Plane

$$a_1x + b_1y + c_1z + d_1 = 0$$
$$a_2x + b_2y + c_2z + d_2 = 0$$

Equation (i) and (ii) will be parallel if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

equation (i) and (ii) will be perpendicular if

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

Example ①: Find the eqn of the plane through the points $(0,-1,2)$, $(2,3,-1)$ and $(-1,0,0)$

Solve: Let the equation of the plane be

$$ax + by + cz + d = 0 \quad \text{-----(i)}$$

Since, equation eqn (i) passes through the points $(0,-1,2)$, $(2,3,-1)$ and $(-1,0,0)$

$$a.0 + b(-1) + 2c + d = 0 \quad \text{-------(ii)}$$
$$a.2 + b(3) + c(-1) + d = 0 \quad \text{-------(iii)}$$
$$a.(-1) + b.0 + c.0 + d = 0 \quad \text{-------(iv)}$$

Now eliminating a, b, c, d from equation (i), (ii), (iii) & (iv) we get,

$$\begin{vmatrix} \overset{(+)}{x} & \overset{(-)}{y} & \overset{(+)}{z} & \overset{(-)}{1} \\ 0 & -1 & 2 & 1 \\ 2 & 3 & -1 & 1 \\ -1 & 0 & 0 & 1 \end{vmatrix} = 0$$

$$\Rightarrow x\begin{vmatrix}-1 & 2 & 1\\ 3 & -1 & 1\\ 0 & 0 & 1\end{vmatrix} - y\begin{vmatrix}0 & 2 & 1\\ 2 & -1 & 1\\ -1 & 0 & 1\end{vmatrix} + z\begin{vmatrix}0 & -1 & 1\\ 2 & 3 & 1\\ -1 & 0 & 1\end{vmatrix} + 1\begin{vmatrix}0 & -1 & 2\\ 2 & 3 & -1\\ -1 & 0 & 0\end{vmatrix} = 0$$

$$\Rightarrow x\left[-1(-1-0) - 2(\overset{3-0}{\cancel{3-0}}) + 1(0+0)\right] - y\left[-2(2+1) + 1(0-1)\right] + z\left[-(-1)(2+1) + 1(0+3)\right] + 1\left[-(-1)(0-1) + 2(0+3)\right]$$

$$\Rightarrow \quad +7y + 6z$$

Example: 2 Find the equation of the plane through the point (4, 0, 1) and parallel to plane $4x+3y-12z+6=0$

Solve: Let the equation of the plane be,

$$ax+by+cz+d=0 \quad \text{------} \; (i)$$

Since equation (i) passes through the point (4, 0, 1)

$$4.a + b.0 + c.1 + d = 0$$

$$\therefore 4a + c + d = 0 \quad \text{------} \; (ii)$$

Given plane $4x+3y-12z+6=0$ — (iii)

According to the question, eqn (i) and eqn (iii) parallel

$$\frac{a}{4} = \frac{b}{3} = \frac{c}{-12} = K \; (\text{say})$$

$$a = 4K, \; b = 3K, \; c = -12K$$

$$(ii) \Rightarrow 4(4K) + (-12K) + d = 0$$

$$\therefore d = \cancel{4} -4K$$

Putting the values of a, b, c and d in eqn (i),

$$4k.x + 3k.y - 12k.z - 4k = 0$$

$\Rightarrow 4x + 3y - 12z - 4 = 0$ which is the required eqn of the plane.

Example: 3 Find the equation of the plane passing through the line of intersection of the plane $2x - y = 0$ and $3z - y = 0$ and perpendicular to the plane $4x + 5y - 3z + 1 = 0$

Solution: Given that, the plane be

$2x - y = 0$ . . . . (i)

$3z - y = 0$ . . . . (ii)

$4x + 5y - 3z + 1 = 0$ . . . . (iii)

The equation of the plane passing through the intersection of the plane (ii) and (iii)

$$2x - y + k(3z - y) = 0$$

$\Rightarrow 2x - y(1 + k) + 3kz = 0$ . . . . (iv)

Since, the plane (iii) and (iv) perpendicular

$$2.4 + (-(1+k).5) + (3k(-3)) = 0$$

$$\Rightarrow 8 - 5 - 5k - 9k = 0$$

$$\Rightarrow k = 3/14$$

Putting the value of k in equation (iv) we get,

$$2x - y\left(1 + \frac{3}{14}\right) + 3 . \frac{3}{14} . z = 0$$

$\Rightarrow 28x - 14y\left(1 + \frac{3}{14}\right) + 9 = 0$

$\Rightarrow 28x - y(14+3) + 9 = 0$

$\therefore 28x - 14y + 9 = 0$, which is the required equation of the plane.

Example: 4 Find the equation of the plane passing through the points $(-1, 1, 1)$ and $(1, -1, 1)$ and perpendicular to the plane $x + 2y + 2z = 5$

Solve: The equation of the plane passing through the point $(-1, 1, 1)$ is,

$$a(x+1) + b(y-1) + c(z-1) = 0 \quad \text{--- (i)}$$

Since the plane (i) passes through the point $(1, -1, 1)$

$a(1+1) + b(-1-1) + c(1-1) = 0$

~~$\Rightarrow 2a - b + c.0 = 0$~~ $\Rightarrow 2a - 2b + c.0 = 0$

$\Rightarrow a - b = 0$ ---- (ii)

According to the question, eqn (i) perpendicular to the plane $x + 2y + 2z = 5$

$a.1 + b.2 + c.2 = 0$

$\Rightarrow a + 2b + 2c = 0$

By cross multiplication, from (ii) and (iii), we get,

$$\frac{a}{-2-0} = \frac{b}{0-2} = \frac{c}{2+1}$$

$$\Rightarrow \frac{a}{-2} = \frac{b}{-2} = \frac{c}{3} = K \quad (\text{say})$$

$$\therefore a = -2K,\ b = -2K,\ c = 3K$$

Putting the value of a, b, c in (i) we get,

$$-2K(x+1) - 2K(y-1) + 3K(z-1) = 0$$

$$\Rightarrow -2(x+1) - 2(y-1) + 3(z-1) = 0$$

$$\Rightarrow -2x - 2 - 2y + 2 + 3z - 3 = 0$$

$\Rightarrow 2x + 2y - 3z + 3 = 0$, which is the required eqn of the plane.