CSE-2 MAT

08.10.2023

Pair of straight Line

- 2nd Degree Homogeneous Equation

$$ax^2+2hxy+by^2=0$$

(i) and (ii) both are straight Line

(i) $\perp$ (ii) $\rightarrow$ if $a+b=0$

(i) $\parallel$ (ii) $\rightarrow$ if $h^2=ab$

$h^2>ab \rightarrow$ line real and different

$h^2<ab \rightarrow$ line imaginatiory

- General equation of the second Degree (Non-homogeneous):

$$ax^2+2hxy+by^2+2gx+2fy+c=0$$

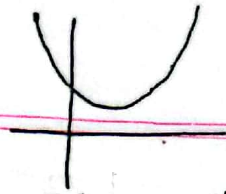
$$\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = abc+2fgh-af^2-bg^2-ch^2$$

(1) two straight Line if $\Delta=0$

$\parallel$ (a) two parallel line if $\Delta=0, h^2=ab$

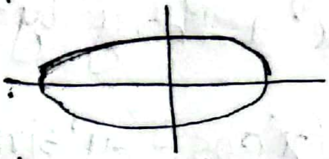
$\perp$ (b) two perpendicular lines if $\Delta=0, a+b=0$

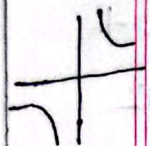
(2) circle, if $\Delta\neq0$, $a=b$, $h=0$



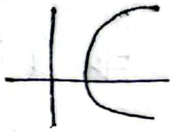
③ parabola, if $\Delta \neq 0$, $h^2 = ab$

④ ellipse, if $\Delta \neq 0$, $h^2 - ab < 0$



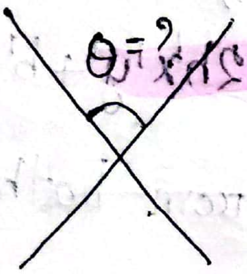


⑤ hyperbola, if $\Delta \neq 0$, $h^2 - ab > 0$ $\longrightarrow$



⑥ rectengular hyperbola if $\Delta \neq 0$, $h^2 - ab > 0$, $a + b = 0$

- $ax^2 + 2hxy + by^2 = 0$, what is the angle between two straight lines?



$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b}$$

▫ $\tan 2\theta = \frac{2h}{a-b}$

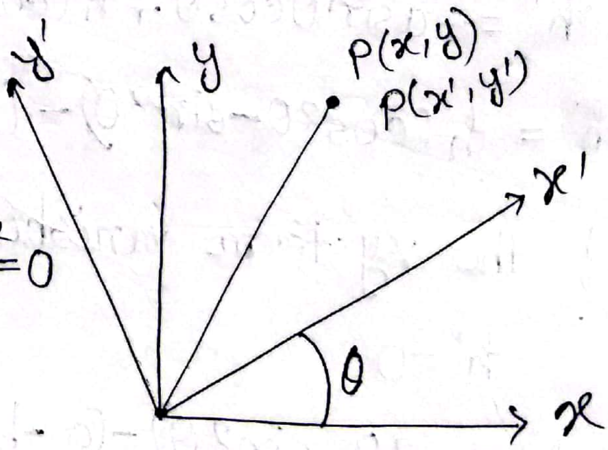
$\Rightarrow \theta = \frac{1}{2}\tan^{-1}\frac{2h}{a-b}$

▫ θ Angle এ Rotate করে

θ

$\boxed{xy}$ $ax^2 + 2hxy + by^2 = 0$

$\boxed{x'y'}$ $a'x'^2 + 2h'x'y' + b'y'^2 = 0$



Invariant:

$a + b = a' + b'$

$ab - h^2 = a'b' - h'^2$

$x = x'\cos\theta - y'\sin\theta$

$y = x'\sin\theta + y'\cos\theta$

$$ax^2 + 2hxy + by^2$$

$\Rightarrow a(x'\cos\theta - y'\sin\theta)^2 + 2h(x'\cos\theta - y'\sin\theta)(x'\sin\theta + y'\cos\theta) + b(x'\sin\theta + y'\cos\theta)^2 = a'x'^2 + 2h'x'y' + b'y'^2$

$\Rightarrow ax'^2\cos^2\theta - 2ax'y'\cos\theta\sin\theta + y'^2 a\sin^2\theta + 2h(x'^2\sin\theta\cos\theta + x'y'\cos^2\theta - x'y'\sin^2\theta - y'^2\sin\theta\cos\theta) + bx'^2\sin^2\theta + 2bx'y'\sin\theta\cos\theta + by'^2\cos^2\theta = a'x'^2 + 2h'x'y' + b'y'^2$

$\Rightarrow$ Equating co-efficient of $x'y'$ from both sides of the equation:

$$(-2a\cos\theta\sin\theta + 2h(\cos^2\theta - \sin^2\theta) + 2b\sin\theta\cos\theta) = 2h'$$

$\Rightarrow h' = -a\sin\theta\cos\theta + h(\cos^2\theta - \sin^2\theta) + b\sin\theta\cos\theta$

$\Rightarrow h' = h(\cos^2\theta - \sin^2\theta) - (a-b)\sin\theta\cos\theta$

If the xy term vanishes, then

$$h' = 0$$

$\Rightarrow h(\cos^2\theta - \sin^2\theta) - (a-b)\sin\theta\cos\theta = 0$

$\Rightarrow h\cos 2\theta - \left(\frac{a-b}{2}\cdot\sin 2\theta\right) = 0$

$\Rightarrow h\cos 2\theta = \frac{a-b}{2}\sin 2\theta$

$$\Rightarrow 2h = (a-b)\frac{\cancel{\sin\theta}\sin 2\theta}{\cos 2\theta}$$

$$\Rightarrow \cancel{\tan 2\theta} \cdot \tan 2\theta = \frac{2h}{a-b}$$

$$\therefore \theta = \frac{1}{2}\tan^{-1}\left(\frac{2h}{a-b}\right)$$

*** find the angle through which the axis are ~~to~~ to be rotated that the expression $ax^2+2hxy+by^2$ is transformed into an expression not containing the xy term.

☐ If the equation $3x^2-16xy+5y^2=0$, then find

(a) angle between the lines

(b) equation of the lines

Given that, $3x^2-16xy+5y^2=0$ ---- (i)

Comparing equation (1) with $ax^2+2hxy+by^2=0$

$$a=3,\ 2h=-16,\ b=5,\ h=-8$$

Let, θ be the angle between the lines

$$\tan\theta = \frac{2\sqrt{h^2-ab}}{a+b}$$

$$\Rightarrow \tan\theta = \frac{2\sqrt{64-15}}{3+5}$$

$\therefore \theta = 60.26°$ (Ans)

ax^2+bx+c

$3x^2-16xy+5y^2$

$\Rightarrow 3x^2-(16y)x+5y^2=0$

$\Rightarrow x = \dfrac{-(-16y) \pm \sqrt{(-16y)^2-4.3.5y^2}}{2.3}$

$\Rightarrow x = \dfrac{16y \pm \sqrt{196y^2}}{6}$

$\Rightarrow x = \dfrac{16y \pm 14y}{6}$

$a = 3$

$b = (-16y)$

$c = 5y^2$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

(+) $x = \dfrac{16y+14y}{6}$

$\Rightarrow x = \dfrac{30y}{6}$

$\therefore x-5y=0$ --- (2)

(−) $y = \dfrac{16y-14y}{6}$

$\Rightarrow 3x-y=0$ ---- (3)

MA110A

10.10.2023

(1) What are represented by the following equation
$12x^2+7xy-10y^2+13x+45y-35=0$

(a) If it will be a straight line than find
(i) the angle between the lines.
(ii) find the equation of each line.

(b) If it will be a curve, then find
(i) find a rotation angle by which the xy term will be eliminated.
(ii) find the standard form

Ans: (a) Given equation,

$12x^2+7xy-10y^2+13x+45y-35=0$ ... (i)

Comparing eqn (1) with $ax^2+2hxy+by^2+2gx+2fy+c=0$

$a=12,\ 2h=7,\ b=-10,\ 2g=13,\ 2f=45,\ C=-35$

$\therefore h=7/2 \qquad \therefore g=13/2 \quad \therefore f=45/2$

equation (1) will be represented pair of straight lines if $\Delta=0$

$\Delta = abc+2fgh-af^2-bg^2-ch^2$

$= 12.(-10).(-35)+2.\frac{45}{2}.\frac{13}{2}.\frac{7}{2}-12\left(\frac{45}{2}\right)^2-(-10)\left(\frac{13}{2}\right)^2$
$-(-35)(7/2)^2$

$=0.$

Since $\Delta=0$, So, eqn (1) represented pair of straight lines

Let θ be the angle between the lines

$$\tan\theta = \frac{2\sqrt{h^2-ab}}{a+b} = \frac{2\sqrt{\frac{49}{4}-120}}{12-10}$$

$$\theta = 85^\circ$$

$$12x^2+7xy-10y^2+13x+45y-35=0$$

Comparing with $ax^2+bx+c=0$, we get :

$$12x^2+x(7y+13)+(-10y^2+45y-35)=0$$

$$\therefore x = \frac{-(7y+13)\pm\sqrt{(7y+13)^2-4.12.(-10y^2+45-35)}}{2.12}$$

$$24x = -(7y+13)\pm\sqrt{529y^2-1978y+1849}$$

$$\Rightarrow 24x = -(7y+13)\pm\sqrt{(23y-43)^2}$$

$$\Rightarrow 24x = -(7y+13)\pm(23y-43)$$

(+) $24x = -7y-13+23y-43$

$\Rightarrow 24x-16y+56=0$

$\Rightarrow 3x-2y+7=0$ - - - - (ii)

(−) $24x = -7y-13-23y+43$

$\Rightarrow 24x+30y-30=0$

$\Rightarrow 4x+5y-5=0$ - - - - (iii)

eq^n (ii) and (iii) are the required straight lines.

(b) It is impossible because both the eq^n ~~are~~ represent straight lines. It cannot be curve

(2) $x^2+2xy+y^2+2x-1=0$

Same as (1)(b) question.

(i) The equation is $x^2+2xy+y^2+2x-1=0$ ....(i)

Comparing eq^n(i) with

$$ax^2+2hxy+by^2+2gx+2fy+c=0$$

$a=1$, $2h=2$, $b=1$, $2g=2$, $2f=0$, $c=-1$

$\therefore h=1$ $\quad\therefore g=1$ $\quad\therefore f=0$

$$\Delta = abc+2fgh-af^2-bg^2-ch^2$$

$$=(1.1.(-1))+2(0.1.1)-(1.0^2)-(1.1^2)-(-1.1^2)$$

$$=-1+0-0-1+1 = -1 \neq 0$$, ~~so the eq^n represent~~

again,

$h^2-ab = 1-1 = 0$, so the eq^n represent parabola.

$$\tan 2\theta = \frac{2h}{a-b} = \frac{2}{1-1} = \infty$$

$\Rightarrow \tan 2\theta = \tan 90°$

$\Rightarrow 2\theta = 90° \quad \therefore \theta = 45°$

$$x = x'\cos 45° - y'\sin 45° = \frac{x'-y'}{\sqrt{2}}$$

$$y = x'\sin 45° + y'\sin 45° = \frac{x'+y'}{\sqrt{2}}$$

$$x'+\frac{1}{2\sqrt2}=4.\frac{1}{4\sqrt2}\left(y'+\frac{5}{4\sqrt2}\right)$$

eq^n (1) $\Rightarrow \left(\frac{x'-y'}{\sqrt2}\right)^2+2\left(\frac{x'-y'}{\sqrt2}\right)\left(\frac{x'+y'}{\sqrt2}\right)+\left(\frac{x'+y'}{\sqrt2}\right)^2+2\left(\frac{x'-y'}{\sqrt2}\right)-1=0$

$\Rightarrow \left(\frac{x'-y'}{\sqrt2}\right)^2+(x'-y')(x'+y')+\frac{(x'+y')^2}{2}+\sqrt2x'-\sqrt2y'-1=0$

$\Rightarrow \frac{(x'-y')^2}{2}+x'^2+\cancel{x'y'}-\cancel{x'y'}-y'^2+\frac{x'^2+2x'y'+y'^2}{2}-\sqrt2x'-\sqrt2y'-1=0$

$\Rightarrow$ ~~$1+x'^2+y'^2+2x'^2+\frac{2x'^2}{2}+\frac{2(2)x'y'}{2}-\frac{2x'}{2}-2y'=0$~~

$\Rightarrow$ ~~$-x'-3y'+2\sqrt2x'^2+2\sqrt2x'y'-1=0$~~ $\times$

$\Rightarrow$ ~~$x'-2\sqrt2x'^2-2\sqrt2x'y'+3y'+1=0$~~

$\Rightarrow \frac{x'^2-2x'y'+y'^2}{2}+x'^2+\frac{x'^2+2x'y'+y'^2}{2}-y'^2-\sqrt2x'-\sqrt2y'-1=0$

$\Rightarrow \frac{x'^2-\cancel{2x'y'}+y'^2+2x'^2+x'^2+\cancel{2x'y'}+y'^2-2y'^2-2\sqrt2x'-2\sqrt2y'-2}{2}=0$

$\Rightarrow 4x'^2-2\sqrt2x'-2\sqrt2y'-2=0$

$\Rightarrow 2x'^2-\sqrt2x'-\sqrt2y'-\frac{1}{2}=0$

$\Rightarrow 2x'^2-\sqrt2x'=\sqrt2y'+1$

$\therefore \Rightarrow \left(x'+\frac{1}{2\sqrt2}\right)^2=4.\frac{1}{4\sqrt2}\left(y'+\frac{5}{4\sqrt2}\right)$

$X^2=4AY$

Which is the standard form of parabola.

$2x'^2+\sqrt2x'=\sqrt2y'+1$

$\Rightarrow 2x^2+\sqrt2x=\sqrt2y+1$

$\Rightarrow x^2+\frac{x}{\sqrt2}=\frac{y}{\sqrt2}+\frac{1}{2}$

$\Rightarrow x^2+2x\frac{1}{2\sqrt2}+\left(\frac{1}{2\sqrt2}\right)^2=y/\sqrt2+\frac12+(1/2\sqrt2)$

(3) Identify the curve $xy = 1$

~~on, Reduce~~ The equation is $xy = 1$ . . . (i)

Comparing eqn (i) with $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

$2h = 1 \quad \therefore h = 1/2, \quad a = 0, \quad b = 0$

$\cancel{\tan 2\theta =} \quad \tan 2\theta = \frac{2h}{a-b} = \frac{1}{0-0}$

$\therefore \theta = 45^\circ$

$x = x'\cos 45^\circ - y'\sin 45^\circ = \frac{x'-y'}{\sqrt{2}}$

$y = x'\sin 45^\circ + y'\cos 45^\circ = \frac{x'+y'}{\sqrt{2}}$

(1) $\Rightarrow \frac{x'-y'}{\sqrt{2}}\left(\frac{x'+y'}{\sqrt{2}}\right) = 1$

$\Rightarrow \frac{x'^2 - y'x' + x'y' - y'^2}{2} = 1$

$\Rightarrow x'^2 - y'^2 = 2$

Removing suffixes, $x^2 - y^2 = 2$

which eqn represent an equidateral hyperbola

(4) $3y^2 - 8xy - 3x^2 - 29x + 3y - 18 = 0$

$\Rightarrow$ If it will be straight lines, then find

(i) each lines

(ii) Also their point of intersection.

(iii) angles between them.

(5) $Px^2 + 4xy + y^2 - 4x - 2y - 3 = 0$

For ~~p~~ What value of P, this equation represent straight line?

$\Delta = 0, \; P = ?$

Same as (4) question.

MAT104
15.10.2023

Rotate the co-ordinate axis to remove the x,y term, then identify the conic.

$x^2 + 4xy - 2y^2 - 6 = 0$ reduce the equation to the standarm form.

Determine the nature of the conic

Ans: Given equation $x^2 + 4xy - 2y^2 - 6 = 0$ - - - (i)

Comparing eqn (1) with

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

$a = 1, \; 2h = 4, \; b = -2, \; c = -6, \; g = 0, \; f = 0$

$\therefore h = 2$

$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$

$= (1)(-2)(-6) + 2.0.0.2 - 1(0^2) - b(0)^2 - (-6)(-2)^2$

$= 12 + 24$

$= 36 \neq 0$

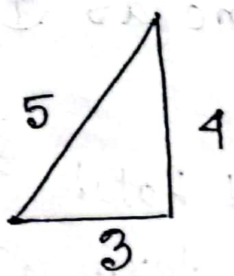
$h^2 - ab = 2^2 - (1)(-2)$

$= 6 > 0$

Since, $\Delta \neq 0$, $h^2 - ab > 0$, So, this equation represent a hyperbola.

the xy term will be vanish if the axis are to be rotated ~~tan 2θ =~~ through an angle θ.

$$\tan 2\theta = \frac{2h}{a-b}$$



$$\Rightarrow \tan 2\theta = 4/3$$

$$\cos 2\theta = \frac{3}{5} \qquad \cancel{\sin 2\theta = \frac{4}{5}}$$

$\therefore 2\cos^2\theta = 1 + \cos 2\theta$

$\Rightarrow \cos^2\theta = \frac{4}{5}$

$\therefore \cos\theta = 2/\sqrt{5}$

$\sin^2\theta = 1 - \cos^2\theta$

$= 1 - \frac{4}{5}$

$= \frac{1}{5}$

$\therefore \sin\theta = 1/\sqrt{5}$

$$x = x'\cos\theta - y'\sin\theta$$

$$x = \frac{2x'}{\sqrt{5}} - \frac{y'}{\sqrt{5}} = \frac{2x' - y'}{\sqrt{5}}$$

$$y = x'\sin\theta + y'\cos\theta$$

$$= \frac{x'}{\sqrt{5}} + \frac{2y'}{\sqrt{5}} = \frac{x' + 2y'}{\sqrt{5}}$$

putting the value in eqn (1)

$$\left(\frac{2x'-y'}{\sqrt{5}}\right)^2 + 4\left(\frac{2x'-y'}{\sqrt{5}}\right)\left(\frac{x'+2y'}{\sqrt{5}}\right) - 2\left(\frac{x'+2y'}{\sqrt{5}}\right)^2 - 6 = 0$$

$$\frac{2x'^2 - 4x'y' + y'^2}{5} + \frac{4(2x'^2 + 4x'y' - x'y' - 2y'^2)}{5} -$$

$$\frac{2(x'^2 + 4x'y' + 4y'^2)}{5} - 6 = 0$$

$$\frac{4x'^2 - 4x'y' + y'^2 + 8x'^2 + 16x'y' - 4x'y' - 8y'^2 - 2x'^2 - 8x'y' - 8y'^2}{5}$$

$$-6 = 0$$

$$\frac{10x'^2 - 15y'^2 - 30}{5} = 0$$

$$\therefore 2x'^2 - 3y'^2 - 6 = 0 \qquad \therefore \frac{x'^2}{3} - \frac{y'^2}{2} = 1$$

~~Test~~ Determine the nature of the following equation. Also ~~d~~ reduce the following equation to the standard form using a suitable rotation and identify the conic.

$$x^2 - 10\sqrt{3}\,xy + 11y^2 + 64 = 0$$

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, comparing we get, $a = 1$, $2h = -10\sqrt{3}$, $b = 11$, $g = 0$, $f = 0$, $c = 64$

$$\Rightarrow h = -5\sqrt{3}$$

$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$

$= (1.11.64) + 0 + 0 + 0 - 64(-5\sqrt{3})^2$

$= 704 - 4800$

$= -4096 \neq 0$

$h^2 - ab = (-5\sqrt{3})^2 - (1.11)$

$= 64 > 0$

Since, $\Delta \neq 0$, $h^2 - ab > 0$, So this equation is representing hyperbola.

$\tan 2\theta = \frac{2h}{a-b}$

$\Rightarrow \tan 2\theta = \frac{2(-5\sqrt{3})}{1-11} = \sqrt{3}$

$\Rightarrow \theta = 30°$

$x = x'\cos 30° - y'\sin 30°$

$x = \frac{x'\sqrt{3}}{2} - \frac{y'}{2} = \frac{x'\sqrt{3} - y'}{2}$

$y = x'\sin 30° + y'\cos 30°$

$= \frac{x'}{2} + \frac{\sqrt{3}y'}{2} = \frac{x' + \sqrt{3}y'}{2}$

$$x^2 - 10\sqrt{3}xy + 11y^2 + 64 = 0$$

$$\Rightarrow \left(\frac{x'\sqrt{3} - y'}{2}\right)^2 - 10\sqrt{3}\left(\frac{x'\sqrt{3} - y'}{2}\right)\left(\frac{x' + \sqrt{3}y'}{2}\right)$$

$$+ 11\left(\frac{x' + \sqrt{3}y'}{2}\right)^2 + 64 = 0$$

$$\Rightarrow \frac{x'^2 3 - 2x'y'\sqrt{3} + y'^2}{4} - \frac{10\sqrt{3}\,(x'^2\sqrt{3} + 3x'y' - x'y' - \sqrt{3}y'^2)}{4}$$

$$+ \frac{11\,(x'^2 + 2\sqrt{3}x'y' + 3y'^2)}{4} + 64 = 0$$

$$\Rightarrow \frac{3x'^2 - 2x'y'\sqrt{3} + y'^2 - 10(3)x'^2 - 30\sqrt{3}x'y' + 10\sqrt{3}x'y' + 30y'^2}{4}$$

$$\cancel{+64 = 0} \quad \frac{+11x'^2 + 22\sqrt{3}x'y' + 33y'^2}{4} + 64 = 0$$

$$\Rightarrow \frac{-16x'^2 + 64y'^2 + 256}{4} = 0$$

$$\Rightarrow 4x'^2 - 16y'^2 - 64 = 0$$

$$\Rightarrow x'^2 - 4y'^2 - 16 = 0$$

$$\Rightarrow \frac{x'^2}{16} - \frac{4y'^2}{16} = 0 \qquad \therefore \frac{x'^2}{16} - \frac{y'^2}{4} = 0$$

Find the value of λ, the following equation represent ~~to~~ two straight lines? Also find there point of intersection and the angle between them.

$$2x^2 - y^2 + xy - 2x - 5y + \lambda = 0$$

Ans: $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

$a = 2,\ 2h = 1,\ b = -1,\ g = \cancel{0}\,-1,\ f = -5/2,\ c = \lambda$

$$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

$$= 2\cdot(-1)\cdot\lambda + 2\left(-\frac{5}{2}\cdot -1\cdot\frac{1}{2}\right) - 2\left(\frac{-5}{2}\right)^2 - (-1)(-1)^2 - \lambda\left(\frac{1}{2}\right)^2$$

$$= -2\lambda + \frac{5}{2} - \frac{25}{2} + 1 - \frac{\lambda}{4} = 0$$

$$\Rightarrow \frac{-8\lambda + 10 - 50 + 4 - \lambda}{4} = 0$$

$$\Rightarrow \frac{-9\lambda + 44}{4} = 0$$

$$\Rightarrow \lambda = -4$$

Quiz: Sunday

1. translation / Shifting
2. Rotation
3. translation + Rotation
4. Nature (pair of straight line, angle, intersection)

$2x-y-4=0$
$x+y+1=0$

$2x^2-y^2+xy-2x-5y-4=0$

$\Rightarrow 2x^2+x(y-2)+(-y^2-5y-4)=0$

$\Rightarrow$

$$x=\frac{-(y-2)\pm\sqrt{(y-2)^2-4.2.(-y^2-5y-4)}}{2\cdot 2}$$

$$=\frac{-y+2\pm\sqrt{y^2-4y+4+8y^2+40y+32}}{4}$$

$$=\frac{(2-y)\pm\sqrt{9y^2+36y+36}}{4}$$

$$=\frac{(2-y)\pm 3y+6}{4}$$

(+) $4x=2-y+3y+6$

$\Rightarrow 4x=2y+8$

$\therefore 2x=y+4$

$\therefore 2x-y-4=0$

(−) $4x=2-y-3y-6$

$\Rightarrow 4x=-4y-4$

$\therefore x=-y-1$

$\therefore x+y+1=0$

$x=1$
$y=-2$

$$\tan\theta=\frac{2\sqrt{h^2-ab}}{a+b}$$

$$\Rightarrow \tan\theta=\frac{2\sqrt{\frac{1}{4}-2.(-1)}}{2-1}=3,$$ which is not possible.

MAT 104

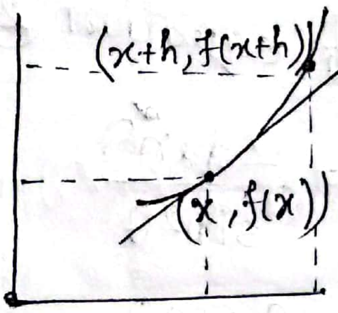
17.10.2023

*** □ Show that, the two straight lines of $x^2(\tan^2\theta + \cos^2\theta) - 2xy\tan\theta + y^2\sin^2\theta = 0$, makes engage with the x-axis such that the difference of theire tangent is 2.

Or, Theory: $\frac{dy}{dx} = \frac{Rise}{Run}$

$= \frac{f(x+h) - f(x)}{x+h-x}$

$= \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$



(I) $\to \alpha$

(II) $\to \beta$

$m = \frac{dy}{dx} = \frac{Rise}{Run}$

• $\tan\alpha - \tan\beta = 2$

এভাবে question থাকতে পারে

ঢাল $= \tan\theta = m$

Ans:

The given equation,

$$x^2(\tan^2\theta + \cos^2\theta) - 2xy\tan\theta + y^2\sin^2\theta = 0$$

$$\Rightarrow y^2\sin^2\theta - 2xy\tan\theta + x^2(\tan^2\theta + \cos^2\theta) = 0$$

$$\Rightarrow y^2 - 2xy\frac{\tan\theta}{\sin^2\theta} + x^2\frac{\tan^2\theta + \cos^2\theta}{\sin^2\theta} = 0 \quad \text{--- (1)}$$

equation (1) represent two straight lines such that, $y = m_1x$ and $y = m_2x$ where m_1 and m_2 are the slope of two lines.

$$(y - m_1x)(y - m_2x) = y^2 - 2xy\frac{\tan\theta}{\sin^2\theta} + x^2\frac{\tan^2\theta + \cos^2\theta}{\sin\theta}$$

$$\Rightarrow y^2 - ym_2x + ym_1x - m_1m_2x^2 = y^2 - 2xy\frac{\tan\theta}{\sin^2\theta} + x^2\frac{\tan^2\theta + \cos^2\theta}{\sin\theta}$$

$\Rightarrow y^2-(m_1+m_2)xy+m_1m_2x^2 = y^2 - 2xy\frac{\tan\theta}{\sin^2\theta} + x^2\frac{\tan^2\theta+\cos^2\theta}{\sin^2\theta}$ ----- (2)

equating the co-efficient of xy and x^2 from both sides of equation (2)

$$m_1+m_2 = \frac{2\tan\theta}{\sin^2\theta}, \quad m_1m_2 = \frac{\tan^2\theta+\cos^2\theta}{\sin^2\theta}$$

$$\Rightarrow m_1+m_2 = \frac{2\sec\theta}{\sin\theta}$$

We know, $(m_1-m_2)^2 = (m_1+m_2)^2 - 4m_1m_2$

$$= \left(\frac{2\sec\theta}{\sin\theta}\right)^2 - 4\left(\frac{\tan^2\theta+\cos^2\theta}{\sin^2\theta}\right)$$

$$= \frac{4\sec^2\theta}{\sin^2\theta} - 4\frac{\tan^2\theta+\cos^2\theta}{\sin^2\theta}$$

$$= \frac{4\sec^2\theta-4\tan^2\theta-4\cos^2\theta}{\sin^2\theta}$$

$$= 4\frac{1-\cos^2\theta}{\sin^2\theta} \quad [\sec^2\theta-\tan^2\theta=1]$$

$$= 4\frac{\sin^2\theta}{\sin^2\theta} = 4 \quad [1-\cos^2\theta=\sin^2\theta]$$

$\therefore m_1-m_2 = 2$

Wrong Answer

□ Prove that $(ax+by)(\alpha x+\beta y)+Kxy-(a+\alpha)x-(b+\beta)y+1=0$ represents a pair of straight lines if $K=(a-\alpha)(b-\beta)$. Find the co-ordinates of their point of intersection. $\cancel{K=\frac{a-\alpha}{b\beta}}$

Ans: $a\alpha x^2+a\beta xy+b\alpha xy+b\beta y^2+Kxy-ax-\alpha x-by-\beta y+1=0$

$\Rightarrow a\alpha x^2+xy(a\beta+b\alpha+K)+b\beta y^2-(a+\alpha)x-(b+\beta)y+1=0$

step ① $\Rightarrow$ Compare with $a_1x^2+2hxy+b_1y^2+2gx+2fy+c_1=0$

$a_1=a\alpha$; $2h=a\beta+b\alpha+K$; $b_1=b\beta$; $\cancel{2g\{-(a+\alpha)\}}$

$2g=-(a+\alpha)$; $2f=(-b+\beta)$; $c_1=1$;

step ② $\Rightarrow$ $\Delta=a_1b_1c_1+2fgh-a_1f^2-b_1g^2-c_1h^2=0$

$\Rightarrow (a\alpha)(b\beta)+2\{-(b+\beta)\}\{-\frac{(a+\alpha)}{2}\}\{\frac{a\beta+b\alpha+k}{2}\}$

$-a\alpha\{\frac{-(b+\beta)}{2}\}^2-b\beta\{\frac{-(a+\alpha)}{2}\}^2-(\frac{a\beta+b\alpha+k}{2})^2$

$\Rightarrow ab\alpha\beta+\frac{(b+\beta)(a+\alpha)(a\beta+b\alpha+k)}{4}-\frac{a\alpha(b+\beta)^2}{4}-\frac{b\beta(a+\alpha)^2}{4}-\frac{(a\beta+b\alpha+k)^2}{4}=0$

$\Rightarrow 4ab\alpha\beta+(b+\beta)(a+\alpha)(a\beta+b\alpha+k)-a\alpha(b+\beta)^2-b\beta(a+\alpha)^2-(\alpha\beta+b\alpha+k)^2=0$

$\Rightarrow 4ab\alpha\beta+(ab+a\beta+b\alpha+\alpha\beta)(a\beta+b\alpha+k)-a\alpha(b^2+2b\beta+\beta^2)-b\beta(a^2+2a\alpha+\alpha^2)-(\alpha^2\beta^2+b^2\alpha^2+k^2+2ab\alpha\beta+2bk\alpha+2ak\beta)=0$

$\Rightarrow \cancel{4ab\alpha\beta}+\cancel{a^2b\beta}+\cancel{ab^2\alpha}+abk+a^2\beta^2+ab\beta\alpha+a\beta k\cancel{+ab\alpha\beta}+\cancel{a\alpha\beta^2}$
$\cancel{b^2\alpha^2}+b\alpha k+a\alpha\beta^2+\cancel{\alpha^2b\beta}+\alpha\beta k-\cancel{a\alpha b^2}-2a\alpha b\beta-\cancel{a\alpha\beta^2}$
$-\cancel{a^2b\beta}-2ab\alpha\beta-\cancel{b\alpha^2\beta}-\alpha^2\beta^2-\cancel{b^2\alpha^2}-k^2-\cancel{2ab\alpha\beta}-2bk\alpha$
$+2ak\beta=0$