Measures of Dispersion

Variation

The deviation of an entry x in a population data set is the difference between the entry and the mean μ of the data set.

$\therefore$ Deviation of $x = x - \mu$

Variance:

The variance is the average of the squared differences of the given values from their arithmetic mean.

As a formula, the variance of population observations $x_1, x_2, \ldots, x_N$, commonly denoted by σ^2 is

$$\sigma^2 = \frac{\Sigma(x_i - \bar{x})^2}{N} \quad \ldots (1)$$

where $\bar{x}$ is the mean of all the observations and N is the total number of observations.

computing variance for frequency distribution,

$$\sigma^2 = \frac{\Sigma f_i (x_i - \bar{x})^2}{N} \quad \text{or,} \quad \sigma^2 = \left[\frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N} \right)^2 \right] \times h^2$$

For grouped data x_i will be the mid value of the i-th class.

$-10, 0, 10, 20, 30$: Mean = 10

$8, 9, 10, 11, 12$: Mean = 10

Standard deviation: The positive square root of the variance is the standard deviation.

That is standard deviation is the positive square root of the mean-square deviations of the observations from their arithmetic mean.

If $x_1, x_2, \ldots, x_N$ be N observations of a variable, then the standard deviation is defined as

$$\sigma = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{N}}$$

For frequency distribution, standard deviation is defined as

$$\sigma = \sqrt{\frac{\Sigma f_i (x_i - \bar{x})^2}{N}} \quad \text{or,} \quad \sigma = \left[\sqrt{\frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2}\right] \times h$$

Coefficient of variance:

A coefficient of variance is computed as a percentage of the standard deviation of the distribution of the mean of the same distribution symbolically

$$C.V = \frac{\sigma}{\bar{x}} \times 100$$

where σ = standard deviation

$\bar{x}$ = mean

Problem: Calculate the variance, Standard deviation and co-efficient of variance for the age distribution given below:

Age	24.5-29.5	29.5-34.5	34.5-39.5	39.5-44.5	44.5-49.5	49.5-54.5
Frequency	3	9	15	12	7	4

Soln:

Age	Frequency f_i	Mid value x_i	$d_i = \frac{x_i - A}{c}$	$f_i d_i$	$f_i d_i^2$
24.5-29.5	3	27	-2	-6	12
29.5-34.5	9	32	-1	-9	9
34.5-39.5	15	37 = A	0	0	0
39.5-44.5	12	42	1	12	12
44.5-49.5	7	47	2	14	28
49.5-54.5	4	52	3	12	36
	$\Sigma f_i = 50$			$\Sigma f_i d_i = 23$	$\Sigma f_i d_i^2 = 97$

Variance,

$$\sigma^2 = \left[\frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2\right] \times h^2$$

$$= \left[\frac{97}{50} - \left(\frac{23}{50}\right)^2\right] \times 5^2$$

$$= 43.21$$

Standard deviation,

$$\sigma = \left[\sqrt{\frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2}\right] \times h$$

$$= \sqrt{43.21} = 6.573$$

Co-efficient of variance:

We know, co-efficient of variance,

$$\frac{\sigma}{\bar{x}} \times 100$$

Now, mean, $\bar{x} = A + \frac{\Sigma f_i d_i}{N} \times h$

$$= 37 + \frac{23}{50} \times 5$$

$$= 39 + 2.3$$

$$= 39.3$$

$$\therefore C.V = \frac{6.573}{39.3} \times 100$$

$$= 16.73\%$$

(i) Calculate Variance, Standard deviation and co-efficient of variance from the following frequency distribution:

Profit	10-20	20-30	30-40	40-50	50-60
No. of companies	8	12	20	6	4

(ii)

profit	0-10	10-20	20-30	30-40	40-50
No of Companies	6	25	36	20	13

(iii)

profit	0-10	10-20	20-30	30-40	40-50	50-60
No of Companies	8	12	20	30	20	10

(iv)

Year	5-10	10-15	15-20	20-25	25-30	30-35	35-40
No of Managers	18	30	46	28	20	12	6