PHY109 Engineering Physics I
Chapter 5 (Electricity)
Part-1 (Electricity-I)

5.1 Electric Charge and the Coulomb's Law

The Coulomb's Law

We know there are two types of electric charges: positive charge and negative charge. We also know that like charges repel and unlike charges attract each other.

The Coulomb's law states that if there two charges q_1 and q_2 separated by a distance r, then the force between these two charges will be given by the following expression

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}. \tag{5.1}$$

The above equation is written in the SI units. The constant quantity ε_0 is known as the permittivity constant and it is given by

$$\varepsilon_0 = 8.85418 \times 10^{-12} \text{ coul}^2/\text{nt} - \text{m}^2.$$

The unit of charge is the *coulomb* (abbr. *coul*) and is defined as "*A coulomb is defined as the amount of charge that flows through a given cross-section of a wire in 1 second if there is a steady current of 1 ampere in the wire*".

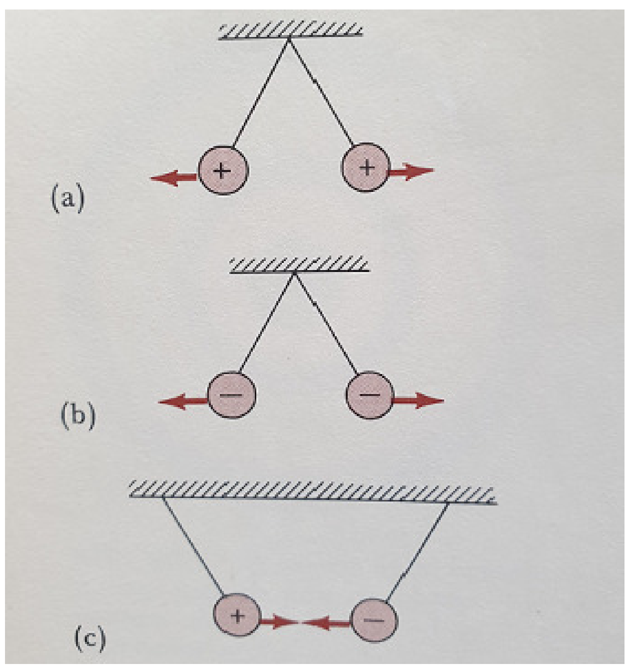

Fig. 5.1 Direction of Coulomb force between two charges. (a) Two positive charges; (b) two negative charges, (c) two unlike charges.

The Coulomb force is a vector quantity and its direction can be found from the following:

Case 1: Let us consider that the two charges q_1 and q_2 are like charges case (a) and case (b) in Fig. 5.1. In this situation, if we sit at the charge q_1, then the direction of **F** will be away from the charge q_2 and will act along the line joining the two charges. Similarly, if we sit at the charge q_2 and looking at the direction of the force **F** then the direction of the force will be away from the charge q_1 and will act along the line joining the two charges.

Case 2: Let us consider that the two charges q_1 and q_2 are unlike charges (case (c) in Fig. 5.1), that is one charge positive and the other charge is negative. In this situation, if we sit at the charge q_1, then the direction of **F** will be toward the charge q_2 and will act along the line joining the two charges. Similarly, if we sit at the charge q_2 and looking at the direction of the force **F** then the direction of the force will be toward the charge q_1 and will act along the line joining the two charges.

Charge is quantized

Experiments show that the electric charge is not continuous but that it is made up of integral multiples of a certain minimum electric charge. This fundamental charge, to which we give the symbol *e*, has the magnitude $e = 1.60210 \times 10^{-19}$ coul.

When a physical property such as charge exists in discrete "packets" rather than in continuous amounts, the property is said to be *quantized*. Quantization is basic to modern physics. The existence of atoms and of particles like the electron and the proton indicates that *mass* is also quantized.

The Electric Field

To define the electric field operationally, we place a small test body carrying a test charge q_0 (assumed positive for convenience) at the point in space that is to be examined, and we measure the electrical force **F** (if any) that acts on this body. The electric field strength **E** at the point is defined as

$$\mathbf{E} = \frac{\mathbf{F}}{q_0}. \tag{5.2}$$

Here **E** is a vector and its direction is the direction of the force **F**, that is, it is the direction in which a resting positive charge placed at the point would tend to move. The unit of the electric field is N/Coul.

Lines of Force

The concept of the electric field as a vector was not appreciated by Michael Faraday, who always thought in terms of *lines of force*. The lines of force still form a convenient way of visualizing electric-field patterns. The relationship between the (imaginary) lines of force and the electric field strength vector is this:

1. The tangent to a line of force at any point gives the direction of **E** at that point.
2. The lines of force are drawn so that the number of lines per unit cross-sectional area is proportional to the magnitude of **E**. Where the lines are close together **E** is large and where they are far apart **E** is small.

Figure 5.2 shows lines of force for different charge configurations.

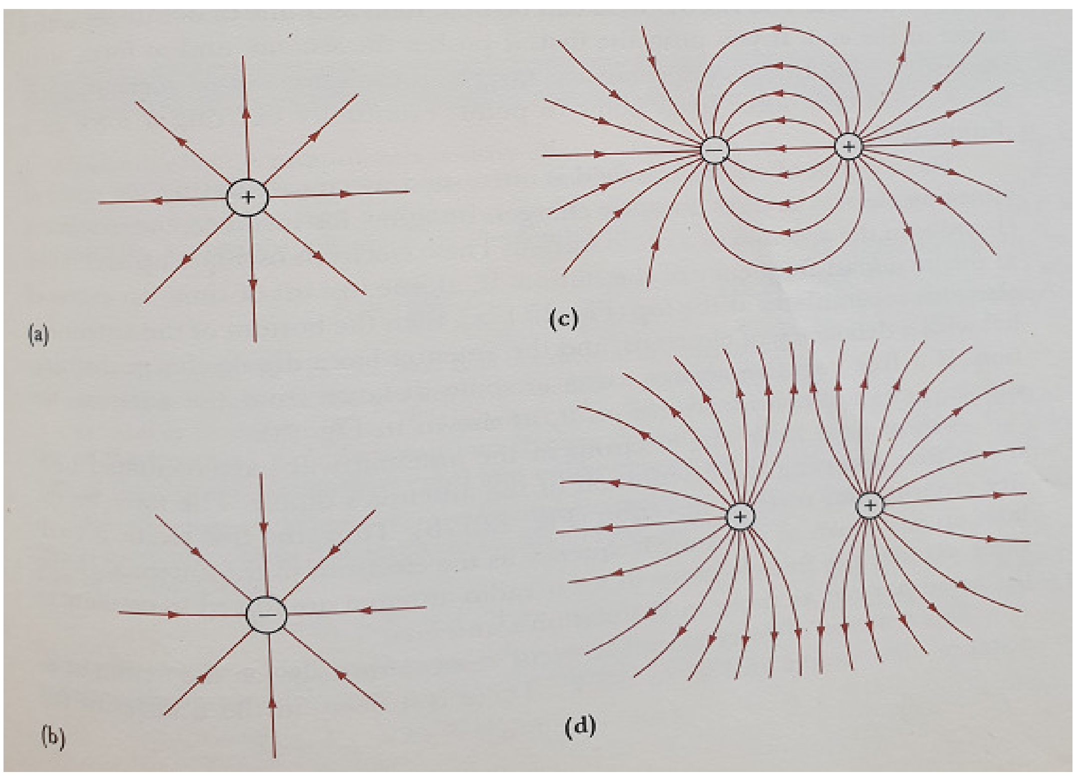

Fig. 5.2 Lines of force showing different situations. (a) Lines of force for a positive point charge, (b) Lines of force for a negative point charge, (c) Lines of force for two nearby opposite charges, and (d) Lines of force between two nearby positive charges.

Calculation of E

Let a test charge q_0 be placed a distance r from a point charge q. The magnitude of the force acting on q_0 is given by the Coulomb's law

$$F = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}.$$

The electric field strength at the site of the test charge is given by

$$E = \frac{F}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}. \tag{5.3}$$

The direction of **E** is on a radial line from q, pointing outward if q is positive and inward if q is negative.

To find **E** for a group of point charges: (a) Calculate $\mathbf{E}_n$ due to each charge at the given point *as if it were the only charge present*. (b) Add these separately calculated fields vectorially to find the resultant field E at the point. In equation form:

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 + \cdots\cdots = \sum \mathbf{E}_n, \quad n = 1, 2, 3, \cdots \tag{5.4}$$

The sum is a vector sum, taken over all the charges.

A Dipole in an Electric Field

Two equal and opposite charges separated by a small distance constitute an electric dipole. If the charges are $+q$ and $-q$ and the distance between them is $2a$, then the electric dipole moment is defined by

$$p = 2aq. \tag{5.5}$$

Electric dipole moment is a vector quantity **p** and its direction is considered to be along the line joining the two charges from negative to the positive charge.

Figure 5.3 shows an electric dipole. The arrangement is placed in a uniform external electric field **E**, its dipole moment **p** making an angle θ with the field. Two equal and opposite forces **F** and –**F** act as shown, where

$$F = qE.$$

The net force is clearly zero, but there is a net torque about an axis through O given by

$$\tau = F(2a\sin\theta) = 2aF\sin\theta.$$

Combining the above two equations, we get

$$\tau = 2aqE\sin\theta = pE\sin\theta. \tag{5.6}$$

Thus an electric dipole placed in an external electric field **E** experiences a torque tending to align it with the field. Equation (5.6) can be written in vector form as

$$\boldsymbol{\tau} = \mathbf{p}\times\mathbf{E}. \tag{5.7}$$

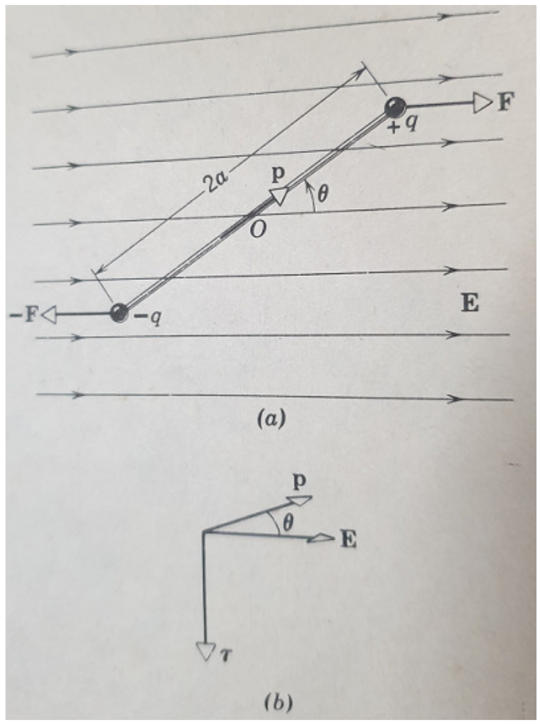

Fig. 5.3: (a) An electric dipole in a uniform external electric field. (b) An oblique view, showing the directions of the vectors τ , **p**, and **E**.

Work (positive or negative) must be done by an external agent to change the orientation of an electric dipole in an external field. This work is stored as potential energy U in the system consisting of the dipole and the arrangement used to set up the external field. If θ in Fig. 5.3a has the initial value θ_0 , the work required to turn the dipole axis to an angle θ is given from

$$W = \int dW = \int_{\theta_0}^{\theta} \tau \, d\theta = U,$$

where τ is the torque exerted by the agent that does the work. Combining the above equation with Eq. (5.6) yields

$$U = \int_{\theta_0}^{\theta} pE \sin\theta \, d\theta = pE \int_{\theta_0}^{\theta} \sin\theta \, d\theta = -pE[\cos\theta - \cos\theta_0].$$

Since we are interested only in changes in potential energy, we choose the reference orientation θ_0 to have any convenient value, in this case we take $\theta_0 = 90^o$. Thus, we have

$$U = -pE\cos\theta \tag{5.8}$$

or, in vector form

$$U = -\mathbf{p}\cdot\mathbf{E}. \tag{5.9}$$

5.2 Electric Flux and The Gauss's Law

Electric Flux

Flux is a property of any vector field; it refers to a hypothetical surface in the field, which may be closed or open. For a flow field the flux (Φ_v) is measured by the number of streamlines that cut through such a surface. For an electric field the flux (Φ_E) is measured by the number of lines of force that cut through such a surface.

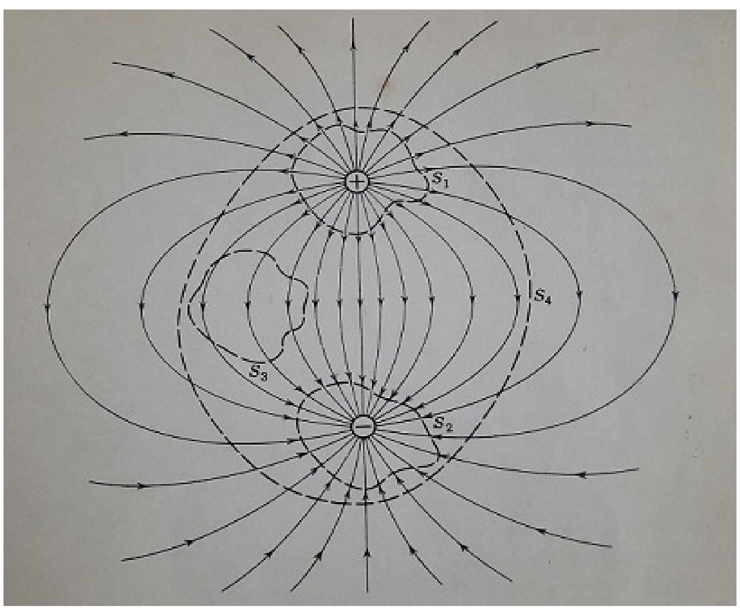

Fig. 5.4 Two equal and opposite charges. The dashed lines represent hypothetical closed surfaces.

For closed surfaces we shall see that Φ_E is positive if the lines of force point outward everywhere and negative if they point inward. Figure 4.4 shows two equal and opposite charges and their lines of force. Curves S_1, S_2, S_3, and S_4 are the intersections with the plane of the figure of four hypothetical closed surfaces. From the statement just given, Φ_E is positive for surface S_1 and negative for S_2.

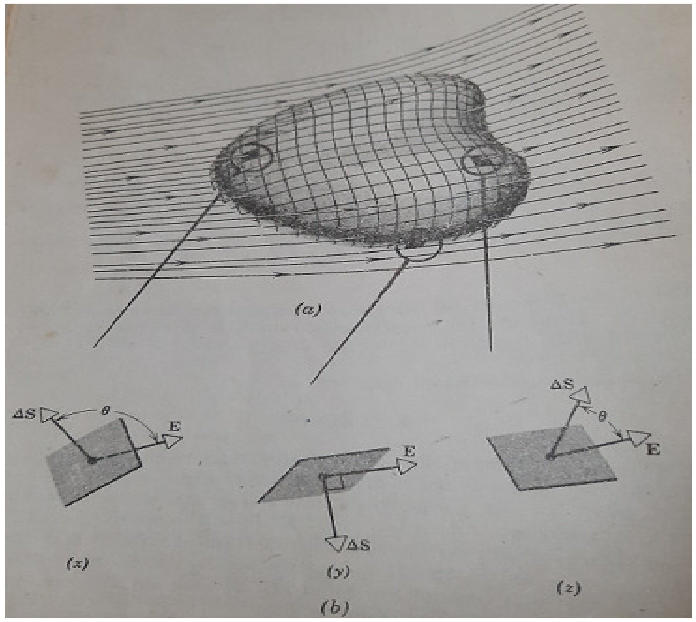

Fig. 5.5 (*a*) A hypothetical surface immersed in an electric field. (*b*) Three elements of areas on this surface, shown enlarged.

To define Φ_E precisely, let us consider Fig. 5.5, which shows an arbitrary closed surface immersed in an electric field. Let the surface be divided into elementary squares ΔS, each of which is small enough so that it may be considered to be plane. Such an element of area can be represented as a vector $\Delta\mathbf{S}$, whose magnitude is the area ΔS; the direction of $\Delta\mathbf{S}$ is taken as the *outward-drawn normal* to the surface.

At every square in Fig. 5.5 we can also construct an electrical field vector **E**. Since the squares have been taken to be arbitrarily small, **E** may taken as constant for all points in a given square. A semiquantitative definition of flux is

$$\Phi_E \cong \sum \mathbf{E}\cdot\Delta\mathbf{S}, \qquad (5.10)$$

which instructs us to add up the scalar quantity $\mathbf{E}\cdot\Delta\mathbf{S}$ for all elements of area into which the surface has been divided. From Eq. (5.10), we see that the unit of electric flux is $\text{newton}-\text{meter}^2/\text{coul}$.

The exact definition of electric flux is the differential limit of Eq. (5.10). Replacing the sum over the surface by an integral over the surface yields

$$\Phi_E = \iint \mathbf{E} \cdot d\mathbf{S} \,. \tag{5.11}$$

The surface integral indicates that the surface in question is to be divided into infinitesimal elements of area $d\mathbf{S}$ is to be evaluated for each element and the sum taken for the entire surface.

The Gauss's Law

Gauss's law, which applies to any closed hypothetical surface (called a Gaussian surface), gives a connection between the electric flux Φ_E for the surface and the net charge q enclosed by that surface. Mathematically, it is

$$\varepsilon_0 \Phi_E = q \,. \tag{5.12}$$

Using Eq. (5.11) for a closed surface, Gauss's law can also be written as

$$\varepsilon_0 \oiint \mathbf{E} \cdot d\mathbf{S} = q. \tag{5.13}$$

Gauss's law can be used to evaluate $\mathbf{E}$ if the charge distribution is so symmetric that by proper choice of a Gaussian surface we can easily evaluate the integral in Eq. (5.13). Conversely, if $\mathbf{E}$ is known for all points in a given closed surface, Gauss's law can be used to compute the charge inside.

Coulomb's Law from the Gauss's Law

Coulomb's law can be deduced from the Gauss's law and symmetry considerations. To do so, let us apply the Gauss's law to an isolated point charge q as in Fig. 5.6. Although Gauss's law holds for any surface whatever, information can most readily be extracted for a physical surface of radius r centered on the charge. The advantage of this surface is that, from symmetry, $\mathbf{E}$ must be normal to it and must have the same (as yet unknown) magnitude for all points on the surface.

In Fig. 5.6 both $\mathbf{E}$ and $d\mathbf{S}$ at any point on the Gaussian surface are directly radially outward. The angle between them is zero and the quantity $\mathbf{E} \cdot d\mathbf{S}$ becomes simply EdS. Gauss's law, Eq. (5.13), thus reduces to

$$\varepsilon_0 \oiint \mathbf{E} \cdot d\mathbf{S} = \varepsilon_0 \oiint E\, dS = q.$$

Because E is constant for all points on the sphere, it can be factored from inside the integral sign, leaving

$$\varepsilon_0 E \oiint dS = \varepsilon_0 E(4\pi r^2) = q$$

or

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}. \tag{5.14}$$

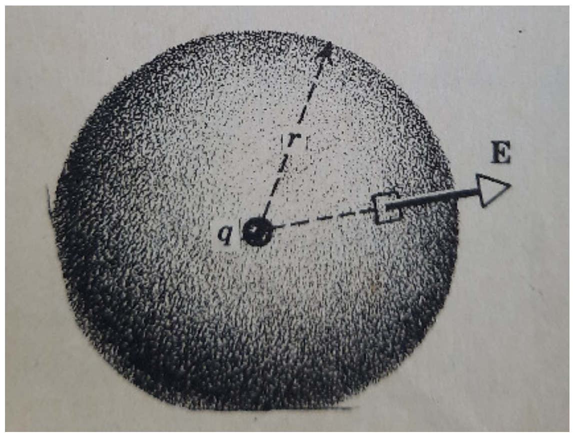

Fig. 5.6 A typical Gaussian surface of radius r surrounding a point charge q.

Equation (5.14) gives the magnitude of the electric field strength **E** at any point a distance r from an isolated point charge q. The direction of **E** is already known from symmetry.
Let us now put a second point charge q_0 at the point at which **E** is calculated. The magnitude of the force that acts on it is

$$F = q_0 E.$$

Combining with Eq. (5.14)(gives

$$F = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}, \tag{5.15}$$

which is precisely the Coulomb's law. Thus we have deduced the Coulomb's law from Gauss's law and considerations of symmetry.

Some Applications of Gauss's Law

Spherically Symmetric Charge Distribution

Figure 5.7 shows a spherical distribution of charge of radius R. The charge density ρ (that is, the charge per unit volume, measured in coul/meter^3) at any point depends only on the distance of the point from the center and not in the direction, a condition called *spherically symmetry*. We have to find expression for E for points (a) outside and (b) inside the charge distribution. We note that the object in the Fig. 5.7 cannot be a conductor, or the excess charge will reside on its surface.

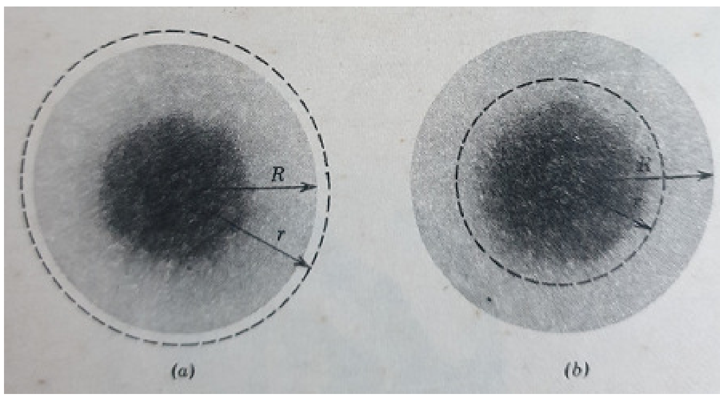

Fig. 5.7 A spherically symmetric charge distribution, showing two Gaussian surfaces. The density of charge, as the shading suggests, varies with distance from the center but not with direction.

Applying the Gauss's law to a physical Gaussian surface of radius r in Fig. 5.7*a* leads to

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}, \tag{5.16}$$

where q is the total charge. Thus for points outside a spherically symmetric distribution of charge, the electric field has the value that it would have if the charge were concentrated at its center.

Figure 5.7*b* shows a spherical Gaussian surface of radius r drawn inside the charge distribution. The Gauss's law gives

$$\varepsilon_0 \oiint \mathbf{E}\cdot d\mathbf{S} = \varepsilon_0 E(4\pi r^2) = q'$$

or

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q'}{r^2},$$

in which q' is that part of q contained within the sphere of radius r. The part of q that lies outside this sphere makes no contribution to **E** at radius r.

We have

$$q' = q\frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} = q\left(\frac{r}{R}\right)^3,$$

where $4\pi R^3/3$ is the volume of the spherical charge distribution. The expression for E then becomes

$$E = \frac{1}{4\pi\varepsilon_0}\frac{qr}{R^3}. \tag{5.17}$$

A Line of Charge

Figure 5.8 shows a section of an infinite rod of charge, the *linear charge density* λ (that is, the charge per unit length, measured in coul/meter) being constant for all points on the line. We have to find an expression for E at a distance r from the line.

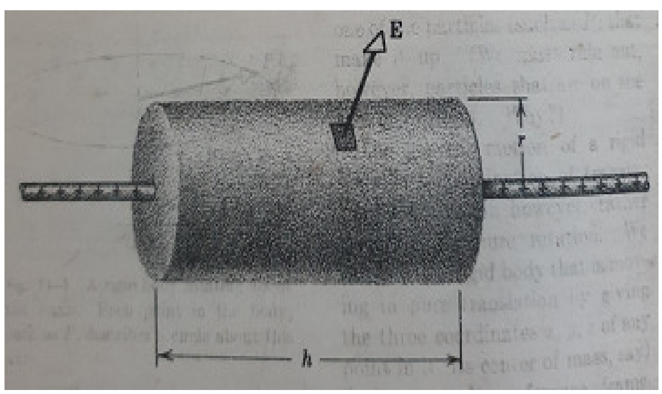

Fig. 5.8 An infinite rod of charge, showing a cylindrical Gaussian surface.

From symmetry, **E** due to a uniform linear charge can only be radially directed. As a Gaussian surface we choose a circular cylinder of radius r and length h, closed at each end by plane caps normal to the axis. E is constant over the cylindrical surface and the flux of **E** through this surface is

$$\Phi_E = \oint\!\!\oint \mathbf{E}\cdot d\mathbf{S} = 0 + \iint E dS + 0 = E\int dS = E(2\pi rh).$$

Thus the Gauss's law gives

$$\varepsilon_0 \oiint \mathbf{E} \cdot d\mathbf{S} = q$$

or

$$\varepsilon_0 E(2\pi rh) = \lambda h$$

or

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}. \tag{5.18}$$

The direction of **E** is radially outward for a line of positive charge.

A Sheet of Charge

Figure 5.9 shows a portion of a thin, nonconducting, infinite sheet of charge, the surface charge density σ (that is, the charge per unit are, measured in coul/meter^2) being constant. We have to find **E** at a distance *r* in front of the charged plane.

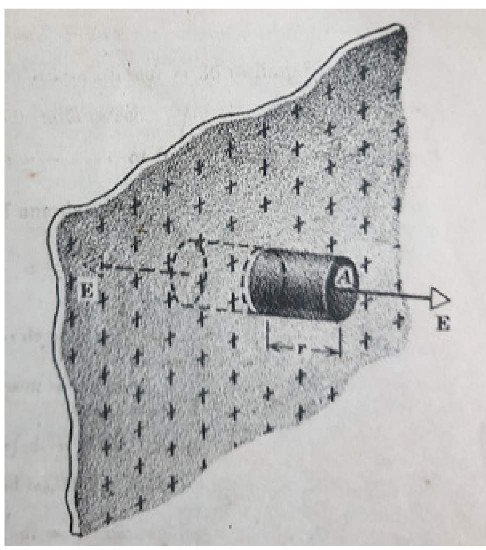

Fig. 5.9 An infinite sheet of charge pierced by a cylindrical Gaussian surface. The cross-section of the cylinder need not be circular, as shown, but can have an arbitrary shape.

A convenient Gaussian surface is a "pill box" of cross-sectional area A and height 2*r*, arranged to pierce the plane as shown. From symmetry, **E** points at right angles to the end caps and away from the plane. Since **E** does not pierce the cylindrical surface, there is no contribution to the flux from this source.

The Gauss's law

$$\varepsilon_0 \oiint \mathbf{E} \cdot d\mathbf{S} = q$$

becomes

$$\varepsilon_0 (EA + EA) = q = \sigma A,$$

where σA is the enclosed charge. This gives

$$E = \frac{\sigma}{2\varepsilon_0}. \tag{5.19}$$

We note that E is the same for all points on each side of the plane. Although an infinite sheet of charge cannot exist physically, this derivation is still useful in that Eq. (5.19) yields substantially correct results for real (not infinite) charge sheets if we consider only points not near the edges whose distance from the sheet is small compared to the dimensions of the sheet.

A Charged Conductor

The direction of **E** *for points close to the surface* is at right angles to the surface, pointing away from the surface if the charge is positive. If **E** were not normal to the surface, it would have a component lying in the surface. Such a component would act on the charge carriers in the conductor and set up surface currents. Since there are no such currents under the assumed electrostatic conditions, **E** must be normal to the surface.

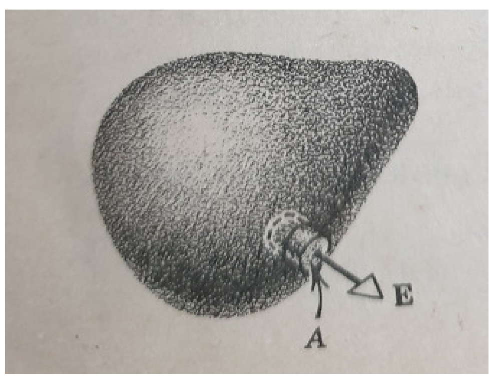

Fig. 5.10 A charged insulated conductor showing a Gaussian surface. The cross-section of the surface need not be circular, as shown, but can have any shape.

The magnitude of **E** can be found from the Gauss's law using a small flat "pill box" of cross-section A as a Gaussian surface. Since **E** equals zero everywhere inside the conductor, the only contribution to the flux is through the plane cap of area A that lies outside the conductor.

The Gauss's law

$$\varepsilon_0 \oiint \mathbf{E} \cdot d\mathbf{S} = q$$

becomes

$$\varepsilon_0 EA = q = \sigma A\ ,$$

where σA is the net charge within the Gaussian surface. This yields

$$E = \frac{\sigma}{\varepsilon_0}. \tag{5.20}$$

Comparison with Eq. (5.19) shows that the electric field here is twice as great near a conductor carrying a charge whose surface charge density is σ as that near a nonconducting sheet with the same surface charge density.

5.3 Electric Potential

The electric field around a charged particle can be described not only by a vector quantity, the electric field strength **E** but also by a scalar quantity, the electric potential V. These quantities are intimately related, and often it is only a matter of convenience which is used in a given problem.

To find the electric potential difference between two points A and B in an electric field, we move a test charge q_0 from A to B, always keeping it in equilibrium, and we measure the work W_{AB} that must be done by the agent moving the charge. The electric potential difference is defined from

$$V_B - V_A = \frac{W_{AB}}{q_0}. \tag{5.21}$$

The work W_{AB} may be (*a*) positive, (*b*) negative, or (*c*) zero. In these cases the electric potential at B will be (*a*) higher, (*b*) lower, or (*c*) the same as the electric potential at A. The SI unit of electric potential is volt where

$$1\ \text{volt} = 1\ \text{joule/coul}.$$

Usually point A is chosen to be at a large (strictly an infinite) distance from all charges, and the electric potential V_A at this infinite distance is arbitrarily taken as zero. This allows us to define the electric potential at a point. Putting $V_A = 0$ in Eq. (5.21) and dropping the subscripts leads to

$$V = \frac{W}{q_0}, \tag{5.22}$$

where W is the work that an external agent must do to move the test charge q_0 from infinity to the point in question.

Work done in Electric Field is Path-independent

It should be mentioned here that both W_{AB} and $V_B - V_A$ in Eq. (5.21) are path independent. We can easily prove that the potential differences are path independent for the special case shown in Fig. 5.11. This figure illustrates the case in which the two points A and B are in a field set up by a spherical charge q; the two points are further chosen, for simplicity, to lie along a radial line. Although our path-independence proof applies only to this special case, it illustrates the general principle involved.

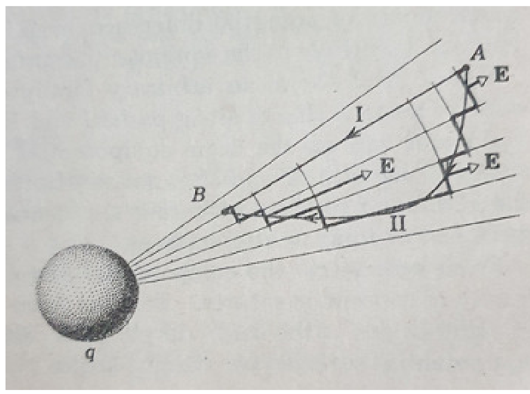

Fig. 5.11 A test charge q_0 is moved from point A to point B in the field of charge q along either of two paths. The open arrows show **E** at three points on path II.

Point A in Fig. 5.11 may be taken as a defined reference point, and we imagine a positive test charge q_0 moved by an external agent from point A to point B. We consider two paths, path I being a radial line between A and B and path II being a completely arbitrary path between these two points. The open arrows on path II show the electric force per unit charge that would act at various points on a test charge q_0.

Path II may be approximated by a broken path made up of alternating elements of arc and of radius. Since the elements can be arbitrarily small, the broken path can be made arbitrarily close to the actual path. On path II the external agent does work only along the radial segments because along the arcs the force **F** and the displacement $d\mathbf{l}$ are at right angles, $\mathbf{F}\cdot d\mathbf{l}$ being zero in such cases. The sum of the work done on the radial segments that make up path II is the same as the work done on path I because each path has the same array of radial segments. Since path II is arbitrary, we have proved that the work done is the same for all paths connecting A and B.

Equipotential Surface

The locus of points, all of which have the same electric potential, is called an *equipotential surface*. A family of equipotential surfaces, each surface corresponding to a different value of the potential, can be used to give a general description of the electric filed in a certain region of space.

No work is required to move a test charge between any two points on an equipotential surface. This follows from Eq. (5.21):

$$V_B - V_A = \frac{W_{AB}}{q_0},$$

because W_{AB} must be zero if $V_A = V_B$. This is true, because of the path independence of potential difference, even if the path connecting A and B does not lie entirely in the equipotential surface.

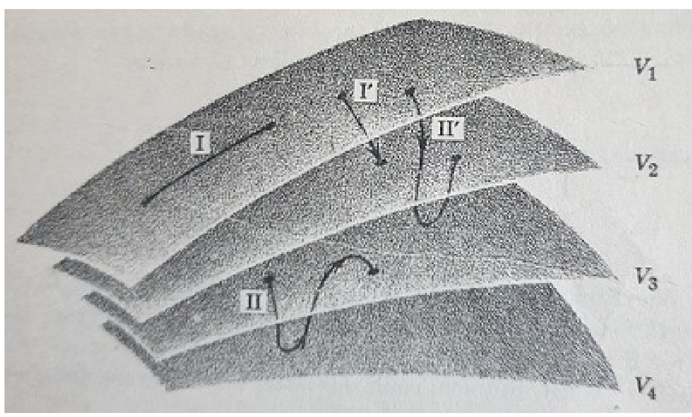

Fig. 5.12 Positions of four equipotential surfaces. The heavy lines show four paths along which a test charge is moved.

Figure 5.12 shows an arbitrary family of equipotential surfaces. The work to move a charge along paths I and II is zero because all these paths begin and end on the same equipotential surface. The work to move a charge along paths I' and II'' is not zero but is the same for each path because the initial and the final potentials are identical; paths I' and II'' connect the same pair of equipotential surfaces.

From symmetry, the equipotential surfaces for a spherical charge are a family of concentric spheres. For a uniform field they are a family of planes at right angles to the field. In all cases (including these two examples) the equipotential surfaces are at right angles to the lines of force and thus to **E**. If **E** were not a right angles to the equipotential surface, it would have a component lying in that surface. Then work has to be done in moving a test charge about on the surface. Work cannot be done if the surface is an equipotential, so **E** must be at right angles to the surface.

Relation Between Electric Potential and Electric Field

Let A and B in Fig. 5.13 be two points in a uniform electric field **E**, set up by an arrangement of charges not shown, and let A be a distance d in the field direction from B. Assuming that a positive test charge q_0 is moved, by an external agent and without acceleration, from A to B along the straight line connecting them.

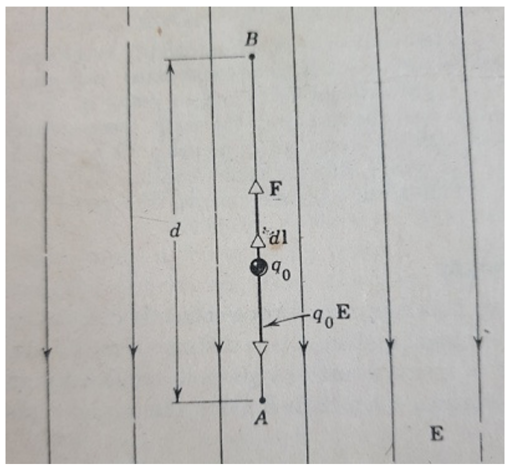

Fig. 5.13 A test charge q_0 is moved from A to B in a uniform electric field **E** by an external agent that exerts a force **F** on it.

The electric force on the charge is $q_0\mathbf{E}$ and points down. To move the charge in the way we have described we must construct this force by applying an external force **F** of the same magnitude that directed upward. The work W_{AB} done by the agent that supplies this force is

$$W_{AB} = Fd = q_0 Ed. \tag{5.23}$$

Thus the relation

$$V_B - V_A = \frac{W_{AB}}{q_0} = Ed. \tag{5.24}$$

This equation shows the connection between potential difference and electric field strength for a simple special case. We note from Eq. (5.24) that another SI unit for **E** is the volt/meter.

In Fig. 5.13 B has a higher potential than A. This is reasonable because an external agent would have to do positive work to push a positive test charge from A to B. Figure 5.13 could be used as it stands to illustrate the act of lifting a stone from A to B in the uniform gravitational field near the earth's surface.

Now we shall discuss the connection between the electric potential and the electric field strength in the more general case when the field is not uniform and in which the test body is moved along a path that is not straight, as in Fig. 5.14

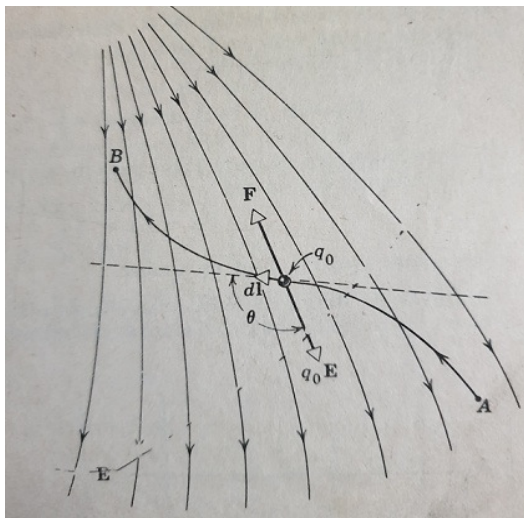

Fig. 5.14 A test charge q_0 is moved from A to B in a nonuniform electric field $\mathbf{E}$ by an external agent that exerts a force $\mathbf{F}$ on it.

The electric field exerts a force $q_0\mathbf{E}$ on the test charge, as shown in Fig. 5.14. To keep the test charge from accelerating, an external agent must apply a force $\mathbf{F}$ chosen to be exactly equal to $-q_0\mathbf{E}$ for all positions of the test body. If the external agent causes the test body to move through a displacement $d\mathbf{l}$ along the path from A to B, the element of work done by the external agent is $\mathbf{F}\cdot d\mathbf{l}$. To find the total work W_{AB} done by the external agent in moving the test charge from A to B, we add up (that is, integrate) the work contributions for all the infinitesimal segments into which the path is divided. This leads to

$$W_{AB} = \int_A^B \mathbf{F}\cdot d\mathbf{l} = -q_0 \int_A^B \mathbf{E}\cdot d\mathbf{l}.$$

Such an integral is called a line integral. We note that we have substituted $-q_0\mathbf{E}$ for its equal, **F**. Substituting this expression for W_{AB} into the following relation

$$V_B - V_A = \frac{W_{AB}}{q_0}$$

leads to

$$V_B - V_A = \frac{W_{AB}}{q_0} = -\int_A^B \mathbf{E}\cdot d\mathbf{l}\,. \tag{5.25}$$

If the point A is taken to be infinitely distant and the potential V_A at infinity is taken to be zero, the above equation gives the potential V at point B, or, dropping the subscript B, we have

$$V = -\int_\infty^B \mathbf{E}\cdot d\mathbf{l}\,. \tag{5.26}$$

Equations (5.25) and (5.26) allow us to calculate the potential difference between any two points (or the potential at any point) if **E** is known at various points in the field.

Electric Potential Due to a Point Charge

Figure 5.15 shows two points A and B near an isolated point charge q. For simplicity we assume that A, B, and q lie on a straight line. Let us compute the potential difference between points A and B, assuming that a test charge q_0 is moved without acceleration along a radial line from A to B.

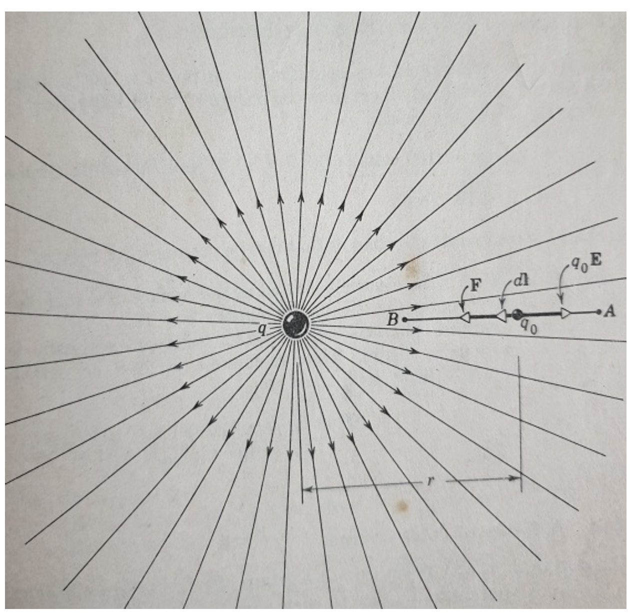

Fig. 5.15 A test charge q_0 is moved by an external agent from A to B in the field set up by a point charge q.

In the Fig. 5.15 **E** points to the right and $d\mathbf{l}$, which is always in the direction of motion, points to the left. Therefore, we have

$$\mathbf{E}\cdot d\mathbf{l} = E\cos 180^{o}\, dl = -Edl.$$

However, as we move a distance dl to the left, we are moving in the direction of decreasing r because r is measured from q as an origin. Thus

$$dl = -dr.$$

Combining we get

$$\mathbf{E}\cdot d\mathbf{l} = Edr.$$

Thus the following relation

$$V_B - V_A = -\int_A^B \mathbf{E}\cdot d\mathbf{l}$$

gives

$$V_B - V_A = -\int_A^B \mathbf{E}\cdot d\mathbf{l} = -\int_A^B E\,dr.$$

Combining with

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

Leads to

$$V_B - V_A = -\frac{q}{4\pi\varepsilon_0}\int_A^B \frac{dr}{r^2} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_B} - \frac{1}{r_A}\right). \tag{5.27}$$

Choosing reference position A to be at infinity (that is, letting $r_A \to \infty$) and choosing $V_A = 0$ at this position, and dropping the subscript B leads to

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}. \tag{5.28}$$

This equation shows clearly that equipotential surfaces for an isolated point charge are spheres concentric with the point charge.

A Group of Point Charges

The potential at any point due to a group of point charges is found by (*a*) calculating the potential V_n due to each charge, as if the other charges were not present, and (*b*) adding the quantities so obtained, or

$$V = \sum_n V_n = \frac{1}{4\pi\varepsilon_0} \sum_n \frac{q_n}{r_n}, \tag{5.29}$$

where q_n is the value of the nth charge and r_n is the distance of this charge from the point in question. The sum is used to calculate V is an algebraic sum and not a vector sum. Herein lies an important computational advantage of potential over electric field strength as electric field strength is a vector quantity.

If the charge distribution is continuous, rather than being a collection of points, the sum in Eq. (5.29) must be replaced by a appropriate integral:

$$V = \int dV = \frac{1}{4\pi\varepsilon 0} \int \frac{dq}{r}, \tag{5.30}$$

Where dq is a differential element of the charge distribution, r is its distance from the point at which V is to be calculated, and dV is the potential it establishes at that point.

Potential Due to a Dipole

Two equal and opposite charges $+q$ and $-q$ separated by a small distance $2a$, constitute an electric dipole. The electric dipole moment **p** has the magnitude $2aq$ and points from the negative charge to the positive charge. Here we derive an expression for the electric potential V at any point of space due to a dipole, provided only that the point is not too close to the dipole.

A point P is specified by giving the quantities r and θ in Fig. 5.16. From symmetry, it is clear that the potential will not change as point P rotates about the z-axis, r and θ being fixed. Thus we need only find $V(r,\theta)$ for any plane containing this axis; the plane of Fig. 5.16 is such a plane. The electric potential is

$$V = \sum_n V_n = V_1 + V_2 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_1} - \frac{q}{r_2}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{r_2 - r_1}{r_1 r_2}\right),$$

which is an exact relationship.

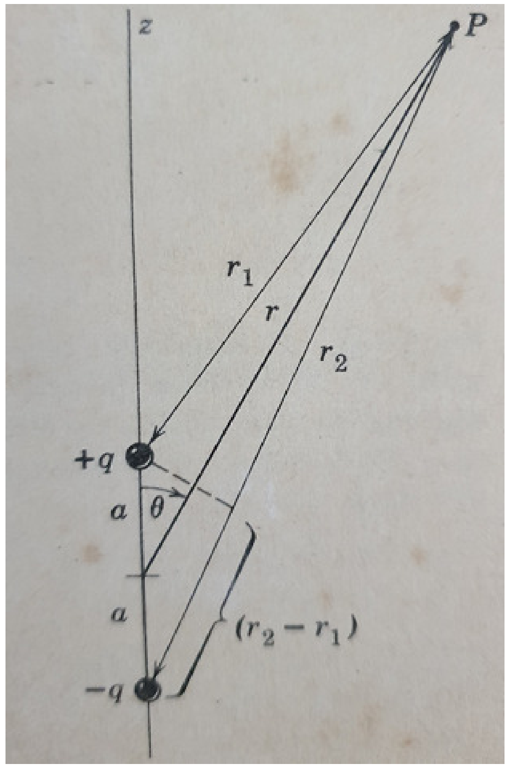

Fig. 5.16 A point P in the field of an electric dipole.

We now limit consideration to points such that $r >> 2a$. These approximate relations then follow from Fig. 5.16:

$$r_2 - r_1 \cong 2a\cos\theta \quad \text{and} \quad r_1 r_2 \cong r^2,$$

and the potential reduces to

$$V = \frac{q}{4\pi\varepsilon_0}\frac{2a\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}, \tag{5.31}$$

In which p (=2aq) is the magnitude of electric dipole moment. We note that V vanishes everywhere in the equatorial plane $(\theta = 90^o)$. This reflects the fact that it takes no work to bring a test charge in from infinity along the perpendicular bisector of the dipole. For a given radius, V has its greatest positive value for $\theta = 0^o$ and its greatest negative value for $\theta = 180^o$. We also note that the potential does not depend separately on q and 2a but only on their product p.

Electrostatic Potential Energy

Let us consider two charges q_1 and q_2 a distance r apart, as in Fig. 5.17. If we increase the separation between them, an external agent must do work that will be positive if the charges are opposite in sign and negative otherwise. The energy represented by this work can be thought of as stored in the system $q_1 + q_2$ as *electric potential energy*. This energy, like all varieties of potential energy, can be transformed into the other forms. If q_1 and q_2, for example, are charges of opposite sign and we release them, they will accelerate toward each other, transforming the stored potential energy into kinetic energy of the accelerating masses.

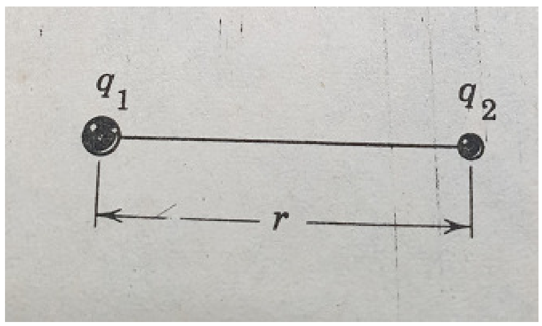

Fig. 5.17 Two charges q_1 and q_2 separated by a distance *r*.

We define the electric potential energy of a system of point charges as the work required to assemble this system of charges by bringing them in from an infinite distance. We assume that the charges are all at rest when they are infinitely separated, that is, they have no initial kinetic energy.

In Fig. 5.17 let us imagine q_2 removed to infinity and at rest. The electric potential at the original site of q_2, caused by q_1, is given by

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r}.$$

If q_2 is moved in from infinity to the original distance *r*, the work required is, from the definition of electrostatic potential:

$$W = Vq_2. \tag{5.32}$$

Combining these two equations and recalling that this work W is precisely the electrostatic potential energy U of the system $q_1 + q_2$ yields

$$U(=W) = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}. \tag{5.33}$$

The subscript of r emphasizes that the distance involved is that between the point charges q_1 and q_2.

Electron-Volt (eV)

In an electric field, let us consider two points A and B separated by a distance *d*. Now if the potential difference between the points A and B is $1V$, i.e., $V_A - V_B = 1\text{V}$, then the work done by an external agent to move an electron from point A at higher potential to point B at lower potential is considered to be 1 electron volt (eV) energy.

We can convert the electron-volt unit of energy into SI unit in the following way:

$$\begin{aligned} 1\text{ eV} &= (\text{charge of an electron})(1\text{ volt poptential difference}) \\ &= \left(1.6\times 10^{-19}\,\text{Coul}\right)(1\text{ volt}) \\ &= 1.6\times 10^{-19}\,\text{J}. \end{aligned}$$

Thus, 1 eV energy is equal to $1.6\times 10^{-19}\,\text{J}$.

Calculation of Electric Field Strength from Electric Potential

As we have seen that V and $\mathbf{E}$ are equivalent descriptions and we have seen how V can be calculated if $\mathbf{E}$ is known:

$$V = -\int_{\infty}^{B} \mathbf{E} \cdot d\mathbf{l}\,.$$

Here we shall develop a formula to calculate $\mathbf{E}$ if V is known. Let us consider Fig. 5.18, it shows the intersection with the plane of the figure of a family of equipotential surfaces. This figure shows that $\mathbf{E}$ at a typical point P is at right angles to the equipotential surface through P, as it must be.

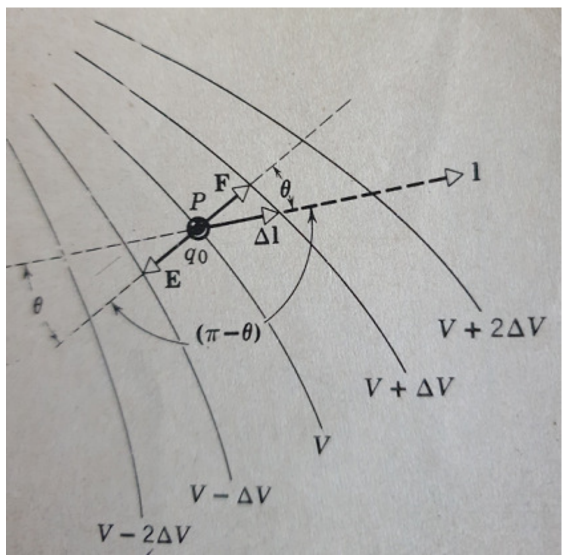

Fig. 5.18 A test charge q_0 is moved from one equipotential surface to another along an arbitrarily selected direction marked **l**.

Let us move a test charge q_0 from P along the path marked $\mathbf{\Delta l}$ to the equipotential surface marked $V + \Delta V$. The work that must be done by the agent exerting the force $\mathbf{F}$ is

$$\Delta W_a = q_0 \Delta V\,.$$

We can also calculate the work from

$$\Delta W = \mathbf{F} \cdot \Delta\mathbf{l},$$

where $\mathbf{F}$ is the force that must be exerted on the charge to overcome exactly the electrical force $q_0\mathbf{E}$. Since $\mathbf{F}$ and $q_0\mathbf{E}$ has opposite signs and are equal in magnitude,

$$\Delta W = -q_0 \mathbf{E} \cdot \Delta\mathbf{l} = -q_0 E \cos(\pi - \theta)\, \Delta l.$$

These two expressions for the ΔW_a and ΔW must be equal, which gives

$$\Delta W_a = \Delta W$$

$$\Rightarrow \qquad q_0 \Delta V = q_0 E \cos\theta \, \Delta l$$

or

$$E\cos\theta = \frac{\Delta V}{\Delta l}. \tag{5.34}$$

Now $E\cos\theta$ is the component of **E** in the direction **–l** in Fig. 5.18; the quantity $-E\cos\theta$, which we call E_l, would then be the component of **E** in the **+l** direction. In the differential limit Eq. (5.34) can then be written as

$$E_l = -\frac{dV}{dl}. \tag{5.35}$$

In words, this equation says: If we travel through an electric field along a straight line and measure V as we go, the rate of change of V with distance that we observe, then changed sign, is the component of **E** in that direction. The minus sign implies that **E** points in the direction of decreasing V, as in Fig. 5.18. Equation (5.35) can also be written as

$$\mathbf{E} = -\nabla V, \tag{5.36}$$

where the operator ∇ is the partial differential operator.

In Cartesian geometry, from Eq. (5.36), we have

$$E_x = -\frac{\partial V}{\partial x}; \quad E_y = -\frac{\partial V}{\partial y}; \quad E_z = -\frac{\partial V}{\partial z}. \tag{5.37}$$

Thus if V is known for all points of space, that is, if the function $V(x,y,z)$ is known, the components of **E**, and thus **E** itself can be found by taking derivatives.

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N.B. This Chapter is prepared for the students mainly based on Physics Part II by David Halliday and Robert Resnick Forty-Second Wiley Eastern Reprint, July 1992