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MAT 110 ASSIGNMENT 02

SET 13

Given function,

$$f(x) = y = 3x^4 - 5x^3 + 6x + 8$$

if we differentiate function f(x) we get,

$$\frac{dy}{dx} = f'(x) = 12x^3 - 15x^2 + 6$$

The function f(x) at the point x=1 is,

$$f(1) = 3(1)^4 - 5(1)^3 + 6(1) + 8$$

$$\Rightarrow f(1) = 12$$

Again the function f'(x) at x=1 is,

$$f'(1) = 12(1)^3 - 15(1)^2 + 6$$

$$\Rightarrow f'(1) = 3$$

Now the equation of the tangent line at x=1 is,

$$f(x) - f(1) = f'(1) \cdot (x - 1)$$

$$\Rightarrow y - 12 = 3 \cdot (x - 1)$$

$$\Rightarrow y - 12 = 3x - 3$$

$$\Rightarrow y = 3x + 9$$

 \therefore the tangent line of y at x=1 is, y = 3x + 9

Given function,

$$x = 2\cos\theta,$$

$$y = 3\sin\theta$$

By differentiating these functions by θ we get,

$$\frac{dy}{d\theta} = 3\cos\theta$$

$$\frac{dx}{d\theta} = -2\sin\theta$$

Now,

$$\frac{dy}{dx} = \frac{dy}{d\theta} \cdot \frac{d\theta}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{3\cos\theta}{-2\sin\theta}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{3}{2}\cot\theta$$

 $\therefore \frac{dy}{dx}$ of the parametric equations is, $-\frac{3}{2}\cot\theta$

Given function,

$$y = \log_e(\frac{2x}{x^2 + 1})$$

$$\Rightarrow y = \ln(\frac{2x}{x^2+1})$$

By differentiating this logarithmic function y we get,

$$\frac{dy}{dx} = \frac{d}{dx} \left(\ln\left(\frac{2x}{x^2 + 1}\right) \right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\frac{2x}{x^2+1}} \cdot \frac{(x^2+1)\frac{d}{dx}(2x) - (2x)\frac{d}{dx}(x^2+1)}{(x^2+1)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(x^2+1)}{2x} \cdot \frac{2(x^2+1) - 2x \cdot 2x}{(x^2+1)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2x^2 + 2 - 4x^2}{2x(x^2 + 1)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2 - 2x^2}{2x(x^2 + 1)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2(1-x^2)}{2x(x^2+1)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1-x^2}{x^3+x}$$

$$\therefore \frac{dy}{dx} = \frac{1-x^2}{x^3+x}$$

Given function,

$$f(x) = 2 + x^2$$

Linear approximation L(x) of the function f(x) at x=3,

$$L(x) = f(3) + f'(3)(x - 3)$$

The value of f(3) is,

$$f(3) = 2 + 3^2$$

$$\Rightarrow f(3) = 2 + 9$$

$$\Rightarrow f(3) = 11$$

The value of f'(3) is,

$$f\prime(x) = 2x$$

$$\therefore f'(3) = 2 \cdot 3$$

$$\Rightarrow f\prime(3) = 6$$

Substituting values of f(3) and f'(3) in L(x) at x=3,

$$L(x) = 11 + 6 \cdot (x - 3)$$

$$\Rightarrow L(x) = 11 + 6x - 18$$

$$\Rightarrow L(x) = 6x - 7$$

Now,

$$L(3.1) = 6 \cdot (3.1) - 7$$

$$\Rightarrow L(3.1) = 18.6 - 7$$

$$\Rightarrow L(3.1) = 11.6$$

Given function,

$$f(x) = x^2 - 5x + 6$$

By differentiating both sides we get,

$$\Rightarrow f'(x) = 2x - 5$$

For extreme values of x,

$$f\prime(x) = 0$$

$$\Rightarrow 2x - 5 = 0$$

$$\Rightarrow x = \frac{5}{2}$$

For the range $(-\infty, 2.5)$,

$$f\prime(1) = 2 - 5$$

$$\Rightarrow f'(1) = -3$$

:
$$f'(1) < 0$$

For the range $(2.5, \infty)$,

$$f\prime(6) = 2 \cdot 6 - 5$$

$$\Rightarrow f'(6) = 12 - 5$$

$$\Rightarrow f'(6) = 7$$

Left side is decreasing and right side is increasing

 \therefore it is a minima.

Given,

The world population in 2000 was $A_o = 6.08$ billion.

The annual increase rate= 1.26 %

We know,

$$x(t) = A_o \cdot e^{kt}$$

$$\Rightarrow x(t) = 6.08 \cdot e^{\frac{1.26}{100} \cdot t}$$

$$\Rightarrow x(t) = 6.08 \cdot e^{0.0126 \cdot t}$$

.: Function to population growth since 2000 is,

$$x(t) = 6.08 \cdot e^{0.0126 \cdot t}$$

40 years from 2000 in 2040 the population will be,

$$x(t) = 6.08 \cdot e^{0.0126 \cdot 40}$$

$$\Rightarrow x(t) = 10.064$$
 billion













