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PROBABILITY DISTRIBUTION OF LEADTIME DEMAND

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OPERATIONS ANALYSIS DEPARTMENT

NAVY FLEET MATERIAL SUPPORT OFFICE

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Report 159

PROBABILITY DISTRIBUTION OF LEADTIME DEMAND

REPORT 159

PROJECT NO. 9322-D75-0154

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ABSTRACT

This study examines 11 probability distributions to determine which distribution best describes demand during leadtime for 1H Cognizance Symbol (Cog) material. Proper selection of the distribution is critical in the accurate calculation of reorder levels. Actual leadtime demand observations were calculated in the study. Histograms, a chi-square goodness-of-fit test and a Mean Square Error (MSE) measure were used to analyze the leadtime demand data.

Histograms of the data suggested the following distributions to describe leadtime demand: Exponential, Gamma, Bernoulli-Exponential, Poisson, Negative Binomial and Geometric. The chi-square goodness-of-fit test indicated that none of these distributions fit the computed leadtime demand data across the entire range of the distribution. However, a relative test of the right hand tails of the distributions, which are most critical in determining reorder levels, indicated that the Bernoulli-Exponential provided the best relative fit for 1H Cog items.

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EXECUTIVE SUMMARY

1. Background. The reorder level calculation in the Uniform Inventory Control Program (UICP) Levels computation (D01) assumes that an item's actual leadtime demand is described by either the Poisson, Negative Binomial, or Normal distribution. The assumption of the most appropriate probability distribution is critical in the accurate calculation of reorder levels. Previous attempts to fit leadtime demand to theoretical probability distributions were restricted by the existing data base to quarterly demand observations. A sufficient data base now exists from which to compute actual leadtime demand observations. This analysis examines the following theoretical probability distributions for possible inclusion in the Levels computation of reorder level: Poisson, Normal, Negative Binomial, Logistic, LaPlace, Gamma, Weibull, Geometric, Exponential, Bernoulli-Exponential and Bernoulli-Lognormal.

2. Objective. To determine the probability distribution that best describes the demand during leadtime for 1H Cognizance Symbol (Cog) material.

3. Approach. The Due-In Due-Out File (DDF) and the Transaction History File (THF) were used to compute the leadtime for each item, and the demands that occurred during that leadtime. These data were then used to produce histograms of the leadtime demand for similar items based upon various grouping criteria. The grouping criteria were MARK, Unit Price, Leadtime Demand, Value of Annual Demand, Requisition Forecast, Leadtime and No Grouping. The histograms were developed and a visual estimate of the distribution that best fit the data was made. In addition to histograms the following statistics were computed: mean, standard deviation, variance and median. These statistics were used to determine the maximum likelihood estimator parameters for the distributions under consideration. The distribution(s) selected were subjected to goodness-of-fit

tests to determine the accuracy of these distribution(s) to describe the histograms under consideration. The goodness-of-fit tests used were the chi-square test and a mean square error measure.

4. Findings. Six distributions were selected for chi-square goodness-of-fit testing. These distributions were: Poisson, Exponential, Gamma, Negative Binomial, Geometric and Bernoulli-Exponential. The chi-square test indicated that none of the distributions fit the data based on the established hypothesis. A mean square error measure was then used to determine the distribution that most closely fit the data in the right hand tail since this is the part of the distribution that is critical when setting the safety level. The Bernoulli-Exponential distribution was selected as having the best relative fit.

5. Recommendation. It is recommended that the Bernoulli-Exponential distribution be adopted as the leadtime demand distribution for 1H Cog items.

I. INTRODUCTION

The Navy Fleet Material Support Office (FMSO) was tasked by reference 1 of APPENDIX A to determine the probability distribution that best describes the demand during leadtime for 1H Cognizance Symbol (Cog) material. Currently, the Uniform Inventory Control Program (UICP) Levels computation (D01) assumes the Poisson, Negative Binomial or Normal distribution describes an item's actual leadtime demand. The assumption of the most appropriate probability distribution is critical in the accurate calculation of reorder levels. The reorder level computation is based on forecasts of the quarterly demand and leadtime, expressed in quarters, and includes a safety level to achieve the acceptable degree of procurement stockout risk. If the probability distribution of an item's leadtime demand is known, the safety level can be accurately determined to achieve that degree of risk.

In the UICP system, items are assigned one of three probability distributions based on their average leadtime demand. The Poisson distribution is used to describe low demand items. The Negative Binomial distribution is used for medium demand items and the Normal distribution is used for high demand items. The criteria used to determine low, medium and high demand items are set by the Inventory Control Points (ICPs). The selection of the most appropriate probability distribution is vital to the calculation of safety level. If the wrong probability distribution is chosen, it will not fit the demand pattern and will result in an inefficient allocation of funds. For example, if too much safety level is allowed, unnecessary costs will be incurred since too much material is being bought. If too little safety level is allowed, the system will be operating at a lower performance level since not enough material is

available. The ultimate goal is to have the best fit possible so that the safety level determined will allow the system to perform at the desired level.

FIGURES I through III demonstrate the possible consequences of using the wrong probability distribution to determine the reorder level. The three distributions that are currently in use in the UICP Levels setting program, Poisson, Negative Binomial and Normal, are shown in these figures. The values on the Y-axis are represented in scientific notation (i.e. 1E-3 = $1*10^{-3}$ = .001).

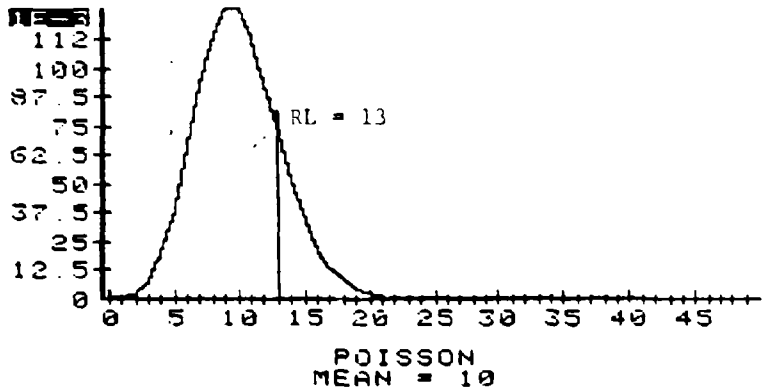


FIGURE I

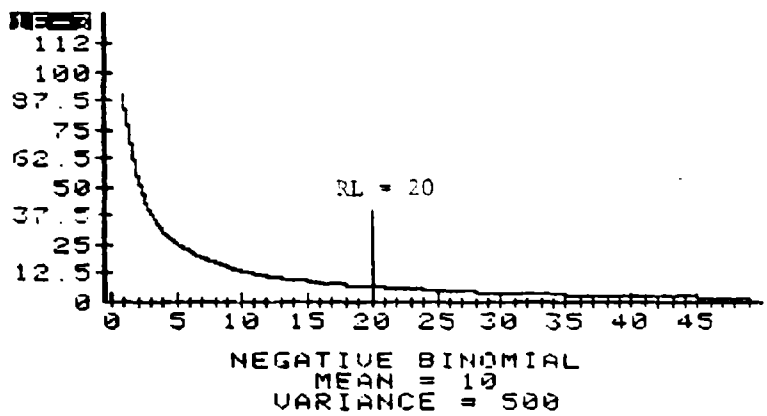


FIGURE II

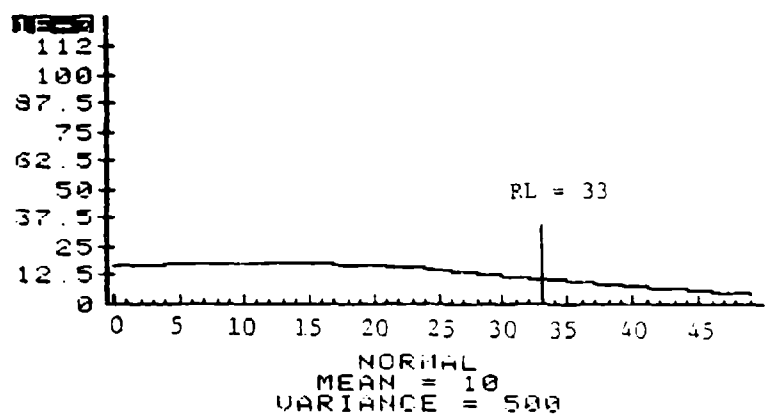


FIGURE III

In the examples above, for the Poisson distribution, the mean is 10, for the Negative Binomial and Normal distributions, the mean is 10 and the variance is 500. The risk (ρ) assigned to each distribution is .15. Using the same mean and variance for each distribution, the Reorder Level (RL) calculated varies widely depending on the distribution selected. The RL calculated using the Poisson, Negative Binomial and Normal distributions are 13, 20 and 33, respectively. Obviously, the selection of the Poisson distribution when the Normal distribution should be used results in a RL which is 20 units less than what is necessary for the desired protection against procurement stockout. Similarly, if the Normal distribution is selected when the Negative Binomial distribu · should be used, unnecessary costs would be incurred because of the incre.. .. RL investment.

The current distributions have been in use since the inventory system was automated. References 2, 3 and 4 of APPENDIX A examined alternate distributions to describe leadtime demand. The distributions examined were compared to the current distributions to determine if they described leadtime demand more accurately. The conclusion reached in reference 2 of APPENDIX A was to continue using the current distributions. Reference 3 of APPENDIX A, however, recommended replacing the Normal distribution for high demand items with either the Bernoulli-Lognormal or the Bernoulli-Exponential distribution. Reference 4 of APPENDIX A suggested the Gamma distribution which can assume various shapes depending on the parameters selected. The current study, drawn from past efforts, used historical data to compute actual leadtimes and to summarize the demands which occurred during that leadtime. In the past, there was not a sufficiently large data base from which to draw the information necessary to compute a true leadtime and the subsequent demands that occurred during

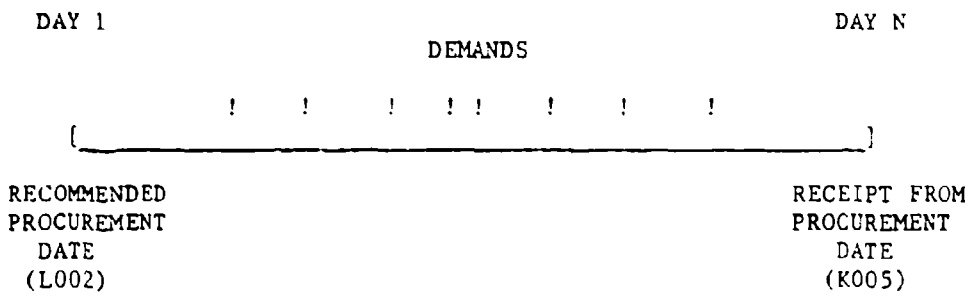
that leadtime. Previous studies relied upon a forecast of the leadtime and a forecast of quarterly demand which, when multiplied together, resulted in the calculation of demand during leadtime.

II. TECHNICAL APPROACH

A. COMPUTATION OF LEADTIME DEMAND. The computation of leadtime demand in previous studies was hindered by the amount and type of data available. Reference 2 of APPENDIX A used 12 quarters of historical stock point demand data. Reference 3 of APPENDIX A used four years of historical daily demand data which were grouped into thirty day "buckets" creating a demand time series of 48 pseudo-monthly demands. Reference 4 of APPENDIX A used Air Force monthly demand data. The demand data used in these three references were insufficient to determine actual leadtime demand observations. The computation of the leadtime for each item was not undertaken in any of the studies. For example, reference 3 of APPENDIX A tried to fit a distribution to the entire time series of demand data without regards to the leadtime and reference 2 of APPENDIX A dealt with this problem by multiplying the forecast of quarterly demand and the forecast of leadtime together in order to compute the leadtime demand. This study determined actual leadtime demand observations based on eight years of demand transactions and procurement initiations. Leadtime demand was computed on an item by item basis using the actual demands and receipts as found in Navy Ships Parts Control Center's (SPCC's) files.

A leadtime for a given National Item Identification Number (NIIN) was computed by using the recommended procurement date (Data Element Number (DEN) L002), located in the Due-In Due-Out File (DDF), as the first day of the

leadtime. The last day of the leadtime was obtained from the transaction date (DEN K005) of a receipt from procurement which is a transaction found in the Transaction History File (THF). In order to ensure that the correct receipt was used, the NIIN, Activity Sequence Code (ASC), and Procurement Instrument Identification Number (PIIN) from the THF were compared with the NIIN, ASC, and PIIN from the DDF; if they matched, the difference between the recommended procurement date and the transaction date was the leadtime for that NIIN computed in days. When the recommended procurement date for a NIIN was found, the leadtime demand for that NIIN was computed by summing the transaction quantities for demand transactions which occurred on or after the recommended procurement date but before the receipt transaction date. FIGURE IV graphically depicts the process described above. (! represents a requisition for one unit.)



Leadtime Demand = 8

FIGURE IV

If a receipt was not found for a NIIN, the leadtime could not be computed and the observation was deleted. The possibility existed for a second leadtime to begin, for the same NIIN, before the first leadtime had ended. When two leadtimes ran concurrently for the same NIIN, they overlapped each other. An

overlapping leadtime or multiple buy outstanding occurred when a second procurement initiation document had a recommended procurement date before the first procurement initiation document was matched to a receipt transaction. The occurrence of overlapping leadtimes during the leadtime demand computation resulted in the demands which occurred during that interval being credited to all the overlapping leadtimes. That is, if a demand was found in an overlapping leadtime, that demand was considered for each overlapping leadtime.

After each leadtime was computed, the length of the leadtime and the total number of demands during that leadtime were recorded. For each NIIN, the mean and standard deviation of leadtime in days and the mean and standard deviation of total leadtime demand were computed.

The ideal inventory system would assign a distribution to each item based on its leadtime demand. Even though there were eight years of data available, the number of leadtime demands associated with each item were insufficient to apply a distribution to each item. Therefore, the items were divided into homogeneous groups based on certain characteristics, since a group of items with similar characteristics should behave in a similar fashion. Using similarly grouped items, a distribution would be hypothesized as fitting the group rather than an individual item. Groups were determined based on one of the following six criteria: MARK, Leadtime Demand Forecast (B011A*B074), Requisition Forecast (A023B), Unit Price (B053), Value of Annual Demand (4*B071A*B053) and Leadtime (B011A). The MARK is based on quarterly demand (B074), replacement price (B055) and value of quarterly demand (B074*B055). Items are divided into one of five MARK categories. Low demand items (B074 $\leq$ .25) are classified as MARK 0. MARK for items which are not classified as MARK 0 are determined by the following matrix:

	B074 ≤ 5	B074 > 5
B055 > $50 or (B055*B074) > $75	MARK III	MARK IV
B055 ≤ $50 and (B055*B074) ≤ $75	MARK I	MARK II

Since the MARK grouping has five categories, the remaining groups were also divided into five categories for the initial evaluation. The breakpoints within each group were selected so that approximately 20% of the total number of leadtimes having certain characteristics would fall between the breakpoints. For example, the breakpoints for Requisition Forecast are 0, .25, 1.0, 3.5 and greater than 3.5; therefore, approximately 20% of the leadtimes have a Requisition Forecast of 0, approximately 20% of the leadtimes have a Requisition Forecast between 0 and .25 and so forth.

B. DATA VALIDATION. An important aspect of this study was the use of historical data to resolve some of the deficiencies that have been a major obstacle in determining the demand which occurred during leadtime. The historical data were derived from two SPCC files, the THF and the DDF. The THF contained demands and receipts from January 1974 to March 1982. The DDF contained procurement initiations from January 1974 to December 1981. Demands from the THF contained a Document Identification Code (DIC) of A0, A4, D7 and DH and the receipts were D4S. The procurement initiations from the DDF contained a DIC of DDS. Additional item information for each NIIN was obtained from the Selective Item Generator (SIG) file of March 1982. The SIG file provides a snapshot of the Master Data File (MDF).

The historical data used in this study required careful validation. Since the data base encompassed an eight year period, there existed a possibility that some of the NIINs on the demand transactions could have changed. If this

had occurred, any leadtime that had started before the NIIN was changed would not have a receipt to end the leadtime since the NIIN was different. Also, the demands for the old NIIN would only be recorded under the old NIIN's leadtime, while the demands for the new NIIN would be ignored. The Old NIIN File (ONF) of March 1983 was used to update the NIINs on both the THF and DDF to prevent inaccurate calculations of leadtime demand.

Before the leadtime demands were computed, a thorough examination was made of the THF and DDF files to remove any records which were determined to be invalid. Records which contained inaccurate or missing NIINs, procurement dates, DICs or requisition quantities were not considered. Records were also dropped if the item was not under SPCC management as of March 1982.

After the leadtime demands were computed, records containing demands of a thousand (1,000) or more during a leadtime were validated. The inclusion of a substantial number of large leadtime demands would tend to skew the distribution to the right and inflate the mean. These leadtime demands were potential outliers and might not be representative. A check of the leadtime demands was made to ensure that only those records with demands that were consistent with not only historical but also forecasted data were retained. Based upon the validation results, approximately 85% of the records that contained leadtime demands of 1,000 or more were dropped from further consideration.

C. DISTRIBUTIONS CONSIDERED. The reorder level calculated in the UICP Levels computation (D01) assumes that an item's actual leadtime demand is described by either the Poisson, Negative Binomial, or Normal distribution. The logical start for an evaluation of the probability distributions used to describe leadtime demand would begin with the three distributions currently implemented.

Previous studies dealing with probability distribuitons used to describe leadtime demand were a valuable source when selecting additional distributions for this study. Reference 2 of APPENDIX A examined the current distributions along with the following four alternate distributions: Logistic, LaPlace, Gamma and Uniform. Both references 2 and 3 of APPENDIX A noted that a significant number of leadtimes have zero demands but only reference 3 of APPENDIX A attempted to address this particular phenomenon. Reference 3 of APPENDIX A found that a compound distribution using a Bernoulli distribution to describe the zero demands and another distribution (e.g., Lognormal or Exponential) to describe demands that are not zero could be used to model leadtime demand. Reference (4) of APPENDIX A recommended the Gamma distribution to describe all leadtime demand. The unique feature of the Gamma distribution was the variety of shapes it could assume with only a change of parameters.

Therefore, the distributions considered in this study were: Poisson, Normal, Negative Binomial, Logistic, LaPlace, Gamma, Weibull, Geometric, Exponential, Bernoulli-Exponential and Bernoulli-Lognormal. Reference 2 of APPENDIX A contains an illustration of the Logistic and LaPlace distributions while the remaining distributions are illustrated in reference 5 of APPENDIX A.

D. EVALUATION PROCEDURES. The first step in deciding whether a particular theoretical distribution represents the observed data is to decide whether the general family; e.g., Exponential, Gamma, Normal or Poisson, of distributions is appropriate, without worrying (yet) about the particular parameter values for the family. Histograms were used to decide whether a particular distribution family was appropriate. After the histograms were analyzed, the values of the parameters for the various distributions were specified using maximum likelihood estimators (MLEs). After the distribution forms were

analyzed and the parameters were estimated, the "fitted" distributions were examined to see if they were in agreement with the observed data using the chi-square goodness-of-fit test. In addition, a relative comparison of the right hand tail of the various distributions was performed using the measure of mean squared error (MSE).

1. Histograms. Histograms are used to hypothesize what family of distributions the observed data comes from. A histogram is a graphical estimate of the plot of the density function corresponding to the distribution of the observed data. Density functions tend to have recognizable shapes. Therefore, a graphical estimate of a density function should provide a good clue to the distributions that might be tried as a model for the data.

To make a histogram, the range of values covered by the observed data is broken up into k disjoint intervals (b_0, b_1), (b_1, b_2), ..., (b_{k-1}, b_k). All the intervals should be the same width, which might necessitate throwing out a few extremely large or small observations to avoid getting an unwieldly looking histogram plot. For $j = 1, 2, \ldots, k$, let q_j be the proportion of the observations that are in the jth interval (b_{j-1}, b_j). Finally, the function h(x) is defined as:

$$h(x) = \begin{cases} 0 \text{ if } x < b_0 \\ q_j \text{ if } b_{j-1} \leq x < b_j \\ 0 \text{ if } b_k \leq x \end{cases}$$

which is plotted as a function of x.

Histograms are applicable to any distribution and provide an easily interpreted visual synopsis of the data. Furthermore, it is relatively easy to

"eyeball" a graph in reference to possible density functions.

2. MLE. After a family of distributions has been hypothesized, the value(s) of its parameter(s) must be specified in order to determine completely the distribution which models the observed data. MLEs were used whenever possible to determine the parameters in this study. The basis for MLEs is most easily understood in the discrete case. Suppose that a discrete distribution has been hypothesized for the observed data which has one unknown parameter θ. Let $p_\theta(x)$ denote the probability mass function for this distribution. Let X_1, X_2 ..., X_n be the actual observation of the observed data. The likelihood function $L(\theta)$ is defined as follows:

$$L(\theta) = p_\theta(X_1)\ p_\theta(X_2)...p_\theta(X_n)$$

$L(\theta)$, which is just the joint probability mass function since the data are assumed to be independent, gives the probability (likelihood) of obtaining the observed data if θ is the value of the unknown parameter. Then, the MLE of the unknown value of θ, which we denote by $\hat{\theta}$, is defined to be the value of θ which maximizes $L(\theta)$; that is, $L(\hat{\theta}) \geq L(\theta)$ for all possible value of θ. Thus, $\hat{\theta}$ "best explains" the data that are observed. MLEs for continuous distributions are defined analogously to the discrete case.

3. Chi-Square Test. After a distribution form for the observed data was hypothesized and its parameters estimated, the "fitted" distributions must be examined to see if it is in agreement with the observed data X_1, X_2, ..., X_n. The question really being asked is this: Is it plausible to have obtained the observed data by sampling from the fitted distribution? If F is the distribution function of the fitted distribution, this question can be

addressed by a hypothesis test with a null hypothesis.

H_0: The X_is are independent identically distributed random variables with distribution function F.

This is a goodness-of-fit test since it tests how well the fitted distribution "fits" the observed data. A chi-square goodness-of-fit test may be thought of as a more formal comparison of a histogram with the fitted density function.

To compute the chi-square test statistic, first divide the entire range of the fitted distribution into k adjacent intervals $[a_0, a_1)$, $[a_1, a_2)$, ..., $[a_{k-1}, a_k)$ where it could be that $a_0 = -\infty$, or $a_k = +\infty$, or both. Then we tally

$$N_j = \text{number of } X_i\text{s in the jth interval } [a_{j-1}, a_j)$$

for $j = 1, 2, \ldots, k$. (Note that $\sum_{j=1}^{k} N_j = n$.) Next, the expected proportion p_j of the X_is that would fall in the jth interval if sampling from the fitted distribution was performed is computed. In the continuous case,

$$p_j = \int_{a_{j-1}}^{a_j} \hat{f}(x)\, dx$$

where $\hat{f}$ is the density function of the fitted distribution. For discrete data

$$p_j = \sum_{(i:a_{j-1} \le x_i < a_j)} \hat{p}(x_i)$$

where $\hat{p}$ is the mass function of the fitted distribution. Finally, the test statistic is:

$$\chi^2 = \sum_{j=1}^{k} \frac{(N_j - np_j)^2}{np_j}$$

Since np_j is the expected number of the n x_is that would fall in the jth interval if H_0 were true, χ^2 is expected to be small if the fit is good. Therefore, H_0 is rejected if χ^2 is too large. To determine if χ^2 is too large, it is compared with the critical point $\chi^2_{v,y}$ for the chi-square distribution with v degrees of freedom where v = k-1 and y = p $\{Y \leq \chi^2_{v,y}\}$. (A chi-square critical point table is available in reference 6 of APPENDIX A.).

The most troublesome aspect of carrying out a chi-square test is choosing the intervals. A common recommendation is to choose the intervals so that the values of np_j are not too small; a widely used rule of thumb (employed in this study) is to select $np_j > 5$ for all j. The reason for this recommendation is that the agreement between the true distribution of χ^2 (for fixed, finite n) and its asymptotic (as $n \to \infty$) chi-square distribution (used to obtain the critical value for a test) is better if the values of np_j are not too small. This contributes to the validity of the test.

4. MSE. The measure of MSE was also used to determine how well the theoretical distributions "fit" the observed data. The chi-square test is a hypothesis test which evaluates the goodness-of-fit for a particular distribution over the entire range of the distribution. The MSE measure was used to

make a relative comparison of the distributions for the right hand tail. The right hand tail of the leadtime demand distribution is critical in determining the reorder point for an item in the UICP.

The MSE was calculated using the following procedure: Given a percentile "p", the reverse cumulative probability function for each distribution was used to calculate a value (x) such that the probability that a leadtime demand is less than or equal to x equaled "p". The calculated values of x were then used to determine the percentage, $\hat{p}$, of the observed leadtime demand which were less than or equal to x. Since the hypothesis is that the observed leadtime demands come from a particular distribution, the expected value of $\hat{p}$ should equal p. The right hand tail of each distribution was evaluated by using every fifth percentile starting with the 50th percentile and ending with the 95th percentile. The mean squared error was computed over the 10 percentiles as follows:

$$MSE = \frac{1}{10} \sum_{i=1}^{10} (p_i - \hat{p}_i)^2$$

where

i = 1 represents 50%	i = 6 represents 75%
i = 2 represents 55%	i = 7 represents 80%
i = 3 represents 60%	i = 8 represents 85%
i = 4 represents 65%	i = 9 represents 90%
i = 5 represents 70%	i = 10 represents 95%

The following example will illustrate the MSE calculation. Assume that the exponential distribution has been hypothesized as the leadtime demand distribution for the group of MARK I items which have an average leadtime demand equal to 8.11 and are distributed as displayed in APPENDIX B. The reverse cumulative probability function for the exponential distribution is:

$$x = B * LN(1-p)$$

where

B is the mean

p is the percentile

For the group of MARK I items, the percentile p and the computed xs are shown in columns one and two of TABLE I, respectively. Using the distribution displayed in APPENDIX B, the third column, $\hat{p}$, is calculated. Column four is calculated using the values in columns one (p) and three ($\hat{p}$) to yield the square error values. The square error values are summed (1920.50) and divided by 10 to calculate the mean square error value of 192.05.

TABLE I

EXAMPLE CALCULATION FOR MARK I ITEMS

p	x	$\hat{p}$	SQUARE ERROR
50	5	74.73	611.57
55	6	76.62	467.42
60	7	78.37	337.46
65	8	80.17	230.13
70	9	81.54	133.17
75	11	84.22	85.01
80	13	86.34	40.20
85	15	87.90	8.41
90	18	89.60	.16
95	24	92.36	6.97

III. FINDINGS

The Findings are divided into the following sections: Leadtime Demand Statistics, Histogram Results, Chi-Square Goodness-Of-Fit Tests and Mean Square Error Results. As discussed in the Technical Approach, since there are not enough leadtime demand observations for each item, the items were divided into homogeneous groups with groups being determined based on one of six criteria. An examination of the data revealed that dividing the data by MARK, Leadtime Demand Forecast and Requisition Forecast provided the most homogeneous groupings and the results are displayed using these three criteria. Each grouping was partitioned into five categories with each category containing approximately 20% of the data.

A. LEADTIME DEMAND STATISTICS. TABLE II displays the following seven statistics computed for the leadtime demand data for each grouping: the number of NIINs, the percentage of the total NIINs, the number of leadtime demand observations, the percentage of the total observations, the mean value of the leadtime demand observations, the variance of the leadtime demand observations and the percent of leadtimes with zero demand. The mean, variance and percent of leadtimes with zero demand are statistics required, depending on the probability distribution, for calculating MLEs. The other four statistics are displayed for general information.

Two general observations about the leadtime demand data can be made based on these statistics. Consistent with other studies, a significant (45%) number of leadtimes have no demand and the large variances indicate that the data encompass a wide range of values.

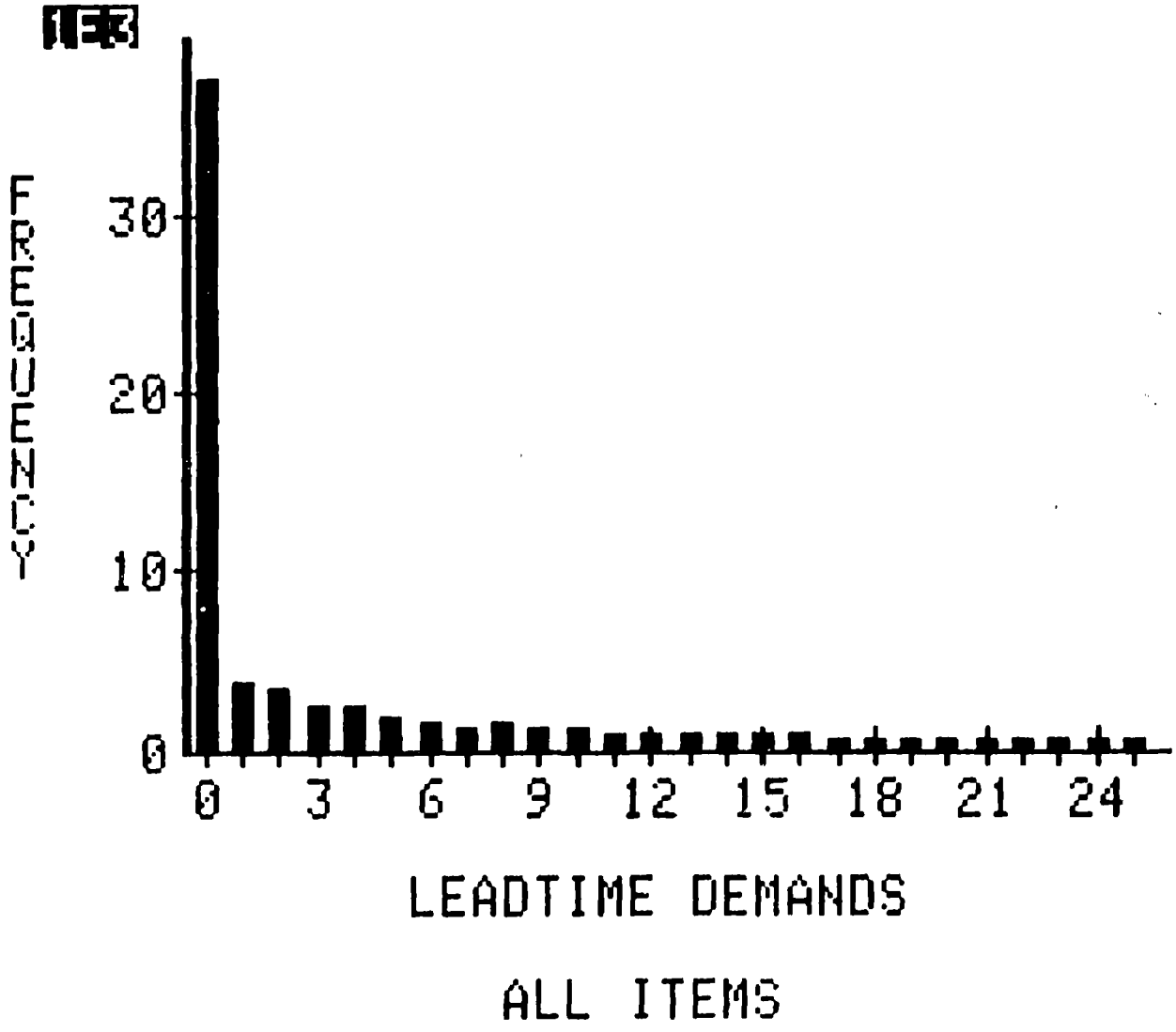
TABLE II

LEADTIME DEMAND STATISTICS

	# NIIN	% OF TOTAL	# OBS	% OF TOTAL	MEAN	VARIANCE	% OF OBS=0
TOTAL	45,701	100	83,704	100	59.17	4,630.53	45
MARK							
0	23,664	52	36,316	43	3.44	760.08	75
1	3,869	8	6,227	8	8.11	913.67	58
2	1,211	3	1,947	2	51.73	16,153.18	34
3	9,485	21	19,148	23	10.27	401.12	26
4	7,472	16	20,066	24	223.25	37,455.60	6
LTDMD							
$x = 0$	15,752	34	23,654	28	5.13	2,109.96	77
$0 < x \leq 2$	10,436	23	16,669	20	2.51	157.22	68
$2 < x \leq 10$	7,840	17	14,875	18	9.14	751.69	36
$10 < x \leq 50$	6,688	15	14,817	18	26.22	2,047.81	15
$x > 50$	4,985	11	13,689	16	311.55	17,490.64	5
RQN FORECAST							
$x = 0$	13,991	31	20,936	25	5.68	2,368.59	75
$0 < x \leq .25$	12,339	27	19,473	23	4.39	816.46	75
$.25 < x \leq 1$	8,621	19	15,631	19	14.46	2,848.34	36
$1 < x \leq 3.5$	6,559	14	14,428	17	41.23	30,075.66	10
$x > 3.5$	4,191	9	13,236	16	296.73	6,649.38	2

B. HISTOGRAM RESULTS. The histograms presented in FIGURES V through XVI are based on all the leadtime demand observations and when the observations are divided into MARK categories. FIGURE V is a graph of the number of leadtime demand observations for 0, 1, 2, ..., 25 demands per leadtime. The X axis is the number of demands observed during a leadtime while the Y axis is the number of leadtimes containing these demands. For example, there are 37,731 observations of zero demand during a leadtime. FIGURE VI is the same graph as FIGURE V except the zero observations were removed. Two graphs of virtually the same data (FIGURES V and VI) are shown to illustrate the impact that the zero lead-

time demand observation has on the shape of the data. Looking at FIGURE V, a distribution resembling the data is hard to define, while FIGURE VI indicates that the data may be exponentially distributed. The remaining FIGURES follow the same pattern of two graphs for each MARK grouping. The first graph contains the zero leadtime demand observations while the second does not. Similar histograms for the Leadtime Demand Forecast and Requisition Forecast groupings are contained in APPENDIX C. Based on the histograms, the following six distributions were selected for chi-square testing: Exponential, Bernoulli-Exponential, Negative Binomial, Poisson, Geometric and Gamma.


1E3
FREQUENCY
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20
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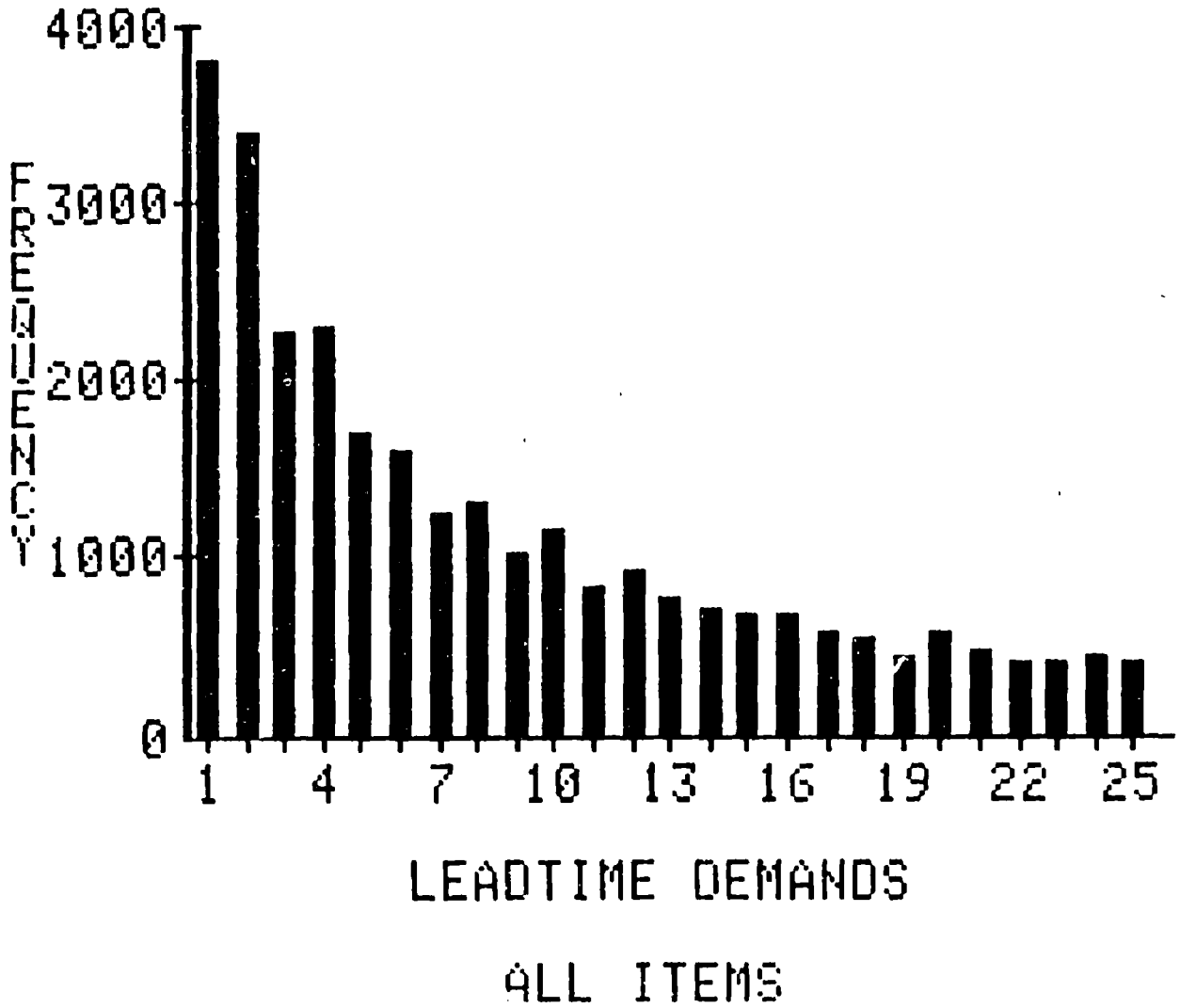
0
0
3
6
9
12
15
18
21
24
LEADTIME DEMANDS
ALL ITEMS

FIGURE V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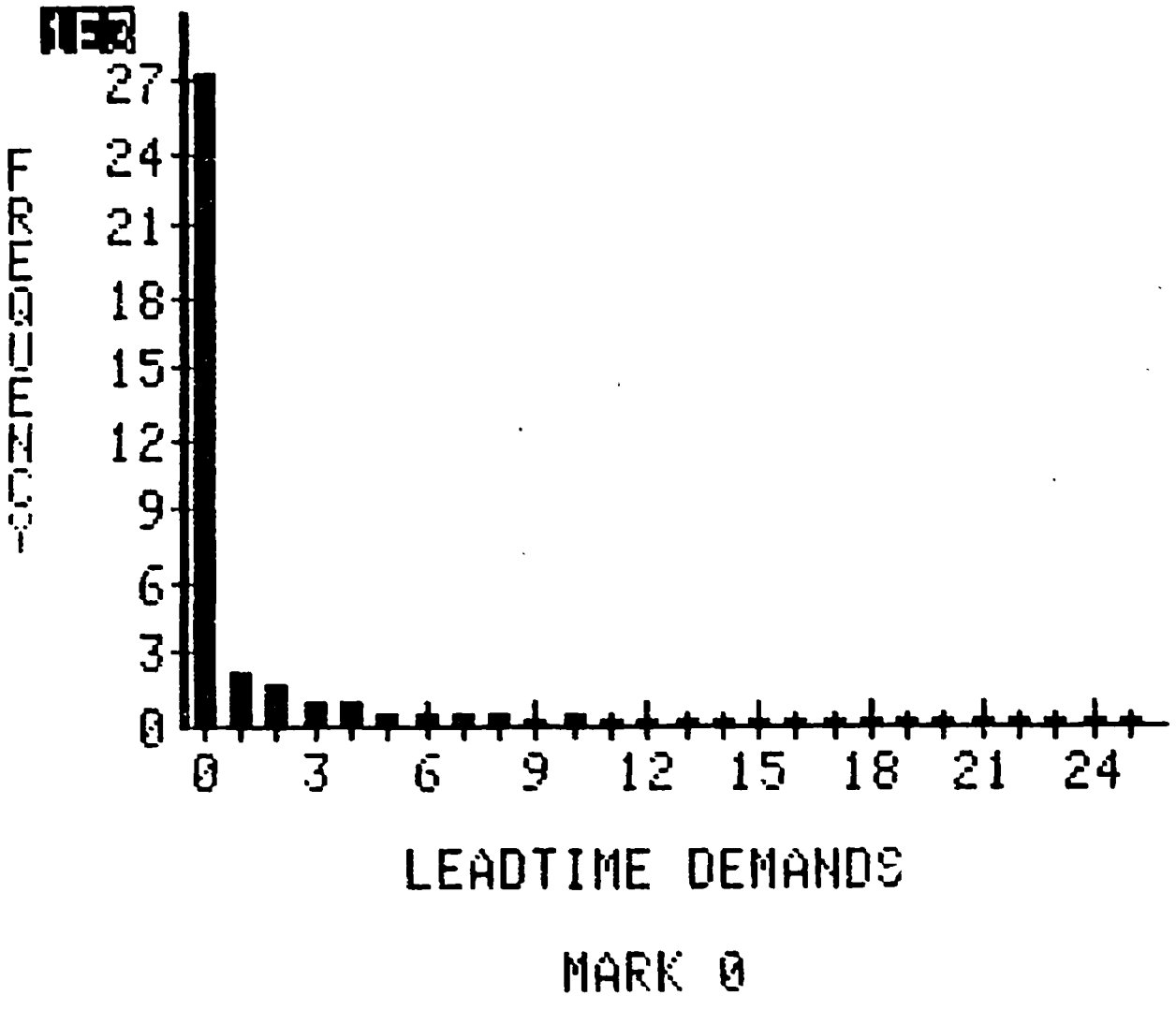
3000
2000
1000
0
FREQUENCY
1
4
7
10
13
16
19
22
25
LEADTIME DEMANDS
ALL ITEMS

FIGURE VI


FREQUENCY
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24
21
18
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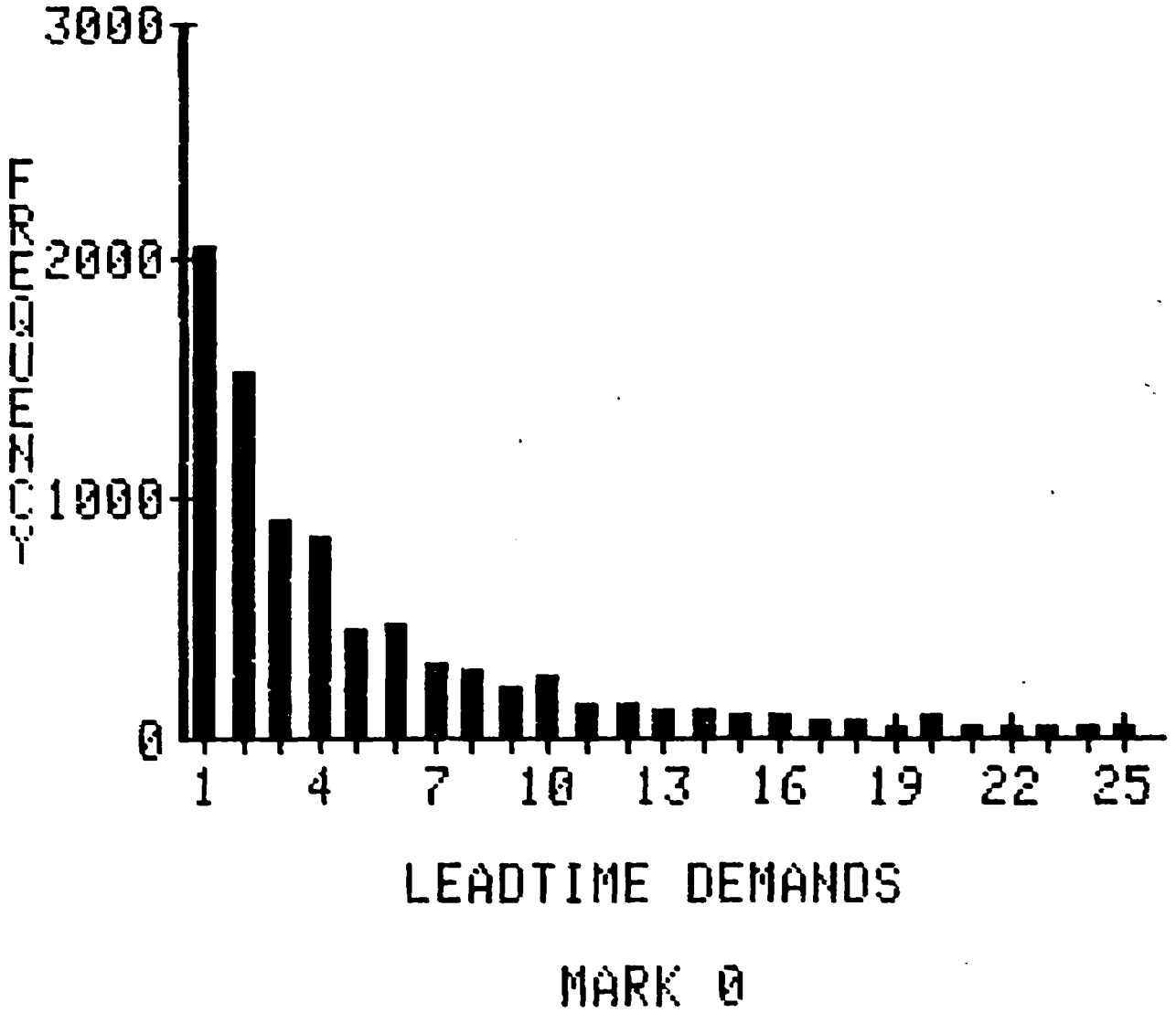
3
6
9
12
15
18
21
24
LEADTIME DEMAND
MARK 0

FIGURE VII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1000
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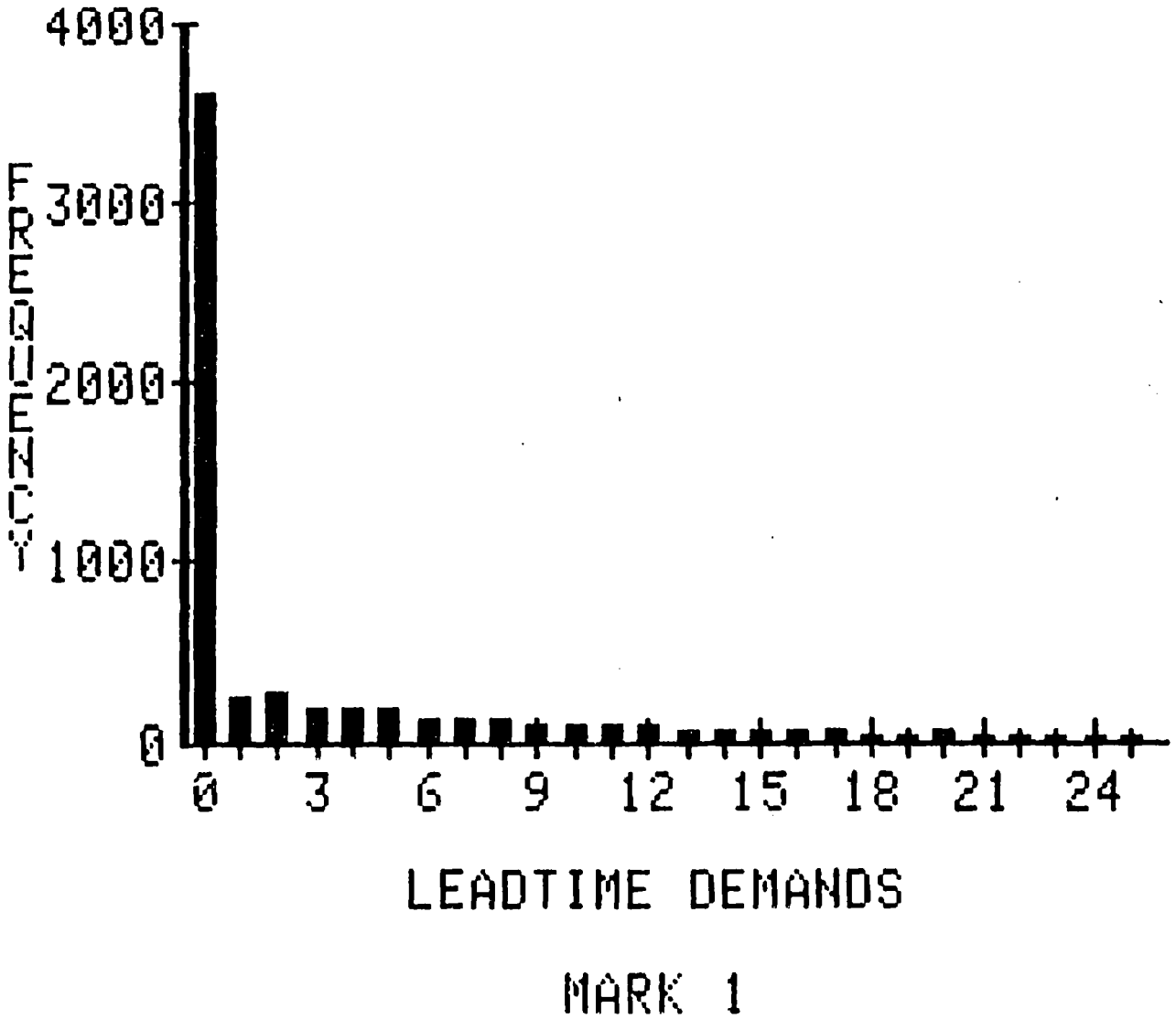
FREQUENCY
1
4
7
10
13
16
19
22
25
LEADTIME DEMANDS
MARK 0

FIGURE VIII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FREQUENCY
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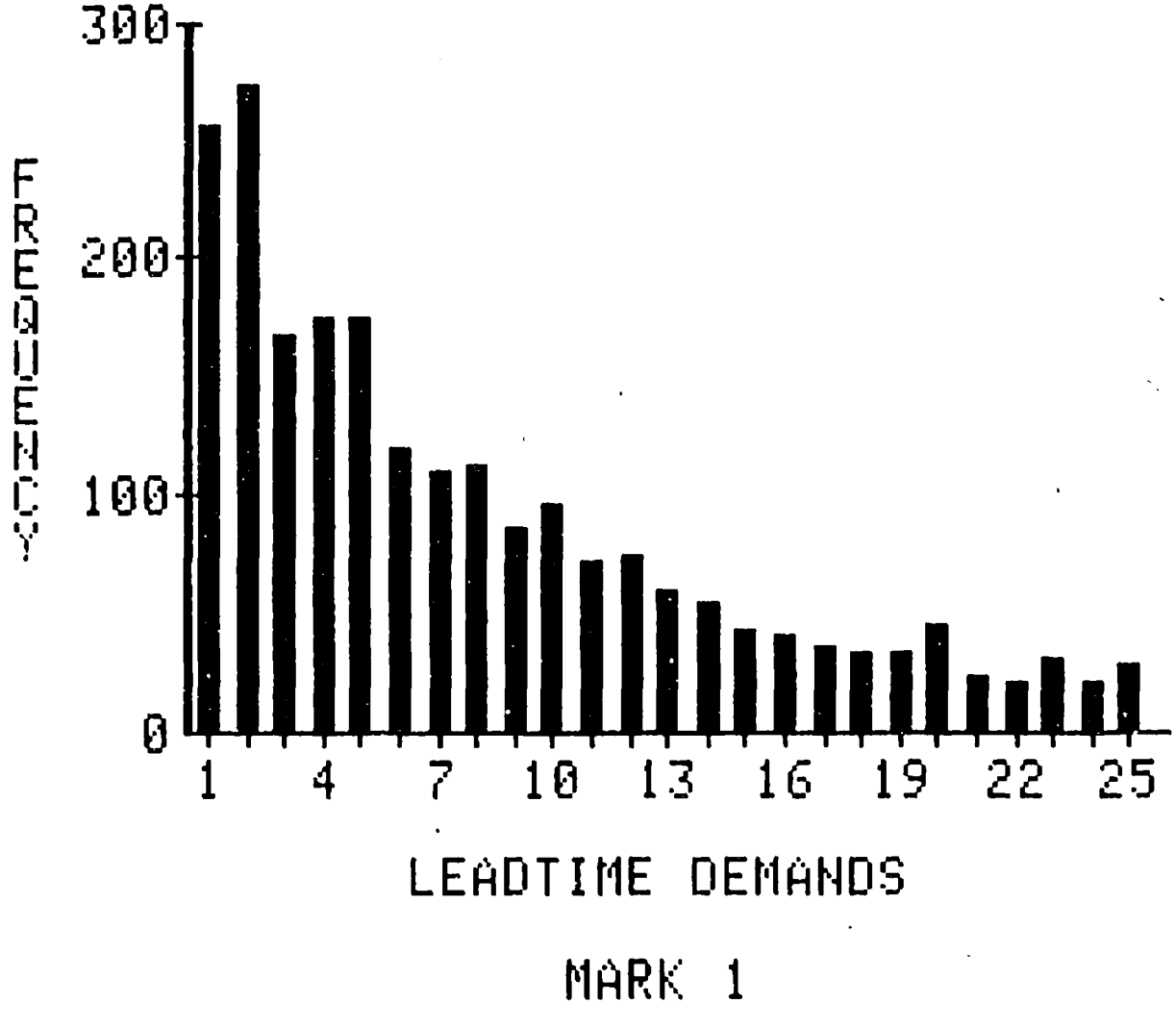
15
18
21
24
LEADTIME DEMANDS
MARK 1

FIGURE IX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FREQUENCY
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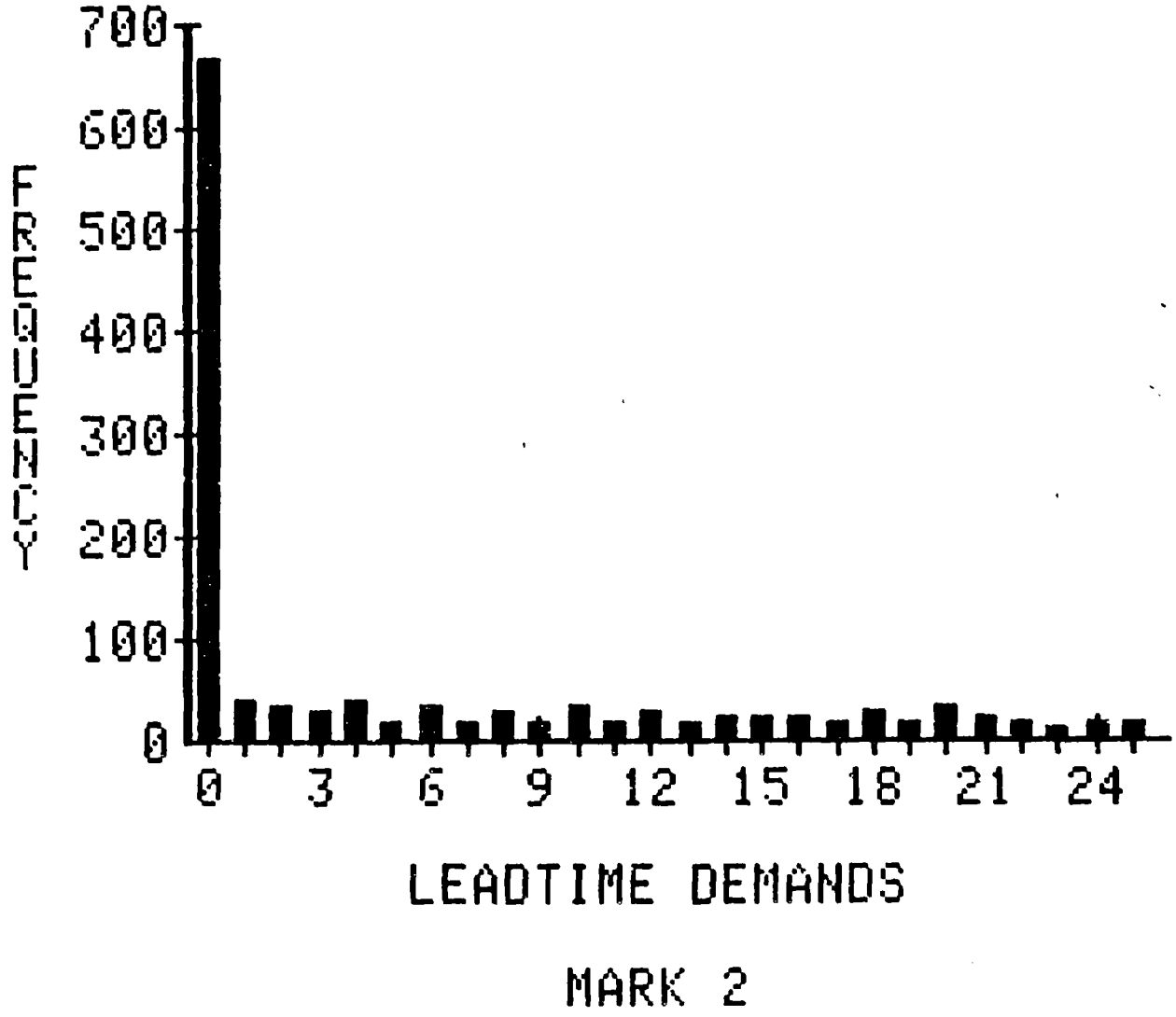
13
16
19
22
25
LEADTIME DEMANDS
MARK 1

FIGURE X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400
300
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FREQUENCY
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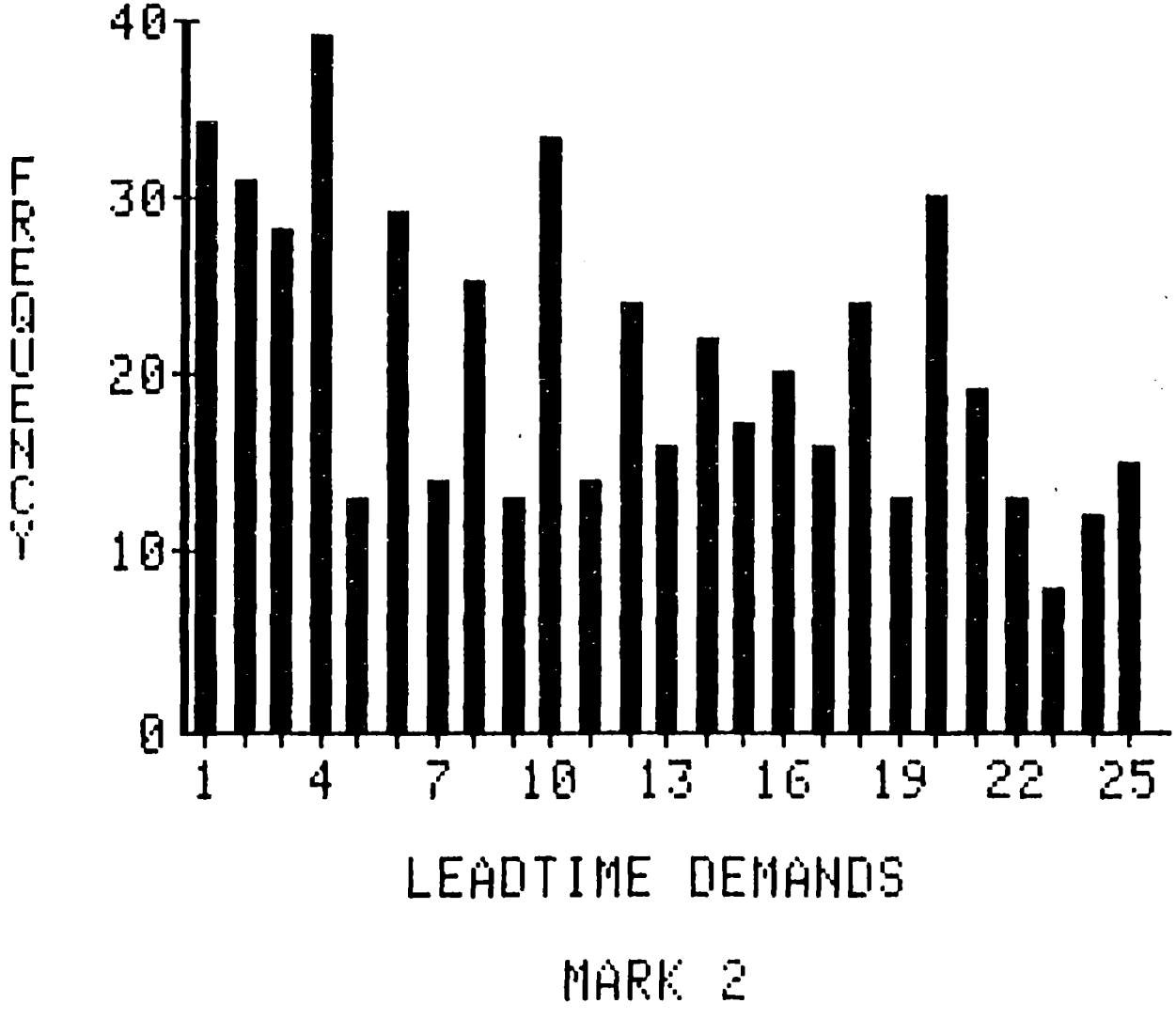
21
24
LEADTIME DEMANDS
MARK 2

FIGURE X1


FREQUENCY
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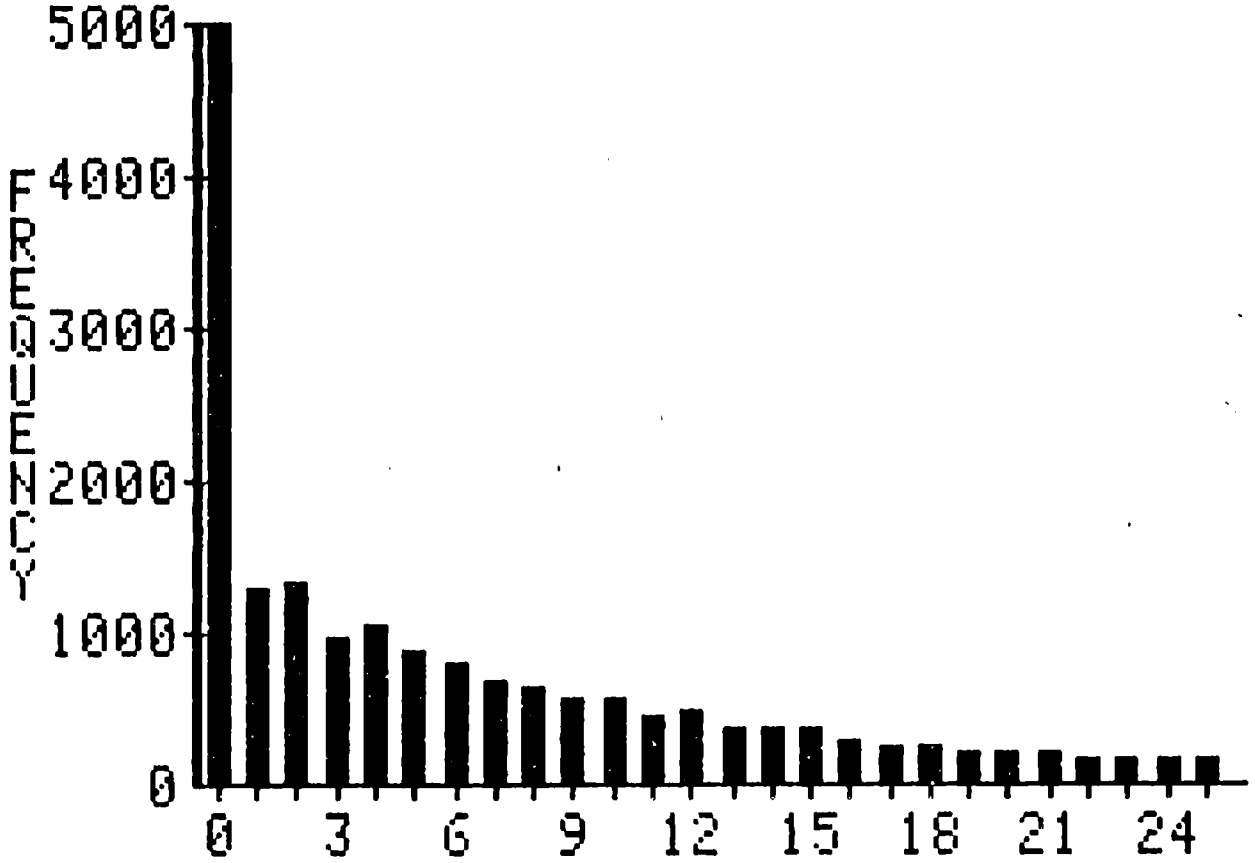
16
19
22
25
LEADTIME DEMANDS
MARK 2

FIGURE XII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3000
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1000
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FREQUENCY
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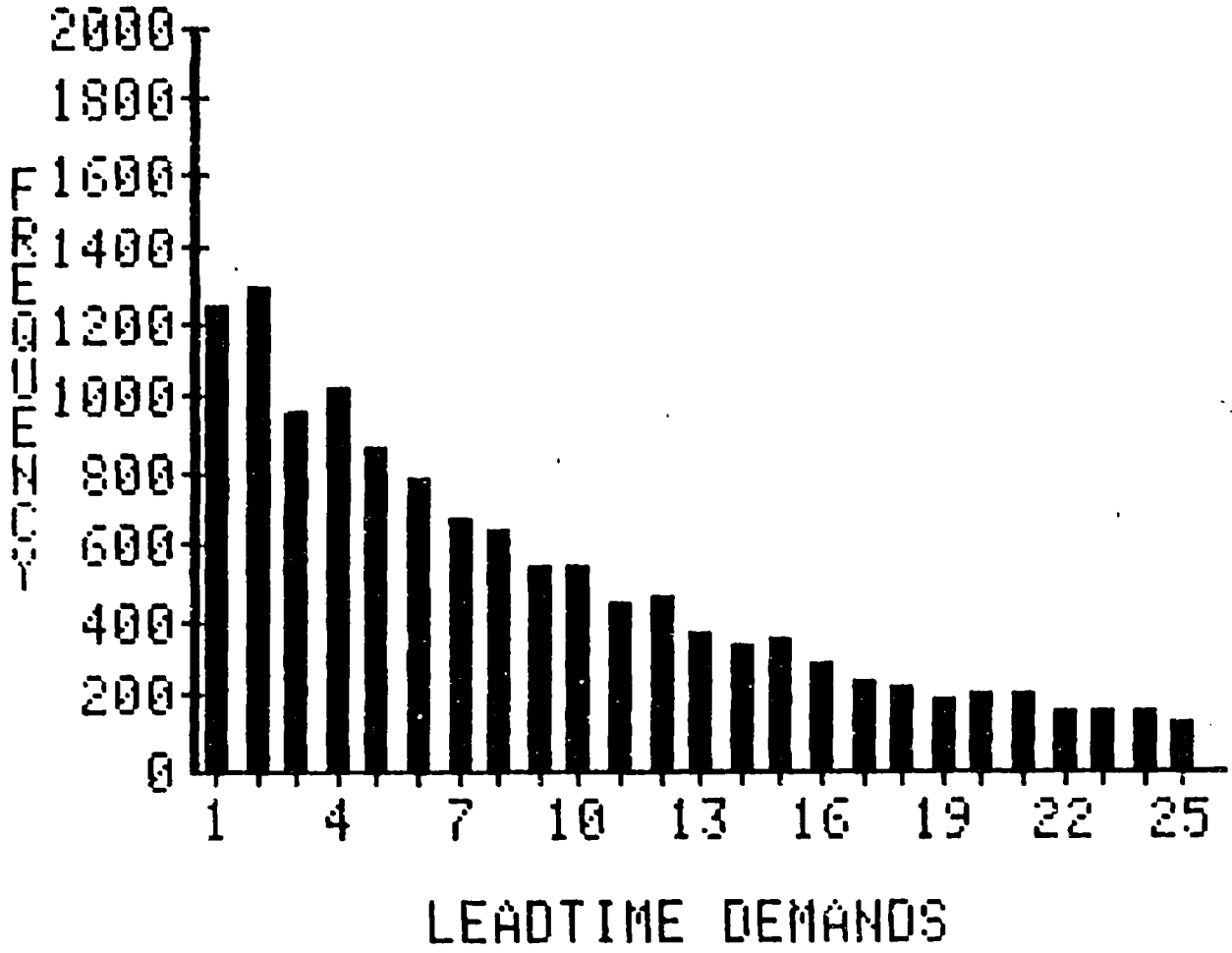
6
9
12
15
18
21
24
LEADTIME DEMANDS
MARK 3

FIGURE XIII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FREQUENCY
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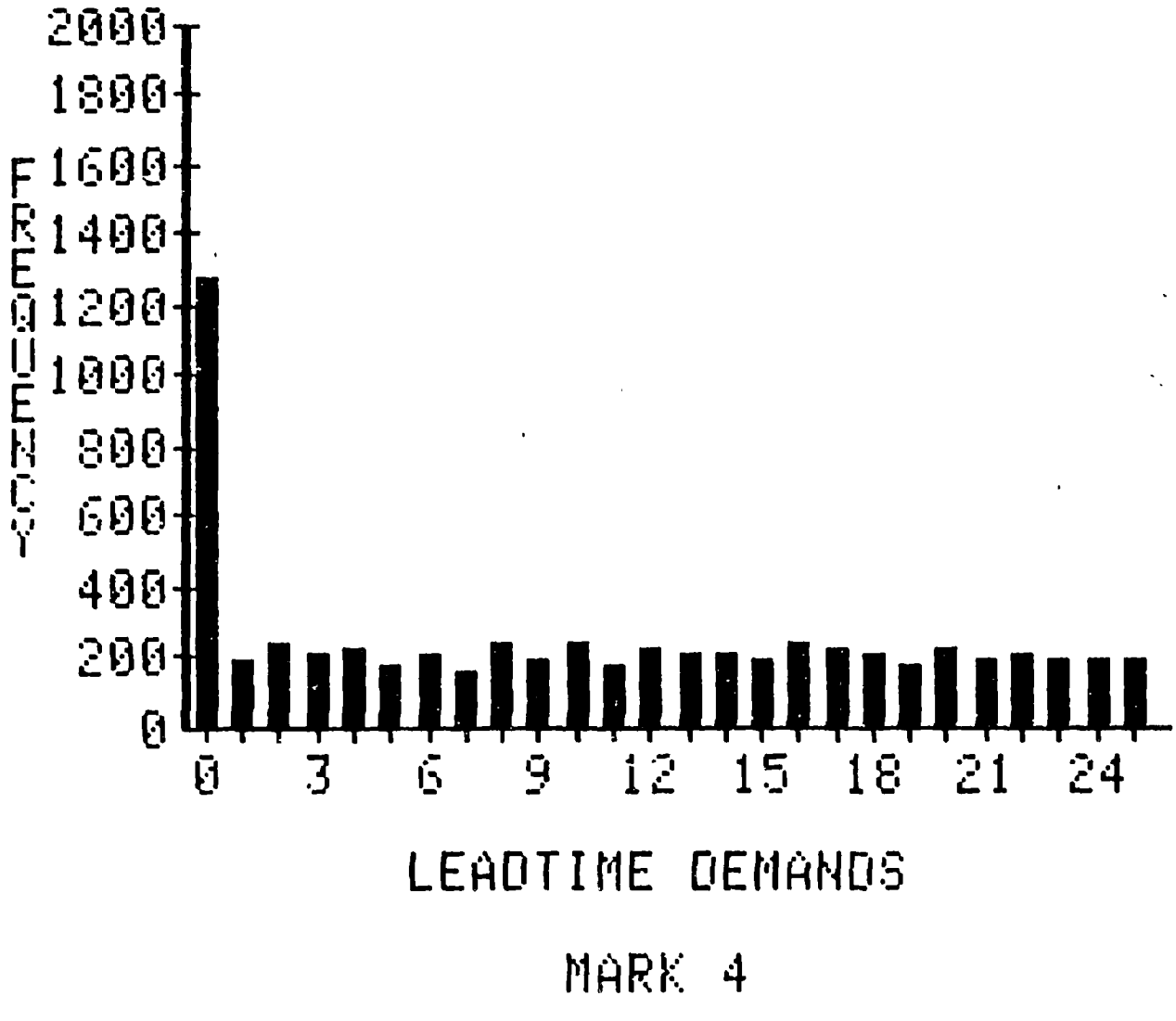
13
16
19
22
25
LEADTIME DEMANDS
MARK 3

FIGURE XIV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800
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200
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FREQUENCY
0
3
6
9
12
15
18
21
24
LEADTIME DEMANDS
MARK 4

FIGURE XV

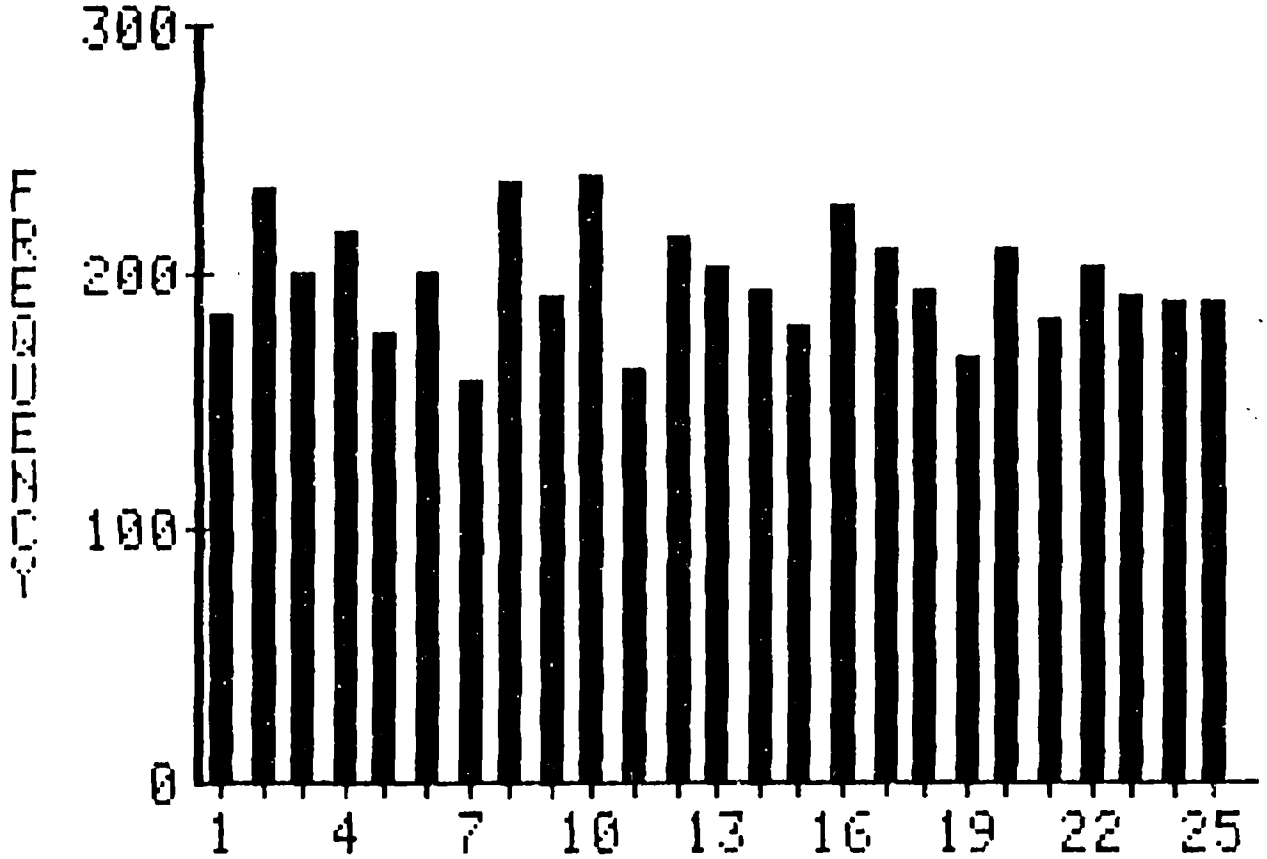


FIGURE XVI

C. CHI-SQUARE GOODNESS-OF-FIT TESTS. The distributions selected for chi-square goodness-of-fit testing are: Exponential, Bernoulli-Exponential, Negative Binomial, Poisson, Geometric and Gamma. The chi-square (χ^2) goodness-of-fit test is a formal comparison of a histogram with the fitted density function. The density functions for the distributions given above are computed and the (χ^2) test is performed as described in the Technical Approach.

TABLE III presents the chi-square test statistics for each grouping of the six distributions being tested. Each distribution contains three columns: degrees of freedom, critical value and computed χ^2 value. The degrees of freedom were determined, as described in the Technical Approach, by combining the expected value of the cells so that each cell would contain at least five observations. When there are 50 data cells, the degrees of freedom are 50-1 or 49, but if several cells had to be combined, then the degrees of freedom are less, for example, with 39 data cells the degrees of freedom are 39-1 or 38. The critical values were obtained from reference 6 of APPENDIX A. The critical value is used to test the hypothesis that the observed leadtime demand data can be described by the distribution being tested. The hypothesis is rejected if the test statistic is larger than the critical value. As shown in TABLE III, none of the computed (χ^2) test statistic values are less than the critical values, therefore, the hypothesis is rejected for all the distributions tested.

The (χ^2) goodness-of-fit test is a hypothesis test which uses the full range of the tested distribution to indicate whether the distribution can describe the observed data. The next step is to use a relative test, based on a MSE measure, to determine the distribution that best fits the data in the right hand tail of the distribution.

TABLE III

CHI-SQUARE STATISTICS

	EXPONENTIAL			BERNOULLI-EXPONENTIAL			NEGATIVE BINOMIAL		
	DEGREES OF FREEDOM	CRITICAL VALUE	COMPUTED χ^2	DEGREES OF FREEDOM	CRITICAL VALUE	COMPUTED χ^2	DEGREES OF FREEDOM	CRITICAL VALUE	COMPUTED χ^2
MARK									
0	28	56.90	166243.00	49	85.40	7147.13	49	85.40	25031.30
1	44	78.70	32478.20	49	85.40	649.29	49	85.40	1240.77
2	49	85.40	22714.60	49	85.40	255.20	46	81.40	357.76
3	49	85.40	20508.30	49	85.40	834.09	49	85.40	2898.19
4	49	85.40	40597.90	49	85.40	8716.34	49	85.40	90643.30
LTDMD									
$x = 0$	38	70.70	151088.00	49	85.40	6373.67	49	85.40	15520.90
$0 < x \leq 2$	19	43.80	281762.00	43	77.40	2487.78	49	85.40	5968.34
$2 < x \leq 10$	49	85.40	146999.00	49	85.40	1608.73	49	85.40	7973.88
$10 < x \leq 50$	49	85.40	13709.60	49	85.40	551.23	49	85.40	3008.29
$x > 50$	49	85.40	25619.90	49	85.40	2647.54	2	13.80	2971710.00
RQN FCST									
$x = 0$	41	74.70	144730.00	49	85.40	5690.91	49	85.40	14021.00
$0 < x \leq .25$	32	62.50	172714.00	49	85.40	4430.10	49	85.40	5994.39
$.25 < x \leq 1.0$	49	85.40	381578.00	49	85.40	5435.34	49	85.40	12130.10
$1.0 < x \leq 3.5$	49	85.40	12533.50	49	85.40	6869.21	49	85.40	39374.00
$x > 3.5$	49	85.40	7096.77	49	85.40	5971.26	*	*	*

*There were only two intervals and only one had an expected value greater than 5.

TABLE III (CONT'D)

CHI-SQUARE STATISTICS

	POISSON			GEOMETRIC			GAMMA		
	DEGREES OF FREEDOM	CRITICAL VALUE	COMPUTED χ^2	DEGREES OF FREEDOM	CRITICAL VALUE	COMPUTED χ^2	DEGREES OF FREEDOM	CRITICAL VALUE	COMPUTED χ^2
MARK									
0	12	32.90	917435.00	31	61.10	98533.50	18	42.30	1015500.00
1	17	40.80	911575.00	46	81.40	16345.70	47	82.70	14666.30
2	15	37.70	240651.00	49	85.40	11017.70	49	85.40	1062.09
3	22	48.30	5846450.00	49	85.40	8090.33	49	85.40	4849.68
4	**	**	**	49	85.40	22866.80	49	85.40	4039.20
LTDMD									
$x = 0$	15	37.70	2530060.00	41	74.70	89530.80	37	69.30	168945.00
$0 < x \leq 2$	9	27.90	139262.00	22	48.30	28565.10	4	18.50	234834.00
$2 < x \leq 10$	20	45.30	2665380.00	49	85.40	12963.80	49	85.40	9993.41
$10 < x \leq 50$	34	65.20	4973440.00	49	85.40	5465.84	49	85.40	1720.50
$x > 50$	**	**	**	49	85.40	13276.10	49	85.40	2845.74
RQN FCST									
$x = 0$	16	39.30	3648120.00	43	77.40	77538.30	42	76.10	127814.00
$0 < x \leq .25$	13	34.50	963628.00	35	66.60	64748.80	28	56.90	165310.00
$.25 < x \leq 1.0$	26	54.10	15562200.00	49	85.40	22552.90	49	85.40	9779.53
$1.0 < x \leq 3.5$	28	56.90	15287700.00	49	85.40	6671.00	49	85.40	2154.43
$x > 3.5$	**	**	**	49	85.40	6223.58	49	85.40	4273.63

**Means are too large for Poisson distribution to handle.

D. MEAN SQUARE ERROR RESULTS. For the findings presented previously, the groupings of MARK, Leadtime Demand Forecast and Requisition Forecast were divided into five categories with each category containing approximately 20% of the data. The initial analysis of MSE also focused on the same five categories. However, reviewing the resulting statistics (TABLE II, the histograms and the initial MSE results) led us to conclude that the categories could be consolidated from five to three without affecting the MSE results. Therefore, for the MSE results shown below, each grouping was consolidated into a low, medium and high demand range. For the MARK grouping the categories were MARK 0 (low demand), MARKs I and III (medium) and MARKs II and IV (high demand). For the Leadtime Demand Forecast grouping, the categories were leadtime demand forecast less than or equal to 2, greater than 2 but less than or equal to 50, and greater than 50. For the Requisition Forecast grouping, the categories were requisition forecast less than or equal to .25, greater than .25 but less than or equal to 3.5, and greater than 3.5.

As discussed in the Technical Approach, the MSE was calculated for the right hand tail of the distribution by using every 5th percentile starting at the 50th percentile and ending at the 95th percentile. The results of these MSE calculations are displayed in TABLE IV. APPENDIX D contains the actual percentages ($\hat{p}$) of observed leadtime demand falling in each interval defined by the tested distribution as 50th percentile, 55th percentile, etc. The Normal distribution was included in the MSE analysis since all of the hypothesized distributions failed the chi-square test and it is a distribution currently being used in the reorder level computations. The Gamma distribution was not included in the MSE analysis because there is no closed form for the Gamma.

The smallest MSE value represents the best fit. The Bernoulli-Exponential distribution provides the best fit in the right hand tail for every group of items except for items with a leadtime demand forecast greater than 50.

TABLE IV

MEAN SQUARE ERROR STATISTICS

	EXP	BEXP	NEGBIN	POISSON	GEOMETRIC	NORMAL
MARK						
0	410.52	152.74	173.85	442.69	448.67	681.97
I&III	48.43	5.38	93.46	151.50	74.03	274.70
II&IV	303.78	287.20	313.83	*	295.04	434.74
LTDMD						
$0 \leq Y \leq 2$	404.76	130.51	167.09	479.95	449.24	297.30
$2 < Y \leq 50$	70.90	38.61	153.49	160.05	74.48	699.62
$Y > 50$	193.47	312.50	379.58	*	317.13	407.25
RQN FCST						
$0 \leq Y \leq .25$	466.07	157.18	175.43	503.51	474.07	699.62
$.25 < Y \leq 3.5$	210.71	179.61	1096.31	294.55	222.97	548.46
$Y > 3.5$	357.71	356.93	411.01	*	358.27	419.56

*Means are too large for Poisson distribution to handle.

IV. SUMMARY AND CONCLUSIONS

In this report, 11 theoretical probability distributions were tested to determine which distribution best describes the demand during leadtime for 1H Cog material. The UICP Levels computation program currently assumes that an item's leadtime demand is described by either the Poisson, Negative Binomial or Normal distributions. In addition to these three distributions, the Exponential, Gamma, Geometric, Logistic, LaPlace, Weibull, Bernoulli-Lognormal and Bernoulli-Exponential distributions were tested. The selection of the most

appropriate probability distribution is vital to the calculation of safety level. Using a distribution to calculate safety level which does not fit the leadtime demand pattern will result in an inefficient allocation of funds.

Previous attempts to fit leadtime demand to theoretical probability distributions were restricted to using quarterly demand observations. In this study, the Due-In Due-Out File and the Transaction History File were used to determine actual leadtime demands for each item. The actual leadtime demands were used to construct histograms to hypothesize what general family of distributions the data comes from; for example, Exponential, Poisson, Normal. After a family of distributions was hypothesized, the value of its parameters were specified using maximum likelihood estimators where possible. The chi-square goodness-of-fit hypothesis test was used to examine whether the hypothesized distributions were in agreement with the observed data. Since the chi-square test measures the fit over the whole distribution, a mean square error measure was used to determine which distribution has the best fit in the right hand tail. The right hand tail is the most important part of the distribution since that is the part of the distribution used to determine safety level.

Ideally, a probability distribution would be fit for each item's leadtime demand observations. However, there were not enough leadtime demand observations for each item. Therefore, the items were divided into homogeneous groups. Groups were determined based on one of the following six criteria: MARK, Forecasted Leadtime Demand, Requisition Forecast, Unit Price, Value of Annual Demand or Leadtime.

None of the theoretical distributions passed the chi-square goodness-of-fit test. However, the Bernoulli-Exponential distribution had the best right hand tail fit.

V. RECOMMENDATIONS

It is recommended that UICP use the Bernoulli-Exponential distribution to model leadtime demand.

APPENDIX A: REFERENCES

1. COMNAVSUPSYSCOM ltr 04A7/JHM of 27 Jun 1980

2. FMSO Operations Analysis Report 120

3. Naval Postgraduate School Thesis, "Distributional Analysis of Inventory Demand Over Leadtime" by Mark Lee Yount, June 1982

4. Naval Postgraduate School Thesis - "An Analysis of Current Navy Procedures for Forecasting Demand with an Investigation of Possible Alternative Techniques" by Edward Joseph Shields, September 1973

5. A. M. Law and W. D. Kelton, Simulation Modeling And Analysis, McGraw-Hill Book Co., 1982

6. A. Hald, Statistical Tables and Formulas, John Wiley & Sons, Inc., 1965

APPENDIX B: DISTRIBUTION OF LEADTIME DEMANDS FOR MARK I ITEMS

Leadtime Demand	Number of Observations	Cumulative Percent of Total Observations
0	3,608	57.94
1	256	62.05
2	273	66.43
3	167	69.11
4	175	71.92
5	175	74.73
6	118	76.62
7	109	78.37
8	112	80.17
9	85	81.54
10	96	83.08
11	71	84.22
12	74	85.41
13	58	86.34
14	54	87.21
15	43	87.90
16	40	88.54
17	34	89.09
18	32	89.60
19	32	90.11
20	45	90.83
21	22	91.18
22	21	91.52
23	31	92.02
24	21	92.36
25	28	92.81
26	19	93.12
27	15	93.36
28	13	93.57
29	17	93.84
30	14	94.06
31	12	94.25
32	18	94.54
33	5	94.62
34	18	94.91
35	10	95.07
36	11	95.25
37	5	95.33
38	6	95.43
39	2	95.46
40	11	95.64

APPENDIX C: HISTOGRAMS

The histograms presented here reflect the data from FIGURES IV through XVI stratified by leadtime demand forecast and requisition forecast vice MARK.

Histograms for Leadtime Demand Forecast Groupings:

Histograms for items with $0 \leq$ Leadtime Demand Forecast ≤ 2 including zero observations C-2

Histograms for items with $0 \leq$ Leadtime Demand Forecast ≤ 2 excluding zero observations C-3

Histograms for items with $2 <$ Leadtime Demand Forecast ≤ 50 including zero observations C-4

Histograms for items with $2 <$ Leadtime Demand Forecast ≤ 50 excluding zero observations C-5

Histograms for items with Leadtime Demand Forecast > 50 including zero observations C-6

Histograms for items with Leadtime Demand Forecast > 50 excluding zero observations C-7

Histograms for Requisition Forecast Groupings

Histograms for items with $0 \leq$ Requisition Forecast $\leq .25$ including zero observations C-8

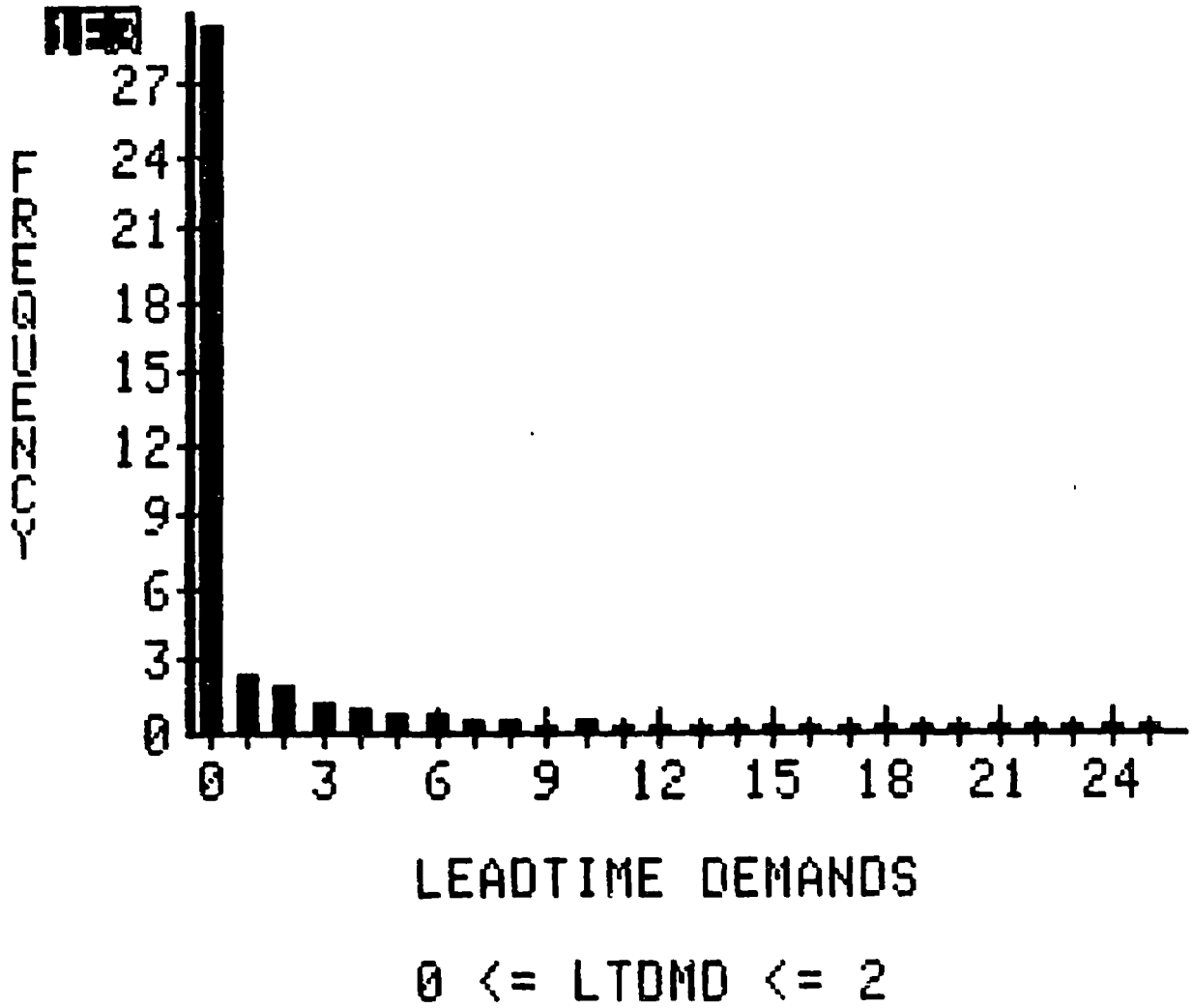
Histograms for items with $0 \leq$ Requisition Forecast $\leq .25$ excluding zero observations C-9

Histograms for items with $.25 <$ Requisition Forecast ≤ 3.5 including zero observations C-10

Histograms for items with $.25 <$ Requisition Forecast ≤ 3.5 excluding zero observations C-11

Histograms for items with Requisition Forecast > 3.5 including zero observations C-12

Histograms for items with Requisition Forecast > 3.5 excluding zero observations C-13


FREQUENCY
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18
15
12
9
6
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LEADTIME DEMANDS
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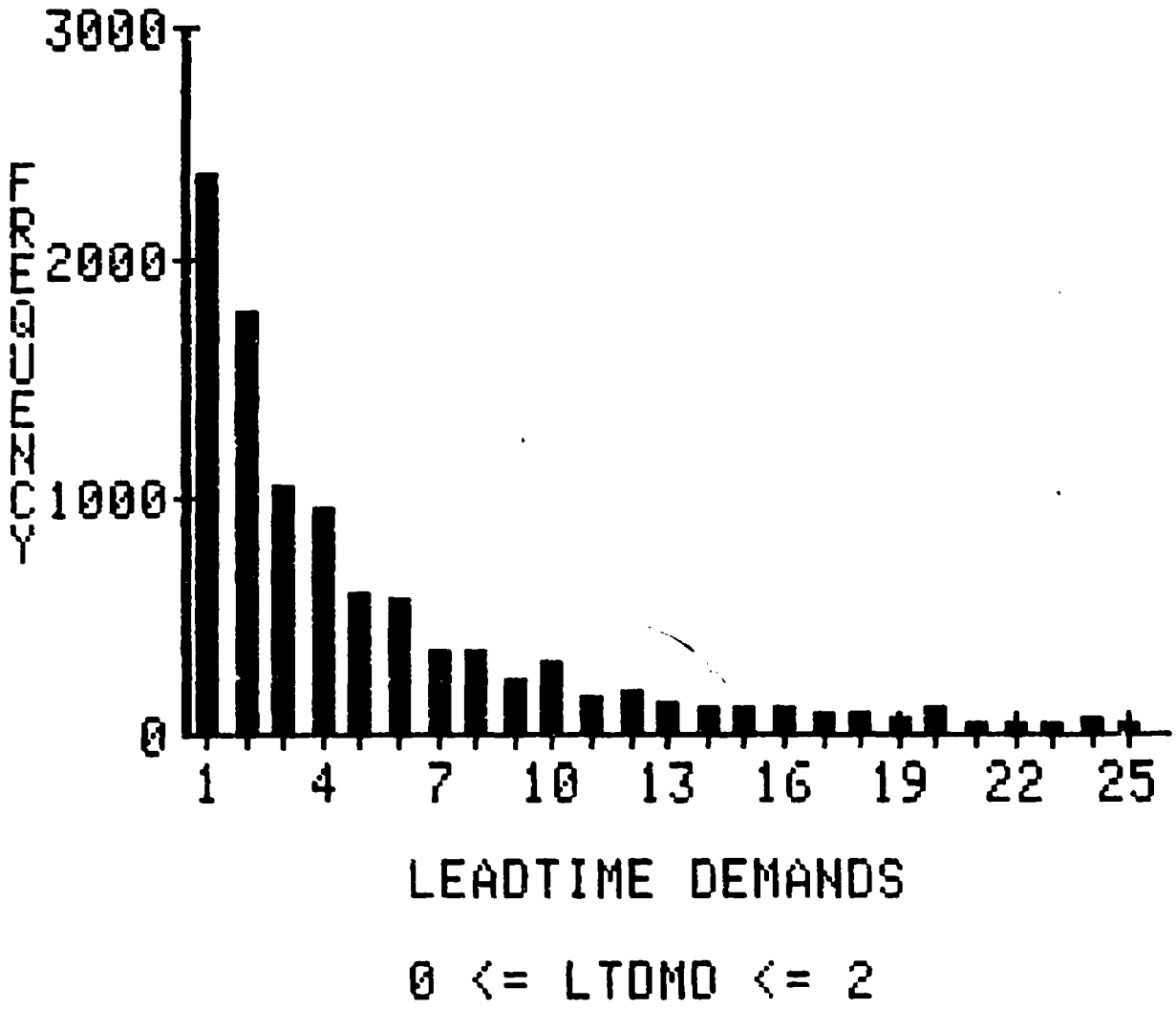
3000
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FREQUENCY
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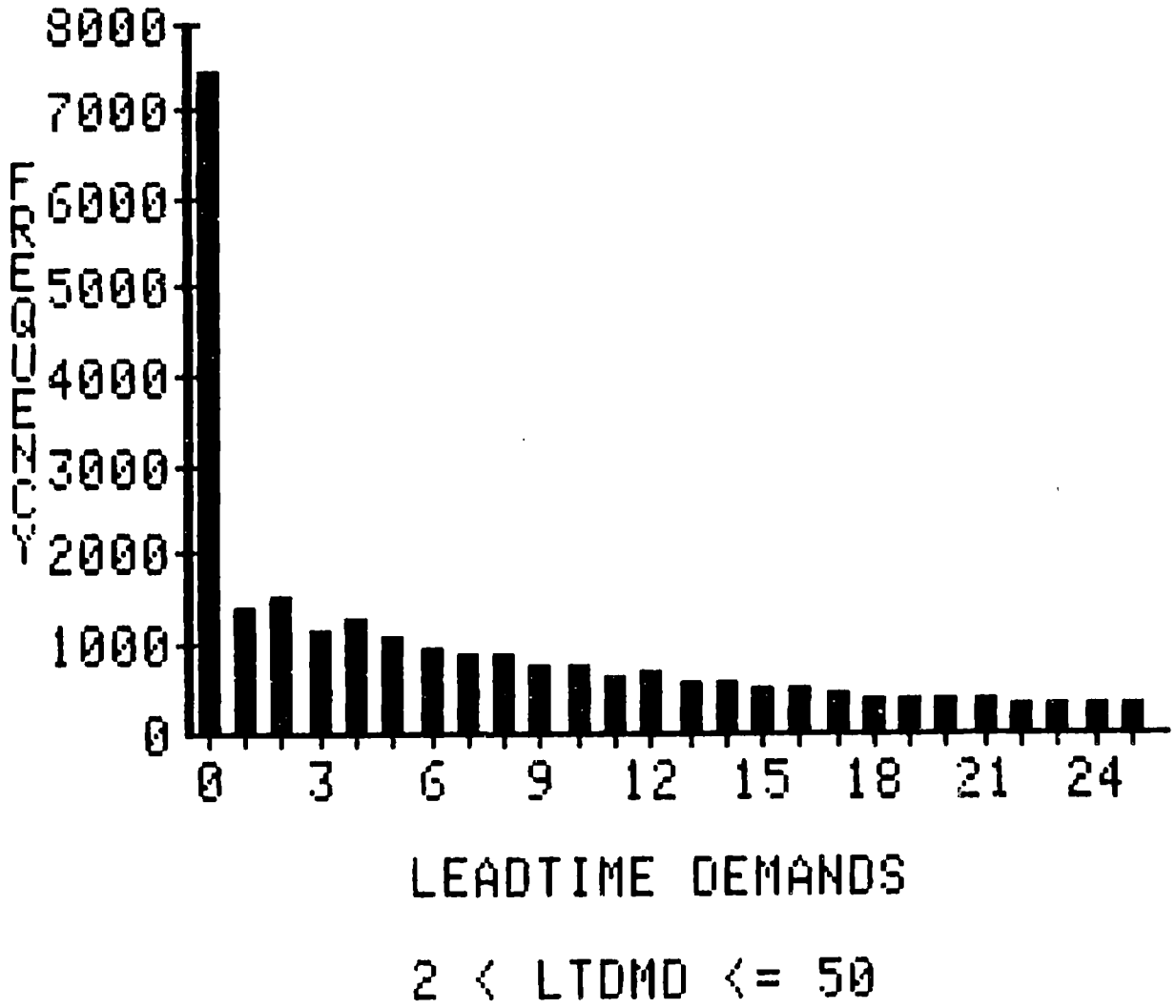
19
22
25
LEADTIME DEMANDS
0 <= LTDMD <= 2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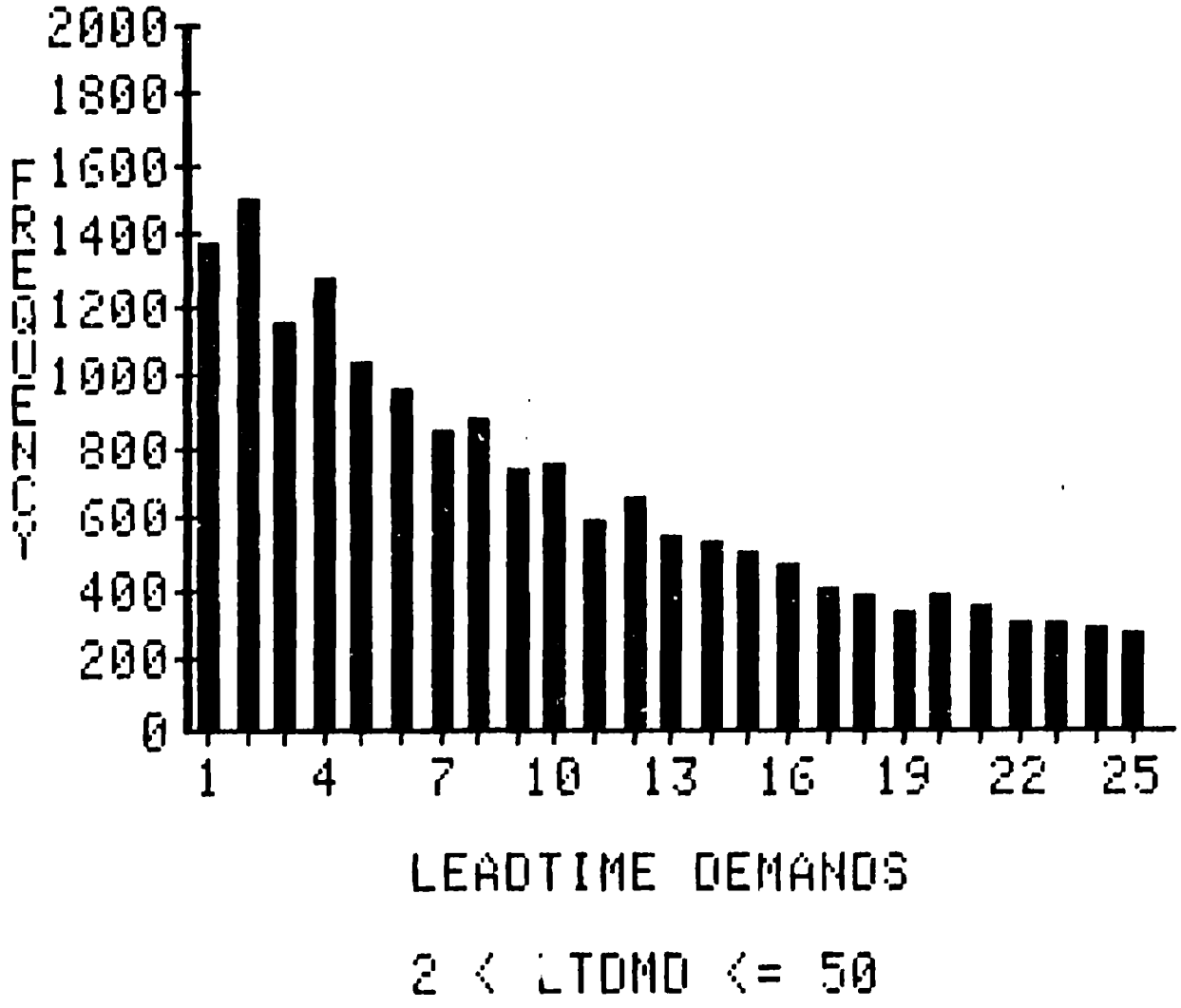
LEADTIME DEMANDS
2 < LTDMD <= 50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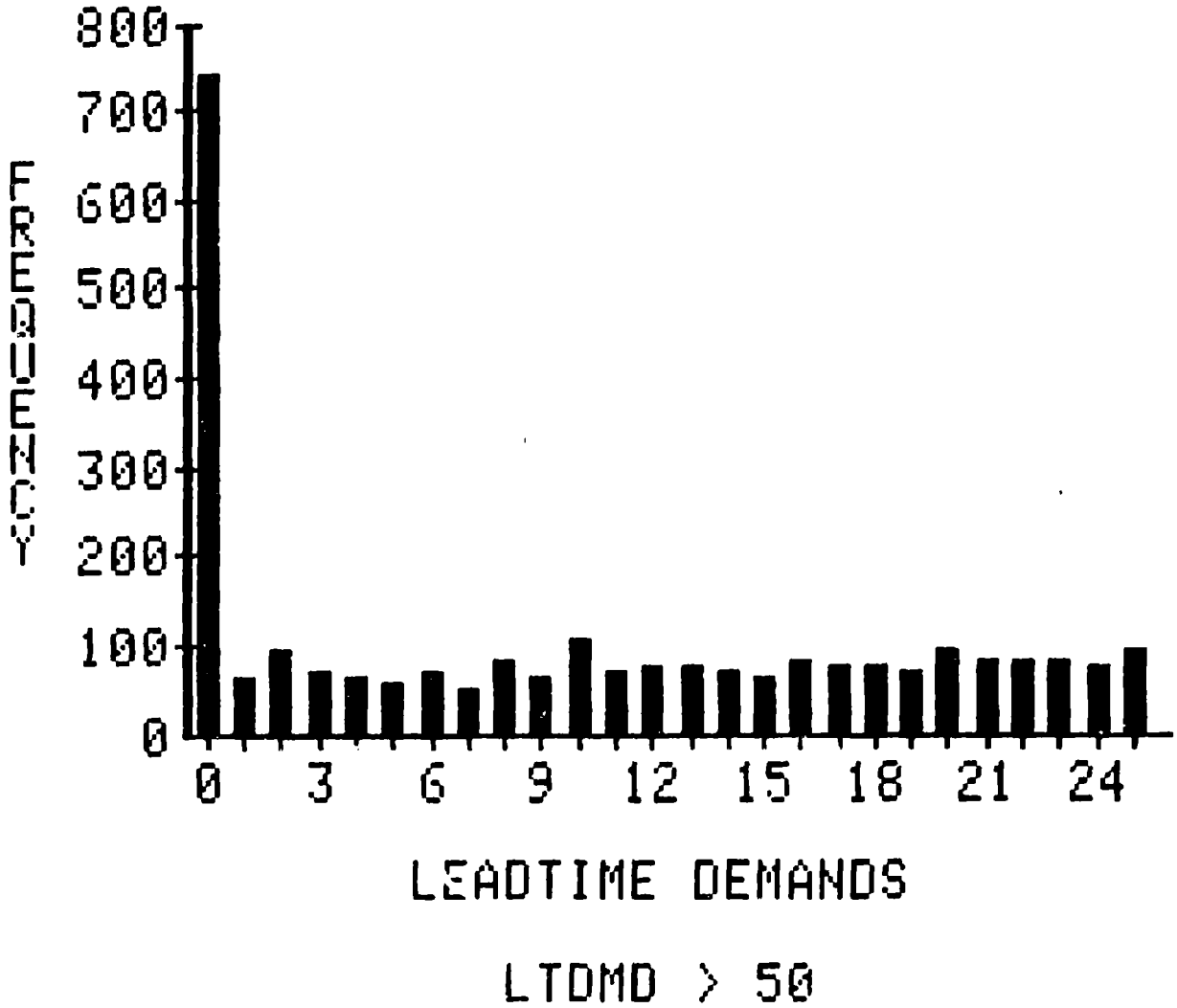
22
25
LEADTIME DEMANDS
2 < LTDMD <= 50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FREQUENCY
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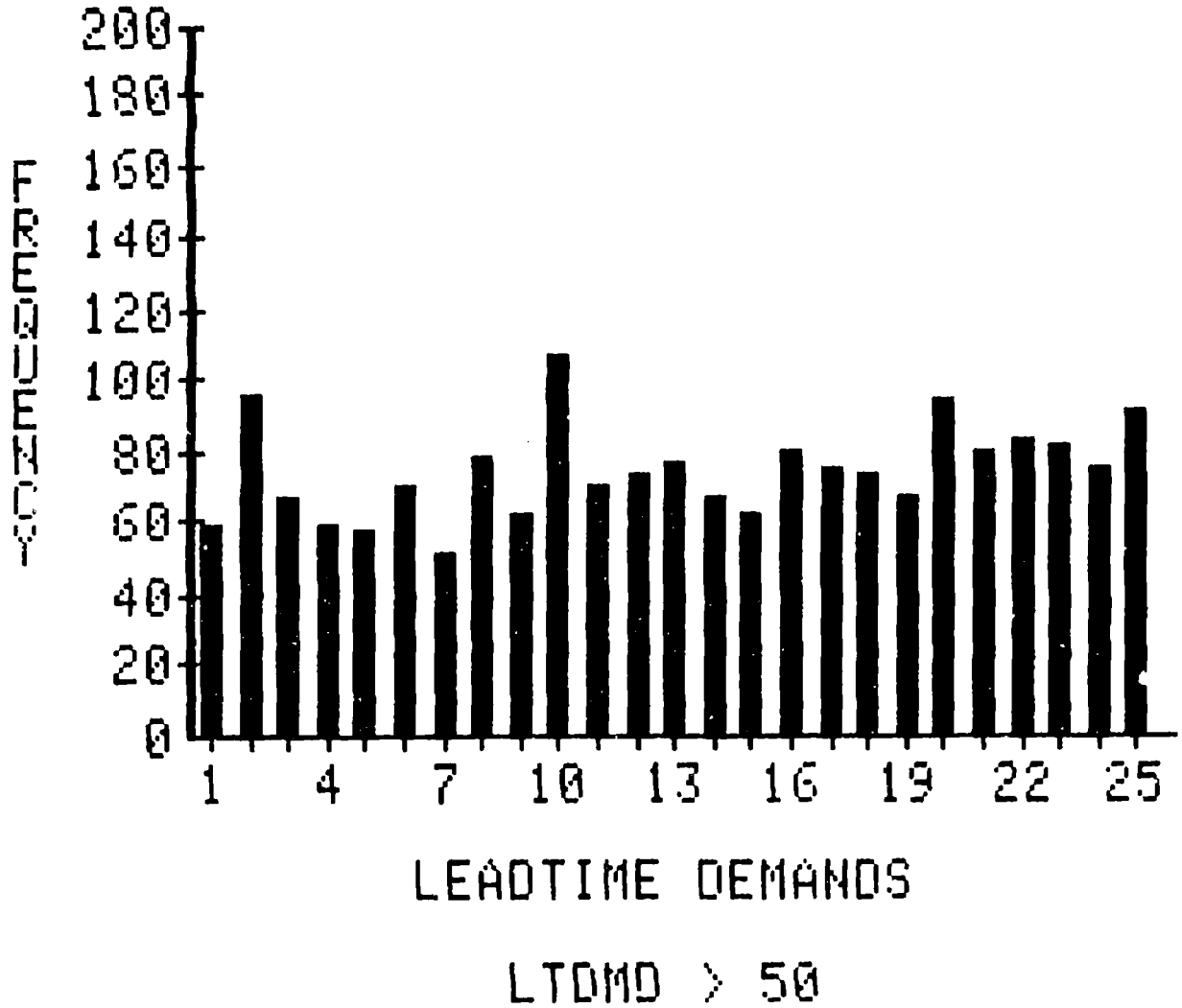
18
21
24
LEADTIME DEMANDS
LTDMD > 50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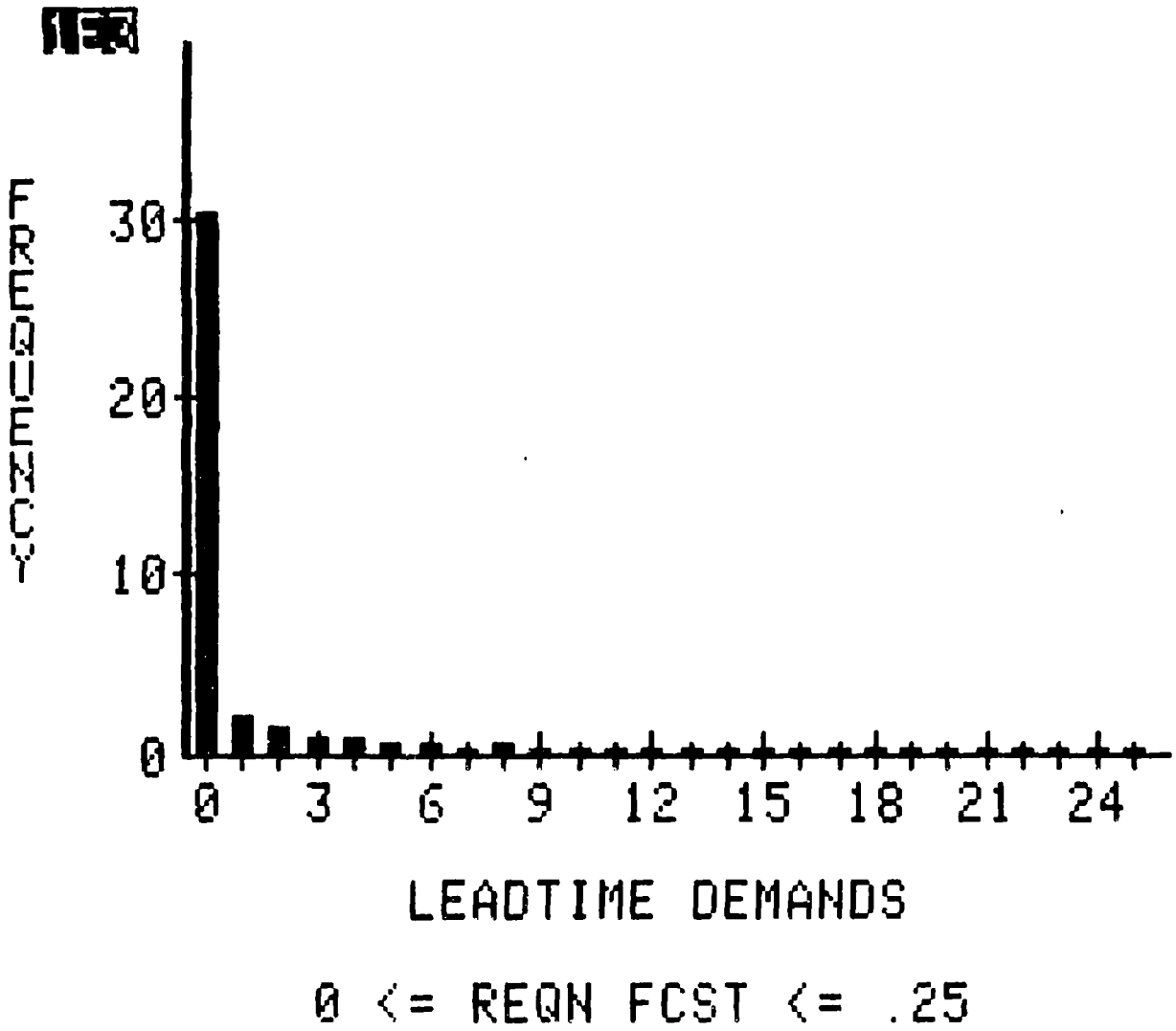
LEADTIME DEMANDS
LTDMD > 50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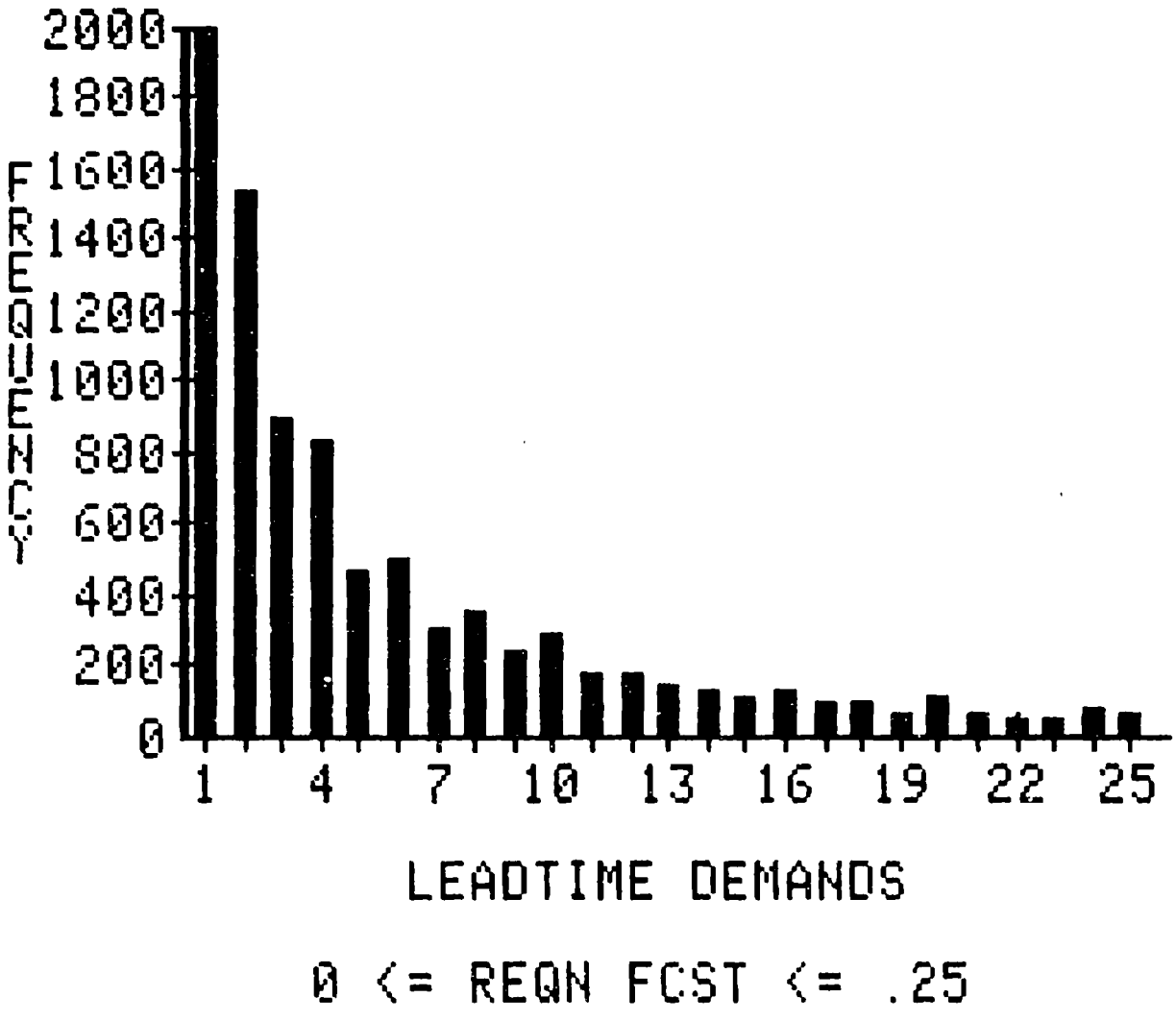
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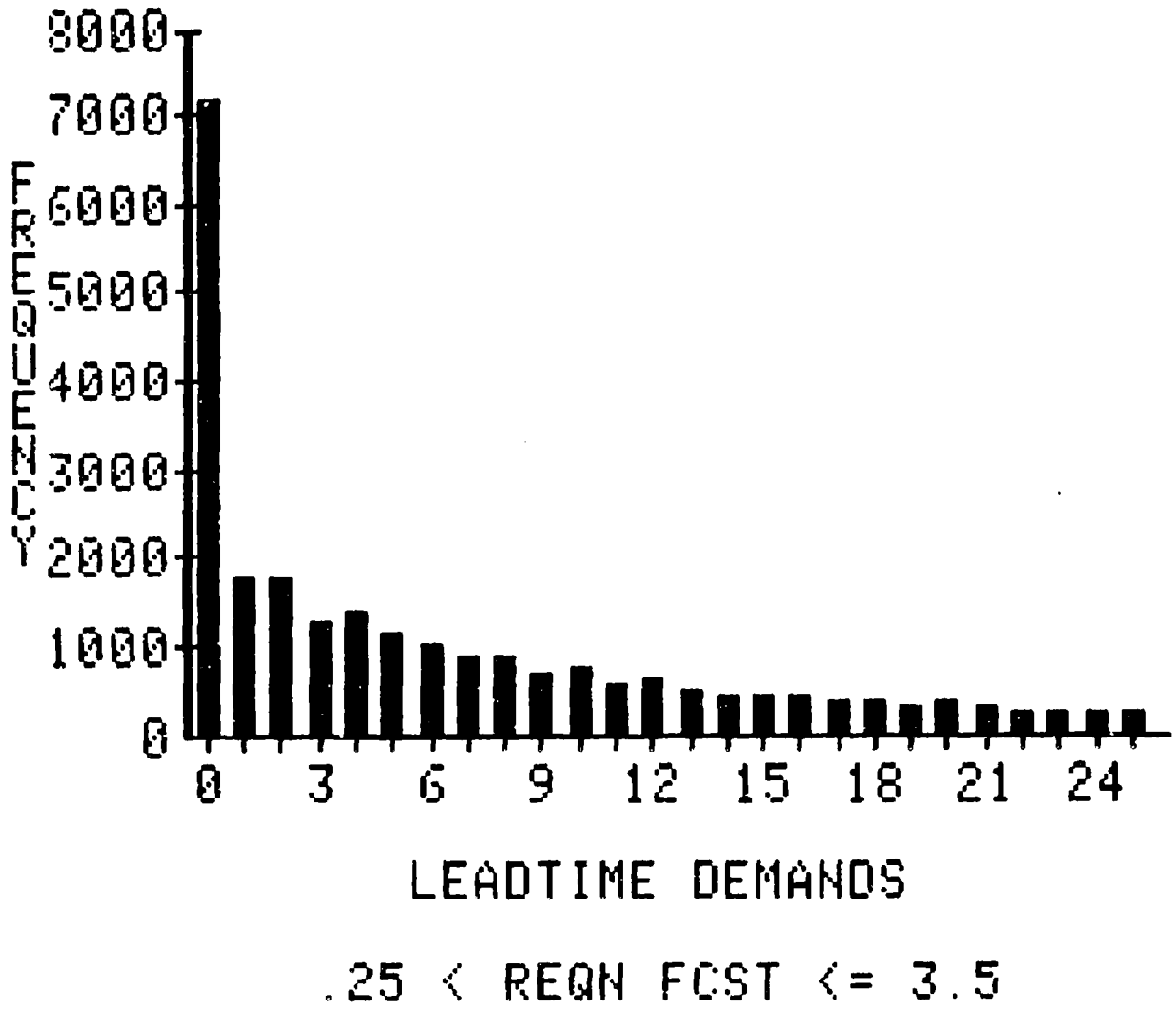
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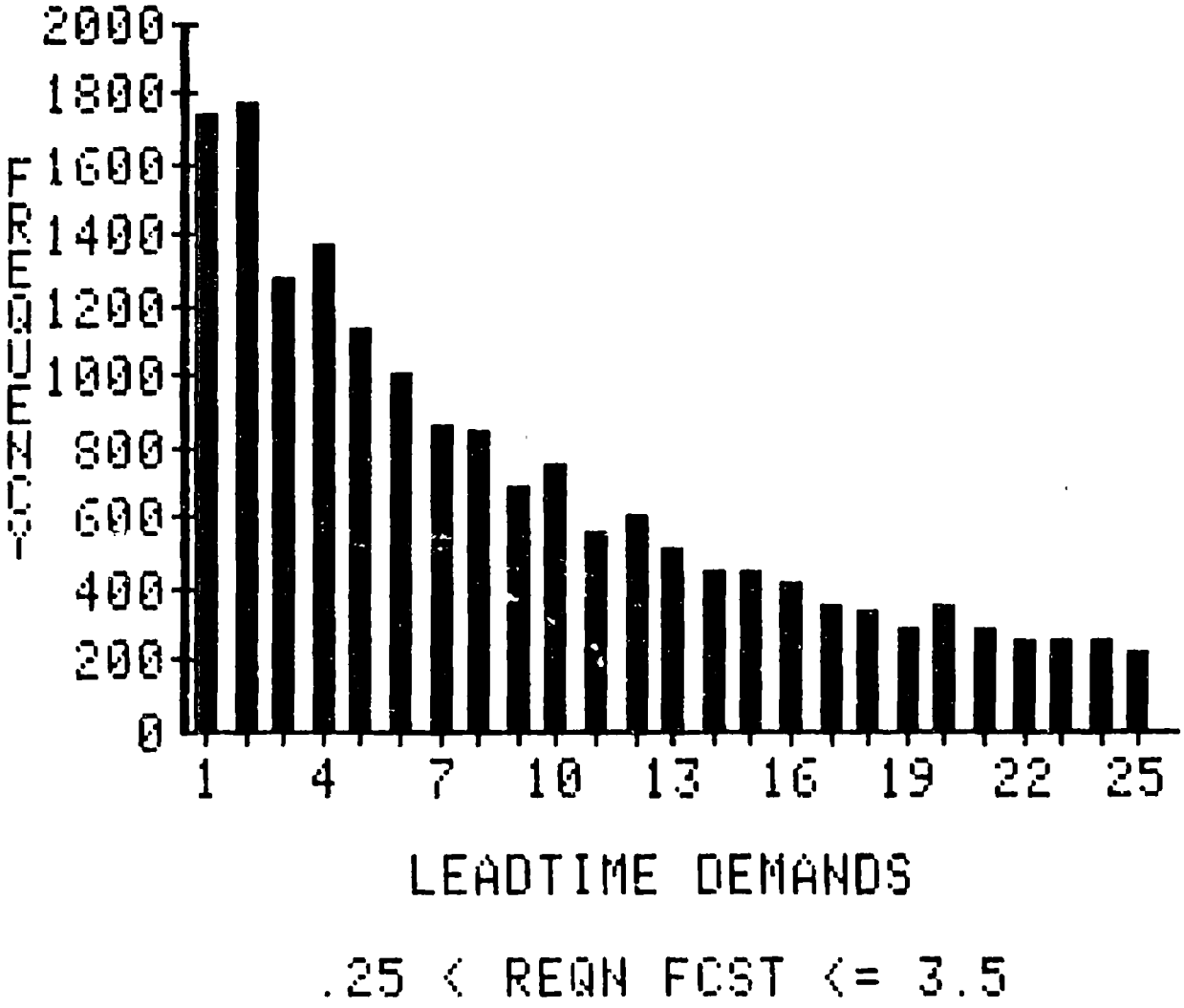
LEADTIME DEMANDS
.25 < REQN FCST <= 3.5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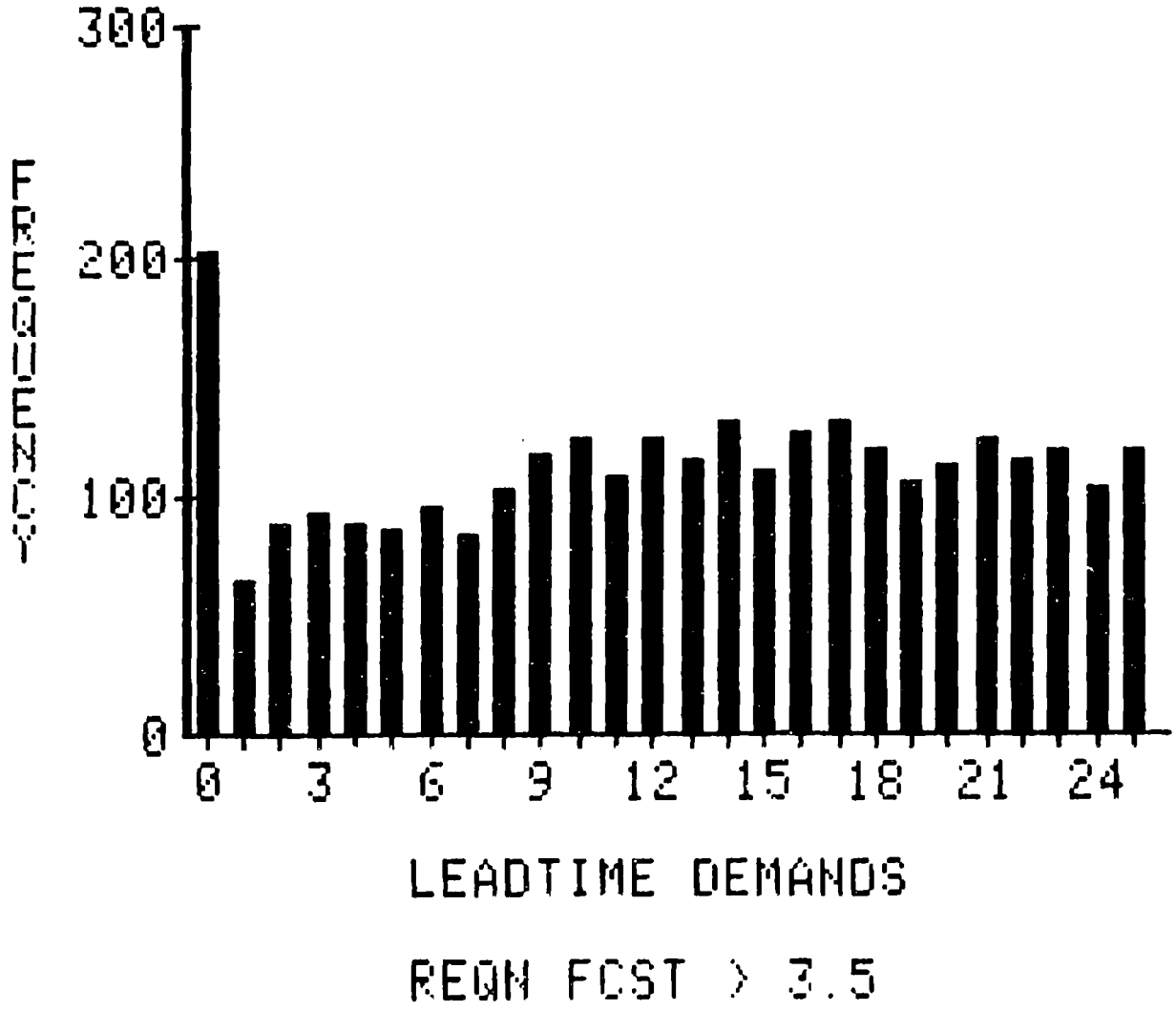
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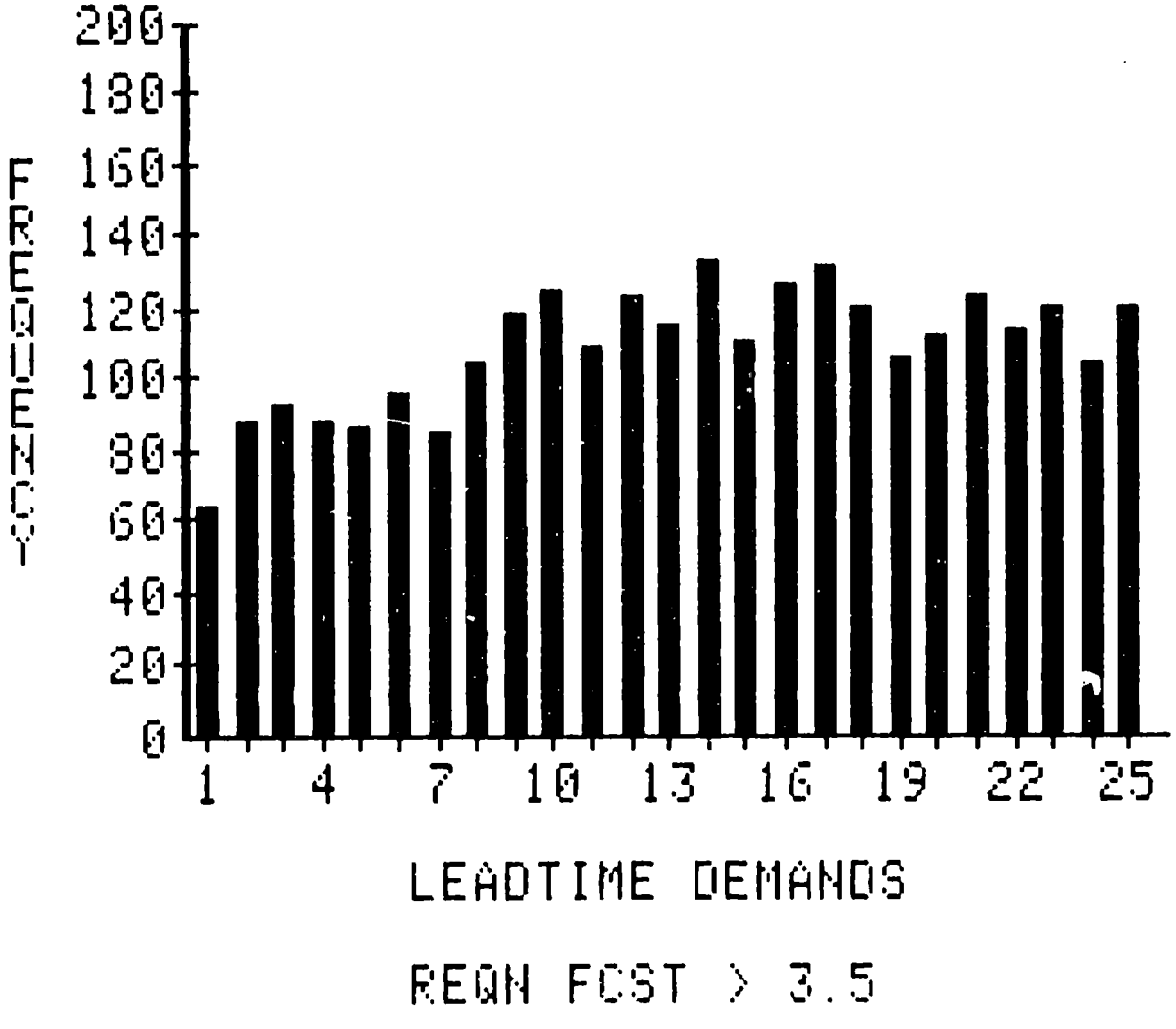
LEADTIME DEMANDS
REQN FCST > 3.5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LEADTIME DEMANDS
REQN FCST > 3.5

APPENDIX D: PERCENTAGE "$\hat{p}$" RESULTS

In calculating the Mean Square Error (MSE) measure, the reverse cumulative probability function for each distribution is used to calculate a value "x" such that the probability that a leadtime demand is less than or equal to "x" equals "p", a given percentile. The calculated values of "x" are then used to determine the percentage, $\hat{p}$, of the observed leadtime demand which are less than or equal to "x". For the 1H data used in this study, APPENDIX D contains the percentage ($\hat{p}$) of the observed leatime demands for the right hand tail percentile (p). The data was grouped based on MARK, Leadtime Demand Forecast and Requisition Forecast. In the table, EXP = Exponential, BEXP = Bernoulli-Exponential and NEGBIN = Negative Binomial.

PERCENTAGE ($\hat{p}$) OF THE OBSERVED LEADTIME DEMAND FOR THE MARK GROUPINGS

	PERCENTILES									
	50	55	60	65	70	75	80	85	90	95
MARK 0										
EXP	84.64	84.64	87.36	87.36	89.63	89.63	90.87	92.19	92.99	95.00
BEXP	74.96	74.96	74.96	74.96	74.96	74.96	87.36	92.99	95.76	97.69
NEGBIN	74.96	74.96	74.96	74.96	74.96	74.96	74.96	74.96	74.96	90.87
POISSON	87.36	87.36	87.36	87.36	89.63	89.63	89.63	90.87	92.19	92.99
GEOMETRIC	84.87	87.36	87.36	89.63	89.63	90.87	92.19	92.99	94.32	95.38
NORMAL	87.26	92.19	95.00	96.34	96.99	97.69	98.08	98.46	98.63	98.91
MARK I&III										
EXP	62.64	65.70	68.66	73.58	75.59	79.29	82.33	85.58	88.96	92.78
BEXP	49.93	54.02	60.60	65.97	70.48	80.82	84.61	88.29	91.91	95.32
NEGBIN	39.71	39.71	45.90	50.32	59.13	68.66	77.65	86.46	93.14	97.17
POISSON	73.58	73.58	73.58	75.59	75.59	77.65	77.65	79.65	80.82	82.33
GEOMETRIC	65.70	68.66	71.08	73.58	77.65	80.82	83.57	86.45	89.67	93.14
NORMAL	71.08	77.65	82.33	85.58	88.29	90.98	92.78	94.08	95.42	96.56
MARK II&IV										
EXP	76.79	79.75	82.37	84.90	87.19	89.29	91.15	92.92	94.89	96.86
BEXP	75.51	79.23	82.06	84.90	87.35	89.54	91.57	93.43	95.39	97.23
NEGBIN	78.47	80.83	83.13	85.55	87.35	89.29	90.92	92.58	94.44	96.22
POISSON	*	*	*	*	*	*	*	*	*	*
GEOMETRIC	76.79	79.75	82.49	84.90	87.25	89.32	91.19	92.94	94.89	96.87
NORMAL	84.09	86.00	87.47	88.91	90.03	91.19	92.20	93.16	94.36	95.67

* Means are too large for Poisson distribution to handle.

PERCENTAGE ($\hat{\rho}$) OF THE OBSERVED LEADTIME DEMAND FOR THE LEADTIME DEMAND FORECAST GROUPINGS

	PERCENTILES									
	50	55	60	65	70	75	80	85	90	95
$0 \leq$ LTDMD ≤ 2										
EXP	83.43	86.03	86.03	88.36	88.36	89.80	91.21	92.07	93.49	95.06
BEXP	73.17	73.17	73.17	73.17	73.17	79.02	88.36	92.92	95.69	97.59
NEGBIN	73.17	73.17	73.17	73.17	73.17	73.17	73.17	73.17	73.17	86.03
POISSON	88.36	88.36	88.36	89.90	89.90	89.90	91.21	91.21	92.07	92.92
GEOMETRIC	86.03	86.03	88.36	88.36	89.80	91.21	92.07	92.93	94.22	95.41
NORMAL	88.36	90.45	92.93	93.98	95.22	97.81	98.19	98.46	98.74	98.99
$2 <$ LTDMD ≤ 50										
EXP	64.67	68.22	71.42	74.04	77.53	80.45	83.45	86.41	89.27	92.54
BEXP	57.98	62.48	68.22	72.77	77.53	81.33	85.34	88.33	91.34	94.41
NEGBIN	29.83	34.87	43.03	49.75	57.98	68.22	78.52	86.89	91.60	96.89
POISSON	72.77	74.04	75.13	75.13	76.39	76.39	77.53	78.52	79.50	81.33
GEOMETRIC	64.67	68.22	71.42	75.13	77.53	81.33	84.12	86.89	89.65	92.92
NORMAL	72.77	78.52	83.45	86.41	88.96	91.08	92.74	94.12	95.23	96.46
LTDMD $>$ 50										
EXP	75.56	75.79	76.90	79.48	82.43	84.79	88.38	90.24	92.08	93.75
BEXP	75.80	79.59	82.99	85.74	88.25	90.75	92.96	94.71	96.51	98.20
NEGBIN	83.60	84.77	85.71	86.82	87.80	88.72	89.82	90.99	92.17	93.83
POISSON	90.08	90.08	90.08	90.08	90.82	90.82	91.71	91.71	92.30	93.07
GEOMETRIC	76.49	80.11	83.15	85.74	88.15	90.56	92.69	94.52	96.37	98.03
NORMAL	84.80	85.77	86.82	87.63	88.30	89.16	90.19	91.10	92.01	93.34

PERCENTAGE ($\hat{p}$) OF THE OBSERVED LEADTIME DEMAND FOR THE REQUISITION FORECAST GROUPINGS

	PERCENTILES									
	50	55	60	65	70	75	80	85	90	95
0 ≤ RQN FCST ≤ .25										
EXP	86.03	88.07	88.07	89.21	90.44	91.17	92.02	92.58	93.0	95.03
BEXP	75.16	75.16	75.16	75.16	75.16	75.16	88.07	93.29	95.73	97.44
NEGBIN	75.16	75.16	75.16	75.16	75.16	75.16	75.16	75.16	75.16	91.17
POISSON	89.20	89.20	89.20	90.44	90.44	90.44	91.17	91.17	92.02	92.58
GEOMETRIC	86.03	88.07	89.21	89.21	90.44	91.17	92.02	93.29	94.12	95.32
NORMAL	89.21	93.29	95.03	96.14	96.91	97.44	97.72	98.05	98.39	98.67
.25 < RQN FCST ≤ 3.5										
EXP	73.80	76.83	80.06	81.77	83.75	85.69	87.32	89.31	91.24	93.15
BEXP	70.20	73.80	78.49	81.25	84.17	86.23	88.35	90.00	92.32	93.71
NEGBIN	23.81	23.81	23.81	23.81	23.81	29.58	39.70	63.56	85.98	96.28
POISSON	81.78	82.29	82.29	82.75	83.29	83.74	84.27	84.69	85.06	85.88
GEOMETRIC	74.74	77.67	80.06	82.23	84.17	85.99	87.58	89.50	91.49	93.30
NORMAL	81.78	89.12	92.67	93.41	94.01	95.14	95.97	96.69	97.30	97.99
RQN FCST > 3.5										
EXP	78.81	82.13	84.62	86.85	89.04	90.92	92.88	94.77	96.46	97.91
BEXP	78.70	82.04	84.60	86.89	89.03	90.96	92.91	94.79	96.48	97.95
NEGBIN	85.73	86.39	86.79	87.42	88.04	88.58	89.20	89.82	90.68	91.91
POISSON	*	*	*	*	*	*	*	*	*	*
GEOMETRIC	78.93	82.19	84.66	86.83	88.99	90.89	92.83	94.73	96.42	97.86
NORMAL	86.09	86.63	87.13	87.67	88.22	88.73	89.34	89.81	90.47	91.59

* Means are too large for Poisson distribution to handle.

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13. ABSTRACT

This study examines 11 probability distributions to determine which distribution best describes demand during leadtime for 1H Cog material. Proper selection of the distribution is critical in the accurate calculation of reorder levels. Actual leadtime demand observations were calculated in the study. Histograms, a chi-square goodness-of-fit test and a mean square error measure were used to analyze the leadtime demand data.

Histograms of the data suggested the following distributions to describe leadtime demand: Exponential, Gamma, Bernoulli-Exponential, Poisson, Negative Binomial and Geometric. The chi-square goodness-of-fit test indicated that none of these distributions from the histograms fit the computed leadtime demand data across the entire range of the distribution. However, a relative test of the right hand tail of the distributions, which are most critical in determining reorder levels, indicated that the Bernoulli-Exponential provided the best relative fit for 11 Cog items.

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