Example:

A bag contains 4 white and 6 black balls. If one ball is drawn at random from the bag, what is the probability that it is i. Black, ii. White, iii. White or black and iv. Red.

Answer:

i. Let A be the event that the ball is black, then the number of outcomes favorable to A is 6. Hence

$P(A) = \frac{m}{n} = \frac{6}{10};$ $m =$ Favorable outcomes of an event A = Number of black balls

$n =$ Total number of outcomes of the experiment = Total number of balls

ii. Let B be the event that the ball is white, and then the number of outcomes favorable to B is 4. Hence

$P(B) = \frac{m}{n} = \frac{4}{10};$ $m =$ Favorable outcomes of an event B = Number of white balls

$n =$ Total number of outcomes of the experiment = Total number of balls

iii. Let C be the event that the ball is black or white and then the number of outcomes favorable to C is 10. Hence

$P(C) = \frac{m}{n} = \frac{10}{10};$ $m =$ Favorable outcomes of an event C = Number of white or white balls

$n =$ Total number of outcomes of the experiment = Total number of balls

iv. Let D be the event that the ball is red, and then the number of outcomes favorable to D is 0. Hence

$P(D) = \frac{m}{n} = \frac{0}{10};$ $m =$ Favorable outcomes of an event D = Number of red balls

$n =$ Total number of outcomes of the experiment = Total number of balls

Problem:

Two balanced dice, one black and one red are thrown and the number of dots on their upper faces are noted, let b be the outcomes of the black die and r be the outcomes of the red die. Now answer the following:

i. List a sample space of the experiment.
ii. What is the probability of throwing a double?
iii. What is the probability that the sum is 5, that is $b + r = 5$?
iv. What is the probability that the sum is even?
v. What is the probability that $r \leq 2$ or $b \leq 3$?
vi. What is the probability that the number on the red die is at least 4 greater than the number on the black dice?

Answer:

i. If two balanced dice, one black and one red are thrown and the number of dots on their upper faces are noted, also if b be the outcomes of the black die and r be the outcomes of the red die. Then the sample space for the given experiment will be as follows:

$r \Rightarrow$ $b \Downarrow$	1	2	3	4	5	6
1	1,1	1,2	1,3			1,6
2	2,1					
3						
4						
5						
6	6,1	6,2				6,6

ii. Let the event A = {the two dice shows the same number}

= {(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)}; There fore $P(A) = \frac{6}{36}$

iii. Let the event B = {The sum of the two dies is 5, that is $b + r = 5$ the two dice shows the same number}

= {(1,4), (2,3), (3,2), (4,1)}

There fore $P(B) = \frac{4}{36}$

iv. Let the event C = {The sum of the two dies is even}

= {(1,1), (1,3), (1,5), (2,2), (2,4), …, … , … ,(6,4), (6,6), }

There fore $P(C) = \frac{18}{36}$

v. Let the event D = {(b,r)| $r \leq 2$ or $b \leq 3$ }

= {(1,1), (1,2), (1,3), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (5,1), (5,2), (6,1), (6,2)}

There fore $P(D) = \frac{24}{36}$

vi. Let the event E = { $r \geq 4 + b$ } = {(1,5), (1,6), (2,6)}

There fore $P(E) = \frac{3}{36}$

Problem:

A card is drawn from a pack of 52 cards. Find the probability that it is i. A red card, ii. A spade, iii. An ace, iv. Not a spade and v. a king or a queen.

Answer:

When a card is drawn from a pack of 52 cards, the total number of equally likely and mutually exclusive outcomes are 52. That is here $n = 52$

i. Let A be the event drawing a red card. There are 26 black card and 26 red cards in a pack and any one of the red cards can be drawn in 26 ways. Hence

$$P(A) = \frac{m}{n} = \frac{26}{52};$$

$m =$ Favorable outcomes of an event A = Number of red cards balls

$n =$ Total number of outcomes of the experiment = Total number of cards

ii. Let B be the event drawing a spade. There are 13 spades. Hence

$$P(B) = \frac{m}{n} = \frac{13}{52};$$

$m =$ Favorable outcomes of an event B = Number of spades

$n =$ Total number of outcomes of the experiment = Total number of cards

iii. Let C be the event drawing an ace. There are 4 spades. Hence

$$P(C) = \frac{m}{n} = \frac{4}{52};$$

$m =$ Favorable outcomes of an event C = Number of ace

$n =$ Total number of outcomes of the experiment = Total number of cards

iv. Let D be the event drawing a card that is not a spade. There are 39 cards that is not spade. Hence

$$P(D) = \frac{m}{n} = \frac{39}{52};$$

$m =$ Favorable outcomes of an event D = Number of cards not spade

$n =$ Total number of outcomes of the experiment = Total number of cards

v. Let E be the event drawing a card will be either king or queen. There are 4 kings and 4 queens. Hence

$$P(D) = \frac{m}{n} = \frac{8}{52};$$

$m =$ Favorable outcomes of an event E = Number of kings and queens

$n =$ Total number of outcomes of the experiment = Total number of cards

Problem:

The following table gives a distribution of weekly wages of 4000 employes of a firm.

Wages in Tk.	Below 500	500-750	750-1000	1000-1250	1250-1500	1500-1750	1750 and above
No. of workers	36	472	1912	800	568	140	72

An individual is selected random. What is the probability that his wage are i) under Tk.750 ii) above Tk. 1250 iii) between Tk. 750 and 1250.

Ans: i) 0.127 ii) 0.195 iii) 0.678

Problem:
A box contains seven balls – two red, three blue and two yellow. Consider an experiment that consists of drawing a ball from the box.

1. What is the probability that the first ball drawn is yellow?
2. What is the probability that the same colored ball is drawn twice without replacement?
3. What is the probability that the same colored ball is drawn twice with replacement?

Answer:

1. P (R) = 2/7
2. P (RR) + P (BB) + P (YY) = [P (R)*P (R)] + [P (B)*P (B)] + [P (Y)*P (Y)] =((2/7 * 1/6) + (3/7 * 2/6) + (2/7 * 1/6)
3. (2/7 * 2/7) + (3/7 * 3/7) + (2/7 * 2/7)

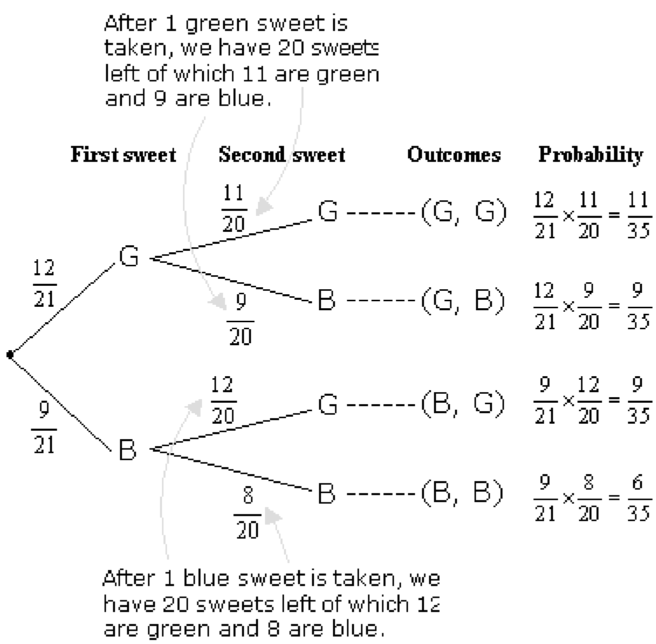
Example:

A jar consists of 21 sweets. 12 are green and 9 are blue. William picked two sweets at random.

a) Draw a tree diagram to represent the experiment.
b) Find the probability that
i) both sweets are blue.
ii) One sweet is blue and one sweet is green.
c) William randomly took a third sweet. Find the probability that:
i) All three sweets are green?
ii) At least one of the sweet is blue?

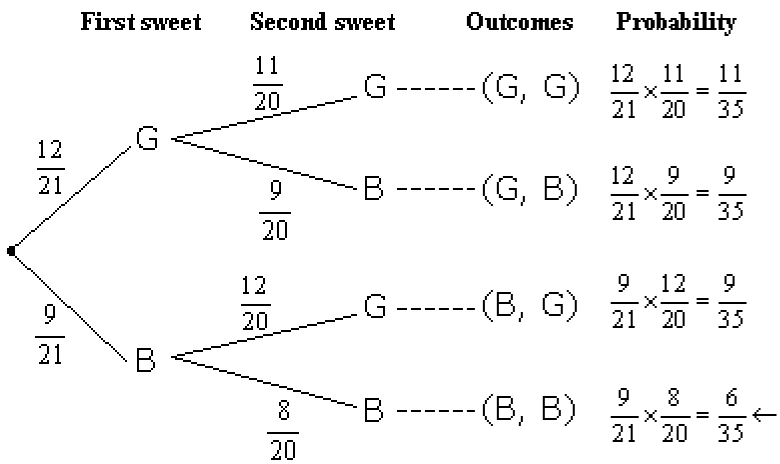
Solution:

a) Although both sweets were taken together it is similar to picking one sweet and then the second sweet without replacing the first sweet.



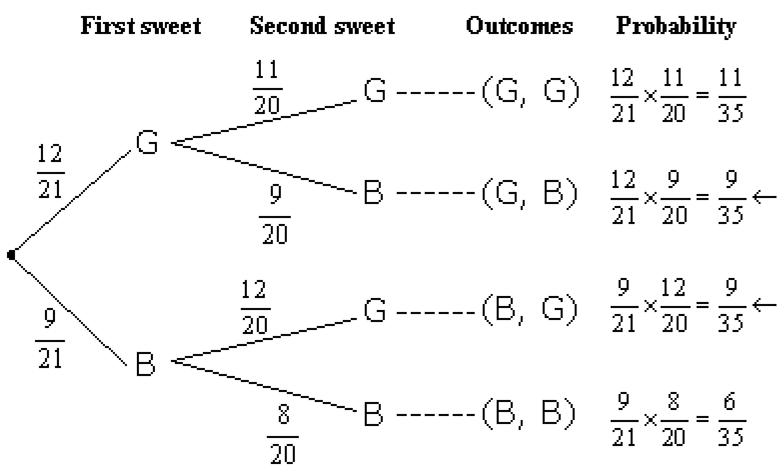
Check that the probabilities in the last column add up to 1.

$$\frac{11}{35}+\frac{9}{35}+\frac{9}{35}+\frac{6}{35}=1$$



b) i) P(both sweets are blue) = P(B, B)

$$=\frac{9}{21}\times\frac{8}{20}=\frac{6}{35}$$



ii) P(one sweet is blue and one sweet is green) = P(G, B) or P(B, G)

$$=\frac{9}{35}+\frac{9}{35}=\frac{18}{35}$$

c) i) P(all three sweets are green) = P(G, G, G)

$$= \frac{12}{21} \times \frac{11}{20} \times \frac{10}{19} = \frac{22}{133}$$

ii) P(at least 1 sweet is blue) = 1 – P(all three sweets are green)

$$= 1 - \frac{22}{133}$$

$$= \frac{111}{133}$$