Bangabandhu Sheikh Mujibur Rahman Science and Technology University

Department of Computer Science and Engineering

2nd Year 1st Semester Final B.Sc. Engineering Examination-2021

Course Code: MAT205 **Course Title: Vector, Matrix and Fourier Analysis**

Total Marks: 60 **Time: 3 (Three) Hours**

N.B.:

i. Answer **SIX** questions taking any **THREE** from each section.

ii. All parts of a question must be answered sequentially.

Section-A

1. a) Define unit vector. Determine a unit vector perpendicular to the plane of $A = 2i - 6j - 3k$ and $B = 4i + 3j - k$. **5**

b) A particle moves along the curve $x = 2t^2, y = t^2 - 4t, z = 3t - 5$, where t is the time. Find the components of its velocity and acceleration at time $t = 1$ in the direction $i - 3j + 2k$. **5**

2. a) Interpret gradient, divergence and curl physically. **3**

b) Prove that $\nabla \times \left(\nabla \times \vec{A}\right) = -\nabla^2 \vec{A} + \nabla\left(\nabla . \vec{A}\right)$. **3**

c) Find the directional derivation of $\varphi = 4xz^3 - 3x^2y^2z$ at (2, –1, 2) in the direction $2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}$. **4**

3. a) Define line integrals, surface integrals and volume integrals. If $\boldsymbol{F} = 3xy\boldsymbol{i} - y^2\boldsymbol{j}$, evaluate $\int_C F \cdot dr$ where C is the curve in the xy plane, $y = 2x^2$, from $(0,0)$ to $(1,2)$. **6**

b) Let $\boldsymbol{F} = 2xz\boldsymbol{i} - x\boldsymbol{j} + y^2\boldsymbol{k}$. Evaluate $\iiint_V F\, dV$ where V is the region bounded by the surfaces $x = 0,\ y = 0,\ y = 6,\ z = x^2,\ z = 4$. **4**

4. a) State and prove Green's theorem in the plane. **6**

b) Using divergence theorem, evaluate $\iint_S F \cdot n\, dS$, where $F = 4xzi - y^2 j + yzk$ and S is the surface of the cube bounded by $x = 0,\ x = 1,\ y = 0,\ y = 1,\ z = 0,\ z = 1$. **4**

Section-B

5. a) Define symmetric matrix, idempotent matrix and normal matrix with example. Show that the matrix $A = \begin{pmatrix} 1 & i \\ -i & -1 \end{pmatrix}$ is unitary. **6**

b) Show that, every square matrix can be uniquely expressed as the sum of a symmetric matrix and a skew-symmetric matrix. **4**

6. a) Find the inverse of the matrix **4**

$A = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ using row operation.

b) Define Rank of a matrix. Determine the rank of the matrix A by reducing it to the normal form **6**

Where $A = \begin{pmatrix} 2 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 3 & 11 & 6 \end{pmatrix}$.

7. a) Define half range Fourier sine and cosine series. **5**

Find the Fourier series of the periodic function x^2 with period $2l$ on the interval $[-l, l]$.

b) Prove that $\int_0^\infty \frac{x \sin mx}{x^2+1} dx = \frac{\pi}{2} e^{-m}$, $m > 0$. **5**

8. a) Find the Fourier transform of $f(x) = e^{-|x|}$ where x belongs to $(-\infty, \infty)$. **4**

b) Use finite Fourier transforms to solve **6**

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial x^2}; \quad U(0,t) = 0; U(\pi,t) = 0; U(x,0) = 2x \text{ where } 0 < x < \pi, t > 0.$$