Topic : Variation of Parameters to solve Linear Second Order Differential Equation

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Introduction:

- There are so many methods to solve linear differential equation , variation of parameter is one of them . This method was invented by astronomer and mathematician Joseph Louis Lagrange. The first order Linear Differential Equation is:

- $$\frac{dy}{dx} + p(x)y = f(x)$$

- This is the standard form of Linear Differential Equation.

Second Order Linear differential Equation:

$$d^2y/dx^2 + P(x)dy/dx + Q(x)y = f(x)$$

The homogeneous part is $d^2y/dx^2 + P(x)dy/dx + Q(x)y$ and nonhomogeneous part is $f(x)$. Here the solution of linear homogeneous differential equation is $y = e^{mx}$

The complementary solution of homogeneous differential equation is $yc = c1y1(x) + c2y2(x)$,When the coefficients are constant.

General Solution:

Complementary solution is : $yc = c1y1(x) + c2y2(x)$

- Now the particular solution is same as the Complementary solution just we have to replace $c1, c2$ by $u1 \; and \; u2$.
- Now, Particular solution is : $yp = u1(x)y1(x) + u2(x)y2(x)$
- The General Solution becomes,
- $$y = yc + yp$$
- So It is the solution of Linear Second Order Differential Equation Using the method Variation Of Parameters.

Completed Example:

$$y'' - 2y' - 15y = 384e^{-t}$$

Second order linear differential equation is,

$$y = y_h + y_p$$

y_h is the solution to the homogeneous .

$y''-2y'-15y$

$m^2-2m-15=0$

$(m-5)(m+3)=0$

$\therefore m_1=5, m_2=-3$

$\therefore y_h = C_1 e^{5t} + C_2 e^{-3t}$

y_p is the particular solution is any function That satisfies the non-homogeneous equation .

$y_p = u_1 v_1 + u_2 v_2$

$u_1 = e^{5t}, u_2 = e^{-3t}, f(t) = 384e^{-t}$

$$= \begin{vmatrix} e^{5t} & e^{-3t} \\ 5e^{5t} & -3e^{-3t} \end{vmatrix} = -8e^{2t}$$

Completed Example:

$v_1 = \int -\frac{-e^{3t}.384e^{-t}}{-8e^{2t}}\, dt$

$= 48\int e^{-6t} dt$

$= -8e^{-6t}$

$V_2 = \int -\frac{e^{5t}.384e^{-t}}{-8e^{2t}}\, dt$

$= 48\int e^{2t} dt$

$= 24e^{2t}$

$y_p = u_1v_1 + u_2v_2$

$= e^{5t} \times -8e^{-6t} + e^{-3t} \times 24e^{2t}$

Second order linear differential equation is

$y = y_h + y_p$

$\therefore y = C_1e^{5t} + C_2e^{-3t} - e^{-t}$

Reference: https://www.assignmentexpert.com/homework-answers/mathematics/differential-equations/question-126299

THANK YOU