PHY109 Engineering Physics I

Chapter 3 (Fluid Mechanics)

Part-1 (Fluid Statics)

3.1 Introduction

What is a Fluid?

A fluid, in contrast to a solid, is a substance that can flow. Fluids conform to the boundaries of any container in which we put them. They do so because a fluid cannot sustain a force that is tangential to its surface. Some materials, such as pitch, take a long time to conform to the boundaries of a container, but they do so eventually; thus, we classify even those materials as fluids.

Density and Pressure

Physical quantities that we find useful, and in those terms we express Newton's laws, are mass and force. With solids, we are more interested in the extended substance and in properties that can vary from point to point in that substance. Then it is more useful to speak of **density** and **pressure** than of mass and force.

Density

To find the density ρ of a fluid at any point, we isolate a small volume element ΔV around that point and measure the mass Δm of the fluid contained within that element. The density of the fluid is then

$$\rho = \frac{\Delta m}{\Delta V}. \tag{3.1}$$

In theory, the density at any point in a fluid is the limit of this ratio as the volume element ΔV at that point is made smaller and smaller. However, in practice, we assume that a fluid sample is large relative to atomic dimensions and thus is smooth (with uniform density), rather than lumpy with atoms. This assumption allows us to write Eq. (3.1) as

$$\rho = \frac{m}{V}, \tag{3.2}$$

where m and V are the mass and volume of the sample.

Density is a scalar quantity and its SI unit is $\mathrm{kg/m^3}$. Table 3.1 shows the densities of some substances and the average densities of some objects. Note that the density of a gas (see Air in the table) varies considerably with pressure, but the density of a liquid (see Water) does not; that is, gases are readily *compressible* but liquids are not.

Pressure

If a force ΔF acts normally into a surface area ΔA of a fluid, then the pressure is defined as

$$p = \frac{\Delta F}{\Delta A}. \tag{3.3}$$

In theory, the pressure at any point in the fluid is the limit of Eq. (3.3) as the surface area ΔA centered on that point, is made smaller and smaller. However, if the force is uniform over a flat area A, we can write Eq. (3.3) as

$$p = \frac{F}{A}, \tag{3.4}$$

where F is the magnitude of the normal force on area A.

Here pressure is a scalar quantity, having no directional properties. The SI unit of pressure is the 'newton per square meter', which is given a special name, the **pascal** (Pa). The pascal is related to some other common (non-SI) pressure units as follows:

$$1\text{ atm} = 1.01\times 10^5\text{ Pa} = 760\text{ torr} = 14.7\text{ lb/in}^2.$$

Table 3.1

Some Densities

Material or Object	Density (kg/m^3)	Material or Object	Density (kg/m^3)
Interstellar space	10^{-20}	Iron	7.9×10^3
Best laboratory vacuum	10^{-17}	Mercury (the metal, not the planet)	13.6×10^3
Air: 20°C and 1 atm pressure	1.21	Earth: average	5.5×10^3
20°C and 50 atm	60.5	core	9.5×10^3
Styrofoam	1×10^2	crust	2.8×10^3
Ice	0.917×10^3	Sun: average	1.4×10^3
Water: 20°C and 1 atm	0.998×10^3	core	1.6×10^5
20°C and 50 atm	1.000×10^3	White dwarf star (core)	10^{10}
Seawater: 20°C and 1 atm	1.024×10^3	Uranium nucleus	3×10^{17}
Whole blood	1.060×10^3	Neutron star (core)	10^{18}

Table 3.2

Some Pressures

	Pressure (Pa)		Pressure (Pa)
Center of the Sun	2×10^{16}	Automobile tire[a]	2×10^5
Center of Earth	4×10^{11}	Atmosphere at sea level	1.0×10^5
Highest sustained laboratory pressure	1.5×10^{10}	Normal blood systolic pressure[a,b]	1.6×10^4
Deepest ocean trench (bottom)	1.1×10^8	Best laboratory vacuum	10^{-12}
Spike heels on a dance floor	10^6		

[a]Pressure in excess of atmospheric pressure. [b]Equivalent to 120 torr on the physician's pressure gauge.

The *atmosphere* (atm) is, as the name suggests, the approximate average pressure of the atmosphere at sea level. The torr named for E. Torricelli, who invented the mercury barometer in 1674) was formally called the *millimeter of mercury* (mm Hg). The pound per square inch is often abbreviated psi. Table 3.2 shows some pressures.

3.2 Fluid Statics

Figure 3.1*a* shows a tank of water—or other liquid—open to the atmosphere. As every diver knows, the pressure *increases* with depth below the air–water interface. The diver's depth gauge, in fact, is a pressure sensor much like that of Fig. 3.1*b*. As every mountaineer knows, the pressure *decreases* with altitude as one ascends into the atmosphere. The pressures encountered by the diver and the mountaineer are usually called *hydrostatic pressures*, because they are due to fluids that are static (at rest). Here we want to find an expression for hydrostatic pressure as a function of depth or altitude.

Pressure Increases with Depth of Fluid

Let us look first at the increase in pressure with depth below the water's surface. We set up a vertical y axis in the tank, with its origin at the air–water interface and the positive direction upward. We next consider a water sample contained in an imaginary right circular cylinder of horizontal base (or face) area A, such that y_1 and y_2 (both of which are *negative* numbers) are the depths below the surface of the upper and lower cylinder faces, respectively.

Figure 3.1*e* shows a free-body diagram for the water in the cylinder. The water is in *static equilibrium;* that is, it is stationary and the forces on it balance. Three forces act on it vertically: Force acts at the top surface of the cylinder and is due to the water above the cylinder (Fig. 3.1*b*). Similarly, force acts at the bottom surface of the cylinder and is due to the water just below the cylinder (Fig. 3.1*c*).The gravitational force on the water in the cylinder is represented by mg, where m is the mass of the water in the cylinder (Fig. 3.1d).The balance of these forces is written as

$$F_2 = F_1 + mg. \tag{3.5}$$

We want to express Eq. (3.5) in terms of pressures. We know that

$$F_1 = p_1 A \qquad \text{and} \qquad F_2 = p_2 A. \tag{3.6}$$

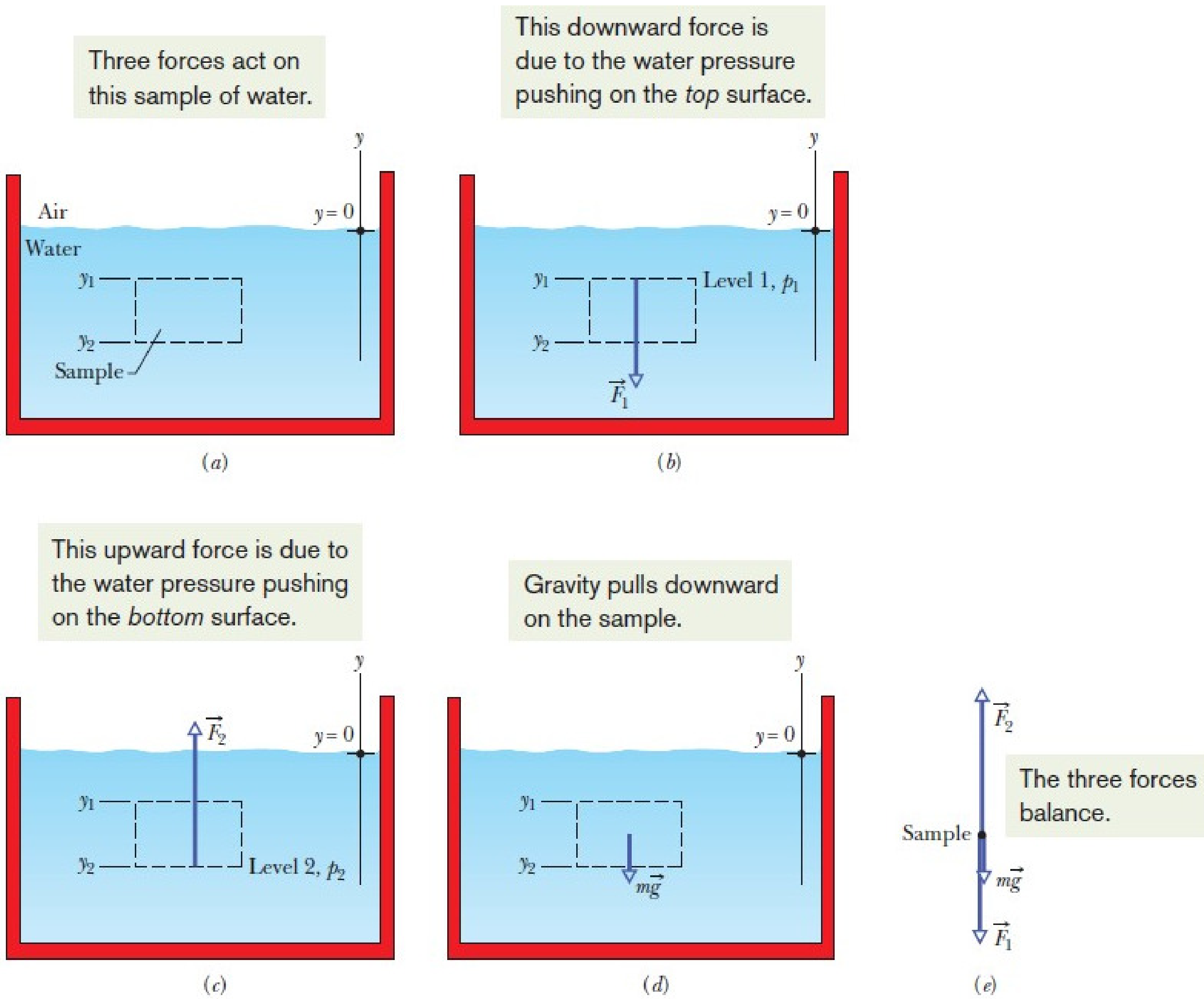

Fig. 3.1 (*a*) A tank of water in which a sample of water is contained in an imaginary cylinder of horizontal base area *A*. (*b*) – (*d*) Force acts at the top surface of the cylinder; force acts at the bottom surface of the cylinder; the gravitational force on the water in the cylinder is represented by . (*e*) A free-body diagram of the water sample.

The mass *m* of the water in the cylinder is $m = \rho V$, where the cylinder's volume *V* is the product of its face area and its height $y_1 - y_2$. Thus, *m* is equal to $\rho A(y_1 - y_2)$. Substituting this and Eq. (3.6) into Eq. (3.5), we find

$$p_2 A = p_1 A + \rho A g(y_1 - y_2)$$

or

$$p_2 = p_1 + \rho g(y_1 - y_2). \qquad (3.7)$$

This equation can be used to find pressure both in liquid (as a function of depth) and in the atmosphere (as a function of altitude or height). For the former case, let us suppose that we seek the pressure *p* at a depth

h below the liquid surface. Then we choose level 1 to be the surface and level 2 to be a distance h below it. Usually p_0 is used for the atmospheric pressure on the surface. We then substitute

$$y_1 = 0,\ p_1 = p_0 \quad \text{and} \quad y_2 = -h,\ p_2 = p$$

Ionto Eq. (3.7), which then becomes

$$p = p_0 + \rho g h \qquad \text{(pressure at depth } h\text{).} \qquad (3.8)$$

We note that the pressure at a given depth in the liquid depends on that depth but not on any horizontal dimension.

Pascal's Principle

When you squeeze one end of a tube to get toothpaste out the other end, you are watching **Pascal's principle** in action. The principle was first stated clearly in 1652 by Blaise Pascal (for whom the unit of pressure is named):

"*A change in the pressure applied to an enclosed incompressible fluid is transmitted undiminished to every portion of the fluid and to the walls of its container*".

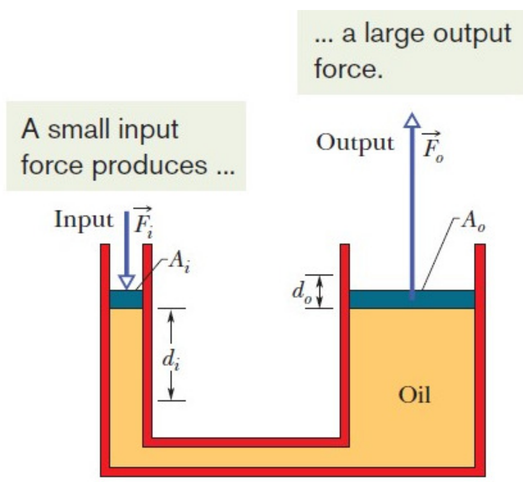

Fig. 3.2 A hydraulic arrangement that can be used to magnify a force. The work done is, however, not magnified and is the same for both the input and output forces.

Figure 3.2 shows how Pascal's principle can be made the basis of a hydraulic lever. In operation, let an external force of magnitude F_i be directed downward on the left-hand (or input) piston, whose surface area is A_i. An incompressible liquid in the device then produces an upward force of magnitude F_0 on the right-hand (or output) piston, whose surface area is A_0. To keep the system in equilibrium, there must be a downward force of magnitude F_0 on the output piston from an external load (not shown). The force F_i applied on the left and its direction is downward with area of cross section A_i while at the output end, the force is F_0 and the area of cross section is A_0. Thus, the pressures at the input end and at the output end are respectively as F_i / A_i and F_0 / A_0. These two pressures must be equal:

$$\frac{F_i}{A_i} = \frac{F_0}{A_0}, \tag{3.9}$$

so

$$F_0 = F_i \frac{A_0}{A_i}. \tag{3.10}$$

Equation (3.10) shows that the output force F_0 on the load must be greater than the input force F_i if $A_0 > A_i$, as is the case in Fig. 3.2. If we move the input piston downward a distance d_i, the output piston moves upward a distance d_0, such that the same volume V of the incompressible liquid is displaced at both pistons. Then

$$V = A_i d_i = A_0 d_0,$$

which we can write as

$$d_0 = d_i \frac{A_i}{A_0}. \tag{3.11}$$

This shows that, if $A_0 > A_i$ the output piston moves a smaller distance than the input piston moves.

We can write the output work as

$$W = F_0 d_0 = \left(F_i \frac{A_0}{A_i}\right)\left(d_i \frac{A_i}{A_0}\right) = F_i d_i, \tag{3.12}$$

which shows that the work W done *on* the input piston by the applied force is equal to the work W done *by* the output piston in lifting the load placed on it. The advantage of a hydraulic lever is this:

> ***"With a hydraulic lever, a given force applied over a given distance can be transformed to a greater force applied over a smaller distance".***

The product of force and distance remains unchanged so that the same work is done. However, there is often tremendous advantage in being able to exert the larger force. Most of us, for example, cannot lift an automobile directly but can with a hydraulic jack, even though we have to pump the handle farther than the automobile rises and in a series of small strokes.

Pascal’s Principle in Biology

Pascal’s principle also has important consequences in biology and medicine. For example, glaucoma, an eye disease, involves Pascal’s principle. A clear fluid called aqueous humor fills two chambers in the front of the eye (Fig. 3.3).

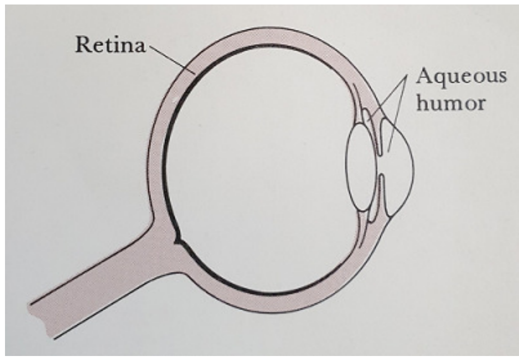

Fig. 3.3 When extra fluid cannot drain out of the eye’s cornea, pressure increases throughout the eye, possibly causing blindness.

In the normal eye new fluid is continually secreted into these chambers while old fluid drains from the eye through small sinus canals. When a person has glaucoma, these drainage canals close, fluid accumulates in the front of the eye, and the pressure inside the eye increases. This pressure increase is transmitted to the retina at the back of the eye and causes degenerative changes in the retina that can eventually lead to blindness.

Archimedes Principle

Fluids exert upward buoyant forces on objects in the fluid. These buoyant forces either completely or partially support the object’s weight. For example, the buoyant of water supports a person’s weight while floating in a swimming pool. The buoyant force of sea water supports the million-pound weight of icebergs. The atmosphere exerts a buoyant force that helps support the weight of a balloon.

To derive an equation for calculating the buoyant force of a fluid on an object, let us consider a metal block immersed in a liquid as shown in Fig. 3.4. The fluid, a liquid in this case, exerts forces on all the six surfaces of the block. The forces exerted on opposite sides of the block cancel each other. However, the upward force F_1 due to the fluid pressure from below the block and the downward force F_2 due to that from above the block do not cancel, since the pressure of the fluid on the bottom is greater than that on the top. The net force of the fluid on the block $F_1 - F_2$, points upward, since F_1 is greater in magnitude than F_2. This net fluid force is called the buoyant force F_B of the fluid, and its magnitude is

$$F_B = F_1 - F_2 = P_1 A - P_2 A = (P_1 - P_2) A, \tag{3.13}$$

where A is the area of the bottom and top surfaces of the block, and P_1 and P_2 are the fluid pressures at the bottom and top of the block, respectively.

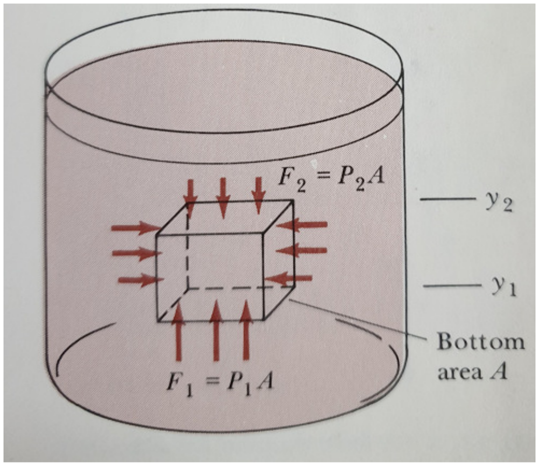

Fig. 3.4 The forces acting on a metal block immersed in a liquid.

The difference in pressure at two different elevations in the fluid is given by

$$P_1 - P_2 = \rho g(y_2 - y_1).$$

Substituting in Eq. (3.13), we find that the buoyant force is

$$F_B = (P_1 - P_2)A = \rho g(y_2 - y_1)A. \tag{3.14}$$

The product $(y_2 - y_1)A$ is the volume V of the block – its height times its area of cross-section – and also equals the space taken up by the block in the fluid. This space is called the volume V of fluid displaced by the object in the fluid. Substituting V in place of $(y_2 - y_1)A$ results in an expression for the buoyant force of the fluid:

$$F_B = \rho g V. \tag{3.15}$$

It is important to note that the density ρ in this equation is the density of the fluid and not of the object in the fluid. The product ρV is the mass of a volume V of the fluid having density ρ. When we multiply mass times g, we convert mass to weight. Thus, $\rho g V$ is the weight of fluid whose volume equals V. Archimedes Principle is the following:

The buoyant force F_B exerted by a fluid on an object in the fluid is equal in magnitude to the weight of fluid displaced by the object.

Any object, totally or partially immersed in a fluid or liquid, is buoyed up by a force equal to the weight of the fluid displaced by the object.

Archimedes' principle allows the buoyancy of any floating object partially or fully immersed in a fluid to be calculated. The downward force on the object is simply its weight. The upward, or buoyant, force on the object is that stated by Archimedes' principle, above. Thus, the net force on the object is the difference between the magnitudes of the buoyant force and its weight.

If this net force is positive, the object rises; if negative, the object sinks; and if zero, the object is neutrally buoyant—that is, it remains in place without either rising or sinking.

In simple words, Archimedes' principle states that, when a body is partially or completely immersed in a fluid, it experiences an apparent loss in weight that is equal to the weight of the fluid displaced by the immersed part of the body(s).

Let us consider a cuboid immersed in a fluid, with one (hence two: top and bottom) of its sides orthogonal to the direction of gravity (assumed constant across the cube's stretch). The fluid will exert a normal force on each face, but only the normal forces on top and bottom will contribute to buoyancy. The pressure difference between the bottom and the top face is directly proportional to the height (difference in depth of submersion). Multiplying the pressure difference by the area of a face gives a net force on the cuboid – the buoyancy, equaling in size the weight of the fluid displaced by the cuboid. By summing up sufficiently many arbitrarily small cuboids this reasoning may be extended to irregular shapes, and so, whatever the shape of the submerged body, the buoyant force is equal to the weight of the displaced fluid. Among completely submerged objects with equal masses, objects with greater volume have greater buoyancy.

Let us suppose a rock's weight is measured as 10N when suspended by a string in a vacuum with gravity acting on it. Suppose that, when the rock is lowered into the water, it displaces water of weight 3N. The force it then exerts on the string from which it hangs would be 10N minus the 3N of buoyant force: 10N – 3N = 7N. Buoyancy reduces the apparent weight of objects that have sunk completely to the sea-floor. It is generally easier to lift an object through the water than it is to pull it out of the water.

For a fully submerged object, Archimedes' principle can be reformulated as follows:

$$\textit{apparent immersed weight} = \textit{weight of object} - \textit{weight displaced fluid}. \tag{3.16}$$

We have

$$\frac{\textit{density of object}}{\textit{density of fluid}} = \frac{\textit{weight of object}}{\textit{weight of displaced fluid}}, \tag{3.17}$$

which can also be written as

$$\frac{\textit{density of object}}{\textit{density of fluid}} = \frac{\textit{weight of object}}{\textit{weight of object} - \textit{apparent immersed weight}}. \tag{3.18}$$

Example: If you drop wood into water, buoyancy will keep it afloat.

Example: A helium balloon in a moving car. When increasing speed or driving in a curve, the air moves in the opposite direction to the car's acceleration. However, due to buoyancy, the balloon is pushed "out of the way" by the air and will drift in the same direction as the car's acceleration.

When an object is immersed in a liquid, the liquid exerts an upward force, which is known as the buoyant force, that is proportional to the weight of the displaced liquid. The resultant force acting on the object, then, is equal to the difference between the weight of the object (downward force) and the weight of displaced liquid (upward force). Equilibrium, or neutral buoyancy, is achieved when these two weights (and thus forces) are equal.

3.3 Surface Tension

Cohesion and Surface Tension

The cohesive forces between liquid molecules are responsible for the phenomenon known as surface tension. The molecules at the surface do not have other like molecules on all sides of them and consequently they cohere more strongly to those directly associated with them on the surface. This forms a surface film which makes it more difficult to move an object through the surface than to move it when it is completely submersed. The cohesive forces between molecules down into a liquid are shared with all neighboring atoms. Those on the surface have no neighboring atoms above and exhibit stronger attractive forces upon their nearest neighbors on the surface. This enhancement of the intermolecular attractive forces at the surface is called surface tension.

Definition of Surface Tension

The surface tension is defined as the force per unit length acting on both sides of an imaginary line drawn on the liquid surface at rest. The direction of the force is tangential to the surface and perpendicular to the line.

If a force F acts perpendicularly on both sides of a line AB of length l drawn over a liquid surface and if the force is tangential to the surface, then surface tension T would be

$$T = \frac{F}{l}. \tag{3.19}$$

The unit of surface tension is $\mathrm{N\,m^{-1}}$.

The surface tension of water is $72\times10^{-3}\,\mathrm{N\,m^{-1}}$. This means that if we imagine a line of length 1m over the water surface, then a force of $72\times10^{-3}\,\mathrm{N}$ acts perpendicularly on both sides of the line and tangentially to the liquid surface.

Molecular Theory of Surface Tension

Surface tension is a molecular phenomenon and it can be explained by molecular theory. Laplace first explained surface tension with the help of molecular theory. Before presenting the molecular explanation of surface tension we must know about the intermolecular forces.

Intermolecular forces are of two types: (a) Cohesive force and (b) Adhesive force.

Cohesive Force: The force of attraction between molecules of same substance is called cohesive force.
Adhesive Force: The force of attraction between molecules of different substances is called adhesive force.

The maximum distance at which the cohesive force between two molecules remains active is called the range of molecular attraction. It is nearly 10^{-10} m. If a sphere is drawn with a molecule as its center and the range of molecular attraction as radius then the sphere is called the influence or range sphere of the molecule. The molecule at the center is only influenced by the other molecules within this influence sphere but not by the molecules outside this sphere. This means this molecule has no cohesive force with any molecule lying outside this sphere.

Let us suppose, in Fig. 3.5, *A*, *B* and *C* are three molecules of a liquid. The molecule *A* is inside the liquid, hence its influence sphere of molecular attraction is inside the liquid. This molecule is equally attracted in all directions by the molecules of the influence sphere. Thus, the resultant cohesive force on it is zero. That is, no cohesive force is acting on it. Therefore, the molecule will remain in the state in which it is situated.

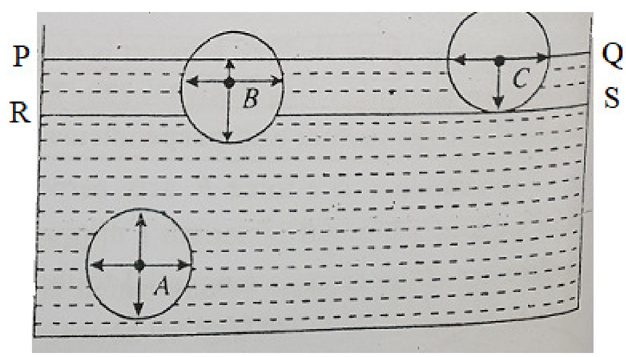

Fig. 3.5 Three liquid molecules *A*, *B*, and *C* are shown along with their influence sphere.

The molecule *B* is in such a position that a portion of its influence sphere is lying outside the liquid. The number of the molecules in the lower half of the influence sphere will be greater than the number of molecules in the upper half. Thus, the downward cohesive force on molecule *B* will be greater than the upward cohesive force. Therefore, the molecule *B* will experience a downward resultant force.

The molecule *C* is situated at the free surface of the liquid. The upper half of the influence sphere of *C* is lying outside the liquid and the lower half of the sphere is lying inside the liquid. Thus, there is no cohesive force on the upper portion, only downward cohesive force is acting on the molecule. Therefore, in this case the molecule *C* will be attracted by the maximum downward force. Thus, among the three molecules situated at different positions, the tendency to go downward is maximum for the molecule *C*.

Now, if we imagine a plane RS parallel to the free surface of the liquid PQ and at a distance equal to the molecular range and inside the liquid; then all the molecules lying between PQ and RS will experience downward tension due to cohesive forces. The magnitude of this downward tension will increase with the distance from RS toward free surface and will be maximum at the free surface.

Now, to bring a molecule above the RS plane from a point inside the liquid, work must be done against the cohesive force and this work will increase the potential energy of the molecule. Thus, the potential energy of the molecules above RS is greater than that of the molecules below RS. But we know that all objects tend to come to lowest potential energy configuration. To minimize the potential energies of all molecules lying between RS and PQ, the area of the free surface must be diminished. Hence, the free surface of the liquid always tries to diminish its area and tries to contract. Thus, the free surface of the liquid acts like a stretched elastic membrane and remain stretched. This tension acts along a tangent to the surface. If we draw a line on the surface of the liquid, then this tension will be perpendicular to the line. The tension arises per unit length of this line is the surface tension.

Surface Energy

The free surface of a liquid acts like a stretched elastic membrane and tends to reach the minimum area by contracting itself. If the free surface of the liquid is to be extended then work must be done against this surface tension. This work is saved as the potential energy at the liquid surface. The measure of the surface energy is the work done against this surface tension to increase the unit area of the free surface at constant temperature.

The amount of work done to increase the surface area of the liquid through unity under isothermal conditions, i.e., potential energy per unit area of the surface is called surface energy.

If the work done to increase area of the free surface of the liquid by an amount ΔA is W, then the surface energy will be

$$E = \frac{W}{\Delta A}. \tag{3.20}$$

The unit of surface energy is $\mathrm{J\,m^{-2} \equiv N\,m^{-1}}$. Thus, the unit of surface energy and the unit of surface tension are the same.

Factors Affecting Surface Tension of a Liquid

1. **Temperature:**
   Surface tension depends on temperature. Surface tension decreases if the temperature of the liquid increases.
2. **Medium above the Liquid:**
   Surface tension depends on the medium touching the free surface of the liquid. For example, if the water surface is in contact with air, then the surface tension of the water is $72\times10^{-3}\,\mathrm{N\,m^{-1}}$, but if vapor is present above the water surface, then the surface tension becomes $70\times10^{-3}\,\mathrm{N\,m^{-1}}$.
3. **Pollution:**
   If oil or fat floats above the liquid, then its surface tension decreases.
4. **Presence of Soluble Material:**
   If some material dissolves in the liquid, then its surface tension changes. If inorganic material such as common salt NaCl dissolves in pure water then its surface tension increases to $84\times10^{-3}\,\mathrm{N\,m^{-1}}$ from $72\times10^{-3}\,\mathrm{N\,m^{-1}}$. But if organic material (for example soap) dissolves then its surface tension decreases to $32\times10^{-3}\,\mathrm{N\,m^{-1}}$ from $72\times10^{-3}\,\mathrm{N\,m^{-1}}$.
5. **Electrification:**
   If the liquid is electrified, then its surface tension decreases.

Angle of Contact

If a solid substance is immersed in a liquid, it is seen that, at the point where the liquid touches the solid, the liquid does not remain horizontal like other places; rather the free surface bends either upwards or downwards. It is observed that, in case the liquid wets the solid (water and glass), the liquid surface rises (Fig. 3.6a). On the other hand, where the liquid does not wet the solid (for example mercury and glass), the liquid surface falls a bit (Fig. 3.6b). If a tangent *CA* is drawn from the point of contact *C* of the liquid with the solid to the curved liquid surface, then the angle made by the tangent with the solid surface *CB* inside the liquid is called the angle of contact. In Fig. 3.6, θ is the angle of contact.

The angle between the tangent to the curved liquid surface at the point of contact and the solid surface inside the liquid is called the angle of contact for a given pair of solid and liquid.

The liquid that wet a solid, the angle of contact becomes acute, i.e., less than 90^o. For glass and pure water the angle of contact is nearly 8^o. The liquid that do not wet a solid, the angle of contact become obtuse, i.e., greater than 90^o. For glass and pure mercury the angle of contact is 139^o.

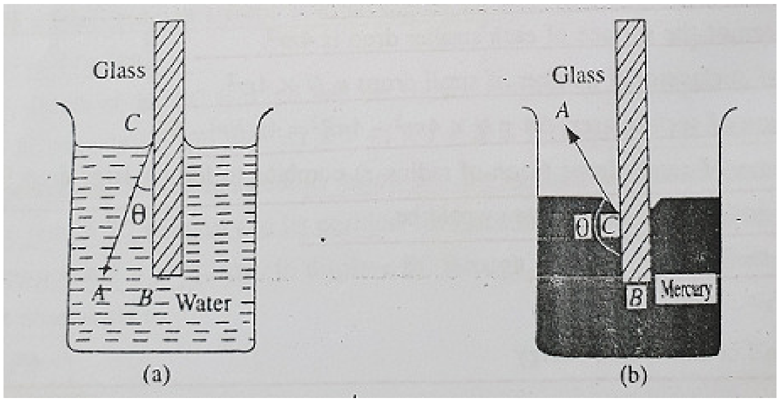

Fig. 3.6 Angle of contact. (a) Glass-water pair, the contact angle is acute. (b) Glass-mercury pair, the contact angle is obtuse.

The angle of contact depends on the following factors

1. Nature of solid and liquid
2. The medium above the free surface of the liquid. For example, if there is air above mercury, then the angle of contact between mercury and glass will be different than if there is water above the mercury.
3. Purity of solid and liquid. If the liquid is not pure and or if any other material is present at solid's surface, then the angle of contact changes. The angle of contact between pure water and clear glass is nearly zero, but if even a small oily material is there in the glass, then the angle of contact increases.

Capillarity

A tube having very small and uniform area of cross section is known as capillary tube. If an end of a capillary glass tube is immersed in a liquid, then some liquid rises or falls from the free surface inside the tube. The liquids which wet the glass tube (e.g., water) for them the free surface of liquid inside the tube rises above the free surface of liquid of the outside vessel (Fig. 3.7a). The liquids which do not wet the glass tube (e.g., mercury) for them the free surface of liquid inside the tube falls below the free surface of liquid of the outside vessel (Fig. 3.7b).

The phenomenon of rise or fall of a liquid in the capillary tube is known as capillarity.

This happens due to surface tension of the liquids. In case of the rise of liquid inside the tube the upper surface of the liquid becomes concave while for the fall the surface becomes convex. The nature of the curvature of the liquid inside the tube depends on the relative magnitude of the adhesive and cohesive forces. The magnitude of the adhesive and cohesive forces depends on the natures of the liquid and solid. The adhesive force of the liquid which wets the solid (e.g., water and glass) is much greater than the adhesive force of the liquid which does not wet the solid (e.g., mercury and glass). Again the cohesive force of water is much less than the cohesive force of mercury.

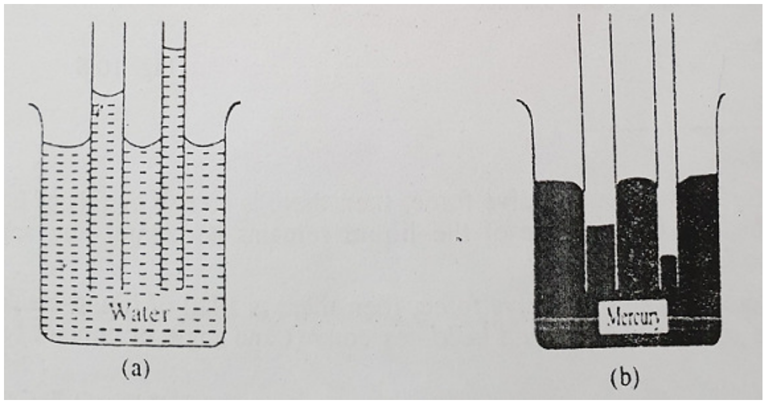

Fig. 3.7 Capillarity action. (a) Glass-water pair, water rises upward. (b) Glass-mercury pair, mercury falls downward.

Capillarity and Determination of Surface Tension of a Liquid

It is seen that if a capillary tube is immersed vertically in water or this kind of liquid (which wets the tube), the liquid rises a little inside the tube and the surface becomes concave. Let the angle of contact between the liquid and the solid be θ (Fig. 3.8). The radius of the tube where the liquid surface touches the tube be *r*. The height of the lower point of the liquid surface inside the tube from the liquid surface outside the tube be *h*. Let the density of the liquid be ρ and the surface tension be *T*.

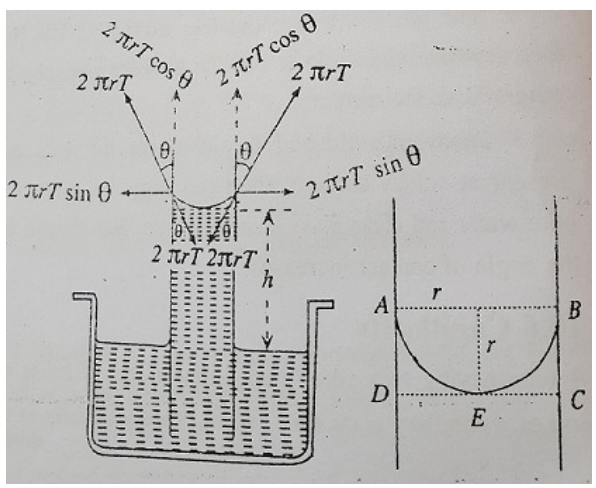

Fig. 3.8 Capillarity action along with forces different forces acting shown.

If a tangent is drawn from the inside wall of the tube and the point of contact of the liquid to the curved liquid surface, then the surface tension would act along the tangent inwards. The circumference of the capillary tube is $2\pi r$, which is the length of the line of the liquid which touches the inner wall of the tube. As a result the wall of the will experience an inward force of magnitude $2\pi rT$ along the tangent. According to the Newton's third law of motion, the wall will exert an equal and opposite force $2\pi rT$ on the liquid directed outwards. If this force is resolved into two perpendicular components, one will be

$2\pi rT\cos\theta$ vertically upwards and another will be $2\pi rT\sin\theta$ perpendicular to it and horizontally outwards (Fig. 3.8). As the components $2\pi rT\sin\theta$ act in pair and oppositely directed along the diameter of the tube, they cancel each other. Hence the net force on the liquid would be $2\pi rT\cos\theta$ vertically upwards. Since, due to the influence of this vertical force the liquid column inside the capillary tube start rising, hence when the weight of the liquid column becomes equal to this vertical force, the liquid column becomes stationary inside the tube. At this state, if the height of the lower end of the liquid column from the outside liquid surface is h, then the volume of the liquid column will be $\pi r^2 h$ plus the volume of the curved portion of the liquid. That is the total volume of the liquid column inside the tube is $\pi r^2 h + V_c$, where V_c is the volume of the curved portion of the liquid. Thus, the weight of the liquid column is $(\pi r^2 h + V_c)\rho g.$ Hence, at equilibrium, $2\pi rT\cos\theta = (\pi r^2 h + V_c)\rho g,$ from which we obtain

$$T = \frac{(\pi r^2 h + V_c)\rho g}{2\pi r\cos\theta}. \tag{3.21}$$

Now, we need to find the volume of the curved portion of the liquid V_c, and which is

$$V_c = volume\ of\ the\ cylinder\ ABCD - volume\ of\ the\ half\ of\ the\ sphere\ AEB$$
$$= \pi r^2 r - \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \pi r^3 - \frac{2}{3}\pi r^3 = \frac{1}{3}\pi r^3.$$

Thus, Eq. (3.19) becomes

$$T = \frac{(\pi r^2 h + \pi r^3/3)\rho g}{2\pi r\cos\theta},$$

or

$$T = \frac{r\rho g}{2\cos\theta}\left(h + \frac{r}{3}\right). \tag{3.22}$$

Equation (3.22) can be used to determine the surface tension of any liquid.

Special Cases:

Now, if the capillary tube is very narrow, i.e., the magnitude of *r* is very small compared to *h*, then the term *r*/3 can be neglected in comparison with *h* and Eq. (3.22) becomes

$$T = \frac{r\rho gh}{2\cos\theta}. \tag{3.23}$$

Furthermore, if we consider pure water as the liquid and with glass, the angle of contact is nearly $\theta = 0^o$, so that $\cos\theta = 1.$ In this case, Eq. (3.23) becomes

$$T = \frac{r\rho gh}{2}. \tag{3.24}$$

Thus, the surface tension of water can be determined from Eq. (3.24).

Some Phenomena Relating to Surface Tension

(a) If we pour some kerosene or other oil on water they spread around over the water surface. It happens, because the surface tension of water is greater than oil and hence water applies a tension on oil and for this the oil spreads around the water surface.

(b) If a painting brush is immersed in water, the fibers of the brush scattered, but when taken out from water the fibers stick with each other. This happens because there is no surface tension inside the water and hence the fibers remain scattered. But, when taken out from water, a thin membrane of water remains attached with the fibers. For surface tension the membrane tends to contracts, hence the fibers come close together.

(c) Using the surface tension property of liquid, turbulent sea can be made calm. If there are much waves in the sea, to make it calm oil is poured in the sea. When wind blows vigorously, the floating oil over the water surface advances with the advancement of waves and clear water remains behind. The surface tension of clear water is larger than the surface tension of oil mixed water; hence the surface tension is greater at behind than in front side. This increased surface tension opposes the formation of big waves, so the sea becomes calm.

(d) We use towel to wipe out water from our body after bath. The small holes in the towel act as capillary tube, thus for the capillary action water from our body comes to towel. Blotting papers also absorb water for capillary.

(e) Water remains cold in clay made container. The reason is that the small holes in the container act as capillary tube. As a result, due to capillary action water comes out to the outer surface and evaporates. The latent heat necessary for evaporation comes from the water in the container, hence water remains cold.

(f) The clay is generally wet, while sand is drier than clay. The reason is that the small holes in the clay act as capillary tube. As a result, due to capillary action water comes out to the outer surface of the clay and it remains wet. The space between the sand particles are comparatively large and do not act as capillary tube. Thus, water remains under sand; it does not come to upper surface. Hence sand is drier than clay.

(g) If we heat one end of a narrow glass tube, the end becomes spherical. When the end of the glass tube is heated the glass of this end melts. This melted glass acts as liquid and due to the surface tension tries to attain minimum surface area. Hence, the end becomes spherical.

(h) The reading in a mercury barometer is always less than the actual reading. Due to the capillary action there is a fall in the mercury column inside the barometer tube. The upper surface of the mercury remains a bit lower than where it is supposed to be. Thus, the mercury barometer gives reading, which is less than the actual reading.

(i) In a clear glass plate water spreads around but mercury becomes a drop. We know that the angle of contact between water and glass is an acute one, whereas the angle of contact between mercury and glass is obtuse one. This happens due to surface tension. To keep the angle of contact with glass an obtuse angle, mercury takes the shape of a sphere; while to keep the angle of contact with glass acute angle water has to spread around.

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