Ch-18

$$(x_0, f_0(x)) \quad (x_1, f_1(x)) \quad (x_2, f_2(x)) \quad (x_3, f_3(x))$$

Cardinal polinomial

$$l_0 = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}$$

$$l_1 = \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}$$

. . . . —

. . . . . =

Now the lagrange polynomial for this —

$$f(x) = l_0(x) f_0(x) + l_1(x) f_1(x) + l_2(x) f_2(x) + l_3 f_2(x)$$

Q. Find cardinal polinomial and lagrange polinomial for —

$$(1/3, 2) \quad (1/4, -1) \quad (1, 7)$$

soln:

cardinal polynomial

$$l_0(x) = \frac{(x-1/4)(x-1)}{(1/3-1/4)(1/3-1)} = \frac{\frac{4x-1}{4} \times (x-1)}{\frac{1}{\cancel{12}_6} \times \frac{-\cancel{2}}{3}}$$

$$= -\frac{1}{4}(4x-1)(x-1) \times 18$$

$$= -\tfrac{9}{2}(4x-1)(x-1)$$

$$l_1(x) = \frac{(x-1/3)(x-1)}{(1/4-1/3)(1/4-1)}$$

$$= \frac{(3x-1)}{3} \times (x-1) \times \frac{\cancel{12}\,4}{-1} \times \frac{4}{-\cancel{3}}$$

$$= \tfrac{16}{3}(3x-1)(x-1)$$

$$l_2(x) = \frac{(x-1/3)(x-1/4)}{(1-1/3)(1-1/4)}$$

$$= \frac{3x-1}{3} \times \frac{4x-1}{\cancel{4}} \times \frac{\cancel{3}}{2} \times \frac{\cancel{4}}{\cancel{3}}$$

$$= \tfrac{1}{6}(3x-1)(4x-1)$$

We Know.

$$f(x) = l_0(x) f_0(x) + l_1(x) f(x) + l_2(x) f_2(x)$$

$$= -2 \times \tfrac{9}{2}(4x-1)(x-1) - 1 \times \tfrac{16}{3}(3x-1)(x-1) + 9 \times \tfrac{1}{6}(3x-1)(4x-1).$$

Ans.

Q. (1, 0), (4, 1.386294), (6, 1.791759)

Soln:

$$l_0(x) = \frac{(x-4)(x-6)}{(1-4)(1-6)} = \frac{1}{15} \times (x^2 - 10x + 24)$$

$$l_1(x) = \frac{(x-1)(x-6)}{(4-1)(4-6)} = -\frac{1}{6} \times (x^2 - 7x + 6)$$

$$l_2(x) = \frac{(x-1)(x-4)}{(6-1)(6-2)} = \frac{1}{20} \times (x^2 - 5x + 4)$$

We Know

$$f(x) = f_0(x)\, l_0(x) + f_1(x)\, l_1(x) + f_2(x)\, l_2(x)$$

$$= 0 + 1.386294 \times \frac{1}{6}(x^2 - 7x + 6) + 1.791759 \times \frac{1}{20}(x^2 - 5x + 4)$$

$$\therefore f(\ln 2) = f(0.693) = -0.231049 \times 1.692249 + 0.0895875 \times 1.015249$$

$$= -0.2854822756$$

// Newton's Divided Difference (Δ)

$$f[x_0, x_1] = \frac{f[x_1] - f[x_0]}{x_1 - x_0}$$

$$f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0}$$

$$f[x_0, x_1, x_2, x_3] = \frac{f[x_1, x_2, x_3] - f[x_0, x_1, x_2]}{x_3 - x_0}$$

$$P(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + \cdots$$

$$a_0 = f(x_0)$$

$$a_1 = f[x_0, x_1]$$

$$a_2 = f[x_0, x_1, x_2]$$

Q.

x	1	3/2	0	2
$f(x)$	3	13/4	3	5/3

x_i	$f(x_i)$	$f[x_i, x_{i+1}]$	$f[x_i, x_{i+1}, x_{i+2}]$	$f[x_i, x_{i+1}, x_{i+2}, x_{i+3}]$
1	3			
		0.5		
3/2	13/4		0.3333	
		0.1667		2
0	3		1.6668	
		-0.6667		
2	5/3			

$$P(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + a_3(x - x_0)(x - x_1)(x - x_2)$$

$$= 3 + 0.5(x - 1) + 0.3333(x - 1)(x - 3/2) + 2(x - 1)(x - 3/2)(x - 0)$$

Ans

Q.

x	1	4	6
$f(x)$	0	1.386294	1.791759

Soln :

x	$f(x)$	$f[x_i, x_{i+1}]$	$f[x_i, x_{i+1}, x_{i+2}]$
1	0		
		0.462	
4	1.386294		-0.0518
		0.203	
6	1.791759		

Now,

$$P(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1)$$

$$= 0 + 0.462(x-1) - 0.0518(x-1)(x-4)$$

Q: Find Newton's forward difference interpolation.

x	0.1	0.2	0.3	0.4	0.5
$y = f(x)$	1.4	1.56	1.76	2	2.28

Soln:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0.1	1.4				
		0.16			
0.2	1.56		0.04		
		0.2		0	
0.3	1.76		0.04		0
		0.24		0	
0.4	2		0.04		
		0.28			
0.5	2.28				

So,

$$y(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + 0 + 0$$

$$p = \frac{x - x_0}{h}$$

$$= \frac{x - 0.1}{0.1}$$

$$= 10x - 1$$

$x_0 = 0.1$

$y_0 = 1.4$

$\Delta y_0 = 0.16$

$\Delta^2 y_0 = 0.04$

$$\therefore\ y(x) = 1.4 + (10x - 1) \times 0.16 + \frac{1}{2}(10x - 1)(10x - 2)\,0.04$$

Ans

5(a) Bckward interpolation:

$p = x$	40	50	60	70	80	90
$t^\circ = y$	184	204	226	250	276	304

Sol^n:

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$
40	189					
50	204	20				
60	226	22	2			
70	250	24	2			
80	276	26	2			
90	304	28	2			

Now,

$$y(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y$$

$$\therefore p = \frac{x - x_n}{h} = \frac{x - 90}{10}$$

$$\therefore y(x) = 304 + \frac{28}{10}(x-90) + \frac{1}{200} \times 2 (x-90)(x-80)$$

$x_0 = 40$

$y_n = 304$

$\nabla y_n = 28$

$\nabla^2 y_n = 2$

$$y(x) = 304 + 2.8(x-90) + 0.01(x-90)(x-80)$$

$$\therefore y(84) = 304 + 2.8(84-90) + 0.01(84-90)(84-80)$$

$$= \cancel{\boxed{290.2}} \quad 287.5$$

Ans

Forward interpolation

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
40	184	20	2	0	0	0
50	204	22	2	0	0	
60	226	24	2	0		
70	250	26	2			
80	276	28				
90	304					

$$P = \frac{x - x_0}{h} = \frac{x - 40}{10}$$

$$y(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0$$

$$= 184 + 2(x-40) + \frac{1}{200}(x-40)(x-41)\times 2$$

$$= 184 + 2(x-40) + 0.01(x-40)(x-41)$$

$$\therefore y(84) = 184 + 2(84-44) + 0.01(84-40)(84-41)$$

$$= 282.92$$

Chapter 21

Q. $\int_0^6 \frac{dx}{1+x^2}$

① Trapizoidal rule (N = 1)
② Simpson's 1/3 rule (N = 2)
③ Simpson 3/8 rule (N = 3)

Soln:

$f(x) = \frac{1}{1+x^2}$; width $h = \frac{b-a}{n} = 1$.

Divide the interval [0, 6] into 6 part

x	0	1	2	3	4	5	6
$y = f(x)$	1	0.5	0.2	0.1	0.0588	0.0385	0.027

① Trapizoidal

$\cancel{f(x)} =$

$$\int_0^6 \frac{dx}{1+x^2} = \frac{h}{2}\left[(y_0 + y_6) + 2(y_1 + y_2 + y_3 + y_4 + y_5)\right]$$

$= 1.4108$

(ii) Simpson 1/3

$$\int_0^6 \frac{dx}{1+x^2} = \frac{h}{3}\left[(y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)\right]$$

$= 1.366$

(iii) Simphson's 3/8 rule

$$\int_0^6 \frac{dx}{1+x^2} = \frac{3h}{8}\left[(y_0+y_6)+3(y_1+y_2+y_4+y_5)+2y_3\right]$$

$$= 1.357.$$

Ans to the

6(a) Q: $\int_0^2 (y^2-(y-2))dy$

(i) Trapezoidal

Here,

$x = f(y) = \{y^2-(y-2)\}$ $\quad w = \frac{2-0}{2} = 1$

Divide the interval [0, 2] into 2 part.

y	0	1	2
x	2	2	4

$$\therefore \int_0^2 \{y^2-(y-2)\}dy = h/2\left[(x_0+x_1)+2.x_2\right]$$

$$= 1/2\left[2+4+2\times 2\right]$$

$$= 5$$

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(11) Multiple Trapezoidal For $n=4$

when $h = \frac{2-0}{4} = 0.5$, $x = f(y) = y^2 - y + 2$

y	0	0.5	1	1.5	2
x	2	1.75	2	2.75	4

$$\therefore \int_0^2 (y^2 - y + 2)\,dy = \frac{h}{2}\left[(x_0 + x_4) + 2(x_1 + x_2 + x_3)\right]$$

$$= \frac{0.5}{2}\left[2 + 4 + 2(1.75 + 2 + 2.75)\right]$$

$$= 4.75$$

$$\text{True error} = \left|\frac{5 - 4.75}{5}\right| \times 100\%$$

$$= 5\%$$

(iii) Simpson's 1/3 for $n = 4$

$h = 0.5$

y	0	0.5	1	1.5	2
x	2	1.75	2	2.75	4

LU: $Ax = B$

$\frac{f''(x_i)}{2!}(x_{i+1} - x_i)^2 + \ldots$

$-x_i)^n + R_n$

$$\therefore \int_0^2 (y^2 - y + 2)\, dy$$

$$= \frac{h}{3}\left[(x_0 + x_4) + 4(x_1 + x_3) + 2x_2\right]$$

$$= \frac{0.5}{3}\left[2 + 4 + 4(1.75 + 2.75) + 2 \times 2\right]$$

$$= \underline{4.667}$$

$$\text{True error} = \left|\frac{5 - 4.667}{5}\right| \times 100\%$$

$$= \underline{6.66\%}$$

Now the Equation is —

$$y = 1.6 - 3.4x + 2x^2$$

5(b) (0, 1) (1, 2) (2, 1) (3, 10)

For cubic

$$Y = a + bx + cx^2 + dx^3 \quad \text{—} (1)$$

And the normal eqn:

$$\Sigma y = an + b\Sigma x + c\Sigma x^2 + d\Sigma x^3$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3 + d\Sigma x^4$$

$$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4 + d\Sigma x^5$$

$$\Sigma x^3 y = a\Sigma x^3 + b\Sigma x^4 + c\Sigma x^5 + d\Sigma x^6$$

x	y	xy	x^2	x^2y	x^3	x^3y	x^4	x^5	x^6
0	1	0	0	0	0	0	0	0	0
1	2	2	1	2	1	2	1	1	1
2	1	2	4	4	8	8	16	32	64
3	10	30	9	90	27	270	81	243	729
6	14	34	14	96	36	280	98	276	794

(0, 1) (1, 2) (2, 1) (3, 10)

We know that
The ~~cubic~~ Quadratic equation $y = a_0 + a_1 x + a_2 x^2$

x	y	xy	x^2	x^2y	x^3	x^4
0	1	0	0	0	0	0
1	2	2	1	2	1	1
2	1	2	4	4	8	16
3	10	30	9	90	27	81
6	14	34	14	96	36	98

$$\Sigma y = a_0 n + a_1 \Sigma x + a_2 \Sigma x^2$$

$$\Sigma xy = a_0 \Sigma x + a_1 \Sigma x^2 + a_2 \Sigma x^3$$

$$\Sigma x^2 y = a_0 \Sigma x^2 + a_1 \Sigma x^3 + a_2 \Sigma x^4$$

From The Table

$$14 = 4a_0 + 6a_1 + 14a_2 \quad \text{— (I)}$$

$$34 = 6a_0 + 14a_1 + 36a_2 \quad \text{— (II)}$$

$$96 = 14a_0 + 36a_1 + 98a_2 \quad \text{— (III)}$$

solving the Three equation we get

$a_0 = 1.6$; $a_1 = -3.4$ $a_2 = 2$

~~Newton Forward Interpolation:~~

Polynomial Regression

Types

linear → $y = ax + b$

Quadratic → $y = ax^2 + bx + c$

Cubic → $y = ax^3 + bx^2 + cx + d$

Quartic → $y = ax^4 + bx^3 + cx^2 + dx + e$

Quintic → $y = ax^5 + bx^4 + cx^3 + dx^2 + ex + f$

$$a = X'B$$

$$X = \begin{bmatrix} n & \Sigma x & \Sigma x^2 \\ \Sigma x & \Sigma x^2 & \Sigma x^3 \\ \Sigma x^2 & \Sigma x^3 & \Sigma x^4 \end{bmatrix} \qquad B = \begin{bmatrix} \Sigma y \\ \Sigma xy \\ \Sigma x^2 y \end{bmatrix}$$

$\therefore f(5) = 5^3 + 2\times5 - 4$

$= 131.$

(iii) For real root $(1, -1)$ and $(2, 8)$ is closed.

x	$f(x)$	$f[\,]$
1	-1	
		9
2	8	

$\therefore f(x) = -1 + 9(x-1)$

$= 9x - 10$

$\therefore 9x - 10 = 0$

$\therefore x = 10/9$

(iv) $g(x) = f(x) + b(x+1)(x-1)(x-2)(x-3)(x-4)$

$\therefore g(5) = 131 + b\times 6\times4\times3\times2\times1$

$\Rightarrow 203 - 131 = 144\,b$

$\Rightarrow b = \frac{72}{144} = \frac{1}{2}$

$\therefore g(x) = x^3 + 2x - 4 + \frac{1}{2}(x+1)(x-1)(x-2)(x-3)(x-4)$

Newton Divided Difference Interpulation:

Q.

x:	-1	1	2	3	4
$f(x)$:	-7	-1	8	29	68

(i) Constract a divided difference Table.

(ii) Find the polynomial of least degree that intercorporates the values in the Table and find $f(5)$.

(iii) Find by linear interpolation of real root.

(iv) Find the polynomial $g(x)$ that takes the value of the above table and $g(5) = 203$.

Soln;

(i)

x	$f(x)$	$f^1[\,]$	$f^2[\,]$	$f^3[\,]$	$f^4[\,]$
-1	-7				
1	-1	3			
2	8	9	2		
3	29	21	6	1	—
4	68	39	9	1	0

(ii) Polynomial: $-7 + 3(x+1) + 2(x+1)(x-1) + 1(x+1)(x-1)(x-2)$

$= -7 + 3x + 3 + \cancel{2x^2} - 2 + x^3 - \cancel{2x^2} - x + 2$

$= x^3 + 2x - 4$

least Squares Regression

The least squares method is a form of mathematical regression analysis used to determine the lime of best fit for a set of Data, providing a visual ~~demon-~~ ~~stratts~~ Demonstration of the relationship between the data points. Each points of data represent the relationship between a known idependent variable and an unknown dependent variable.

1. It is a statistical procedure to find the best fit for a set of data points.

2. It works by minimizing the sum of the offsets or residuals of points from the plotted curve.

3. It is used to predict the behavior of dependent variable.

Advantage:

(i) Easy to Apply and understand

(ii) Highlight relationship between two variable

(iii) Can be used to make predictions about future performance.

Disadvantage:

(i) Only highligh relationship between two variable.
(ii) Doesn't account for outliers.
(iii) May be skewed if data is not evenly distributed.

Curve Fitting:

Two main methods:

1. least Square Regression:
   - Function is best fit to Data.
   - Does not necessary pass through point
   - Used for experimental data.
   - Can develop model for analysis/Design.

2. Interpolation