

$$(x_0, f_0(x)) \quad (x_1, f_1(x)), \quad (x_2, f_2(x)) \quad (x_3, f_3(x))$$

Cardinal polynomial

$$l_0 = \frac{(x - x_1)(x - x_2)(x - x_3)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)}$$

$$l_1 = \frac{(x - x_0)(x - x_2)(x - x_3)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)}$$

$$\dots = \frac{(x - x_0)(x - x_1)(x - x_3)}{(x_n - x_0)(x_n - x_1)(x_n - x_3)}$$

Now the lagrange polynomial for this -

$$f(x) = l_0(x) f_0(x) + l_1(x) f_1(x) + l_2(x) f_2(x) + l_3 f_3(x)$$

Q. Find cardinal polynomial and lagrange polynomial for -

$$\text{Soln: } (y_3, 2) \quad (y_4, -1) \quad (1, 2)$$

+ (cardinal polynomial)

$$l_0(x) = \frac{(x_1 - x_3)(x - x_1)}{(y_3 - y_4)(y_3 - 1)} = \frac{\frac{4n-1}{4} \times (n-1)}{\frac{1}{4} \times \frac{-2}{3}}$$

$$= -\frac{1}{4} (n-1)(n-1) \times 18$$

$$= \frac{1}{2} (n-1)(n-1)$$

$$\begin{aligned}
 I_1(n) &= \frac{(n-1/3)(n-1)}{(1/4-1/3)(1/4-1)} \\
 &= \frac{(3n-1)}{3} n(n-1) x \cancel{\frac{1}{(1-\frac{1}{3})(1-\frac{1}{4})}} \cancel{x} \cancel{\frac{4}{(1-\frac{1}{4})(1-\frac{1}{3})}} \\
 &= \frac{16}{3} (3n-1)(n-1) x^2
 \end{aligned}$$

$$l_2(n) = \frac{(n - 1)_3}{(n - 1)_9}$$

$$= 1 + \left(1 - \frac{1}{3}\right) r \left(1 + \frac{1}{2} \frac{1}{9}\right) \text{ open括号} \quad \text{and then multiply}$$

$$\text{Ansatz: } x = \frac{3m+1}{3} = x^{\frac{1}{3}} \cdot \frac{3m+1}{1} + x^{\frac{2}{3}} \cdot \frac{1}{2} + x^{\frac{4}{3}} \cdot \frac{1}{3}$$

$$S_{\text{loop}} = \frac{1}{6} (3n-1) (4n-1) \text{ with } n = 10$$

We know.

$$f(n) = 1(n) f_0(n) + 1(n) f_1(n) + 2(n) f_2(n)$$

$$= -2 \times \frac{9}{2} (4n-1)(n-1) + 1 \times \frac{16}{3} (3n-1)(n-1) +$$

$$\left( \frac{1}{6} \times \frac{1}{2} \right) (3m-1)(4m-1)$$

-Ang

$$Q. (1, 0) \Delta (9, -1.386294), (6, 1.181759)$$

$$\text{Soln: } [e^x] = [e^x]^T, [e^x + e^{-x}]$$

$$l_0(n) = \frac{(n-4)(n-6)}{(n-1)(n-2)(n-3)} = \frac{1}{15} \times (n^2 - 10n + 24)$$

$$l_1(n) = \frac{(n-5)(n-6)}{(n-1)(n-2)} = \frac{1}{6} \times (n^2 - 7n + 6)$$

$$[e^x] = [e^x]^T, [e^x + e^{-x}]$$

$$l_2(n) = \frac{(n-1)(n-9)}{(n-1)(n-2)} = \frac{1}{20} \times (n^2 - 5n + 4)$$

We know

$$f(x) = f_0(n) l_0(n) + f_1(n) l_1(n) + f_2(n) l_2(n)$$

$$= 0 + 1.386294 \times \frac{1}{15} (n^2 - 7n + 6) + 1.181759 \times \frac{1}{6} (n^2 - 5n + 4)$$

$$\therefore f(0.693) = f(0.693) = -0.231049 \times 1.628294 + 0.08958795 \times 1.015249$$

$$= \underline{\underline{-0.2854822856}}$$

" Newton's Divided Difference  $f(x)$

$$f[x_0, x_1] = \frac{f[x_1] - f[x_0]}{(x_1 - x_0)}$$

$$(x + x_1 - x_0) \times \cancel{x} = \cancel{(x - x_0)(x_1 - x_0)} : (x)$$

$$f[x_0, x_1, x_2] = \frac{f[x_2, x_1] - f[x_0, x_1]}{(x_2 - x_1)(x_1 - x_0)}$$

$$(x + x_2 - x_1) \times \cancel{x} = \cancel{(x - x_1)(x_2 - x_1)} : (x)$$

$$f[x_0, x_1, x_2, x_3] = \frac{f[x_3, x_2, x_1] - f[x_0, x_1, x_2]}{(x_3 - x_2)(x_2 - x_1)(x_1 - x_0)}$$

$$P(n) = a_0 + a_1(n - x_0) + a_2(n - x_0)(n - x_1) + \dots$$

$$a_0 = f(x_0)$$

$$a_1 = f[x_0, x_1]$$

$$+ a_2 = f[x_0, x_1, x_2]$$

$$+ a_3 = f[x_0, x_1, x_2, x_3]$$

$$\frac{dP}{dx} = a_1 + 2a_2 + 3a_3$$

Q.

$x_i$	$i$	$3/2$	$0$	$2$
$f(x_i)$	3	$3/4$	3	$15/3$

Q.

$x_i$	$f(x_i)$	$f[x_i, x_{i+1}]$	$f[x_i, x_{i+1}, x_{i+2}]$	$f[x_i, x_{i+1}, x_{i+2}, x_{i+3}]$
1	3	0.5	0	1
$3/2$	$3/4$	0.1667	0.3333	2
0	3	-0.6667	-1.6668	3
2	$15/3$			4

$$P(n) = a_0 + a_1(n - 1) + a_2(n - 1)(n - 3/2) + a_3(n - 1)(n - 3/2)(n - 0)$$

$$(1 - n)(1 - n) \neq (1 - n)^2 a_3(n - 1)(n - 0)$$

$$= 3 + 0.5(n - 1) + 0.3333(n - 1)(n - 3/2)$$

$$\neq 2(n - 1)(n - 3/2)(n - 0)$$

Ans

Q.

$n$	1	2	3
$f(n)$	0	1.386294	1.391759

D

Soln:

$f[n_i, n_{i+1}, n_{i+2}]$	$f[n_i]$	$f[n_i, n_{i+1}]$	$f[n_i, n_{i+1}, n_{i+2}]$
1	0	0.462	?
4	1.386294	0.203	-0.0518
6	1.391759	0.222	?

Now,

$$P(X) = a_0 + a_1(n - n_0) + a_2(n - n_0)(n - n_1) \\ (n - n_0)(n - n_1) = (0 + 0.462(n-1) + 0.0518(n-1)(n-4))$$

$$(e^{\lambda} - \lambda)(1 - \lambda) e^{(\lambda - \lambda)^2} + (1 - \lambda) e^{\lambda} + \dots$$

$$(e^{\lambda} - \lambda)(e^{\lambda} - \lambda)^2 + \dots$$

Ans

Q. Find Newton's forward difference interpolation.

$x$	0.1	0.2	0.3	0.4	0.5
$y = f(x)$	1.4	1.56	1.76	2	2.28

Sol:

$x$	$y$	$\Delta y$	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0.1	1.4	0.16			
0.2	1.56	0.2	0.04	0	0
0.3	1.76	0.24	0.09	0	0
0.4	2	0.28	0.09	0	
0.5	2.28				

$$y(n) = y_0 + P \Delta y_0 + \frac{P(P-1)}{2!} \Delta^2 y_0 + 0 + 0$$

$$\begin{aligned} P &= \frac{n - n_0}{h} \\ &= \frac{n - 0.1}{0.1} \\ &= 10n - 1 \end{aligned}$$

$$\left| \begin{array}{l} n_0 = 0.1 \\ y_0 = 1.4 \\ \Delta y_0 = 0.16 \\ \Delta^2 y_0 = 0.04 \end{array} \right.$$

$$y(n) = 1.4 + (10n - 1) \times 0.16 + \frac{1}{2} (10n - 1) (10n - 2) 0.07$$

8.0	1.0	8.0	8.0	1.0	5
28.2	8	28.1	28.1	28.1	(0.07)

Ans

Am

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$$0 \neq 0 + \frac{(\beta - 1)g}{12} + \beta g^2 + \alpha f = (\beta)g$$

1.0 - 0.05

$$P \cdot j = \omega$$

$$M \cdot 0 = 0$$

P.O. - 8A

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$$\text{f} \circ \text{g} \rightarrow \text{h}$$

4 - 3601

5(a) Backward interpolation:

$p = n$	40	50	60	70	80	90
$t^p, y$	189	209	226	250	276	304

Soln:

$x$	$y$	$\nabla y$	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$
40	189					
50	209	20				
60	226	2.2	0.2			
70	250	2.4	0.2	0.08	0.01	0.001
80	276	2.6	0.2	0.08	0.01	0.001
90	304	2.8	0.2	0.08	0.01	0.001

No. of

$$y(x) = y_n + p \nabla y_n + \frac{p(p+1)}{2!} \nabla^2 y_n$$

$$\therefore p = \frac{x - x_n}{h} = \frac{n - 90}{10} = 9$$

$$\therefore y(x) = 304 + \frac{2.8}{10} (n - 90) + \frac{1}{200} \times 2(n - 90)(n - 89)$$

$$y_9 = 304$$

$$\nabla y_n = 2.8$$

$$\nabla^2 y_n = 2$$

$$y(x) = 309 + 2.8(x - 90) + 0.01(x - 90)(x - 89)$$

$$\therefore y(89) = 309 + 2.8(89 - 90) + 0.01(89 - 90)(89 - 89)$$

$$= \boxed{290.2} . 287.5$$

Ans

11.02

### # Forward interpolation

$x$	$y$	$\Delta y$	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
40	184	20	2	0	0	0	0
50	209	22	2	0	0	0	0
60	226	24	2	0	8	0	0
70	250	26	2				
80	276	28					
90	304						

$$PDE = \Delta^6 y$$

$$P = \frac{x - x_0}{h} = \frac{90 - 40}{10} = 5$$

$$y = y_0 + \frac{(x-x_0)}{10} y_1 + \frac{(x-x_0)(x-90)}{2!} y_2 + \frac{(x-x_0)(x-90)(x-80)}{3!} y_3 + \frac{(x-x_0)(x-90)(x-80)(x-70)}{4!} y_4 + \frac{(x-x_0)(x-90)(x-80)(x-70)(x-60)}{5!} y_5 + \frac{(x-x_0)(x-90)(x-80)(x-70)(x-60)(x-50)}{6!} y_6$$

19 अक्टूबर

$$\text{प्रयोग } 1 \text{ लोगिसिक्युट } ① \\ P y(x) = y_0 + P \Delta y_0 + \frac{P(P-1)}{2!} \Delta^2 y_0$$

$$= 184 + 2(x-40) + \frac{1}{2 \cdot 0.01} (x-40)(x-41) \times 0.01 \\ = 184 + 2(x-40) + 0.01(x-40)(x-41)$$

$\therefore P = \frac{P-d}{n} = 0.01 \text{ लोगिसिक्युट } ②$

$$\therefore y(84) = 184 + 2(84-40) + 0.01(84-40)(84-41) \\ = 282.92$$

3	2	4	3	1	0	3
180.0	2880.0	8820.0	1.0	3.0	2.0	0.0

लोगिसिक्युट ①

$$\left[ (v^k + v^l + v^m + v^n + v^o) s + (v^k + v^l) \right] \frac{s}{s-1} = \frac{mb^3}{s-1}$$

80 DP. 1

लोगिसिक्युट ②

$$\left[ (v^k + v^l) s + (v^k + v^l + v^m) s^2 + (v^k + v^l) \right] \frac{s}{s-1} = \frac{mb^3}{s-1}$$

33 DP. 1

## Chapter 21

$$Q. \int_0^6 \frac{dx}{1+x^2}$$

- ① Trapezoidal rule  $N=1$
- ② Simpson's  $\frac{1}{3}$  rule  $N=2$
- ③ Simpson  $\frac{3}{8}$  rule  $N=3$

Soln:

$$(y_0 - y_1)(x_1 - x_0) + (y_1 - y_2)(x_2 - x_1) + \dots + (y_{n-1} - y_n)(x_n - x_{n-1})$$

$$f(x) = \frac{1}{1+x^2} \quad \text{width } h = \frac{b-a}{n} = 1$$

Divide the interval  $[0, 6]$  into 6 parts

x	0	1	2	3	4	5	6
y = f(x)	1	0.5	0.2	0.1	0.0588	0.0385	0.022

### ① Trapezoidal

$$f(x) =$$

$$\int_0^6 \frac{dx}{1+x^2} = \frac{h}{2} \left[ (y_0 + y_6) + 2(y_1 + y_2 + y_3 + y_4 + y_5) \right]$$

$$= 1.4108$$

### ② Simpson's $\frac{1}{3}$

$$\int_0^6 \frac{dx}{1+x^2} = \frac{h}{3} \left[ (y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4) \right]$$

$$= 1.366$$

(iii) Simpson's  $\frac{1}{3}$  rule

$$\int_0^6 \frac{dx}{1+x^2} = \frac{3h}{8} [(y_0 + y_6) + 3(y_1 + y_2 + y_4 + y_5) + 2(y_3)]$$

$$= 1.357.$$

Ans to the

Q(1)  $\int_0^2 (y^2 - (y-2)) dy$

① Trapezoidal

Here,

$$x = f(y) = \{y^2 - (y-2)\} \quad w = \frac{2-0}{2} = 1$$

Divide the interval  $[0, 2]$  into 2 part

y	0	1	2
x	2	2	4

$$\begin{aligned} \therefore \int_0^2 (y^2 - (y-2)) dy &= \frac{1}{2} [x_0 + x_1] + 2 \cdot x_2 \\ &= \frac{1}{2} [2 + 4 + 2 \times 2] \\ &= 5 \end{aligned}$$

⑪ Multiple Trapezoidal Form.  $n=4$

When  $h = \frac{2-0}{4} = 0.5$ ,  $x = f(y) = y^{\sqrt{y+2}}$

$y$	0	0.5	1	1.5	2
$x$	2	1.75	2	2.75	4

$$\therefore \int_0^2 (y^{\sqrt{y+2}}) dy = \frac{h}{2} \left[ (x_0 + x_4) + 2(x_1 + x_2 + x_3) \right]$$

$$= \frac{0.5}{2} [2 + 4 + 2(1.75 + 2 + 2.75)]$$

$$= 4.75$$

+ 50% error in  $[0, 2]$  because of subdividing

$$\text{True error} = \left| \frac{5 - 4.75}{5} \right| \times 100\%$$

2	1.75	2	2.75
7.5%			

$$\left[ 2^{\sqrt{2}} + (1.75 + 2.75) \right] \frac{1}{2} = \left[ 2^{\sqrt{2}} + (1.75 + 2.75) \right] \frac{1}{2}$$

$$\left[ 2^{\sqrt{2}} + 4.5 \right] \frac{1}{2} =$$

2.5

(iii) Simpson's  $\frac{1}{3}$  for  $n=4$

$$h = 0.5$$

$y$	0	0.5	1	1.5	2
$x$	2	1.75	2	2.25	2

$$\text{LHS: } Ax=B$$

$$\frac{f''(x_i)}{2!} (x_{i+1} - x_i) + \dots - x_i)^n + R_n$$

$$\therefore \int_0^2 (y - y+2) dy$$

$$= \frac{h}{3} \left[ (n_0 + n_4) + 4(n_1 + n_3) + 2n_2 \right]$$

$$= \frac{0.5}{3} \left[ 2 + 4 + 4(1.75 + 2.25) + 2 \times 2 \right]$$

$$= \underline{4.667}$$

$$\text{True error} = \left| \frac{5 - 4.667}{5} \right| \times 100\%$$

$$= \underline{6.66\%}$$

Now the Equation is —

$$y = 1.6 + 3.4x^2 + 2x^3$$

5(b)  $(0, 1), (1, 2), (2, 4), (3, 10)$  + O.P. → 32

For Cubic:  $y = a + bx + cx^2 + dx^3$  → ①

$$y = a + bx + cx^2 + dx^3 \rightarrow ①$$

And the normal eqn:

$$\sum y = an + b\sum x + c\sum x^2 + d\sum x^3$$

$$\sum xy = a\sum x + b\sum x^2 + c\sum x^3 + d\sum x^4$$

$$\sum x^2 y = a\sum x^2 + b\sum x^3 + c\sum x^4 + d\sum x^5$$

$$\sum x^3 y = a\sum x^3 + b\sum x^4 + c\sum x^5 + d\sum x^6$$

$x$	$y$	$xy$	$x^2$	$x^3$	$x^4$	$x^5$	$x^6$
0	1	0	0	0	0	0	0
1	2	2	1	2	1	1	1
2	4	8	4	8	16	32	64
3	10	30	9	90	27	243	729
6	14	34	14	96	36	280	794

~~(0, 1) (1, 2) (2, 1) (3, 10)~~ not in ascending order

We know that

The ~~Quadratic~~ equation  $y = a_0 + a_1x + a_2x^2$

x	y	$xy$	$x^2$	$x^3$	$x^4$
0	1	0	0	0	0
1	2	2	1	1	1
2	1	2	4	8	16
3	10	30	9	27	81
6	14	34	14	96	36
					98

$$\sum y = a_0 n + a_1 \sum x + a_2 \sum x^2$$

$$\sum xy = a_0 \sum x + a_1 \sum x^2 + a_2 \sum x^3$$

$$\sum x^2y = a_0 \sum x^2 + a_1 \sum x^3 + a_2 \sum x^4$$

From The Table

$$14 = 4a_0 + 14a_1 + 98a_2 \quad \text{--- (I)}$$

$$34 = 6a_0 + 14a_1 + 36a_2 \quad \text{--- (II)}$$

$$96 = 14a_0 + 36a_1 + 98a_2 \quad \text{--- (III)}$$

Solving the three equations we get —

$$a_0 = 1.6 ; a_1 = -3.9 ; a_2 = 2$$

## Newton Forward Interpolation:

### Polynomial Regression

Types

$$\text{linear} \rightarrow y = an + b$$

$$\text{Quadratic} \rightarrow y = an^2 + bn + c$$

$$\text{Cubic} \rightarrow y = an^3 + bn^2 + cn + d$$

$$\text{Quartic} \rightarrow y = an^4 + bn^3 + cn^2 + dn + e$$

$$\text{Quintic} \rightarrow y = an^5 + bn^4 + cn^3 + dn^2 + en + f$$

$$a = x' B$$

$$X = \begin{bmatrix} n & \sum x & \sum x^2 \\ \sum x & \sum x^2 & \sum x^3 \\ \sum x^2 & \sum x^3 & \sum x^4 \end{bmatrix}$$

$$B = \begin{bmatrix} \sum y \\ \sum xy \\ \sum x^2y \end{bmatrix}$$

$$\begin{aligned} \text{(i)} & \rightarrow 1000 + 1000 + 100 = 1200 \\ \text{(ii)} & \rightarrow 1000 + 1000 + 100 = 1200 \\ \text{(iii)} & \rightarrow 1000 + 1000 + 100 = 1200 \\ & \rightarrow \text{The three equations coincide with each other.} \end{aligned}$$

$$\therefore f(5) = 5^3 + 2 \times 5 - 9$$

$$= 131$$

(iii) For real root  $(1, -1)$  and  $(2, 8)$  is closed.

$$\begin{array}{ccccccc} x & f(x) & f'(x) \\ 1 & 1 & 1 \\ 2 & 8 & 9 \end{array}$$

$$\therefore f(x) = b - 1 + 9(n-1)$$

$$\therefore g(n) = 131 + b(n+1)(n-1)(n-2)(n-3)(n-4)$$

$$\therefore g_{n-10} = 0$$

$$\therefore n = 10$$

$$\therefore g(n) = f(n) + b(n+1)(n-1)(n-2)(n-3)(n-4)$$

$$\therefore g(5) = 131 + b \times 6 \times 4 \times 3 \times 2 \times 1$$

$$\Rightarrow g(5) = 131 + 144b$$

$$\Rightarrow b = \frac{722}{144} = \frac{18}{36} = \frac{1}{2}$$

$$\therefore g(n) = n^3 + 2n^2 - 9 + \frac{1}{2}(n+1)(n-1)(n-2)(n-3)(n-4)$$

$$= n^3 + 2n^2 - 9n + 2n^2 - 8n + 2 + 10n^2 - 30n + 24 - 24$$

$$= n^3 + 10n^2 - 38n + 24$$

## Newton Divided Difference Interpolation:

Q.  $n: -1 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5$

$f(x): -7 \quad -1 \quad 8 \quad 29 \quad 68$

(i) Construct a divided difference Table.

(ii) Find the polynomial of least degree that incorporates the values in the Table and find  $P_5$ .

(iii) Find by linear interpolation of real root.

(iv) Find the polynomial  $g(n)$  that takes the value of the above Table and  $g(5) = 203$ .

Soln:

①

$$(1-x)(x-1)(x-2)(x-3)(x-4)(x-5) = 0 \quad (i)$$

$$1 \quad -14846432 \text{ and } -1484 = (3)P_5$$

$$2 \quad 8 \quad 9 \quad \text{dP}_5 \text{ at } x=5 = 508$$

$$3 \quad 29 \quad 21 \quad 68 \quad 1 \quad -$$

$$4 \quad 68 \quad 39 \quad 68 \quad 1 \quad 0$$

$$(ii) \text{ Polynomial: } -7 + 3(n+1) + 2(n+1)(n-1) + 1(n+1)(n-1)(n-2) = (5)P_5$$

$$= -7 + 3n + 3 + 2n^2 - 2 + n^3 - 2n^2 - n + 2$$

$$= n^3 + 2n^2 - n - 4$$

## Least Squares Regression

The least squares method is a form of mathematical regression analysis used to determine the line of best fit for a set of data, providing a visual demonstration of the relationship between the data points. Each point of data represent the relationship between a known independent variable and an unknown dependent variable.

1. It is a statistical procedure to find the best fit for a set of data points.

2. It works by minimizing the sum of the offsets or residuals of points from the plotted curve.

3. It is used to predict the behavior of dependent variable.

### Advantage:

- ① Easy to Apply and understand
- ② Highlight relationship between two variable
- ③ Can be used to make predictions about future performance.

## Disadvantages:

- ① Only highlight relationship between all data information to next point.
- ② Two variables have direct relationship.
- ③ Doesn't account for outliers.
- ④ May be skewed if data is not evenly distributed.

## Curve Fitting: used to predict values from

Two main methods:

### 1. Least Square Regression:

feed all kind of distributional data to fit L.S.

- Function is best fit to Data.

- Does not necessarily pass through point.

- Used for experimental data.

- Can develop model for analysis/design.

### 2. Interpolation of how it fits.

- follows for bridge

## Interpolation

Interpolation has higher at point ①

lower and required additional function ②

function considering some of known and new ③

- differentiation and ④