A Generic Solver for Unconstrained Control Problems with Integral Functional Objectives

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Unconstrained Control Problem (UCP)

- Given an objective $\mathcal{J}$, we aim to find controllers $u_m : \mathbb{R} \to \mathbb{R}$ to minimize

$$\min \ \mathcal{J}[U] = \int L_m(u_m(y_m), y_m) \ dy_m + \mathcal{R}_m[U_{-m}]$$

where $U = \{u_0(y_0), u_1(y_1), \ldots, u_{M-1}(y_{M-1})\}$ is the set of controllers.

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- Several control problems can be written in this form, including the Witsenhausen's counterexample.

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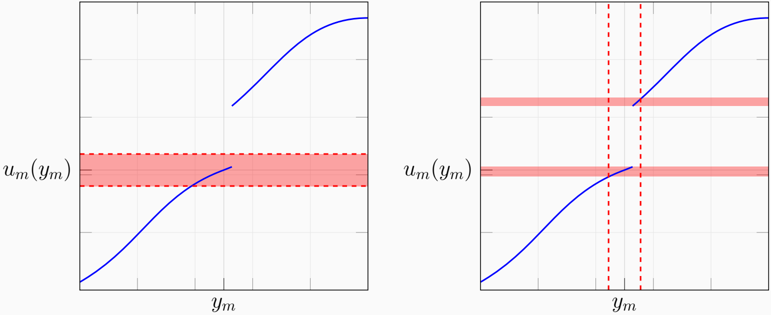
- Faster computation
  $\Rightarrow$ GPU-accelerated parallel computation.

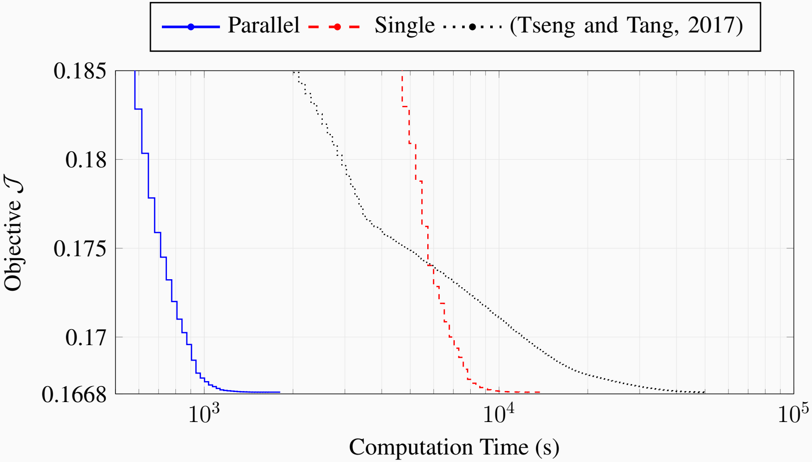


(a) Local Update

(b) Partial Exhaustion

Figure 1: Candidate sets, marked by shaded areas.

Convergence Time



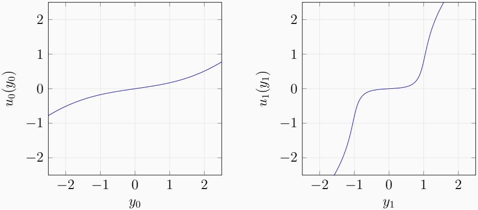


Figure 2: Zero-delay source-channel coding.

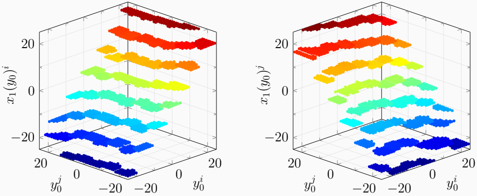


Figure 3: 2-dimensional Witsenhausen's counterexample.