# STA201 Assignment Solution 3-5 (Fall 2022)

**1. (a)** In a simultaneous throw of a pair of fair 6-sided dice, find the probability of getting:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | (1, 1) | (1, 2) | (1, 3) | (1, 4) | (1, 5) | (1, 6) |
| 2 | (2, 1) | (2, 2) | (2, 3) | (2, 4) | (2, 5) | (2, 6) |
| 3 | (3, 1) | (3, 2) | (3, 3) | (3, 4) | (3, 5) | (3, 6) |
| 4 | (4, 1) | (4, 2) | (4, 3) | (4, 4) | (4, 5) | (4, 6) |
| 5 | (5, 1) | (5, 2) | (5, 3) | (5, 4) | (5, 5) | (5, 6) |
| 6 | (6, 1) | (6, 2) | (6, 3) | (6, 4) | (6, 5) | (6, 6) |

(i) A sum of 8

For this event, E = { (2, 6), (3, 5), (4, 4), (5, 3), (6, 2) }

So, the probability of getting 8 as the sum, P(E) = $\frac{5}{36}$

(ii) A doublet (two dice landing on the same value)

For this event, E = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) }

So, the probability of getting a doublet, P(E) = $\frac{6}{36}$

(iii) A sum greater than 5

For this event, E = {(1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5), (3, 6), (4,2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6,4), (6,5), (6,6)}

So, the probability of getting a sum greater than 5, P(E) = $\frac{26}{36} = \frac{13}{18}$

(iv) An odd number on one and an even number on the other

For this event,

E = {(1,2), (1,4), (1,6), (2,1), (2, 3), (2, 5), (3,2), (3,4), (3,6), (4, 1), (4, 3), (4,5), (5, 2), (5, 4), (5, 6), (6, 1), (6, 3), (6, 5)}

So, the probability of getting an odd number on one and an even number on the other

P(E) = $\frac{18}{36} = \frac{1}{2}$

**(b)** A bag contains 30 balls numbered 1 through 30. Suppose drawing an even numbered ball is considered a 'Success'. Two balls are drawn from the bag with replacement. Find the probability of getting:

(i) Two successes

There are 15 even numbers from 1 to 30 and so the probability of selecting a ball having odd number in the first draw is, A = $\frac{15}{30}$.
Since two balls are drawn with replacement, for the second selection there are 15 even numbered balls. So the probability of selecting a ball having even number in the second draw is, B = $\frac{15}{30}$.
Here, the events A & B are independent.
Thus, the probability of getting two successes is,
P(E) = $\frac{15}{30} \times \frac{15}{30}$ = 0.25

(ii) Exactly one success

One even numbered ball can be selected either in the first draw or in the second draw.
Thus, the probability of getting exactly one success is,
P(E) = $\left(\frac{15}{30} \times \frac{15}{30}\right) + \left(\frac{15}{30} \times \frac{15}{30}\right)$ = 0.50

(iii) At least one success

It can happen during the draw that two even numbered balls are selected.Also, it can occur that one even numbered ball can be selected either in the first draw or in the second draw. Thus, the probability of getting at least one success,
P(E) = $\left(\frac{15}{30} \times \frac{15}{30}\right) + \left(\frac{15}{30} \times \frac{15}{30}\right) + \left(\frac{15}{30} \times \frac{15}{30}\right)$ = 0.75

(iv) No successes

Odd, no even numbered can be selected during the drawn and so, the probability of getting no successes is,
P(E) = $\frac{15}{30} \times \frac{15}{30}$ = 0.25

**2.** Assume that the chances of a patient suffering from high blood pressure is 60%. It is also assumed that a course of meditation reduces the risk of high blood pressure by 45% and prescription of certain drugs reduces its chances by 55%. At a time, a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options, the patient selected at random does not suffer from high blood pressure. Find the probability that the patient chose a course of meditation?

Let us define the events,
$A_1$: Person is treated with meditation
$A_2$: Person is treated with drugs
B: Person has high blood pressure

We have to find the probability that the patient followed a course of meditation given that the patient selected at random does not suffer from high blood pressure.

$$P(A_1|B') = \frac{P(B'|A_1)*P(A_1)}{P(B')}$$

It is given that, meditation and drug has equal probabilities,

$$P(A_1) = 0.5,\ P(A_2) = 0.5$$

We know the chances of having high blood pressure without any treatment is 60%

Meditation reduces the risk by 45% so the risk becomes 55% of the original.
Therefore, the probability of having high blood pressure if treated with meditation,
$P(B|A_1) = 0.60 \times 0.55 = 0.33$

The drug reduces the risk by 55% so the risk becomes 45% of the original.
Therefore, the probability of having high blood pressure, if treated with drugs,
$P(B|A_2) = 0.60 \times 0.45 = 0.27$

Therefore,
$P(B) = P(B|A_1) * P(A_1) + P(B|A_2) * P(A_2) = (0.33 ⍰ 0.5) + (0.27 ⍰ 0.5) = 0.3$

Finally,
$P(A_1|B') = \frac{P(B'|A_1)*P(A_1)}{P(B')} = \frac{(1-P(B|A_1))*P(A_1)}{1-P(B)} = \frac{(1-0.33)*0.5}{1-0.3} = \frac{67}{140} = 0.4786$

**3.** A discrete random variable $X$ has the following probability mass function

$$P(X = x) = \begin{cases} 2kx & x = 2, 4, 6 \\ k(x+2) & x = 8 \\ 0 & otherwise \end{cases}$$

where $k$ is a constant

a. Show that $k = \frac{1}{34}$
b. Find the exact value of $P(4 < x \leq 8)$
c. Find the exact value of $P(2 < x < 4)$
d. What is the expected value of the random variable $X$?
e. What is the variance of the random variable $X$?
f. Determine $Var(5 - 3X)$

**ANSWER:**

a) We know,
$\sum f(x) = 1$
Here,
$\sum f(x) = (2k \times 2) + (2k \times 4) + (2k \times 6) + (k \times 8 + 2) = 34k$
From the formula we can write,
$34k = 1$
$\therefore k = \frac{1}{34}$ [SHOWED]

b)

| $x$ | 2 | 4 | 6 | 8 |
|---|---|---|---|---|
| $P(X=x)$ | 4/34 | 8/34 | 12/34 | 10/34 |

$$\therefore P(4 < x \le 8) = \frac{12}{34} + \frac{10}{34} = \frac{22}{34} = \frac{11}{17}$$

c) $P(2 < x < 4) = 0$

d) Expected value of $X$,
$E(X) = \sum x \times f(x) = 2 \times \frac{4}{34} + 4 \times \frac{8}{34} + 6 \times \frac{12}{34} + 8 \times \frac{10}{34} = 5.647$

e) We know,
$Var(X) = E(X^2) - [E(X)]^2$
Here,
$E(X^2) = \sum x^2 \times f(x) = 2^2 \times \frac{4}{34} + 4^2 \times \frac{8}{34} + 6^2 \times \frac{12}{34} + 8^2 \times \frac{10}{34} = 35.765$
Therefore,
$Var(X) = E(X^2) - [E(X)]^2 = 35.765 - 5.647^2 = 3.876$

f) $Var(5 - 3X)$
$= Var(-3x)$
$= (-3)^2 \times Var(x)$
$= 9 \times 3.876 = 34.884$

**4.** When traveling from Bangladesh to Vietnam, travelers need to first land at Kuala Lumpur, and then get on a connecting flight to Vietnam. The total time in transit Y in hours can be shown to have the following:

$$f(Y = y) = \begin{cases} \frac{1}{20} y & 0 < y \le 4 \\ \frac{1}{30}(10 - y) & 4 < y \le 10 \\ 0 & otherwise \end{cases}$$

(i) What is the probability that total transit time is at most 6 hours?

$f(Y \le 6) = \int_0^4 \frac{1}{20} y dy + \int_4^6 \frac{1}{3} - \frac{1}{30} y dy$

$= \frac{1}{20} \times \left[\frac{y^2}{2}\right]_0^4 + \frac{1}{3} \times [y]_4^6 - \frac{1}{30} \times \left[\frac{y^2}{2}\right]_4^6$

$= \frac{11}{15} = 0.733$

(ii) What is the probability that the transit time is either less than 3 hours or more than 7 hours?

$$f(Y < 3) + f(Y > 7) = \int_0^3 \frac{1}{20} y dy + \int_7^{10} \frac{1}{3} - \frac{1}{30} y dy$$

$$= \frac{1}{20} \times \left[\frac{y^2}{2}\right]_0^3 + \frac{1}{3} \times [y]_7^{10} - \frac{1}{30} \times \left[\frac{y^2}{2}\right]_7^{10}$$

$$= \frac{3}{8} = 0.375$$

(iii) What is the expected total transit time for travelers going from Bangladesh to Vietnam?

$$E(Y) = \int_0^4 y \times \frac{1}{20} y dy + \int_4^{10} y \times \left(\frac{1}{3} - \frac{1}{30} y\right) dy$$

$$= \frac{1}{20} \times \left[\frac{y^3}{3}\right]_0^4 + \frac{1}{3} \times \left[\frac{y^2}{2}\right]_4^{10} - \frac{1}{30} \times \left[\frac{y^3}{3}\right]_4^{10}$$

$$= \frac{14}{3} = 4.667$$

(iv) Determine the standard deviation in the total transit time.

We Know,

$$E(Y^2) = \int_0^4 y^2 \times \frac{1}{20} y dy + \int_4^{10} y^2 \times \left(\frac{1}{3} - \frac{1}{30} y\right) dy$$

$$= \frac{1}{20} \times \left[\frac{y^4}{4}\right]_0^4 + \frac{1}{3} \times \left[\frac{y^3}{3}\right]_4^{10} - \frac{1}{30} \times \left[\frac{y^4}{4}\right]_4^{10}$$

$$= 26$$

$$Var(Y) = E(Y^2) - [E(Y)]^2$$

$$= 26 - \frac{14}{3}^2 = \frac{38}{9}$$

$$\therefore SD(Y) = \sqrt{Var(Y)} = \sqrt{\frac{38}{9}} = 2.055$$

**5.** In each of 4 races, the Democrats have a 60% chance of winning. Assuming that the races are independent of each other, what is the probability that:

(i) The Democrats will win 0 races, 1 race, 2 races, 3 races, or all 4 races?

Here, n=4, p=0.60, q= 0.40

$4c_0 * p^0 * q^{4-0}$ = 0.0256

$4c_1 * p^1 * q^{4-1}$ = 0.1536

$4c_2 * p^2 * q^{4-2}$ = 0.3456

$4c_3 * p^3 * q^{4-3}$ = 0.3456

$4c_4 * p^4 * q^{4-4}$ = 0.1296

(ii) The Democrats will win at least 1 race.

P(at least 1) = P(X ≥ 1) = 1 - P(none) = 1 - P(0) = 0.9744. Or,
P(1) + P(2) + P(3) + P(4) = 0.9744.

(iii) The Democrats will win a majority of the races.

P(Democrats will win a majority) = P(X ≥ 3) = P(3) + P(4) = 0.3456 +0 .1296 = 0.4752.

**6.** Suppose on average, Nepal experiences 6 earthquakes per year.

a. What is the mean number of earthquakes in Nepal in the first four month of a year?
b. What is the probability that there'll be 7 earthquakes in Nepal in the next two years?
c. What is the probability that there'll be at least 9 earthquakes in Nepal in 2021?

**ANSWER:**

a) Average number of earthquakes per 12 months, $\lambda_{12} = 6$

So, average number of earthquakes per 4 months, $\lambda_4 = \frac{6}{12} \times 4 = 2$

b) $\lambda_{12} = 6$

$\therefore \lambda_{24} = 6 \times 2 = 12$

$Let, X = Number\ of\ earthquakes$

$\therefore P(X = 7) = e^{-\lambda_{24}} \cdot \frac{\lambda_{24}^x}{x!} = e^{-12} \cdot \frac{12^7}{7!}$

c) $\lambda_{12} = 6$

$Let, X = Number\ of\ earthquakes$

$\therefore P(X \geq 9) = 1 - P(X < 9) = 1 - P(X \leq 8) = 1 - \left(\sum_{x=0}^{8} e^{-6} \cdot \frac{6^x}{x!}\right) = 0.1528$