Supplementary Material S1

Algorithms and Mathematical Models of SAIM Engine v9.3

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1 S1.1 Overview of the Computational Pipeline

The Spinal Active Inference Model (SAIM) Engine v9.3 is a computational pipeline designed to quantify neuro-somatic reorganization. It integrates multi-modal physiological signals—Electroencephalography (EEG), Acceleration (ACC), Photoplethysmography (PPG), and Hemodynamics (fNIRS)—into a unified state space based on the Free Energy Principle [1].

2 S1.2 Hemodynamic Processing (fNIRS)

To estimate neurovascular flexibility, we apply the Modified Beer-Lambert Law (MBLL) [2] to raw optical data obtained from the Muse S headband.

2.1 S1.2.1 Sensor Mapping and Wavelengths

The device utilizes optical sensors at the frontal region. We utilize specific emitter-detector pairs corresponding to the "Left Inner" and "Right Inner" positions to capture prefrontal cortex hemodynamics.

- **Wavelengths (λ):** $\lambda_{Red} = 660\,\text{nm}$, $\lambda_{IR} = 850\,\text{nm}$.
- **Extinction Coefficients (ε):** Based on Prahl's standard extinction table [3] [mM^{-1} cm^{-1}]:

$$\mathbf{E} = \begin{bmatrix} \varepsilon_{HbO}^{\lambda_{Red}} & \varepsilon_{HbR}^{\lambda_{Red}} \\ \varepsilon_{HbO}^{\lambda_{IR}} & \varepsilon_{HbR}^{\lambda_{IR}} \end{bmatrix} = \begin{bmatrix} 0.087 & 0.730 \\ 0.052 & 0.032 \end{bmatrix} \tag{1}$$

2.2 S1.2.2 Concentration Calculation

Optical Density changes (ΔOD) are calculated relative to the baseline of each session window. Using a fixed Differential Pathlength Factor ($DPF = 6.0$) appropriate for the adult forehead [4, 5], the concentration changes of Oxygenated Hemoglobin (ΔHbO) are derived by solving:

$$\begin{bmatrix} \Delta\text{HbO}(t) \\ \Delta\text{HbR}(t) \end{bmatrix} = \mathbf{E}^{-1} \cdot \begin{bmatrix} \frac{\Delta OD_{Red}(t)}{DPF} \\ \frac{\Delta OD_{IR}(t)}{DPF} \end{bmatrix} \tag{2}$$

2.3 S1.2.3 HEMO Index (Neurovascular Flexibility)

The HEMO index quantifies the *capacity* of the vascular system to respond to neural demand. It is calculated as the sigmoid-normalized volatility of ΔHbO:

$$HEMO = \frac{1}{1 + \exp\left(-\left(\ln(\sigma_{\Delta HbO} \cdot 1000 + \epsilon) - 2.0\right)\right)} \tag{3}$$

where $\sigma_{\Delta HbO}$ is the standard deviation of ΔHbO within a 10-second window.

3 S1.3 Neuro-Somatic Metrics

In addition to hemodynamics, the engine computes three other domain-specific metrics.

3.1 S1.3.1 Prediction Error (PE)

PE represents the instability of the internal model, proxied by the volatility of Alpha-band EEG power (P_α), reflecting the minimization of surprisal [1].

$$PE = \frac{\text{std}(\text{detrend}(P_\alpha))}{\text{mean}(|P_\alpha|) + \epsilon} \tag{4}$$

3.2 S1.3.2 Micro-Kinematic Stability (SOM)

SOM quantifies the stillness of the body, reflecting descending inhibitory control. Calculated from 3-axis accelerometer data (x, y, z):

$$SOM = \frac{1}{1 + \text{std}\left(\sqrt{x^2 + y^2 + z^2}\right)} \tag{5}$$

3.3 S1.3.3 Autonomic Flexibility (AUT)

AUT proxies the system's ability to shift autonomic states, calculated via the Shannon entropy (H) of the heart rate (HR) distribution:

$$AUT = \frac{H(\text{HR})}{\ln(N_{bins}) + \epsilon} \tag{6}$$

4 S1.4 Integration Logic (The SAIM Core)

To evaluate the "Destruction and Reorganization" hypothesis, we integrate these metrics into a single state variable.

4.1 S1.4.1 Bias Neutralization (Equal Weighting)

To prevent analyzer bias, we employ an Equal Weighting strategy. The Integration Score (S_{int}) is the mean of all available functional metrics ($M \in \{FSI, SOM, AUT, HEMO\}$):

$$S_{int} = \frac{1}{|M|} \sum_{m \in M} m \tag{7}$$

4.2 S1.4.2 Neural Complexity Index (NCI)

NCI represents the system's global integrity. It is defined as the Integration Score penalized by Prediction Error (Cost):

$$NCI = \frac{1}{1 + \exp\left(-(S_{int} - 0.5 \cdot PE)\right)} \tag{8}$$

5 S1.5 Statistical Validation

All time-series visualizations utilize Robust Z-scores based on the Median and Median Absolute Deviation (MAD) [6] to mitigate physiological artifacts.

References

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