

## Binomial Distribution

**Problem:** Suppose that the probability that an adult person is suffering from diabetes in a population is 0.10. Suppose that we randomly select a sample of 6 persons.

- (1) Find the probability distribution of the random variable (X) representing the number of persons with diabetes in the sample.
- (2) Find the expected number of persons with diabetes in the sample (mean of X).
- (3) Find the variance of X.
- (4) What is the probability that there will be exactly 2 persons with diabetes?
- (5) What is the probability that there will be at most 2 persons with diabetes?
- (6) What is the probability that there will be at least 4 persons with diabetes?

### **Solution:**

We are interested in the following random variable:

$X$  = the number of persons with diabetes in the sample of 6 persons.

Notes:

- (i) Bernoulli trial: diagnosing whether a person has diabetes or not. There are two outcomes for each trial:
- (ii) Number of trials = 6, or  $n=6$ .
- (iii) Probability of success:  $P(S) = p = 0.10$ .
- (iv) Probability of failure:  $P(F) = q = 1 - p = 0.9$ .
- (v) The trials are independent because of the fact that the result of each person does not affect the result of any other person since the selection was made at random.

The random variable,  $X$ , has a binomial distribution with parameters:  $n=6$  and  $p=0.10$ , that is:

$$X \sim \text{Binomial}(n, p),$$

$$X \sim \text{Binomial}(6, 0.10).$$

The possible values of  $X$  are  $X=x$ , where

$$x = 0, 1, 2, 3, 4, 5, 6$$

Try yourself

More problems on Binomial:

- Light bulbs are tested for their life-span. It is found that 4% of the light bulbs are rejected. A random sample of 15 bulbs is taken from stock and tested. The random variable X is the number of bulbs that are rejected.
  - Give four reasons why X will have a binomial distribution.
  - Use a formula to find the probability that 2 light bulbs in the sample are rejected.
- Based on previous results a marksman firing at a target hits the bull's eye with one in every four shots. If he fires at the target 7 times what is the probability that he will hit the bull's eye:
  - Four times
  - Never
  - At least five times
- If 95% of households have a TV and 8 houses are surveyed, what is the probability that more than 6 have a TV?

### Poisson Distribution

- Use the Poisson probability formula  $P(X = x) = \frac{e^{-\lambda} \lambda^x}{x!}$  to find the following probabilities for the distribution X:
  - $P(X = 2)$  when  $\lambda = 3$
  - $P(X = 1)$  when  $\lambda = 0.5$
  - $P(X = 0)$  when  $\lambda = 1.2$
- Use the Poisson Distribution tables to find the following probabilities:
  - $P(X = 5)$  when  $\lambda = 3$
  - $P(X = 1)$  when  $\lambda = 0.7$
  - $P(X < 3)$  when  $\lambda = 2$
- A stunt person injures himself an average of three times a year. Use the Poisson probability formula to calculate the probability that he will be injured:
  - 4 times a year
  - Less than twice this year.
  - More than three times this year.
  - Once in the six months.
- Occasionally, a machine producing steel tools needs to be reset. The random variable Y is the number of resets in a month and is modelled by a Poisson distribution. The mean number of resets needed per month has been found to be 6. Find the probability that:
  - 7 resets per month are needed.
  - Fewer than 3 resets per month are needed.
  - More than 4 resets per month are needed.

**Problem:** Suppose that the number of accidents per day in a city has a Poisson distribution with average 2 accidents.

- (1) What is the probability that in a day:
  - (i) The number of accidents will be 5?
  - (ii) The number of accidents will be less than 2?
- (2) What is the probability that there will be 6 accidents in 2 days?
- (3) What is the probability that there will be no accidents in an hour?

### Normal Distribution:

**Problem 1:** Let  $Z$  be a standard normal random variable. Find the probability that  $Z$  will be

a. Less than 1.5	b. Greater than 2.4	c. Between 1.5 and 2.14
d. Less than - 1.32	e. Greater than -2.52	f. Between - 2.52 and 1.64
g. Between - 2.52 and - 1.5	h. Exactly 2	

**Answer:**

a. 0.9332	b. 0.0082	c. 0.0506	d. 0.0934
e. .9941	f. 0.9436	g. 0.0609	h. 0

### Problem 2:

Suppose that the average temperature in July in a certain region is a normal random variable with parameters  $\mu = 90$  (measured in degrees Fahrenheit) and  $\sigma = 5$ . Find the probability that in a given year the average temperature in July in that region will be

- a. Above  $100^{\circ} F$
- b. Below  $95^{\circ} F$
- c. Between  $85^{\circ} F$  and  $95^{\circ} F$
- d. Below  $88^{\circ} F$

**Answer:**

- a. 0.00228
- b. 0.8413
- c. 0.6826
- d. 0.3446

**Problem 3:**

A certain type of insect survives on the average 3.0 years, with a variance of 0.25 year. Assuming that the lives of the insect are normally distributed, find the probability that a given insect will survive less than 2.3 years.

(Answer: 0.0808)

**Problem 4:**

The grade - point average score of 80 student of Department of Computer Science of Dhaka University in their term final examination was found to follow approximately a normal distribution with a mean of 2.1 and a standard deviation 0.6. How many of the students are expected to have a score between 2.5 and 3.5?

(Answer: 20)

**Problem 5:**

The Philips Bangladesh manufactures electric bulbs that have length of life that is normally distributed with mean equal to 800 hours and standard deviation of 40 hours. Find the probability that a bulb burns between 778 and 834 hours.

(Answer: 0.5111)