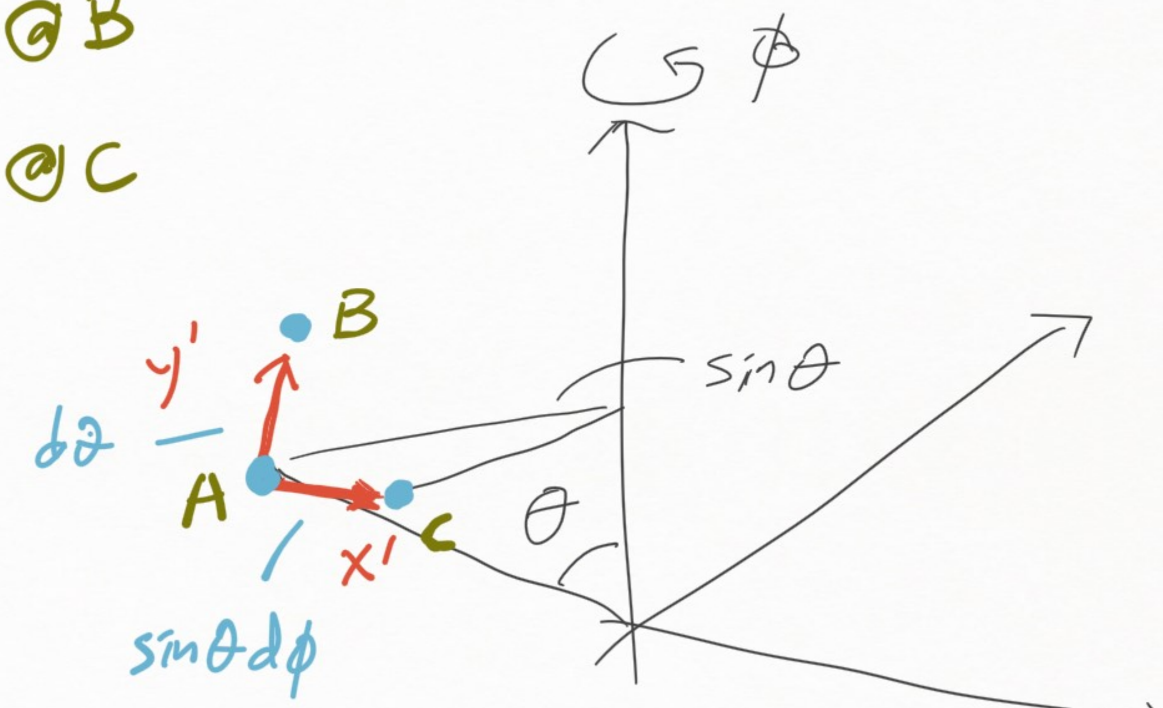
$f(\theta, \phi) = \sin^2\theta \cos\phi$ @A

$f(\theta + d\theta, \phi)$ @B

$f(\theta, \phi + d\theta)$ @C



distance

A − C : $\sin\theta d\phi$

A − B : $d\theta$

Differentiate w.r.t. local coordinates x', y'

$$\frac{\partial f}{\partial x'} = \lim_{d\phi \to 0} \frac{\sin^2\theta \cos(\phi + d\phi) - \sin^2\theta \cos(\phi)}{\sin\theta \, d\phi}$$

$$= -\sin\theta \sin\phi$$

$$\frac{\partial^2 f}{\partial x'^2} = \lim_{d\phi \to 0} \frac{-\sin\theta \sin(\phi + d\phi) + \sin\theta \sin\phi}{\sin\theta \, d\phi}$$

$$= -\cos\phi$$

$$\frac{\partial f}{\partial y'} = \lim_{d\theta \to 0} \frac{\sin^2(\theta + d\theta)\cos\phi - \sin^2\theta\cos\phi}{d\theta}$$

2sin(theta)cos(theta)cos(phi)

$$= \cancel{2\sin\theta\sin\theta\cos\phi} = \sin 2\theta \cos\phi$$

$$\frac{\partial^2 f}{\partial y'^2} = \lim_{d\theta \to 0} \frac{\sin 2(\theta + d\theta)\cos\phi - \sin 2\theta\cos\phi}{d\theta}$$

$$= 2\cos 2\theta \cos\phi$$

Therefore

$$\nabla_L^2 f = 2\cos 2\theta \cos\phi - \cos\phi$$

$$= (2\cos 2\theta - 1)\cos\phi$$

local 2D coordinates

use point cloud to compute $\nabla^2 f$ for all points and compare to this answer

report L_∞ norm.