Solutions of the Problems of Chapter 3

PHY109

3.6 Relevant Problems

Problem 3.1

A living room has floor dimensions of 3.5 m and 4.2 m and a height of 2.4 m.

(a) What does the air in the room weigh when the air pressure is 1.0 atm?

(b) What is the magnitude of the atmosphere's downward force on the top of your head, which we take to have an area of $0.040\ \text{m}^2$?

Solution

(a) We have

$$\begin{aligned} mg &= (\rho V)g \\ &= (1.21\ \text{kg/m}^3)(3.5\ \text{m} \times 4.2\ \text{m} \times 2.4\ \text{m})(9.8\ \text{m/s}^2) \\ &= 418\ \text{N} \approx 420\ \text{N}. \qquad \text{(Answer)} \end{aligned}$$

This is the weight of about 110 cans of Pepsi.

(b) Although air pressure varies daily, we can approximate that p = 1.0 atm. Then

$$\begin{aligned} F = pA &= (1.0\ \text{atm})\left(\frac{1.01 \times 10^5\ \text{N/m}^2}{1.0\ \text{atm}}\right)(0.040\ \text{m}^2) \\ &= 4.0 \times 10^3\ \text{N}. \qquad \text{(Answer)} \end{aligned}$$

This large force is equal to the weight of the air column from the top of your head to the top of the atmosphere.

Problem 3.2

How large a force is needed on a small piston of area 2 cm^2 in order to support a 1000 N weight resting on a piston of area 20 cm^2?

Solution

We are given that $A_i = 2\ \text{cm}^2$, $A_0 = 20\ \text{cm}^2$, $F_0 = 1000\ \text{N}$, and are asked to find F_i. We know

$$F_i = F_0 \frac{A_i}{A_0} = (1000\ \text{N})\frac{(2\ \text{cm}^2)}{(20\ \text{cm}^2)} = 100\ \text{N}. \qquad \text{(Answer)}$$

Problem 3.3
A crown is found to weigh 25.0 N when suspended in air and 22.6 N when suspended in water. Calculate the density of the crown to see if it is made of gold, as claimed by its donor. The density of water is 1000 kg/m^3.

Solution
The weight of the crown is related to its density ρ and its volume V by

$$w = mg = \rho V g.$$

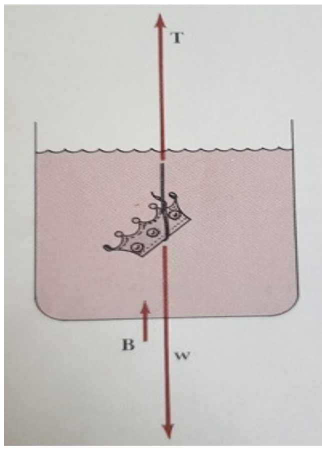

Fig. p3.3

We know the crown's weight (25.0 N) but not its volume; hence we cannot determine its density. We learn more by considering the forces as shown in the figure that act on the crown when in water: B is the upward buoyant force of the water on the crown, w is the crown's weight (25.0 N), and T is the upward tension force of the string that is attached from the crown to the spring balance. According to the scale, T = 22.6 N. Since the crown hangs at rest, these three factors must add to zero:

$$T + B - w = 0.$$

We know

$$B = \rho_f g V_f ,$$

where V_f is the volume of displaced water, which equals the volume of the crown (V). Thus $V_f = V$. Since the crown is completely immersed, and ρ_f is the density of water, we have after substitution:

$$T + \rho_f g V - w = 0$$

or

$$V = \frac{w - T}{\rho_f g}.$$

Therefore,

$$\rho = \frac{w}{gV} = \frac{w}{g}\left(\frac{\rho_f g}{w - T}\right) = \left(\frac{w}{w - T}\right)\rho_f = \left(\frac{25.0\ \text{N}}{25.0\ \text{N} - 22.6\ \text{N}}\right)\left(1000\ \text{kg/m}^3\right) = 10400\ \text{kg/m}^3. \quad \text{(Answer)}$$

Problem 3.4

A large drop of water is formed from 2500 small drops of water; each of diameter 10^{-6} m. What is the amount of energy released?

Solution

We know if the change in area is $\varDelta$A, then the amount of energy releases W is

$$W = \varDelta A\ T = 4\pi(Nr^2 - R^2)T.$$

Here,

Radius of the small drop, $r = \dfrac{(1)(10^{-6}\ \text{m})}{2} = 0.5\times10^{-6}$ m.

Surface tension, $T = 72\times10^{-3}\,\text{N m}^{-1}$.

Number of drop, $N = 2500$.

Radius of the large drop, $R = ?$

Released energy, $W = ?$

Now

$$(2500)\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi R^3$$

from which, we obtain

$$R = 13.57\,r = (13.57)(0.5\times10^{-6}\ \text{m}) = 6.785\times10^{-6}\ \text{m}.$$

Thus,

$$W = (4)(3.14)\left\{(2500)(0.5\times10^{-6}\ \text{m})^2 - (6.785\times10^{-6}\ \text{m})^2\right\}(72\times10^{-3}\,\text{Nm}^{-1}) = 5.24\times10^{-10}\ \text{J}. \quad \text{(Answer)}$$

Problem 3.5

A capillary tube of radius 0.1mm is dipped into oil of density $800\ \text{kg m}^{-3}$. The surface tension of the oil is $60\times10^{-3}\ \text{N m}^{-1}$ and angle of contact 20^o. How much height will the oil rise in the tube?

Solution

We know that

$$T = \frac{h\rho g r}{2\cos\theta}.$$

Here,

Surface tension, $T = 60\times10^{-3}\,\text{N m}^{-1}$.

Density of oil, $\rho = 800\ \text{kg m}^{-3}$.

Angle of contact, $\theta = 20^o$.

Radius of the tube, r = 0.1mm = 10^{-4} m.

Density of mercury, $\rho = 13.6\times10^{-3}\ \text{kg m}^{-3}$.

Acceleration due to gravity, $g = 9.8\ \text{ms}^{-2}$.

Height, $h = ?$

From the formula stated above, we have

$$h = \frac{2T\cos\theta}{\rho g r} = \frac{(2)(60\times10^{-3}\,\text{Nm}^{-1})(\cos 20^o)}{(800\ \text{kgm}^{-3})(9.8\ \text{ms}^{-2})(10^{-4}\ \text{m})} = 0.1438\ \text{m} = 1.438\ \text{mm}. \quad \text{(Answer)}$$

Problem 3.6

A hydraulic can crusher shown in the figure. The large piston has an area of $8\ \text{m}^2$ and exerts a force of magnitude $2\times10^6\ \text{N}$ on the cans. Calculate the magnitude of the force exerted by the small piston whose area is $10\ \text{cm}^2$ on the fluid. Do not ignore the fact that the large piston is 1 m higher than the small piston.

Solution

The pressure P_2 in the fluid at the top of the crusher is

$$P_2 = \frac{F_2}{A_2} = \frac{2.0\times10^6\ \text{N}}{8.0\ \text{m}^2} = 2.5\times10^5\ \text{N/m}^2.$$

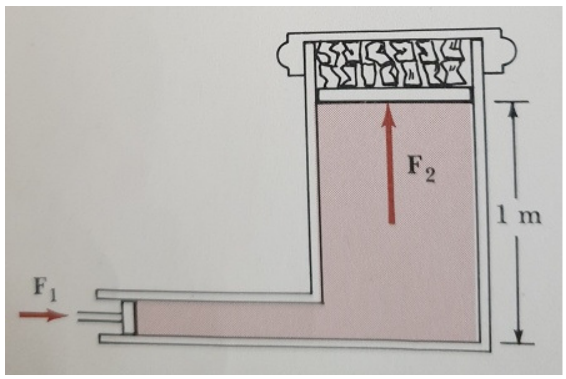

Fig. p3.6

The pressure on the fluid at the small piston is

$$\begin{aligned} P_1 &= P_2 + \rho g(y_2 - y_1) \\ &= 2.5\times10^5\ \text{N/m}^2 + (1000\ \text{kg/m}^2)(9.8\ \text{m/s}^2)(1.0\ \text{m} - 0) \\ &\approx 2.6\times10^5\ \text{N/m}^2. \end{aligned}$$

Since the small piston has an area of $10\ \text{cm}^2 (10^{-3}\ \text{m}^2)$, the magnitude of the force F_i on this piston is

$$F_1 = (2.6\times10^5\ \text{N/m}^2)(10^{-3}\ \text{m}^2) = 260\ \text{N}. \qquad \text{(Answer)}$$

Problem 3.7

Figure below shows how the stream of water emerging from a faucet "necks down" as it falls. This change in the horizontal cross-sectional area is characteristic of any laminar (nonturbulant) falling stream because the gravitational force increases the speed of the stream. Here the indicated cross-sectional areas are $A_0 = 1.2\ \text{cm}^2$ and $A = 0.35\ \text{cm}^2$. The two levels are separated by a vertical distance $h = 45\ \text{mm}$. What is the volume flow rate from the tap?

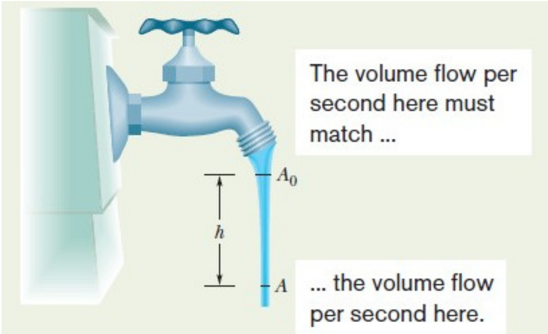

Fig. p3.5

Solution

We have from the continuity equation

$$A_0 v_0 = Av,$$

where v_0 and v are the water speeds at the levels corresponding to A_0 and A. Because the water is falling freely with acceleration g, we can write

$$v^2 = v_0^2 + 2gh.$$

Eliminating v from the above two equations, we get

$$v_0 = \sqrt{\frac{2ghA^2}{A_0^2 - A^2}}$$

$$= \sqrt{\frac{(2)(9.8\ \text{m/s}^2)(0.045\ \text{m})(0.35\times10^{-4}\ \text{m}^2)^2}{(1.2\times10^{-4}\ \text{m}^2)^2 - (0.35\times10^{-4}\ \text{m}^2)^2}}$$

$$= 0.286\ \text{m/s}.$$

The volume flow rate R_v is then

$$R_v = A_0 v_0 = (1.2\times10^{-4}\ \text{m}^2)(0.286\ \text{m/s}) = 0.3432\times10^{-4}\ \text{m}^3/\text{s} = 34.32\ \text{cm}^3/\text{s}. \quad \text{(Answer)}$$

Problem 3.8
Water flows through a pipe of radius 0.10 m at a speed of 5.0 m/s.

(a) Calculate the flow rate in this pipe.
(b) The water goes down a hill with a vertical drop of 4.0m and then flows along a pipe of radius 0.080 m. Calculate the pressure of the fluid in the small pipe minus the pressure in the large pipe. The density of water is $1000\ \text{kgm}^{-3}$.

Solution
(a) The situation described in the problem is shown in the figure above. The flow rate is

$$R_v = A_1 v_1 = (\pi r_1^2) v_1 = (3.14)(0.10\ \text{m})^2 (5.0\ \text{m/s}) = 0.16\ \text{m}^3/\text{s}.$$

(b) We wish to find the pressure difference, $p_2 - p_1$, using the Bernoulli's equation. The speed of the fluid at position 2 can be determined using the continuity equation:

$$A_1 v_1 = A_2 v_2$$

or

$$v_2 = \frac{A_1}{A_2} v_1 = \frac{\pi (0.100\ \text{m})^2}{\pi (0.018\ \text{m})^2}(5.0\ \text{m/s}) = 7.8\ \text{m/s}.$$

From Bernoulli's equation, we find that

$$\begin{aligned} p_2 - p_1 &= \frac{1}{2}\rho v_1^2 - \frac{1}{2}\rho v_2^2 + \rho g(y_1 - y_2) \\ &= \frac{1}{2}(1000\ \text{kg/m}^3)(5.0\ \text{m/s})^2 - \frac{1}{2}(1000\ \text{kg/m}^3)(7.8\ \text{m/s})^2 \\ &\qquad + (1000\ \text{kg/m}^3)(9.8\ \text{m/s})(4.0\ \text{m}) \\ &= +21000\ \text{N/m}^2. \qquad \text{(Answer)} \end{aligned}$$

Problem 3.9

(a) Calculate the speed with which water flows from a hole in the dam of a large irrigation canal. The hole is 0.80 m below the surface of the water.

(b) If the hole has a radius of 2.0 cm, what is the flow rate of water from the hole?

Solution

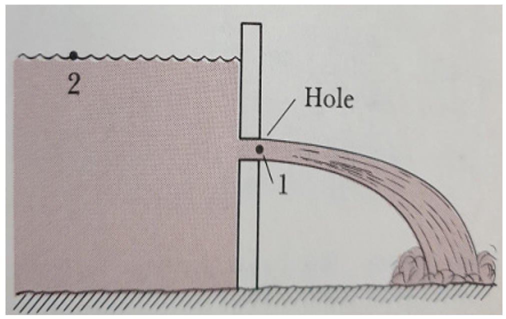

Fig. p3.9

(a) We choose 1 as the place where the water leaves the hole (see the figure). Position 2 can be chosen at a place where the pressure, elevation, and speed of the water are known. A convenient place is at the top of the water behind the dam. There is so much water behind the dam that its level moves downward very slowly. Thus, the water speed at position 2 is nearly zero ($v_2 \cong 0$). Since position 2 is 0.80 m higher than position 1, $y_2 - y_1 = 0.80\,\text{m}$. The water pressure at both positions 1 and 2 is atmospheric pressure: $p_1 = p_2 = p_{atm}$.

We next substitute all of this information into the Bernoulli's equation:

$$p_2 - p_1 = \frac{1}{2}\rho v_1^2 - \frac{1}{2}\rho v_2^2 + \rho g(y_1 - y_2)$$

or

$$p_{atm} - p_{atm} = \frac{1}{2}\rho\left(v_1^2 - 0^2\right) + \rho g(y_1 - y_2).$$

Solving for v_1, we find that

$$v_1 = \sqrt{2g(y_2 - y_1)}.$$

This is actually called **Torricelli's theorem**. Substituting for g and $(y_2 - y_1)$, we find that

$$v_1 = \sqrt{2(9.8\text{ m/s}^2)(0.80\text{ m})} = 4.0\text{ m/s}.$$

(b) The flow rate of the water from the hole is determined using

$$R_v = A_1 v_1 = \pi r_1^2 v_1 = (3.14)(0.020\text{ m})^2(4.0\text{ m/s}) = 5.0\times10^{-3}\text{ m}^3\text{/s}. \qquad \text{(Answer)}$$

Problem 3.10
Suppose that the speed of air over the top of the wing of a airplane is 180 m/s, that the air speed under the wing is 165 m/s, and that the density of air at the altitude where the plane flies is $0.80\ \text{kg/m}^3$.

(a) Calculate the pressure difference between the bottom and top of the wing.

(b) If the plane has a mass of 2.0×10^4 kg, what must be the area of the wings so that the lift can support the airplane's weight?

Solution

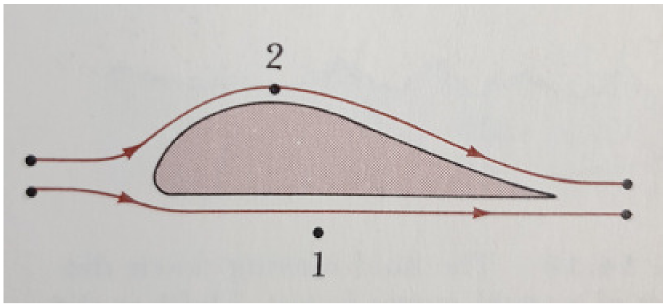

Fig. p3.10

(a) Let us take position 1 under the wing and position 2 over the wing (figure given above). We will assume that the difference in gravitational potential energies is negligible because the vertical separation of the points is very small, as is the air density $(y_1 - y_2 \cong 0)$. We find, then using Bernoulli's equation:

$$p_2 - p_1 = \frac{1}{2}\rho v_1^2 - \frac{1}{2}\rho v_2^2 + \rho g(y_1 - y_2)$$

or

$$p_1 - p_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 + 0$$

$$= \frac{1}{2}(0.80\ \text{kg/m}^3)(180\ \text{m})^2 - \frac{1}{2}(0.80\ \text{kg/m}^3)(165\ \text{m})^2$$

$$= 2070\ \text{N/m}^2.$$

(b) The net lift force on the wing is

$$F_1 - F_2 = (p_1 - p_2)A.$$

This must balance the airplane's weight, $w = mg$. Thus,

$$mg = (p_1 - p_2)A.$$

Rearranging the above equation to find the area of the wing, we find that

$$A = \frac{mg}{p_1 - p_2} = \frac{(2.0\times 10^4\ \text{kg})(9.8\ \text{m/s}^2)}{2070\ \text{N/m}^2} = 95\ \text{m}^2. \qquad \text{(Answer)}$$

Problem 3.11

A air bubble of radius 10^{-5} m is rising through water. The coefficient of viscosity of water is $10^{-3}\,\mathrm{Nsm^{-2}}$ and its density is $10^3\,\mathrm{kgm^{-3}}$. The density of air is negligible in comparison to water. Determine the terminal velocity of the bubble.

Solution

For upward velocity of the air bubble inside water viscous drag force must be equal to the upward resultant force:

$$\mathrm{F}_D = \text{weight of the removed water} - \text{weight of the bubble}$$

or

$$6\pi\eta r v_T = \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \rho_a g.$$

Here,

Radius, $r = 10^{-5}$ m,

Density of water, $\rho = 10^3\,\mathrm{kg\,m^{-3}}$,

Density of air, ρ_a = negligible ≈ 0,

Acceleration due to gravity, $g = 9.8\ \mathrm{ms^{-2}}$,

Coefficient of viscosity of water, $\eta = 10^{-3}\,\mathrm{Nsm^{-2}}$,

Upward velocity or Terminal velocity, v_T = ?

The above equation can be approximately written as

$$6\pi\eta r v_T = \frac{4}{3}\pi r^3 \rho g$$

from which we obtain

$$v_T = \frac{2r^2\rho g}{9r} = \frac{(2)(10^{-5}\ \mathrm{m})^2(10^3\,\mathrm{kg\,m^{-3}})(9.8\ \mathrm{ms^{-2}})}{(9)(10^{-3}\,\mathrm{Nsm^{-2}})} = 2.18\times10^{-4}\,\mathrm{ms^{-1}}. \quad \text{(Answer)}$$

Problem 3.12

Estimate the air drag force on a 1130 kg compact car when moving at a speed of 27 m/s (60 mph). The cross-sectional area of the car is roughly $2.0\ \mathrm{m^2}$ and the drag coefficient C_D is approximately 0.5 for a well-designed car. The density of air is $1.3\ \mathrm{kg/m^3}$.

Solution

We know from the formula for drag force

$$F_D = \frac{1}{2}C_D \rho A v_T^2.$$

Thus we have

$$F_D = \frac{1}{2}(0.5)(1.3\ \mathrm{kg/m^3})(2.0\ \mathrm{m^2})(27\ \mathrm{m/s})^2 = 470\ \mathrm{kg\ m/s^2} = 470\ \mathrm{N}. \quad \text{(Answer)}$$

Problem 3.13

Estimate the terminal speed of a Ping-Pong ball weighing 2.2×10^{-2} N whose radius is 1.9×10^{-2} m. The ball experiences a turbulent drag force with a drag coefficient of 0.6.

Solution

The ball reaches terminal speed when its weight is balanced by the upward drag force:

$$w = F_D.$$

But we know that

$$F_D = C_D \rho A v^2 / 2,$$

where $v = v_T$, v_T being the terminal speed of the ball.

Thus,

$$w = C_D \rho A v_T^2 / 2$$

or

$$v_T = \sqrt{\frac{2w}{C_D \rho A}} = \sqrt{\frac{2w}{C_D \rho \pi r^2}} = \sqrt{\frac{(2)(2.2\times10^{-2}\ \text{N})}{(0.6)(1.3\ \text{kg/m}^3)(3.14)(1.9\times10^{-2}\ \text{m})^2}} = 7.1\ \text{m/s.} \qquad \text{(Answer)}$$

Problem 3.14

A hollow spherical iron shell floats almost completely submerged in water. If the outer diameter is 2.0m and the relative density of iron is 7.8, find the inner diameter.

Solution

Since the iron shell floats almost completely submerged, then

$$\text{weight of a body} = \text{weight of liquid displaced} = \frac{4}{3}\pi R^3 \rho_0, \qquad \text{(p3.14a)}$$

where ρ_0 is the density of water and R is the outer radius.

Therefore, density of iron

$$\rho_F = \text{Relative density of iron} \times \rho_0 = 7.8\rho_0. \qquad \text{(p3.14b)}$$

Let the inner radius be R_0.

Volume of the shell is

$$= \frac{4}{3}\pi\left(R^3 - R_0^3\right) \qquad \text{(p3.14c)}$$

Thus

$$\rho_F = \frac{\text{mass}}{\text{volume}} = 7.8\rho_0$$

or

$$\frac{(4\pi R^3/3)\rho_0}{4\pi\left(R^3 - R_0^3\right)/3} = 7.8\rho_0,$$

from which we obtain for R = 1.0 ft to get

$$\frac{1}{1 - R_0^3} = 7.8$$

or

$$R_0 = 0.955\ \text{ft.}$$

Thus, the inner diameter is

$$2R_0 = 1.91\ \text{ft.} \quad \text{(Answer)}$$

Problem 3.15
A block of wood floats in water with two-thirds of its volume submerged. In oil it has 0.90 of its volume submerged. Find the density of the wood and the oil.

Solution
Let L be the thickness of wood of the block, A its cross-section area and ρ be its density.

$$\text{Weight of block} = LA\rho. \qquad \text{(p3.15a)}$$

$$\text{Weight of water displaced} = \frac{2}{3}LA\rho_0. \qquad \text{(p3.15b)}$$

According to Archimedes Principle,

$$\text{Weight of block} = \text{Weight of water displaced}. \qquad \text{(p3.15c)}$$

Therefore,

$$LA\rho = \frac{2}{3}LA\rho_0,$$

from which we obtain

$$\rho = \frac{2}{3}\rho_0 = \left(\frac{2}{3}\right)(1.0\ \text{gm.cm}^{-3}) = 0.67\ \text{gm.cm}^{-3}. \qquad \text{(p3.15d)}$$

$$\text{Weight of oil displaced} = 0.9LA\rho_1, \qquad \text{(p3.15e)}$$

Where ρ_1 is the density of oil.
Again, by Archimedes Principle, we get

$$LA\rho = 0.9LA\rho_1,$$

from which we obtain

$$\rho_1 = \frac{\rho}{0.9} = \frac{0.67}{0.9} = 0.74\ \text{gm.cm}^{-3}. \quad \text{(Answer)}$$

Problem 3.16
A block of wood weighs 8.0kg and has a relative density of 0.60. It is to be loaded with lead so that it will float in water with 0.90 of its volume immersed. What weight of lead is needed (a) if the lead is on top of the wood? (b) if the lead is attached below the wood?

Solution

(a) Weight of the floating body = Weight of the liquid displaced.
Now

$$\begin{aligned}\text{density of wood} &= (\text{relative density})(\text{density of water}) \\ &= (0.6)(62.4\ \text{lb.ft}^{-3}) \\ &= 37.4\ \text{lb.ft}^{-3}.\end{aligned}$$

The volume of wooden block,

$$\text{V} = \frac{\text{mass}}{\text{density}} = \frac{8\ \text{lb}}{37.4\ \text{lb.ft}^{-3}} == 0.214\ \text{ft}^3.$$

$$\text{Volume of water displaced} = 0.9V = (0.9)(0.214\ \text{ft}^3) = 0.193\ \text{ft}^3.$$

$$\text{Weight of water displaced} = (0.193\ \text{ft}^3)(62.4\ \text{lb.ft}^{-3}) = 12\ \text{lb}.$$

Therefore,

$$\text{Weight of lead} + \text{weight of wood} = 12\ \text{lb}.$$

$$\text{Weight of lead} = 12\ \text{lb} - 8\ \text{lb} = 4\ \text{lb}. \quad \text{(Answer)}$$

(b) Density of lead $= 705$ lb.ft^{-3}.

Let

$$W_0 = \text{weight of lead in air.}$$

Volume of lead

$$V = \frac{W_0}{705}.$$

Loss of weight of lead = Weight of water displaced

$$= \left(\frac{W_0}{705}\right)(62.4)$$

$$= 0.09 \text{ lb.}$$

Weight of lead in water $= W_0 - 0.09W_0 = 0.91W_0$lb.

Weight of leadn in water + Weight of wood = Weight of water displaced.

Therefore,

$$0.91W_0 + 8 \text{ lb} = 12 \text{ lb},$$

from which we obtain

$$W_0 = 4.4 \text{ lb.} \quad \text{(Answer)}$$

Problem 3.17

(a) Consider a container of fluid subject to a vertical upward acceleration a. Show that the pressure variation with depth in the fluid is given by

$$p = \rho h(g + a),$$

where h is the depth and ρ is the density.

(b) Show also that if the fluid as a whole undergoes a vertical downward acceleration a, the pressure at a depth h is given by

$$p = \rho h(g - a).$$

(c) What is the state of affairs in free fall?

Solution

(a) For the vertical upward acceleration, the weight of liquid is increased since $g \to g + a.$ (Answer)

(b) Hence, the formula $p = \rho hg$ must be replaced by $p = \rho h(g + a).$ (Answer)

(c) For the vertical downward direction, $g \to g - a.$ (Answer)

Hence, the formula is modified as $p = \rho h(g - a).$ (Answer)

(d) In free fall, $a = g$ and $p = 0.$ (Answer)

Problem 3.18

What amount of work is done when a water sphere of diameter 2mm breaks up into 10^6 drops? [Surface tension of water $T = 72\times10^{-3}$ N m^{-1}]

Solution

We know if the change in area is ΔA, then

$$W = T\,\Delta A = 4\pi(Nr^2 - R^2)T. \qquad \text{(p3.18a)}$$

Here,

Radius of the large drop, $R = 10^{-3}$ m,

Surface tension, $T = 72\times10^{-3}$ N/m,

Number of drops, $N = 10^6$,

Radius of the small drop, $r = ?$

Work done, $W = ?$

We have

$$106\times\frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3 \quad \Rightarrow \quad R = 10^2 r. \qquad \text{(p3.18b)}$$

Therefore,

$$r = \frac{R}{100} = \frac{10^{-3}\text{ m}}{100} = 10^{-5}\text{ m}.$$

Therefore,

$$W = (4)(3.14)\left((10^6)(10^{-5}\text{ m}) - (10^{-3}\text{ m})\right)(72\times10^{-3}\text{ N m}^{-1}) = 8.95\times10^{-5}\text{ J.} \quad \text{(Answer)}$$

Problem 3.19

A soap bubble of radius 0.01m is expanded to a radius of 0.1m. Calculate the work done in the process. [Surface tension of soap solution $T = 26\times10^{-3}$ N m^{-1}]

Solution

We know if the change in area is ΔA, then

$$W = T\,\Delta A. \qquad \text{(p3.19a)}$$

Here,

Increased radius, $r_2 = 0.1\text{ m} = 10^{-1}$ m,

Initial radius, $r_1 = 0.01\text{ m} = 10^{-2}$ m,

Surface tension, $T = 26\times10^{-3}$ N.m^{-1},

Work done, $W = ?$

The soap bubble has two surfaces, so the increase of area is

$$\Delta A = 2(4\pi)\left(r_2^2 - r_1^2\right). \qquad \text{(p3.19b)}$$

Substituting the parameters in Eq. (p3.19b), we get

$$\Delta A = (8)(3.14)[(10^{-1}\text{ m})^2 - (10^{-2}\text{ m})^2] = 2.487\times10^{-1}\text{ m}^2.$$

Therefore, work done is obtained from Eq. (p3.19a)

$$W = (26\times10^{-3}\text{ N.m}^{-1})(2.487\times10^{-1}\text{ m}^2) = 6.47\times10^{-3}\text{ J.} \quad \text{(Answer)}$$

Problem 3.20

Calculate the error in reading of a mercury barometer whose tube has a diameter 4mm. The angle of contact of mercury with glass is 140^o, surface tension of mercury is $T = 465\times10^{-3}$ Nm^{-1} and density of mercury is $\rho = 13.6\times10^{-3}$ kg m^{-3}.

Solution

Let the rise of mercury in the capillary tube be *h*.
We know

$$T = \frac{h\rho g r}{2\cos\theta}. \qquad \text{(p3.20a)}$$

Here,

Surface tension of mercury, $T = 465\times10^{-3}\ \mathrm{N/m}$,

Angle of contact, $\theta = 140^o$,

Radius of the tube, $r = \frac{4}{2}\ \mathrm{mm} = 2\times10^{-3}\ \mathrm{m}$,

Density of mercury, $\rho = 13.6\times10^{3}\ \mathrm{kg.m^{-3}}$,

Acceleration due to gravity, $g = 9.8\ \mathrm{m.s^{-2}}$,

Height, $h = ?$

Substituting the above parameters into Eq. (p3.20a), we obtain

$$h = \frac{2T\cos\theta}{\rho g r} = \frac{(2)(465\times10^{-3}\ \mathrm{N/m})(\cos140^o)}{(13.6\times10^{3}\ \mathrm{kg.m^{-3}})(9.8\ \mathrm{m.s^{-2}})(2\times10^{-3}\ \mathrm{m})} = -2.67\times10^{-3}\ \mathrm{m} = 0.267\ \mathrm{cm}. \quad \text{(Answer)}$$

Problem 3.21

A capillary tube of diameter 0.5mm is vertically dipped into a liquid of density 0.8×10^{-3} kg m^{-3} and it wets the tube. The liquid rises 3.06cm of height in the tube. Calculate the surface tension of the liquid.

Solution

We know the formula for surface tension

$$T = \frac{h\rho g r}{2\cos\theta}. \qquad \text{(p3.21a)}$$

If θ is very small, then $\cos\theta = 1,$ and the above formula reduces to

$$T = \frac{h\rho g r}{2}. \qquad \text{(p3.21b)}$$

Here

Radius of the tube, $r = \frac{0.5\ \mathrm{mm}}{2} = 0.25\times10^{-3}\ \mathrm{m}$,

Density of the liquid, $\rho = 0.8\times10^{3}\ \mathrm{kg.m^{-3}}$,

Acceleration due to gravity, $g = 9.8\ \mathrm{m.s^{-2}}$,

Rise of liquid in the tube, $h = 3.06\ \mathrm{cm} = 3.06\times10^{-2}\ \mathrm{m}$,

Surface tension, $T = ?$

Substituting the above parameters in Eq. (p3.21b), we obtain

$$T = \frac{(3.06\times10^{-2}\ \mathrm{m})(0.8\times10^{3}\ \mathrm{kg.m^{-3}})(9.8\ \mathrm{m.s^{-2}})(0.25\times10^{-3}\ \mathrm{m})}{2} = 3.0\times10^{-2}\ \mathrm{Nm^{-1}}. \quad \text{(Answer)}$$

Problem 3.22

Models of torpedoes are sometimes tested in a pipe of flowing water, much as a wind tunnel is used to test model airplanes. Consider a circular pipe of internal diameter 10inch and a torpedo model, aligned along the axis of the pipe, with a diameter of 2inch. The torpedo is to be tested with water flowing past it at 8ft/s. (a) With what speed must the water flow in the unconstructed part of the pipe? (b) What will the pressure difference be between the constricted and unconstructed parts of the pipe?

Solution

(a) We know the formula $A_1 v_1 = A_2 v_2$.

For the torpedo model, the cross-sectional area through which water follows is

$$A_2 = \frac{\pi}{4}\left(d_1^2 - d_2^2\right) = \frac{\pi}{4}\left[\left(\frac{10}{12}\text{ ft}\right)^2 - \left(\frac{2}{12}\text{ ft}\right)^2\right] = \frac{\pi}{6}\text{ft}^2. \qquad \text{(p3.22a)}$$

Cross-sectional area of the circular pipe is

$$A_1 = \frac{\pi}{4}d_1^2 = \frac{\pi}{4}\left(\frac{10}{12}\text{ ft}\right)^2 = \frac{25\pi}{144}\text{ft}^2. \qquad \text{(p3.22b)}$$

Speed of water in the torpedo, $v_2 = 8.0$ ft/s.

Speed of water in the unconstructed part of the pipe,

$$v_1 = \frac{A_2 v_2}{A_1} = (8.0\text{ ft/s})\frac{(\pi/6\text{ ft}^2)}{(25\pi/144\text{ ft}^2)} = 7.68\text{ ft/s.} \quad \text{(Answer)}$$

(b) Pressure difference,

$$\Delta p = p_2 - p_1 = \frac{1}{2}\rho(v_2^2 - v_1^2) = \frac{1}{2}\left(\frac{62.4}{32}\text{ slug/ft}^3\right)\left[(8\text{ ft/s})^2 - (7.68\text{ ft/s})^2\right] = 4.89\text{ lb/ft}^2. \quad \text{(Answer)}$$

Problem 3.23

How much work is done by pressure on forcing 50 m^3 of water through a 1.50cm diameter pipe if the difference in pressure at the two ends of the pipe is 15 kg m^{-2}?

Solution

The required work done,

$$\begin{aligned} W &= (\text{pressure difference})(\text{volume of fluid}) \\ &= \left(15\text{ kg.m}^{-2} \times 9.8\text{ m.s}^{-2}\right)\left(50\text{ m}^3\right) \\ &= 7.35\text{ kJ.} \qquad \text{(Answer)} \end{aligned}$$

Problem 3.24

Water falls from a height of 60m at the rate of $10\ \text{m}^3 s^{-1}$ and drive a water turbine. What is the maximum power that can be developed by this turbine?

Solution

The work done is

$$W = mg(y_2 - y_1) = mgh, \qquad \text{(p3.24a)}$$

where

$$m = \text{(volume)(density)} = (10\ \text{m}^3)\left(997\ \text{kg.m}^{-3}\right) = 9970\ \text{kg}. \qquad \text{(p3.24b)}$$

Therefore, the work is from Eq. (p3.24a)

$$W = (9970\ \text{kg})(9.8\ \text{ms}^{-2})(60\ \text{m}) = 5.86\times10^6\ \text{J}. \qquad \text{(p3.24c)}$$

Therefore, the required power is

$$P = \frac{W}{t} = \frac{5.86\times10^6\ \text{J}}{60\ \text{s}} = 9.77\times10^4\ \text{Watt.} \quad \text{(Answer)}$$

Problem 3.25

By applying the Bernoulli's equation and the equation of continuity to points 1 and 2 of the Venturimeter of Fig. p3.25, show that the speed of flow at the entrance is

$$v = a\sqrt{\frac{2(\rho' - \rho)gh}{\rho(A^2 - a^2)}}.$$

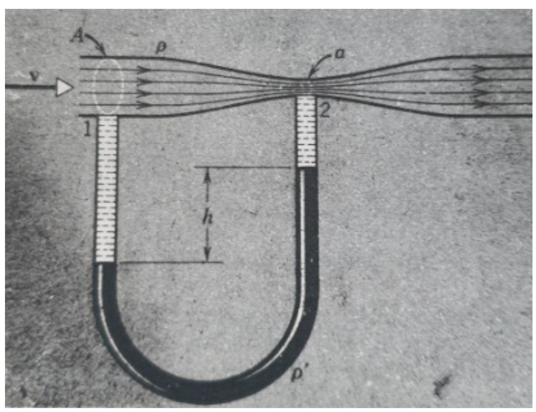

Fig. p3.25

Solution

Let us apply the Bernouli's equation

$$p + \frac{1}{2}\rho v^2 + \rho g y = C, \qquad \text{(p3.25a)}$$

to two points 1 and 2. Thus, we have

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2, \qquad \text{(p3.25b)}$$

where we have ignored the gravitational potential energy term, as the center of the cross-sectional areas *A* and *a* are at the same horizontal level.

Thus,

$$\Delta p = p_1 - p_2 = \frac{1}{2}\rho(v_2^2 - v_1^2). \qquad \text{(p3.25c)}$$

But we know the equation of continuity

$$A_1 v_1 = A_2 v_2. \qquad \text{(p3.25d)}$$

In our case, Eq. (p3.25d) becomes

$$Av_1 = av_2$$

or

$$v_2 = \frac{Av_1}{a}. \qquad \text{(p3.25e)}$$

Substituting Eq. (p3.25e) into Eq. (p3.25c), we obtain

$$\Delta p = \frac{1}{2}\rho\left[\left(\frac{Av_1}{a}\right)^2 - v_1^2\right] = \frac{1}{2}\rho\frac{v_1^2}{a^2}\left[A^2 - a^2\right]. \qquad \text{(p3.25f)}$$

But the pressure difference shown in the manometer can be written as

$$\Delta p = hg\rho' - hg\rho. \qquad \text{(p3.25g)}$$

Equating Eqs. (p3.25f) and (p3.25g) and setting $v_1 = v$, we have

$$\Delta p = \frac{1}{2}\rho\frac{v^2}{a^2}\left[A^2 - a^2\right] = hg\rho' - hg\rho\ ,$$

from which we readily obtain

$$v = a\sqrt{\frac{2(\rho' - \rho)gh}{\rho(A^2 - a^2)}}. \qquad \text{(Showed)}$$

Problem 3.26

A venturimeter has a pipe diameter of 10.0cm and a throat diameter of 5.0cm. If the water pressure in the pipe is $5.0\ \text{kg}\,\text{cm}^{-2}$ and in the throat is $3.0\ \text{kg}\,\text{cm}^{-2}$, determine the rate of flow of water.

Solution

We know the difference in pressures in a venturimeter,

$$\Delta p = p_1 - p_2 = \frac{1}{2}\rho v^2\left[\frac{A^2}{a^2} - 1\right]. \qquad \text{(p3.26a)}$$

We have

$$\frac{A}{\text{a}} = \frac{\pi(d_1/2)^2}{\pi(d_2/2)^2} = \left(\frac{d_1}{d_2}\right)^2 = \left(\frac{10.0\ \text{cm}}{5.0\ \text{cm}}\right)^2 = 4.0. \qquad \text{(p3.26b)}$$

Substituting the given values of different quantities, we obtain

$$\left[\frac{5.0\ \text{kg}}{(0.01\ \text{m})^2} - \frac{5.0\ \text{kg}}{(0.01\ \text{m})^2}\right](9.8\ \text{m.s}^{-2}) = \frac{1}{2}(997\ \text{kg.m}^{-3})v^2\left[(4.0)^2 - 1\right]. \qquad \text{(p3.26c)}$$

From Eq. (p3.26c), we get

$$v = 1.35\ \text{m/s}.$$

The volume flux

$$R = Av = \pi\left(\frac{d_1}{2}\right)^2 v = (3.14)\left(\frac{10.0}{2}\ \text{m}\right)^2(1.35\ \text{m/s}) = 106\ \text{m}^3\text{s}^{-1}. \quad \text{(Answer)}$$

Problem 3.27

Figure p3.27 shows water discharging from an orifice in a large tank at a distance h below the water surface. (a) Apply the Bernoulli's equation to a streamline connecting points 1, 2, and 3, and show that the speed of efflux is

$$v = \sqrt{2gh}\,.$$

This is known as Torricelli's law. (b) If the orifice were curved directly upward, how high would the liquid stream rise? (c) How would viscosity or turbulence affect the analysis?

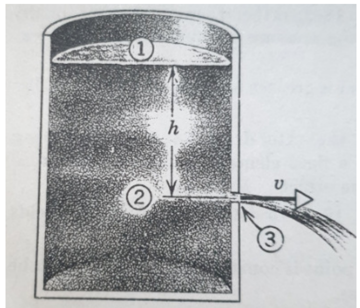

Fig. p3.27

Solution

(a) Bernoulli's equation yields

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2 = p_3 + \frac{1}{2}\rho v_3^2 + \rho g y_3\,. \qquad \text{(p3.27a)}$$

We note that

$$y_3 = y_2 \quad \text{and} \quad y_1 - y_2 = h. \qquad \text{(p3.27b)}$$

Furthermore

$$v_1 = v_2 \approx 0; \quad v_3 = v. \qquad \text{(p3.27c)}$$

Since the orifice is exposed to the atmosphere,

$$p_3 = p_1 = p_0 \text{ (atmospheric pressure).} \qquad \text{(p3.27d)}$$

Then Eq. (p3.27a) becomes

$$p_0 + 0 + \rho g y_1 = p_2 + 0 + \rho g y_2 = p_0 + \frac{1}{2}\rho v^2 + \rho g y_2\,,$$

from which we obtain

$$\rho g y_1 = \frac{1}{2}\rho v^2 + \rho g y_2\,. \qquad \text{(p3.27e)}$$

From Eq. (p3.27e), we obtain

$$v = \sqrt{2g(y_1 - y_2)} = \sqrt{2gh}\,. \qquad \text{(Answer)} \qquad \text{(p3.27f)}$$

(b) The height H to which the liquid stream rises is determined by the equation

$$0 = v^2 - 2gH,$$

where v is the initial speed. Then

$$H = \frac{v^2}{2g} = h. \qquad \text{(Answer)}$$

(c) Turbulence causes vortices and pressure is reduced in this region. They take away some energy and therefore offer resistance to the flow from point 2 to 3. The same thing happens due to existence of viscosity. Consequently the speed of efflux will be lower than that given by v. (Answer)

Problem 3.28

A tank is filled with water to a height H. A hole is punched in one of the walls at a depth h below the water surface (Fig. p3.28). (a) Find the distance x from the foot of the wall at which the stream strikes the floor. (b) Could a hole be punched at another depth so that this second stream would have the same range? If so, at what depth?

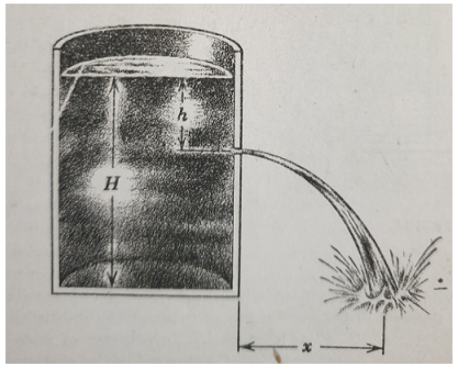

Fig. p3.28

Solution

(a) The efflux velocity which has horizontal direction is given by

$$v=\sqrt{2gh}\,. \qquad \text{(p3.28a)}$$

Distance x is traversed in time t such that

$$x=vt=\sqrt{2gh}\,t. \qquad \text{(p3.28b)}$$

The vertical height $(H-h)$ is traversed in the same time t, since the initial vertical component of velocity is zero.

$$S=H-h=0+\frac{1}{2}gt^2,$$

from which we obtain

$$t=\sqrt{2(H-h/g}\,. \qquad \text{(p3.28c)}$$

Eliminating t from Eqs. (p3.28b) and (p3.28c) to find

$$x=2\sqrt{h(H-h)}\,. \qquad \text{(p3.28d)}$$

(b) Let the second hole be punched at some other depth y. Then,

$$v=\sqrt{2gy}\,.$$

Proceeding as above, we obtain

$$t=\sqrt{2(H-h)/g}\,.$$

We have

$$x=vt=2\sqrt{y(H-y)}=2\sqrt{h(H-h)}$$

or

$$yH-y^2=hH-h^2$$

or

$$(y-h)(H-y-h)=0,$$

from which we obtain $y=h,$ as before.

The other solution is $y=H-h.$ That is at a height h above the bottom, the second stream would give the same range. (Answer)

Problem 3.29
The upper surface of water in a standpipe is a height H above level ground. At what depth h should a small hole be put to make the emerging horizontal water stream strike the ground at the maximum distance from the base of the standpipe? What is the maximum distance?

Solution
From the figure below, the horizontal range is

$$x = 2\sqrt{h(H-h)}. \qquad \text{(p3.29a)}$$

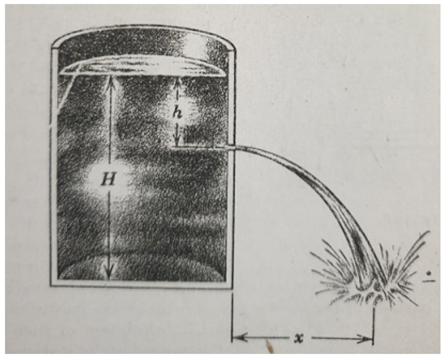

Fig. p3.29

In order to find maximum range, we differentiate x with respect to h and set it to zero:

$$\frac{dx}{dh} = \frac{2(H-h)}{\sqrt{h(H-h)}} = 0,$$

from which we obtain $H - 2h = 0$ or

$$h = H/2. \qquad \text{(p3.29b)}$$

Therefore, the maximum range is, by substituting (p3.29b) into (p3.29a)

$$x_{\max} = 2\sqrt{\frac{H}{2}\left(H - \frac{H}{2}\right)} = H. \quad \text{(Answer)}$$

Problem 3.30
A Pitot tube is mounted on an airplane wing to determine the speed of the plane relative to the air. The tube contains alcohol and indicates a level difference of 12.45cm. What is the plane's speed relative to the air?

Solution
In the case of a Pitot tube, the speed of gas flowing is given by

$$v = \sqrt{2gh\rho'/\rho}. \qquad \text{(p3.30a)}$$

Here,

$\rho' = 789\ \text{kg.m}^{-3}$ (density of alcohol)

$\rho = 1.204\ \text{kg.m}^{-3}$ (density of air at NTP

$h = 70.1245\ \text{m}$ (p3.30b)

$g = 9.8\ \text{m.s}^{-2}$

Thus, the plane's speed relative to the air

$$v = \sqrt{(2)(9.8\ \text{m.s}^{-2})(70.1245\ \text{m})(789\ \text{kg.m}^{-3})/1.204\ \text{kg.m}^{-3}} = 39.23\ \text{m.s}^{-1}. \quad \text{(Answer)}$$

Problem 3.31
If the speed of flow past the lower surface of a wing is 106.68m/s, what speed of flow over the upper surface will give a lift of $97.634\ \mathrm{kg\,m^{-2}}$?

Solution
The pressure difference is obtained from the formula

$$\Delta p = p_2 - p_1 = \frac{1}{2}\rho(v_1^2 - v_2^2). \qquad \text{(p3.31a)}$$

Substituting the values of the given parameters in the above equation, we have

$$\Delta p = \frac{1}{2}(1.2754\ \mathrm{kg.m^{-3}})\left[v_1^2 - (106.68\ \mathrm{m.s^{-1}})^2\right]. \qquad \text{(p3.31b)}$$

We know

$$\text{Lift} = (\Delta p)(Area) = (\Delta p)(1.0\ \mathrm{m^2}). \qquad \text{(p3.31c)}$$

Since, the Lift is given as $97.634\ \mathrm{kg\,m^{-2}} \equiv (97.634\ \mathrm{kg\,m^{-2}})(9.8\ \mathrm{m.s^{-2}}) = 956.8132\ \mathrm{N},$ we have from Eq. (p3.31a)

$$956.8132\ \mathrm{N} = \frac{1}{2}(1.2754\ \mathrm{kg.m^{-3}})\left[v_1^2 - (106.68\ \mathrm{m.s^{-1}})^2\right](1.0\ \mathrm{m^2}). \qquad \text{(p3.31d)}$$

Solving Eq. (p3.31d) for v_1, we obtain

$$v_1 \approx 113.50\ \mathrm{m.s^{-1}}. \quad \text{(Answer)}$$

Problem 3.32
Determine the force required to drive a copper sheet of area $10^{-2}\,\mathrm{m^2}$ horizontally with a velocity $3\times10^{-2}\ \mathrm{m\,s^{-1}}$ through oil of thickness 2×10^{-3} m. The coefficient of viscosity of the oil is $1.55\ \mathrm{Ns\,m^{-2}}$.

Solution
We know that the expression for the force is

$$F = \eta A\frac{dv}{dy}, \qquad \text{(p3.32a)}$$

where

The coefficient of viscosity, $\eta = 1.55\ N\ \mathrm{Nsm^{-2}}$,

Area, $A = 10^{-2}\ \mathrm{m^2}$,

Velocity gradient, $\dfrac{dv}{dy} = \dfrac{3\times10^{-2}\ \mathrm{m\,s^{-1}}}{2\times10^{-3}\ \mathrm{m}} = 15\ \mathrm{s^{-1}}$,

Force, $F = ?$

Substituting above parameter values in Eq. (p3.32a), we obtain

$$F = (1.55\ N\ \mathrm{Nsm^{-2}})(10^{-2}\ \mathrm{m^2})(15\ \mathrm{s^{-1}}) = 0.2325\ \mathrm{N}. \quad \text{(Answer)}$$

Problem 3.33

A rain drop is falling through the air with a terminal velocity 1.2×10^{-2} m s^{-1}. The coefficient of viscosity of air is 1.8×10^{-5} Ns m^{-2}. What is the diameter of the drop?

Solution

We know that if the radius of the rain drop is *r*, then

$$v = 2r^2(\rho_s - \rho_f)g/9\eta, \tag{p3.33a}$$

or

$$r = \sqrt{9\eta v/2(\rho_s - \rho_f)g}, \tag{p3.33b}$$

where

Terminal speed, $v = 1.2\times10^{-2}$ m s^{-1},

Coefficien t of viscosi ty, $\eta = 1.8\times10^{-5}$ Ns m^{-2},

Acceleration due to gravity, $g = 9.8$ m.s^{-2},

Density of water, $\rho_s = 1.0\times10^3$ kg.m^{-3},

Density of air, $\rho_f = 1.21$ kg.m^{-3},

Diameter, $d = ?$

Using the above parameters in Eq. (p3.33b), we obtain

$$r = \sqrt{\frac{(9)(1.8\times10^{-5}\ \text{Ns m}^{-2})(1.2\times10^{-2}\ \text{m s})}{2(1.0\times10^3\ \text{kg.m}^{-3} - 1.21\ \text{kg.m}^{-3})(9.8\ \text{m.s}^{-2})}} = 9.97\times10^{-6}\ \text{m}.$$

Therefore, diameter is

$$d = 2r = 2\times9.97\times10^{-6}\ \text{m} = 1.99\times10^{-5}\ \text{m}. \quad \text{(Answer)}$$

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