PHY109 Engineering Physics I

Chapter 3 (Fluid Mechanics)

Part-2 (Fluid Dynamics)

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3.4 Fluid Dynamics

Fluid dynamics is a sub-discipline of fluid mechanics that deals with fluid flow in motion. There are many branches in fluid dynamics, aerodynamics and hydrodynamics few among the popularly known fluid mechanics. It involves in a wide range of applications such as calculating force & moments, determining the mass flow rate of petroleum through pipelines, predicting weather patterns, understanding nebulae in interstellar space and modeling fission weapon detonation.

Fluid dynamics is the study of the movement of fluids, including their interactions as two fluids come into contact with each other. In this context, the term "fluid" refers to either liquid or gases. It is a macroscopic, statistical approach to analyzing these interactions at a large scale, viewing the fluids as a continuum of matter and generally ignoring the fact that the liquid or gas is composed of individual atoms.

Fluid dynamics is one of the two main branches of *fluid mechanics*, with the other branch being *fluid statics,* the study of fluids at rest. (Perhaps not surprisingly, fluid statics may be thought of as a bit less exciting most of the time than fluid dynamics.) Every discipline involves concepts that are crucial to understanding how it operates. Here are some of the main ones that you'll come across when trying to understand fluid dynamics.

Basic Fluid Principles

The fluid concepts that apply in fluid statics also come into play when studying fluid that is in motion. Pretty much the earliest concept in fluid mechanics is that of buoyancy, discovered in ancient Greece by Archimedes.

As fluids flow, the density and pressure of the fluids are also crucial to understanding how they will interact. The viscosity determines how resistant the liquid is to change, so is also essential in studying the movement of the liquid. Here are some of the variables that come up in these analyses:

- Bulk viscosity: μ
- Density: ρ
- Kinematic viscosity: $v = \mu / \rho$

Flow

Since fluid dynamics involves the study of the motion of fluid, one of the first concepts that must be understood is how physicists quantify that movement. The term that physicists use to describe the physical properties of the movement of fluid is *flow*. Flow describes a wide range of fluid movement, such blowing through the air, flowing through a pipe, or running along a surface. The flow of a fluid is classified in a variety of different ways, based upon the various properties of the flow.

Steady vs. Unsteady Flow

If the movement of fluid does not change over time, it is considered a *steady flow*. This is determined by a situation where all properties of the flow remain constant with respect to time or alternately can be talked about by saying that the time-derivatives of the flow field vanish. (Check out calculus for more about understanding derivatives.)

A *steady-state flow* is even less time-dependent because all of the fluid properties (not just the flow properties) remain constant at every point within the fluid. So if you had a steady flow, but the properties of the fluid itself changed at some point (possibly because of a barrier causing time-dependent ripples in some parts of the fluid), then you would have a steady flow that is *not* a steady-state flow.

All steady-state flows are examples of steady flows, though. A current flowing at a constant rate through a straight pipe would be an example of a steady-state flow (and also a steady flow). If the flow itself has properties that change over time, then it is called an *unsteady flow* or a *transient flow*. Rain flowing into a gutter during a storm is an example of unsteady flow. As a general rule, steady flows make for easier problems to deal with than unsteady flows, which are what one would expect given that the time-dependent changes to the flow don't have to be taken into account, and things that change over time are typically going to make things more complicated.

Laminar Flow vs. Turbulent Flow

A smooth flow of fluid is said to have *laminar flow*. Flow that contains seemingly chaotic, non-linear motion is said to have *turbulent flow*. By definition, a turbulent flow is a type of unsteady flow.

Both types of flows may contain eddies, vortices, and various types of recirculation, though the more of such behaviors that exist the more likely the flow is to be classified as turbulent.
The distinction between whether a flow is laminar or turbulent is usually related to the *Reynolds number* (*Re*). The Reynolds number was first calculated in 1951 by physicist George Gabriel Stokes, but it is named after the 19th-century scientist Osborne Reynolds.

The Reynolds number is dependent not only on the specifics of the fluid itself but also on the conditions of its flow, derived as the ratio of inertial forces to viscous forces in the following way:

$$Re = \text{Inertial force} / \text{Viscous forces}$$

$$Re = (\rho\, V\, dV/dx) / (\mu\, d^2V/dx^2)$$

The term dV/dx is the gradient of the velocity (or first derivative of the velocity), which is proportional to the velocity (V) divided by L, representing a scale of length, resulting in $dV/dx = V/L$. The second derivative is such that $d^2V/dx^2 = V/L^2$.

Substituting these in for the first and second derivatives results in:

$$Re = (\rho\ V\ V/L)\ /\ (\mu\ V/L^2)$$

$$\mathrm{Re} = (\rho\ V\ L)\ /\ \mu$$

You can also divide through by the length scale L, resulting in a *Reynolds number per foot*, designated as $Re\,f = V\ /\ \nu$.

A low Reynolds number indicates smooth, laminar flow. A high Reynolds number indicates a flow that is going to demonstrate eddies and vortices and will generally be more turbulent.

Pipe Flow vs. Open-Channel Flow

Pipe flow represents a flow that is in contact with rigid boundaries on all sides, such as water moving through a pipe (hence the name "pipe flow") or air moving through an air duct.

Open-channel flow describes flow in other situations where there is at least one free surface that is not in contact with a rigid boundary. (In technical terms, the free surface has 0 parallel sheer stress.) Cases of open-channel flow include water moving through a river, floods, water flowing during rain, tidal currents, and irrigation canals. In these cases, the surface of the flowing water, where the water is in contact with the air, represents the "free surface" of the flow.

Flows in a pipe are driven by either pressure or gravity, but flows in open-channel situations are driven solely by gravity. City water systems often use water towers to take advantage of this, so that the elevation difference of the water in the tower (the *hydrodynamic head*) creates a pressure differential, which is then adjusted with mechanical pumps to get water to the locations in the system where they are needed.

Compressible vs. Incompressible Fluids

Gases are generally treated as compressible fluids because the volume that contains them can be reduced. An air duct can be reduced by half the size and still carry the same amount of gas at the same rate. Even as the gas flows through the air duct, some regions will have higher densities than other regions.

As a general rule, being incompressible means that the density of any region of the fluid does not change as a function of time as it moves through the flow. Liquids can also be compressed, of course, but there's more of a limitation on the amount of compression that can be made. For this reason, liquids are typically modeled as if they were incompressible.

Applications of Fluid Dynamics

Two-thirds of the Earth's surface is water and the planet is surrounded by layers of atmosphere, so we are literally surrounded at all times by fluids, almost always in motion. Thinking about it for a bit, this makes it pretty obvious that there would be a lot of interactions of moving fluids for us to study and understand scientifically. That's where fluid dynamics comes in, of course, so there's no shortage of fields that apply concepts from fluid dynamics.

This list is not at all exhaustive, but provides a good overview of ways in which fluid dynamics show up in the study of physics across a range of specializations:

- ***Oceanography, Meteorology, & Climate Science*** - Since the atmosphere is modeled as fluids, the study of atmospheric science and ocean currents, crucial for understanding and predicting weather patterns and climate trends, relies heavily on fluid dynamics.
- ***Aeronautics*** - The physics of fluid dynamics involves studying the flow of air to create drag and lift, which in turn generate the forces that allow heavier-than-air flight.
- ***Geology & Geophysics*** - Plate tectonics involves studying the motion of the heated matter within the liquid core of the Earth.
- ***Hematology & Hemodynamics*** **-** The biological study of blood includes the study of its circulation through blood vessels, and the blood circulation can be modeled using the methods of fluid dynamics.
- ***Plasma Physics*** - Though neither a liquid nor a gas, plasma often behaves in ways that are similar to fluids, so can also be modeled using fluid dynamics.
- ***Astrophysics & Cosmology*** - The process of stellar evolution involves the change of stars over time, which can be understood by studying how the plasma that composes the stars flows and interacts within the star over time.
- ***Traffic Analysis*** - Perhaps one of the most surprising applications of fluid dynamics is in understanding the movement of traffic, both vehicular and pedestrian traffic. In areas where the traffic is sufficiently dense, the whole body of traffic can be treated as a single entity that behaves in ways that are roughly similar enough to the flow of a fluid.

Fluid dynamics is also sometimes referred at as *hydrodynamics*, although this is more of a historical term. Throughout the twentieth century, the phrase "fluid dynamics" became much more commonly used. Technically, it would be more appropriate to say that hydrodynamics is when fluid dynamics is applied to liquids in motion and *aerodynamics* is when fluid dynamics is applied to gases in motion.

However, in practice, specialized topics such as hydrodynamic stability and magneto-hydrodynamics use the "hydro-" prefix even when they are applying those concepts to the motion of gases.

Continuity Equation

Continuity equation represents that the product of cross-sectional area of the pipe and the fluid speed at any point along the pipe is always constant. This product is equal to the volume flow per second or simply the flow rate.

$$R = Av = \text{constant}. \tag{3.25}$$

where

- R is the volume flow rate
- A is the flow area
- v is the flow velocity

Assumption of Continuity Equation

Following are the assumptions of continuity equation:

- The tube is having a single entry and single exit
- The fluid flowing in the tube is non-viscous
- The flow is incompressible
- The fluid flow is steady

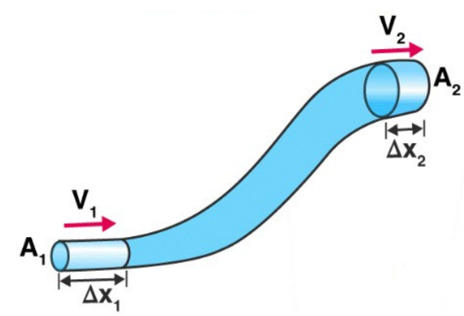

Fig. 3.9 Derivation of the Continuity Equation.

Now, consider the fluid flows for a short interval of time in the tube. So, assume that short interval of time as Δt. In this time, the fluid will cover a distance of Δx_1 with *a* velocity v_1 at the lower end of the pipe. At this time, the distance covered by the fluid will be:

$$\Delta x_1 = v_1 \, \Delta t.$$

Now, at the lower end of the pipe, the volume of the fluid that will flow into the pipe will be:

$$V = A_1 \Delta x_1 = A_1 v_1 \, \Delta t.$$

It is known that mass is the product of the density (ρ) and the volume. So, the mass of the fluid in Δx_1 region will be:

$$\Delta m_1 = density \times volume$$

or

$$\Delta m_1 = \rho A_1 v_1 \, \Delta t. \tag{3.26}$$

Now, the mass flux has to be calculated at the lower end. Mass flux is simply defined as the mass of the fluid per unit time passing through any cross-sectional area. For the lower end with cross-sectional area A_1, mass flux will be:

$$\frac{\Delta m_1}{\Delta t} = \rho A_1 v_1. \tag{3.27}$$

Similarly, the mass flux at the upper end will be:

$$\frac{\Delta m_2}{\Delta t} = \rho A_2 v_2. \tag{3.28}$$

Here, v_2 is the velocity of the fluid through the upper end of the pipe i.e. through Δx_2, in Δt time and A_2, is the cross-sectional area of the upper end.

In this, the density of the fluid between the lower end of the pipe and the upper end of the pipe remains the same with time as the flow is steady. So, the mass flux at the lower end of the pipe is equal to the mass flux at the upper end of the pipe i.e. by equating Eqs. (3.27) and (3.28), we obtain

$$\rho A_1 v_1 = \rho A_2 v_2. \tag{3.29}$$

This can be written in a more general form as:

$$\rho A v = \text{constant}.$$

The equation proves the law of conservation of mass in fluid dynamics.

Thus, Eq. (3.29) can be written as:

$$A_1 v_1 = A_2 v_2.$$

This equation can be written in general form as:

$$A v = \text{constant}.$$

Now, if R is the volume flow rate, the above equation can be expressed as:

$$R = A v = \text{constant}.$$

This was the derivation of the continuity equation.

Bernoulli's Equation

Bernoulli's principle formulated by Daniel Bernoulli states that as the speed of a moving fluid increases (liquid or gas), the pressure within the fluid decreases. Although Bernoulli deduced the law, it was Leonhard Euler who derived Bernoulli's equation in its usual form in the year 1752.

Bernoulli's principle states that

"*The total mechanical energy of the moving fluid comprising the gravitational potential energy of elevation, the energy associated with the fluid pressure and the kinetic energy of the fluid motion, remains constant*".

Bernoulli's Principle

Bernoulli's equation formula is a relation between pressure, kinetic energy, and gravitational potential energy of a fluid in a container. The formula for Bernoulli's principle is given as:

$$p + \frac{1}{2}\rho v^2 + \rho g h = \text{constant} \,, \tag{3.30}$$

where

- p is the pressure exerted by the fluid
- v is the velocity of the fluid
- ρ is the density of the fluid
- h is the height of the container

Bernoulli's equation gives great insight into the balance between pressure, velocity, and elevation.

Derivation of Bernoulli's Equation

Consider a pipe with varying diameter and height through which an incompressible fluid is flowing. The relationship between the areas of cross sections A, the flow speed v, height from the ground y, and pressure p at two different points 1 and 2 is given in the Fig. 3.10 below.

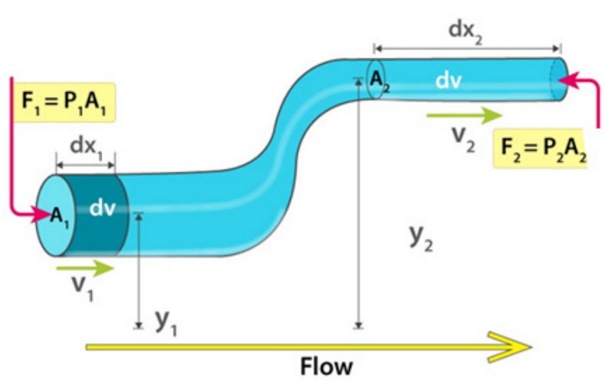

Fig. 3.10 Derivation of the Bernoulli's Equation.

Assumptions:

- The density of the incompressible fluid remains constant at both the points.
- Energy of the fluid is conserved as there are no viscous forces in the fluid.

First we apply energy conservation in the form of the work-kinetic energy theorem,

$$W = \Delta K \ , \tag{3.31}$$

which tells us that the change in the kinetic energy of our system must equal the net work done on the system. The change in kinetic energy results from the change in speed between the ends of the tube and is

$$\Delta K = K_2 - K_1 = \frac{1}{2}m_2 v_2^2 - \frac{1}{2}m_1 v_1^2 = \frac{1}{2}\rho \Delta V\left(v_2^2 - v_1^2\right), \tag{3.32}$$

in which $m_1 = m_2 = \Delta m (= \rho \Delta V)$ is the mass of the fluid that enters at the input end and leaves at the output end during a small time interval Δt.

The work done on the system arises from two sources. The work W_g done by the gravitational force $(\Delta m\, g)$ on the fluid of mass Δm during the vertical lift of the mass from the input level to the output level is

$$W_g = -\Delta m\, g(y_2 - y_1) = -\rho\, g\, \Delta V(y_2 - y_1)\ . \tag{3.33}$$

This work is negative because the upward displacement and the downward gravitational force have opposite directions.

Work must also be done on the system (at the input end) to push forward the fluid that is located ahead of the emerging fluid. In general, the work done by a force of magnitude F, acting on a fluid sample contained in a tube of area A to move the fluid through a distance Δx is $F\Delta x = (pA)(\Delta x) = p(A\,\Delta x) = p\,\Delta V$.

The work done on the system is

$$W_p = \mathbf{F}_1 \cdot \mathbf{\Delta x}_1 + \mathbf{F}_2 \cdot \mathbf{\Delta x}_2 = F_1\, \Delta x_1 \cos 0^o + F_2\, \Delta x_2 \cos 180^o = F_1\, \Delta x_1 - F_2\, \Delta x_2,$$

which, by using $A_1\, \Delta x_1 = A_2\, \Delta x_2 = \Delta V$, can also be written as

$$W_p = p_1\, A_1\, \Delta x_1 - p_2\, A_2\, \Delta x_2 = p_1 \Delta V - p_2\, \Delta V = -(p_2 - p_1)\Delta V. \tag{3.34}$$

The work-kinetic energy theorem then gives

$$W = W_g + W_p = \Delta K.$$

Substituting Eqs. (3.32), (3.33) and (3.34) in the above equation, yields

$$-\rho\, g\, \Delta V(y_2 - y_1) - (p_2 - p_1)\Delta V = \frac{1}{2}\rho \Delta V\left(v_2^2 - v_1^2\right).$$

Thus, after a slight rearrangement, we have

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2. \tag{3.35}$$

The above equation says that

$$p + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}.$$

This is the Bernoulli's equation.

3.5 Viscosity

We know that both liquid and gas are known as fluid. When the molecules of a fluid flow parallel to the direction of motion of the fluid, then this motion is called streamline motion. If the molecules do not advance parallel to the direction of motion of the fluid, then the motion is called turbulent motion. How much resistive a fluid is while flowing is measured by its viscosity. In the case of flow honey is more resistive than water; hence honey is more viscous than water. The viscosity of a fluid is similar to the friction of two solid bodies. Fluid does not have any definite shape because their intermolecular forces are negligible. A fluid, which follows above a horizontal surface, may be imagined as divided into many layers. The layer attached to the lower surface is stationary and other layers are moving. The relative velocity varies with the distance from the lower surface. The greater the distance, the greater is the relative velocity. At time of flow a layer in the fluid makes a friction with the adjacent layer and it opposes the relative motion of the layer. As a result different layers move with different velocities. The friction between different layers of a fluid is known as viscosity. It may be defined as follows:

The property of fluid (liquid or gas) by virtue of which it opposes the relative motion between its different layers is called viscosity.

As friction opposes the relative motion of two solid bodies, viscosity opposes the relative motion of two layers of a fluid and tries to retard its motion. Hence, viscosity is also called internal friction. Viscous force does not act in case of static fluid, but acts only when the fluid is in motion. The difference between the frictional force and the viscous force is that while the magnitude of the frictional force does not depend on the area of the contact surface, the magnitude of the viscous force depends on the area of the layers of the fluid. Further, the viscous force depends on the velocities of the layers of the fluid and the distance from the static layer.

Coefficient of Viscosity

Let us consider a fluid, which flows in layers. We consider two parallel layers of the fluid such that area of each layer is A and the distance between the layers is dy. The velocities of these two layers are v and $v + dv$ respectively. Then the derivative of velocity with respect to distance is dv/dy, which is also known as the velocity gradient. For the difference of velocities between two layers of a fluid a force acts opposite to the direction of the flow due to the viscosity of the fluid. Newton had enunciated a law regarding the magnitude of the force. This is known as the Newton's law of viscosity.

Newton's Law of Viscosity

If there is a relative velocity between two layers of a fluid, then the tangential viscous force (F), which acts opposite to the flow, is directly proportional to the surface area (A) and the velocity gradient (dv/dy) of the layers at constant temperature.

Thus,

$F \propto A,$ when dv/dy is constant and $F \propto dv/dy,$ when A is constant.

Equating the above two relations, we have

$$F \propto A\frac{dv}{dy}, \quad \text{when bot } A \text{ and } dv/dy \text{ change,}$$

or

$$F = \eta\, A\frac{dv}{dy}. \tag{3.36}$$

Here η is the proportionality constant and is known as the coefficient of viscosity. Its magnitude depends on the nature of the fluid and temperature.

Coefficient of Viscosity

The tangential force required maintaining unit velocity gradient between two layers of a fluid of unit area at constant temperature is called the coefficient of viscosity of the fluid.

The unit of the coefficient of viscosity is $\mathrm{N\,s\,m^{-2}}$. The coefficient of viscosity of water is $10^{-3}\,\mathrm{N\,s\,m^{-2}}$ means that if two layers of water of $1\,\mathrm{m}^2$ area are at a distance 1 m from each other, then $10^{-3}\,\mathrm{N}$ force is necessary to maintain a relative velocity $1\,\mathrm{ms}^{-1}$ between them.

Effect of Temperature and Pressure on Viscosity

There are effects of temperature and pressure on the viscosity of fluids.

Effects of Temperature on Viscosity

(a) Viscosity of Liquids

The effect of temperature on the viscosity of liquids is found from many experiments. It is found that the magnitude of the coefficient of viscosity of water at $80^o\mathrm{C}$ is one-third of that of at $10^o\mathrm{C}$. But, a definite relation between the temperature and the coefficient of viscosity of liquid is not found. Different scientists have given different formulas. One such formula is

$$\log\eta = A + \frac{B}{T}. \tag{3.37}$$

Here η is the coefficient of viscosity of the liquid, T is the Kelvin temperature and A and B are two constants.

(b) Viscosity of Gases

The viscosity of gases increase with the increase of temperature. The coefficient of viscosity of a gas is directly proportional to the square root of its Kelvin temperature:

$$\eta \propto \sqrt{T}. \tag{3.38}$$

Effects of Pressure on Viscosity

(a) Viscosity of Liquid

Viscosity of liquids increases with the increase of pressure. The effect of pressure on mineral oil is very much noticeable.

(b) Viscosity of Gas

On the basis of kinetic theory of gas, Maxwell states that there is no effect of pressure on the viscosity of gas and it is applicable for a large range of pressure. But, in case of low pressure exception is also seen.

Drag Force and Stokes' Law

Here we discuss the viscous friction force that opposes the motion of an object relative to a fluid. Examples include the resistive force of water on a swimmer and the resistive force of air on a car, bicycle, or skydiver. These viscous friction forces are often called **drag forces**.

A Ping-Pong ball dropped from a building is illustrated in Fig. 3.11. If the ball falls slowly, laminar flow occurs. Streamlines of air pass the ball as it falls. As the ball's speed increases, the flow of air past the ball becomes turbulent. The onset of turbulence is predicted by a different form of the Reynolds number:

$$\mathrm{Re} = \frac{vL\rho}{\eta}, \tag{3.39}$$

where v is the object's speed, L is the object's length, ρ is the density of the fluid, and η is the fluid's viscosity. This number is used for an entirely different purpose than the usual Reynolds number and the two should not be confused. When Re, given by Eq. (3.39), is less than about one, laminar flow occurs past an object moving through a fluid, and when Re is greater than one, the flow past the object is turbulent. The drag force acting on an object moving relative to a fluid depends on whether the flow past the object is laminar or turbulent.

If the flow is laminar, the drag force increases approximately in proportion to the object's speed relative to the fluid:

$$F_D = Dv, \tag{3.40}$$

where D is a constant. For the situation where a circular object of radius r falls at speed v through a liquid with viscosity η, the drag force for laminar flow is

$$F_D = 6\pi\eta r v. \tag{3.41}$$

This equation is called **Stoke's law**.

On the other hand, if the fluid flow past the moving object is turbulent, the drag force increases approximately in proportion to the square of the object's speed relative to the fluid:

$$F_D = kv^2, \tag{3.42}$$

where k is a constant. For instance, for objects moving through air with turbulent flow,

$$F_D \cong \frac{1}{2} C_D \rho A v^2, \tag{3.43}$$

where ρ is the density of air, C_D is a unitless number called the drag coefficient, and A is the cross-sectional area of the object as seen along its line of motion.

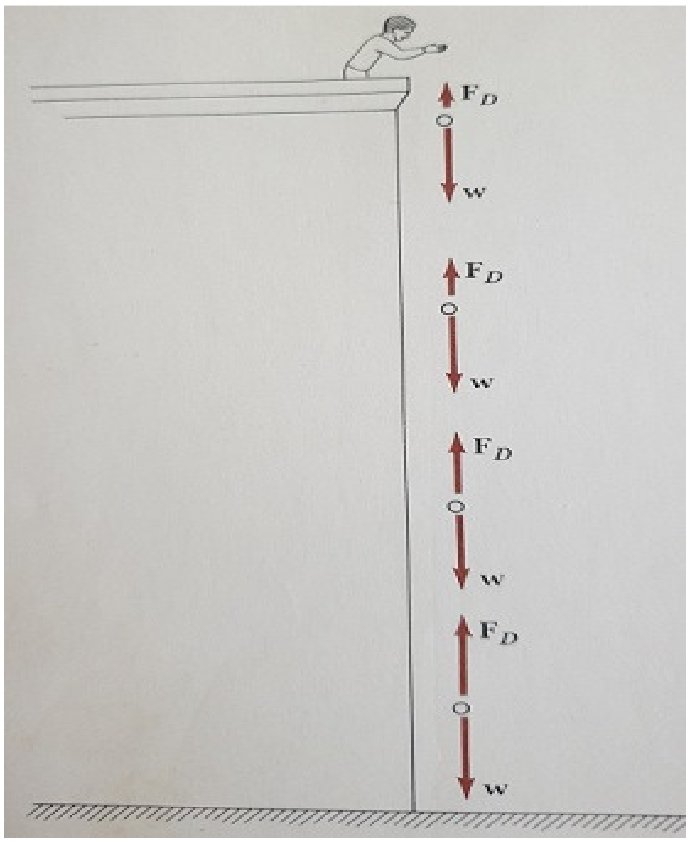

Fig. 3.11 As a ball falls faster, the magnitude of the drag force increases. Eventually, F_D and w are equal in magnitude and the ball falls at a constant terminal speed v_T .

Proof of the Stokes' Law

The drag force F_D will act opposite to the direction of motion of the body. We shall now prove the Stokes' law by dimensional analysis.

It can be reasonably assumed that, the opposing force F_D will depend on radius r of the body (in the form of a sphere), velocity v of the body and the coefficient of viscosity of the liquid η. We can write the relationship between them with the following equation:

$$F_D = kr^x \eta^y v^z. \tag{3.44}$$

Here, k is a dimensionless quantity and x, y, and z are unknown indices. The dimension of both sides of Eq. (3.44) must be equal. Hence

$$[F_D] = [r^x][\eta^y][v^z]$$

or

$$MLT^{-2} = (L)^x (ML^{-1}T^{-1})^y (LT^{-1})^z$$

or

$$MLT^{-2} = M^y L^{x-y+z} T^{-y-z}.$$

Equating the indices of M, L and T on both sides, we obtain

$$1 = y,$$
$$1 = x - y + z,$$
$$-2 = -y - z.$$

Solving these equations, we obtain

$$x = 1,\ \ y = 1 \text{ and } z = 1.$$

Putting these values of x, y and z into Eq. (3.42), we obtain

$$F_D = k\eta rv.$$

From advanced mathematical analysis, it can be shown that $k = 6\pi.$ Hence,

$$F_D = 6\pi\eta rv,$$

which is nothing but the Stokes' law. This equation was first deduced by Stokes; hence it is known as the Stokes' equation. It is to be remembered that

(1) Stokes' equation is only applicable in case of infinite extent of fluid.

(2) If the body moves very rapidly and if the flow of fluid is not streamline, then the equation will not be good enough.

Terminal Speed

An object falling through a fluid eventually reaches a speed where the force pulling it in one direction (for example, the weight force pulling it down) is balanced by the opposing drag force. The net force on the object is zero; it no longer accelerates and continues to move at a constant speed called its **terminal speed** v_T. The terminal speed of a skydiver is approximately 54 m/s, whereas that of a falling raindrop is about 12 m/s. A red blood cell falls in a beaker of water with a terminal speed of about 10^{-7} m/s.

Expression for Terminal Speed

From Stokes' law it is clear that, the opposing force on a body is directly proportional to its speed. If $v = 0$, then $F_D = 0$, if v increases F_D also increases. Thus, we see that if a sphere falls under the influence of gravity through a viscous fluid, initially its speed will increase due to acceleration due to gravity, but simultaneously the opposing force acting on it also increase and hence the net acceleration will decrease. At one instant, the net acceleration on it will be zero. Then the body will fall with constant speed. This speed is called the terminal speed.

Now, let us deduce an expression for the terminal speed.
Let us consider a sphere falling through a viscous liquid. The forces acting on the sphere are:

(a) Downward force, i.e., the weight of the sphere W_g,

(b) Upward force, i.e., buoyancy F_B and

(c) Upward resistive force, i.e., viscous dragging force F_D.

Initially, the downward force W_g is greater than the upward force $F_B + F_D$. When the speed of the sphere increases, the viscous dragging force also increases, and at one instant $F_B + F_D$ becomes equal to W_g. Then the sphere moves downwards and no net force acts on it and its speed attains a constant maximum value. This is the terminal speed v_T. Now, if ρ_F and ρ_B are the densities of the fluid and the body, then

$$F_B = \frac{4}{3}\pi r^3 \rho_F g \quad \text{and} \quad W_g = \frac{4}{3}\pi r^3 \rho_B g.$$

If η be the coefficient of viscosity of the fluid, then $F_D = 6\pi\eta r v$.

According to the Stokes' law, if the sphere attains terminal speed $v = v_T$, then

$$F_B + F_D = W_g,$$

or

$$\frac{4}{3}\pi r^3 \rho_F g + 6\pi\eta r v_T = \frac{4}{3}\pi r^3 \rho_B g,$$

or

$$6\pi\eta r v_T = \frac{4}{3}\pi r^3 \rho_B g - \frac{4}{3}\pi r^3 \rho_F g,$$

or

$$v_T = \frac{2r^2(\rho_B - \rho_F)g}{9\eta}. \quad (3.45)$$

Few Phenomena Regarding Viscosity

(1) The speed of hot water is greater than cold water. The reason is that the motion of flow of liquid depends on the viscous property of the liquid. When water is heated its coefficient of viscosity decreases, hence its speed increases.

(2) When a raindrop falls freely, its speed should be very high, but it is not the case. The reason is that, when the raindrop falls through the atmosphere due to gravity, its speed increases, but at the same time due to viscosity the opposing force of atmosphere also increases. At one instant, the net acceleration of the drop becomes zero. Then the drop falls with constant speed. This speed is the terminal speed. Thus, the raindrop attains the terminal speed, it does not gain higher speed.

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