Ans to Q 1

(a)

Given, $x_1 + 6x_2 + 2x_3 = 10$

$3x_1 + 2x_2 + x_3 = 6$

$4x_1 + 5x_2 + 2x_3 = 9$

$\therefore Ax = b$

$$= \begin{bmatrix} 1 & 6 & 2 \\ 3 & 2 & 1 \\ 4 & 5 & 2 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 6 \\ 9 \end{bmatrix}$$

$$A \times x = b$$

(b)

We have,

$$A_{(1)} = \begin{bmatrix} 1 & 6 & 2 \\ 3 & 2 & 1 \\ 4 & 5 & 2 \end{bmatrix}$$

Here,

$$m_{21} = \frac{A_{21}}{A_{11}} = \frac{3}{1} = 3$$

$$m_{31} = \frac{A_{31}}{A_{11}} = \frac{4}{1} = 4$$

$$F^{(1)} = \begin{bmatrix} 1 & 0 & 0 \\ -m_{21} & 1 & 0 \\ -m_{31} & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix}$$

Now, $A_2 = F^{(1)} \times A_{(1)}$

$$A_2 = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 6 & 2 \\ 3 & 2 & 1 \\ 4 & 5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 6 & 2 \\ 0 & -16 & -5 \\ 0 & -19 & -6 \end{bmatrix}$$

Now,

$$m_{32} = \frac{-19}{-16} = \frac{19}{16}$$

$$f^{(2)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -m_{32} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{19}{16} & 1 \end{bmatrix}$$

$$\underline{\underline{C}}$$

Unit lower triangular matrix,

$$L = \begin{pmatrix} 1 & 0 & 0 \\ m_{21} & 1 & 0 \\ m_{31} & m_{32} & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 4 & \frac{19}{16} & 1 \end{pmatrix}$$

d

$$U = A_3 = F^{(2)} \times A(2)$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{19}{16} & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 6 & 2 \\ 0 & -16 & -5 \\ 0 & -19 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 6 & 2 \\ 0 & -16 & -5 \\ 0 & 0 & -\frac{1}{16} \end{bmatrix}$$

Now, $Ly = b$

$$\Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 4 & \frac{19}{16} & 1 \end{pmatrix} \times \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 10 \\ 6 \\ 9 \end{pmatrix}$$

Using ...ward substitution:

$$y_1 = 10 \quad \text{—} \ ①$$

$$3y_1 + y_2 = 6 \Rightarrow y_2 = 6 - (3 \times 10)$$

$$\Rightarrow y_2 = -24 \quad \text{—} \ ②$$

$$4y_1 + \frac{19}{16} y_2 + y_3 = 9$$

$$\Rightarrow y_3 = 9 - \frac{19}{16} \times (-24) - 4 \times 10$$

$$\Rightarrow y_3 = -5/2 = -2.5 \quad — (3)$$

Now,

$$Ux = y$$

$$\Rightarrow \begin{bmatrix} 1 & 6 & 2 \\ 0 & -16 & -5 \\ 0 & 0 & -1/16 \end{bmatrix} \times \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 10 \\ -24 \\ -2.5 \end{pmatrix}$$

Using backward substitution:

$$(1) \quad -\frac{1}{16} x_3 = -5/2 \Rightarrow x_3 = 40 \quad — (1)$$

$$(2) \quad -16x_2 - 5x_3 = -24 \Rightarrow x_2 = \frac{-24 + 5 \times 40}{-16}$$

$$x_2 = -11 \quad — (2)$$

$$(3) \quad x_1 + 6x_2 + 2x_3 = 10$$

$$\Rightarrow x_1 = 10 - 6 \times (-11) - 2 \times 40 = -4 \quad — (3)$$

Solution: $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -4 \\ -11 \\ 40 \end{pmatrix}$.

Ans to Q 2

(a)

$$\begin{bmatrix} 0 & 6 & 2 \\ 3 & 2 & 1 \\ 4 & 5 & 2 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 6 \\ 9 \end{bmatrix}$$

$$A \times x = b$$

(b)

Matrix A has a pivoting problem. To apply gaussian elimination / find roots we need the diagonal values of a matrix to be no zero. In both upper or lower triangular matrix a_{11}, a_{22}, a_{33} are nonzero and here we can see a_{11} is zero which causes the pivoting problem.

C

$$Aug(A) = \left[\begin{array}{ccc|c} 0 & 6 & 2 & 10 \\ 3 & 2 & 1 & 6 \\ 4 & 5 & 2 & 9 \end{array}\right]$$

Swap $R_1 \rightleftharpoons R_2$

$$\left[\begin{array}{ccc|c} 3 & 2 & 1 & 6 \\ 0 & 6 & 2 & 10 \\ 4 & 5 & 2 & 9 \end{array}\right]$$

$$m_{31} = \frac{a_{31}}{a_{11}} = \frac{4}{3}$$

$$r_3' = r_3 - m_{31} \times r_1$$

$$\left[\begin{array}{ccc|c} 3 & 2 & 1 & 6 \\ 0 & 6 & 2 & 10 \\ 0 & \frac{7}{3} & \frac{2}{3} & 1 \end{array}\right]$$

$$m_{32} = \frac{a_{32}}{a_{22}} = \frac{7/3}{6} = \frac{7}{18}$$

$$r_3' = r_3 - m_{32} \times r_2$$

$$\left[\begin{array}{ccc|c} 3 & 2 & 1 & 6 \\ 0 & 6 & 2 & 10 \\ 0 & 0 & -1/9 & -26/9 \end{array}\right]$$

upper triangular matrix = $\begin{bmatrix} 3 & 2 & 1 \\ 0 & 6 & 2 \\ 0 & 0 & -1/9 \end{bmatrix}$

d

Using backward substitution:

① $-\frac{1}{9}x_3 = -26/9 \Rightarrow x_3 = -\frac{26}{9} \times -9$

$= 26$

② $6x_2 + 2x_3 = 10$

$\rightarrow x_2 = \frac{10 - 2 \times 26}{6} = -7$

③ $3x_1 + 2x_2 + x_3 = 6$

$\rightarrow x_1 = \frac{6 - 2 \times (-7) - 26}{3}$

$= -2$

$\therefore x_1 = -2,\ x_2 = -7,\ x_3 = 26$