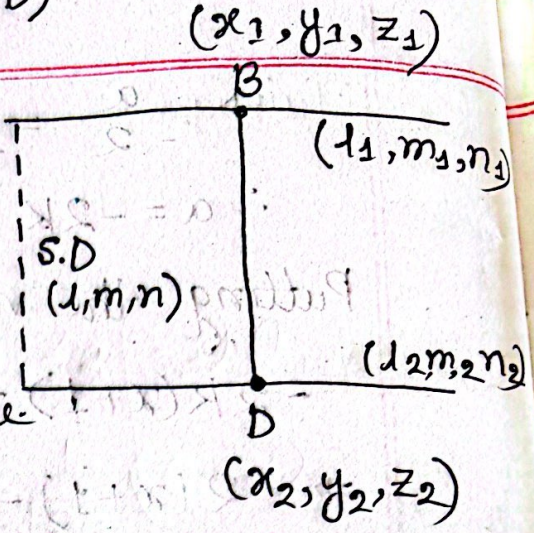
Shortest Distance (S.D)



দুটো লাইন দুটো Plane এ অবস্থিত কেন parallel (non-intersecting) দুটো লাইনের উপর perpendicular distance এর যে ছায়া, সেই ছায়াকে Shortest Distance বলে। / Perpendicular line এর projection ই Shortest Distance.

$$S.D = l(x_2 - x_1) + m(y_2 - y_1) + n(z_2 - z_1)$$

- দুটো লাইনের Direction of cosine যদি কোনো লাইনের উপর perpendicular হয়:

$$SD \perp (i) \quad ll_1 + mm_1 + nn_1 = 0$$

$$SD \perp (ii) \quad ll_2 + mm_2 + nn_2 = 0$$

$$\frac{l}{m_1 n_2 - m_2 n_1} = \frac{-m}{n_1 l_2 - n_2 l_1} = \frac{n}{m_2 l_1 - m_1 l_2}$$

→ the two lines are,

$$\frac{x - x_1}{l_1} = \frac{y - y_1}{m_1} = \frac{z - z_1}{n_1} \quad \text{------(i)}$$

$$\frac{x - x_2}{l_2} = \frac{y - y_2}{m_2} = \frac{z - z_2}{n_2} \quad \text{------(ii)}$$

(1) Find the shortest distance between the lines,

$$\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1} \quad \& \quad \frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$$

Given that, $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ ----- (i)

$$\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4} \quad \text{----- (ii)}$$

(i) $\Rightarrow (x_1,y_1,z_1)=(3,8,3)$

$(l_1,m_1,n_1)=(3,-1,1)$

(ii) $\Rightarrow (x_2,y_2,z_2)=(-3,-7,6)$

$(l_2,m_2,n_2)=(-3,2,4)$

let, l,m,n be the d.c of S.D line. Since S.D line is perpendicular to (i) and (ii) then,

$$3l-m+n=0$$
$$-3l+2m+4n=0$$

By cross multiplication,

$$\frac{l}{-4-2}=\frac{-m}{12+3}=\frac{n}{6-3}$$

$$\Rightarrow \frac{l}{-6}=\frac{m}{-15}=\frac{n}{3}=\frac{\sqrt{l^2+m^2+n^2}}{\sqrt{(-6)^2+(-15)^2+3^2}}$$

$$\Rightarrow \frac{l}{-6}=\frac{m}{-15}=\frac{n}{3}=\frac{1}{3\sqrt{30}}$$

$$\therefore \frac{l}{-2}=\frac{m}{-5}=\frac{n}{1}=\frac{1}{\sqrt{30}}$$

$\therefore l = \frac{-2}{\sqrt{30}}, m = \frac{-5}{\sqrt{30}}, n = \frac{1}{\sqrt{30}}$

$S.D = l(x_2 - x_1) + m(y_2 - y_1) + n(z_2 - z_1)$

$= \frac{-2}{\sqrt{30}}(-3-3) + \frac{-5}{\sqrt{30}}(-7-8) + \frac{1}{\sqrt{30}}(6-3)$

$= \frac{12+75+3}{\sqrt{30}} = \frac{90}{\sqrt{30}} = 3\sqrt{30}$ (Ans)

(2) $\frac{x+1}{2} = \frac{y-5}{3} = \frac{z-7}{-1}$ & $\frac{x+4}{5} = \frac{y-1}{-3} = \frac{z-3}{1}$

(3) $\frac{x+3}{2} = \frac{y+15}{3} = \frac{z-7}{-3}$ & $\frac{x+1}{4} = \frac{y+1}{5} = \frac{z+1}{-1}$

(4) $\frac{2x+3}{-2} = \frac{y-6}{3} = \frac{2z-6}{3}$ & $\frac{x}{2} = \frac{y-6}{-2} = \frac{z-3}{1}$

$\frac{x+3/2}{-1}$ $\frac{z-3}{3/2}$