Logistic Regression

Multivariate analysis

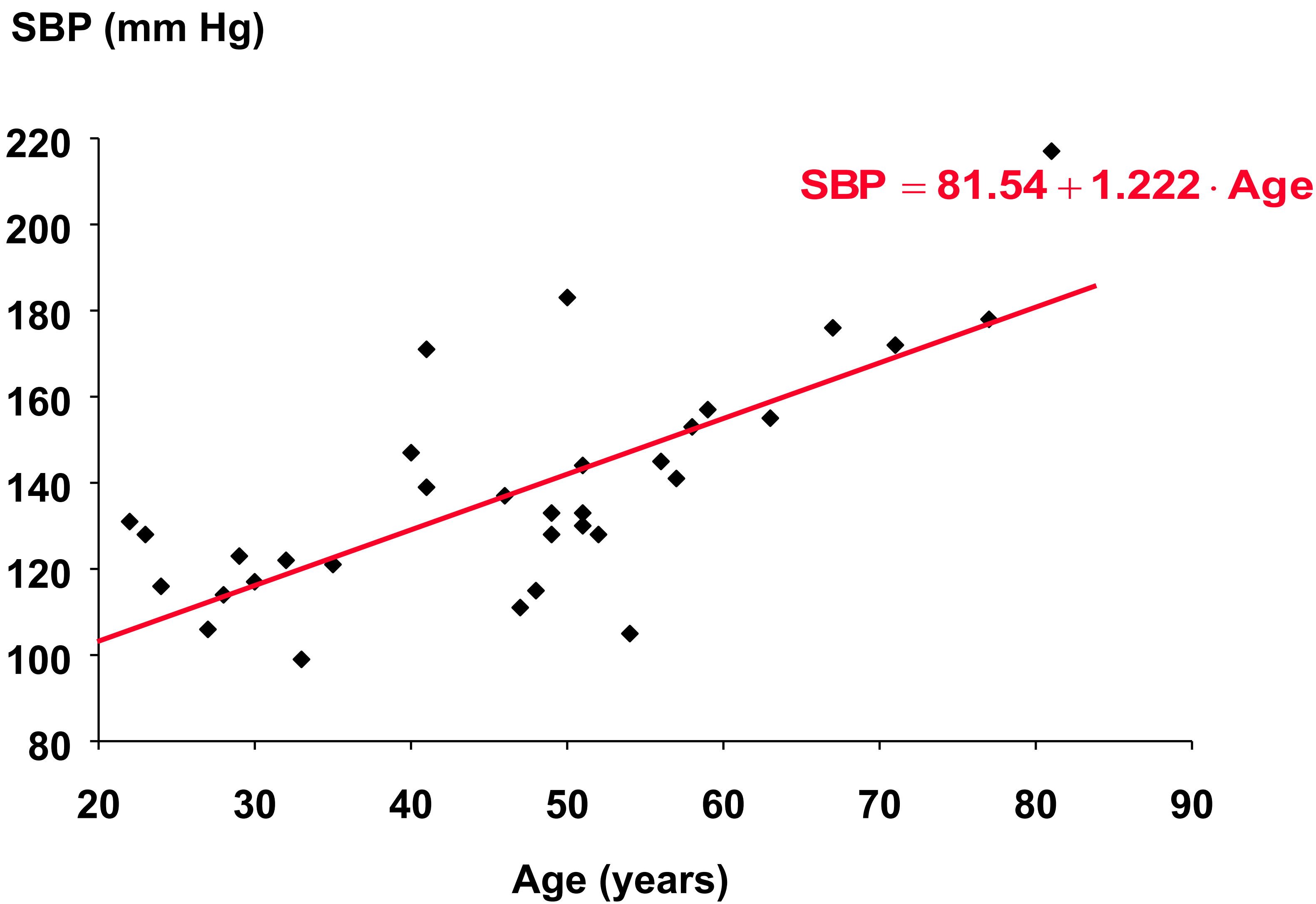
- Multiple models
  - Linear regression
  - Logistic regression

- Choice of the tool according to the objectives, the study, and the variables

Simple linear regression

Table 1 Age and systolic blood pressure (SBP) among 33 adult women

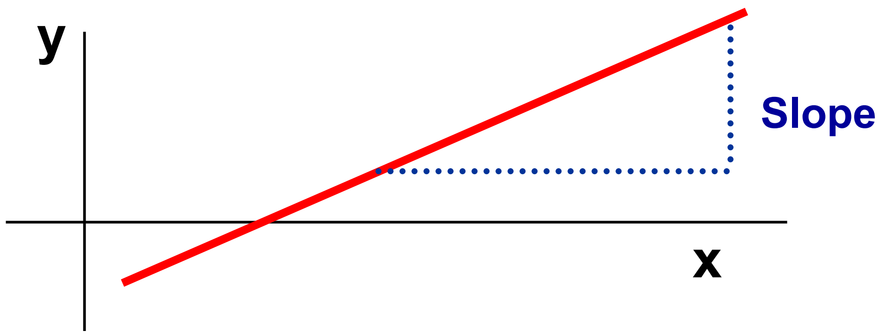
Age	SBP	Age	SBP	Age	SBP
22	131	41	139	52	128
23	128	41	171	54	105
24	116	46	137	56	145
27	106	47	111	57	141
28	114	48	115	58	153
29	123	49	133	59	157
30	117	49	128	63	155
32	122	50	183	67	176
33	99	51	130	71	172
35	121	51	133	77	178
40	147	51	144	81	217



adapted from Colton T. Statistics in Medicine. Boston: Little Brown, 1974

Simple linear regression

- Relation between 2 continuous variables (SBP and age)



$$y = \alpha + \beta_1 x_1$$

- Regression coefficient β_1
  - Measures association between y and x
  - Amount by which y changes on average when x changes by one unit
  - Least squares method

Multiple linear regression

- **Relation between a continuous variable and a set of i continuous variables**

$$y = \alpha + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_i x_i$$

- **Partial regression coefficients β_i**
  - **Amount by which y changes on average when x_i changes by one unit and all the other x_is remain constant**
  - **Measures association between x_i and y adjusted for all other x_i**

- **Example**
  - **SBP *versus* age, weight, height, etc**

Multiple linear regression

$$y = \alpha + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_i x_i$$

Predicted	Predictor variables
Response variable	Explanatory variables
Outcome variable	Covariables
Dependent	Independent variables

Logistic regression (1)

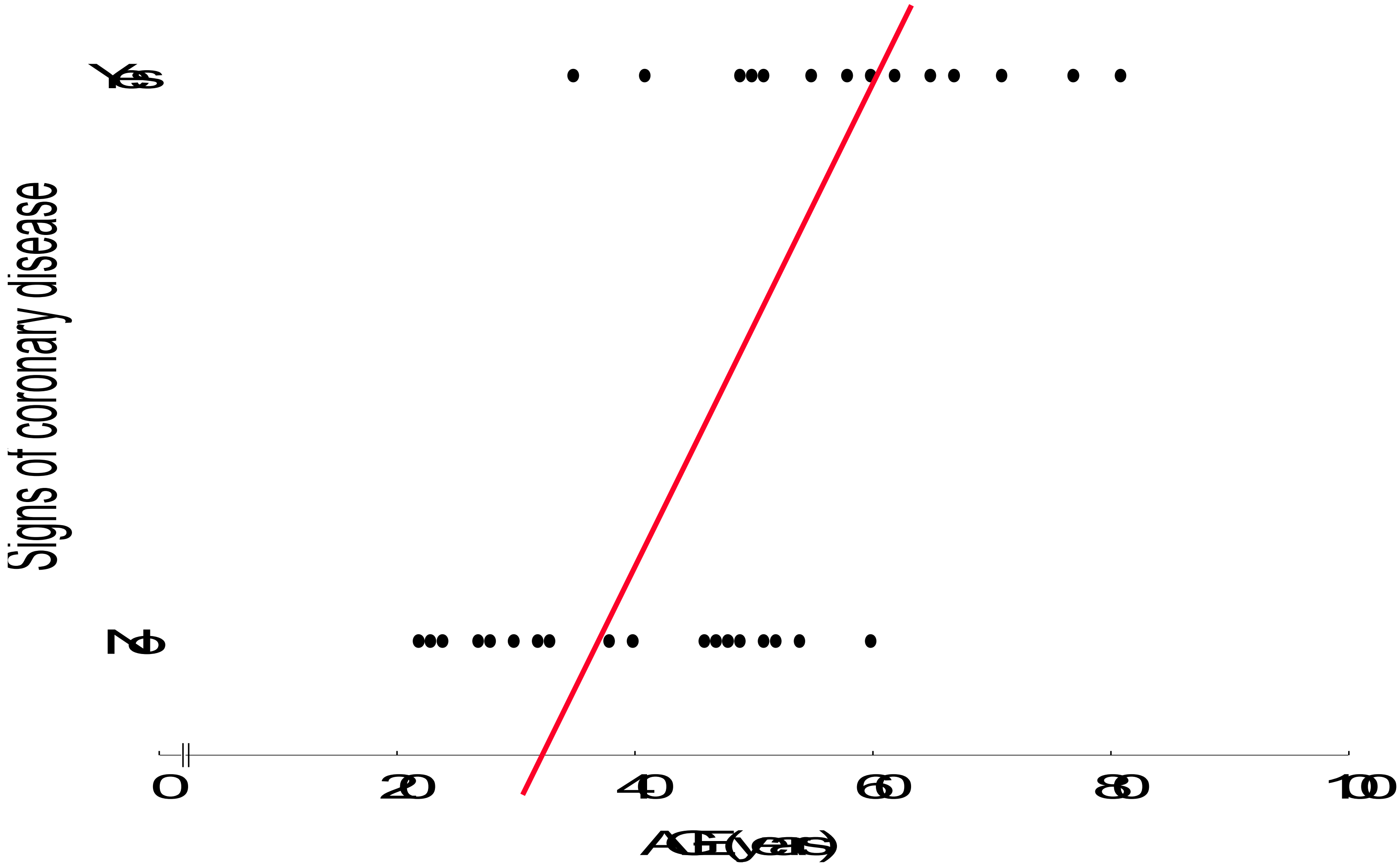
Table 2 Age and signs of coronary heart disease (CD)

Age	CD	Age	CD	Age	CD
22	0	40	0	54	0
23	0	41	1	55	1
24	0	46	0	58	1
27	0	47	0	60	1
28	0	48	0	60	0
30	0	49	1	62	1
30	0	49	0	65	1
32	0	50	1	67	1
33	0	51	0	71	1
35	1	51	1	77	1
38	0	52	0	81	1

How can we analyse these data?

- Compare mean age of diseased and non-diseased
  - Non-diseased: 38.6 years
  - Diseased: 58.7 years ($p<0.0001$)
- Linear regression?

Dot-plot: Data from Table 2

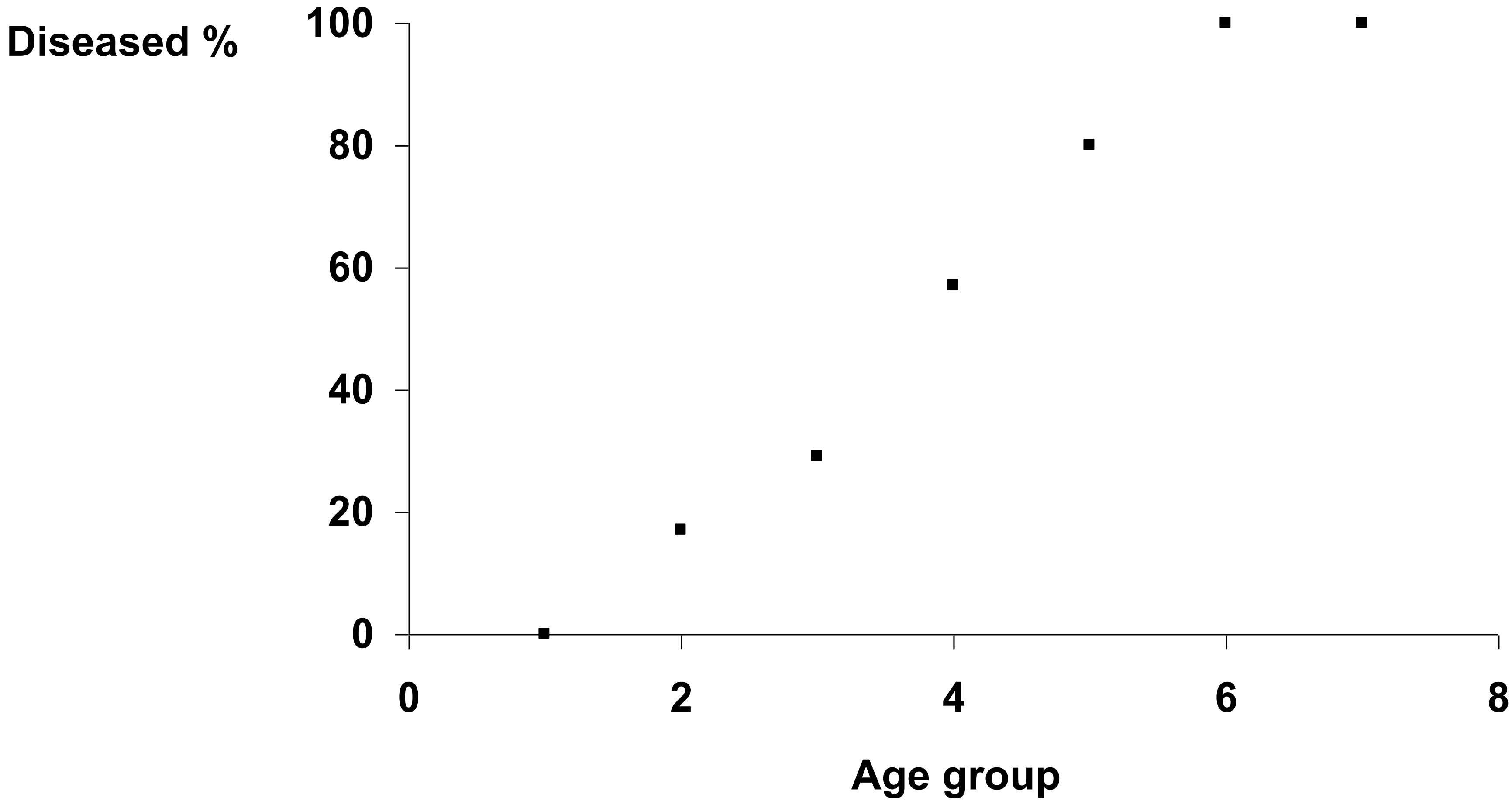


Logistic regression (2)

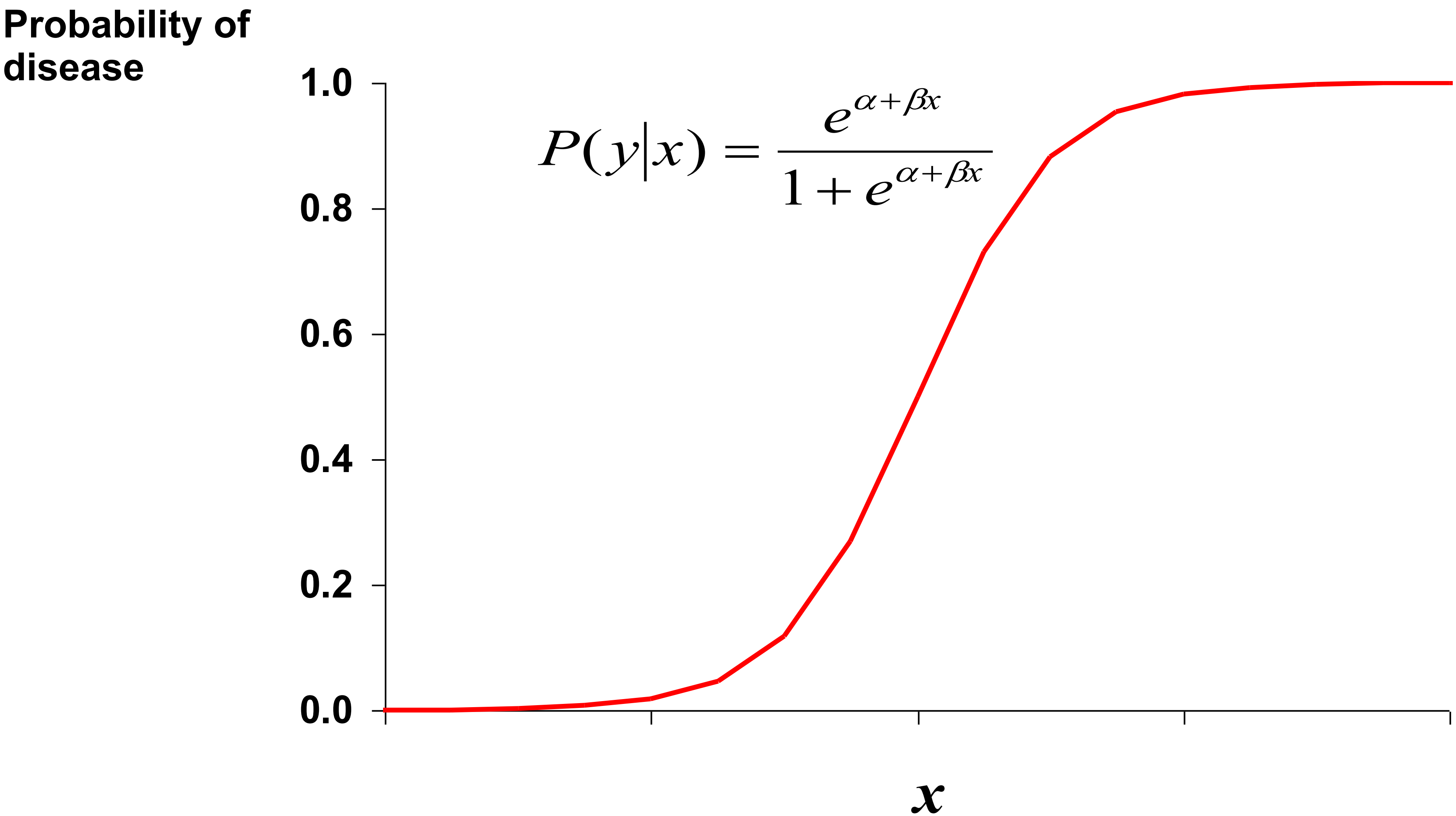
Table 3   Prevalence (%) of signs of CD according to age group

Age group	# in group	Diseased	
		#	%
20 - 29	5	0	0
30 - 39	6	1	17
40 - 49	7	2	29
50 - 59	7	4	57
60 - 69	5	4	80
70 - 79	2	2	100
80 - 89	1	1	100

Dot-plot: Data from Table 3



Logistic function (1)



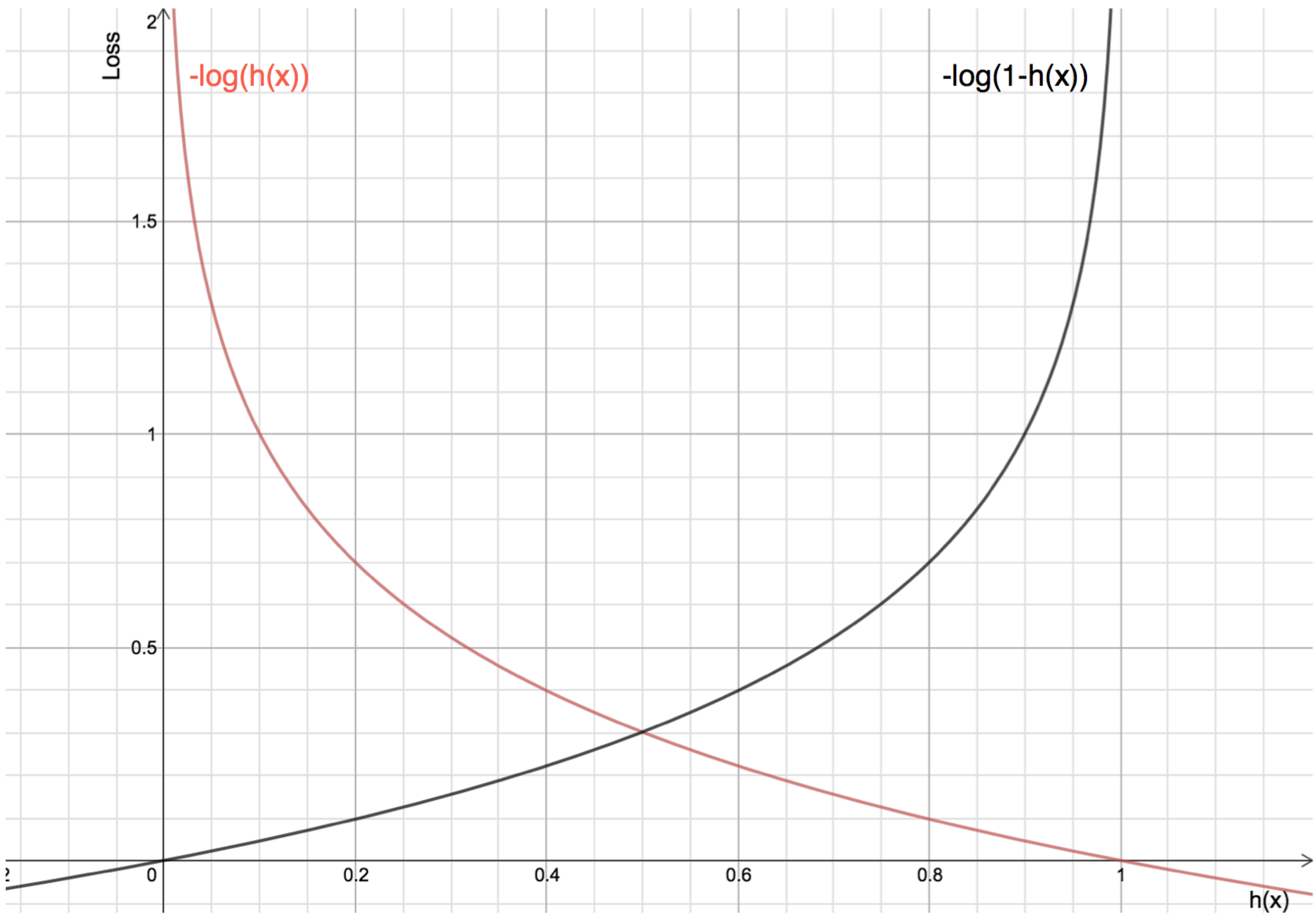
Transformation

$$P(y|x) = \frac{e^{\alpha+\beta x}}{1+e^{\alpha+\beta x}}$$

$$\frac{P(y|x)}{1-P(y|x)}$$

$$\underbrace{\ln\left[\frac{P(y|x)}{1-P(y|x)}\right]}_{\text{logit of } P(y|x)} = \alpha + \beta x$$

- ✓ α = log odds of disease in unexposed
- ✓ β = log odds ratio associated with being exposed
- ✓ e^{β} = odds ratio



Loss
-log(h(x))
-log(1-h(x))
2
1.5
1
0.5
0
0.2
0.4
0.6
0.8
1
h(x)

Fitting equation to the data

- **Linear regression: Least squares**
- **Logistic regression: Maximum likelihood**
- **Likelihood function**
  - **Estimates parameters α and β**
  - **Practically easier to work with log-likelihood**

$$L(\mathrm{B}) = \ln\left[l(\mathrm{B})\right] = \sum_{i=1}^{n} \left\{ y_i \ln\left[\pi(x_i)\right] + (1 - y_i) \ln\left[1 - \pi(x_i)\right] \right\}$$

Maximum likelihood

- Iterative computing
  - Choice of an arbitrary value for the coefficients (usually 0)
  - Computing of log-likelihood
  - Variation of coefficients' values
  - Reiteration until maximisation (plateau)

- Results
  - Maximum Likelihood Estimates (MLE) for α and β
  - Estimates of P(y) for a given value of x

Multiple logistic regression

- More than one independent variable
  - Dichotomous, ordinal, nominal, continuous …

$$\ln\left(\frac{P}{1-P}\right) = \alpha + \beta_1 x_1 + \beta_2 x_2 + \ldots \beta_i x_i$$

- Interpretation of β_i
  - Increase in log-odds for a one unit increase in x_i with all the other x_is constant
  - Measures association between x_i and log-odds adjusted for all other x_i