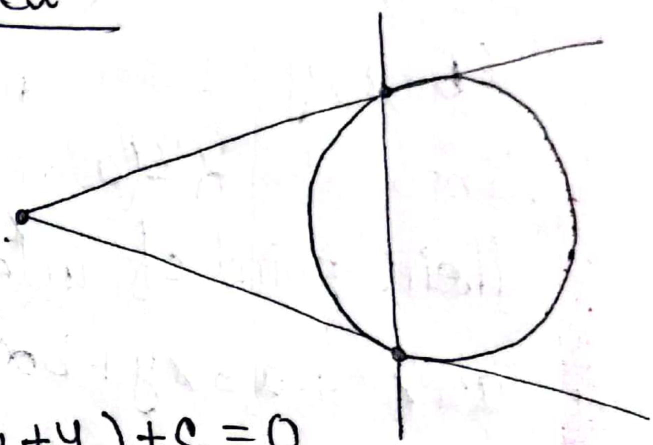
Chord of contact

circle

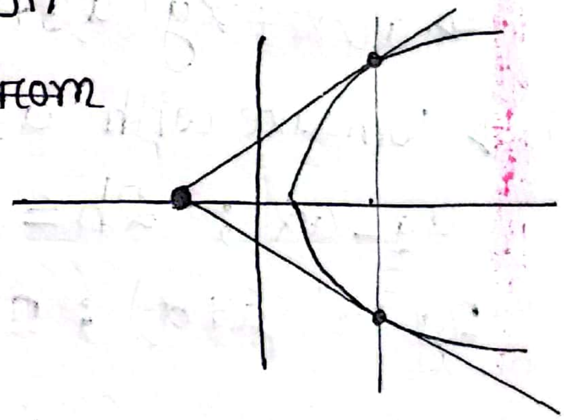
$x^2+y^2+2gx+2fy+c=0$



Equation of chord of contact from (x_1, y_1)

$$xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0$$

~~Equation of chord of contact from (x_1, y_1)~~ Elipse:

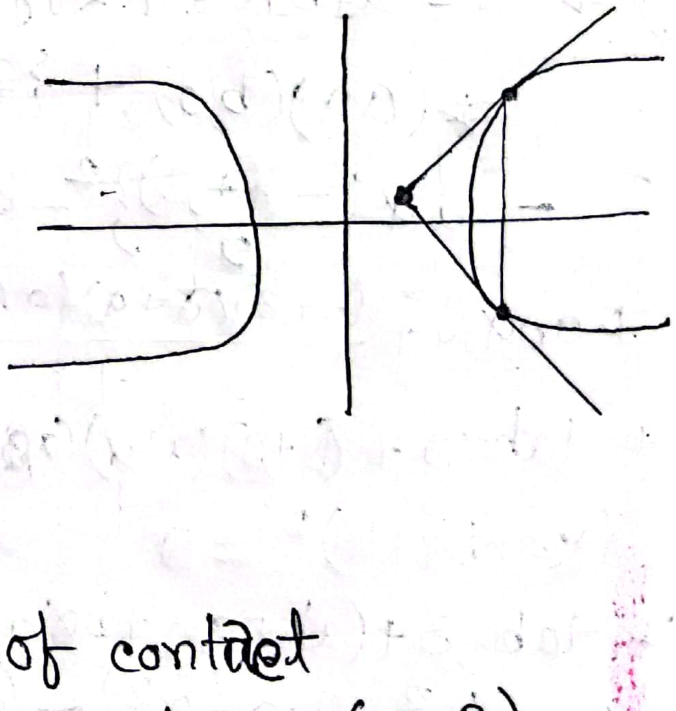


$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$\cancel{xx_1}$

$$\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$$

Parabola: $y^2=4ax$



$$yy_1=2a(x+x_1)$$

Hyperbola: $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$

$$\frac{xx_1}{a^2}-\frac{yy_1}{b^2}=1$$

□ Find equation of chord of contact

$x^2+y^2+4x-6y-12=0$ from $(2, 3)$

$2g=4$, $2f=-6$, $c=-12$

$\therefore g=2$, $f=-3$, $c=-12$

$(4,5)$ $\frac{x^2}{9}+\frac{y^2}{4}=1$

$(2,1)$ $x^2=2y$

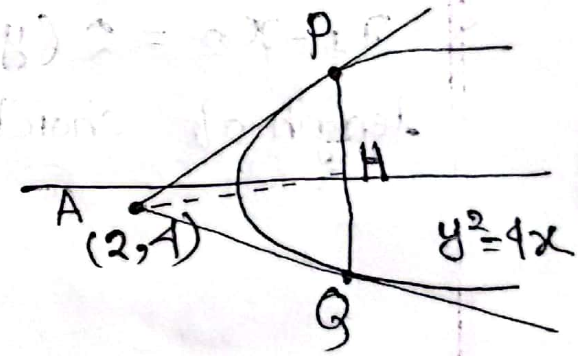
$xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0$

$\Rightarrow 2x+3y+2(x+2)-3(y-3)-12=0$

$\Rightarrow 2x+3y+2x+4-3y+9-12=0$

$\Rightarrow 4x-17=0$

□ Find the area of triangle by tangent and chord of contact from $(2,4)$ to $y^2=4x$



Solution: $y^2=4x$ ------ (1)

$\Rightarrow y^2=4ax$

$\therefore a=1$

Chord of contact:

$yy_1=2a(x+x_1)$

2 $\Rightarrow 4y=2a(2+x)$

$\therefore x=2(y-1)$ ------ (11)

$y^2=4.2(y-1)$

$\Rightarrow y^2-8y+8=0$

$\therefore y_1+y_2=8$

$y_1y_2=8$

$ax^2+bx+c=0$

$\alpha,\beta \rightarrow$ মূল

$\alpha+\beta=-b/a$

$\alpha\beta=\frac{c}{a}$

$(y_1-y_2)^2=(y_1+y_2)^2-4y_1y_2$

$\Rightarrow (y_1-y_2)^2=32$

Length of chord, $PQ=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

$x-2y+2=0 \rightarrow$ PQ line

$(2,4)$ $\quad x-2y+2=0$

$$= \left| \frac{2-8+2}{\sqrt{1^2+(-2)^2}} \right| = \frac{4}{\sqrt5} = AH$$

perpendicular distance:

$$AH = \left| \frac{\cancel{yy_1 - 2a(x+x_1)}}{\cancel{\sqrt{y_1^2+4a^2}}} \right|$$

$$\left| \frac{ax_1+by_1+c}{\sqrt{a^2+b^2}} \right|$$

$x_1 = 2(y_1-1)$

$x_2 = 2(y_2-1)$

$x_1-x_2 = 2(y_1-y_2)$

length of chord $= PQ = \sqrt{(x-x_1)^2+(y-y_1)^2}$

$$= \sqrt{(2(y_1-y_2))^2+(y-y_2)^2}$$

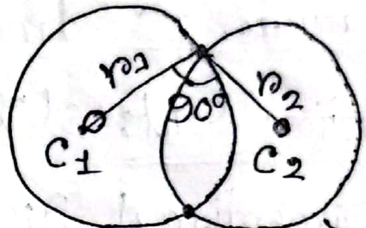
$$= \sqrt{5(y_1-y_2)^2}$$

$$\cancel{= 5\times 36} = \sqrt{160}$$

Area $= \frac{1}{2}\times PQ \times AH = \frac{1}{2}\times\sqrt{160}\times\frac{4}{\sqrt5} = 8\sqrt2$

Orthogonal circle

~~একটি~~ দুটো circle ২টি Point এ cut করলে কেন্দ্র c_1, c_2 থেকে



Point এর উপর স্পর্শক অঙ্কন করলে যদি দুটি tangent এর মধ্যবর্তী কোণ 90° হলে তারা Orthogonal circle.

$$x^2+y^2+2gx+2fy+c_1=0$$

center$(-g,-f)$, $r_1=\sqrt{g^2+f^2-c_1}$

$$x^2+y^2+2g'x+2f'y+c_2=0$$

$(-g',-f')$, $r_2=\sqrt{g'^2+f'^2-c_2}$

পিথাগোরাসের সূত্রে,

$$(c_1c_2)^2=r_1^2+r_2^2$$

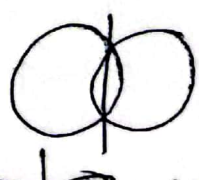
$$\Rightarrow \sqrt{(-g+g')^2+(-f+f')^2}^{\,2}=\left(\sqrt{g^2+f^2-c_1}\right)^2+\left(\sqrt{g'^2+f'^2-c_2}\right)^2$$

$$\Rightarrow (-g+g')^2+(-f+f')^2=g^2+f^2-c_1+g'^2+f'^2-c_2$$

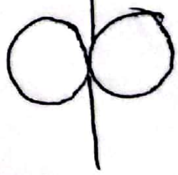
$$\Rightarrow 2gg'+2ff'=c_1+c_2$$

□ একটি বৃত্ত S_1, অপরটি S_2, common chord কত?

- common chord $S_1-S_2=0$

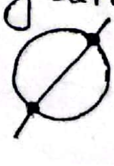


- common tangent /radical axis/touching line $S_1-S_2=0$



(touching)

\# $L_1=0$, $S_1=0$
(Line)



$S_1+KL_1=0$

Intersecting point দিয়ে যায়,

$S_1 + KS_2 = 0 \rightarrow$ The tangents at each point of intersection pass through the centers of the other circle.

angle of intersection, $\cos\theta = \dfrac{r_1^2 + r_2^2 - (C_1C_2)^2}{2r_1r_2}$

□ Show that the circles $x^2+y^2-8x+6y-23=0$ and $x^2+y^2-2x-5y+16=0$ are orthogonal.

Given equations,

$$x^2+y^2-8x+6y-23=0 \quad \text{------ (i)}$$

$$x^2+y^2-2x-5y+16=0 \quad \text{----- (ii)}$$

from equation (i),

$2g=-8$, $2f=6$, $c_1=-23$

$\therefore g=-4$, $f=3$, $c_1=-23$

from equation (ii)

$2g'=-2$, $2f'=-5$, $c_2=16$

$g'=-1$, $f'=-\frac{5}{2}$, $c_2=16$

Eqn (1) & (2) orthogonaly cut if, $2gg'+2ff'=c_1+c_2$

Now, $2gg'+2ff' = 2(-4)(-1)+2(3)(-5/2) = -7$

$c_1+c_2 = -23+16 = -7$

Since, $2gg'+2ff' = c_1+c_2$

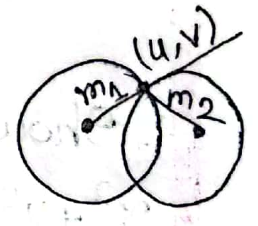
eq^n (i) and eq(ii) are orthogonal.

If the circles $x^2+y^2=1$ and $(x-\cancel{y})^2+(y-1)^2=r^2$ are orthogonal to each other at point (u,v) then prove that $u+v=1$.

Given that, $x^2+y^2=1 \; ....(1)$

$$\cancel{(x-y)^2+(y-1)^2} \quad (x-1)^2+(y-1)^2=r^2$$

Differentiate both sides w.r.to x, we get,



$$2x+2y\frac{dy}{dx}=0$$

$$\Rightarrow \frac{dy}{dx}=-x/y=m_1 \text{, which passes through } (u,v)$$

$$\therefore m_1=-\frac{u}{v}$$

Again,

$$2(x-1)+2(y-1).\frac{dy}{dx}=0$$

$$\Rightarrow \frac{dy}{dx}=\frac{-(x-1)}{(y-1)} \text{ which passes through } (u,v)$$

$$\Rightarrow m_2=-\frac{u-1}{v-1}=\frac{1-u}{v-1}$$

According to the condition;

□ prove that, two circles $x^2+y^2+2ax+c^2=0$ and $x^2+y^2+2by+c^2=0$, touch if $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c^2}$

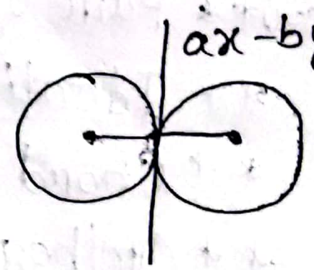
Let, $S_1 = x^2+y^2+2ax+c^2=0$ ---- (I)

$S_2 = x^2+y^2+2by+c^2=0$ ---- (II)

The radical axis of the two circle if $S_1-S_2=0$

$\cancel{x^2}+\cancel{y^2}+2ax+\cancel{c^2}-\cancel{x^2}-\cancel{y^2}-2by-\cancel{c^2}=0 \Rightarrow ax-by=0$ .. (iii)

center of (I), $c_1=(-a,0)$



radius $=\sqrt{g^2+f^2-c^2}$

$=\sqrt{a^2-c^2}$

perpendicular distance from center $(-a,0)$ to $ax-by=0$ is,

center $=(-g,-f)$
radius $=\sqrt{g^2+f^2-c^2}$

$$=\left|\frac{a(-a)-b.0}{\sqrt{a^2+b^2}}\right| = \left|\frac{-a^2}{\sqrt{a^2+b^2}}\right| = \frac{a^2}{\sqrt{a^2+b^2}}$$

According to the condition,

perpendicular distance $=r_1$

$$\Rightarrow \frac{a^2}{\sqrt{a^2+b^2}}=\sqrt{a^2-c^2}$$

$a^2 = (\sqrt{a^2-c^2})(\sqrt{a^2+b^2})$

$\Rightarrow a^4 = (a^2-c^2)(a^2+b^2)$

$\Rightarrow a^4 = a^4 + a^2b^2 - a^2c^2 - b^2c^2$

$\Rightarrow a^2c^2 + b^2c^2 = a^2b^2$

$\therefore \frac{1}{b^2} + \frac{1}{a^2} = \frac{1}{c^2}$ [Divided by abc]

Mid 1 $\boxed{\text{30 Marks}}$ $\boxed{12\cdot10\cdot2023}$

Translation

Rotation

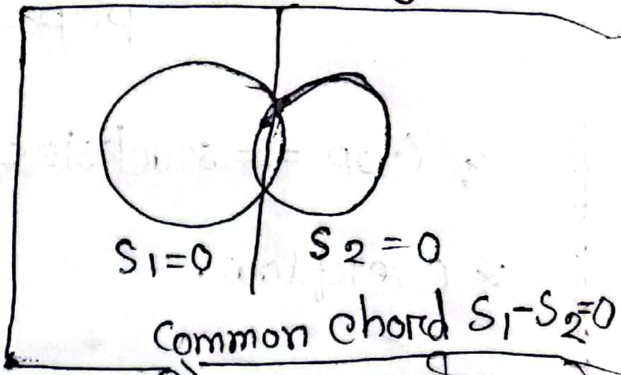
$ax^2+bx+c=0$ ** Pair of straight line (General eq^n of 2nd Degree)

** Identification (standard form)

** Chord of contact (triangle, area ***)

** Orthogonal circle

Find the eq^n of the circle whose diameter is the common chord of the circle $x^2+y^2+2x+3y+1=0$ ~~47~~ & $x^2+y^2+4x+3y+2=0$



S_1-S_2 (Common chord)

$\Rightarrow -2x-1=0$

$\Rightarrow 2x+1=0$ ---- (i)

$\Rightarrow S_1+KS_2=0$ (দুটো বৃত্ত দিয়ে যায় আরেকটি বৃত্তের সমীকরণ)

$= x^2(1+K)+y^2(1+K)+x(2+4K)+y(3+3K)+1+2K=0$

$= x^2+y^2+2x\frac{1+2K}{1+K}+3y+\frac{1+2K}{1+K}=0$ ---- (ii)

center$\left(-\frac{1+2K}{1+K}, -\frac{3}{2}\right)$, putting in eq^n (i)

$2\left\{-\frac{1+2K}{1+K}\right\}+1=0$

$\Rightarrow -2-4K+1+K=0$

$\Rightarrow -3K-1=0$

$\Rightarrow K=-\frac{1}{3}$, putting in eq 2,

$x^2+y^2+2x\frac{1+2(-\frac{1}{3})}{1+(-\frac{1}{3})}+3y+\frac{1+2(-\frac{1}{3})}{1+(-\frac{1}{3})}=0$

$\Rightarrow x^2+y^2+\frac{2x-\frac{4x}{3}}{1-\frac{1}{3}}+3y+\frac{1-\frac{2}{3}}{1-\frac{1}{3}}=0$

$\therefore x^2+y^2+x+3y+\frac{1}{2}=0$