Discrete Mathematics

Pigeonhole principle

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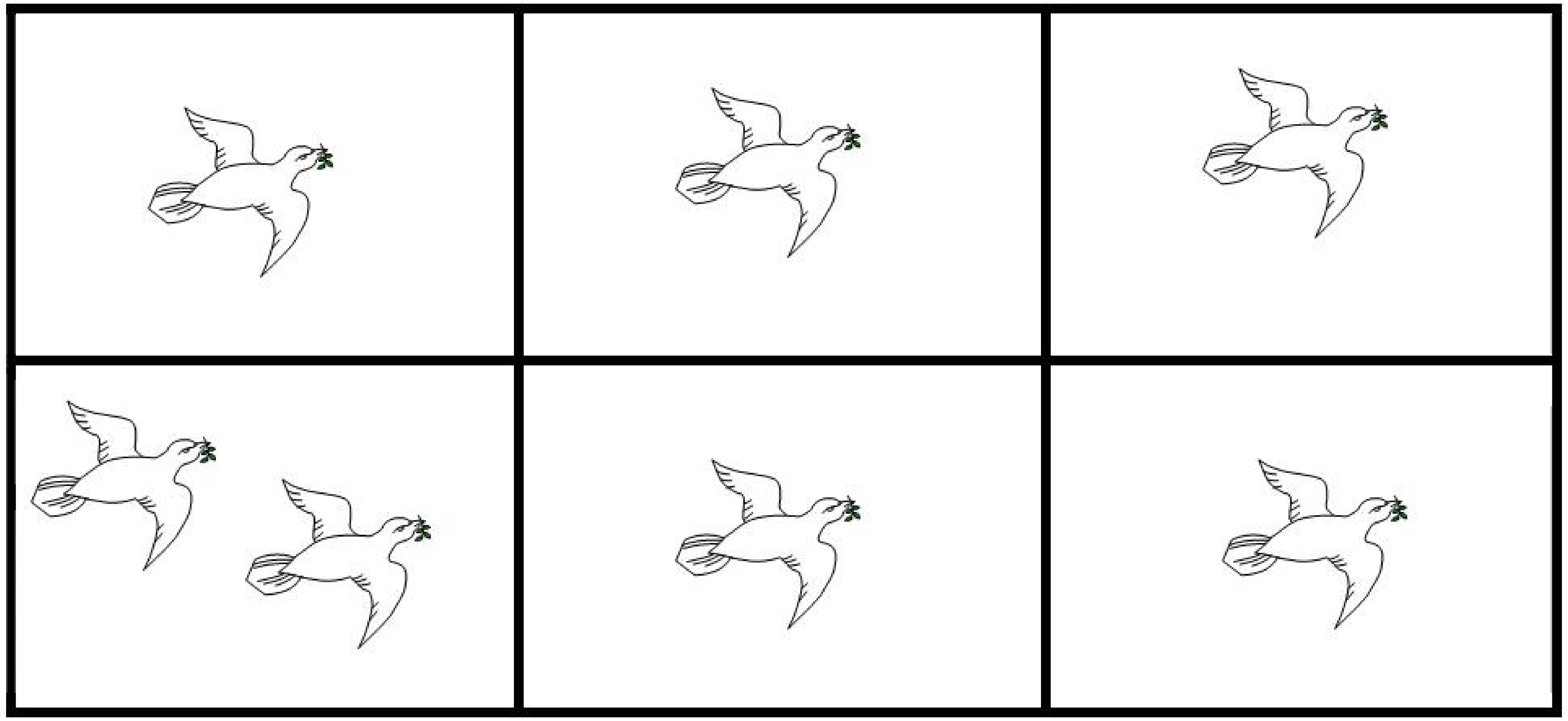
PIGEONHOLE PRINCIPLE

Suppose a flock of pigeons fly into a set of pigeonholes to roost

If there are more pigeons than pigeonholes, then there must be at least 1 pigeonhole that has more than one pigeon in it



PIGEONHOLE PRINCIPLE

If k+1 or more objects are placed into k boxes, then there is at least one box containing two or more objects.

Examples:

In a group of 367 people, there must be two people with the same birthday.

- There are 366 possible birthdays

In a group of 27 English words, at least two words must start with the same letter.

- There are only 26 letters

GENERALIZED PIGEONHOLE PRINCIPLE

If *N* objects are placed into *k* boxes, then there is at least one box containing $\lceil \frac{n}{k} \rceil$ objects.

Recall that the ceiling function, $\lceil \; \rceil$, tells us to round up to the nearest whole integer.

PR.1: PIGEONHOLE PRINCIPLE

Among 100 people, what is the number of people that must be born on the same month?

How many students in a class must there be to ensure that 6 students get the same grade (one of A, B, C, D, or F)?

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- There are at least $\left\lceil \frac{100}{12} \right\rceil = 9$ born on the same month

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- The "boxes" are the grades. Thus, $k = 5$
- Thus, we set $\left\lceil \frac{n}{5} \right\rceil = 6$
- Lowest possible value for n is 26

PIGEONHOLE PRINCIPLE

A bowl contains 10 red and 10 yellow balls

a) How many balls must be selected to ensure 3 balls of the same color?

- One solution: consider the "worst" case
  - Consider 2 balls of each color
  - You can't take another ball without hitting 3
  - Thus, the answer is 5
- Via generalized pigeonhole principle
  - How many balls are required if there are 2 colors, and one color must have 3 balls?
  - How many pigeons are required if there are 2 pigeon holes, and one must have 3 pigeons?
  - number of boxes: $k = 2$
  - We want $\lceil N/k \rceil = 3$
  - What is the minimum N?
  - $N = 5$

PIGEONHOLE PRINCIPLE

A bowl contains 10 red and 10 yellow balls

b) How many balls must be selected to ensure 3 yellow balls?

- Consider the "worst" case
  - Consider 10 red balls and 2 yellow balls
  - You can't take another ball without hitting 3 yellow balls
  - Thus, the answer is 13

QUESTIONS 1A, 1B,1C

a. What's the minimum number of students, each of whom comes from one of the 50 states must be enrolled in a university to guarantee that there are at least 100 who come from the same state?

b. There are 38 different time periods during which classes at a university can be scheduled. If there are 677 different classes, how many different rooms will be needed?

c. A computer network consists of six computers. Each computer is directly connected to zero or more of the other computers. Show that there are at least two computers that are directly connected to the same number of other computers.

QUESTIONS 1A, 1B,1C

a. What's the minimum number of students, each of whom comes from one of the 50 states must be enrolled in a university to guarantee that there are at least 100 who come from the same state?

$$50 * 99 + 1 = 4951$$

$$\lceil 4951/50 \rceil = 100$$

b. There are 38 different time periods during which classes at a university can be scheduled. If there are 677 different classes, how many different rooms will be needed?

$$\lceil 677/38 \rceil = 18$$

c. A computer network consists of six computers. Each computer is directly connected to zero or more of the other computers. Show that there are at least two computers that are directly connected to the same number of other computers.

The number of boxes, k, is the number of computer connections

- This can be 1, 2, 3, 4, or 5

The number of pigeons, N, is the number of computers

- That's 6

By the generalized pigeonhole principle, at least one box must have $\lceil N/k \rceil$ objects

- $\lceil 6/5 \rceil = 2$
- In other words, at least two computers must have the same number of connections

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