# Shihab Muhtasim

STUDENT ID: 21301610

MAT 110
ASSIGNMENT 01
SET 15

f(x) will be continuous at x=0 if left hand limit = right hand limit,  $\lim_{x\to 0} f(x) = f(0)$  and f(0) is defined.

Now, L.H.L=
$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} x + 2 = 0 + 2 = 0$$
  
R.H.L= $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x^2 - 1 = 0 - 1 = -1$   
 $\therefore L.H.L \neq R.H.L$ 

Hence, the first condition of continuity is not fulfilled. f(x) is not continuous at x=0

 $\lim_{x\to 1} f(x)$  will exist if left hand limit = right hand limit,  $\lim_{x\to 1} f(x) = f(1)$  and f(1) is defined.

Now, L.H.L=
$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} x + 5 = 1 + 5 = 6$$

R.H.L=
$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x^3 - 4x = 1 - 4 = -3$$

$$\therefore L.H.L \neq R.H.L$$

Hence,  $\lim_{x\to 1} f(x)$  does not exist

Evaluating,

$$\begin{split} &\frac{d^2}{dx^2} \Big( \ln \left( \frac{x^4 + 2x^3}{x^2 + 1} \right) \Big) \\ &\Rightarrow \frac{d}{dx} \Big( \frac{d}{dx} \Big( \ln \left( \frac{x^4 + 2x^3}{x^2 + 1} \right) \Big) \\ &\Rightarrow \frac{d}{dx} \Big[ \Big( \frac{x^2 + 1}{x^4 + 2x^3} \Big) \cdot \frac{d}{dx} \Big( \frac{x^4 + 2x^3}{x^2 + 1} \Big) \Big] \\ &\Rightarrow \frac{d}{dx} \Big[ \Big( \frac{x^2 + 1}{x^4 + 2x^3} \Big) \cdot \frac{(x^2 + 1) \frac{d}{dx} (x^4 + 2x^3) - (x^4 + 2x^3) \frac{d}{dx} (x^2 + 1)}{(x^2 + 1)^2} \Big] \\ &\Rightarrow \frac{d}{dx} \Big[ \Big( \frac{x^2 + 1}{x^4 + 2x^3} \Big) \cdot \frac{(x^2 + 1)(4x^3 + 6x^2) - (x^4 + 2x^3)(2x)}{(x^2 + 1)^2} \Big] \\ &\Rightarrow \frac{d}{dx} \Big[ \frac{(x^2 + 1)(4x^3 + 6x^2) - 2x(x^4 + 2x^3)}{(x^4 + 2x^3)(x^2 + 1)} \Big] \\ &\Rightarrow \frac{d}{dx} \Big[ \frac{4x^3 + 6x^2}{x^4 + 2x^3} - \frac{2x}{x^2 + 1} \Big] \\ &\Rightarrow \frac{d}{dx} \Big[ \frac{4x + 6}{x^2 + 2x} - \frac{2x}{x^2 + 1} \Big] \\ &\Rightarrow \frac{d}{dx} \Big[ \frac{4x + 6}{x^2 + 2x} \Big) - \frac{d}{dx} \Big( \frac{2x}{x^2 + 1} \Big) \\ &\Rightarrow \frac{(x^2 + 2x) \frac{d}{dx} (4x + 6) - (4x + 6) \frac{d}{dx} (x^2 + 2x)}{(x^2 + 2x)^2} - \frac{(x^2 + 1) \frac{d}{dx} (2x) - (2x) \frac{d}{dx} (x^2 + 1)}{(x^2 + 1)^2} \\ &\Rightarrow \frac{(x^2 + 2x) \cdot 4 - (4x + 6) \cdot (2x + 2)}{(x^2 + 2x)^2} - \frac{(x^2 + 1) \cdot 2 - 2x \cdot (2x)}{(x^2 + 1)^2} \\ &\Rightarrow \frac{4x^2 + 8x - 8x^2 - 20x - 12}{(x^2 + 2x)^2} - \frac{2x^2 + 2 - 4x^2}{(x^2 + 1)^2} \end{aligned}$$

$$\Rightarrow -\frac{4x^2 + 12x + 12}{(x^2 + 2x)^2} + \frac{2x^2 - 2}{(x^2 + 1)^2}$$

$$\Rightarrow \frac{2x^2 - 2}{(x^2 + 1)^2} - \frac{4x^2 + 12x + 12}{(x^2 + 2x)^2}$$

$$\Rightarrow \frac{(2x^2 - 2)(x^2 + 2x)^2 - (4x^2 + 12x + 12)(x^2 + 1)^2}{(x^2 + 1)^2(x^2 + 2x)^2}$$

$$\Rightarrow \frac{(2x^2 - 2)(x^4 + 4x^3 + 4x^2) - (4x^2 + 12x + 12)(x^4 + 2x^2 + 1)}{x^2(x^2 + 1)^2(x + 2)^2}$$

$$\Rightarrow \frac{2x^6 + 8x^5 + 8x^4 - 2x^4 - 8x^3 - 8x^2 - 4x^6 - 12x^5 - 12x^4 - 8x^4 - 24x^3 - 24x^2 - 4x^2 - 12x - 12}{x^2(x^2 + 1)^2(x + 2)^2}$$

$$\Rightarrow \frac{-2x^6 - 4x^5 - 14x^4 - 32x^3 - 36x^2 - 12x - 12}{x^2(x^2 + 1)^2(x + 2)^2}$$

Given, 
$$(x - y)^2 = x + y - 1$$
  

$$\Rightarrow x^2 - 2xy + y^2 = x + y - 1$$

$$\Rightarrow \frac{d}{dx}(x^2 - 2xy + y^2) = \frac{d}{dx}(x + y - 1)$$

$$\Rightarrow 2x - 2y + 2y\frac{dy}{dx} - 2x\frac{dy}{dx} = 1 + \frac{dy}{dx} - 0$$

$$\Rightarrow 2y\frac{dy}{dx} - 2x\frac{dy}{dx} - \frac{dy}{dx} = 1 - 2x + 2y$$

$$\Rightarrow \frac{d}{dx}(2y - 2x - 1) = 2y - 2x + 1$$

$$\Rightarrow \frac{d}{dx} = \frac{2y - 2x + 1}{2y - 2x - 1}$$

$$\therefore \frac{d}{dx} = \frac{2y - 2x + 1}{2y - 2x - 1}$$

We have to evaluate,

$$\frac{d}{dx}(\sin^2\frac{(2x)}{(x+1)})$$
Let,
$$a = \frac{2x}{x+1}$$

$$\Rightarrow a = 2x(x+1)^{-1}1$$

$$\Rightarrow \frac{d}{dx}a = \frac{d}{dx}2x(x+1)^{-1}1$$

$$\Rightarrow \frac{d}{dx}(a) = 2(x+1)^{-1}1 + 2x(-1)(x+1)^{-2}2$$

$$\Rightarrow \frac{d}{dx}a = \frac{2}{x+1} - \frac{2x}{(x+1)^2}$$

$$\Rightarrow \frac{d}{dx}a = \frac{2(x+1)-2x}{(x+1)^2}$$

$$\Rightarrow \frac{d}{dx}a = \frac{2x+2-2x}{(x+1)^2}$$

$$\Rightarrow \frac{d}{dx}a = \frac{2}{(x+1)^2}$$

Evaluating,

$$\frac{d}{dx}(\sin^2\frac{(2x)}{(x+1)}) 
= \frac{d}{dx}(\sin^2 a) 
= 2\sin a\cos a\frac{d}{dx}a 
= \sin 2a\frac{d}{dx}a 
= \sin(\frac{2\cdot 2x}{x+1}) \cdot \frac{2}{(x+1)^2} 
= \frac{2\sin(\frac{4x}{x+1})}{(x+1)^2} 
We get,  $\frac{d}{dx}(\sin^2\frac{(2x)}{(x+1)}) = \frac{2\sin(\frac{4x}{x+1})}{(x+1)^2}$$$

## Ans to the question no 06 (a)

Given,

$$P(t) = \frac{M}{1 + Ae^{-}kt}$$

$$\lim_{t \to \infty} P(t) = \lim_{t \to \infty} \frac{M}{1 + Ae^{-}kt}$$

$$\Rightarrow \lim_{t \to \infty} P(t) = \lim_{t \to \infty} \frac{M}{1 + \frac{A}{e^{k}t}}$$

$$\Rightarrow \lim_{t \to \infty} P(t) = \frac{M}{1 + \frac{A}{e^{k}\infty}}$$

$$\Rightarrow \lim_{t \to \infty} P(t) = \frac{M}{1 + \frac{A}{\infty}}$$

$$\Rightarrow \lim_{t \to \infty} P(t) = \frac{M}{1} \Rightarrow \lim_{t \to \infty} P(t) = M$$

The answer M refers to the maximum population size that can be carried and when t tends to infinity it means that the carrying capacity becomes the maximum population size that can be supported. That's why the answer M is to be expected

# Ans to the question no 06 (b)

Given,

$$P(t) = \frac{M}{1 + Ae^{-}kt}$$

$$\lim_{M \to \infty} P(t) = \lim_{M \to \infty} \frac{M}{1 + Ae^{-}kt}$$

$$\Rightarrow \lim_{M \to \infty} P(t) = \lim_{t \to \infty} \frac{M}{1 + (\frac{M - P_o}{P_o})e^{-}kt}$$

$$\Rightarrow \lim_{M \to \infty} P(t) = \lim_{M \to \infty} \frac{MP_o}{P_o(1 - e^{-}kt) + Me^{-}kt}$$

$$\Rightarrow \lim_{M \to \infty} P(t) = \lim_{M \to \infty} \frac{P_o}{e^{-}kt}$$

$$\Rightarrow \lim_{M \to \infty} P(t) = P_o e^{kt}$$

The result is  $P_0e^{kt}$  which is an exponential function

















