Final Examination : Questions for CSE330. All Sections.

Department of Computer Science & Engineering
BRAC University
Summer Semester
Date : September 06, 2022
Time : One hour 50 minutes

Faculty Name (Initial) : ____________ Student ID# : _________________ Section#: ______

Instructions:

- There are six question. **Answer any four questions**. Total marks 60.
- Use pencil for your answers. No break for bathroom/freshroom is allowed. **Must use your own calculator**. Cell phones must be turned off (Not in vibration mode). We assume that you know how to use scientific calculator of model CASIO fx-991 ES or equivalent.
- Return this question along with your answer script.
- All examinees must abide by the 'Regulations of Students Conduct' of Brac university.

Read carefully the questions below and answer properly:

1. [**CO-3**] A function is given by $f(x) = x^2 + x - 6$. Three fixed point functions, $g_1(x) = 6 - x^2$, $g_2 = \sqrt{6-x}$ and $g_3(x) = \sqrt{x+10} - 2$ are constructed from $f(x)$. Based on these answer the following:
   (a) (4.5+1.5 marks) **Calculate** the convergence rate (or ratio) λ for these three fixed point functions, and **state** if these are diverging, linear or superlinear. Note that you need to find the roots first to answer this question.
   (b) (3 marks) **Construct** a fixed point function (x) that is superlinear, and let's call it $g_4(x)$.
   (c) (3+1+1+1 marks) Use $g_4(x)$ and $x_0 = 0$ to **evaluate** the iterated values of x_k up to $k = 4$. Keep up to 6 decimal places. **Decide** which root $g_4(x)$ is converging to and **explain** why the iteration did not converges to the other root. Also **estimate** the actual error up to six decimal places.

2. [**CO-3**] A linear system is described by the following equations:

$$\begin{aligned} 2x_1 + 6x_2 - 9x_3 &= 15 \\ 2x_1 + 4x_2 - 6x_3 &= 10 \\ -2x_1 - 3x_2 + 4x_3 &= -6 \end{aligned}$$

   Based on these equations, answer the questions below.
   (a) (3 marks) From the given linear equations, **identify** the matrices A, x and b such the the linear system can be expressed as a matrix equation.
   (b) (2 marks) **Examine** if the matrix A has any pivoting problem? **Explain** why or why not?
   (c) (6 marks) **Write** down the Augmented matrix, Aug(A), from the given linear system, and **evaluate** the upper triangular matrix U. Note that you have to show the row multipliers m_{ij} for each step as necessary.
   (d) (4 marks) Using the upper triangular matrix found in the previous question, **compute** the solution of the given linear system by Gaussian elimination method.

3. [**CO-3**] A linear system is described by the following equations:

$$\begin{aligned} 4x_2 + 2x_3 &= 1 \\ 2x_1 + 3x_2 + 5x_3 &= 0 \\ 3x_1 + x_2 + x_3 &= 11 \end{aligned}$$

   Based on these equations, answer the questions below.
   (a) (3 marks) From the given linear equations, **identify** the matrices A, x and b such the the linear system can be expressed as a matrix equation.

(b) (3 marks) **Compute** the Frobenius matrices $F^{(1)}$ and $F^{(2)}$ for this system.

(c) (3 marks) **Evaluate** the unit lower triangular matrix L, and the upper triangular matrix U. **Show** that $\det A = \det L \times \det U$.

(d) (6 marks) Now **compute** the solution of the given linear system using LU-decomposition method. Use the matrices L and U found in the previous question. Show your works.

4. [**CO-4**] Answer the following questions:

(a) (6 marks) **Show** that the following set

$$S = \left\{ \frac{1}{\sqrt{6}} (1, 2, 1)^{\mathrm{T}},\ \frac{1}{\sqrt{30}} (1, 2, -5)^{\mathrm{T}},\ \frac{1}{\sqrt{45}} (6, -3, 0)^{\mathrm{T}} \right\}$$

is an orthonormal set of vectors.

(b) (2+1 marks) Now, consider the function $f(x) = \sin x$, and the data points at $x_0 = 3$, $x_1 = -7$ and $x_2 = 5$. **Identify** the matrices V and b. Keep up to 3 decimal places for all evaluated values.

(c) (6 marks) **Determine** the best fit polynomial of degree one using the Discrete Square Approximation method for the data given in Part-(b). Keep up to 3 decimal places for all evaluated values.

5. [**CO-4**] A student has decided to setup an ice-cream cart as a relaxation after the stressful final exam. The ice-cream cart contains two flavors, vanilla and chocolate. Let x_1 and x_2 are the number of scoops of vanilla and chocolate ice creams respectively taken to prepare the ice-creams. The total number of scoop is 30. On the first day, one scoop of vanilla cost Tk.10 and one scoop of chocolate costs Tk.20, and you earned Tk.400. But on the next day, one scoop of vanilla cost Tk.12, and no change in chocolate price, and you earned Tk.440. In the following, this overdetermined system will be solved by using the QR Decomposition Method by answering the following step by step:

(a) (3 marks) **Write** down the linear equations that relate the variable x_1 and x_2.

(b) (1+1/2+1/2 marks) **Identify** the matrices A, x and b so that the equations in the previous question can be expressed in the standard matrix equation form $Ax = b$.

(c) (4+3 marks) From matrix A in the previous question, **compute** the matrices Q and R such that $A = QR$, where the symbols have their usual meanings.

(d) (3 marks) **Evaluate** $Q^{\mathrm{T}}b$, and finally **solve** the system by evaluating x (that is , **evaluate** x_1 and x_2).

6. [**CO-3**] Consider the function $f(x) = e^{0.5x} + \dfrac{1}{30}x^2$ which is continuous on the interval $[0, 2]$. Answer the following questions:

(a) (3 marks) **Calculate** the exact value of integration $I(f)$.

(b) (5+1 marks) **Evaluate** the approximate value of the integration using Composite Newton Cotes formula with 4 segments $C_{1,4}$. Then, **calculate** the relative error in percentage using part (a) and (b).

(c) (3 marks) For the Newton-Cotes formula with $n = 2$, **show** that one of weight function/factors is given as $\sigma_2 = \dfrac{b - a}{6}$, where a and b are the lower and upper limits of the integral.

(d) (3 marks) **Evaluate** the approximate value of the integral of $f(x)$ using Simpson's rule.