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MAT 110

ASSIGNMENT 03

SET 7

Ans to the question no 01

Given function,

$f(x) = \sin x + 2\cos x$

By differentiating this function we get,

$f'(x) = \cos x - 2\sin x$

$f''(x) = -\sin x - 2\cos x$

$f'''(x) = -\cos x + 2\sin x$

Now, at x=0,

$f(0) = \sin 0 + 2\cos 0 = 2$

$f'(0) = \cos 0 - 2\sin 0 = 1$

$f''(0) = -\sin 0 - 2\cos 0 = -2$

$f'''(0) = -\cos 0 + 2\sin 0 = -1$

We know, $P_n(0) = \frac{f^0(x)}{0!}x^0 + \frac{f'(x)}{1!}x^1 + \frac{f''(x)}{2!}x^2 + \frac{f'''(x)}{3!}x^3 + ...$

According to Taylor series around x=0,

$P_3(0) = \frac{f^0(0)}{0!}x^0 + \frac{f'(0)}{1!}x^1 + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3$

$\Rightarrow P_3(0) = \frac{2}{0!}(x-0)^0 + \frac{1}{1!}(x-0)^1 + \frac{-2}{2!}(x-0)^2 + \frac{-1}{3!}(x-0)^3$

$\Rightarrow P_3(0) = 2 + (x-0) - (x-0)^2 - \frac{x^3}{6}$

$\Rightarrow P_3(0) = 2 + x - x^2 - \frac{1}{6}x^3$

$\therefore P_3(0) = 2 + x - x^2 - \frac{1}{6}x^3$

Ans to the question no 02

We know,

$P_n(0) = \frac{f^0(x)}{0!}x^0 + \frac{f'(x)}{1!}x^1 + \frac{f''(x)}{2!}x^2 + \frac{f'''(x)}{3!}x^3 + ...$

Given function,

$f(x) = \ln(e^{2x} + e^{-2x})$

By differentiating we get,

$f'(x) = \frac{1}{e^{2x}+e^{-2x}} \cdot (2e^{2x} - 2e^{-2x}) = \frac{2(e^{4x}-1)}{e^{4x}+1}$

$f''(x) = \frac{2\cdot 4e^{4x}(e^{4x}+1) - 4e^{4x}2(e^{4x}-1)}{(e^{4x}+1)^2}$

$\Rightarrow f''(x) = \frac{8e^{4x}(e^{4x}+1) - 8e^{4x}(e^{4x}-1)}{(e^{4x}+1)^2}$

$\Rightarrow f''(x) = \frac{8e^{4x}(e^{4x}-e^{4x}+2)}{(e^{4x}+1)^2}$

$\Rightarrow f''(x) = \frac{16e^{4x}}{(e^{4x}+1)^2}$

Putting x=0 in f(x) and its derivatives we get,

$f(0) = \ln(e^{2\cdot 0} + e^{-2\cdot 0}) = \ln 2$

$f'(0) = \frac{2(e^{4\cdot 0}-1)}{e^{4\cdot 0}+1} = \frac{0}{2} = 0$

$f''(0) = \frac{16e^{4\cdot 0}}{(e^{4\cdot 0}+1)^2} = \frac{16}{4} = 4$

Therefore,

$P_o = \frac{f^0(0)x^0}{0!} = \ln 2$

$P_1 = \frac{f^0(0)x^0}{0!} + \frac{f'(0)x^1}{1!} = \ln 2 + 0 = \ln 2$

$P_2 = \frac{f^0(0)x^0}{0!} + \frac{f'(0)x^1}{1!} + \frac{f''(0)x^2}{2!} = \ln 2 + 2x^2$

Ans to the question no 03

Given multi-variable function,

$f(x, y) = (x^2 + 2y^2x + 3x + y^2)^2$

Derivatives of f(x,y) with respect to x considering other variables constant,

$f_x = 2(x^2 + 2y^2x + 3x + y^2)(2x + 2y^2 + 3)$

$f_{xx} = 2\{2(x^2 + 2y^2x + 3x + y^2) + (2x + 2y^2 + 3)^2\}$

$\Rightarrow f_{xx} = 2\{(2x^2 + 4xy^2 + 6x + 2y^2) + (4x^2 + 4y^4 + 9 + 8xy^2 + 12y^2 + 12x)\}$

$\Rightarrow f_{xx} = 2(6x^2 + 12xy^2 + 18x + 14y^2 + 4y^2 + 9)$

$\therefore f_{xx} = 2(6x^2 + 12xy^2 + 18x + 14y^2 + 4y^2 + 9)$

Similarly,

$f_y = 2(x^2 + 2y^2x + 3x + y^2)(4xy + 2y)$

$f_{yy} = 2\{(4x + 2)(x^2 + 2y^2x + 3x + y^2) + (4xy + 2y)^2\}$

$\Rightarrow f_{yy} = 2\{(4x^3 + 8x^2y^2 + 12x^2 + 4xy^2 + 2x^2 + 4xy^2 + 6x + 2y^2) + (16x^2y^2 + 4y^2 + 16xy^2)\}$

$\Rightarrow f_{yy} = 2(4x^3 + 24x^2y^2 + 14x^2 + 24xy^2 + 6x + 6y^2)$

$\therefore f_{yy} = 2(4x^3 + 24x^2y^2 + 14x^2 + 24xy^2 + 6x + 6y^2)$

Ans to the question no 04

Given function,

$f(x, y) = \ln((x^2 + 3y + xy)^2)$

Where $x(t) = 2t + 3, y(t) = t^2 + 3t$

Chair rule for partial derivatives,

$\frac{df}{dt} = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dt}$

Now,

$\frac{\partial f}{\partial x} = \frac{1}{(x2+3y+xy)^2} \cdot 2(x^2 + 3y + xy) \cdot (2x + y) = \frac{2(2x+y)}{(x^2+3y+xy)}$

$\frac{dx}{dt} = 2$

$\frac{\partial f}{\partial y} = \frac{1}{(x2+3y+xy)^2} \cdot 2(x^2 + 3y + xy) \cdot (3 + x) = \frac{2(x+3)}{(x^2+3y+xy)}$

$\frac{dy}{dt} = 2t + 3$

Imposing these values on the chain rule formula,

$\frac{df}{dt} = \frac{2 \cdot 2(2x+y)}{(x^2+3y+xy)} + \frac{2(x+3)(2t+3)}{(x^2+3y+xy)}$

Since we have to evaluate this expression when t=0,

$\frac{4(2x+y)}{(x^2+3y+xy)} + \frac{2(x+3)(2 \cdot 0+3)}{(x^2+3y+xy)}$

$= \frac{8x+4y}{(x^2+3y+xy)} + \frac{6x+18}{(x2+3y+xy)}$

$= \frac{14x+4y+18}{(x^2+3y+xy)}$

$\therefore \frac{df}{dt} = \frac{14x+4y+18}{(x^2+3y+xy)}$ when t=0

Ans to the question no 05

Given function,

$y^2 \sin(x^3) + ze^{3x} - \cos(z^2) = z^4 - y^2 + x^2$

Imposing partial differentiation on both sides with respect to x ,

$\frac{\partial}{\partial x}\{y^2 \sin(x^3) + ze^{3x} - \cos(z^2)\} = \frac{\partial}{\partial x}(z^4 - y^2 + x^2)$

$\Rightarrow y^2 \cos(x^3) \cdot 3x^2 + \frac{\partial z}{\partial x}e^{3x} + z \cdot 3e^{3x} + \sin(z^2)2z\frac{\partial z}{\partial x} = 4z^3\frac{\partial z}{\partial x} + 2x$

$\Rightarrow \frac{\partial z}{\partial x}(e^{3x} + 2z\sin(z^2) - 4z^3) = 2x - z \cdot 3e^{3x} - 3x^2y^2\cos(x^3)$

$\Rightarrow \frac{\partial z}{\partial x} = \frac{2x-3ze^{3x}-3x^2y^2\cos(x^3)}{(e^{3x}+2z\sin(z^2)-4z^3)}$

$\therefore \frac{\partial z}{\partial x} = \frac{2x-3ze^{3x}-3x^2y^2\cos(x^3)}{(e^{3x}+2z\sin(z^2)-4z^3)}$

Similarly partial differentiation on both sides with respect to y,

$\frac{\partial}{\partial y}\{y^2 \sin(x^3) + ze^{3x} - \cos(z^2)\} = \frac{\partial}{\partial y}(z^4 - y^2 + x^2)$

$\Rightarrow 2y\sin(x^3) + \frac{\partial z}{\partial y}e^{3x} + \sin(z^2)2z\frac{\partial z}{\partial y} = 4z^3\frac{\partial z}{\partial y} - 2y$

$\Rightarrow \frac{\partial z}{\partial y}(e^{3x} + 2z\sin(z^2) - 4z^3) = -2y - 2y\sin(x^3)$

$\Rightarrow \frac{\partial z}{\partial y} = \frac{-2y(1+\sin x^3)}{(e^{3x}+2z\sin(z^2)-4z^3)}$

$\therefore \frac{\partial z}{\partial y} = \frac{-2y(1+\sin x^3)}{(e^{3x}+2z\sin(z^2)-4z^3)}$

Ans to the question no 06

Given function,

$h(x, y, z) = y^3 z^3 \cos(x^4) + x^3 \sin(z^2)$

We have to use partial differentiation to find h_{yzx}.

Imposing partial differentiation of $h(x, y, x)$ with respect to y,

$h_y = 3y^2 z^3 \cos(x^4)$

Again partial differentiation of h_y with respect to z,

$h_{yz} = 3y^2 3z^2 \cos(x^4) = 9y^2 z^2 \cos(x^4)$

Finally partial differentiation of h_{yz} with respect to x,

$h_{yzx} = 9y^2 z^2 \{-\sin(x^4)(4x^3)\} = 36x^3 y^2 z^2 \{-\sin(x^4)\}$

evaluating the expression of h_{yzx} at point (0,1,1) we get,

$h_{yzx} = 36 \cdot 0^3 \cdot 1^2 \cdot 1^2 \{-\sin(0^4)\} = 0$