**Test Yourself 1**

The following data represents Annual wages of two Factories X and Y for the given information

I. Determine range and coefficient of range. (in '000 Tk)

II. Determine the quartile deviation and coefficient of Co-efficient of quartile deviation.

Table 1: Annual wages of Factory X workers (in '000 Tk)

| | | | | | |
|---|---|---|---|---|---|
| 91 | 70 | 74 | 79 | 86 | 93 |
| 60 | 71 | 76 | 79 | 87 | 96 |
| 112 | 72 | 127 | 79 | 87 | 62 |
| 68 | 72 | 77 | 79 | 90 | 76 |
| 69 | 73 | 77 | 85 | 48 | 157 |

Table 2: Annual wages of Factory Y workers (in '000 Tk)

| | | | | | |
|---|---|---|---|---|---|
| 97 | 78 | 85 | 92 | 97 | 105 |
| 72 | 79 | 85 | 92 | 97 | 107 |
| 112 | 79 | 87 | 92 | 97 | 72 |
| 113 | 80 | 90 | 96 | 68 | 75 |
| 78 | 82 | 90 | 97 | 100 | |

Hints:

1. Find the lowest and highest value for each table.
2. Calculate the quartiles ($Q1$, $Q2$, $Q3$) for each table.

I. Determine range and coefficient of range. (in ‘000 Tk)

For table 1:

R = 157-48 = 109

Coefficient of Range = $\frac{157-48}{157+48}$ = 0.5317

For table 2:

R = 113-68 = 45

Coefficient of Range = $\frac{113-68}{113+68}$ = 0.2486

II. Determine the quartile deviation and coefficient of Co-efficient of quartile deviation.

For table 1:

First Quartile $Q_1$:

$\frac{in}{4} = \frac{1*30}{4}$ = 7.5, not an integer

$Q_1$= 72

Second Quartile $Q_2$ :

$\frac{in}{4} = \frac{2*30}{4}$ = 15, an integer

$Q_2 = \frac{1}{2}[77 + 79] = 78$

Third Quartile $Q_3$:

$\frac{in}{4} = \frac{3*30}{4}$ = 22.5, not an integer

$Q_3$= 87

QD = $\frac{Q3-Q1}{2} = \frac{87-72}{2}$ = 7.5

Coefficient of QD = $\frac{Q3-Q1}{Q3+Q1} = \frac{87-72}{87+72}$ = 0.09434

For table 2:

First Quartile $Q_1$:

$\frac{in}{4} = \frac{1*29}{4}$ = 7.25, not an integer

$Q_1 = 79$

Second Quartile $Q_2$ :

$\frac{in}{4} = \frac{2*29}{4} = 14.5$, not an integer

$Q_2 = 90$

Third Quartile $Q_3$:

$\frac{in}{4} = \frac{3*29}{4} = 21.75$, not an integer

$Q_3 = 97$

QD $= \frac{Q3-Q1}{2} = \frac{97-79}{2} = 9$

Coefficient of QD $= \frac{Q3-Q1}{Q3+Q1} = \frac{97-79}{97+79} = 0.10227$

**Test Yourself 2**

A population of 10 students got the marks in the examination as given in the table below. Find the variance and Standard Deviation of the given data.

13 15 14 16 2 8 9 23 28 12

Answer:

| $x_i$ | $x_i^2$ |
|---|---|
| 13 | 169 |
| 15 | 225 |
| 14 | 196 |
| 16 | 256 |
| 2 | 4 |
| 8 | 64 |
| 9 | 81 |
| 23 | 529 |
| 28 | 784 |
| 12 | 144 |
| $\sum x_i = 140$ | $\sum x_i^2 = 2452$ |

Variance, $\boldsymbol{\sigma}^2 = \frac{1}{N}\left[\sum_{i=1}^{N} x_i^2 - \frac{\left(\sum_{i=1}^{N} x_i\right)^2}{N}\right] = \frac{1}{10}\left[2452 - \frac{(140)^2}{10}\right] = 49.2$

SD = 7.01427

**Test Yourself 3**

A population of 40 students got marks in the examination as given in the table below. Find the variance and Standard Deviation of the given data.

| $Xi$ | 15 | 20 | 25 | 30 | 35 |
|---|---|---|---|---|---|
| $fi$ | 6 | 8 | 15 | 7 | 4 |

| $x_i$ | $f_i$ | $f_i * x_i^2$ | $f_i * x_i$ |
|---|---|---|---|
| 15 | 6 | 1350 | 90 |
| 20 | 8 | 3200 | 160 |
| 25 | 15 | 9375 | 375 |
| 30 | 7 | 6300 | 210 |
| 35 | 4 | 4900 | 140 |
| | | $\sum f_i * x_i^2 = 25125$ | $\sum f_i * x_i = 975$ |

$$\sigma^2 = \frac{1}{N}\left[\sum_{i=1}^{N} f_i x_i^2 - \frac{\left(\sum_{i=1}^{N} f_i x_i\right)^2}{N}\right] = \frac{1}{40}\left[25125 - \frac{(975)^2}{40}\right] = 33.9844$$

SD = 5.8296

**Test Yourself 4**

| $x_i$ | $x_i^2$ |
|---|---|
| 13 | 169 |
| 15 | 225 |
| 14 | 196 |
| 16 | 256 |
| 2 | 4 |
| 8 | 64 |
| 9 | 81 |
| 23 | 529 |
| 28 | 784 |
| 12 | 144 |
| $\sum x_i = 140$ | $\sum x_i^2 = 2452$ |

$s^2 = \frac{1}{n-1}\left[\sum_{i=1}^{N} x_i^2 - \frac{\left(\sum_{i=1}^{N} x_i\right)^2}{n}\right] = \frac{1}{9}\left[2452 - \frac{(140)^2}{10}\right] = 54.66667$

$\mathrm{SD} = 7.39369$

**Test Yourself 5**

A sample of 40 students got marks in the examination as given in the table below. Find the variance and Standard Deviation of the given data.

| $Xi$ | 15 | 20 | 25 | 30 | 35 |
|---|---|---|---|---|---|
| $fi$ | 6 | 8 | 15 | 7 | 4 |

| $x_i$ | $f_i$ | $f_i * x_i^2$ | $f_i * x_i$ |
|---|---|---|---|
| 15 | 6 | 1350 | 90 |
| 20 | 8 | 3200 | 160 |
| 25 | 15 | 9375 | 375 |
| 30 | 7 | 6300 | 210 |
| 35 | 4 | 4900 | 140 |
| | | $\sum f_i * x_i^2 = 25125$ | $\sum f_i * x_i = 975$ |

$s^2 = \frac{1}{n-1}\left[\sum_{i=1}^{N} f_i x_i^2 - \frac{\left(\sum_{i=1}^{N} f_i x_i\right)^2}{n}\right] = \frac{1}{39}\left[25125 - \frac{(975)^2}{40}\right] = 34.8558$

SD = 5.9039

**Test Yourself 6**

An Advertising company is looking for a group of extras to shoot a sequence for a movie. The ages of the first 20 candidates to be interviewed are

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 50 | 56 | 44 | 49 | 52 | 57 | 56 | 57 | 56 | 59 |
| 54 | 55 | 61 | 60 | 51 | 59 | 62 | 52 | 54 | 49 |

| $x_i$ | $x_i^2$ |
|---|---|
| 50 | 2500 |
| 56 | 3136 |
| 44 | 1936 |
| 49 | 2401 |
| 52 | 2704 |
| 57 | 3249 |
| 56 | 3136 |
| 57 | 3249 |
| 56 | 3136 |
| 59 | 3481 |
| 54 | 2916 |
| 55 | 3025 |
| 61 | 3721 |
| 60 | 3600 |
| 51 | 2601 |
| 59 | 3481 |
| 62 | 3844 |
| 52 | 2704 |
| 54 | 2916 |
| 49 | 2401 |
| $\sum x_i = 1093$ | $\sum x_i^2 = 60137$ |

$s^2 = \frac{1}{n-1}\left[\sum_{i=1}^{N} x_i^2 - \frac{\left(\sum_{i=1}^{N} x_i\right)^2}{n}\right] = \frac{1}{19}\left[60137 - \frac{(1093)^2}{20}\right] = 21.29211$

$\mathrm{SD} = 4.6143$

As the director suggested that a standard deviation of 3 years would be accepted.

So this group of extras will not qualify.

**Test Yourself 7**

In a University, students can take any number of courses per semester. For two samples of 30 students each. The data of how many courses one takes is given below:

| Course Number | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| Sample 1 | 2 | 5 | 10 | 12 | 1 |
| Sample 2 | 1 | 6 | 8 | 13 | 2 |

For which sample of students, the relative variability of course numbers is higher?

**Sample 1:**

| $x_i$ | $f_i$ | $f_i * x_i^2$ | $f_i * x_i$ |
|---|---|---|---|
| 2 | 2 | 8 | 4 |
| 3 | 5 | 45 | 15 |
| 4 | 10 | 160 | 40 |
| 5 | 12 | 300 | 60 |
| 6 | 1 | 36 | 6 |
| | | $\sum f_i * x_i^2 = 549$ | $\sum f_i * x_i = 125$ |

$s_1^2 = \frac{1}{n-1}\left[\sum_{i=1}^{N} f_i x_i^2 - \frac{\left(\sum_{i=1}^{N} f_i x_i\right)^2}{n-1}\right] = \frac{1}{29}\left[549 - \frac{(125)^2}{30}\right] = 0.971264$

$SD_1 = 0.985527$

$AM_1 = 4.1667$

$CV_1 = \frac{SD}{AM} = 0.2365$

**Sample 2:**

| $x_i$ | $f_i$ | $f_i * x_i^2$ | $f_i * x_i$ |
|---|---|---|---|
| 2 | 1 | 4 | 2 |
| 3 | 6 | 54 | 18 |
| 4 | 8 | 128 | 32 |
| 5 | 13 | 325 | 65 |
| 6 | 2 | 72 | 12 |
| | | $\sum f_i * x_i^2 = 583$ | $\sum f_i * x_i = 129$ |

$s_2^2 = \frac{1}{n-1}\left[\sum_{i=1}^{N} f_i x_i^2 - \frac{\left(\sum_{i=1}^{N} f_i x_i\right)^2}{n-1}\right] = \frac{1}{29}\left[25125 - \frac{(975)^2}{30}\right] = 0.9759$

$SD_2 = 0.9879$

$AM_2 = 4.3$

$CV_2 = \frac{SD}{AM} = 0.2297$

Here,

$CV_1 > CV_2$

So, the relative variability is higher in sample 1.

**Test Yourself 8**

From the analysis of monthly wages paid to employees in two service organizations X and Y, the following results were obtained:

| | Organization X | Organization Y |
|---|---|---|
| Number of wage-earners | 550 | 650 |
| Average monthly wages | 5000 | 4500 |
| Variance of the distribution of wages | 900 | 1600 |

a. Which organization pays a larger amount as monthly wages?

Organization X pays: 550 * 5000 = 2750000

Organization Y pays: 650 * 4500 = 2925000

Organization Y pays larger amount as monthly wages.

b. Determine the combined variance of all the employees taken together?

For organization X:

$\mu_1 = 5000$

$Variance, \sigma_1^2 = 900$

$SD, \sigma_1 = 30$

$n_1 = 550$

For organization Y:

$\mu_2 = 4500$

$Variance, \sigma_2^2 = 1600$

$SD, \sigma_2 = 40$

$n_2 = 650$

Now,

$$\mu_{12} = \frac{n_1\mu_{1+}\ n_2\mu_2}{n_1+n_2} = \frac{550*5000+650*4500}{550+650} = 4729.16667$$

$$d_1 = \mu_{12} - \mu_1 = -270.83333$$

$$d_2 = \mu_{12} - \mu_2 = 229.16667$$

Combined Variance, $\sigma_{12}{}^2 = \frac{n_1\,(\sigma_1^2+d_1^2)+n_2\,(\sigma_2^2+d_2^2)}{n_1+n_2} = 63345.13144$

**Test Yourself 9**

For a group of 50 male workers, the mean and standard deviation of their monthly wages are tk. 6300 and tk. 600 respectively. For a group of 40 female workers, these are tk. 5400 and tk. 600 respectively. Find the standard deviation of monthly wages for the combined group of workers.

For Group 1:

$\mu_1 = 6300$

$SD, \sigma_1 = 600$

$n_1 = 50$

For Group 2:

$\mu_2 = 5400$

$SD, \sigma_2 = 600$

$n_2 = 40$

Now,

$$\mu_{12} = \frac{n_1\mu_{1+} n_2\mu_2}{n_1+n_2} = 5900$$

$$d_1 = \mu_{12} - \mu_1 = -400$$

$$d_2 = \mu_{12} - \mu_2 = 500$$

Combined Standard deviation, $\sigma_{12} = \sqrt{\frac{n_1(\sigma_1^2+d_1^2)+n_2(\sigma_2^2+d_2^2)}{n_1+n_2}} = 748.3315$

**Test Yourself 10**

1. If for a distribution Mean =18, Median= 32, and Mode= 36, the distribution is negatively skewed. [Mean < Median < Mode]

2. If for a distribution Mean = 20, Median= 26.4, and SD= 3.3, the distribution is negatively skewed.

$Sk_p = \frac{3(Mean-Median)}{SD}$ = -5.818<0 [Negatively skewed]

3. If for a distribution, Mean = 35.6, Mode = 24, and SD= 5.2, what is the skewness coefficient of the distribution?

$Sk_p = \frac{Mean-Mode}{SD}$ = 2.2307>0. The distribution is positively skewed.

**Test Yourself 11**

Construct a box plot for the data given below and hence comment on the skewness of the distribution:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 99 | 75 | 84 | 33 | 45 | 66 | 97 | 69 | 55 | 61 |
| 72 | 91 | 74 | 93 | 54 | 76 | 62 | 91 | 77 | 68 |

Minimum value = 33

Maximum value = 99

Median = 73

First Quartile $Q_1$:

$\frac{in}{4} = \frac{1*20}{4} = 5$, an integer

$Q_1 = \frac{1}{2}[61 + 62] = 61.5$

Second Quartile $Q_2$:

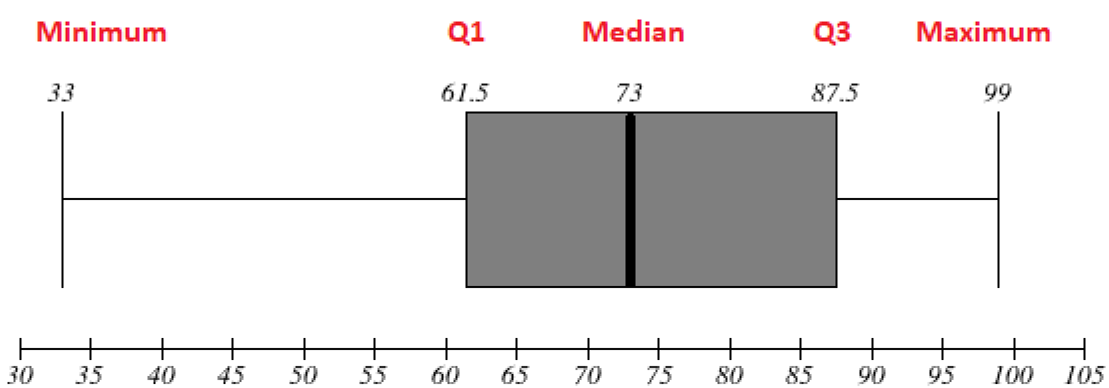
$\frac{in}{4} = \frac{2*20}{4} = 10$, an integer

$Q_2 = \frac{1}{2}[72 + 74] = 73$

Third Quartile $Q_3$:

$\frac{in}{4} = \frac{3*20}{4} = 15$, an integer

$Q_3 = \frac{1}{2}[84 + 91] = 87.5$



Using Bowley's coefficient for Skewness we get

$Sk_b = \frac{(Q_3 - Q_2) - (Q_2 - Q_1)}{Q_3 - Q_1} = 0.1154 > 0$, the distribution is positively skewed.