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MAT 110

ASSIGNMENT 01

SET 15

Ans to the qr no 1

Given, $$f(x) = \begin{cases} x+2; & x<0 \\ x^2-1; & 0 \le x \le 2 \\ 2x+1; & x>2 \end{cases}$$

Now, $f(x) = x^2 - 1$

$\Rightarrow f(0) = 0^2 - 1 = -1$

Therefore, $f(x)$ is defined at $x = 0$

$$\text{L.H.L} = \lim_{x \to 0^-} x+2$$
$$= 0+2$$
$$= 2$$

$$\text{R.H.L} = \lim_{x \to 0^+} x^2 - 1$$
$$= 0^2 - 1$$
$$= -1$$

$$\text{L.H.L} \neq \text{R.H.L}$$

So, $\lim_{x \to 0} f(x)$ does not exist

Therefore, $f(x)$ is not continuous at $x = 0$

Ans to the qr no 2

Given,

$$f(x) = \begin{cases} x^2; & x < -1 \\ x+5; & -1 \le x \le 1 \\ x^3 - 4x; & x > 1 \end{cases}$$

Now,

$$f(x) = x+5$$

$$f(1) = 1+5 = 6$$

Therefore, $f(x)$ is defined at $x = 1$

$$\text{L.H.L} = \lim_{x \to 1^-} x+5$$

$$= 1+5$$

$$= 6$$

$$\text{R.H.L} = \lim_{x \to 1^+} x^3 - 4x$$

$$= 1^3 - 4.1$$

$$= 1 - 4$$

$$= -3$$

$\therefore$ L.H.L $\neq$ R.H.L

Therefore, $\lim_{x \to 1} f(x)$ does not exist

Ans to the qr no 3

Evaluating, $\frac{d}{dx^2}\left(\ln\left(\frac{x^4+2x^3}{x^2+1}\right)\right)$

$$= \frac{d}{dx}\left\{\frac{d}{dx}\left(\ln\left(\frac{x^4+2x^3}{x^2+1}\right)\right)\right\}$$

$$= \frac{d}{dx}\left\{\frac{x^2+1}{x^4+2x^3}\cdot\frac{d}{dx}\left(\frac{x^4+2x^3}{x^2+1}\right)\right\}$$

$$= \frac{d}{dx}\left\{\frac{x^2+1}{x^4+2x^3}\cdot\frac{(x^2+1)(4x^3+6x^2)-(x^4+2x^3)2x}{(x^2+1)^2}\right\}$$

$$= \frac{d}{dx}\left\{\left(\frac{x^2+1}{x^4+2x^3}\right)\cdot\frac{4x^5+6x^4+4x^3+6x^2-2x^5-4x^4}{(x^2+1)^2}\right\}$$

$$= \frac{d}{dx}\left(\frac{2x^5+2x^4+4x^3+6x^2}{x^6+x^4+2x^5+2x^3}\right)$$

$$= \frac{\begin{array}{l}6x^6+4x^5+4x^4+12x^5+8x^4+8x^3+6x^4+4x^3+4x^2+12x^3\\ +8x^2+8x-8x^6-12x^5-4x^4-4x^3-8x^5-12x^4-4x^2-16x^4\\ -24x^3-8x-24x^3-36x^2-12x-12\end{array}}{\{x(x^2+1)(x+2)\}^2}$$

$$= \frac{-2x^6-4x^5-14x^4-32x^3-36x^2-12x-12}{x^2(x+2)^2(x^2+1)^2}$$

Ans to the qr no 4

Given,

$$(x-y)^2 = x+y-1$$

$$\Rightarrow x^2 - 2xy + y^2 = x+y-1$$

$$\Rightarrow \frac{d}{dx}(x^2-2xy+y^2) = \frac{d}{dx}(x+y-1)$$ [multiplying $\frac{d}{dx}$ on both sides]

$$\Rightarrow 2x - 2y + 2y\frac{dy}{dx} - 2x\frac{dy}{dx} = 1 + \frac{dy}{dx} - 0$$

$$\Rightarrow 2y\frac{dy}{dx} - 2x\frac{dy}{dx} - \frac{dy}{dx} = 1 - 2x + 2y$$

$$\Rightarrow \frac{dy}{dx}(2y-2x-1) = 2y-2x+1$$

$$\Rightarrow \frac{dy}{dx} = \frac{2y-2x+1}{2y-2x-1}$$

$$\therefore \frac{dy}{dx} = \frac{2y-2x+1}{2y-2x-1}$$

Ans to the qr no 5

Given,

$$\frac{d}{dx} \sin^{-1}\left(\frac{2x}{x+1}\right)$$

Let,

$$a = \frac{2x}{x+1}$$

$$\Rightarrow a = 2x(x+1)^{-1}$$

$$\Rightarrow \frac{da}{dx} = \frac{d}{dx} 2x(x+1)^{-1}$$

$$\Rightarrow \frac{da}{dx} = 2(x+1)^{-1} + 2x(-1)(x+1)^{-2} . 1$$

$$\Rightarrow \frac{da}{dx} = \frac{2}{x+1} - \frac{2x}{(x+1)^2}$$

$$\Rightarrow \frac{da}{dx} = \frac{2(x+1) - 2x}{(x+1)^2}$$

$$\Rightarrow \frac{da}{dx} = \frac{2(x+1) - 2x}{(x+1)^2}$$

$$\Rightarrow \frac{da}{dx} = \frac{2}{(x+1)^2}$$

Evaluating,

$$\frac{d}{dx}\left(\sin^2\left(\frac{2x}{x+1}\right)\right)$$

$$\Rightarrow \frac{d}{dx}\left(\sin^2 a\right) \qquad \left[\because a = \frac{2x}{x+1}\right]$$

$$= 2\sin a \cos a \frac{d(a)}{dx}$$

$$= \sin 2a \frac{d(a)}{dx}$$

$$= \sin\left(\frac{2 \times 2x}{x+1}\right) \cdot \frac{2}{(x+1)^2}$$

$$= \frac{2\sin\left(\frac{4x}{x+1}\right)}{(x+1)^2}$$

we get, $\frac{d}{dx}\left(\sin^2\left(\frac{2x}{x+1}\right)\right) = \frac{2\sin\left(\frac{4x}{x+1}\right)}{(x+1)^2}$

Ans to the q no 6 (a)

Given, $P(t) = \dfrac{M}{1+Ae^{-kt}}$

$\lim_{t \to \infty} P(t)$

$= \lim_{t \to \infty} \dfrac{M}{1+Ae^{-kt}}$

$= \lim_{t \to \infty} \dfrac{M}{1+A/e^{kt}}$

$= \cancel{\lim_{t \to \infty}} \dfrac{M}{1+A/e^{k\infty}}$

$= \dfrac{M}{1+\frac{A}{\infty}}$

$= M/1$

$= M$

$\therefore \lim_{t \to \infty} P(t) = M$

The answer M refers to the maximum population size that can be carried and when t tends to infinity it means that the carrying capacity becomes the maximum population size that can be supported. That's why the answer M is to b expected.

Ans to the qr no 6 (b)

Given, $p(t) = \frac{M}{1 + Ae^{-kt}}$

$$\lim_{M \to \infty} p(t)$$

$$= \lim_{M \to \infty} \frac{M}{1 + Ae^{-kt}}$$

$$= \lim_{M \to \infty} \frac{M}{1 + \frac{M - P_0}{P_0} \cdot e^{-kt}} \qquad \left[\because A = \frac{M - P_0}{P_0} \right]$$

$$= \lim_{M \to \infty} \frac{M}{\frac{P_0 + (M - P_0) e^{-kt}}{P_0}}$$

$$= \lim_{M \to \infty} \frac{M P_0}{P_0 (1 - e^{-kt}) + M e^{-kt}}$$

$$= \lim_{M \to \infty} \frac{P_0}{e^{-kt}} \qquad [\text{L's Hospital rule}]$$

$$= P_0 e^{kt}$$

The result is $P_0 e^{kt}$ which is an exponential function.