$$\cancel{\hat{B}\hat{C}\hat{A}\hat{D}} + \cancel{\hat{C}\hat{B}\hat{A}\hat{D}}$$

$$= \hat{A}\hat{B}\hat{C}\hat{D} + \hat{C}\hat{B}\hat{D}\hat{A} - \hat{B}\hat{C}\hat{D}\hat{A} - \hat{A}\hat{C}\hat{B}\hat{D}$$

04/04/2024

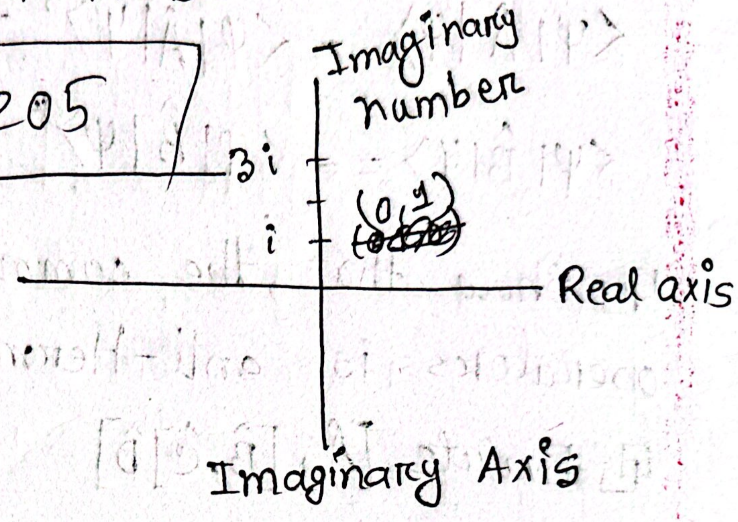
$\boxed{\text{MAT205}}$

$\boxed{\text{Complex Number}}$

$x^2 = -2$

$x^2 + 2 = 0$

$i = \sqrt{-1}$



$= 2 + 3i$

$z = a + ib \rightarrow b \in \mathbb{R}$ (Real Number)

(real) $a \in \mathbb{R}$

imaginary unit

$|i| = 1$

- Every real number is complex number
- Every complex number is subset of Real Number.

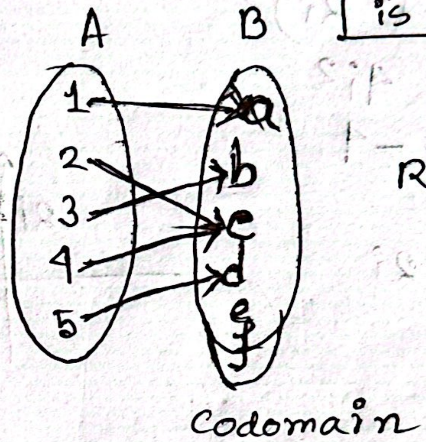
$z = x + iy.$

- $z = (x, y)$
- $z = (x + iy)$
- $(x, 0) = x$
- $(0, y) = iy$
- $(0, 1) = i$
- $R(z) = x$
- $Im(z) = y$

- $z = iy$ (Purely imaginary number)
- $z = x$ (Purely Real Number)

$f(x) = H$

If Range, codomain are same than it is an onto function



Range $f(x) = \{a, b, c, d\}$

$z_1 = (x_1, y_1)$

$z_2 = (x_2, y_2)$

$\therefore z_1 + z_2 = (x_1 + x_2, y_1 + y_2)$

$= (x_1 + x_2) + i(y_1 + y_2)$

$= (x_1 + iy_1) + (x_2 + iy_2)$

$\therefore z_1 z_2 = (x_1, y_1)(x_2, y_2)$

$= (x_1 x_2 - y_1 y_2, x_1 y_2 + x_2 y_1)$

$\therefore z_1 z_2 = (x_1, y_1)(x_2, y_2)$

$\therefore = x_1(x_2 + iy_2) + iy_1 (x_1 + iy_2)$

$= x_1 x_2 + ix_1 y_2 + iy_1 x_1 + i^2 y_1 y_2$

$= x_1 x_2 - y_1 y_2 + i(x_1 y_2 + y_1 x_2)$

$(\sqrt{2} - i) - i(1 - i\sqrt{2}) = ?$

~~$1\sqrt{2}$ ...~~

$1(\sqrt{2} - i) - i(1 - i\sqrt{2})$

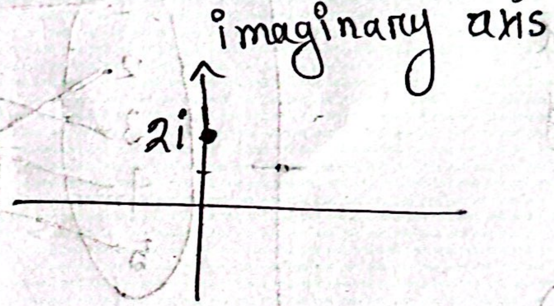
$\Rightarrow \sqrt{2} - i - i + i^2\sqrt{2}$

$\Rightarrow \sqrt{2} - 2i - \sqrt{2} = -2i$

$(1-i)^4 = ?$

$$\left((1-i)^2\right)^2 = (1-2i+i^2)^2$$
$$= (1-2i-1)^2$$
$$= (-2i)^2$$
$$= 4i^2$$
$$= -4$$

$z_1 = 2i$



18.04.202

MAT 205

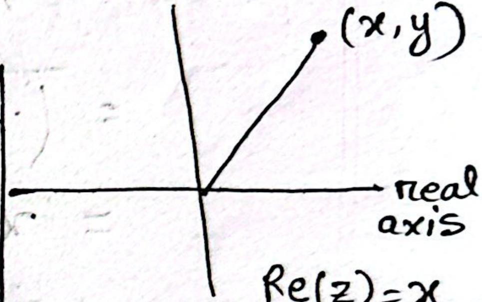
After vacation

Book: Complex variables & Applications by Ruel V. Churchill.

$z = (x, y)$
$= x + iy$

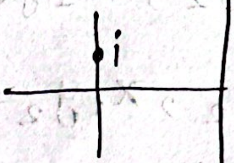
$i = \sqrt{-1}$
$i^2 = -1$

$|i| = 1$

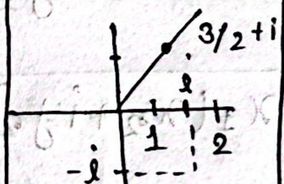


$Re(z) = x$
$Im(z) = y$

$z_1 = zi$
$z_1 + z_2 = \frac{3}{2} + i$

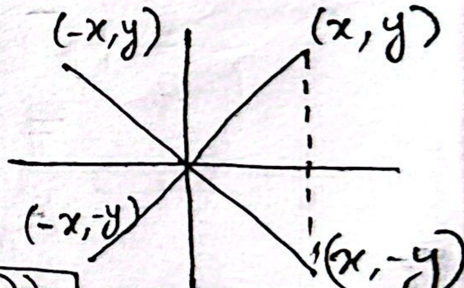


$z_2 = \frac{3}{2} - i$
$Re(z_2) = \frac{3}{2}$
$Im(z) = -1$



Complex Conjugate:

$z = (x, y)$ $\quad \bar{z} = (x, -y)$
$= x + iy$ $\quad = x - iy$



$z = x + iy$
$\bar{z} = x - iy$
$z = x - iy$
$\bar{z} = x + iy$

$$\boxed{\begin{aligned} z_1 + z_2 &= (x_1 + x_2 - (y_1 + y_2)) \\ &= x - y_1 + (x_2 - y_2) \end{aligned}}$$

$z_1 = (x_1, y_1)$
$z_2 = (x_2, y_2)$

$\overline{z_1 + z_2} = \overline{(x_1 + x_2, y_1 + y_2)}$

$\overline{z_1 + z_2 + \cdots + z_{100}} = \overline{z_1} + \overline{z_2} + \cdots + \overline{z_{100}}$

Show that, $\overline{z_1 + z_2} + \overline{z_1} + \overline{z_2}$

$\overline{z_1 . z_2} = ?$

$$z_1 = (x_1, y_1), \quad z_2 = (x_2, y_2)$$

$$\overline{z_1 z_2} = \overline{(x_1 + iy_1)(x_2 + iy_2)}$$

$$= \overline{x_1 x_2 - y_1 y_2 + i(x_1 y_2 + x_2 y_1)}$$

$$= \overline{(x_1 x_2 - y_1 y_2 ,\ x_1 y_2 + x_2 y_1)}$$

$$= x_1 x_2 - y_1 y_2 ,\ x_1 y_2 + x_2 y_1$$

$$= \overline{x_1 (x_2 + iy_2) + y_1 (ix_2 - y_2)}$$

$$= \overline{x_1 (x_2 + iy_2) + iy_1 (x_2 + iy_2)}$$

$$= \overline{x_1 (x_2 + iy_2)} + \overline{iy_1} \overline{(x_2 + iy_2)}$$

$$= x_1 (x_2 - iy_2) - iy_1 (x_2 - iy_2)$$

$$= (x_1 - iy_2)(x_2 - iy_2)$$

$$= \overline{z_1} \cdot \overline{z_2}$$

$$\overline{z_1 z_2} = \overline{z_1}\ \overline{z_2}$$

$$\overline{z_1 - z_2} = \overline{z_1 + (-z_2)}$$

$$= \overline{z_1} + \overline{(-z_2)}$$

$$= \overline{z_1} - \overline{z_2}$$

$$\overline{(z_1 / z_2)} = \frac{\overline{z_1}}{\overline{z_2}}$$

$$z = x + iy$$

$$\overline{z} = x - iy$$

$$z + \overline{z} = 2x$$

$$= 2\,\mathrm{Re}(z)$$

$$z - \overline{z} = 2iy$$

$$= 2\,\mathrm{Im}(z)$$

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rotation: $2n\pi \pm \frac{\pi}{2}$, $n = 0, 1, 2, \dots$

Absolute value / Modulus:

$|z| = \sqrt{x^2+y^2}$

$\boxed{2x\pi \pm \frac{\pi}{2}} \rightarrow$ principle Arg

$Arg(z)$

$arg(z) \rightarrow$ general statement

$z = 3+4i$

$\tan\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53.13$

$z_1 = 2+3i$, $|z_1| = \sqrt{13}$

$z_2 = 4+4+2i$, $|z_2| = \sqrt{20}$

$|z_1| < |z_2|$

$|z|^2 = x^2+y^2 = (Re(z))^2 + (Im(z))^2$

$$\left.\begin{aligned} |z_1+z_2| &\leq |z_1|+|z_2| \\ |z_1-z_2| &\geq \big||z_1|-|z_2|\big| \end{aligned}\right\} \text{Triaglular inequality}$$

$z = x+iy$

$z = 3+4i$

$|z| = \sqrt{9+16} = 5$

$z = x+yi$

$x = r\cos\theta$

$y = r\sin\theta$

$|z| = \sqrt{x^2+y^2} = \sqrt{r^2}$

$\theta = \tan^{-1}(y/x)$

Polar Form

$z = x+iy$

$x = r\cos\theta$, $y = r\sin\theta$

$z = x\cos\theta + i\sin\theta = r\cos\theta + i\sin\theta = re^{i\theta}$

$z_1+z_2 = r_1+e^{i\theta_2}+r_2e^{i\theta_2} = (r_1\cos\theta_1 + i\sin\theta_2) + (r_2\cos\theta_2 + i\sin\theta_2)$

$= x_1+iy_1+x_2+iy_2 = r_1\cos\theta_1 + r_2\cos\theta_2 + (r_1\sin\theta_1 + r_2\sin\theta_2)$

$z_1z_2 = r_1e^{i\theta_1}r_2e^{i\theta_2} = r_1r_2e^{i(\theta_1+\theta_2)}$

$|z_1z_2| = r_1r_2$

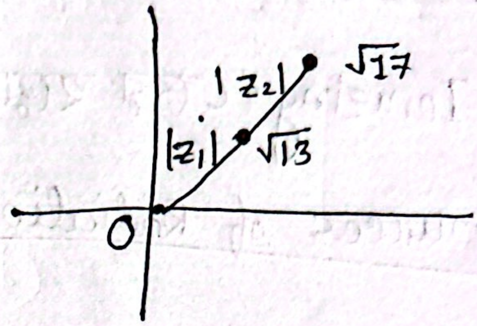
$arg(z_1z_2) = \theta_1+\theta_2 = arg(z_1) + arg(z_2)$

C.W 21/04/2024

MAT205

(New book)

- $z = -3+2i$, $z_2 = 1+4i$



$|z_1| = \sqrt{9+4} = \sqrt{13}$

$|z_2| = \sqrt{1+16} = \sqrt{17}$

$\therefore z_1$ is closer to the origin.

- $|z-1+3i| = 2$

$\Rightarrow |x+iy-1+3i| = 2$

$\Rightarrow |(x-1)+i(y+3)| = 2$

$\Rightarrow \sqrt{(x-1)^2+(y+3)^2} = 2$

$\Rightarrow (x-1)^2+(y+3)^2 = 2^2$

This represents the equation of a circle whose radius is 2 and center at $(1,-3)$

Modulas (chapter-1) (Page-128)

1, 3, 4, 5, 6

= (Circle)

(5) $|z+i| \leq 3$

$\leq r^2$ (Interior to the circle)

$\Rightarrow |x+i(y+1)| \leq 3$

$r^2 \geq$

$\Rightarrow \sqrt{x^2+(y+1)^2} \leq 3$

$\Rightarrow x^2+(y+1)^2 \leq 3^2$, which repnents the set of i.e. all points ~~inside~~ interior and

HW - 1, 4, 5, 6

upto the circle with cente $(0,-1)$ and radius 3.

$z\bar{z} = (x+iy)(x-iy)$
$= x^2 - i^2y^2 = x^2+y^2 = |z|^2$

$|z_1 z_2| = |z_1||z_2|$

$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$

$\frac{-1+3i}{2-i} = \frac{(-1+3i)(2+i)}{(2-i)(2+i)} = \frac{-2-i+6i+3i^2}{2^2-i^2}$

$= \frac{5i-2-3}{2^2+1} = \frac{5i-5}{5} = i-1$

$\overline{iz} = \bar{i}\bar{z} = -i(x-iy) = -ix+i^2y = -y-xi$

$\overline{iz} = \bar{i}\bar{z} = -i\bar{z}$

Show that, $\overline{(2+i)^2} = 3-4i$

$\overline{(2+i)^2} \cancel{= (2-i)^2} = (\overline{2-i})^2$

$= \overline{4+4i+i^2}$

$= 4-4i+(-1)$

$= 3-4i$

(Showed) SAAD

Page-33 (complex conjugate)

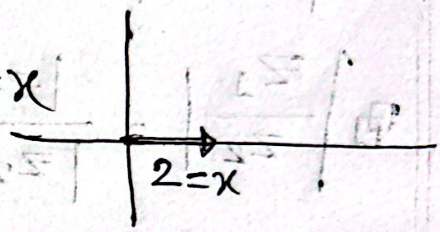
Exercise 1, 2, 7, 8

2| Sketch the set of points determind by the condition $Re(\bar{z}-i)=2$

$z = x+iy$
$\bar{z} = x-iy$

$= x-iy-i = x-i(y+1)$

$2 = Re(\bar{z}-i) = Re(x-i(y+1)) = x$



$|2z-i|=4$

$= |2(x+iy)-i|$

$= |2x+2iy-i|$

$= |2x+i(2y-1)| = 4$

$= \sqrt{(2x)^2+(2y-1)^2} = 4$

$= 4x^2+4y^2-4y+1 = 16$

$\Rightarrow 4x^2+4y^2-4y = 15$

$\Rightarrow x^2+y^2-y = \frac{15}{4}$

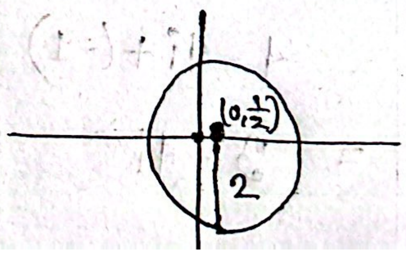
$\Rightarrow x^2+y(y-1) = \frac{15}{4}$

$\Rightarrow x^2+\left(y-\frac{1}{2}\right)^2 = \frac{15}{4}+\frac{1}{4}$

$x^2+\left(y-\frac{1}{2}\right)^2 = 4$

$\therefore$ radius $= 2$

$\therefore$ center $\left(0,\frac{1}{2}\right)$



□ $|Re(2+\bar{z}+z^3)| \leqslant 4$ when $|z| \leqslant 1$

$(x+iy)^3 = x^3+3x^2 \cdot iy+3x(iy)^2+(iy)^3$

$= x^3+i3x^2y-3xy^2-iy^3$

$= x^3-3xy^2+i(3x^2y-y^3)$

$Re(2+x-iy+x^3-3xy^2+i(3x^2y-y^3))$

$= Re((2+x+x^3-3xy^2+i(3x^2y-y^3-y))$

$= 2+x+x^3-3xy^2$

$= |2+x+x^3-3xy^2|$

$\leqslant |2+x+x^3-3x(1-x^2)|$

$= |2+x+x^3-3x+3x^3|$

$= |2-2x+4x^3|$

$= |2-2x(1-2x^2)|$

$|z| \leqslant 1$

$(x^2+y^2)^{\frac{1}{2}} \leqslant 1$

$y^2 \leqslant 1-x^2$

$y \leqslant |1-x^2|$

$|2+x+x^3-3xy^2| \leqslant 2+|x|+|x|^3+3|x||y|^2$

$\leqslant 2+|x|+|x|^3+3|x|(1-|x|^2)$

$\leqslant 2+|x|+|x|^3+3|x|-3|x|^3$

$= 2+4|x|-2|x|^3$

$\leqslant 2+4-2 \quad (|x| \leqslant 1)$

$= 4$

$|xy| = |x||y|$

formula:

$|x+y| \leqslant |x|+|y|$

$|x-y| \leqslant |x|-|y|$

• Genetic effect (can not be repaired)

28/04/24

Homework

Ex-1: $Arg(z_1 z_2) = Arg(-i) = -\frac{\pi}{2}$

$arg(z_1 z_2) = arg(z_1) + arg(z_2)$

$Arg(z_1 z_2) + 2\pi = -\frac{\pi}{2} + 2\pi = \frac{3\pi}{2}$

Ex-2: $z = \frac{-2}{1+\sqrt{3}i}$

$arg\, z = arg(-2) - arg(1+\sqrt{3}i)$

$arg\, z$ is $\frac{2\pi}{3}$ $[-\pi < z \leq z]$

Online Class

(7) $\left|\frac{z_1+z_2}{z_3+z_4}\right| \leq \frac{|z_1|+|z_2|}{||z_3|-|z_4||}$

We know, $|z_1+z_2| \leq |z_1|+|z_2|$

$|z_1+z_2| \geq ||z_1|-|z_2||$

$\Rightarrow \frac{1}{|z_1+z_2|} \leq \frac{1}{||z_1|-|z_2||}$

(4) $z = x+iy$ | $Re(z) = x$, $Im(z) = y$

$Re(z) \leq |Re(z)| \leq |z|$ | $|Re(z)| = |x|$

$x \leq |x| \leq \sqrt{x^2+y^2}$ | $|z| = \sqrt{x^2+y^2}$

8) $|Re(2+\bar{z}+z^3)| \leq |2+\bar{z}+z^3| \leq 2+|\bar{z}|+|z^3|$

$$= 2+|z|+|z|^3$$

$$\leq 2+1+1^3 = 4$$

We know, $|z_1+z_2+z_3| \leq |z_1|+|z_2|+|z_3|$

Slide-25 argument

r = modulus

θ = argument

$$z = x+iy$$

$$z = r\cos\theta + ir\sin\theta$$

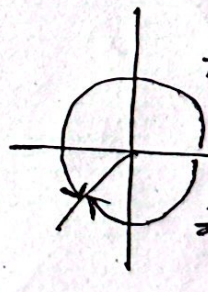
$$= re^{i\theta} \quad [e^{i\theta} = \cos\theta + i\sin\theta] \text{ exponential form}$$

$$r = \sqrt{x^2+y^2} = |z|$$

Arg = principal argument

$$-\pi < \text{Arg} \leq \pi$$

$$\arg = \text{Arg} + 2\pi n \quad [n = 0, 1, 2 \ldots]$$



$\pi + \frac{\pi}{4} = \frac{5\pi}{4}$,

$2\pi - \frac{5\pi}{4} = -\frac{3\pi}{4}$

Example-1 : $z = -1-i$

$$r = \sqrt{(-1)^2+(-1)^2} = \sqrt{2}$$

$$\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\left(\frac{-1}{-1}\right) = \tan^{-1}(1) = \tan^{-1}\tan\frac{\pi}{4} = \frac{\pi}{4}$$

$$\text{Arg}(z) = -\frac{3\pi}{4}$$

$$\arg = \text{Arg}(z) + 2n\pi = -\frac{3\pi}{4} + 2n\pi \quad [n = 0, 1, 2 \ldots]$$

Example: $z = -1 - i$

$$z = re^{i\theta} = \sqrt{2}\, e^{i\left(-\frac{3\pi}{4}\right)} \rightarrow \text{principal argument}$$

$$= \sqrt{2}\, e^{i\left(-\frac{3\pi}{4}\right)} = \sqrt{2}\, e^{i\left(\left(-\frac{3\pi}{4}\right) \pm 2n\pi\right)}$$

$n = 0, 1, 2, \ldots$

(General Argument)

$$e^{i\pi} = \cos\pi + i\sin\pi = -1$$

$$e^{-i\pi/2} = \cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right) = -i$$

page-19 Example-1, 2, 3 [H.W]

*** page-25

Exercise-(1-4)

Example-3

(complex number-page-25)

Next Sunday
05/05/24

28/01/2021

Roots of Complex Number

We know, $z = re^{i(\theta+2K\pi)}$

$z = re^{i\theta}$

$z_0 = r_0 e^{i\theta_0}$

$x^2 = 1$

$x = \pm(1)^{1/2} = \pm 1$

n^{th} root of a complex number: $(\sqrt[n]{z_0})^n = (z)^n$

$z^n = z_0$

$z = z_0^{1/n}$

$$\boxed{\begin{aligned}\sqrt[n]{z_0} &= z_0^{1/n}\\ \sqrt[3]{z_0} &= z_0^{1/3}\end{aligned}}$$

$z_1 = r_1 e^{i\theta_1}$ $\quad r_1 = r_2$

$z_2 = r_2 e^{i\theta_2}$ $\quad \theta_1 = \theta_2 + 2K\pi \; (K = \pm 1, \pm 2 \ldots)$

To find the n^{th} root of a given complex number z_0.

Solution: The n^{th} root of a complex number z_0 be z then, $z^n = z_0$

We know, $z = re^{i\theta}$ and $z_0 = r_0 e^{i\theta_0}$

Then $z^n = (re^{i\theta})^n = r^n e^{i\theta n}$

from ①, we get

$r^n e^{i\theta n} = r_0 e^{i\theta_0}$

which implies $r^n = r_0$ and $n\theta = \theta_0 + 2K\pi \; (K = 0, \pm 1, \pm 2, \ldots)$

$r = r_0^{1/n}$ and $\theta = \frac{\theta_0}{n} + \frac{2K\pi}{n}$

Scanned with CamScanner

Principal argument

Reference Argument $z_0^{1/3}$ = 3rd root of z_0

$z_0^{1/5}$ = 5th root of z_0

$$z = \sqrt[n]{r_0}\, e^{i\left(\boxed{\frac{\theta_0}{n}} + \boxed{\frac{2k\pi}{n}}\right)} \quad (k = 0, \pm 1, \pm 2, \dots \infty)$$

To find the distinct value $k = 0, 1, 2, \dots (n-1)$

$\cancel{k=0 \; e^{i\frac{\theta_0}{n}}}$

$z^n = 1$

$z_0 = 1$

<u>Example-1:</u> To find the n^{th} root of unity.

$$1^{1/n} = 1 e^{i\cdot\frac{(0+2k\pi)}{n}} \quad \begin{pmatrix} k = 0, 1, 2, \dots \\ \dots n-1 \end{pmatrix}$$

$$= e^{\frac{2k\pi}{n} i}$$

$\omega_n^{1/n}$

$|e^{i\theta}| = \sqrt{\cos^2\theta + \sin^2\theta}$

$= 1$

$i = 1 e^{i\pi/2}$

i

ω_0, ω_1, ω_2, ω_3, ω_4, ⋮, ω_{n-1}

$n = 3,\ k = 0, 1, 2$

$\omega_0 = 1,\ \omega_1 = e^{\frac{2\pi}{3} i}$

$= \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$

$= -\frac{1}{2} + i\frac{\sqrt{3}}{2}$

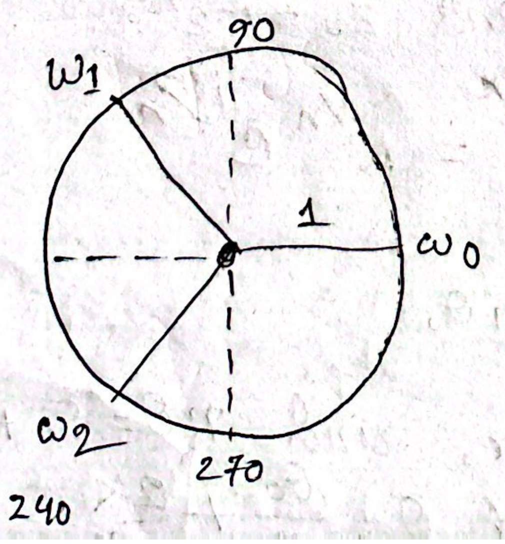
$= \frac{1}{2}\left(-1 + i\frac{\sqrt{3}}{2}\right)$

$\omega_2 = e^{\frac{4\pi}{3} i}$

$= \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}$

$= -\frac{1}{2} - i\frac{\sqrt{3}}{2}$

$= -\frac{1}{2}(-1 - i\sqrt{3})$



$z_0 = -8i$

$(-8i)^{1/3} = 8e^{-i\left(\frac{\pi}{2}+2K\pi\right)}$

$z = 2e^{i\left(-\frac{\pi}{6}+\frac{2K\pi}{3}\right)}$ $(K=0,1,2)$

$c_0 = 2e^{-i\frac{\pi}{6}}$

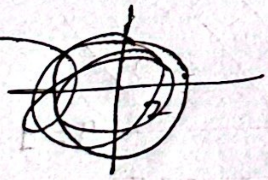
$c_1 = 2e^{i\left(-\frac{\pi}{6}+\frac{2\pi}{3}\right)}$

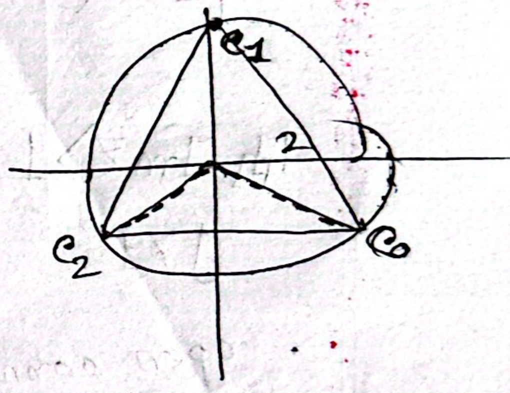
$= 2e^{i\frac{\pi}{2}} = 2\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right) = 2i$

$c_2 = 2e^{i\left(-\frac{\pi}{6}+\frac{4\pi}{3}\right)} = 2e^{i\frac{7\pi}{6}}$

$= 2\left(\cos\frac{7\pi}{6}+i\sin\frac{7\pi}{6}\right) = 2\left(+\frac{\sqrt{3}}{2}-\frac{1}{2}i\right) = (\sqrt{3}-i)$

$= -(\sqrt{3}+i)$

$re^{i\theta}$ 

$re^{i\theta}$ 

***Example-3, Exercise-(1-4)
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Quiz - Complex number থেকে উপর পর্যন্ত (5th May)

$\boxed{\text{Page-31}}$ Domain : Open connected region

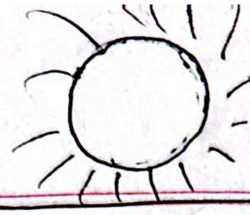
$\boxed{\text{Ex-1}}$ $(0,1)$ $[0,1]$ (close)

0,1 বাদে (open)

(a) $|z-2+i|<1 = |z-(2-i)| \leq 1$

Interior of the circle $|z|<1 = |z-0|<1$

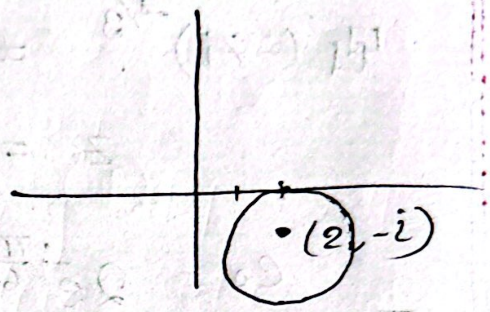
$|z| \geq 1$



$|z| \leq 1$ with boundary

$(2,-i)$

(a). The set consists of points which ~~lines~~ lies interior as well as on a circle centered at $(2,-i)$ radius 1

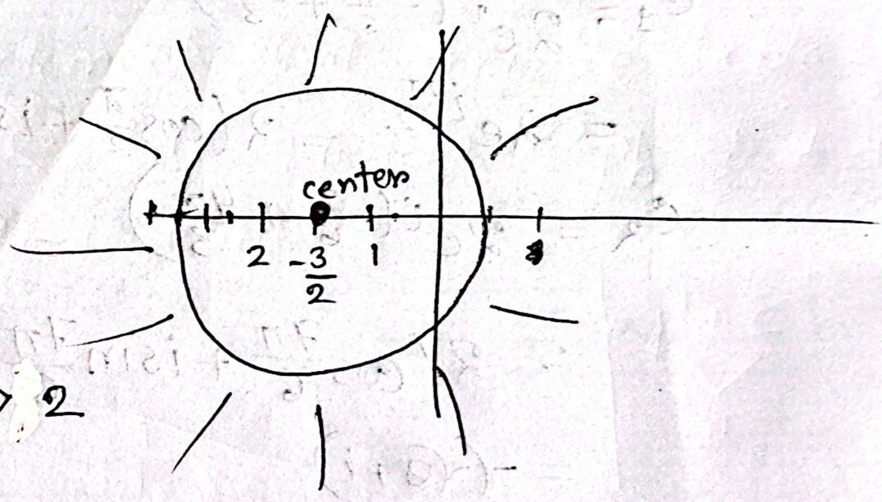


(b) $|2z+3| > 4$

$\Rightarrow 2\left|z+\frac{3}{2}\right| > 4$

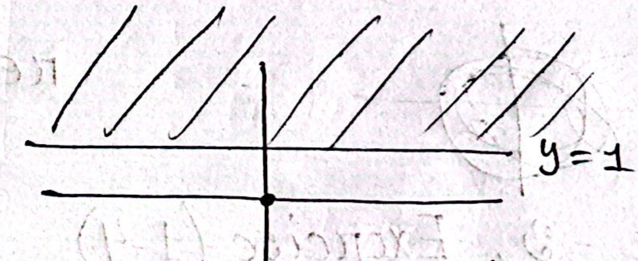
$\Rightarrow \left|z+\frac{3}{2}\right| > 2$

$\Rightarrow \left|z-\left(-\frac{3}{2}\right)\right| > 2$



□ $\operatorname{Im} z > 1$

$y > 1$



• open connected region, so this is domain

□

~~** page~~