CSE 230

Assignment 2

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sec : 08

Lecture 4

Ans to the Q no 1

Expansion to do of $\left(2x^3 + \frac{1}{x}\right)^{29}$

Now, general term $= {}^{29}C_r (2x^3)^{29-r} \left(\frac{1}{x}\right)^r$

$= {}^{29}C_r \; 2^{29-r} \; x^{87-4r}$

Here, $x^{47} = x^{87-4r}$

$\Rightarrow 47 = 87 - 4r$

$\Rightarrow r = 40/4 = 10$

$\therefore$ coefficent of $x^{47} = {}^{29}C_{10} \cdot 2^{29-10}$

$= 1.05 \times 10^{13}$ (Ans)

Now, since the answer $({}^{29}C_{10} \cdot 2^{29-10})$ has a form like ${}^{29}C_a \cdot b^k$, the value of $(a+b+k)$,

$(a+b+k) = 10 + 2 + (29-10) = 31$, (Ans)

Ans to the qr no 2

Given, $a = \left(y^2 + \frac{2}{y}\right)^{21}$

$b = \left(y + \frac{1}{3}\right)^{37}$

$\therefore$ Third term for $a = {}^{21}C_2 (y^2)^{21-2} \left(\frac{2}{y}\right)^2$

Third term for $b = {}^{37}C_2 (y)^{37-2} \left(\frac{1}{3}\right)^2$

Now,

$${}^{21}C_2 (y^2)^{19} \left(\frac{2}{y}\right)^2 = {}^{37}C_2 (y)^{35} \left(\frac{1}{3}\right)^2$$

$$\Rightarrow \frac{21!}{2!\ 19!} \cdot y^{38} \cdot \frac{4}{y^2} = \frac{37!}{2!\ 35!} \frac{y^{35}}{9}$$

$$\Rightarrow \frac{y^{36}}{y^{35}} = \frac{37!\ 2!\ 19!}{21!\ 2!\ 35!} \times \frac{1}{36}$$

$$\Rightarrow y = \frac{37}{420}$$

$\therefore y = \frac{37}{420}$ (Ans)

Ans to the qr no 3

Given, $(z^3+3z+1)^6$

General term in it's expansion $=$

$$\frac{6!}{p!\,q!\,r!}(z^3)^p(3z)^q.(1)^r$$

Now, possible values of p, q, r for z^4,

i) when $p=1, q=1, r=4$,

coefficient (i) for $z^4 = \frac{6!}{1!\,1!\,4!}(1)^1.(3)^1.(1)^4$

$= 90$

ii) when, $p=0, q=4, c=2$,

coefficient for (ii) $z^4 = \frac{6!}{0!\,4!\,2!}.(1)^0(3)^4(1)^2$

$= 1215$

$\therefore$ coefficient of $z^4 = 1215+90 = 1305$.

(Ans)

Ans to the qr no 4

Given, $(370a + 285b + 99c)^{11}$

General term in its expansion =

$$\frac{11!}{p!\,q!\,r!} \cdot (370a)^p (285b)^q (99c)^r$$

if, $p = 5$, $q = 3$, $r = 2$

coefficient of $a^5 b^3 . c^2$ will be =

$$\frac{11!}{5!\,3!\,2!} \cdot (370)^5 (285)^3 (99)^2$$

$$= 4.36120 \times 10^{28}$$

$\therefore$ coefficient of $a^5 b^3 c^2 = 4.36120 \times 10^{28}$

(Ans)

Lecture - 7

Answer to the qr no 1

When two dice are tossed, number of events, $n(S) = 6 \times 6 = 36$

Now, events where the sum is a prime number greater than 3, $E = \{(1,4), (2,3), (1,6), (3,2), (2,5), (3,4), (4,1), (4,3), (5,2), (5,6), (6,1), (6,5)\}$

$\therefore n(E) = 12$

$\therefore$ Probability, $P(E) = \frac{n(E)}{n(S)} = \frac{12}{36} = \frac{1}{3}$

$\therefore$ Probability that total score is a prime number greater than 3 is $1/3$ (Ans)

Ans to the qr no 2

2 bulbs out of 20 can be chosen in $^{20}C_2$ ways.

2 bulbs out of $(20-4) = 16$ non defective bulbs can be chosen $^{16}C_2$ ways.

∴ Probability of choosing no defective bulb if 2 bulbs are chosen at random $= \frac{^{16}C_2}{^{20}C_2}$

$= \frac{12}{19}$

∴ Probability that at least one of the two bulbs is defective $= 1 - \frac{12}{19}$

$= \frac{7}{19}$ (Ans)

Ans to the Q no 3

spades in a deck = 13

Remaining spades in deck after drawing 3 spades = $(13-3) = 10$

Remaining cards in deck after drawing 3 spades = $(52-3) = 49$

∴ Probability that the fourth card drawn is a spade = $10/49$

∴ Probability that the next (fourth) card is not a spade = $1 - \frac{10}{49} = \frac{39}{49}$ (Ans)

Ans to the q no 4

4 ETS big books and 5 barrons books can be arranged in $\frac{(5+4)!}{5!\ 4!}$

$= 126$ ways.

Books of same publisher can be put in 2 ways.

∴ probability that the books of the same publisher will be put together is $= \frac{2}{126} = \frac{1}{63}$ (Ans)

Lecture - 8

Ans to the Q no 1

We know,

variance, $\sigma^2 = E(x)^2 - (E(x)^2)$

Now, the dice has faces labled from 1 to n and x can be any number from 1 to n;

$$E(x) = \frac{1}{n} \times 1 + \frac{1}{n} \times 2 + \frac{1}{n} \times 3 + \dots + \frac{1}{n} \times n$$

$$= (1+2+3+\dots+n) \times \frac{1}{n}$$

$$= \frac{n(n+1)}{2} \times \frac{1}{n} = \frac{n+1}{2}$$

$$E(x)^2 = \frac{1}{n} \times 1^2 + \frac{1}{n} \times 2^2 + \frac{1}{n} \times 3^2 + \dots + \frac{1}{n} \times n^2$$

$$= (1^2 + 2^2 + 3^2 + \dots + n^2) \times \frac{1}{n}$$

$$= \frac{n(n+1)(2n+1)}{6} \times \frac{1}{n}$$

$$= \frac{(n+1)(2n+1)}{6}$$

$\therefore$ varience, $\sigma^2 = \dfrac{(n+1)(2n+1)}{6} - \left(\dfrac{n+1}{2}\right)^2$

$= \dfrac{n+1}{2}\left(\dfrac{2n+1}{3} - \dfrac{n+1}{2}\right)$

$= \dfrac{n^2-1}{12}$ (Ans)

Ans to the or no 2

The random variables possible for a toss of n sided die are 1, 2, 3, 4 . . . . n.

Furthermore, the outcome of a toss can only be one of the faces (top face after die has landed). So the event $X = i$ nautually exclusive from the event $X = J$ if $i \neq J$.

$\therefore Pr(X \leq n) = Pr(X=1) + Pr(X=2) + \ldots + Pr(X=(n-1)) + Pr(X=n)$

$\therefore Pr(X \leq n) = 1.$ (Ans)

Ans to the Q no 3

Number of events when two 10 sided dice are rolled, $n(S) = 10 \times 10 = 100$

Events where sum is a prime number, E =

$\{(1,1), (1,2), (1,4), (1,6), (1,10), (2,1), (2,3), (2,5),$
$(2,9), (3,2), (3,4), (3,8), (3,10), (4,3), (4,1), (4,7),$
$(4,9), (5,2), (5,6), (5,8), (6,1), (6,5), (6,7),$
$(7,4), (7,6), (7,10), (8,3), (8,5), (8,9), (9,2), (9,4),$
$(9,8), (9,10), (10,1), (10,3), (10,7), (10,9)\}$

$\therefore n(E) = 37$

$\rightarrow$ probability that sum of two numbers is a prime $= \frac{37}{100}$ (Ans)

Ans to the qr no 4

Number of events when two n sided dice rolled $= n \times n = n^2$

probability of red dice landing a number divisible by 3 $= \frac{n}{3}$

probability of blue dice landing a number divisible by 2 $= \frac{n}{2}$

$\therefore$ probability of red dice landing a number not divisible by 3 $= 1 - \frac{\left(\frac{n}{3}\right)}{n}$

$$= \frac{n - \frac{n}{3}}{n}$$

$\therefore$ probability of blue dice landing a number not divisible by 2 $= 1 - \left(\frac{n}{2}\right)$

$$= \frac{n - \frac{n}{2}}{n}$$

$\therefore$ probability of red dice not landing a number divisible by 3 and blue one not landing a number divisible by 2 is =

$$\frac{n-\frac{n}{3}}{n} \times \frac{n-\frac{n}{2}}{n}$$

$$= \frac{3n-n}{3} \times \frac{2n-n}{2} \times \frac{1}{n^2}$$

$$= \frac{6n^2-2n^2-3n^2+n^2}{6} \times \frac{1}{n^2}$$

$$= \frac{3n^2}{6} \times \frac{1}{n^2}$$

$$= \frac{1}{3} \text{ (Ans)}$$