Midterm Examination : Questions for CSE330. All Sections.
Department of Computer Science & Engineering
BRAC University
Fall Semester
Date : November 08, 2022
Time : One hour 10 minutes

Faculty Name (Initial) : _____ Student ID# : _____ Section#: _____

Instructions:

- There are four questions. **Answer any three questions**. Total marks 30.
- Use pencil for your answers. No break for bathroom/freshroom is allowed. **Must use your own calculator**. Cell phones must be turned off (Not in vibration mode). We assume that you know how to use scientific calculator of model CASIO fx-991 ES or equivalent.
- Return this question along with your answer script.
- All examinees must abide by the 'Regulations of Students Conduct' of Brac university.

Read carefully the questions below and answer properly (All are CO1 and CO2):

1. Answer the following questions:
   (a) (5 marks) [**CO-1, CO-2**] Consider the quadratic equation $x^2 - 12x + 5 = 0$. Explain how the loss of significance occurs in finding the roots of the quadratic equation if we restrict to 4 significant figures. Discuss how to avoid this and find the roots.
   (b) [**CO-3, CO-4**] Given $\beta = 2$, $m = 3$, $e_{\min} = -2$ and $e_{\max} = 1$. Using the normalized convention, answer the following:
      i. (2 marks) Evaluate the Machine Epsilon.
      ii. (3 marks) Calculate the minimum and maximum positive number representable by this system.

2. Consider the function $f(x) = e^x + e^{-x}$ and the nodes at -1, 0, and 1. Now answer the following using 3 significant figures:
   (a) (1 mark) [**CO-1**] Write down the matrices b and V used in Vandermonde method.
   (b) (2 marks) [**CO-3**] Compute the determinant of the Vandermonde matrix V.
   (c) (3 marks) [**CO-3**] Using The results of the previous two parts, calculate the Taylor coefficients a_0, a_1 and a_2; and finally find the interpolation polynomial.
   (d) (4 marks) [**CO-3**] Evaluate the upper bound of the error for the given function for the interval $[-1.1, 1.1]$.

3. Consider the function $f(x) = e^x + e^{-x}$ and the nodes at -1, 0, and 1. Now answer the following using 3 significant figures:
   (a) (4.5 marks) [**CO-4**] Evaluate the Lagrange bases for the given function and nodes.
   (b) (2.5 marks) [**CO-3**] Compute the Lagrange interpolation polynomial for the given function, and express the result in the natural basis.
   (c) (3 marks) [**CO-4**] Evaluate the relative error in percentage form at $x = 1.5$.

4. Consider the following data set:

x	2.2	2.4	2.6	2.8
$f(x)$	20.05	22.56	25.79	29.14

   Using these data values, answer the following questions:
   (a) (2 marks) [**CO-3**] Compute $f'(2.4)$ using the central difference method.
   (b) (2 marks) [**CO-4**] Evaluate the truncation error for $f(x) = 12 \ln x$ at 2.3 using $h = 0.1$ in forward difference method.
   (c) (4+2 marks) [**CO-3**] Deduce an expression for $D_h^{(1)}$ from D_h by replacing h with $h/3$ using the Richardson extrapolation method. Then calculate the upper bound of error of $D_h^{(1)}$, if $f(x) = \sin x$, $x_0 = 1$ and $h = 0.1$.