Set

Ans to the qr no 1

Given expression,

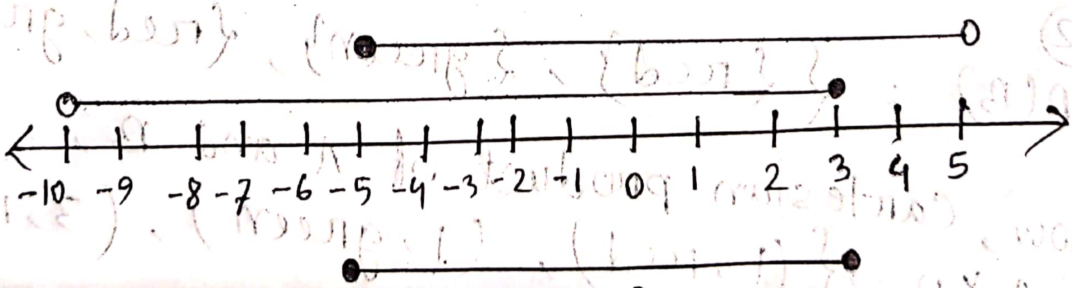
$(-10, 3] \cap [-5, 5)$

$= \{x \in R : -10 < x \leq 3\} \cap \{x \in R : -5 \leq x < 5\}$

$= \{x \in R : -5 \leq x \leq 3\}$

set buider notation expression $= \{x \in R : -5 \leq x \leq 3\}$

Number line:



$-5 \leq x \leq 3$

Ans to the qr no 2

Given, $A = \{1, 3, 5\}$

$B = \{red, green\}$

since $|A| = 3$, there are $2^3 = 8$ subsets in power set of A, P(A).

(1)

$P(A) = \{ \{1\}, \{3\}, \{5\}, \{1,3\}, \{1,5\}, \{3,5\}, \{1,3,5\}, \emptyset \}$

since $|B| = 2$, there are $2^2 = 4$ subsets, in P(B), power set of B.

(2)

$P(B) = \{ \{red\}, \{green\}, \{red, green\}, \emptyset \}$

Now, cartesian product of A and B,

(3) $A \times B = \{ (1, red), (1, green), (3, red), (3, green), (5, red), (5, green) \}$

(4)

Cardinality of $A \times B$, $|A \times B| = 6$

Ans to the qr no 3

De Morgan law : $A' \cap B' = (A \cup B)'$

Now,

$$A' \cap B' = \{x : x \in (A' \cap B')\}$$
$$= \{x : x \notin A \text{ and } x \notin B\}$$
$$= \{x : x \notin (A \cup B)\}$$
$$= \{x : x \in (A \cup B)'\}$$
$$= (A \cup B)'$$

Again, $(A \cup B)' = \{x : x \in (A \cup B)'\}$
$$= \{x : x \notin (A \cup B)\}$$
$$= \{x : x \notin A \text{ and } x \notin B\}$$
$$= \{x : x \in A' \text{ and } x \in B'\}$$
$$= \{x : x \in A' \cap B'\}$$
$$= A' \cap B'$$

$\therefore A' \cap B' \subseteq (A \cup B)'$ and $(A \cup B)' \subseteq A' \cap B'$

$\therefore A' \cap B' = (A \cup B)'$

Ans to the qr no 4

Given,

Total members, $|U| = 105$

Travellers who visited India, $|I| = 50$

Travellers who visited Nepal, $|N| = 30$

Travellers who visited Bhutan, $|B| = 20$

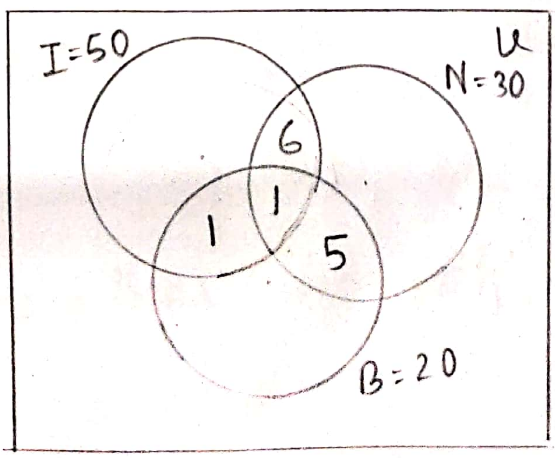
visited both India and Nepal, $|I \cap N| = 6$

visited both India and Bhutan, $|I \cap B| = 1$

visited both Nepal and Bhutan, $|N \cap B| = 5$

visited all countries, $|I \cap N \cap B| = 1$

Total travellers who visited at least one country, $|I \cup N \cup B|$



from Inclusion - exclusion principal,

$$|I \cup B \cup N| = |I| + |N| + |B| - |I \cap N| - |I \cap B| - |N \cap B| + |I \cap B \cap N|$$

$$= 50 + 30 + 20 - 6 - 1 - 5 + 1$$

$$= 89$$

$\therefore$ Total members, $|U| = 105$

Members who visited at least one country,

$$|I \cup B \cup N| = 89$$

$\therefore$ Travellers who did not visit any country,

$$|I \cup B \cup N|^c = |U| - |I \cup B \cup N|$$

$$= 105 - 89 = 16$$

$\therefore$ 16 travellers did not visit any country.

Ans to the Qn no 5

Given relation = {(1,2), (5,6), (8,6), (7,2), (9,2), (8,6)}

Since every set has a unique element there can be only one pair of (8,6).

$\therefore R = \{(1,2), (5,6), (8,6), (7,2), (9,2)\}$

Domain	Range
1	2
5	6
8	6
7	2
9	2

In order to be a function, every domain can have only one range and two diffrent domain can have same range.

Here, every domain is diffrent in this relation. So, we get only one range for every diffrent domain.

$\therefore$ This relation is a function.

Ans to the Q no 6

Given, $f(x) = \cos(4x-1)$

Here, $f(x) = y$

$\Rightarrow x = f^{-1}(y)$ - - - (1)

Now, $y = \cos(4x-1)$

$\Rightarrow 4x-1 = \cos^{-1}(y)$

$\Rightarrow x = \dfrac{\cos^{-1}(y)+1}{4}$

$\Rightarrow f^{-1}(y) = \dfrac{\cos^{-1}(y)+1}{4}$

$\Rightarrow f^{-1}(x) = \dfrac{\cos^{-1}(x)+1}{4}$

Here, For this $f^{-1}(x)$ function will be defined for all the values of x for which $\cos^{-1}(x)$ is defined.

$\therefore$ Domain of $f^{-1}(x)$, $D_{f^{-1}} = \{x \in R : -1 \leq x \leq 1\}$

$\therefore$ Range of $f(x)$, $R_f = \{y \in R : -1 \leq y \leq 1\}$

$= [-1, 1]$

Again, $f(x) = \cos(4x-1)$

Here, $f(x)$ is defined for all the values of x that defines $\cos(4x-1)$.

$\therefore$ $\cos(4x-1)$ is defined for all real numbers.

$\therefore$ Domain of $f(x)$, $D_f = \{x : x \in R\}$, $(-\infty, \infty)$

Ans to the qr no 7

Given, $f(x) = \log(x^2 - 3)$

Since log is undefined for negative value or zero of x,

Domain contains values, $x^2 - 3 > 0$

$\Rightarrow x^2 > 3$

$\Rightarrow x > \pm\sqrt{3}$

$\therefore x > \sqrt{3}$ or $x < -\sqrt{3}$

$\therefore$ Domain of $f(x)$, $D_f = \{x : x \in R, x > \sqrt{3}, x < -\sqrt{3}\}$

$= (-\infty, \sqrt{3}) \cup (\sqrt{3}, \infty)$

Ans to the Q no 8

Given,

$$f(x) = \frac{x-2}{x^2-8x+8}$$

Domain will be all the values of x for which,

$$x^2-8x+8 \neq 0$$

Comparing this equation with $ax^2+bx+c \neq 0$,

$$x_1 \neq \frac{-(-8) \pm \sqrt{(-8)^2-4.1.8}}{2(1)}$$

$$\Rightarrow x \neq \frac{8 \pm 4\sqrt{2}}{2} = 4 \pm 2\sqrt{2}$$

$\therefore x \neq 4+2\sqrt{2}$ and $x \neq 4-2\sqrt{2}$

$$D_f = \{x : x \in R, x \neq 4+2\sqrt{2}, x \neq 4-2\sqrt{2}\}$$

Therefore, The domain of $f(x)$, $(-8, -4) \cup (-4+\infty)$ is not correct.

Ans to the qno 9

Since there are total 3 pairs of siblings. To arrange them in the second row, we have to pick one of the siblings from each pair in $3P_3 = 6$ ways.

Again, for picking one of the siblings from each pair there are $= 2^3 = 8$ ways.

$\therefore$ Arrangement in second row $= (6 \times 8) = 48$ ways

Now, In third row only two places are left for the siblings to sit fullfilling the condition that siblings can't sit beside.

$\therefore$ Total seat arrangment $= (48 \times 2) = 96$ ways.

Ans to the qr no 10

From given data we see;

$5 = 5^1$, has total two divisor's $(1+1) = 2$

$6 = 2^1 \times 3^1$, has four divisors $(1+1)+(1+1) = 4$

$16 = 2^4$, has total five divisor $(4+1) = 5$

$\therefore$ The summation of the exponents of each number's prime number products, plus 1 is the amount of divisors a number has.

Here, $n = P_1^{\infty_1} \times P_2^{\infty_2} \times P_3^{\infty_3} \times \dots \times P_{k-1}^{\infty_{k-1}} \times P_k^{\infty_k}$

Number of divisors of n is,

$\infty_1 + 1 + \infty_2 + 1 + \infty_3 + 1 + \dots \infty_{k-1} + 1 + \infty_k + 1$

since, total exponents $= k$

Divisors of $n = \infty_1 + \infty_2 + \infty_3 + \dots \infty_{k-1} + \infty_k + k$

Ans to the qr no 11

For a 5 digit positive integer to be divisible by 5,

First digit can be filled in 9 ways

Last digit can be filled in 2 ways

∴ 5 digit numbers amount that are divisible by 5 $= 9 \times 10 \times 10 \times 10 \times 2$

$= 18000$

Now, Excluding one number, 6 from the first four digits we can get all the five digit numbers divisible by 5 and has no 6 in it $= 8 \times 9 \times 9 \times 9 \times 2$

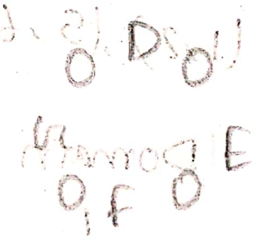
$= 11664$

∴ 5 digit numbers divisible by 5 and have at least one 6 digit $= 18000 - 11664$

$= 6336$

Ans to the Qr no 12

In a round table A, B, C can't sit next to each other. So they have to sit between D, E, F, G. So they will have four empty seats.



A, B, C can seat in the four empty seats in = $^4P_3$ ways.

Again, D, E, F, G can arrange in $(4-1)!$ ways

$\therefore$ Total seatable ways = $(4-1)! \times {}^4P_3$

= 144 ways