PHY 112

MIDTERM EXAM

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sec: 8

Ans to the qr no 1

a

Given, $q_1 = 32 \times 10^{-6}$ C, $P_1\,(0, 12)$

$q_3 = 4 \times 10^{-6}$ C, $P_3\,(-22, 0)$

$\vec{P_1} = 0\hat{i} + 12\hat{j}$

$\vec{P_3} = -22\hat{i} + 0\hat{j}$

$\vec{r}_{P_1, P_3} = \vec{P_3} - \vec{P_1}$

$= -22\hat{i} - 12\hat{j}$

$= -0.22\hat{i} - 0.12\hat{j}$

b

Given, $q_2 = 20\times10^{-6}\,C$

$$\vec{P_2} = -12\hat{i} + 30\hat{j}$$

$$\vec{r}_{P_3,P_2} = \vec{P_2} - \vec{P_3}$$

$$= -12\hat{i} + 22\hat{i} + 30\hat{j}$$

$$= 10\hat{i} + 30\hat{j}$$

$$= 0.1\hat{i} + 0.3\hat{j}$$

$$\vec{F}_{P_3,P_2} = \frac{k\, q_2\, q_3}{\left(|\vec{r}_{P_3,P_2}|\right)^3} \times \vec{r}_{P_3,P_2}$$

$$= \frac{9\times10^{9}\times20\times10^{-6}\times4\times10^{-6}}{\sqrt{(0.1)^2+(0.3)^2}} \times (0.1\hat{i}+0.3\hat{j})$$

$$= \frac{0.72}{0.316}\times(0.1\hat{i}+0.3\hat{j})$$

$$= 2.2768\,(0.1\hat{i}+0.3\hat{j})$$

3

$~~0.2276~~\hat{i}$

$= 0.0276\hat{i} + 0.08304\hat{j}$

C

$\vec{r}_{1,2} = \vec{P_2} - \vec{P_1}$

$= -12\hat{i} + 30\hat{j} - 12\hat{j}$

$= -0.12\hat{i} + 0.18\hat{j}$

$|\vec{r}_{12}| = \sqrt{(0.12)^2 + (0.18)^2} = 0.216$

$\vec{F}_{1,2} = \frac{k q_1 q_2}{(r_{12})^3} \times \vec{r}_{12}$

$= \frac{9\times10^9 \times 32\times10^{-6} \times 20\times10^{-6}}{(0.216)^3} \times \vec{r}_{12}$

$= 571.559 \, (-0.12\hat{i} + 0.18\hat{j})$

$= -68.587\hat{i} + 102.880\hat{j}$

$$\vec{F}_{net,2} = \vec{F}_{1,2} + \vec{F}_{3,2}$$

$$= (0.0276 - 68.587)\hat{i} + (0.08304 + 102.880)\hat{j}$$

$$= -68.5594\hat{i} + 102.963\hat{j}$$

d

$r_{2,4} = 0.15\ m$

If $F = 0$,

$$F = \frac{k\, q_4\, q_2}{r^2}$$

$$\Rightarrow 0 = \frac{9\times10^{9} \times q_4 \times 20}{(0.15)^2}$$

$$\Rightarrow q_4 = 0\ \mu C$$

d

$\vec{F}_{net,2} = \sqrt{(-68.559)^2 + (102.963)^2}$

$= 123.7001$

$r_{2,4} = 0.15 \text{ m}$

Now, $F = \dfrac{k q_4 q_2}{r_{24}^2}$

$\Rightarrow 123.700 = \dfrac{9\times10^9 \times q_4 \times 20\times10^{-6}}{(0.15)^2}$

$\Rightarrow q_4 = \dfrac{123.7 \times (0.15)^2}{9\times10^9 \times 20\times10^{-6}}$

$\Rightarrow q_4 = 1.54625 \times 10^{-5}$ C

$\Rightarrow q_4 = 15.4624$ C

Ans to the Q no 3

a

Given,

$Q = 35 \times 10^{-6}$ C , $Q' = 23 \times 10^{-6}$ C

$R_2 = 0.11$ m , $R_1 = 0.05$ m

Charge on inner surfac, $= Q_{in}$

$Q_{in} + Q' = 0$

$\Rightarrow Q_{in} = -Q' = -23 \times 10^{-6}$ C

Charge on the outer surface $= Q_{out}$

$Q_{out} + Q_{in} = Q$

$$\Rightarrow Q_{out} = Q - Q_{in}$$
$$= Q - (-Q')$$
$$= Q + Q'$$
$$= 35 \times 10^{-6} + 23 \times 10^{-6}$$
$$= 5.8 \times 10^{-5} \text{ C}$$

$\therefore$ charge density of outer surface,

$$\sigma Q_{out} = \frac{Q_{out}}{4\pi r_2^2} = \frac{5.8 \times 10^{-5}}{4\pi (0.11)^2}$$

$$\doteq 3.8144 \times 10^{-4} \ C/m^2$$

$\therefore$ charge density on inner surface,

$$\sigma Q_{in} = \frac{Q_{in}}{4\pi r_1^2}$$

$$= \frac{-23 \times 10^{-6}}{4\pi (0.05)^2}$$

$$\doteq -7.3211 \times 10^{-4} \ C/m^2$$

b

Given, $r_a = 0.17\,m$, $r_b = 0.2\,m$

charge, $q = 35\times10^{-6}\,C$, $q' = 23\times10^{-6}\,C$

Let charge on inner surface = Q_{in}

Since its a conducting shell,

Q_{in} + total charge enc = 0

$\Rightarrow Q_{in} + q + q' = 0$

$\Rightarrow Q_{in} = -(q + q')$

$= -(35\times10^{-6} + 23\times10^{-6})$

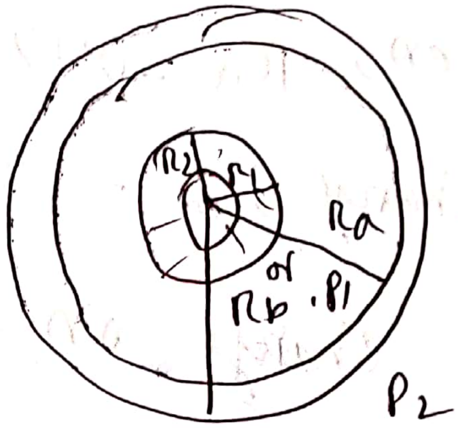
$= -5.8\times10^{-6}\,C$

charge density on new spherical shell of the inner surface,

$\sigma_{inner} = \frac{Q_{in}}{4\pi r_a^2} = \frac{-5.8\times10^{-5}}{4\pi(0.17)^2}$

$= -1.597\times10^{-4}\,C/m^2$

C.



$r_a = 0.17\ m$

$r_b = 0.2\ m$

$r_1 = 0.05\ m$

$r_2 = 0.11\ m$

Given, $r_{P_1} = 0.14\ m$

$r_{P_2} = 0.24\ m$

$\therefore\ r_1 < r_{P_1} < r_a \ \therefore\ Q_{enc} = q + q'$

$= (35 + 23) \times 10^{-6}$

$= 5.8 \times 10^{-5}\ C$

$$\oint E\,dA = \frac{Q_{enc}}{\epsilon_0}$$

$$\Rightarrow E_{r_{P_1}} 4\pi r_{P_1}^2 = \frac{5.8 \times 10^{-5}}{\epsilon_0}$$

$$\Rightarrow E_{P_1} = \frac{5.8 \times 10^{-5}}{4\pi\epsilon_0 (0.14)^2} = 26.608 \times 10^{6}\ N/C$$

Again $r_{P_2} > r_b$. $Q_{enc} = \sigma + \sigma'$

$$= (35+23) \times 10^{-6}$$

$$= 5.8 \times 10^{-5} \, C$$

$$\Rightarrow \oint E_{rP_2} \cdot dA = \frac{Q_{enc}}{\epsilon_0}$$

$$\Rightarrow E_{rP_2} A = \frac{Q_{enc}}{\epsilon_0}$$

$$\Rightarrow E_{rP_2} = \frac{Q_{enc}}{\epsilon_0 A} = \frac{5.8 \times 10^{-5}}{4\pi\epsilon_0 (0.24)^2}$$

$$= 9.054 \times 10^{6} \, N/C$$

$$E_{rP_1} = 26.608 \times 10^{6} \, N/C$$

$$E_{rP_2} = 9.054 \times 10^{6} \, N/C$$

d

The spheres are conducting so the E inside sphere is 0. And will be maximum on the surface as $E \propto \frac{1}{r^2}$

so, if $r < r_1$, $E = \frac{q'}{4\pi\epsilon_0 r^2}$

if $r_1 < r < R_2$; $E = 0$

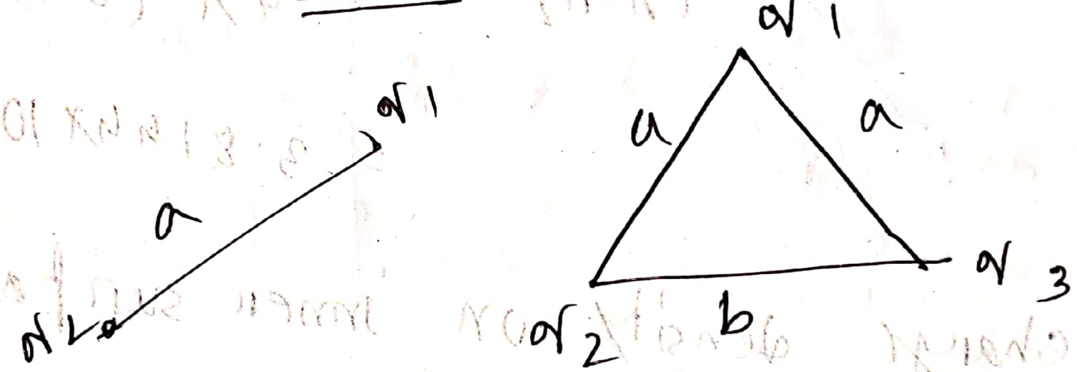
if $R_2 < r < r_a$, $E = \frac{q + q'}{4\pi\epsilon_0 r^2}$

if $r_a < r < r_b$; $E = 0$

if $r > r_b$; $E = \frac{q + q'}{4\pi\epsilon_0 r^2}$

Ans to the q no 4

$$\underline{\underline{a}}$$



$q_1 = 28 \times 10^{-6}$ C

$q_2 = 23 \times 10^{-6}$ C

$q_3 = 21 \times 10^{-6}$ C

$a = 0.16$ m, $b = 0.05$ m

To bring q_1 and q_2 together,

$$\text{work done} = k\frac{q_1 q_2}{a}$$

$$= \frac{9 \times 10^{9} \times 28 \times 10^{-6} \times 23 \times 10^{-6}}{0.16}$$

$$= 36.225 \text{ J}$$

To bring q_3 and the pair of q_1, q_2 together, work done =

$$\frac{k\,q_1 q_3}{a} + \frac{k\,q_2 q_3}{b}$$

$$= \frac{21\times10^{-6}\times 9\times10^{9}\times 28\times10^{-6}}{0.16} + \frac{9\times10^{9}\times 23\times10^{-6}\times 21\times10^{-6}}{0.05}$$

$$= 33.075 + 86.94$$

$$= 120.015\ J$$

$\Rightarrow$ Total work done $= \dfrac{k\,q_1 q_2}{a} + \dfrac{k\,q_1 q_3}{a} + \dfrac{k\,q_2 q_3}{b}$

$$= 36.225 + 120.015$$

$$= 156.24\ J$$

$\Rightarrow$ Total work $= 156.24\ J$

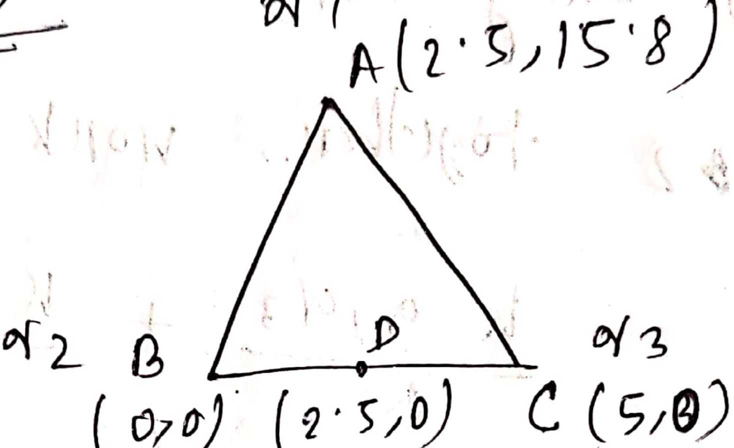
(b)

Here,

$AD^2 = AB^2 - BD^2$

$\sqrt{16^2 - 2.5^2}$

$= 15.8$



$\therefore P = \frac{2.5+0+5}{3}, \frac{15.8+0+0}{3}$

$= (2.5, 5.27)$

$AP = 15.8 - 5.27 = 10.53 = 0.1053\ m$

$BP = \sqrt{(0-2.5)^2 + (0-5.27)^2}$

$= \sqrt{2.5^2 + 5.27^2} = 5.8$

$= 0.058\ m$

$CP = 0.058\ m$

potential at $P = V_{q_1} + V_{q_2} + V_{q_3}$

$$= \frac{Kq_1}{AP} + \frac{Kq_2}{BP} + \frac{Kq_3}{CP}$$

$$= K\left(\frac{28\times10^{-6}}{0.1053} + \frac{23\times10^{-6}}{0.058} + \frac{21\times10^{-6}}{0.058}\right)$$

$$= \cancel{10^9}\ 9\times10^9 \times 1.0245\times10^{-3}$$

$$= 9220748.6\ V$$

e

All of the charges will be enclosed by sphere. Hence,

$$\phi_E = \frac{Q_{net}}{\varepsilon_0}$$

$$= \frac{q_1 + q_2 + q_3}{\varepsilon_0}$$

$\cancel{\Phi}$ $\cancel{\phi}$

$$\phi_E = \frac{28 \times 10^{-6} + 23 \times 10^{-6} + 21 \times 10^{-6}}{8.854 \times 10^{-12}}$$

$$\cancel{\phi_E = 8.1319 \times 10^{-18}} \ Nm^2/C$$

$$\phi_E = 8131917.777 \ Nm^2/C$$