07/03/24

Linear combination

$\{v_1, v_2, \ldots\ldots v_k\}$

$c_1v_1 + c_2v_2 + \cdots + c_kv_k = 0$

$c_1 = c_2 = c_3 = \cdots\cdots = c_k = 0$ [Linear independent]

Example 1: A Linearly independent set

$c_1 = 3,\ c_2 = 1,\ c_3 = -1$

$c_1v_1 + c_2v_2 + c_3v_3 = 0$

$\Rightarrow c_1(2,-1,0,3) + c_2(1,2,5,-1) + c_3(7,-1,5,8) = 0$

$\Rightarrow (2c_1 + c_2 + 7c_3), \{-c_1 + 2c_2 - c_3\}, \{0c_1 + 5c_2 + 5c_3\}, \{3c_1 - c_2 + 8c_3\} = 0$

$$\left.\begin{aligned} 2c_1 + c_2 + 7c_3 &= 0 \\ -c_1 + 2c_2 - c_3 &= 0 \\ 5c_2 + 5c_3 &= 0 \\ 3c_1 - c_2 + 8c_3 &= 0 \end{aligned}\right\}$$

$$\begin{aligned} 2c_1 + c_2 + 7c_3 &= 0 \\ 5c_2 + 5c_3 &= 0 \\ 5c_2 + 5c_3 &= 0 \\ 5c_2 + 5c_3 &= 0 \end{aligned}$$

$$\begin{aligned} 2c_1 + c_2 + 7c_3 &= 0 \\ 5c_2 + 5c_3 &= 0 \end{aligned}$$

$c_3 = 5$ (assume)

$c_2 = -5$

$c_1 = \cancel{2c_1} - 35$

~~$c_1(1,0,0)$~~

polynomial

$P_n \rightarrow$ polynomial of degree n

Example 4 : Determining

$v_1 = (1,-2,3)$

$v_2 = (5,6,-1)$

$v_3 = (3,2,1)$

$2c_1 \quad 10c_2 + 6c_3$
$6c_2 + 2c_3$
$-15c_2 - 9c_3$
$-c_3 + c_3$

$c_1 v_1 + c_2 v_2 + c_3 v_3 = 0$

$c_1(1,-2,3) + c_2(5,6,-1) + c_3(3,2,1) = 0$

$\Rightarrow (c_1 + 5c_2 + 3c_3, \ -2c_1 + 6c_2 + 2c_3, \ 3c_1 - c_2 + c_3) = 0$

$c_1 + 5c_2 + 3c_3 = 0$

$-2c_1 + 6c_2 + 2c_3 = 0$

$3c_1 - c_2 + c_3 = 0$

$c_1 + 5c_2 + 3c_3 = 0 \quad -(i)$

$16c_2 + 8c_3 = 0 \quad -(ii)$

$-16c_2 + 8c_3 = 0 \quad -(iii)$

infinite solution, so its linearly dependent.

$c_1 + 5c_2 + 3c_3 = 0$

$16c_2 + 8c_3 = 0$

Limpet
Glimepiride

BASIS AND DIMENSION

একটা basis র মধ্যে যতগুলো linearly independent variable থাকে সে কয়টা dimension.

i) linearly independent variable

ii) $(x,y,z) = c_1 v_1 + c_2 v_2 + c_3 v_3$; $(x,y,z) \in \mathbb{R}^3$

$S = (v_1, v_2, v_3) \Rightarrow$ basis

Example 3 Demonstrating That a set vectors is a basis

$c_1 + 2c_2 + 3c_3 = 0$

$2c_1 + 9c_2 + 3c_3 = 0$

$c_1 + 0c_2 + 4c_3 = 0$

$c_1 + 2c_2 + 3c_3 = 0$

$5c_2 - 3c_3 = 0$

$-c_2 + c_3 = 0$

$c_1 + 2c_2 + 3c_2 = 0$

$5c_2 - 3c_3 = 0$

$2c_3 = 0$

$c_1 = c_2 = c_3 = 0$

These vectors are linearly independent.

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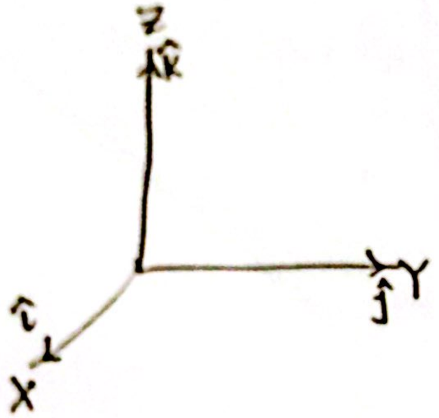
03/03/24
Quiz

1. System of linear eq^n
2. Gaussian elimination
3. Reduced row eschelon form

Vector space: in which all elements are vector.



$2\hat{i}+3\hat{j}+5\hat{k}$

analytical
geometric

$A=(2,4)$, $B=(4,8)$

$B=2A$

A_1 , A_2 , A_3

$c_1A_1 + c_2A_2 + A_3c_3 = 0$

$c_1=c_2=c_3=0$ linear independent.

$\Rightarrow c_1A + c_2B = 0$

$\Rightarrow c_1(2,4) + c_2(4,8) = 0$

$\Rightarrow (2c_1, 4c_1) + (4c_2, 8c_2) = 0$

$\Rightarrow (2+4c_2, 4c_1+8c_2) = 0$

$2c_1 + 4c_2 = 0$

$4c_1 + 8c_2 = 0 \Rightarrow c_1 = -2c_2$

$2(-2c_2) + 4c_2 = 0$

~~a+~~ 4+4

25/02/24

Vector Space Axioms:

যোগ করলে একই নাম্বারের মধ্যে থাকে → closed under addition

even + even = even, odd number is not closed under addition.

odd × odd = odd [closed under multiplication]

V: is a non empty set. V is called a vector space (V.S.) If it satisfies the following conditions

A1: $v_1, v_2 \in V \Rightarrow v_1 + v_2 \in V$ [closed under addition]

A2: $v_1 + v_2 = v_2 + v_1 \ \forall \ v_1, v_2 \in V$ [commutative law]

A3: $(v_1+v_2)+v_3 = v_1 + (v_2+v_3) \ \forall \ v_1, v_2, v_3 \in V$ [associative la

A4: For each $v \in V \ \exists \ 0 \in V$ such that

$v+0 = 0+v = v$

A5: For each $v \in V$, $\exists \ (-v) \in V$ such that

$v + (-v) = (-v) + v = 0$

multiplication:

~~M_1 : $\forall\, v_1, v_2 \in V$, $v_2 v_1$, $v_2 \in V$ [closed under multiplication~~ ভেক্টর

~~M_2~~: M_1: $\forall\, v \in V$ and $k \in F$, $kv \in V$ [closed under scaler multiplication.

M_2: $k(u+v) = ku + kv$ [distributive law]

M_3: $(k+m)v = kv + mv$ [distributive Law]

স্কেলার

M_4; $k(mv) = (km)v$

$V = \mathbb{R}$

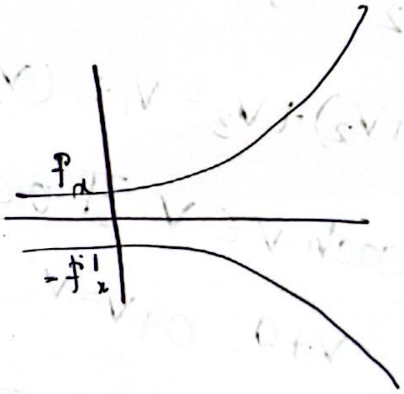
M_5: $1 \times v = v$ $[1 \in v]$

$\mathbb{R}^n = \mathbb{R} \times \mathbb{R} \times \cdots \times \mathbb{R}$

$= (x_1, x_2, x_3, \cdots x_n)$

$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $B = \begin{pmatrix} x & y \\ z & w \end{pmatrix}$

Example-4 বাদ



5.2

subspace — subset + Origin pass করতে হবে
↓
vector space (0, itself) → every vector has two subspace

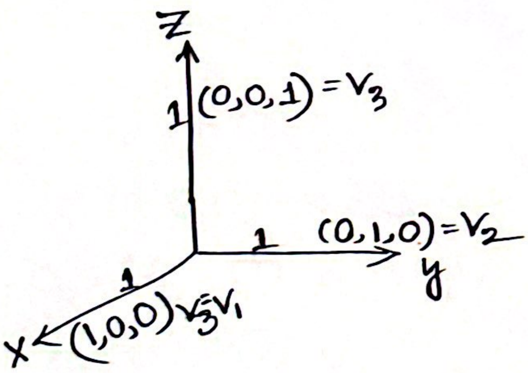
example 2: Lines through the origin are subspace.

03/03/24

Linear combination:

$$W = k_1 v_1 + k_2 v_2 + \dots\dots k_r v_r$$

$$k_1, k_2 \dots\dots k_r \in F \rightarrow \text{scalar field}$$



$W = (a, b, c)$

$a v_1 + b v_2 + c v_3$

$= (a, 0, 0) + (0, b, 0) + (0, 0, c)$

$= (a, b, c)$

Example-09

$$(9,2,7) = c_1 (16, 2, -1) + c_2 (6, 4, 2)$$

$$(9, 2, 7) = (c_1, 2c_1, -c_1) + (6c_2, 4c_2, 2c_2)$$

$$\Rightarrow (9, 2, 7) = (c_1 + 6c_2, 2c_1 + 4c_2, -c_1 + 2c_2)$$

$c_1 + 6c_2 = 9$

$2c_1 + 4c_2 = 2$

$-c_1 + 2c_2 = 7$

$$\left(\begin{array}{cc|c} 1 & 6 & 9 \\ 2 & 4 & 2 \\ -1 & 2 & 7 \end{array}\right)$$

$$= \left[\begin{array}{cc|c} 1 & 6 & 9 \\ 0 & -8 & -16 \\ 0 & 8 & 16 \end{array}\right] = \left[\begin{array}{cc|c} 1 & 6 & 9 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{array}\right]$$

$$= \left[\begin{array}{cc|c} 1 & 6 & 9 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right]$$

$c_2 = 2$

$c_1 = -3$

$c_1 + 6c_2 = 9$

$c_1 = 9 - 12$

$(9, 2, 7) = -3U + 2V$

showed

$(4, -1, 8) = c_1 (1, 2, -1) + c_2 (6, 4, 2)$

$(4, -1, 8) = (c_1, 2c_1, -c_1) + (6c_2, 4c_2, 2c_2)$

$(4, -1, 8) = (c_1 + 6c_2, 2c_1 + 4c_2, -c_1 + 2c_2)$

$c_1 + 6c_2 = 4$

$2c_1 + 4c_2 = -1$

$-c_1 + 2c_2 = 8$

$$\left[\begin{array}{cc|c} 1 & 6 & 4 \\ 2 & 4 & -1 \\ -1 & 2 & 8 \end{array}\right] -2$$

$$= \left[\begin{array}{cc|c} 1 & 6 & 4 \\ 0 & -8 & -9 \\ 0 & 8 & 12 \end{array}\right] = \left[\begin{array}{cc|c} 1 & 6 & 4 \\ 0 & \cancel{8}\ 1 & 9/8 \\ 0 & 1 & 3/2 \end{array}\right]$$

$$= \left[\begin{array}{cc|c} 1 & 6 & 4 \\ 0 & 1 & 9/8 \\ 0 & 0 & -3/8 \end{array}\right]$$ no solution

Space Spanned

$(1,3,5)$

$$\left.\begin{array}{l} c_1 = (1,0,0) \approx 1(1,0,0) \\ c_2 = (0,3,0) \approx 3(0,1,0) \\ c_3 = (0,0,9) \approx 9(0,0,9) \end{array}\right\} R_3 \text{ 생 subspace}$$

Example 12

$$c_1 V_1 + c_2 V_2 + c_3 V_3 = (x, y, z)$$

$c_1 + c_2 + 2c_3 = x, \quad c_1 + \ldots \ 3 = y$

$2c_1 + c_2 + 3c_3 = z$

04/03/24

Linear combination

$\{v_1, v_2, \ldots\ldots v_k\}$

$c_1v_1 + c_2v_2 + \ldots + c_kv_k = 0$

$c_1 = c_2 = c_3 = \ldots\ldots = c_k = 0$ [Linear independent]

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$\Rightarrow (2c_1 + c_2 + 7c_3), \{-c_1 + 2c_2 - c_3\}, \{0c_1 + 5c_2 + 5c_3\}, \{3c_1 - c_2 + 8c_3\} = 0$

$$\left.\begin{aligned} 2c_1 + c_2 + 7c_3 &= 0 \\ -c_1 + 2c_2 - c_3 &= 0 \\ 5c_2 + 5c_3 &= 0 \\ 3c_1 - c_2 + 8c_3 &= 0 \end{aligned}\right\}$$

$2c_1 + c_2 + 7c_3 = 0$

$5c_2 + 5c_3 = 0$

$5c_2 + 5c_3 = 0$

$5c_2 + 5c_3 = 0$

$2c_1 + c_2 + 7c_3 = 0$

$5c_2 + 5c_3 = 0$

$c_3 = s$ (assume)

$c_2 = -s$

$c_1 = \cancel{2c_1} -3s$

~~$c_1(1,0,0)$~~

polynomial

$P_n \to$ polynomial of degree n

Example 4: Determining

$v_1 = (1,-2,3)$

$v_2 = (5,6,-1)$

$v_3 = (3,2,1)$

$2c_1 \quad 10c_2 + 6c_3$
$6c_2 + 2c_3$
$-15c_2 - 9c_3$
$-c_3 + c_3$

$c_1 v_1 + c_2 v_2 + c_3 v_3 = 0$

$c_1(1,-2,3) + c_2(5,6,-1) + c_3(3,2,1) = 0$

$\Rightarrow (c_1 + 5c_2 + 3c_3, \quad -2c_1 + 6c_2 + 2c_3, \quad 3c_1 - c_2 + c_3) = 0$

$c_1 + 5c_2 + 3c_3 = 0$

$-2c_1 + 6c_2 + 2c_3 = 0$

$3c_1 - c_2 + c_3 = 0$

$c_1 + 5c_2 + 3c_3 = 0$ — (i)

$16c_2 + 8c_3 = 0$ — (ii)

$-16c_2 \overset{+8}{\cancel{+}} 8c_3 = 0$ — (iii)

infinite solution, so its linearly dependent.

$c_1 + 5c_2 + 3c_3 = 0$

$16c_2 + 8c_3 = 0$

BASIS AND DIMENSION

একটা basis র মধ্যে যতগুলো linearly independent variable থাকে সে কয়টা dimension.

1) linearly independent variable

11) $(x,y,z) = c_1 v_1 + c_2 v_2 + c_3 v_3$; $(x,y,z) \in R^3$

$S = (v_1, v_2, v_3) \Rightarrow$ basis

Example 3 Demonstrating That a set vectors is a basis

$c_1 + 2c_2 + 3c_3 = 0$

$2c_1 + 9c_2 + 3c_3 = 0$

$c_1 + 0c_2 + 4c_3 = 0$

Exercise set 5.3 $\boxed{1,2,3}$

→ 378 slide page

$c_1 + 2c_2 + 3c_3 = 0$

$5c_2 - 3c_3 = 0$

$-c_2 + c_3 = 0$

$c_1 + 2c_2 + 3c_2 = 0$

$5c_2 - 3c_3 = 0$

$2c_3 = 0$

$c_1 = c_2 = c_3 = 0$

These vectors are linearly independent.

31/03/24
Mid Exam

10/03/24

$(a,b,c) = c_1v_1 + c_2v_2 + c_3v_3$

0 না হলে basis করে না

Example 8 some finit and Infinite

$\dim(R^n) = n$

$\dim(P_n) = n+1$

$\dim(M_{mn}) = mn$

** Example ~~10~~ 10 Dimention of a solution space

$x_1 = -s-t$

$x_2 = s$

$x_3 = -t$

$x_4 = 0$

$x_5 = t$

5.4
2,12-17
399 pg

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} -s-t \\ s \\ -t \\ 0 \\ t \end{pmatrix} = \begin{pmatrix} -s \\ s \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -t \\ 0 \\ -t \\ 0 \\ t \end{pmatrix}$$

$$= s\begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} -1 \\ 0 \\ -1 \\ 0 \\ 1 \end{pmatrix}$$

ভেক্টর

যতগুলো ভেক্টর
ততগুলো dimension

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LINEAR TRANSFORMATION

$T: V \to W$ condition 2types of condition

$T(u+v) = Tu + Tv$

$T(\alpha u) = \alpha Tu$

$T: V \to \{0\}$
$T(u) = 0 \;\forall\; u \in V$
Zero Transformation

$T(u+v) = T(v) = 0$
$= 0 + 0$
$\boxed{= Tu + Tv}$

$T(\alpha u) = 0$
$= \alpha . 0$
$\boxed{= \alpha . Tu}$

Identity Operator:

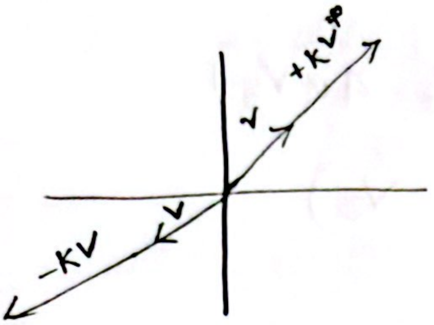
$T \cdot V \to V$
$Tu = u$

$T(v+\omega) = V + W$
$= TV + TW$

$T(\alpha V) = \alpha V = \alpha . TV$

Dilation and contraction:

$T(v) = kv$



Exercise set 8.1

$$T(x_1, x_2, x_3) = (2x_1 - x_2 + x_3,\ x_2 - 4x_3)$$

$T(u+v)$ $\quad u \in \mathbb{R}^3$

$\quad v \in \mathbb{R}^3$

$u = (u_1 + u_2 + u_3)$

$v = (v_1,\ v_2 + v_3$

$u + v = u_1 + v_1,\ u_2 + v_2,\ u_3 + v_3)$

$$T(u+v) = T(\underbrace{u_1 + v_1}_{x_1},\ \underbrace{u_2 + v_2}_{x_2},\ \underbrace{u_3 + v_3}_{x_3})$$

$$= (2(u_1 + v_1) - (u_2 + v_2) + (u_3 + v_3),\ u_2 + v_2 - 4(u_3 + v_3))$$

$$= (2u_1 + 2v_1 - u_2 - v_2 + u_3 + v_3,\ u_2 + v_2 - 4u_3 - 4v_3$$

$$= (2u_1 - u_2 + u_3,\ u_2 - 4u_3) + (2v_1 - v_2 + v_3,\ v_2 - 4v_3)$$

$$= T(u_1, u_2, u_3) + T(v_1, v_2, v_3)$$

$$= Tu + Tv$$

$\alpha V = (\alpha V_1, \alpha V_2, \alpha V_3)$

$T(\alpha V) = T(\alpha V_1, \alpha V_2, \alpha V_3)$

$= \cancel{\phi}\,(2\alpha V_1 - \alpha V_2 + \alpha V_3, \alpha V_2 - 4\alpha V_3)$

$= \alpha\,(2V_1 - V_2 + V_3, V_2 - V_3)$

$= \alpha T\,(V_1, V_2, V_3)$

$= \alpha TV$

1.

$T(x_1, x_2) = (x_1 + 2x_2, 3x_1 - x_2)$

$T(u+v) \cdot u \in \mathbb{R}^2$
$v \in \mathbb{R}^2$

$u = (u_1, u_2)$, $v = \cancel{(u}\,(v_1, v_2)$

$u+v = u_1 + v_1 + u_2 + v_2$

$T(u+v) = T\,(\underbrace{u_1+v_1}_{x_1}, \underbrace{u_2+v_2}_{x_2})$

$= (u_1+v_1 + 2(u_2+v_2), 3(u_1+v_1) - (u_2+v_2))$

$= \cancel{u_1 v_1 + 2(u_2 v_2)}$

$= (u_1 + v_1 + 2u_2 + 2v_2, 3u_1 + 3v_1 - u_2 - v_2)$

$= (u_1 + 2u_2, 3u_1 - u_2) + (v_1 + 2v_2, 3v_1 - v_2)$

$= T(u_1, u_2) \cancel{+3} + T(v_1, v_2)$

$= TU + TV$

$\cancel{\text{क}}\ \alpha v = \underline{(\alpha v_1, \alpha v_2 \cancel{+\alpha v_3})}$ $= (\alpha v_1, \alpha v_2)$

$T(\alpha v) = T(\alpha v_1, \alpha v_2)$

$= (\alpha v_1 + 2(\alpha v_2), 3\alpha v_1 - \alpha v_2)$

$= \alpha(v_1 + 2v_2, 3v_1 - v_2)$

$= \cancel{\alpha T (v_1, v_2, v_3)}$ $\alpha T(v_1, v_2)$

$= \alpha T v$ $= \alpha T v$

6. $T : M_{nn} \to R$, where $T(A) = tr(A)$

$T(A) = tr(A)$

$T(A+B) = tr(A+B)$

$= tr(A) + tr(B)$

$\boxed{\text{590 page}}$

5, 6, 7, 8

$T(\alpha A) = tr(\alpha A)$

$= \alpha\, tr(A)$

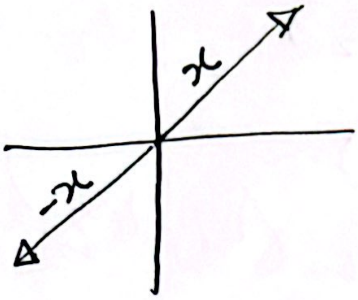
$= \cancel{\alpha t} \alpha T(A)$

7. F হবে T এর জায়গায়।

14/03/24

Eigen values & Eigenvectors

Figure 7.1.1



λx

$$Ax = \lambda x$$
$$Ax - \lambda x = 0$$
$$(A - \lambda I)x = 0$$

Example 1:

$$A = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}, \quad x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$Ax \quad \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \end{bmatrix} = \underbrace{3}_{\lambda} \underbrace{\begin{bmatrix} 1 \\ 2 \end{bmatrix}}_{x}$$

$$Ax = \lambda x$$

This is an eigenvector

Example 2:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & -17 & 8 \end{bmatrix}, \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$Ax = \lambda x$$
$$\Rightarrow (A - \lambda I)x = 0 \text{ ———— } (1)$$

$$P(\lambda) = |A - \lambda I|$$

$$= \begin{vmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ 4 & -17 & 8-\lambda \end{vmatrix}$$

$$\cdot \text{Det} = -\lambda\left[(8-\lambda)(-\lambda) + 17\right] - 1(-4)$$

$$= \cancel{-\lambda\left[-8\lambda + \lambda^2 + 17\right] + 4}$$

$$= \lambda^2(8-\lambda) - 17\lambda + 4$$

$$= 8\lambda^2 - \lambda^3 - 17\lambda + 4$$

$$= -\lambda^3 + 8\lambda^2 - 17\lambda + 4$$

Therefore, The characteristic equation of A is

$$P(\lambda) = 0$$

$$\Rightarrow -\lambda^3 + 8\lambda^2 - 17\lambda + 4 = 0$$

$$\cancel{\Rightarrow -\lambda^3 + 8\lambda^2} \quad \Rightarrow \lambda^3 - 8\lambda^2 + 17\lambda - 4 = 0$$

$$\cancel{\Rightarrow \lambda(\lambda^2 + 8\lambda) - \lambda(17} \quad \Rightarrow \lambda^3 - 4\lambda^2 - 4\lambda^2 + 16\lambda + \lambda - 4 = 0$$

$$\Rightarrow \lambda^2(\lambda - 4) - 4\lambda(\lambda - 4) + (\lambda - 4)$$

$$\Rightarrow (\lambda - 4)(\lambda^2 - 4\lambda + 1)$$

$$\Rightarrow (\lambda - 4)\,\cancel{(\lambda +}\,(\lambda - 2 - \sqrt{3})(\lambda - 2 + \sqrt{3}) = 0$$

$$\Rightarrow \lambda - 4 = 0, \quad \lambda = 2 + \sqrt{3}, \quad \lambda = 2 - \sqrt{3}$$

$$\Rightarrow \lambda = 4$$

$\lambda_1 = 4$

$\lambda_2 = 2+\sqrt{3}$

$\lambda_3 = 2-\sqrt{3}$

triangular matrix diagonal গুলাই হলো eigen value

trace of the matrix = sum of the eigen value

"Complex EigenValue"

$$\lambda^2+1=0$$

$$\lambda = \pm\sqrt{-1}$$

$$\lambda = \pm i$$

× ~~ext~~

+ example-5

542 page

EigenVectors

$$(A-\lambda I)x=0$$

$$\begin{bmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ 4 & -17 & 8-\lambda \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

put $\lambda = 4$ then we have,

$$\begin{bmatrix} -4 & 1 & 0 \\ 0 & -4 & 1 \\ 4 & -17 & 4 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$$

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$$\begin{bmatrix} -4 & 1 & 0 \\ 0 & 1 & -\frac{1}{4} \\ 0 & -16 & 4 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$R_3 - 16R_1$

$$= \begin{bmatrix} -4 & 1 & 0 \\ 0 & 1 & -\frac{1}{4} \\ 0 & 1 & -\frac{1}{4} \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$= \begin{bmatrix} -4 & 1 & 0 \\ 0 & 1 & -\frac{1}{4} \\ 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$-4x + y = 0$

$y - \frac{1}{4}z = 0$

let, $z = s$

$y = \frac{1}{4}s$

$\left.\begin{array}{l} -4x + \frac{1}{4}s = 0 \\ \Rightarrow x = \frac{1}{16}s \end{array}\right\}$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = s \begin{pmatrix} \frac{1}{16} \\ \frac{1}{4} \\ 1 \end{pmatrix}$$

let, $s = 1$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{1}{16} \\ \frac{1}{4} \\ 1 \end{pmatrix}$ is the eigenvectorc corrcesponding to $\lambda = 4$

$Ax = \lambda x$

$A(Ax) = A(\lambda x)$

$\Rightarrow A^2x = \lambda(Ax)$

$\Rightarrow A^2x = \lambda(\lambda x)$

$\Rightarrow A^2x = \lambda^2 x$ [powers]

$\Rightarrow A^n x = \lambda^n x$

I) Gauss elimination law

II) Matrix inverse

III) linear transformation

IV) Linear independent, dependent

V) Linear combination

V) Eigen value & eigen vector

VI) Identify linear or non-linear

IV, V → definition