PHY109 Engineering Physics I
Chapter 5
Part-3 (Relevant Problems)

5.10 Some Relevant Problems

Problem 5.1 *An Electric Dipole*. The figure shows a positive and a negative charge of equal magnitude q placed a distance $2a$ apart, a configuration called an electric dipole. What is the field **E** due to these charges at point P, a distance r along the perpendicular bisector of the line joining the charges? You can assume $r >> a$.

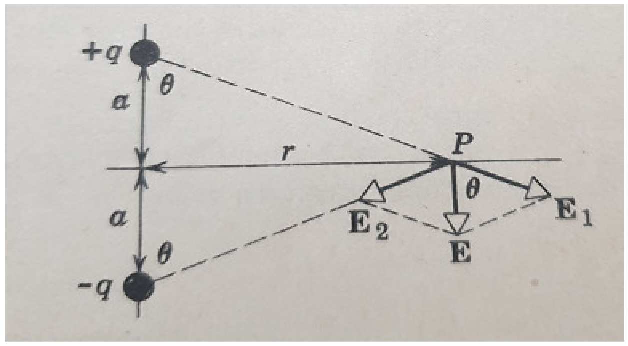

Fig. p5.1a

Solution
The resultant field is

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2,$$

where

$$E_1 = E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{a^2 + r^2}.$$

The vector sum of $\mathbf{E}_1$ and $\mathbf{E}_2$ point vertically downward and has the magnitude

$$E = 2E_1 \cos\theta.$$

From the Fig. p5.1a, we see that

$$\cos\theta = \frac{a}{\sqrt{a^2 + r^2}}.$$

Thu we have

$$E = \frac{2}{4\pi\varepsilon_0}\frac{q}{(a^2 + r^2)}\frac{a}{\sqrt{a^2 + r^2}} = \frac{1}{4\pi\varepsilon_0}\frac{2aq}{(a^2 + r^2)^{3/2}}.$$

If $r >> a$, we can neglect a in the denominator; the above equation then reduces to

$$E \approx \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3},$$

where $p = 2aq$ is known as the dipole moment.

Problem 5.2: Figure below shows a charge $q_1 = +1.0\times10^{-6}$ coul 10.0 cm from a charge $q_2 = +2.0\times10^{-6}$ coul. At what point on the line joining the two charges is the electric field strength zero?

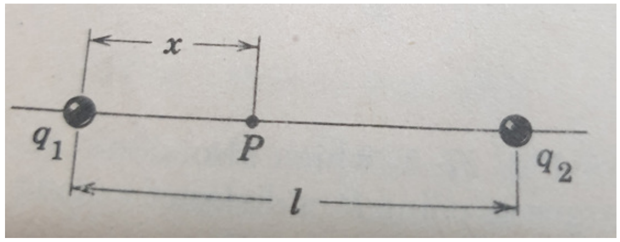

Fig. p5.2a

Solution

The point must lie between the charges because only here do the forces exerted by q_1 and q_2 on a test charge oppose each other. If $\mathbf{E}_1$ is the electric field strength due to q_1 and $\mathbf{E}_2$ that due to q_2, we must have

$$E_1 = E_2$$

or

$$\frac{1}{4\pi\varepsilon_0}\frac{q_1}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{(l-x)^2},$$

where x is the distance from q_1 and l equals 10 cm. Solving for x, we obtain

$$x = \frac{l}{1+\sqrt{q_2/q_1}} = \frac{10\text{ cm}}{1+\sqrt{2}} = 4.1\text{ cm}.$$

Problem 5.3: Figure below shows a ring of charge q and of radius a. Calculate $\mathbf{E}$ for points on the axis of the ring a distance x from its center.

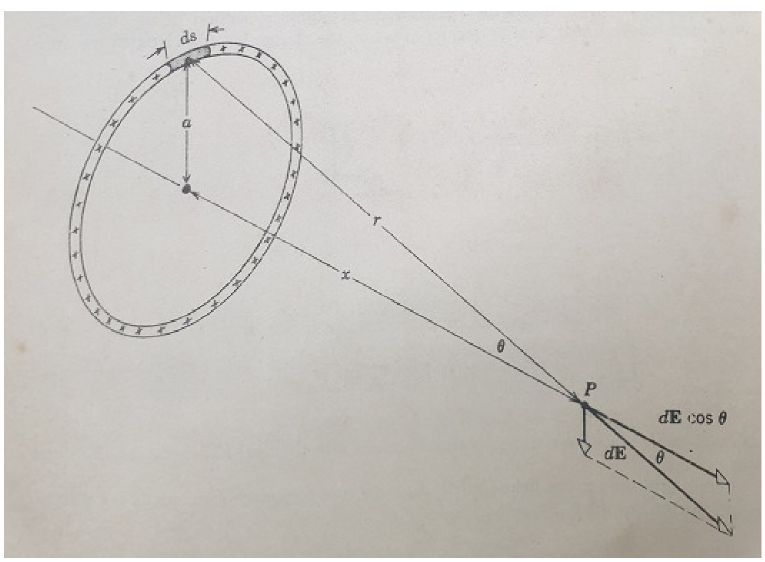

Fig. 5.3a

Solution

Let us consider a differential element of the ring of length *ds*, located at the top of the ring in the given figure. It contains an element of charge given by

$$dq = q\frac{ds}{2\pi a},$$

where $2\pi a$ is the circumference of the ring. The element sets up a differential electric field $d\mathbf{E}$ at point P. The resultant field $\mathbf{E}$ at point P is found by integrating the effects of all the elements that make up the ring. From symmetry this resultant field must lie along the ring axis. Thus only the component of $d\mathbf{E}$ parallel to this axis contributes to the final result. The component perpendicular to the axis is cancelled out by an equal but opposite component established by the charge element on the opposite side of the ring.

Thus the general vector integral

$$\mathbf{E} = \int d\mathbf{E}$$

becomes a scalar integral

$$E = \int dE\cos\theta.$$

We have

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\left(\frac{qds}{2\pi a}\right)\frac{1}{a^2 + x^2}.$$

From the given figure

$$\cos\theta = \frac{x}{\sqrt{a^2 + x^2}}.$$

We note that, for a given point P, x has the same value for all charge elements and is not a variable, thus we obtain

$$E = \int dE\cos\theta = \int \frac{1}{4\pi\varepsilon_0}\left(\frac{qds}{2\pi a}\right)\frac{x}{(a^2 + x^2)^{3/2}}$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{qx}{2\pi a}\right)\frac{1}{(a^2 + x^2)^{3/2}}\int ds$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{qx}{2\pi a}\right)\frac{1}{(a^2 + x^2)^{3/2}}2\pi a$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{qx}{(a^2 + x^2)^{3/2}}.$$

Special Cases

1. At $x = 0$, $E = 0$.
2. At $x >> a$,

$$E \approx \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}.$$

Problem 5.4: *Line of Charge*. Figure given below shows a section of an infinite line of charge whose linear charge density (that is, the charge per unit length measured in coul/meter) has the constant value λ. Calculate the electric field **E** a distance *y* from the line.

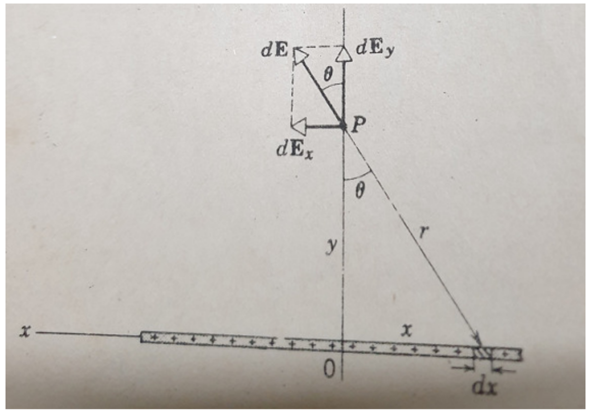

Fig.5.4a

Solution

The magnitude of the field contribution DE due to charge element dq $(= \lambda\, dx)$ is given by

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{y^2+x^2}.$$

The vector $d\mathbf{E}$ as the figure shows, has the components

$$dE_x = -dE\sin\theta \quad \text{and} \quad dE_y = dE\cos\theta .$$

The minus sign in front of dE_x indicates that $d\mathbf{E}_x$ points in the negative *x*-direction. Thus

$$E_x = \int dE_x = -\int_{x=-\infty}^{x=+\infty} \sin\theta\, dE \quad \text{and} \quad E_y = \int dE_y = \int_{x=-\infty}^{x=+\infty} \cos\theta\, dE .$$

E_x must be zero because every charge element on the right has a corresponding element on the left such that their field contributions in the *x*-direction cancel. Thus **E** points entirely in the *y*-direction. Because the contributions to E_y from the right- and left-hand halves of the rod are equal, we can write

$$E = Ey = 2\int_{x=0}^{x=+\infty} \cos\theta\, dE .$$

Substituting the expression for *dE*, we obtain

$$E = \frac{\lambda}{2\pi\varepsilon_0}\int_{x=-\infty}^{x=+\infty} \cos\theta \frac{dx}{y^2+x^2} .$$

The relation between *x* and θ is

$$x = y\tan\theta .$$

Therefore,

$$E = \frac{\lambda}{2\pi\varepsilon_0 y}\int_{\theta=0}^{\theta=\pi/2} \cos\theta\, d\theta = \frac{\lambda}{2\pi\varepsilon_0 y}.$$

Problem 5.5: A particle of mass *m* and charge *q* is placed at rest in a uniform electric field (figure given below) and released. Describe its motion.

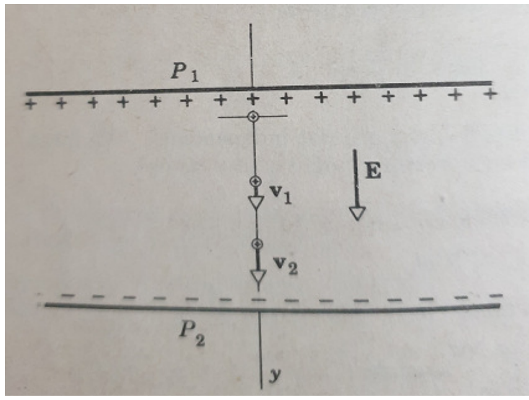

Fig.5.5a

Solution

The motion resembles that of a body falling in the earth's gravitational field. The constant acceleration is given by

$$a = \frac{F}{m} = \frac{qE}{m}.$$

The equations for uniformly accelerated motion then apply. With $u_0 = 0$, they are

$$u = at = \frac{qEt}{m},$$

$$y = \frac{1}{2}at^2 = \frac{qEt^2}{2m},$$

and

$$u^2 = 2ay = \frac{2qEy}{m}.$$

The kinetic energy attained after moving a distance *y* is found from

$$K = \frac{1}{2}mu^2 = \frac{1}{2}m\left(\frac{2qEy}{m}\right) = qEy.$$

Problem 5.6: *Deflecting an Electron Beam*. Figure given below shows an electron of mass m and charge e projected with speed u_0 at right angles to a uniform field **E**. Describe its motion.

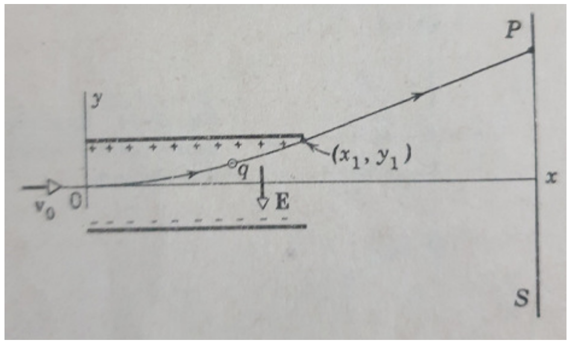

Fig.5.6a

Solution

The motion is like that of a projectile fired horizontally in the earth's gravitational field. The horizontal (x) and the vertical (y) motions being given by

$$x = u_0 t \quad \text{and} \quad y = \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{F}{m}\right)t^2 = \frac{1}{2}\left(\frac{eE}{m}\right)t^2 = \frac{eEt^2}{2m}.$$

Eliminating t from the above two equations, we get the equation of motion of the trajectory as $y = eE/2mu_0^2 x^2$.

When the electron emerges from the plates in the figure, it travels (neglecting gravity) in a straight line tangent to the parabola at the exit point. We can let it fall on a fluorescent screen S placed some distance beyond the plates. Together with other electrons following the same path, it will then make itself visible as a small luminous spot; this is the principle of the electrostatic *cathode-ray oscilloscope*.

Problem 5.7: An electric dipole consists of two opposite charges of magnitude $q = 1.0\times10^{-6}$ coul separated by d=2.0cm. The dipole is placed in an external electric field of magnitude $E = 1.0\times10^5$ nt/coul.

(a) What maximum torque does the field exert on the dipole?
(b) How much work must an external agent do to turn the dipole end for end, starting from a position of alignment $\theta = 0$?

Solution

(a) The maximum torque is found by putting $\theta = 90^o$. Thus

$$\tau = pE\sin\theta = qd\sin\theta = (1.0\times10^{-6}\text{ coul})(0.020\text{ meter})(1.0\times105\text{ nt/coul})(\sin 90^o)$$
$$= 2.0\times10^{-3}\,\text{nt}-\text{m}.$$

(b) The work is the difference in potential energy U between the positions $\theta = 180^o$ and $\theta = 0^o$. Thus

$$W = U(\theta = 180^o) - U(\theta = 0^o) = (-pE\cos 180^o) - (-pE\cos 0^o)$$
$$= 2pE = 2qdE$$
$$= (2)(1.0\times10^{-6}\text{ coul})(0.020\text{ meter})(1.0\times105\text{ nt/coul})$$
$$= 4.0\times10^{-3}\,\text{joule}.$$

Problem 5.8: A thin nonconducting rod of finite length l carries a total charge q, spread uniformly along it. Show that E at point P on the perpendicular bisector in the figure below is given by

$$E = \frac{q}{2\pi\varepsilon_0 y}\frac{1}{\sqrt{l^2+4y^2}}.$$

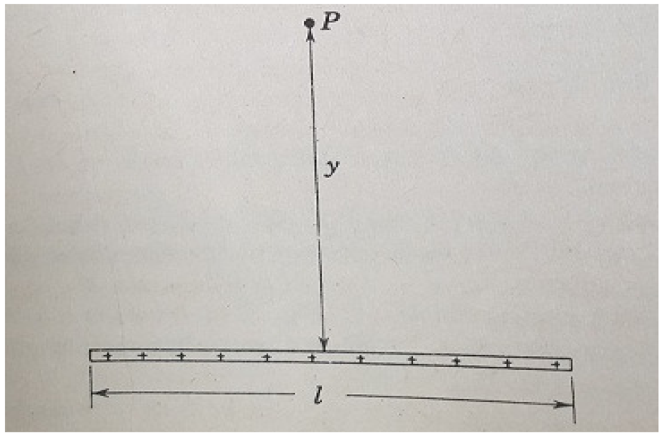

Fig.5.8a

Solution

Let λ be the charge of the rod per unit length. We consider a small element dx of the rod at a distance x from the middle point of the rod. Therefore the charge of the element is $dq = \lambda dx$. The differential electric field strength due to this charge element at point P is

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{x^2+y^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{x^2+y^2}.$$

It can be shown that the horizontal components will be cancelled out and only the vertical components will survive. Thus the total electric field along the perpendicular to the length of the wire is given by

$$E = \int dE_y = \int dE\frac{y}{\sqrt{x^2+y^2}} = \int \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{x^2+y^2}\frac{y}{\sqrt{x^2+y^2}} = \frac{\lambda y}{4\pi\varepsilon_0}\int_{-l/2}^{l/2}\frac{dx}{(x^2+y^2)^{3/2}}.$$

By substituting $x = y\tan\theta$, we have $dx = y\sec^2\theta\, d\theta$ and the limits change to $\theta = \tan^{-1}(-l/2y)$ and $\theta = \tan^{-1}(l/2y)$ correspond to $x = -l/2$ and $x = l/2$ respectively. Therefore, the integral becomes

$$E = \frac{\lambda y}{4\pi\varepsilon_0}\int_{\tan^{-1}(-l/2y)}^{\tan^{-1}(l/2y)}\frac{y\sec^2\theta\, d\theta}{(y^2\tan^2\theta + y^2)^{3/2}} = \frac{\lambda}{4\pi\varepsilon_0 y}\int_{\tan^{-1}(-l/2y)}^{\tan^{-1}(l/2y)}\cos\theta\, d\theta = \frac{\lambda}{4\pi\varepsilon_0 y}\left[\sin\theta\right]_{\tan^{-1}(-l/2y)}^{\tan^{-1}(l/2y)}$$

$$= \frac{\lambda}{4\pi\varepsilon_0 y}\left[\sin\tan^{-1}\left(\frac{l}{2y}\right) - \sin\tan^{-1}\left(-\frac{l}{2y}\right)\right]$$

$$= \frac{\lambda}{4\pi\varepsilon_0 y}\left[\sin\sin^{-1}\left(\frac{l}{\sqrt{l^2+(2y)^2}}\right) + \sin\sin^{-1}\left(\frac{l}{\sqrt{l^2+(2y)^2}}\right)\right]$$

$$= \frac{\lambda}{4\pi\varepsilon_0 y}\left[\frac{2l}{\sqrt{l^2+(2y)^2}}\right] = \frac{q}{2\pi\varepsilon_0 y}\frac{1}{\sqrt{l^2+(2y)^2}}.$$

Problem 5.9: The figure given below shows a hypothetical cylinder of radius R immersed in a uniform electric field **E**, the cylinder axis being parallel to the field. What is Φ_E for the closed surface?

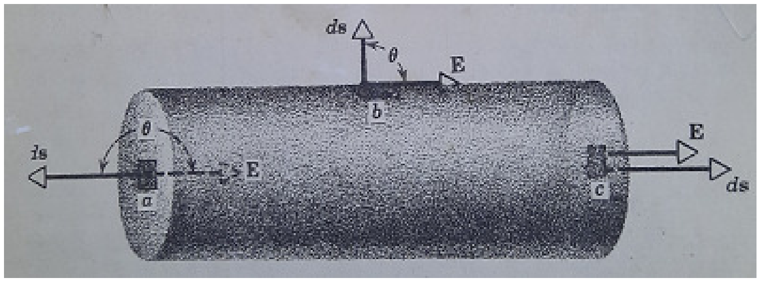

Fig.5.9a

Solution

The flux Φ_E can be written as the sum of three terms, an integral over (a) the left cylinder cap, (b) the cylinder surface, and (c) the right cap. Thus

$$\Phi_E = \oint \mathbf{E}\cdot d\mathbf{S} = \int_{(a)} \mathbf{E}\cdot d\mathbf{S} + \int_{(b)} \mathbf{E}\cdot d\mathbf{S} + \int_{(c)} \mathbf{E}\cdot d\mathbf{S}.$$

For the left cap, the angle θ for all points is 180^o, E has a constant value, and the vector $d\mathbf{S}$ are all parallel. Thus

$$\int_{(a)} \mathbf{E}\cdot d\mathbf{S} = \int E\cos 180^o\, dS = -E\int dS = -ES,$$

where $S\ (=\pi r^2)$ is the cap area. Similarly, for the right cap

$$\int_{(c)} \mathbf{E}\cdot d\mathbf{S} = \int E\cos 0^o\, dS = +E\int dS = +ES,$$

the angle θ for all points being zero here. Finally, for the cylinder wall,

$$\int_{(b)} \mathbf{E}\cdot d\mathbf{S} = \int E\cos 90^o\, dS = 0.$$

The angle θ for all points being 90^o on the cylindrical surface, for which $\cos 90^o = 0.$ Thus

$$\Phi_E = -ES + 0 + ES = 0.$$

Problem 5.10: In the figure given below let a test charge q_0 be moved without acceleration from A to B over the path shown. Compute the potential difference between A and B.

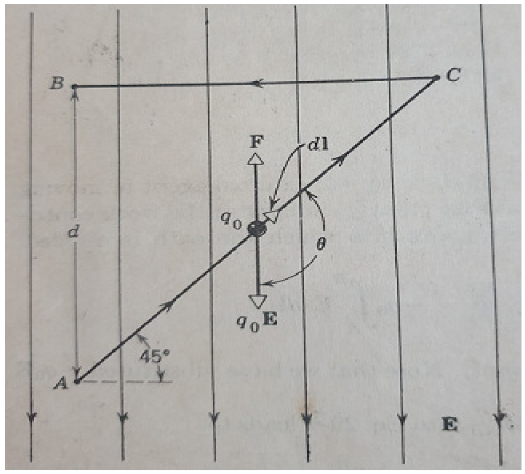

Fig.5.10a

Solution

For the path AC we have the angle between **E** and $d\mathbf{l}$ is $\theta = 135^o$, and we have

$$V_C - V_A = -\int_A^C \mathbf{E}\cdot d\mathbf{l} = -\int_A^C E\cos 135^o\, dl = \frac{E}{\sqrt{2}}\int_A^C dl = \frac{E}{\sqrt{2}}\sqrt{2}\,d = Ed.$$

Points B and C have the same potential because no work is done in moving a charge between them, **E** and $d\mathbf{l}$ being at right angles for all points on the line CB. In other words, B and C lie on the same equipotential surface at right angles to the lines of force. That is, $V_B - B_C = 0$. Thus

$$V_B - V_A = (V_C - V_A) + (V_B - V_C) = Ed + o = Ed.$$

This is the same value and can be derived for the direct path connecting A and B, a result to be expected because the potential difference between two points is path independent.

Problem 5.11: *A charged disk*. Find the electric potential for points on the axis of a uniformly charged disk as shown in the figure below whose surface charge density is σ.

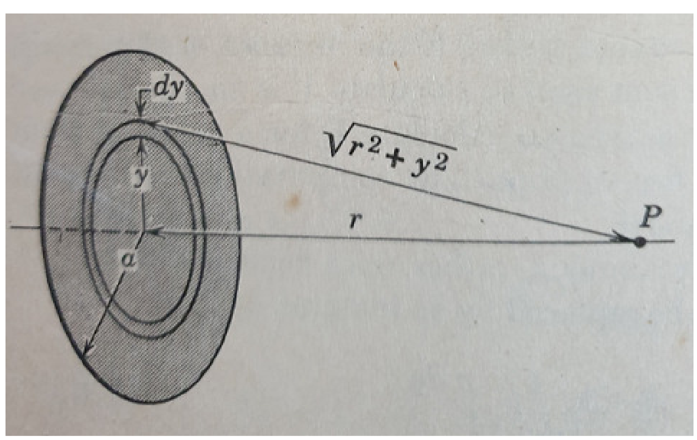

Fig.5.11a

Solution

Let us consider a charge element dq consisting of a flat circular strip of radius y and width dy. We have

$$dq = \sigma(2\pi y)(dy),$$

where $(2\pi y)(dy)$ is the area of the strip. All parts of this charge element are the same distance $r'(=\sqrt{y^2+r^2})$ from axial point P so that their contribution dV to the electric potential at P is given by

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r'} = \frac{1}{4\pi\varepsilon_0}\frac{\sigma 2\pi y\, dy}{\sqrt{y^2+r^2}}.$$

The potential V is found by integrating over all the strips into which the disk can be divided or

$$V = \int dV = \frac{\sigma}{2\varepsilon_0}\int_0^a (y^2+r^2)^{-1/2}\, y\, dy = \frac{\sigma}{2\varepsilon_0}(\sqrt{a^2+r^2} - r).$$

This general result is valid for all values of r. In the special case of $r >> a$ the quantity $\sqrt{a^2+r^2}$ can be approximated as

$$\sqrt{a^2+r^2} = r\left(1+\frac{a^2}{r^2}\right)^{1/2} = r\left(1+\frac{1}{2}\frac{a^2}{r^2}+\cdots\right) \cong r + \frac{a^2}{2r},$$

In which the quantity in parentheses in the second member of this equation has been expanded by the Binomial theorem. This equation means that V becomes

$$V = \frac{\sigma}{2\varepsilon_0}(r+\frac{a^2}{2r}-r) = \frac{\sigma\pi a^2}{4\pi\varepsilon_0 r} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r},$$

where $q\ (=\sigma\pi a^2)$ is the total charge on the disk.

Problem 5.12: *An electric quadrupole*. An electric quadrupole, of which the figure given below is an example, consists of two electric dipoles so arranged that they almost, but not quite, cancel each other in their electric effects at distant points. Calculate V for points on the axis of this quadrupole.

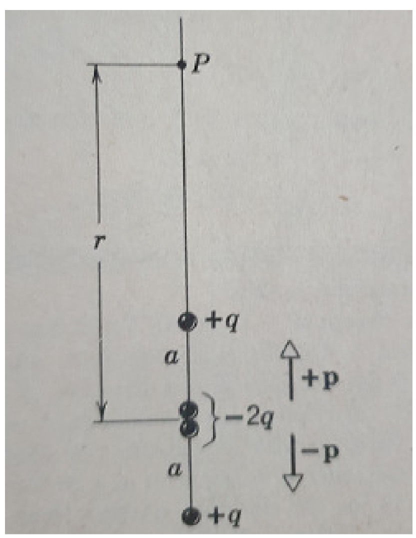

Fig.5.12a

Solution
We have the total electric potential

$$V = \sum_n V_n = V_1 + V_2 + V_3 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r-a} - \frac{2q}{r} + \frac{q}{r+a}\right)$$
$$= \frac{q}{4\pi\varepsilon_0}\frac{2a^2}{(r-a)(r)(r+a)}.$$

Assuming $r >> a$ allows us to put $a = 0$ in the sum and difference terms in the denominator, yielding

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q^3}{r^3},$$

where $Q\ (= 2qa^2)$ is the electric quadrupole moment of the charge assembly of the figure. We note that V varies (a) as $1/r$ for a point charge, (b) $1/r^2$ for a dipole, and (c) $1/r^3$ for a quadrupole.

Problem 5.13: For the charge configuration of the figure given below, show that $V(r)$ for points on the vertical axis, assuming $r >> a$, is given by

$$V(r) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r} + \frac{2aq}{r^2}\right).$$

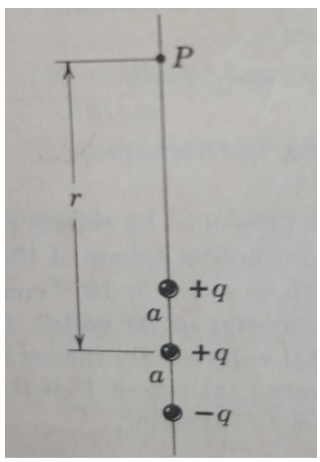

Fig.5.13a

Solution
The total electrostatic potential at point P is

$$V = \sum_n V_n = V_1 + V_2 + V_3$$
$$= \frac{1}{4\pi\varepsilon 0}\left(\frac{q}{r-a}\right) + \frac{1}{4\pi\varepsilon 0}\left(\frac{q}{r}\right) + \frac{1}{4\pi\varepsilon 0}\left(\frac{-q}{r+a}\right)$$
$$= \frac{q}{4\pi\varepsilon 0}\left(\frac{1}{r-a} + \frac{1}{r} - \frac{1}{r+a}\right)$$
$$= \frac{q}{4\pi\varepsilon 0}\left(\frac{1}{r} + \frac{2a}{r^2 - a^2}\right)$$
$$= \frac{1}{4\pi\varepsilon 0}\left(\frac{q}{r} + \frac{2aq}{r^2 - a^2}\right).$$

Problem 5.14:

(a) Show that the electric potential at a point on the axis of a ring of charge of radius *a*, is given by

$$V(z) = \frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{z^2 + a^2}}.$$

(b) Find al so the electric field at that point.

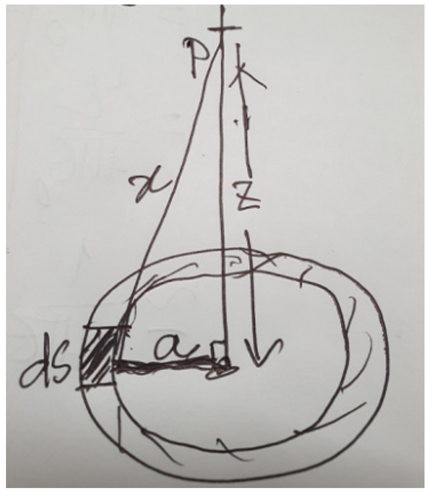

Fig.5.14a

Solution

(a) We consider a small arc element *ds* of the ring. If λ be the charge per unit length of the ring, then the charge of the arc element of length *ds* is $dq = \lambda ds$. Thus the electric potential at point *P* due to this charge element *ds* is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{x} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda ds}{\sqrt{z^2 + a^2}}.$$

The total electric potential at point *P* due to the whole ring is obtained by integrating *dV*:

$$V = \int dV = \frac{\lambda}{4\pi\varepsilon_0}\int_0^{2\pi a}\frac{ds}{\sqrt{z^2 + a^2}} = \frac{\lambda}{4\pi\varepsilon_0}\frac{1}{\sqrt{z^2 + a^2}}\int_0^{2\pi a} ds$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\lambda(2\pi a)}{\sqrt{z^2 + a^2}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{z^2 + a^2}}.$$

(b) We have the component of the electric field strength along the axis of the ring is obtained by differentiating *V* with respect to *z* and taking a negative sign:

$$E_z = -\frac{\partial V(z)}{\partial z} = -\frac{q}{4\pi\varepsilon_0}\left(-\frac{1}{2}\right)\left(z^2 + a^2\right)^{-3/2}(2z)$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{qz}{\left(z^2 + a^2\right)^{3/2}}.$$

Problem 5.15: *The electric field* **E** *for a Dipole*. Figure below shows a (distant) point *P* in the field of a dipole located at the origin of an *xy*-plane. The electric potential *V* is given by the following expression:

$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}.$$

Calculate **E** as a function of position.

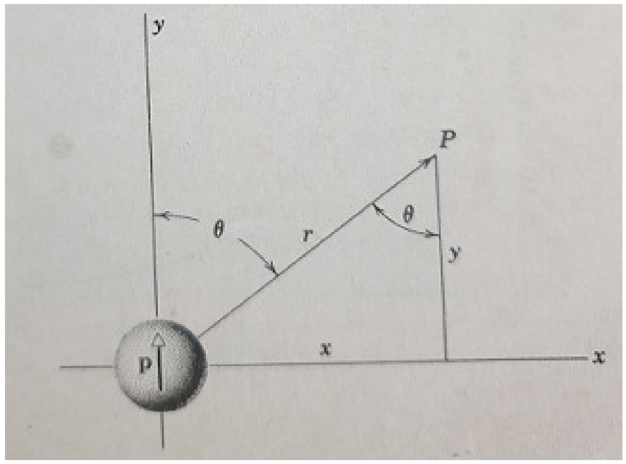

Fig.5.15a

Solution

From symmetry, **E**, for points in the plane of the figure lies in this plane. Thus it can be expressed in terms of its components E_x and E_y. Let us first express the potential function in rectangular coordinates *x* and *y* rather than polar coordinates, making use of

$$r = (x^2 + y^2)^{1/2} \quad \text{and} \quad \cos\theta = \frac{y}{(x^2 + y^2)^{1/2}}.$$

Thus

$$V(x, y) = \frac{p}{4\pi\varepsilon_0}\frac{y}{(x^2 + y^2)^{3/2}}.$$

Therefore the *x*-component of the electric field strength is obtained from

$$E_x = -\frac{\partial V(x, y)}{\partial x} = \frac{3p}{4\pi\varepsilon_0}\frac{xy}{(x^2 + y^2)^{5/2}}.$$

Similarly the *y*-component of the electric field strength is

$$E_y = -\frac{\partial V(x, y)}{\partial y} = -\frac{p}{4\pi\varepsilon_0}\frac{x^2 - 2y^2}{(x^2 + y^2)^{5/2}}.$$

Problem 5.16: A capacitor C_1 is charged to a potential difference V_0. This charging battery is then removed and the capacitor is connected as in the figure given below to an uncharged capacitor C_2.

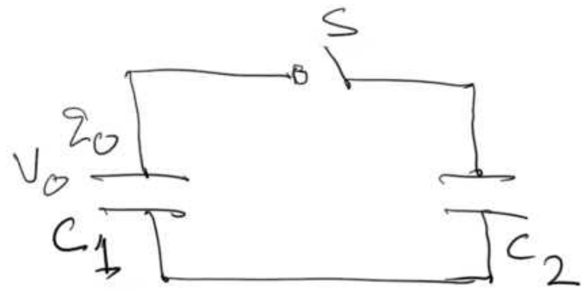

Fig.5.16a

(a) What is the final potential difference V across the combination?
(b) What is the stored energy before and after the in the figure is thrown?

Solution

(a) The original charge q_0 is now shared by the two capacitors. Thus

$$q_0 = q_1 + q_2.$$

Applying the relation $q = CV$ to each of the terms yields

$$C_1 V_0 = C_1 V + C_2 V$$

$\Rightarrow$

$$V = V_0 \frac{C_1}{C_1 + C_2}.$$

(b) The initial stored energy is

$$U_0 = \frac{1}{2} C_1 V_0^2.$$

The final stored energy is

$$U = \frac{1}{2} C_1 V^2 + \frac{1}{2} C_2 V^2$$

$$= \frac{1}{2}(C_1 + C_2)\left(V_0 \frac{C_1}{C_1 + C_2} \right)^2$$

$$= \left(\frac{C_1}{C_1 + C_2} \right) U_0.$$

We see from the above expression for the final potential energy that $U < U_0$. The missing energy appears as heat in the connecting wires as the charges move through them.

Problem 5.17: A parallel plate capacitor has plates with area *A* and separation *d*. A battery charges the plates to a potential difference V_0. The battery is then disconnected, and a dielectric slab of thickness *d* is introduced. Calculate the stored energy both before and after the slab is introduced and account for any difference.

Solution

The energy U_0 before introducing the dielectric slab is

$$U_0 = \frac{1}{2} C_0 V_0^2 \ .$$

After the slab is introduced, we have

$$C = \kappa C_0 \quad \text{and} \quad V = V_0 / \kappa$$

and thus

$$U = \frac{1}{2} C V^2 = \frac{1}{2} \kappa C_0 \left(\frac{V_0}{\kappa} \right)^2 = \frac{1}{\kappa} U_0 .$$

The energy after the slab is introduced is less than by a factor $1/\kappa$. The missing energy would be apparent to the person who inserted the blab.

Problem 5.18

Resistors in Series. Resistors in series are connected so that there is only one conducting path through them as in the figure given below. What is the equivalent resistance *R* of the series combination?

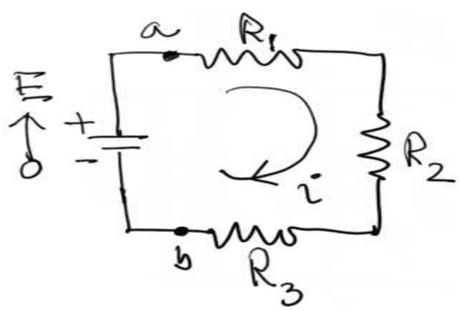

Fig.5.18a

Solution

The equivalent resistance is the single resistance *R* which, substituted for the series combination between the terminals *ab*, will leave the current *i* unchanged.

Applying the loop theorem (going clockwise from *a*) yields

$$-iR_1 - iR_2 - iR_3 + E = 0$$

or

$$i = \frac{E}{R_1 + R_2 + R_3} .$$

For the equivalent resistance *R*

$$i = E / R$$

or

$$R = R_1 + R_2 + R_3 .$$

The extension to more than three resistors is clear.

Problem 5.19:

Resistors in Parallel: The figure below shows three resistors connected across the same seat of emf. Resistances across which the identical potential difference is applied are said to be in parallel. What is the equivalent resistance R of this parallel combination?

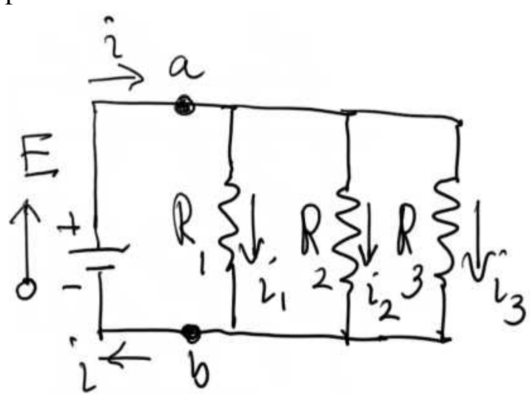

Fig.5.19a

Solution

The equivalent resistance of that single resistance which, substituted for the parallel combination between termionals *ab*, would leave the current i unchanged.

The currents in the three branches are

$$i_1 = \frac{V}{R_1}, \qquad i_2 = \frac{V}{R_2}, \qquad \text{and} \qquad i_3 = \frac{V}{R_3},$$

where V is the potential difference appears between points a and b which is actually the seat's emf E. The total current i is found by applying the junction theorem to junction a:

$$i = i_1 + i_2 + i_3 = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right).$$

If the equivalent resistance is used instead of the parallel combination, we have

$$i = \frac{V}{R}.$$

Combining these two equations gives

$$\frac{V}{R} = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

which implies

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}.$$

This formula can easily be extended to more than three resistances. We note that the equivalent resistance of a parallel combination is less than any of the resistances that make up it.

Problem 5.20
After how many time constants will the energy stored in the capacitor in the figure below reach one-half its equilibrium value when the switch S is attached to a?

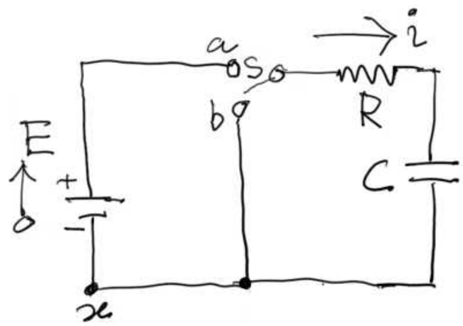

Fig.5.20a

Solution
The energy is given by the following formula

$$U = \frac{1}{2C} q^2 ,$$

The equilibrium energy is

$$U(t \to \infty) \equiv U_\infty = \frac{1}{2C}(CE)^2 = \frac{1}{2}CE^2 .$$

We know

$$q(t) = CE\left(1 - e^{-t/RC}\right).$$

Therefore

$$U = \frac{1}{2C}\left(CE\left[1 - e^{-t/RC}\right]\right)^2 = U_\infty \left(1 - e^{-t/RC}\right)^2$$

which gives

$$U == U_\infty \left(1 - e^{-t/RC}\right)^2$$

Substituting $U = U_\infty / 2$ yields

$$\frac{1}{2} = \left(1 - e^{-t/RC}\right)^2 .$$

By solving, we get

$$t = 1.22\, RC = 1.22\, t_{RC} .$$

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