BRAC UNIVERSITY

Inspiring Excellence

MAT216: Mathematics IV: Linear Algebra and Fourier Analysis
Assignment 3
Fall 2022
Total Marks: 25

1) Show the derivation of the formula of least square method.

2) i) Let S be the subspace with basis $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ -4 \end{bmatrix}, \begin{bmatrix} 5 \\ -6 \\ 7 \\ 8 \end{bmatrix} \right\}$.

   Find the basis of the subspace $S^{\perp}$ orthogonal to S.

   ii) Find the orthogonal projection matrix P onto the plane $x + y - z = 0$.

3) Let B = $\{v_1, v_2, v_3\}$ be a set of vectors (which may or may not be a basis of $R^3$) , where $v_1$ = . $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$, $v_2 = \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix}$, $v_3 = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$.

   You have to perform the **Gram Schmidt Method** on this set of vectors. Which means, you will try to convert this set of vectors into an *orthogonal* set $\{w_1, w_2, w_3\}$, and then finally into an *orthonormal* set $\{u_1, u_2, u_3\}$.

   The first part is done for your convenience –

   $w_1 = v_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

   $u_1 = \left(\frac{1}{\sqrt{1^2+(-1^2)+0^2}}\right) \times \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.707 \\ -0.707 \\ 0 \end{bmatrix}$

   Let $u_2 = \begin{bmatrix} u2x \\ 0.408 \\ u2z \end{bmatrix}$ and $u_3 = \begin{bmatrix} 0 \\ u3y \\ u3z \end{bmatrix}$

   Find the value of $u2x, u2z, u3y$ and $u3z$ ?

4) Consider the following system of linear equations-

$$x + y = 4, \; x + 2y = 6, \; x + 4y = 11$$

First of all, write the system in the form $Ax=b$.

There is no pair of $(x,y)$ that solves this particular system, as the system is inconsistent.

You have to use the **Least Square Method** to find the best approximation of $x$ and $y$.