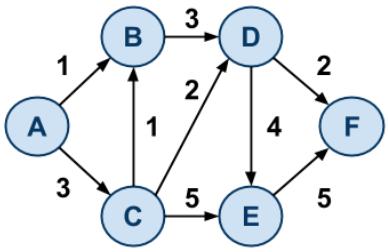
CSE221: Algorithms

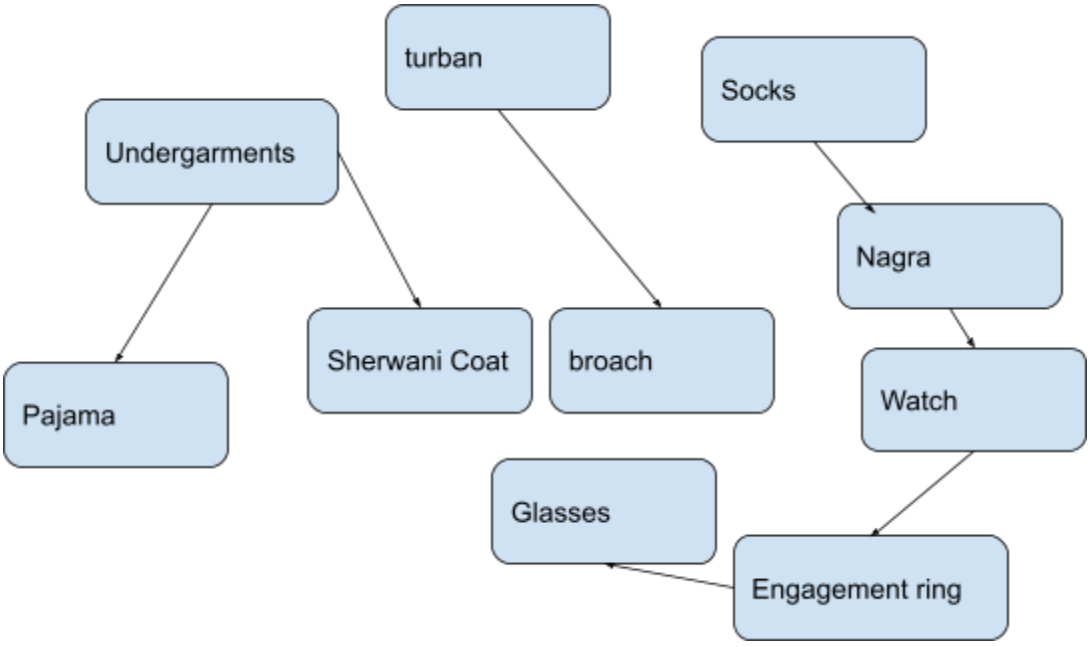
Assignment 2

1. **Graph Basics [30 points]**
    a. **[12 points]** Define the following with an example-
        i. Simple graph & multigraph
        ii. Undirected graph & directed graph
        iii. Cyclic graph & acyclic graph
        iv. (Edge) weighted graph & vertex weighted graph
        v. Sparse graph & dense graph
        vi. Connected graph & disconnected graph
        vii. Connected component
        viii. Complete graph
        ix. Subgraph & Clique
        x. Bipartite graph
        xi. Regular graph
        xii. DAG
    b. **[3 points]** Define the following terms related to Graph with an example for each-
        i. Degree of a graph
        ii. Degree-sum theorem
        iii. Path & Cycle of a graph



    c. **[5 points]** Consider the above graph as an undirected graph. How many MSTs are possible for the graph? Draw all of them.
    d. **[5+5=10 points]** Consider the graph above.
        i. Draw the adjacency matrix and the adjacency list considering it as an undirected graph.
        ii. Draw the adjacency matrix and the adjacency list considering it as a directed graph
2. **Graph Traversal [20 points]**
    a. **[5 points]** What is the difference between back and cross edge? Define with an example.
    b. **[5 points]** Explain one practical application of Edge classification of DFS.
    c. **[8 points]** Let's say, Anis is getting married. He came out of the shower and saw all the dresses that he needs to get dressed is on the bed. Now he needs to find a system to wear them. HE made up some directed Acyclic graph in his mind

using the dresses. It is directed because in some cases he can not choose any order to dress. As an example, he can not wear undergarments after his pants. So in his mind, the graph looks like this:



Help him to find an order to put those dresses on(Topological sort). (Show the simulation step by step).

d. **[2 points]** Do you think all of your classmates suggest the same ordering? Explain why you believe the answer is yes or no.

3. **[15 points]** The Economic condition in BD is not good, even in DK. To reduce the operating costs, the government of DK has decided to optimize road lighting. Till now every road was lighted all night long, which costs a lot of electricity bill. To save money, they decided to no longer illuminate every road but to switch off the road lighting of some streets. To make sure that the inhabitants of DK still feel safe, they want to optimize the lighting in such a way, **that after darkening some junctions at night, there will still be at least one illuminated path from every junction in DK to every other junction.** What is the maximum amount of money the government of DK can save, without making their inhabitants feel unsafe?

   a. **[7 points]** Write **a pseudo code** for this problem with a proper algorithm given the conditions with the following input & output. The algorithm should not take more than **O(n log m) or O(n log n)** time.

Sample input: 7 11 0 1 7 0 3 5 1 2 8 1 3 9 1 4 7	**Sample output:** 51

2 4 5 **3 4 15** **3 5 6** **4 5 8** **4 6 9** **5 6 11** **Input Explanation:** The first line starts with two numbers 7 **and 11**, the number of nodes/junctions in DK, and the number of roads/edges in DK. Then follow 11 integer triples **x, y, z** specifying that there will be a bidirectional road between x and y with total cost z.	`//hint: 90-39 =51` **Output Explanation:** For the test case print one line containing the maximum amount the government can save.

b. **[5+1=6 points]** Draw an undirected weighted graph from the above input and **show the simulation** of how you got the output step by step. Mention **an alternative algorithm** name that can also be used to solve the above problem.

c. **[2 points]** Show the **time complexity breakdown** of your pseudo code.

4. **MST [10 points]**
   a. **[5 points]** You are given a simple connected undirected graph with N nodes(or vertices) for computing a Minimum Spanning Tree (MST) of the graph. Write down the upper and lower bound of the possible number of edges of that MST.
   b. **[5 points]** Now consider the same for a "simple disconnected undirected graph". What is the lower and upper bound now?
5. **[20 points (+ 5 bonus)]** Look at the following implementation of Dijkstra's Shortest Path Algorithm:

```
Initialize-Single-Source(G, s) {
   for each vertex v in V(G)
      d[v] ← ∞
      π[v] ← NIL
   d[s] ← 0
}

Relax(u, v) {
   //update only if we found a strictly shortest path
   if d[v] > d[u] + w(u,v)
      d[v] ← d[u] + w(u,v)
      π[v] ← u
      Q.push(v)
}
Dijkstra(G, w, s)  {
   Initialize-Single-Source(G, s)
   S ← Ø
```

```
    Q ← s
    while Q ≠ Ø do
        u ← Extract-Min(Q)
        S ← S U {u}
        for each vertex v in Adj[u] do
            If d[v] = ∞
                Relax(u, v)

}
```

a. **[2.5*4=10 points]** Assuming 'A' as our *source* node, we run the above algorithm on the following graph. Now, write down the distance values for each node after each iteration of the **`while Q ≠ Ø`** loop**.**

Follow this format:

Iteration: 1
Node Being Processed: A
d[A]=?, d[B]=?, d[C]=?, d[D]=?

Iteration:2
.....

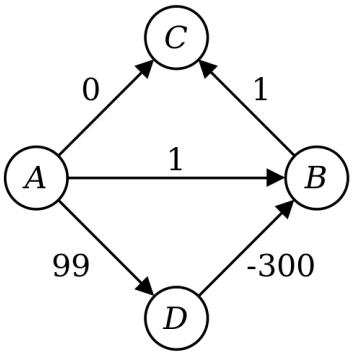


Figure for Question 5

b. **[10 points]** The final distance values stored in **d[]** are not correct. Why so? Explain.

c. **[5 points bonus]** If not, can you make a little change to the above code to correct the output? [Hint: you can remove something!]