Final Examination : Questions for CSE330. All Sections.

Department of Computer Science & Engineering
BRAC University
Summer Semester
Date : September 06, 2022
Time : One hour 50 minutes

Faculty Name (Initial) : ____________ Student ID# : _________________ Section#: ______

Instructions:

- There are six question. **Answer any four questions**. Total marks 60.
- Use pencil for your answers. No break for bathroom/freshroom is allowed. **Must use your own calculator**. Cell phones must be turned off (Not in vibration mode). We assume that you know how to use scientific calculator of model CASIO fx-991 ES or equivalent.
- Return this question along with your answer script.
- All examinees must abide by the 'Regulations of Students Conduct' of Brac university.

Read carefully the questions below and answer properly:

1. [**CO-3**] A function is given by $f(x) = x^2 - x - 2$. Three fixed point functions, $g_1(x) = x^2 - 2$, $g_2 = \sqrt{x+2}$ and $g_3(x) = \sqrt{6-x} + 2$ are constructed from $f(x)$. Based on these answer the following:
   (a) (4.5+1.5 marks) **Calculate** the convergence rate (or ratio) λ for these three fixed point functions, and **state** if these are diverging, linear or superlinear. Note that you need to find the roots first to answer this question.
   (b) (3 marks) **Construct** a fixed point function (x) that is superlinear, and let's call it $g_4(x)$.
   (c) (3+1+1+1 marks) Use $g_4(x)$ and $x_0 = 0$ to **evaluate** the iterated values of x_k up to $k = 4$. Keep up to 6 decimal places. **Decide** which root $g_4(x)$ is converging to and **explain** why the iteration did not converges to the other root. Also**estimate** the actual error up to six decimal places.

2. [**CO-3**] A linear system is described by the following equations:

$$\begin{aligned} 4x_2 + 2x_3 &= 1 \\ 2x_1 + 3x_2 + 5x_3 &= 0 \\ 3x_1 + x_2 + x_3 &= 11 \end{aligned}$$

   Based on these equations, answer the questions below.
   (a) (3 marks) From the given linear equations, **identify** the matrices A, x and b such the the linear system can be expressed as a matrix equation.
   (b) (2 marks) **Examine** if the matrix A has any pivoting problem? **Explain** why or why not?
   (c) (6 marks) **Write** down the Augmented matrix, Aug(A), from the given linear system, and **evaluate** the upper triangular matrix U. Note that you have to show the row multipliers m_{ij} for each step as necessary.
   (d) (4 marks) Using the upper triangular matrix found in the previous question, **compute** the solution of the given linear system by Gaussian elimination method.

3. [**CO-3**] A linear system is described by the following equations:

$$\begin{aligned} 2x_1 + 6x_2 - 9x_3 &= 15 \\ 2x_1 + 4x_2 - 6x_3 &= 10 \\ -2x_1 - 3x_2 + 4x_3 &= -6 \end{aligned}$$

   Based on these equations, answer the questions below.
   (a) (3 marks) From the given linear equations, **identify** the matrices A, x and b such the the linear system can be expressed as a matrix equation.

(b) (3 marks) **Compute** the Frobenius matrices $F^{(1)}$ and $F^{(2)}$ for this system.

(c) (3 marks) **Evaluate** the unit lower triangular matrix L, and the upper triangular matrix U. **Show** that $\det A = \det L \times \det U$.

(d) (6 marks) Now **compute** the solution of the given linear system using LU-decomposition method. Use the matrices L and U found in the previous question. Show your works.

4. [**CO-4**] Answer the following questions:

(a) (6 marks) **Show** that the following set

$$S = \left\{ \frac{1}{\sqrt{5}} (2, -1, 0)^{\mathrm{T}} , \ \frac{1}{\sqrt{30}} (1, 2, -5)^{\mathrm{T}} , \ \frac{1}{\sqrt{24}} (2, 4, 2)^{\mathrm{T}} \right\}$$

is an orthonormal set of vectors.

(b) (2+1 marks) Now, consider the function $f(x) = \sin x$, and the data points at $x_0 = 4$, $x_1 = 9$ and $x_2 = -6$. **Identify** the matrices V and b. Keep up to 3 decimal places for all evaluated values.

(c) (6 marks) **Determine** the best fit polynomial of degree one for the data points in Part-(b) using the Discrete Square Approximation method. Keep up to 3 decimal places for all evaluated values.

5. [**CO-4**] A test has been offered where there are a total of 40 questions and total marks is 100, and there are two sets of question. Set-1 contains x_1 number of 2-marks questions and x_2 number of 4-marks questions. Set-2 contains x_1 number of 3-marks questions and x_2 number of 1-mark questions. In the following, this overdetermined system will be solved by using the QR Decomposition Method by answering the following step by step:

(a) (3 marks) **Write** down the linear equations that relate the variable x_1 and x_2.

(b) (1+1/2+1/2 marks) **Identify** the matrices A, x and b so that the equations in the previous question can be expressed in the standard matrix equation form $Ax = b$.

(c) (4+3 marks) From matrix A in the previous question, **compute** the matrices Q and R such that $A = QR$, where the symbols have their usual meanings.

(d) (3 marks) **Evaluate** $Q^{\mathrm{T}} b$, and finally **solve** the system by evaluating x (that is , **evaluate** x_1 and x_2).

6. [**CO-3**] Consider the function $f(x) = e^{0.5x} + \frac{1}{30} x^2$ which is continuous on the interval $[-2, 0]$. Answer the following questions:

(a) (3 marks) **Calculate** the exact value of integration $I(f)$.

(b) (5+1 marks) **Evaluate** the approximate value of the integration using Composite Newton Cotes formula with 4 segments $C_{1,4}$. Then, **calculate** the relative error in percentage using part (a) and (b).

(c) (3 marks) For the Newton-Cotes formula with $n = 2$, **show** that one of weight function/factors is given as $\sigma_0 = \frac{b-a}{6}$, where a and b are the lower and upper limits of the integral.

(d) (3 marks) **Evaluate** the approximate value of the integral of $f(x)$ using Simpson's rule.