- Course Title : Differential Equations and Special Functions
- Course Code: MAT102
- Section-3
- Lecture-8 (March 13, 2023) [It is the first class after MID-I exam]

Today's Lecture Topics:

- Ordinary Differential Equations

- First order: Non-exact differential equations
- Linear differential equations (second or higher order) with constant coefficients

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■ **First order differential equations**

➢ **Non-Exact differential equations**

■ **If** $\frac{1}{M}\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right)=\begin{cases} f(y) \\ constant \end{cases}$ **then** $exp(\int f(y)dy)$ or $exp(\int Constant\ dy)$ **is an integrating factor of** $Mdx+Ndy=0$

Example: 1 Solve $\left(2xy^4e^y+2xy^3+y\right)dx$
$+\left(x^2y^4e^y-x^2y^2-3x\right)dy=0$

Solution: Given equation $\left(2xy^4e^y+2xy^3+y\right)dx$
$+\left(x^2y^4e^y-x^2y^2-3x\right)dy=0 ------(1)$

Comparing (1) with $Mdx+Ndy=0$**, we have**

M=$\left(2xy^4e^y+2xy^3+y\right)$**and** $\mathrm{N}=\left(x^2y^4e^y-x^2y^2-3x\right)$

■ **First order differential equations**

➢ **Non-Exact differential equations**

Example: 1 Solve $\left(2xy^4e^y + 2xy^3 + y\right)dx$
$$+\left(x^2y^4e^y - x^2y^2 - 3x\right)dy = 0$$

Solution:

$$M = \left(2xy^4e^y + 2xy^3 + y\right) \text{ and } N = \left(x^2y^4e^y - x^2y^2 - 3x\right)$$

$$\therefore \frac{\partial M}{\partial y} = 8xy^3\ e^y + 2xy^4e^y + 6xy^2 + 1 \ \ and\ \ \frac{\partial N}{\partial x} = 2x\ y^4e^y\ - 2xy^2 - 3$$

$$\therefore \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = 2x\ y^4e^y\ - 2xy^2 - 3 - 8xy^3\ e^y - 2xy^4e^y - 6xy^2 - 1$$
$$= -8xy^3\ e^y - 8xy^2 - 4 = -4(2xy^3e^y + 2\ xy^2 + 1)$$

■ First order differential equations

➢ Non-Exact differential equations

Example: 1 Solve $\left(2xy^4e^y + 2xy^3 + y\right)dx + \left(x^2y^4e^y - x^2y^2 - 3x\right)dy = 0$

Solution:

$$\therefore \ \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = -4(2xy^3e^y + 2\ xy^2 + 1) = -\frac{4}{\mathrm{y}}((2xy^4e^y + 2\ xy^3 + \mathrm{y})$$

$$= -\frac{4\mathrm{M}}{\mathrm{y}} \ \Rightarrow \ \frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) = -\frac{4}{\mathrm{y}};$$ **Therefore,** $I.F. = e^{\int -\frac{4}{\mathrm{y}}dy} = e^{-4logy} = \frac{1}{y^4}$

Multiplying the given equation by $I.F$**, we get**

$$\left(2xe^y + \frac{2x}{y} + \frac{1}{y^3}\right)dx + \left(x^2e^y - \frac{x^2}{y^2} - \frac{3x}{y^4}\right)\mathrm{dy} = 0$$ **which must be an exact.**

■ **First order differential equations**

➢ **Non-Exact differential equations**

Example: 1 Solve $\left(2xy^4e^y + 2xy^3 + y\right)dx + \left(x^2y^4e^y - x^2y^2 - 3x\right)dy = 0$

Solution:

Multiplying the given equation by $I.F$, we get

$\left(2xe^y + \frac{2x}{y} + \frac{1}{y^3}\right)dx + \left(x^2e^y - \frac{x^2}{y^2} - \frac{3x}{y^4}\right)\text{dy} = 0$ **which is exact.**

$\therefore \int M_1 dx = \int \left(2xe^y + \frac{2x}{y} + \frac{1}{y^3}\right)dx = x^2\, e^y + \frac{x^2}{y} + \frac{x}{y^3}$ **(treating 'y' as constant) and** $\int N_1 dy = \int \left(x^2e^y - \frac{x^2}{y^2} - \frac{3x}{y^4}\right)dy = x^2\, e^y + \frac{x^2}{y} + \frac{x}{y^3}$ **(treating 'x' as a constant)**$= 0$ $[\, omitting\ x^2\, e^y + \frac{x^2}{y} + \frac{x}{y^3}$ **as it is already obtained in** $\int M_1 dx\,]$ $\therefore$ **The solution is** $x^2\, e^y + \frac{x^2}{y} + \frac{x}{y^3} = C\ (\text{Ans.})$

First order differential equations

- **Exercise: 9** [Solve the non-exact differential equations]

1. $(xy^2 - x^2)\,dx + (3x^2y^2 + x^2y - 2x^3 + y^2)dy = 0$

   [Ans. $e^{6y}\left(\frac{1}{2}x^2y^2 - \frac{1}{3}x^3 + \frac{1}{6}y^2 - \frac{1}{18}y + \frac{1}{108}\right) = c$]

2. $(xy^3 + y)\,dx + 2\,(x^2y^2 + x + y^4)\,dy = 0.$

   [Ans. $3x^2y^4 + 6xy^2 + 2y = c$]

3. $(y^4 + 2y)\,dx + (xy^3 + 2y^4 - 4x)\,dy = 0$

   [Ans. $x\,\{y + (2/y^2)\} + y^2 = c$]

■ **Linear differential equations (second or higher order) with constant coefficients**

➢ **Definition: A linear differential equation of order $'n'$ with constant coefficient can be expressed in the following form**

$$a_0 \frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_{n-1} \frac{dy}{dx} + a_n y = F(x) \text{ -------- } (1)$$

where $F(x)$ is a nonhomogeneous term and a_0, a_1, …, a_{n-1}, a_n are real constants.

If $F(x) = 0$, then Eq. (1) is called a homogeneous differential equation.

If $F(x) \neq 0$, then Eq. (1) is called a nonhomogeneous differential equation.

■ Linear differential equations (second or higher order) with constant coefficients

➢ Homogeneous equations

$$a_0\frac{d^ny}{dx^n}+a_1\frac{d^{n-1}y}{dx^{n-1}}+\cdots+a_{n-1}\frac{dy}{dx}+a_ny=0\;-------\;-(2)$$

Let $y = e^{mx}$ be a trial solution of the homogeneous equation (2). Then we can write

$$y'=me^{mx},y''=m^2e^{mx},y'''=m^3e^{mx},y^{(n)}=m^ne^{mx}.$$

Substituting these in (2), we get

$$a_0m^ne^{mx}+a_1m^{n-1}e^{mx}+\cdots+a_{n-1}me^{mx}+a_ne^{mx}=0$$

- **Linear differential equations (second or higher order) with constant coefficients**

- **Homogeneous equations**

$$\Rightarrow e^{mx}\left(a_0 m^n + a_1 m^{n-1} + \cdots + a_{n-1} m + a_n\right) = 0$$

Since $e^{mx} \neq 0$, we obtain following polynomial equation of m unknown:

$$a_0 m^n + a_1 m^{n-1} + \cdots + a_{n-1} m + a_n = 0 \quad -------(3)$$

This equation is called the auxiliary equation (A.E.) or the characteristic equation of the given differential equation (2).

■ **Linear differential equations (second or higher order) with constant coefficients**

➢ **Homogeneous equations**

Auxiliary equation: $a_0 m^n + a_1 m^{n-1} + \cdots + a_{n-1} m + a_n = 0 \quad ---(3)$

Case 1: Real and distinct roots

Suppose the roots of (3) are the n distinct real numbers $m_1, m_2, \ldots, m_n$. Then $e^{m_1 x}, e^{m_2 x}, e^{m_3 x}, \ldots, e^{m_n x}$ are n distinct solutions of (2) which are linearly independent.

Therefore, the general solution of Eq.(2) is
$y = c_1 e^{m_1 x} + c_2 e^{m_2 x} + c_3 e^{m_3 x} + \cdots + c_n e^{m_n x}$ **where $c_1, c_2, \ldots, c_n$ are arbitrary constants.**

- **Linear differential equations (second or higher order) with constant coefficients**

➢ **Homogeneous equations**

➢ **Example 1: Solve** $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0 \text{ or } (D^2 - 3D + 2)y = 0 \ \ where$

D **stands for** d/dx **and** D^2 **stands for** d^2/dx^2

Solution: Given differential equation is

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0 \text{ or } (D^2 - 3D + 2)y = 0 \quad ---------(1)$$

Let $y = e^{mx}$ **be a trial solution of the equation (1). Then auxiliary equation of the equation (1) is**

$$m^2 - 3m + 2 = 0 \Rightarrow m^2 - 2m - m + 2 = 0 \Rightarrow (m-2)(m-1) = 0$$

$$\therefore m = 2, 1 \text{ (Real and distinct roots)}$$

Therefore, the general solution of (1) is $y = c_1e^x + c_2e^{2x}$ $(Ans.)$

➢ ☞ Instructions:

The following slides are given to all of you in advance for your convenience, and please check and try to understand the given slides (lecture martials) before joining the next class.

If you do so, then it would be easier for you to understand the next class. Besides, you will feel comfortable in asking relevant questions in the classroom, if any!

- **Linear differential equations (second or higher order) with constant coefficients**

- **Homogeneous equations**
- **Example 2: Solve** $\frac{d^2y}{dx^2}+5\frac{dy}{dx}+6y=0\ or\ (D^2+5D+6)y=0\ \ where$

D **stands for** d/dx **and** D^2 **stands for** d^2/dx^2

Solution: Given differential equation is

$$\frac{d^2y}{dx^2}+5\frac{dy}{dx}+6y=0\ or\ (D^2+5D+6)y=0\ \ --------(1)$$

Let $y=e^{mx}$ **be a trial solution of the equation (1). Then the auxiliary equation of the equation (1) is**

$$m^2+5m+6=0 \Rightarrow m^2+3m+2m+6=0 \Rightarrow (m+3)(m+2)=0$$

$$\therefore m=-2,-3\ (\text{Real and distinct roots})$$

Therefore, the general solution of (1) is $y=c_1e^{-2x}+c_2e^{-3x}\ (Ans.)$

■ **Linear differential equations (second or higher order) with constant coefficients**

➢ **Homogeneous equations**

$$a_0 \frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_{n-1} \frac{dy}{dx} + a_n y = 0 \text{ ------------------ } (2)$$

$$\Downarrow$$

Auxiliary equation: $a_0 m^n + a_1 m^{n-1} + \cdots + a_{n-1} m + a_n = 0$ ---- (3)

<u>Case 2</u>: Repeated real roots

Suppose the roots of (3) are the n repeated real numbers $m, m, \ldots, m$

Then the general solution of (2) is

$$y = \left(c_1 + c_2 x + c_3 x^2 + \cdots + c_n x^{n-1}\right) e^{mx}$$

where $c_1, c_2, \ldots, c_n$ are arbitrary constants.

■ **Linear differential equations (second or higher order) with constant coefficients**

➢ **Homogeneous equations**

$$a_0\frac{d^ny}{dx^n}+a_1\frac{d^{n-1}y}{dx^{n-1}}+\cdots+a_{n-1}\frac{dy}{dx}+a_ny=0 -------------(2)$$

$$\Downarrow$$

Auxiliary equation: $a_0m^n+a_1m^{n-1}+\cdots+a_{n-1}m+a_n=0 \quad ----(3)$

<u>Case 2 (i)</u>: Certain number of repeated real roots and the rest of the roots are distinct

Now if there are the k repeated real roots *m*, *m*, … , *m* and the rest of the roots are distinct then the general solution of eq. (2) is

$$y=\left(c_1+c_2x+c_3x^2+\cdots+c_kx^{k-1}\right)e^{mx}+c_{k+1}e^{m_{k+1}x}+c_{k+2}e^{m_{k+2}x}+\cdots+c_ne^{m_nx}$$

■ **Linear differential equations (second or higher order) with constant coefficients**

➢ **Homogeneous equations**

➢ **Example 1: Solve** $\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 18y = 0$

or $\left(D^3 - 4D^2 - 3D + 18\right)y = 0$

Solution: Given differential equation is

$$\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 18y = 0 \text{ --------------- } -(1)$$

Let $y = e^{mx}$ **be a trial solution of the equation (1). Then, the auxiliary equation of (1) is** $m^3 - 4m^2 - 3m + 18 = 0$

$\Rightarrow m^3 - 3m^2 - m^2 + 3m - 6m + 18 = 0$

$\Rightarrow m^2(m-3) - m(m-3) - 6(m-3) = 0 \Rightarrow (m-3)\left(m^2 - m - 6\right) = 0$

$\Rightarrow (m-3)(m-3)(m+2) = 0 \therefore m = 3, 3, -2$ **(Repeated real roots)**

Therefore, the general solution of (1) **is** $y = (c_1 + c_2x)e^{3x} + c_3e^{-2x}$

■ **Linear differential equations (second or higher order) with constant coefficients**

➢ **Homogeneous equations**

➢ **Example 2: Solve** $\frac{d^3y}{dx^3} - 7\frac{d^2y}{dx^2} + 16\frac{dy}{dx} - 12y = 0$

or $\left(D^3 - 7D^2 + 16D - 12\right)y = 0$

Solution: Given differential equation is

$$\frac{d^3y}{dx^3} - 7\frac{d^2y}{dx^2} + 16\frac{dy}{dx} - 12y = 0 ----------------(1)$$

Let $y = e^{mx}$ be a trial solution of the equation (1). Then, the auxiliary equation of (1) is $m^3 - 7m^2 + 16m - 12 = 0$

$\Rightarrow m^3 - 2m^2 - 5m^2 + 10m + 6m - 12 = 0$

$\Rightarrow m^2(m-2) - 5m(m-2) + 6(m-2) = 0 \Rightarrow (m-2)\left(m^2 - 5m + 6\right) = 0$

$\Rightarrow (m-2)(m-2)(m-3) = 0 \therefore m = 2, 2, 3$ **(Repeated real roots)**

Therefore, the general solution of (1) is $y = (c_1 + c_2x)e^{2x} + c_3e^{3x}$

- **Linear differential equations (second or higher order) with constant coefficients**

- **Homogeneous equations**

$$a_0 \frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_{n-1} \frac{dy}{dx} + a_n y = 0 ------------- -(2)$$

Auxiliary equation: $a_0 m^n + a_1 m^{n-1} + \cdots + a_{n-1} m + a_n = 0 \ --- -(3)$

Case 3: Conjugate complex roots

Suppose the auxiliary equation has two complex conjugate roots $a + ib$ and $a - ib$ which are non-repeated.

So, the general solution can be written as

$$y = k_1 e^{(a+ib)x} + k_2 e^{(a-ib)x} = k_1 e^{ax+ibx} + k_2 e^{ax-ibx}$$

$$= k_1 e^{ax}(\cos b\,x + i \sin b\,x) + k_2 e^{ax}(\cos b\,x - i \sin b\,x)$$

$$= e^{ax}[(k_1 + k_2)\cos b\,x + i(k_1 - k_2)\sin b\,x] \therefore y = e^{ax}(c_1 \cos b\,x + c_2 \sin b\,x)$$

where $c_1 = k_1 + k_2$ *and* $c_2 = i(k_1 - k_2)$ **are new arbitrary constants.**

- **Linear differential equations (second or higher order) with constant coefficients**

- **Homogeneous equations**
- **Example 1: Solve** $\frac{d^2y}{dx^2}+4y=0\ or\ (D^2+4)y=0$

Solution: Given differential equation is

$$\frac{d^2y}{dx^2}+4y=0\ or\ (D^2+4)y=0 \text{------------------}(1)$$

Let $y=e^{mx}$ **be a trial solution of the equation** (1)**. Then the auxiliary equation of the equation** (1) **is**

$$m^2+4=0 \Rightarrow m^2=-4 \Rightarrow m^2=4i^2$$

$$\therefore m=\pm 2i=0\pm 2i\ \ (i.e.,0+2i,0-2i)$$

Therefore, the general solution of (1) is

$$y=e^{0x}(c_1\cos 2x+c_2\sin 2x)=c_1\cos 2x+c_2\sin 2x\ (Ans.)$$

■ **Linear differential equations (second or higher order) with constant coefficients**

➢ **Homogeneous equations**

$$a_0 \frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_{n-1} \frac{dy}{dx} + a_n y = 0 \text{ --------------- } (2)$$

Auxiliary equation of (2): $a_0 m^n + a_1 m^{n-1} + \cdots + a_{n-1} m + a_n = 0 \text{ -- } (3)$

Case 4: Conjugate complex roots (repeated)

(i) Suppose the roots of auxiliary equation (3) are $a \pm ib$ (occur twice).

So, the general solution of (2) can be written as

$y = e^{ax}[(c_1 + c_2 x) \cos b\, x + (c_3 + c_4 x) \sin bx]$ **containing four arbitrary constants.**

■ **Linear differential equations (second or higher order) with constant coefficients**

➢ **Homogeneous equations**

$$a_0 \frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_{n-1} \frac{dy}{dx} + a_n y = 0 \text{ ---------------- } (2)$$

Auxiliary equation of (2): $a_0 m^n + a_1 m^{n-1} + \cdots + a_{n-1} m + a_n = 0 \;\; --(3)$

Case 4: Conjugate complex roots (repeated)

(ii) Suppose the roots of auxiliary equation (3) are $a \pm ib$ (occur thrice).

So, the general solution of (2) can be written as

$y = e^{ax}\left[\left(c_1 + c_2 x + c_3 x^2\right) \cos b\, x + \left(c_4 + c_5 x + c_6 x^2\right) \sin bx\right]$ **containing six arbitrary constants.**

■ **Linear differential equations (second or higher order) with constant coefficients**

➢ **Homogeneous equations**

$$a_0 \frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_{n-1} \frac{dy}{dx} + a_n y = 0 -------------(2)$$

Auxiliary equation of (2): $a_0 m^n + a_1 m^{n-1} + \cdots + a_{n-1} m + a_n = 0 \quad --(3)$

Case 4: Conjugate complex roots (repeated)

(iii) Suppose the roots of auxiliary equation (3) are $a \pm ib$ (occur k times)

So, the general solution of (2) can be written as

$y = e^{ax}\left[\left(c_1 + c_2 x + c_3 x^2 + \cdots + c_k x^{k-1}\right) \cos b\, x + \left(c_{k+1} + c_{k+2} x + c_{k+3} x^2 + \cdots + c_{2k} x^{k-1}\right) \sin bx\right]$ **where** $c_1, c_2, \ldots \ldots, c_{2k}$ **are arbitrary constants.**

- **Linear differential equations (second or higher order) with constant coefficients**

- **Homogeneous equations**
- **Example 1: Solve $[(D^2+1)^3(D^2+D+1)^2]y=0$**

Solution: Given differential equation is

$$[(D^2+1)^3(D^2+D+1)^2]y=0 \text{ ------------- } (1)$$

Let $y=e^{mx}$ be a trial solution of the equation (1). Then the auxiliary equation of the equation (1) is

$$(m^2+1)^3(m^2+m+1)^2=0$$

$$\Rightarrow m = 0 \pm i \text{ (thrice) and}$$

$$-\frac{1}{2} \pm i\frac{\sqrt{3}}{2} \text{ (twice)}$$

- **Linear differential equations (second or higher order) with constant coefficients**

- **Homogeneous equations**

- **Example 1: Solve $[(D^2+1)^3(D^2+D+1)^2]y=0$**

Solution: Given differential equation is

$$(D^2+1)^3(D^2+D+1)^2y=0 \text{ ------------- } (1)$$

The roots of auxiliary equation of the equation (1) are

$$m=0\pm i \text{ (thrice) and } -\frac{1}{2}\pm i\frac{\sqrt{3}}{2} \text{ (twice)}$$

Therefore, the general solution of (1) is

$$y = e^{0x}\left[\left(c_1+c_2x+c_3x^2\right)cosx+\left(c_4+c_5x+c_6x^2\right)sinx\right] + e^{-\frac{x}{2}}\left[(c_7+c_8x)cos\frac{\sqrt{3}}{2}x+(c_9+c_{10}x)sin\frac{\sqrt{3}}{2}x\right](Ans.)$$

■ **Linear differential equations (second or higher order) with constant coefficients**

➢ **Homogeneous equations**

➢ **Example 2: Solve** $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 25y = 0, y(0) = -3, y'(0) = -1$.

Solution: Given differential equation is

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 25y = 0 \quad ---------------------(1)$$

Let $y = e^{mx}$ **be the trail solution of** (1)**. Then the auxiliary equation of (1) is** $m^2 - 6m + 25 = 0$

$$\Rightarrow m^2 - 2.m.3 + 9 + 16 = 0 \Rightarrow (m-3)^2 = -16$$

$$\Rightarrow (m-3)^2 = 16i^2$$

$$\Rightarrow (m-3)^2 = (4i)^2$$

$$\Rightarrow m - 3 = \pm 4i$$

$$\therefore m = 3 \pm 4i$$

■ **Linear differential equations (second or higher order) with constant coefficients**

➢ **Homogeneous equations**

➢ **Example 2: Solve** $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 25y = 0, y(0) = -3, y'(0) = -1.$

Solution: Given differential equation is

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 25y = 0 \text{ ------------------------ } -(1)$$

The roots of auxiliary equation of (1) is $m = 3 \pm 4i$. **Therefore the general solution of (1) is**

$$y = e^{3x}(c_1 \cos 4x + c_2 \sin 4x) \text{ ------------------------ } -(2)$$

$$\therefore y'$$
$$= 3e^{3x}(c_1 \cos 4x + c_2 \sin 4x) + e^{3x}(-4c_1 \sin 4x + 4c_2 \cos 4x) - -(3)$$

When $x = 0, y = -3$, **we can write from (2) is as follows**

$$-3 = e^0(c_1 \cos 0 + c_2 \sin 0) \therefore c_1 = -3$$

■ **Linear differential equations (second or higher order) with constant coefficients**

➢ **Homogeneous equations**

➢ **Example 2: Solve** $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 25y = 0, y(0) = -3, y'(0) = -1.$

Solution:

$$\therefore y' = 3e^{3x}(c_1 \cos 4x + c_2 \sin 4x) + e^{3x}(-4c_1 \sin 4x + 4c_2 \cos 4x) - -(3)$$

Again, $x = 0, y' = -1$, **we can write from (3) is as follows**

$$-1 = 3e^0(c_1 \cos 0 + c_2 \sin 0) + e^0(-4c_1 \sin 0 + 4c_2 \cos 0)$$

$$\Rightarrow -1 = 3c_1 + 4c_2 \Rightarrow -1 = -9 + 4c_2(\text{ as } c_1 = -3) \Rightarrow 4c_2 = 8 \therefore c_2 = 2$$

Therefore, the general solution of the given differential equation is

$$y = e^{3x}(-3 \cos 4x + 2 \sin 4x) \text{ (Ans.)}$$

■ Linear differential equations (second or higher order) with constant coefficients

HOMOGENEOUS LINEAR EQUATIONS WITH CONSTANT COEFFICIENTS → **Even numbered problems**

Solve the following differential equations :

1. (a) $(D^3 + 6D^2 + 11D + 6)y = 0$ **Ans.** $y = c_1 e^{-x} + c_2 e^{-2x} + c_3 e^{-3x}$

(b) $d^2y/dx^2 + 2(dy/dx) + 5y = 0$ **Ans.** $y = e^{-x}(c_1 \cos 4x + c_2 \sin 4x)$

(c) $d^3y/dx^3 - 6(d^2y/dx^2) + 9(dy/dx) = 0$ **Ans.** $y = c_1 + (c_2 + xc_3)e^{3x}$

2. $(D^3 + 6D^2 + 12D + 8)y = 0.$ **Ans.** $y = (c_1 + c_2x + c_3x^2)e^{-2x}$

3. $(d^2y/dx^2) + 2p(dy/dx) + (p^2 + q^2)y = 0.$ **Ans.** $y = e^{-px}(c_1 \cos qx + c_2 \sin qx)$

4. $(D^4 - 2D^3 + 5D^2 - 8D + 4)y = 0.$ **Ans.** $y = (c_1 + c_2x)e^x + c_3 \cos 2x + c_4 \sin 2x$

5. $(D^4 + D^2 + 1)y = 0.$

Ans. $y = e^{x/2}[c_1 \cos(x\sqrt{3}/2) + c_2 \sin(x\sqrt{3}/2)] + e^{-x/2}[c_3 \cos(x\sqrt{3}/2) + c_4 \sin(x\sqrt{3}/2)]$

6. $(D^4 + 4D^3 - 5D^2 - 36D - 36)y = 0.$ **Ans.** $y = c_1e^{-3x} + c_2e^{3x} + (c_3 + c_4x)e^{-2x}$

7. $(D^4 - 7D^3 + 18D^2 - 20D + 8)y = 0.$ **Ans.** $y = c_1e^x + (c_2 + c_3x + c_4x^2)e^{2x}$

8. $(D^2 \pm w^2)y = 0,\ w \neq 0.$ **Ans.** $y = c_1 \cos wx + c_2 \sin wx + c_3 e^{wx} + c_4 e^{-wx}$

9. $\{D^3 + D^2(2\sqrt{3} - 1) + D(3 - 2\sqrt{3}) - 3\}y = 0$ **Ans.** $y = c_1 e^x + (c_2 + c_3x)e^{-x\sqrt{3}}$

10. (a) $(D^5 - 13D^3 + 26D^2 + 82D + 104)y = 0$

Ans. (a) $y = c_1 e^{-4x} + e^{-x}(c_2 \cos x + c_3 \sin x) + e^{3x}(c_4 \cos 2x + c_5 \sin 2x)$

(b) $(D^6 + 9D^4 + 24D^2 + 16)y = 0$ (b) $y = c_1 \cos x + c_2 \sin x + (c_3 + c_4x)\cos 2x + (c_5 + c_6x)\sin 2x$

Thank you for your attendance and attention