Teorema I

$$\boxed{\mathbf{V}^{(1)}_{\mathrm{ext}}(\mathbf{r}) \rightarrow \mathbf{H}^{(1)} \rightarrow \mathbf{\Psi}^{(1)}}$$

$$\boxed{\mathbf{V}^{(2)}_{\mathrm{ext}}(\mathbf{r}) \rightarrow \mathbf{H}^{(2)} \rightarrow \mathbf{\Psi}^{(2)}}$$

$$\boxed{\rho_0(\mathbf{r})}$$

$$\langle \mathbf{\Psi}^{(1)} | \mathbf{H}^{(1)} | \mathbf{\Psi}^{(1)} \rangle < \langle \mathbf{\Psi}^{(2)} | \mathbf{H}^{(1)} | \mathbf{\Psi}^{(2)} \rangle$$

$$\langle \mathbf{\Psi}^{(1)} | \mathbf{H}^{(1)} | \mathbf{\Psi}^{(1)} \rangle = \mathbf{E}^{(1)}$$

$$\begin{aligned} \langle \mathbf{\Psi}^{(2)} | \mathbf{H}^{(1)} | \mathbf{\Psi}^{(2)} \rangle &= \langle \mathbf{\Psi}^{(2)} | \mathbf{H}^{(2)} | \mathbf{\Psi}^{(2)} \rangle + \langle \mathbf{\Psi}^{(2)} | \mathbf{H}^{(1)} - \mathbf{H}^{(2)} | \mathbf{\Psi}^{(2)} \rangle \\ &= \mathbf{E}^{(2)} + \int \mathbf{d}^3\mathbf{r}[\mathbf{V}^{(1)}_{\mathrm{ext}}(\mathbf{r}) - \mathbf{V}^{(2)}_{\mathrm{ext}}(\mathbf{r})]\rho_0(\mathbf{r}) \end{aligned}$$

$$\mathbf{E}^{(1)} < \mathbf{E}^{(2)} + \int \mathbf{d}^3\mathbf{r}[\mathbf{V}^{(1)}_{\mathrm{ext}}(\mathbf{r}) - \mathbf{V}^{(2)}_{\mathrm{ext}}(\mathbf{r})]\rho_0(\mathbf{r})$$

$$\langle \mathbf{\Psi}^{(2)} | \mathbf{H}^{(2)} | \mathbf{\Psi}^{(2)} \rangle < \langle \mathbf{\Psi}^{(1)} | \mathbf{H}^{(2)} | \mathbf{\Psi}^{(1)} \rangle$$

$$\langle \mathbf{\Psi}^{(2)} | \mathbf{H}^{(2)} | \mathbf{\Psi}^{(2)} \rangle = \mathbf{E}^{(2)}$$

$$\begin{aligned} \langle \mathbf{\Psi}^{(1)} | \mathbf{H}^{(2)} | \mathbf{\Psi}^{(1)} \rangle &= \langle \mathbf{\Psi}^{(1)} | \mathbf{H}^{(1)} | \mathbf{\Psi}^{(1)} \rangle + \langle \mathbf{\Psi}^{(1)} | \mathbf{H}^{(2)} - \mathbf{H}^{(1)} | \mathbf{\Psi}^{(1)} \rangle \\ &= \mathbf{E}^{(1)} + \int \mathbf{d}^3\mathbf{r}[\mathbf{V}^{(2)}_{\mathrm{ext}}(\mathbf{r}) - \mathbf{V}^{(1)}_{\mathrm{ext}}(\mathbf{r})]\rho_0(\mathbf{r}) \end{aligned}$$

$$\mathbf{E}^{(2)} < \mathbf{E}^{(1)} - \int \mathbf{d}^3\mathbf{r}[\mathbf{V}^{(1)}_{\mathrm{ext}}(\mathbf{r}) - \mathbf{V}^{(2)}_{\mathrm{ext}}(\mathbf{r})]\rho_0(\mathbf{r})$$

$$\mathbf{E}^{(1)} + \mathbf{E}^{(2)} < \mathbf{E}^{(2)} + \mathbf{E}^{(1)}$$

Teorema II

$$\mathbf{V_{ext}^{(1)}(r)} \rightarrow \mathbf{H^{(1)}} \rightarrow \mathbf{\Psi^{(1)}} \rightarrow \rho_{\mathbf{0}}^{(\mathbf{1})}$$

La energía es : $\langle \mathbf{\Psi^{(1)}} | \mathbf{H^{(1)}} | \mathbf{\Psi^{(1)}} \rangle = \mathbf{E^{(1)}}$

Considerando una densidad de carga distinta $\rho^{(\mathbf{2})}$ y su función de onda correspondiente $\mathbf{\Psi^{(2)}}$

Luego:

$$\mathbf{E^{(1)}} = \langle \mathbf{\Psi^{(1)}} | \mathbf{H^{(1)}} | \mathbf{\Psi^{(1)}} \rangle < \langle \mathbf{\Psi^{(2)}} | \mathbf{H^{(1)}} | \mathbf{\Psi^{(2)}} \rangle$$