Out-of-sample forecasting of housing bubble tipping points

(Ardila, Sanadgol, Sornette)

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ECE478: Financial Signal Processing

Definitions

- Out-of-sample/in-sample
- Financial bubble
- Tipping point

Deficit in the Literature

- Ad-hoc rules to estimate large scale deviations from equilibrium relations
- Very limited use of formal statistical tests
    - When they are employed, assume rather than examine the validity of the tests

Objective

- Examine the effectiveness of 5 statistical indicator tests in forecasting tipping points of:
  1. Housing bubbles
  2. Systemic financial crises
- The statistical tests examined:
  - Log Periodic Power Law Singularity Model: supLPPLS
  - Power Law Model: supPL
  - Kim-Busetti-Taylor statistic: supKBT
  - Busetti-Taylor statistic: supBT
  - Phillips statistic using Dickey-Fuller t-statistics: supDF

Dataset

- Quarterly times series data between 1975 and 2013 from 18 countries
- Existing benchmarks: price-to-income and price-to-rent time series

Challenges in Identifying Bubbles in the Data

1. No theoretical or empirical definition of a bubble

2. Hindsight may muddle bubble identification

Ex: Crash does not mean a bubble preceded but may be interpreted as such

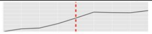
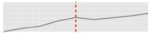
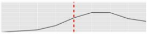
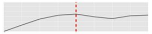
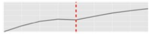
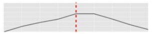
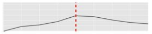
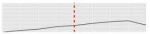
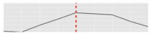
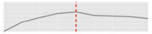
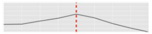
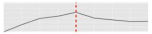
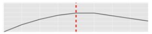
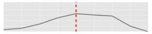
Ex: No strong correction after a price boom may lead scholars to conclude that the boom was not a bubble but driven by an unrecognized fundamental factor

Bubble Identification Method

- Fundamental Analysis of Prices - method of evaluating a security in an attempt to assess the intrinsic value, helps determine whether an asset is overvalued/undervalued
- Compile the results from 19 single-country, 3 bi-country, 3 multi-country studies based on fundamental analysis of prices

- More objective benchmark because internal macroeconomic conditions of the countries are considered in fundamental-based studies

Bubble Identification Method

Country	Peak t_p	$\ln p_t$ $[t_p - 4, t_p + 4]$	Reference
Australia	2003		Glindro et al. (2011) Berry and Dalton (2004)
Canada	2008		Walks (2014)
Denmark	2006		Sørensen (2013) Dam et al. (2011)
France	2008		Antipa and Lecat (2010)
Ireland	2002 2007	 	Connor et al. (2012) Stevenson (2008) Hott and Monnin (2008)
Japan	1991		Hott and Monnin (2008) Barsky (2009)
Netherlands	2006		Hott and Monnin (2008) Francke et al. (2009)
Norway	1989		Anundsen and Jansen (2011) Jacobsen and Naug (2005)
S. Africa	2008		Das et al. (2011)
S. Korea	1991		Glindro et al. (2011) Kim and Min (2011)
Spain	1992 2007	 	Ayuso and Restoy (2006) Neal and García-Iglesias (2013) Antipa and Lecat (2010)
Sweden	1990		Hort (1998) Andreas Claussen (2013) Sørensen (2013)

Bubble Peak = Peak of Overvaluation Period

- Local maxima is NOT bubble peak because fundamental factors may adjust
- Bubbles may not always burst (Netherland, Denmark - prices continue to rise)
- Asymmetric behavior

Theory of Rational Expectations Bubbles (REB)

- Log price time series:

  $\ln(p_t)$ where $t = 1, \ldots [\tau T], [\tau T]+1, \ldots T$ and $0<\tau<1$ and $[\tau T]$ is the starting period in which the bubble is detected

Theory of Rational Expectations Bubbles (REB)

- No arbitrage condition for the price of an asset:

$$P_t = \frac{1}{1+R} E_t(P_{t+1} + D_{t+1})$$

- P_t = asset price at time t; D_t = dividend received from the asset; 1/(1+R) is the discounted factor; E_t(-) is the expectation conditioned on the information at time t/adapted to the filtration

Theory of Rational Expectations Bubbles (REB)

- Log linear approximation f_hat of an equation f around the point x*:

$$\hat{x} = log(x) \; - log(x*)$$

$$\hat{f}(\hat{x}) \; = f(x*) \; + \; (x*)f'(x*)\hat{x}$$

Source: http://www.econ.boun.edu.tr/hatipoglu/ec504/loglin.pdf (Andrew Clausen)

Theory of Rational Expectations Bubbles (REB)

- Yields:

$$\ln P_t = p_t^f + b_t$$

$$p_t^f = \frac{\kappa - \gamma}{1 - \varrho} + (1 - \varrho) \sum_{t=1}^{\infty} \varrho^i E_t(d_{t+1+i}) \qquad b_t = \lim_{i \to \infty} \varrho^i E_t(p_{t+i})$$

$$\kappa = -\ln\varrho - (1 - \varrho)\ln(\tfrac{1}{\varrho} - 1) \qquad E_t(b_{t+i}) = \tfrac{1}{\varrho} b_t = (1 + exp(\overline{d - p}))\, b_t$$

$$\gamma = \ln(1 + R)$$

$$\varrho = \frac{1}{1 + exp(\overline{d - p})}$$

$\overline{d - p)}$ = average log dividend price ratio

Theory of Rational Expectations Bubbles (REB)

- If b_t=0:

$$ln\, P_t = p_t^f + b_t$$

$$p_t^f = \frac{\kappa-\gamma}{1-\varrho} + (1-\varrho)\sum_{t=1}^{\infty} \varrho^i E_t(d_{t+1+i})$$

Theory of Rational Expectations Bubbles (REB)

- If b≠0:

$$ln\, P_t = p_t^f + b_t$$

$$E_t(b_{t+i}) = \frac{1}{\varrho} b_t = (1 + exp(\overline{d - p}))\, b_t$$

Statistical Tests: (1)supLPPLS (2)supPL

- Based on detection of **transient super-exponential trend** in the log prices
- Super-exponential: f(g)f(h) ≤ f(g+h); for all g and h strictly positive
- Non-nested hypothesis test of model selection
  - Null hypothesis: non-explosive process → stationary AR(1) process in the log returns
  - Alternative hypothesis: super-exponential trend → fitted super-exponential trend in the subsample between [τT] and T
  - Uses the model:

$$\Delta ln\, p_t = \varrho \Delta ln\, p_{t-1} + \alpha_{sexp} \Delta ln\, \hat{p}_{t\, sexp} + \varepsilon_t$$

Statistical Tests: (1)supLPPLS (2)supPL

- Two different super-exponential specifications to obtain $\Delta ln\, \hat{p}_{t\, sexp}$:
  1. Log Periodic Power Law Singularity (**LPPLS**):

$$ln\, p_{t\, sexp} = A + (t_c - t)^{m}[B + C cos(\omega ln(t_c - t) - \varphi)]$$

where 0<m<1, B<0, 3<w<15, and $|C|\,(w^2 + m^2)^{0.5} \leq |B|\, m$

  2. Simplification of #1 to exclude the log periodic oscillation (**PL**):

$$ln\, p_{t\, sexp} = A + (t_c - t)^{m}$$

where 0<m<1, B<0

Statistical Tests: (3)supKBT (4)supBT

- Based on detection of **non-stationarity** in Δp_t
  - If d_t is stationary (as it is for real estate assets) AND there is no bubble, Δp_t should be stationary by REB model
- To test for non-stationarity, a linear equation is set up that is stationary until the bubble and has a unit root nonstationarity afterwards (using an indicator function to achieve this)
- The non-stationarity corresponds to explosive behavior in the time series

Statistical Tests: (3)supKBT (4)supBT

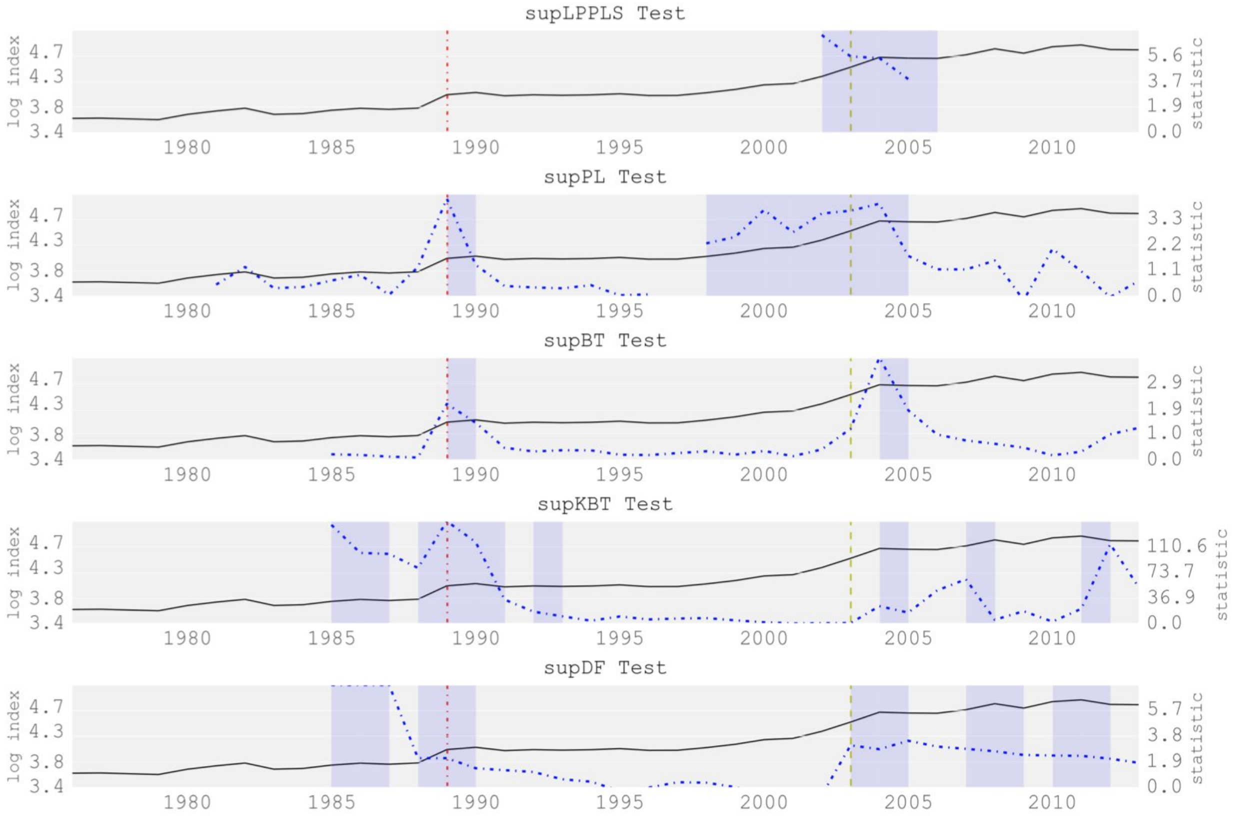
- Two different statistics are applied to the residuals of the LS to determine whether to reject null hypothesis
  1. Kim-Busetti-Taylor statistic (**KBT**)
  2. Busetti-Taylor statistic (**BT**)

Statistical Tests: (5)supDF

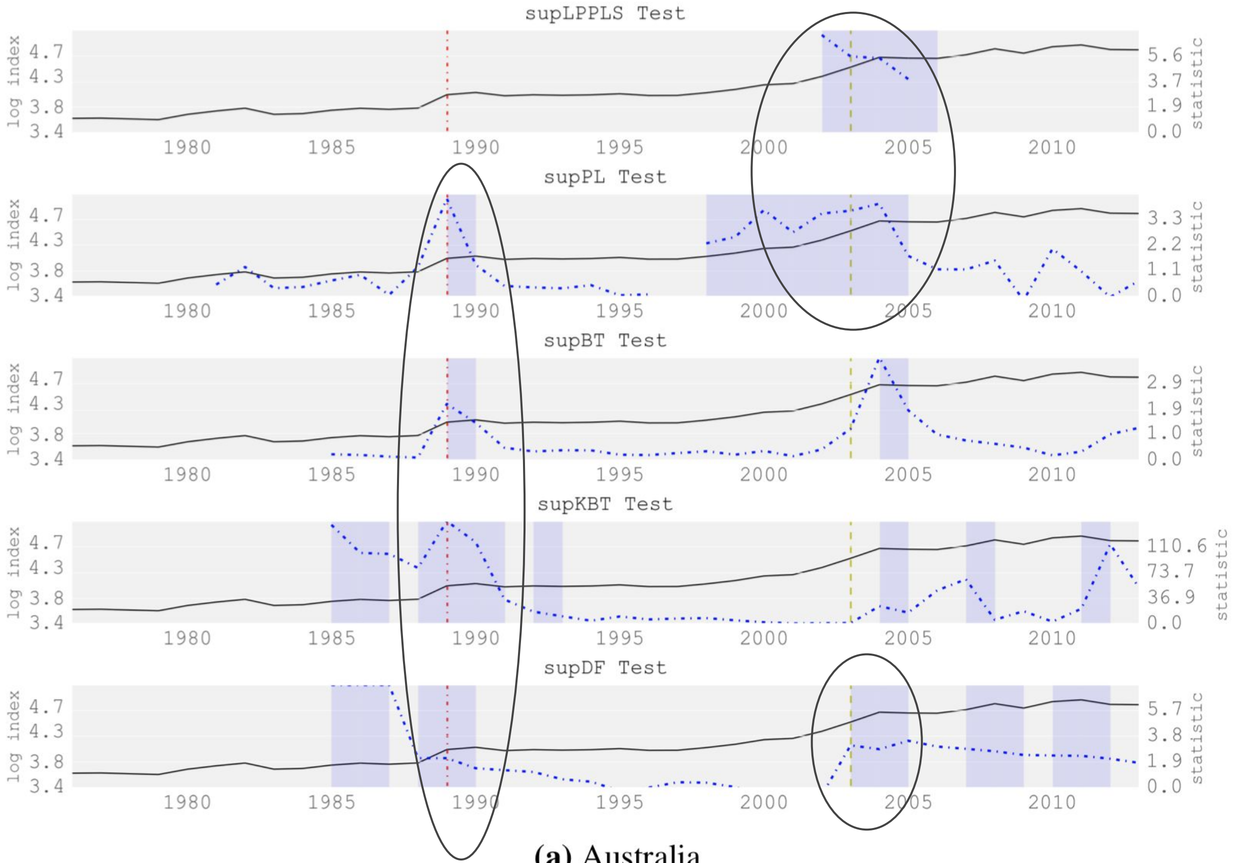
- Based on detection of **explosiveness** in p_t regardless of whether d_t is stationary or integrated process
- Directly test the data (supKBT and supBT perform LS on an extrapolated model) via a time-varying AR model and recursive Dickey-Fuller t-statistics

Experiment

- Apply the 5 statistics to test for bubbles in the log of house prices for each country in the data set
- 15,000 Monte Carlo simulations on rolling window of 60 quarters/15 years to calculate critical values for hypothesis testing
- Bubble signal is significant at a 5%


supLPPLS Test
supPL Test
supBT Test
supKBT Test
supDF Test
log index
statistic
4.7
4.3
3.8
3.4
1980
1985
1990
1995
2000
2005
2010
5.6
3.7
1.9
0.0
3.3
2.2
1.1
2.9
1.0
110.6
73.7
36.9
5.7
3.8

(a) Australia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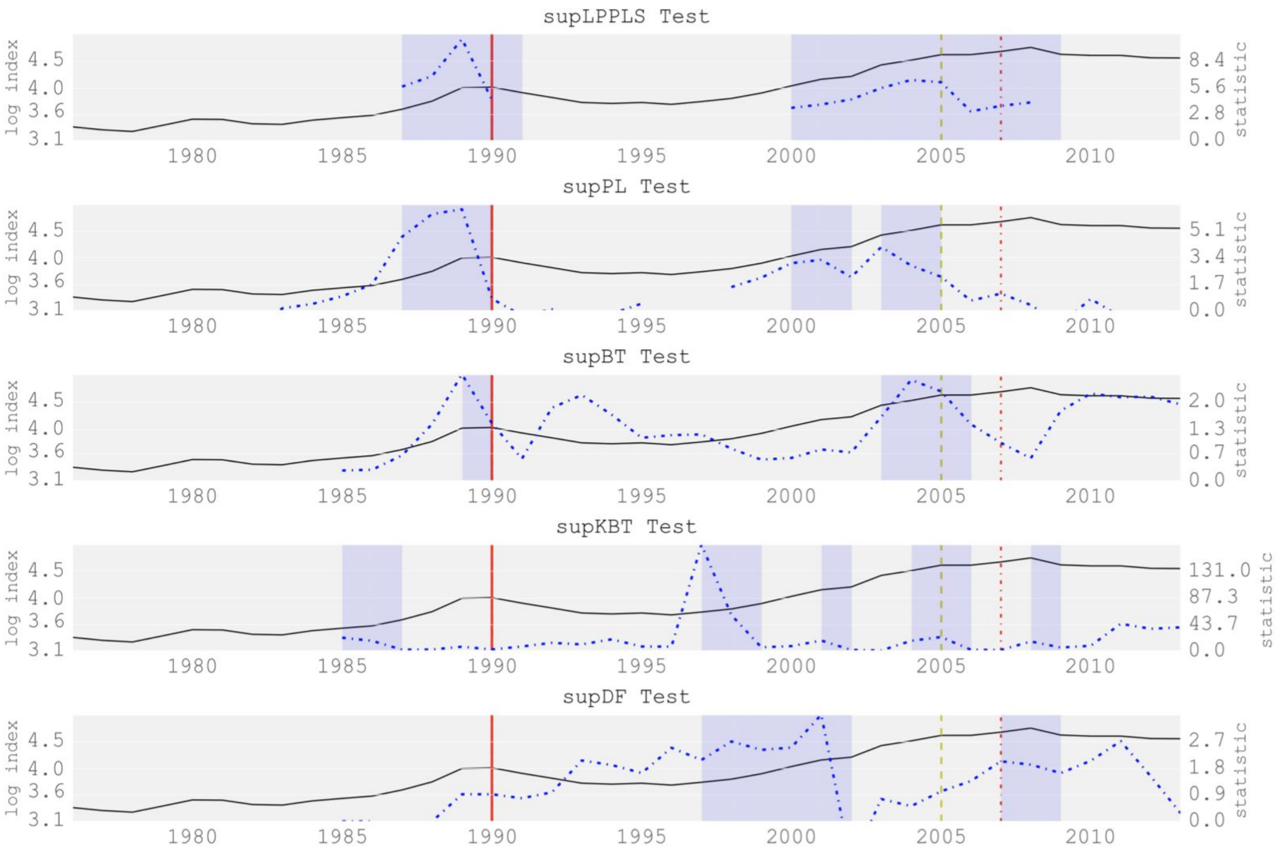
(a) Australia


supLPPLS Test
supPL Test
supBT Test
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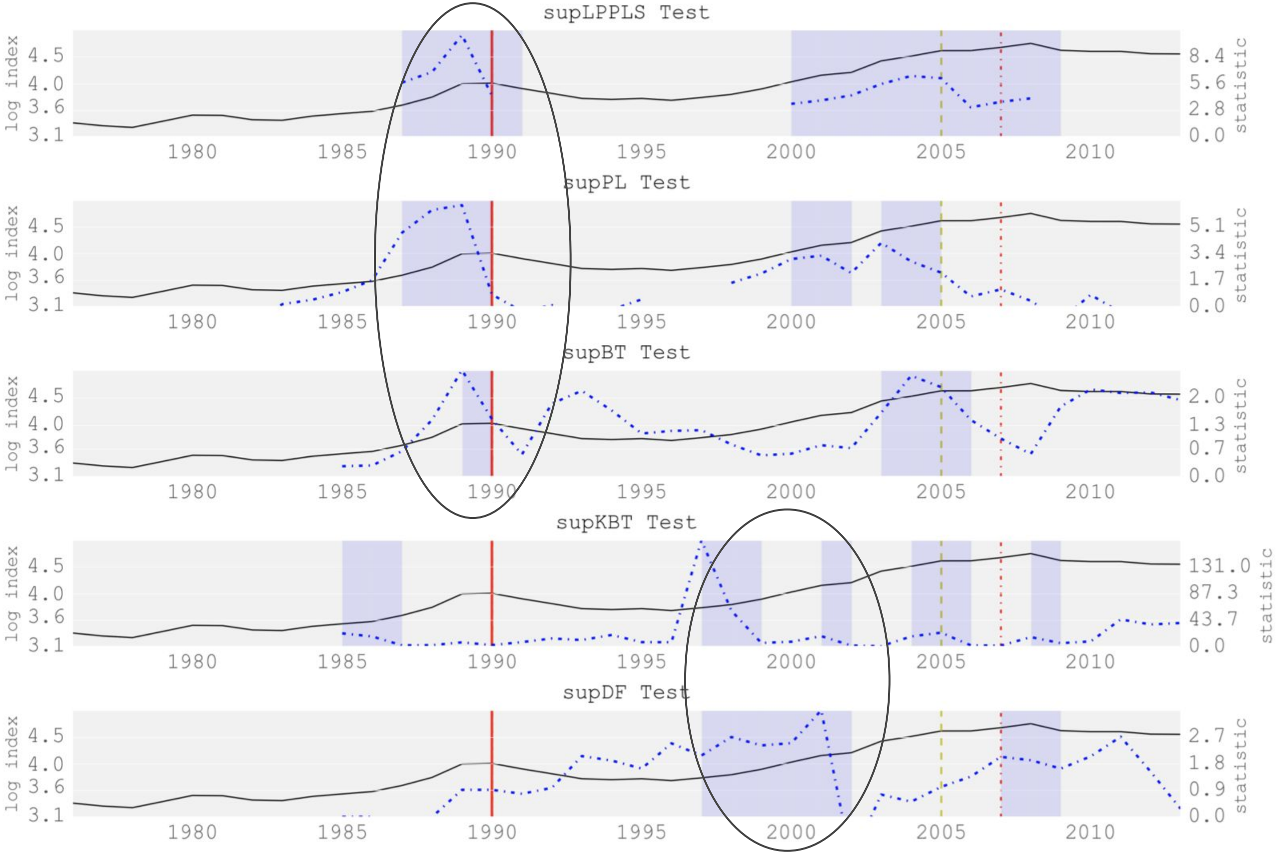
(b) UK


supLPPLS Test
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supBT Test
supKBT Test
supDF Test
log index
statistic
1980
1985
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1995
2000
2005
2010

(b) UK

How to evaluate the statistical indicators

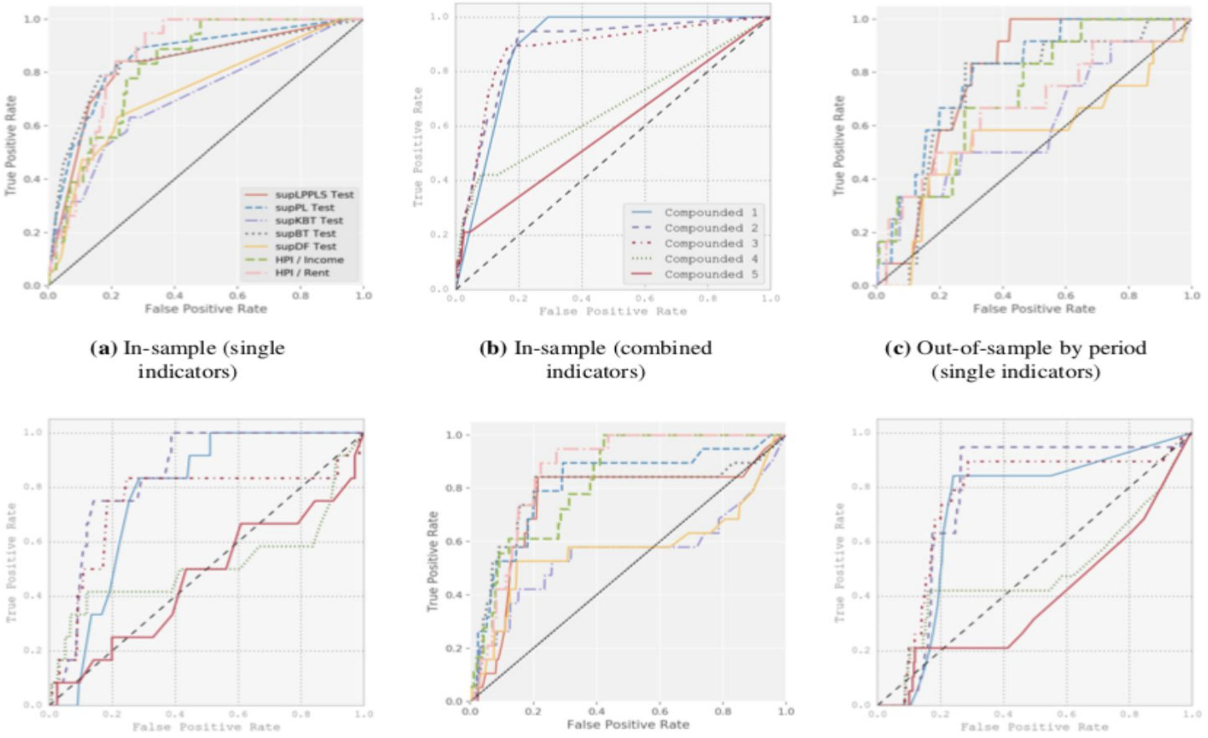
Yearly Logit Regressions:

$$y_{i,t}^{bubble} = \Lambda\left(\sum_{k=0}^{MAXLAG=3} \beta_k BI^{j}_{i,t-k-1} \right)$$

Where i = country, k = time lag, j = bubble indicator, Λ = logit function, $y_{i,t}^{bubble}$ = 1 if there is a tipping point at time t in country i and 0 otherwise, BI = 1 if the jth indicator detected a bubble and 0 otherwise

How to evaluate the statistical indicators

- Then, the quality of the model is assessed via the significance of the Area under the Receiver Operating Characteristics Curve (AUROC)
- Good forecast system = area under the curve greater than 0.5
- Mann-Whitney U statistic



(a) In-sample (single indicators)

(b) In-sample (combined indicators)

(c) Out-of-sample by period (single indicators)

(d) Out-of-sample by period (combined indicators)

(e) Out-of-sample by country (single indicators)

(f) Out-of-sample by country (combined indicators)

Figure 2. Receiver operating characteristic (ROC) curves derived from bubble logit regressions (22). The independent variables are the bubble indicators derived from log prices (see equations (21) and (24)). The dependent variable $y_{i,t}^{bubble}$ is set to 1 if a bubble peaked at time t in country i.

Results: In-Sample

- AUROC all above 0.7 with 99% significance level
- Best: supPL followed by supBT, supLPPLS
- supKBT and supDF also good

Results: Out-of-Sample

- Not surprisingly, performance of all drops from an average of 0.79 to 0.69(period) and 0.65(country)
- By period: best is supPL followed by supLPPLS and then supBT
- By country: supPL, supLPPLS, supBT relevant but significance levels drop

Results: Out-of-Sample Explanation

- supBT > supKBT
  - Same data generating process
  - Highlights impact of a test's statistical power
- supLPPLS, supPL perform well
  - Reveals key features of bubbles
    - Bubbles are fundamentally different market regimes driven by transient non-linearity, super-exponential growth pattern

Issue in interpretation: joint-hypothesis problem

Overall: bubble tests have forecasting skill associated with end of bubble, robust because rely on few parameters

Should the tests be applied directly to prices or some other measure?

- Price-to-income and price-to-rent ratios
  - Reflect misalignment of prices with the fundamental value
- Performance and statistical significance drops substantially when applied to ratios
  - supKBT and supDF not in-sample significant at all anymore
  - Only supLPPLS is out-of-sample by period significant
  - Only supPL is out-of-sample by country significant
- Conclusion: housing bubbles occur as part of a broader macroeconomic phenomenon, fundamentals themselves may be going through unstable period

Is there an improvement when indicators are combined?

- Better, more robust performance
- In line with economic and pattern recognition literature that compound predictors boosts performance

Applying the same tests to systemic financial crises

- Out-of-sample not significant at all for single and combined indicators
- Clear evidence that bubbles and crises are different phenomena
- Should be treated and analyzed differently

Sensitivity of results to rolling window length

- Stationary time series: longer window, increase performance
- Non-stationary time series (like real estate prices): longer window, complicates the results
- supLPPLS, supPL, supBT low sensitivity
- supKBT, supDF highly sensitive

Conclusions

- Derived five indicators by applying statistical tests to log housing prices
- Assessed the indicator in-sample and out-of-sample performance using logit regressions
- Tests contain significant information related to the end of bubble period
- Best tests: supPL and supBT
- Combining indicators is best, apply directly on data

Contributions of the Paper

- Can be used in broader forecast system
- Bubble tests based on price dynamics, specifically super-exponential growth patterns, are sound tools to identify bubble regimes
- Highlights the need to understand theory behind explosiveness in price dynamics and the consequences