Parameter Estimation

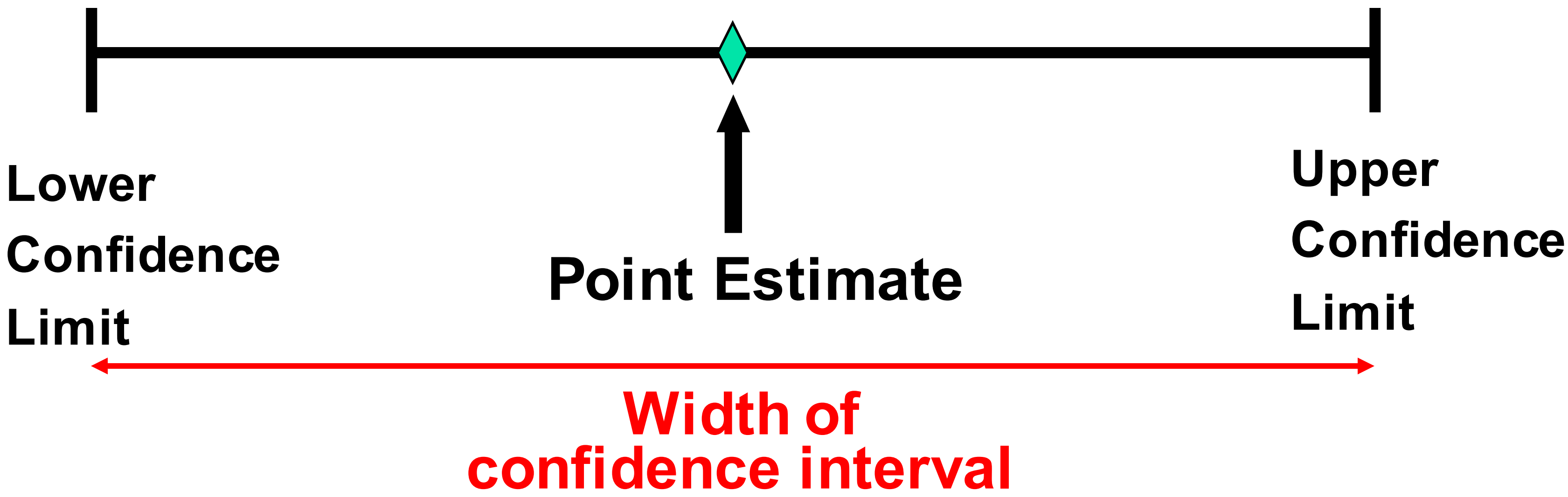
Dr. Md. Israt Rayhan
Professor
Institute of Statistical Research and Training (ISRT)
University of Dhaka
Email: israt@isrt.ac.bd

Definitions

- An estimator of a population parameter is
  - a random variable that depends on sample information . . .
  - whose value provides an approximation to this unknown parameter
- A specific value of that random variable is called an estimate

Point and Interval Estimates

- A point estimate is a single number,
- a confidence interval provides additional information about variability



Point Estimates

We can estimate a Population Parameter …		with a Sample Statistic (a Point Estimate)
Mean	μ	$\bar{x}$

Unbiasedness

- A point estimator $\hat{\theta}$ is said to be an unbiased estimator of the parameter θ if the expected value, or mean, of the sampling distribution of $\hat{\theta}$ is θ,

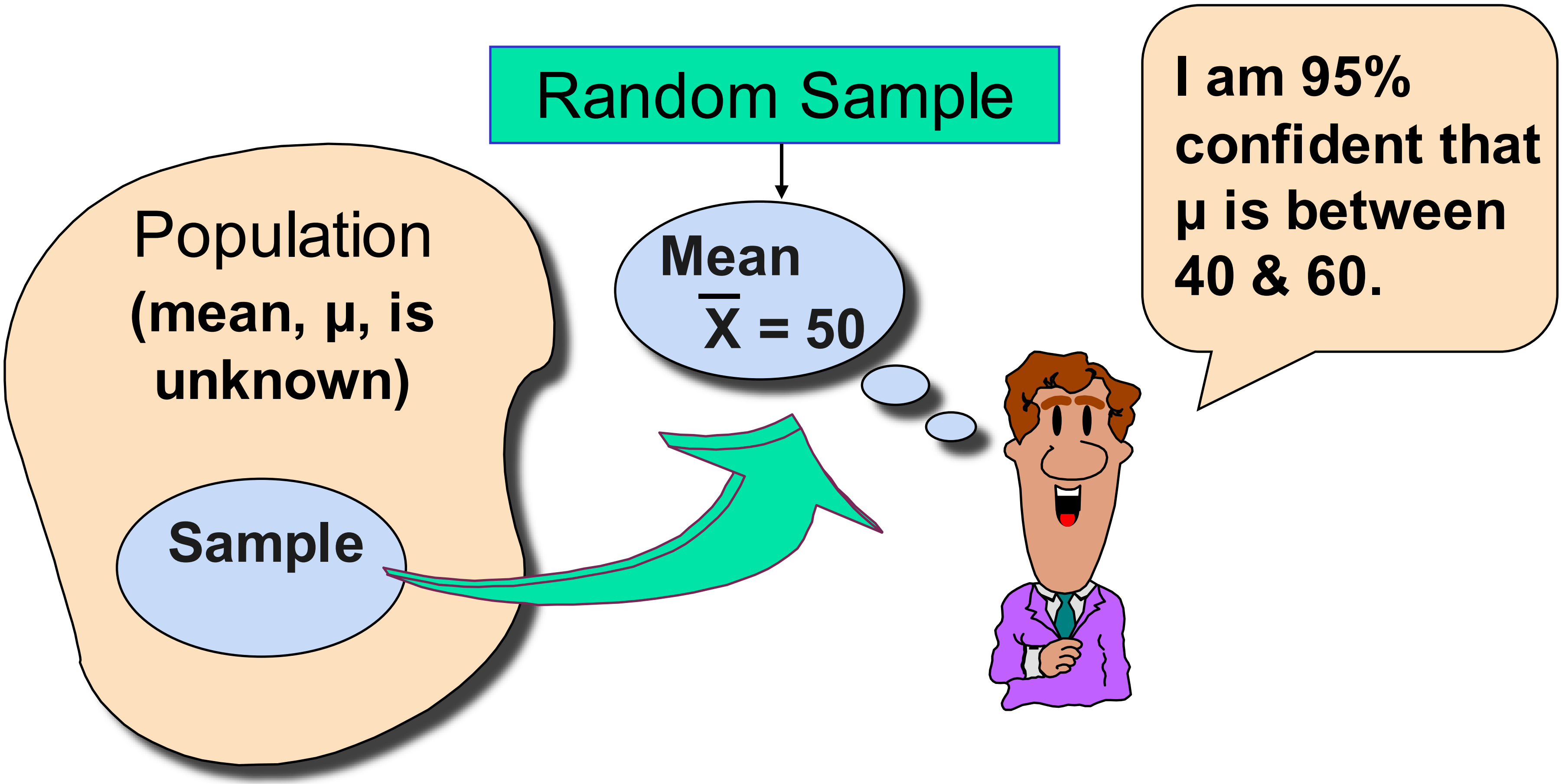
$$\boxed{E(\hat{\theta}) = \theta}$$

- Examples:
  - The sample mean is an unbiased estimator of μ
  - The sample variance is an unbiased estimator of σ^2
  - The sample proportion is an unbiased estimator of P

Confidence Intervals

- How much uncertainty is associated with a point estimate of a population parameter?
- An interval estimate provides more information about a population characteristic than does a point estimate
- Such interval estimates are called confidence intervals

Estimation Process



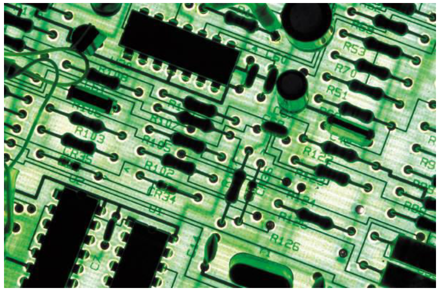
Confidence Level, $(1-\alpha)$

(continued)

- Suppose confidence level = 95%
- Also written $(1 - \alpha) = 0.95$
- A relative frequency interpretation:
  - From repeated samples, 95% of all the confidence intervals that can be constructed will contain the unknown true parameter
- A specific interval either will contain or will not contain the true parameter
  - No probability involved in a specific interval

Example

- A sample of 11 newly born baby, from a large normal population, has a mean height of 2.20 feet. We know from past testing that the population standard deviation is .35 foot.
- Determine a 95% confidence interval for the true mean height of the newly born baby.



Example

(continued)

- A sample of 11 newly born baby, from a large normal population, has a mean height of 2.20 feet. We know from past testing that the population standard deviation is .35 foot.

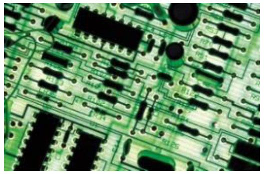
- Solution:

$$\bar{x} \pm z \frac{\sigma}{\sqrt{n}}$$

$$= 2.20 \pm 1.96\,(.35/\sqrt{11})$$

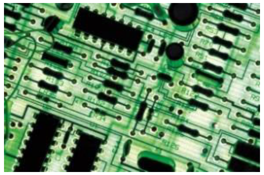
$$= 2.20 \pm .2068$$

$$\boxed{1.9932 < \mu < 2.4068}$$



Interpretation

- We are 95% confident that the true mean is between 1.9932 and 2.4068 feet
- Although the true mean may or may not be in this interval, 95% of intervals formed in this manner will contain the true mean



Student's t Distribution

- Consider a random sample of n observations
  - with mean $\bar{x}$ and standard deviation s
  - from a normally distributed population with mean μ

- Then the variable

$$\boxed{t = \frac{\bar{x} - \mu}{s/\sqrt{n}}}$$

follows the Student's t distribution with (n - 1) degrees of freedom

Student's t Distribution

- The t is a family of distributions
- The t value depends on degrees of freedom (d.f.)
  - Number of observations that are free to vary after sample mean has been calculated

$$\boxed{d.f. = n - 1}$$

Example

A random sample of n = 25 has $\bar{x}$ = 50 and s = 8. Form a 95% confidence interval for μ

- d.f. = n – 1 = 24, so $t_{n-1,\alpha/2} = t_{24,.025} = 2.0639$

The confidence interval is

$$\bar{x} - t_{n\text{-}1,\alpha/2}\frac{S}{\sqrt{n}} < \mu < \bar{x} + t_{n\text{-}1,\alpha/2}\frac{S}{\sqrt{n}}$$

$$50 - (2.0639)\frac{8}{\sqrt{25}} < \mu < 50 + (2.0639)\frac{8}{\sqrt{25}}$$

$$\boxed{46.698 < \mu < 53.302}$$