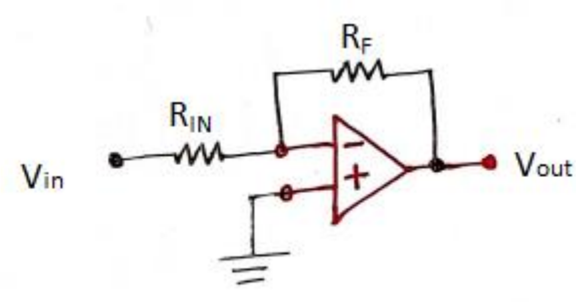
Proof of $V_{OUT} = -\frac{R_F}{R_{IN}} V_{IN}$ **, without considering the input voltages as equal.**



The op-amp's transfer function is:

$$V_{OUT} = G \times (V_+ - V_-)$$

where G is the op-amp's gain. I could leave it as G, but we'll use a real value instead. 100000 is a typical value. The actual value shouldn't matter, 200000 would also be a possibility, that means that we'll probably have to get rid of it during the calculation.
V_+V+ is 0V, so

$$V_{OUT} = -100000 \times V_-$$

An assumption we *can* make is that the input current is negligible. It's one of the ideal op-amp's axioms, and a datasheet will confirm it. Then, according to KCL:

$$\frac{V_{IN} - V_{-}}{R_{IN}} = \frac{V_{-} - V_{OUT}}{R_F}$$

or

$$\left(\frac{1}{R_F} + \frac{1}{R_{IN}}\right) V_{-} = \frac{V_{IN}}{R_{IN}} + \frac{V_{OUT}}{R_F}$$

then

$$V_{-} = V_{IN}\left(\frac{R_F}{R_{IN} + R_F}\right) + V_{OUT}\left(\frac{R_{IN}}{R_{IN} + R_F}\right)$$

Filling this in in our transfer function:

$$V_{OUT} = -100000\left(V_{IN}\left(\frac{R_F}{R_{IN} + R_F}\right) + V_{OUT}\left(\frac{R_{IN}}{R_{IN} + R_F}\right)\right)$$

Looks nasty, but we'll be alright in a minute!

$$V_{OUT}\left(1 + 100000\left(\frac{R_{IN}}{R_{IN} + R_F}\right)\right) = -100000 \times V_{IN}\left(\frac{R_F}{R_{IN} + R_F}\right)$$

Now you see why we like to use the 100000 value: you can easily see that the "1" can be neglected. If G isn't much greater than 1, the whole reasoning becomes invalid.

$$V_{OUT} = \frac{-100000 \times V_{IN}\left(\frac{R_F}{R_{IN} + R_F}\right)}{100000\left(\frac{R_{IN}}{R_{IN} + R_F}\right)}$$

Now we can cancel a lot, including the op-amp's gain factor, and what remains is

$$V_{OUT} = -\frac{R_F}{R_{IN}} V_{IN}$$

That's the inverting amplifier's transfer function!

If you replace the V_{OUT} in the equation for V_{-} by this value you'll find

$$V_{-} = 0V$$

So, the input voltages are indeed equal, but only as a consequence of the proof.