right] \quad \left\{R_3'' = -3R_2'+2R_3'\right\}$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 2 & -7 & -17 \\ 0 & 0 & -1 & -3 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 2 & -7 & -17 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad R_3''' = (-1)R$$

$x+y+2z=9$:
$2y-7z=-17$
$z=3$

Backward substitution :-

$2y=-17+21$
$\Rightarrow 2y=4$
$\therefore y=2$

$x=9-y-2z$
$=9-2-2\cdot 3$
$=9-2-6=1$

$\therefore (x,y,z)=(1,2,3)$

This is the solution of unique ~~solutio~~ case.

⊞ 11 $\left.\begin{array}{l} x-y=3 \\ 2x-2y=K \end{array}\right\} \left[\begin{array}{cc|c} 1 & -1 & 3 \\ 2 & -2 & K \end{array}\right]$

$\Rightarrow \left[\begin{array}{cc|c} 1 & -1 & 3 \\ 0 & 0 & -6+K \end{array}\right]$ $R_2' = -2R_1+R_2$

If $K\neq 6$, ~~there will be~~ then the system does not have any solution.

If $K=6$, than $\left[\begin{array}{cc|c} 1 & -1 & 3 \\ 0 & 0 & 0 \end{array}\right] \rightarrow x-y=3$

then, we have $(2-1)=(\text{variable}-\text{eqn})=1=\text{free variable}$

let it be, $y\in\mathbb{R}$ then $x=3+y$ $[y=t\in\mathbb{R}]$

$\therefore x=3+t$

$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3+t \\ t \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} + \begin{pmatrix} t \\ t \end{pmatrix}$

$(x,y) \Rightarrow (3+t,t)$

$= \begin{pmatrix} 3 \\ 0 \end{pmatrix} + t\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$

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As, there is only one equation for 2 variable thus, there is no unique solution.

From Above, We ~~can~~ can say, the system has only two solutions → no solution, infinite solution.

-02-24

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Reduced Row echelon form

Leading ~~up~~ উপাদান গুলো 1 হতে হবে, Leading উপাদান এর উপরের সারির উপাদানটি 0 হবে,

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 2 & -7 & -17 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad [R_1' = R_1 + (-2)R_3 \quad R_2' = R_2 + 7R_3]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & 2 & 0 & 4 \\ 0 & 0 & 1 & 3 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad \left[R_2'' = \frac{R_2'}{2}\right]$$

$$[R_1''' = R_1'' + (-1)R_2'']$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad Ex. \rightarrow 4, 6$$

$$x_1 = 1 - 2x_2 + x_4 - x_5$$

$$= 1 - 2 \cdot \frac{1}{3}(1 + 7x + t) + s - t$$

$$= 1 - 2/3 - \frac{1}{3} - \frac{2t}{3} + s - t$$

$$= 1/3 - \frac{11}{3}s - \frac{5}{3}t$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 1/3 & -11/3\,s & -5/3\,t \\ 1/3 & 7/3\,s & +1/3\,t \\ 1 & -7s & 0 \\ 0 & s & 0 \\ & & \end{pmatrix}$$

$$= \begin{pmatrix} 1/3 \\ 1/3 \\ 1 \\ 0 \\ 0 \end{pmatrix} + s \begin{pmatrix} -11/3 \\ 7/3 \\ 7 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} -5/3 \\ 1/3 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$x_1 + 2x_2$ $\qquad$ $x_4 + x_5 = 1$

$\Rightarrow 3x_2 + x_3$ $\qquad$ $-x = 2$

$x_3 + 7x_4 = 1$

$$\left[\begin{array}{ccccc|c} 1 & 2 & 0 & -1 & 1 & 1 \\ 0 & 3 & 1 & 0 & -1 & 2 \\ 0 & 0 & 1 & 7 & 0 & 1 \end{array}\right]$$

$(5 - 3) = 2$ Free Variables)

Unknown – eqns

~~(5~~ Let $x_4 = s$

$x_5 = t$ $\quad s, t \in \mathbb{R}$

$x_3 = 1 - 7x_4 = 1 - 7s$ $\qquad s \in \mathbb{R}$

We have to take x_4 and x_5 as free variables

As x_1, x_2 and x_3 are in leading variable.

$3x_x = 2 - x_3 + x_5 = 2(1 - 7s) + t$

$= 1 + 7s + t$

$\therefore x_2 = 1/3(1 + 7s + t)$

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AB
BA

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 3 & 5 \\ 7 & 9 & 1 \\ 3 & 5 & 2 \end{bmatrix}$$

$$\underset{3\times3}{AB} = \begin{bmatrix} 1+14+9 & 3+18+15 & 5+2+6 \\ 4+35+18 & 12+45+30 & 20+5+12 \\ 7+56+27 & 21+72+45 & 35+8+18 \end{bmatrix} = \begin{bmatrix} 24 & 36 & 13 \\ 57 & 87 & 37 \\ 90 & 138 & 61 \end{bmatrix}$$

□ $A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 6 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & 1 & 4 & 3 \\ 0 & -1 & 3 & 1 \\ 2 & 7 & 5 & 2 \end{bmatrix}$

$$AB = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 6 & 0 \end{bmatrix} \begin{bmatrix} 4 & 1 & 4 & 3 \\ 0 & -1 & 3 & 1 \\ 2 & 7 & 5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+0+8 & 1+(-2)+28 & 4+6+20 & 3+2+8 \\ 8+0+0 & \cancel{8+0+0} \; 2-6+0 & 8+18+0 & 6+6+0 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & \cancel{26}\; 27 & 30 & 13 \\ 8 & -4 & 26 & 12 \end{bmatrix} = 2\times4$$

Additive Identity = 0.
Multiplicative Identity = 1

linear combination

P-40-43
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$$[c_1 \quad c_2 \quad c_3][c] = [B]$$

$$\Rightarrow 2[c_1] + (-1)c_2 + 3c_3 = [B]$$

$$a_1c_1 + a_2c_2 + \ldots + a_nc_n = b$$

- Transpose Matrix: P-50

row → colume

$$\begin{bmatrix} 1 & 2 & 4 \\ 2 & 6 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 \\ 2 & 6 \\ 4 & 0 \end{bmatrix}$$

horizontal vertical

matrix এর trace = তার Transpose এর trace

- $AB = C$
  $\Rightarrow B = A^{-1}C$

$AA^{-1} = I$ (Identity Matrix)

- Inverse Matrix
  - square matrix হতে হবে
  - Determinant $\neq 0$

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$$

$$[A \mid I] = \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 1 & 3 & 0 & 1 \end{array}\right] = \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & -1 & 1 \end{array}\right]$$

$$= \left[\begin{array}{cc|cc} 1 & 0 & 3 & -2 \\ 0 & 1 & -1 & 1 \end{array}\right]$$

$$\therefore A^{-1} = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$$

Prove :

$$AA^{-1} = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3-2 & -2+2 \\ 3-3 & -2+3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$|I_3| = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1(0-0) - 0(1-0) + 0(0-$$

$$= 1$$

(i) যদি Determinant $= 0$ হয় তাহলে তা singular matr

(ii) Inverse ২প্রকার;

(a) Additive

(2) Multiplicative

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$[A|I] \longrightarrow [I|A^{-1}]$

$\therefore AA^{-1} = I = A^{-1}A$

$(A^{-1})^T = (A^T)^{-1}$

$A^{-1} = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$

$A^T = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$; $(A^T)^{-1} = \begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix}$ $\quad (A^{-1})^T = \begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix}$

~~$(A^T)^{-1} \cdot A^T = I$~~

~~$\Rightarrow A^{-1} =$~~

$\boxed{\text{Example } 1}$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$$

The matrix must be non-singular. Let us check

$$A = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{vmatrix} = 1(40-0) - 2(16-3) + 3(0-5)$$
$$= 40 - 26 - 15 = -1 \neq 0$$

$\therefore$ A has an inverse.

$$[A|I] = \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 3 & 0 & 1 & 0 \\ 1 & 0 & 8 & 0 & 0 & 1 \end{array}\right]$$

$R_2' = -2R_1 + R_2$
$R_3' = (-1)R_1 + R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & -2 & 5 & -1 & 0 & 1 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & -1 & -5 & 2 & 1 \end{array}\right] \quad [R_3'' = 2R_2' + R_3']$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right] \quad R_3''' = (-1)R_3''$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & -14 & 6 & 3 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right] \quad \begin{array}{l} R_1^{IV} = (-3)R_3''' + R_1''' \\ R_2^{IV} = 3R_3''' + R_2''' \end{array}$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \overset{-40}{\cancel{-14}} & \overset{16}{\cancel{6}} & \overset{9}{\cancel{3}} \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right] \quad R_1^{V} = (-2)R_2^{IV} + R_1^{IV}$$

$$A^{-1} = \begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix}$$

$$A \cdot A^{-1} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix} \begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix}$$

Raya

$$\Rightarrow \begin{bmatrix} \cancel{16}-40+26+15 & 16-10-6 & 9-6-3 \\ \cancel{16-10-6} & & \\ \cancel{9-6-3} & 32-25-6 & 18-15-3 \\ -80+65+15 & & \\ -40+0+40 & 18+0+(-16) & 9+0+(-8) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

□ $Ax = b$

$x = A^{-1}b$ ***

↓

left side of b.

$$\begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix} \begin{pmatrix} 5 \\ 3 \\ 17 \end{pmatrix}$$

$$x_1 + 2x_2 + 3x_3 = 5$$
$$2x_1 + 5x_2 + 3x_3 = 3$$
$$x_1 + \quad + 8x_3 = 17$$

$$\Rightarrow x = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

Example 2 (P-98)

Solve the system a, b

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$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

যদি leading element non-zero আছে

$$\left[\begin{array}{ccc|c} 0 & 1 & 2 & 3 \\ 4 & 5 & 6 & 2 \\ 1 & 2 & 4 & 3 \end{array}\right] \quad R_1 \sim R_3 \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 4 & 3 \\ 4 & 5 & 6 & 2 \\ 0 & 1 & 2 & 3 \end{array}\right]$$

↳ Interchange

Triangular Matrix

(i) Upper triangular Matrix $\rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$

(ii) lower triangular Matrix $\rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{bmatrix}$

~~Trans~~ Inverse of diagonal Matrix ঃ

$$\begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix}^T \rightarrow \begin{bmatrix} 1/d_1 & 0 & 0 \\ 0 & 1/d_2 & 0 \\ 0 & 0 & 1/d_3 \end{bmatrix}$$

(All diagonal element must be non-zero)

$A = (a_{ij})_{i,j} = 1, 3$

$a_{ij} \begin{cases} 0 \text{ if } i > j \\ \text{nonzero otherwise} \end{cases}$ $A = \begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix}$

$$A^2 = \begin{bmatrix} d_1^2 & 0 & 0 \\ 0 & d_2^2 & 0 \\ 0 & 0 & d_3^2 \end{bmatrix}$$

Identity matrix এর Inverse matrix = Identity Matrix

symmetrix Matrix $= \begin{bmatrix} 1 & \boxed{2} & 3 \\ \boxed{2} & 4 & \boxed{5} \\ 3 & 5 & 6 \end{bmatrix}$

Anti symmetric $= \begin{bmatrix} 1 & 2 & 3 \\ -2 & 4 & 5 \\ -3 & -5 & 6 \end{bmatrix}$

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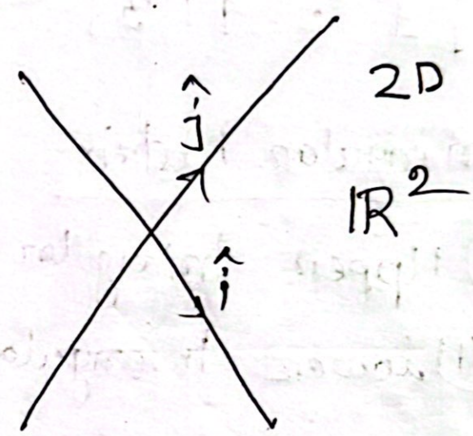
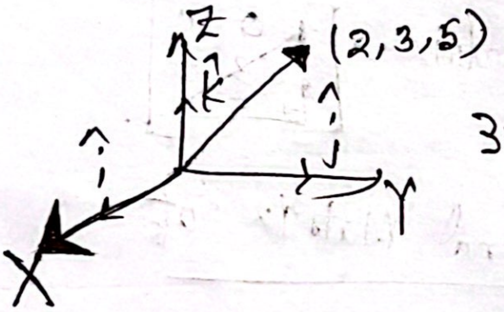
MAT205 03 March 2024 → Quiz 1 → (I) System of linear Eqns
(II) Gaussian Elimination
(III) Reduced Row echelon form

Vector ($\mathbb{R}$)

$A = 2\hat{i} + 3\hat{j} + 5\hat{k}$, $B = 4\hat{i} + 5\hat{j} + 7\hat{k}$



Vector space:



$A = (2, 3, 5)$, $B(4, 5, 7)$

$A + B = (6, 8, 12)$

A_1, A_2, A_3 যদি হয়,

$c_1A_1 + c_2A_2 + c_3A_3 = 0$

যদি $c_1 = c_2 = c_3 = 0$ তাহলে linearly Independent.

$A = (2, 4)$
$B = (4, 8)$

$B = 2A$

$c_1A + c_2B = 0$

$\Rightarrow c_1(2,4) + c_2(4,8) = 0$

$\Rightarrow (2c_1, 4c_1) + (4c_2, 8c_2) = 0$

$\Rightarrow (2c_1 + 4c_2, 4c_1 + 8c_2) = 0$

$2c_1 + 4c_2 = 0$

$4c_1 + 8c_2 = 0$

$\Rightarrow c_1 = -2c_2$

$\therefore c_1 = -2$ Not linearly Independent

$c_2 = 1$

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$(4,-1,8) = c_1(1,2,-1) + c_2(6,4,2)$

$(4,-1,8) = (c_1+6c_2,\ 2c_1+4c_2,\ -c_1+2c_2)$

$$\left(\begin{array}{cc|c} 1 & 6 & 4 \\ 2 & 4 & -1 \\ -1 & 2 & 8 \end{array}\right)$$

linear system:

$c_1 + 6c_2 = 4$

$2c_1 + 4c_2 = -1$

$-c_1 + 2c_2 = 8$

$$\Rightarrow \left(\begin{array}{cc|c} 1 & 6 & 4 \\ 0 & -8 & -9 \\ 0 & 8 & 12 \end{array}\right) \quad \begin{array}{l} (-2R_1+R_2) \\ (R_1+R_3) \end{array}$$

$$\Rightarrow \left(\begin{array}{cc|c} 1 & 6 & 4 \\ 0 & 2 & \frac{9}{4} \\ 0 & 2 & 3 \end{array}\right)$$

$$\Rightarrow \left(\begin{array}{cc|c} 1 & 6 & 4 \\ 0 & 1 & 9/8 \\ 0 & 0 & 3/8 \end{array}\right)$$

$0c_1 + 0c_2 = 3.$

There is no solution.

$c_2 = 9/8$

$c_1 + 6c_2 = 4$

$\Rightarrow c_1 = 4 - \left(\cancel{6}^{3} \cdot \frac{9}{\cancel{8}_{4}}\right)$

$\Rightarrow c_1 = 4 - \frac{27}{4}$

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Example 12 : Three Vectors that do not span R^3

$$c_1v_1 + c_2v_2 + c_3v_3 = (x, y, z)$$

$$c_1 + c_2 + 2c_3 = x$$

$$c_1 + c_3 = y$$

$$2c_1 + c_2 + 3c_3 = z$$

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MAT 209

A linear independent set

$= \{v_1, v_2, v_3 \ldots\ldots, v_k\}$

$c_1 v_1 + c_2 v_2 + \ldots\ldots + c_k v_k = 0$

$c_1 = c_2 = c_3 = \ldots = c_k = 0.$

p. 370 Example-1 : $c_1 = 3, c_2 = 1, c_3 = -1$

$c_1 v_1 + c_2 v_2 + c_3 v_3 = 0.$

$\Rightarrow c_1 (2, -1, 0, 3) + c_2 (1, 2, 5, -1) + c_3 (7, -1, 5, 8) = 0$
$(0, 0, 0, 0)$

$\Rightarrow (2c_1 + c_2 + 7c_3, -c_1 + 2c_2 - c_3, 0c_1 + 5c_2 + 5c_3, 3c_1 - c_2 + 8c_3) = 0$

$$\left.\begin{aligned} 2c_1 + c_2 + 7c_3 &= 0 \\ -c_1 + 2c_2 - c_3 &= 0 \\ 5c_2 + 5c_3 &= 0 \\ 3c_1 - c_2 + 8c_3 &= 0 \end{aligned}\right\} \quad \begin{aligned} 2c_1 + c_2 + 7c_3 &= 0 \\ 5c_2 + 5c_3 &= 0 \\ 5c_2 + 5c_3 &= 0 \\ 5c_2 + 5c_3 &= 0 \end{aligned}$$

$$\begin{cases} 2c_1 + c_2 + 7c_3 = 0 \\ 5c_2 + 5c_3 = 0 \end{cases}$$

$c_3 = s$ (Assume)

$\therefore c_2 = -s$

$\therefore c_1 = -\left(\frac{-s + 7s}{2}\right) = \frac{-6s}{2} = -3s.$

Linearly dependent

Example-4

linearly dependent

$c_1v_1+c_2v_2+c_3v_3 = c_1(1,-2,3)+c_2(5,6,-1)+$

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$c_3(3,2,1)$

$\Rightarrow (c_1+5c_2+3c_3, -2c_1+6c_2+2c_3, 3c_1-c_2+c_3)$

$c_1+5c_2+3c_3=0$

$-2c_1+6c_2+2c_3=0$

$3c_1-c_2+c_3=0$

$$\begin{bmatrix} 1 & 5 & 3 \\ -2 & 6 & 2 \\ 3 & -1 & 1 \end{bmatrix}$$

Gaussian Elimination

~~$c_1 = -(5c_2+3c_3)$~~

~~$-2(-5c_2-3c_3)+6c_2+2c_3=0$~~

~~$\Rightarrow 10c_2+6c_3+6c_2+2c_3=0$~~

$c_1+5c_2+3c_3=0$

$16c_2+8c_3=0$

$\Rightarrow c_3=-2$

$c_2=1$

$c_1=1$

5.4
Basis and Dimension

$v_1+v_2-2v_3=0$

** Theorem 5.3.3. P-375

একটি Basis এর মধ্যে ৪টি Linear

P-383

linearly independent,

vector space এর span করে যা কিনা (x,y,z).

Basis of R_3

$(1,0,0)$

$(0,1,0)$

$(0,0,1)$

P-288

Ex-3 : Demonstrating that a set of vectors is a Basis.

$$\left.\begin{aligned} c_1+2c_2+3c_3&=0\\ 2c_1+9c_2+3c_3&=0\\ c_1+0c_2+4c_3&=0 \end{aligned}\right\} \xrightarrow[\text{iii}]{\text{Gausisian Elimination}} \boxed{\begin{aligned} c_1+2c_2+3c_3&=0\\ 5c_2-3c_3&=0\\ \cancel{2}c_2+c_3&=0 \end{aligned}}$$

$$\Rightarrow \boxed{\begin{aligned} c_1+2c_2+3c_3&=\\ 5c_2-3c_3&=0\\ 2c_3&=0 \end{aligned}}$$

$$c_1=c_2=c_3=0$$

20/03/202

MID – 31st March
(Linear Algebra)

Basis and Dimension

Ex-5.3 — 1, 2, 3 (Page 378) ***

$\boxed{\text{Example - 3}}$ ~~Example~~ Demonstating a set of vectors is a Basis.

$$c_1 + 2c_2 + 3c_3 = a$$
$$2c_1 + 9c_2 + 3c_3 = b$$
$$c_1 \quad + 4c_3 = c$$

$$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 9 & 3 \\ 1 & 0 & 4 \end{vmatrix} = 1(36-0)-2(8-3)+3(0-9) = 36-10+-27 = -1 \neq 0$$

If it's determinant $= 0$ then it doesn't span.

$\boxed{\text{Example-10}}$ $x_1 = -s-t$, $x_2 = s$, $x_3 = -t$.
$x_4 = 0$, $x_5 = t$

P-393

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} \Rightarrow \begin{pmatrix} -s-t \\ s \\ -t \\ 0 \\ t \end{pmatrix} \Rightarrow \begin{pmatrix} -s \\ s \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -t \\ 0 \\ -t \\ 0 \\ t \end{pmatrix} \Rightarrow s\begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} -1 \\ 0 \\ -1 \\ 0 \\ 1 \end{pmatrix}$$

Basis এর মধ্যে যতটি vector থাকবে তাকে dimension of space বলা হয়।

*** 2, 12, 17 — 5.4 — P-397

Linear Transformation

$f(x) = 2x+1$

$f(x) = 2x+1$

Ch-8

V.S = Vector space

$T: V \longrightarrow W$

T is a transformation from V.S V to V.S W

$T(u+v) = Tu + Tv$

$T(\alpha u) = \alpha Tu$

Zero transformation $T: V \longrightarrow \{0\}$ P-57

$T(u) = 0 \ \forall \ u \in V$

linear proved

$T(u+v) = 0 = 0 + 0 = Tu + Tv$

$T(\alpha u) = \alpha \cdot 0 = \alpha \cdot Tu$

Identity Operator: same input, output

linear proved

$T: V \longrightarrow V$

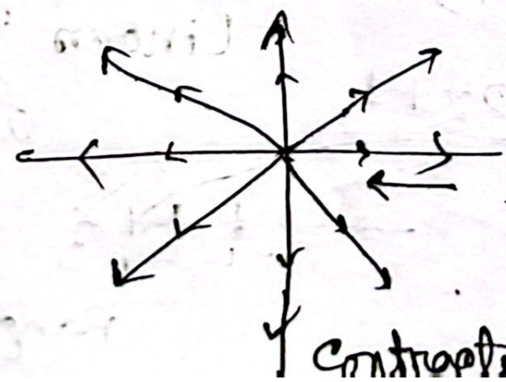
$Tu = u$

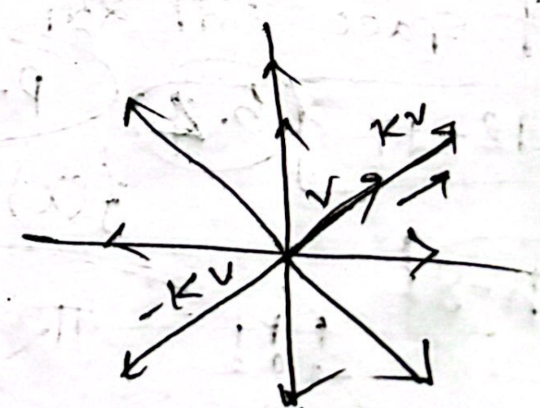
$T(v+w) = v+w = Tv + Tw$

$T(\alpha v) = \alpha v = \alpha \cdot Tv$

Dilation and Contraction Operator:

$T(v) = kv$


Contraction



Dialation

$\alpha v = (\alpha v_1, \alpha v_2, \alpha v_3)$

Ex-8.1 (p-589)

② $T(x_1, x_2, x_3) = (2x_1 - x_2 + x_3, x_2 - 4x_3)$

$T(u+v)$ $u \in \mathbb{R}^3$
$v \in \mathbb{R}^3$

$u = (u_1, u_2, u_3)$, $v = (v_1, v_2, v_3)$

$\therefore u+v = (u_1+v_1, u_2+v_2, u_3+v_3)$

$\therefore T(u+v) = T(u_1+v_1, u_2+v_2, u_3+v_3)$

$= \{2(u_1+v_1) - (u_2+v_2) + (u_3+v_3), (u_2+v_2) - 4(u_3+v_3)\}$

$= (2u_1 + 2v_1 - u_2 - v_2 + u_3 + v_3, u_2 + v_2 - 4u_3 - 4v_3)$

$= (2u_1 - u_2 + u_3, u_2 - 4u_3) + (2v_1 - v_2 + v_3, v_2 - 4v_3)$

$= T(u_1, u_2, u_3) + T(v_1, v_2, v_3)$

$= Tu + Tv$

$\therefore T(\alpha v) = T(\alpha v_1, \alpha v_2, \alpha v_3)$

$= (2\alpha v_1 - \alpha v_2 + \alpha v_3, \alpha v_2 - 4\alpha v_3)$

$= \alpha(2v_1 - v_2 + v_3, v_2 - 4v_3)$

$= \alpha T(v_1, v_2, v_3)$

$= \alpha Tv$

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*** Problem 3, 4, 6, 7, 5, 8 $\quad$ p-589

(1) $T(x_1, x_2) = (x_1 + 2x_2, 3x_1 - x_2)$

$T(u+v) = T(u_1+v_1, u_2+v_2)$

$= (u_1+v_1+2(u_2+v_2), 3(u_1+v_1)-(u_2+v_2))$

$= (u_1+v_1+2u_2+2v_2, 3u_1+3v_1-u_2-v_2)$

$= (u_1+2u_2, 3u_1-u_2) + (v_1+2v_2, 3v_1-v_2)$

$= T(u_1, u_2) + T(v_1, v_2)$

$= Tu + Tv$

$T(\alpha u) = T(\alpha v_1 + \alpha v_2)$

$= (\alpha v_1 + 2\alpha v_2, 3\alpha v_1 - \alpha v_2)$

$= \alpha(v_1+2v_2, 3v_1 - v_2)$

$= \alpha T(v_1, v_2)$

$= \alpha Tv$

(6) $T(A) = tr(A)$

$T(A+B) = tr(A+B)$

$= tr(A) + tr(B)$

$= T(A) + T(B)$

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$$T(\alpha A) = tr(\alpha A)$$
$$= \alpha \, tr(A)$$
$$= \alpha \, T(A)$$

$$\alpha A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \alpha A \begin{bmatrix} \alpha & 0 & 0 \\ 0 & 2\alpha & 3\alpha \\ 4\alpha & 5\alpha & 6\alpha \end{bmatrix}$$

$$Tr(\alpha A) = \alpha \, Tr(A)$$

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1/10/2

MAT205

Eigen values & Eigen vectors

Page-538

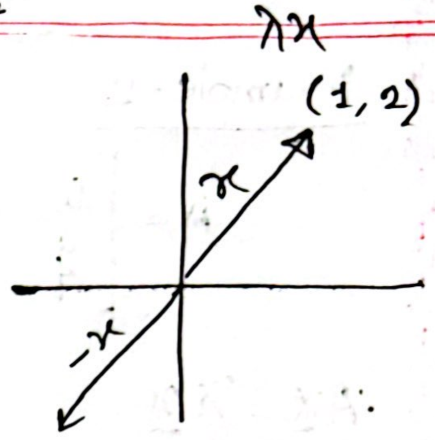

Figure 7.1.1

$$Ax = \underbrace{\lambda}_{\downarrow} x$$

λ এর মানগুলোকে Eigen Value বলে

Eigen Value বসানোর ফলে যে মানগুলো আসবে তাকে Eigen vector বলে।

$$Ax - \lambda x = 0$$

$$\Rightarrow Ax - \lambda I x = 0$$

$$\Rightarrow (A - \lambda I)x = 0$$

$$\Rightarrow |A - \lambda I| = 0$$

Example-1 : Eigen Vector of 2x2 Matrix

$$A = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}, \quad x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$Ax = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \end{bmatrix} = \underset{\text{Eigen Value}}{3}\begin{bmatrix} 1 \\ 2 \end{bmatrix} \rightarrow \text{Eigen vector}$$

$Ax = \lambda x$ $\quad \therefore \lambda = 3 =$ Eigen Value

- □ charasteristic polynomial $= \det(\lambda I - A)$
- □ charasteristic equation $= \det(\lambda I - A) = 0$

Example-2

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & -17 & 8 \end{bmatrix}, \quad \underline{\lambda} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$A\underline{x} = \lambda \underline{x}$

$\Rightarrow (A - \lambda I)x = 0 \; \ldots \ldots (1)$

$P(\lambda) = |A - \lambda I|$

$$\lambda I = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

the characteristics polynomial of A

$$\begin{vmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ 4 & -17 & 8-\lambda \end{vmatrix}$$

$= -\lambda\left((8-\lambda)(-\lambda) + 17\right) - 1(0-4)$

$= -\lambda(-8\lambda + \lambda^2 + 17) + 4$

$= 8\lambda^2 - \lambda^3 - 17\lambda + 4$

$\cancel{= -\lambda^3 + 8\lambda^2}$ Therefore, the charasteristic equation of A is,

$P(\lambda) = 0$

$\therefore -\lambda^3 + 8\lambda^2 - 17\lambda + 4 = 0$

$\Rightarrow \lambda^3 - 8\lambda^2 + 17\lambda - 4 = 0$

$\Rightarrow \cancel{\lambda^3}\ \lambda^2(\lambda-4) - 4\lambda(\lambda-4) + (\lambda-4) = 0$

$\Rightarrow (\lambda-4)(\lambda^2 - 4\lambda + 1) = 0$

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Trace of mantrix = sum of eigun value

$\lambda^2+1=0.\quad \therefore \lambda=-1$

~~$(\lambda-4)($~~

either $\lambda-4=0$ $\therefore \lambda=4$ | or $\lambda^2-4\lambda+1=0$

$\cancel{\Rightarrow \lambda^2-2\lambda-2\lambda+1=0}$

$\cancel{\Rightarrow \lambda(\lambda-2)-2(\lambda-}$

$\lambda=\frac{4\pm\sqrt{16-4.1.1}}{2.1}$

$=\frac{4+2\sqrt{3}}{2}=2\pm\sqrt{3}$

$\lambda_1=4$

$\lambda_2=2+\sqrt{3}$

$\lambda_3=2-\sqrt{3}$

Ex-3 Upper Triangular/Lower Triangular Matrix এর diagonal eliment হবে eigen value.

☐ Complex Eigenvalues:-

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$(A-\lambda I)x=0 \quad \text{(i)}$

$$\begin{bmatrix}-\lambda & 1 & 0\\ 0 & -\lambda & 1\\ 4 & -17 & 8-\lambda\end{bmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$$

$\lambda^2+1=0$

$\lambda^2=-1$

$\lambda=\pm\sqrt{-1}$

$=\pm i$

Put $\lambda=4$, the we have,

$$\begin{bmatrix}-4 & 1 & 0\\ 0 & -4 & 1\\ 4 & -17 & 4\end{bmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$$

$$=\begin{bmatrix}-4 & 1 & 0\\ 0 & 1 & -\frac{1}{4}\\ 0 & -16 & 4\end{bmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$$

(gaussian elimination

$$= \begin{bmatrix} -4 & 1 & 0 \\ 0 & 1 & -\frac{1}{4} \\ 0 & 1 & -\frac{1}{4} \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$= \begin{bmatrix} -4 & 1 & 0 \\ 0 & 1 & -1/4 \\ 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$-4x + y = 0$

$y - \frac{1}{4}z = 0$

Let, $z = s$.

$y = \frac{1}{4}s$

$x = \frac{1}{16}s$

$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = s \begin{pmatrix} \frac{1}{16} \\ \frac{1}{4} \\ 1 \end{pmatrix}$$

Let $s = 1$

$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1/16 \\ 1/4 \\ 1 \end{pmatrix}$$ is the eigen vector corresponding to $\lambda = 4$.

* $\boxed{\text{Example-5}}$ same

Page - 542.

$Ax = \lambda x$

$A(Ax) = A(\lambda x)$

$\Rightarrow A^2 x = \lambda(Ax)$

$\Rightarrow \lambda . \lambda x = \lambda^2 x$

$\Rightarrow A^n x = \lambda^n x$