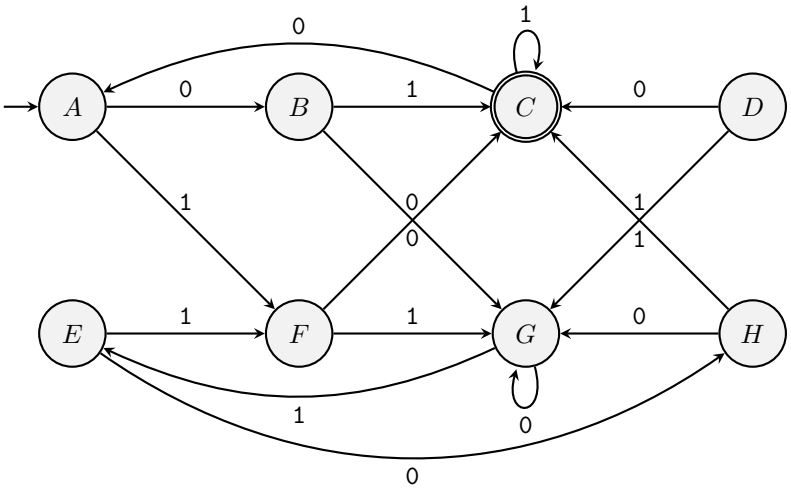
Final Exam
Total marks: 50
Duration: 95 minutes CSE331: Automata and Computability

There are a total of six problems. You have to solve **any five** of them.

Problem 1 (CO1): DFA Minimization (10 points)

Minimize the following DFA. You must **show** all the steps of the minimization algorithm.



Problem 2 (CO3): Nonregular Languages (10 points)

Let $\Sigma = \{0, 1\}$. Consider the following language.

$$L = \{w \in \Sigma^* : w = 0^n 1^m 0^k \text{ where } n = m \text{ or } m \neq k\}$$

(a) Use the pumping lemma to **demonstrate** that L is not regular. (8 points)

(b) **Find** a string $w \in L$ such that there exist $x, y, z \in \Sigma^*$ with $y \neq \varepsilon$ such that $w = xyz$ and $xy^i z \in L$ for all $i \geq 0$. Does this contradict the pumping lemma? (2 points)

Problem 3 (CO2): Derivations, Parse Trees and Ambiguity (10 points)

Take a look at the grammar below and solve the following problems.

$$A \to A1 \mid 0A1 \mid 01$$

(a) **Give** a leftmost derivation for the string 001111. (3 points)

(b) **Sketch** the parse tree corresponding to the derivation you gave in (a). (2 points)

(c) **Demonstrate** that there are two more parse trees (apart from the one you already found in (b)) for the same string. (4 points)

(d) **Find** a string w of length six such that w has exactly one parse tree in the grammar above. (1 point)

Problem 4 (CO2): Chomsky Normal Form (10 points)

Convert the following grammar into Chomsky Normal Form. You must show work. Here `a`, `b`, `c` are terminals and the rest are variables.

$$S \to \texttt{b}X\texttt{a}Y|ZX\texttt{b}$$
$$X \to \texttt{a}Y \mid \texttt{b}Y \mid Y$$
$$Y \to X \mid \texttt{c} \mid \varepsilon$$
$$Z \to Z\texttt{a}X$$

Problem 5 (CO4): The CYK Algorithm (10 points)

Apply the CYK algorithm to determine whether the string `abcaa` can be derived in the following grammar. You must show the entire CYK table. Here `a`, `b`, `c` are terminals and the rest are variables.

$$S \to CA$$
$$A \to AA \mid AD \mid \texttt{a}$$
$$B \to AB \mid BC \mid \texttt{b}$$
$$C \to CA \mid BC \mid \texttt{c}$$
$$D \to \texttt{a}$$

Problem 6 (CO2): Models Recognizing CFLs (10 points)

Let $\Sigma = \{\texttt{0}, \texttt{1}\}$. Consider the following language.

$$L = \{w \in \Sigma^* : w = 0^n 1^n \text{ where } n \text{ is odd}\}$$

(a) **Construct** a context-free grammar that generates L. (5 points)

(b) **Design** a pushdown automaton that recognizes L. (5 points)