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Midterm 2

Practice Version

CS 166

Winter 2024

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Instructions

- Wait until instructed to turn over the cover page.
- There are 4 questions on the test. Circle the problem number on the top right of the page for the 3 problems you will solve.
- If one is taking more than 15 minutes, move on to another problem.
- For every question, simplify your answer as much as possible. We will accept your answer if we are able to plug it into a graphing calculator.
- Please circle your final answer(s).

※ Problem 1: The No Cloning Theorem and Quantum Money

1.1

For each of the following sets of single qubit states, decide whether or not there exists a unitary that can clone them.

1. $\{|+\rangle, |-\rangle\}$
2. $\{|\pi/4\rangle, |3\pi/4\rangle\}$

1.2

Design a circuit that clones a state from the following set of single qubit states.

$$\{\frac{\sqrt{3}}{2}|0\rangle + i\frac{1}{2}|1\rangle, \frac{i}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle\} \quad (1)$$

1.3

Consider a bank that is using Weisner's quantum money scheme with 4 bit serial numbers. They use a function that maps to an 8 bit string, and they use the following rules to assign the qubit states.

$$00 \rightarrow |+\rangle \quad 01 \rightarrow |1\rangle \quad 10 \rightarrow |0\rangle \quad 11 \rightarrow |-\rangle \quad (2)$$

1. What is the state of the bill for the serial number 1001, if $f(1001) = 10100011$?
2. If you submit this bill to the bank, which basis will they measure the third qubit in?

1.4

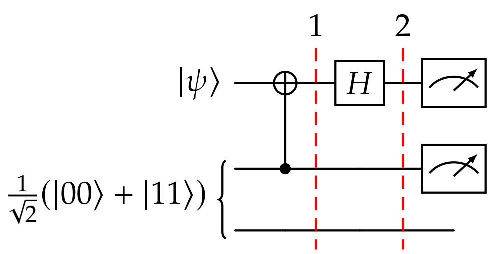
Consider a bank that is using Weisner's quantum money scheme using a 1 bit serial number. They are equally likely to print a bill in one of the four states $\{|0\rangle, |1\rangle, |+\rangle, |-\rangle\}$. Suppose you measure your bill from this bank in the $\{|\pi/8\rangle, |5\pi/8\rangle\}$ basis, and clone it in this basis. What is the probability that both of the bills pass the verification procedure?

※ Problem 2: Quantum Teleportation

Recall the circuit from the quantum teleportation protocol, where Alice's goal is to send her state

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle \tag{3}$$

to Bob. In class we measured Alice's qubits in the standard basis. Let's see what would happen if she measures in the Hadamard basis.



1. What is Bob's state after each measurement result of Alice's qubits in the Hadamard basis?

   - $|{+}{+}\rangle$
   - $|{+}{-}\rangle$
   - $|{-}{+}\rangle$
   - $|{-}{-}\rangle$

2. For each of the measurement results, what operation should Bob apply to Alice's qubits?

   - $|{+}{+}\rangle$
   - $|{+}{-}\rangle$
   - $|{-}{+}\rangle$
   - $|{-}{-}\rangle$

※ Problem 3: Hidden Variables and CHSH

Consider the following "modified" CHSH game.

- Alice and Bob will be separated with no way to communicate with each other.
- Charlie will flip two coins, and send a bit to Alice and Bob depending on the outcomes. He sends a 0 if tails, and a 1 if heads.
- After receiving the random bit x, Alice sends back an answer bit a to Charlie.
- After receiving the random bit y, Bob sends back an answer bit b to Charlie.
- Alice and Bob win against Charlie if the answer bits and random bits satisfy

$$a + y \pmod 2 = bx. \tag{4}$$

1. If Charlie flipped heads for both Alice and Bob's bits, list **all the ways** for Alice and Bob to respond so that they win.

Alice and Bob decide on the following strategy. They will share a bell pair in the $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ state, and will perform the following measurements depending on Charlie's random bits.

Alice	Bob
If $x = 0$, measure in $\{|0\rangle, |1\rangle\}$ basis.	If $y = 0$, measure in $\{|+\rangle, |-\rangle\}$ basis.
If $x = 1$, measure in $\{|\pi/8\rangle, |5\pi/8\rangle\}$ basis.	If $y = 1$, measure in $\{|-\pi/8\rangle, |3\pi/8\rangle\}$ basis.
If the outcome is $|0\rangle$ or $|\pi/8\rangle$, output $a = 0$.	If the outcome is $|+\rangle$ or $|-\pi/8\rangle$, output $b = 0$.
If the outcome is $|1\rangle$ or $|5\pi/8\rangle$, output $a = 1$.	If the outcome is $|-\rangle$ or $|5\pi/8\rangle$, output $b = 1$.

2. If Charlie send both Alice and Bob 0, what is the probability that Alice and Bob win using this strategy?
3. Would this version of the CHSH game be a viable way to detect a pseudorandom number generator?

❈ Problem 4

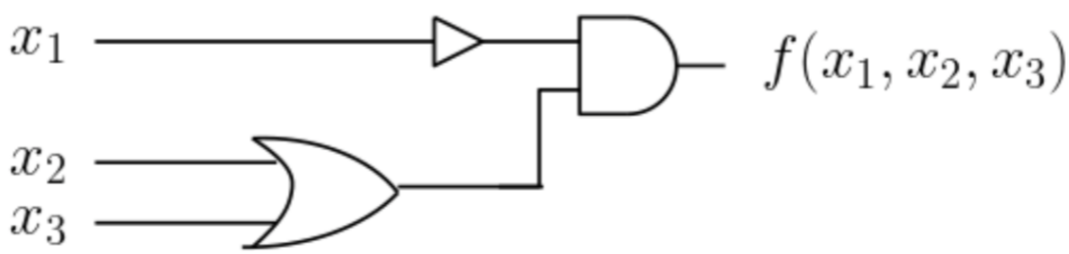
1. The subset sum problem asks the following: Given a list of n integers, all in the range -200 to 200, is there a subset of the integers such that they sum to 0?

   Determine if this problem is in P or NP. Explain your answer.

2. There are three necessary conditions for a set of quantum gates to be universal. Specifically, the following three things must be realizable using the gates. Give an example of a gate which is able to achieve each of the following.

   - Create superposition
   - Create entanglement
   - Create relative phase

3. Consider the following classical circuit which takes three input bits x_1, x_2, x_3, and outputs a bit $f(x_1, x_2, x_3)$. If we were to simulate this using a quantum computer, we need to implement it in a reversible way. Draw the quantum circuit that computes $f(x_1, x_2, x_3)$.



❇ Extra space

If you have time and room, draw Alice and Bob.