Shihab Muhtasim

STUDENT ID: 21301610

MAT 110

ASSIGNMENT 03

SET 7

Ans to the question no 01

Given function,

$f(x) = \sin x + 2\cos x$

By differentiating this function we get,

$f'(x) = \cos x - 2\sin x$

$f''(x) = -\sin x - 2\cos x$

$f'''(x) = -\cos x + 2\sin x$

Now, at x=0,

$f(0) = \sin 0 + 2\cos 0 = 2$

$f'(0) = \cos 0 - 2\sin 0 = 1$

$f''(0) = -\sin 0 - 2\cos 0 = -2$

$f'''(0) = -\cos 0 + 2\sin 0 = -1$

We know, $P_n(0) = \frac{f^0(x)}{0!}x^0 + \frac{f'(x)}{1!}x^1 + \frac{f''(x)}{2!}x^2 + \frac{f'''(x)}{3!}x^3 + \ldots..$

According to Taylor series around x=0,

$P_3(0) = \frac{f^0(0)}{0!}x^0 + \frac{f'(0)}{1!}x^1 + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3$

$\Rightarrow P_3(0) = \frac{2}{0!}(x-0)^0 + \frac{1}{1!}(x-0)^1 + \frac{-2}{2!}(x-0)^2 + \frac{-1}{3!}(x-0)^3$

$\Rightarrow P_3(0) = 2 + (x-0) - (x-0)^2 - \frac{x^3}{6}$

$\Rightarrow P_3(0) = 2 + x - x^2 - \frac{1}{6}x^3$

$\therefore P_3(0) = 2 + x - x^2 - \frac{1}{6}x^3$

Ans to the question no 02

We know,

$P_n(0) = \frac{f^0(x)}{0!}x^0 + \frac{f'(x)}{1!}x^1 + \frac{f''(x)}{2!}x^2 + \frac{f'''(x)}{3!}x^3 + ...$

Given function,

$f(x) = \ln(e^{2x} + e^{-2x})$

By differentiating we get,

$f'(x) = \frac{1}{e^{2x}+e^{-2x}} \cdot (2e^{2x} - 2e^{-2x}) = \frac{2(e^{4x}-1)}{e^{4x}+1}$

$f''(x) = \frac{2 \cdot 4e^{4x}(e^{4x}+1) - 4e^{4x}2(e^{4x}-1)}{(e^{4x}+1)^2}$

$\Rightarrow f''(x) = \frac{8e^{4x}(e^{4x}+1) - 8e^{4x}(e^{4x}-1)}{(e^{4x}+1)^2}$

$\Rightarrow f''(x) = \frac{8e^{4x}(e^{4x}-e^{4x}+2)}{(e^{4x}+1)^2}$

$\Rightarrow f''(x) = \frac{16e^{4x}}{(e^{4x}+1)^2}$

Putting x=0 in f(x) and its derivatives we get,

$f(0) = \ln(e^{2 \cdot 0} + e^{-2 \cdot 0}) = \ln 2$

$f'(0) = \frac{2(e^{4 \cdot 0}-1)}{e^{4 \cdot 0}+1} = \frac{0}{2} = 0$

$f''(0) = \frac{16e^{4 \cdot 0}}{(e^{4 \cdot 0}+1)^2} = \frac{16}{4} = 4$

Therefore,

$P_o = \frac{f^0(0)x^0}{0!} = \ln 2$

$P_1 = \frac{f^0(0)x^0}{0!} + \frac{f'(0)x^1}{1!} = \ln 2 + 0 = \ln 2$

$P_2 = \frac{f^0(0)x^0}{0!} + \frac{f'(0)x^1}{1!} + \frac{f''(0)x^2}{2!} = \ln 2 + 2x^2$

Ans to the question no 03

Given multi-variable function,

$f(x,y) = (x^2 + 2y^2x + 3x + y^2)^2$

Derivatives of f(x,y) with respect to x considering other variables constant,

$f_x = 2(x^2 + 2y^2x + 3x + y^2)(2x + 2y^2 + 3)$

$f_{xx} = 2\{2(x^2 + 2y^2x + 3x + y^2) + (2x + 2y^2 + 3)^2\}$

$\Rightarrow f_{xx} = 2\{(2x^2 + 4xy^2 + 6x + 2y^2) + (4x^2 + 4y^4 + 9 + 8xy^2 + 12y^2 + 12x)\}$

$\Rightarrow f_{xx} = 2(6x^2 + 12xy^2 + 18x + 14y^2 + 4y^2 + 9)$

$\therefore f_{xx} = 2(6x^2 + 12xy^2 + 18x + 14y^2 + 4y^2 + 9)$

Similarly,

$f_y = 2(x^2 + 2y^2x + 3x + y^2)(4xy + 2y)$

$f_{yy} = 2\{(4x + 2)(x^2 + 2y^2x + 3x + y^2) + (4xy + 2y)^2\}$

$\Rightarrow f_{yy} = 2\{(4x^3 + 8x^2y^2 + 12x^2 + 4xy^2 + 2x^2 + 4xy^2 + 6x + 2y^2) + (16x^2y^2 + 4y^2 + 16xy^2)\}$

$\Rightarrow f_{yy} = 2(4x^3 + 24x^2y^2 + 14x^2 + 24xy^2 + 6x + 6y^2)$

$\therefore f_{yy} = 2(4x^3 + 24x^2y^2 + 14x^2 + 24xy^2 + 6x + 6y^2)$

Ans to the question no 04

Given function,

$f(x, y) = \ln((x^2 + 3y + xy)^2)$

Where $x(t) = 2t + 3, y(t) = t^2 + 3t$

Chair rule for partial derivatives,

$\frac{df}{dt} = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dt}$

Now,

$\frac{\partial f}{\partial x} = \frac{1}{(x2+3y+xy)^2} \cdot 2(x^2 + 3y + xy) \cdot (2x + y) = \frac{2(2x+y)}{(x^2+3y+xy)}$

$\frac{dx}{dt} = 2$

$\frac{\partial f}{\partial y} = \frac{1}{(x2+3y+xy)^2} \cdot 2(x^2 + 3y + xy) \cdot (3 + x) = \frac{2(x+3)}{(x^2+3y+xy)}$

$\frac{dy}{dt} = 2t + 3$

Imposing these values on the chain rule formula,

$\frac{df}{dt} = \frac{2 \cdot 2(2x+y)}{(x^2+3y+xy)} + \frac{2(x+3)(2t+3)}{(x^2+3y+xy)}$

Since we have to evaluate this expression when t=0,

$\frac{4(2x+y)}{(x^2+3y+xy)} + \frac{2(x+3)(2 \cdot 0+3)}{(x^2+3y+xy)}$

$= \frac{8x+4y}{(x^2+3y+xy)} + \frac{6x+18}{(x2+3y+xy)}$

$= \frac{14x+4y+18}{(x^2+3y+xy)}$

$\therefore \frac{df}{dt} = \frac{14x+4y+18}{(x^2+3y+xy)}$ when t=0

Ans to the question no 05

Given function,

$y^2 \sin(x^3) + ze^{3x} - \cos(z^2) = z^4 - y^2 + x^2$

Imposing partial differentiation on both sides with respect to x ,

$\frac{\partial}{\partial x}\{y^2 \sin(x^3) + ze^{3x} - \cos(z^2)\} = \frac{\partial}{\partial x}(z^4 - y^2 + x^2)$

$\Rightarrow y^2 \cos(x^3) \cdot 3x^2 + \frac{\partial z}{\partial x}e^{3x} + z \cdot 3e^{3x} + \sin(z^2)2z\frac{\partial z}{\partial x} = 4z^3\frac{\partial z}{\partial x} + 2x$

$\Rightarrow \frac{\partial z}{\partial x}(e^{3x} + 2z\sin(z^2) - 4z^3) = 2x - z \cdot 3e^{3x} - 3x^2y^2\cos(x^3)$

$\Rightarrow \frac{\partial z}{\partial x} = \frac{2x - 3ze^{3x} - 3x^2y^2\cos(x^3)}{(e^{3x} + 2z\sin(z^2) - 4z^3)}$

$\therefore \frac{\partial z}{\partial x} = \frac{2x - 3ze^{3x} - 3x^2y^2\cos(x^3)}{(e^{3x} + 2z\sin(z^2) - 4z^3)}$

Similarly partial differentiation on both sides with respect to y,

$\frac{\partial}{\partial y}\{y^2 \sin(x^3) + ze^{3x} - \cos(z^2)\} = \frac{\partial}{\partial y}(z^4 - y^2 + x^2)$

$\Rightarrow 2y\sin(x^3) + \frac{\partial z}{\partial y}e^{3x} + \sin(z^2)2z\frac{\partial z}{\partial y} = 4z^3\frac{\partial z}{\partial y} - 2y$

$\Rightarrow \frac{\partial z}{\partial y}(e^{3x} + 2z\sin(z^2) - 4z^3) = -2y - 2y\sin(x^3)$

$\Rightarrow \frac{\partial z}{\partial y} = \frac{-2y(1 + \sin x^3)}{(e^{3x} + 2z\sin(z^2) - 4z^3)}$

$\therefore \frac{\partial z}{\partial y} = \frac{-2y(1 + \sin x^3)}{(e^{3x} + 2z\sin(z^2) - 4z^3)}$

Ans to the question no 06

Given function,

$h(x, y, z) = y^3 z^3 \cos(x^4) + x^3 \sin(z^2)$

We have to use partial differentiation to find h_{yzx}.

Imposing partial differentiation of $h(x, y, x)$ with respect to y,

$h_y = 3y^2 z^3 \cos(x^4)$

Again partial differentiation of h_y with respect to z,

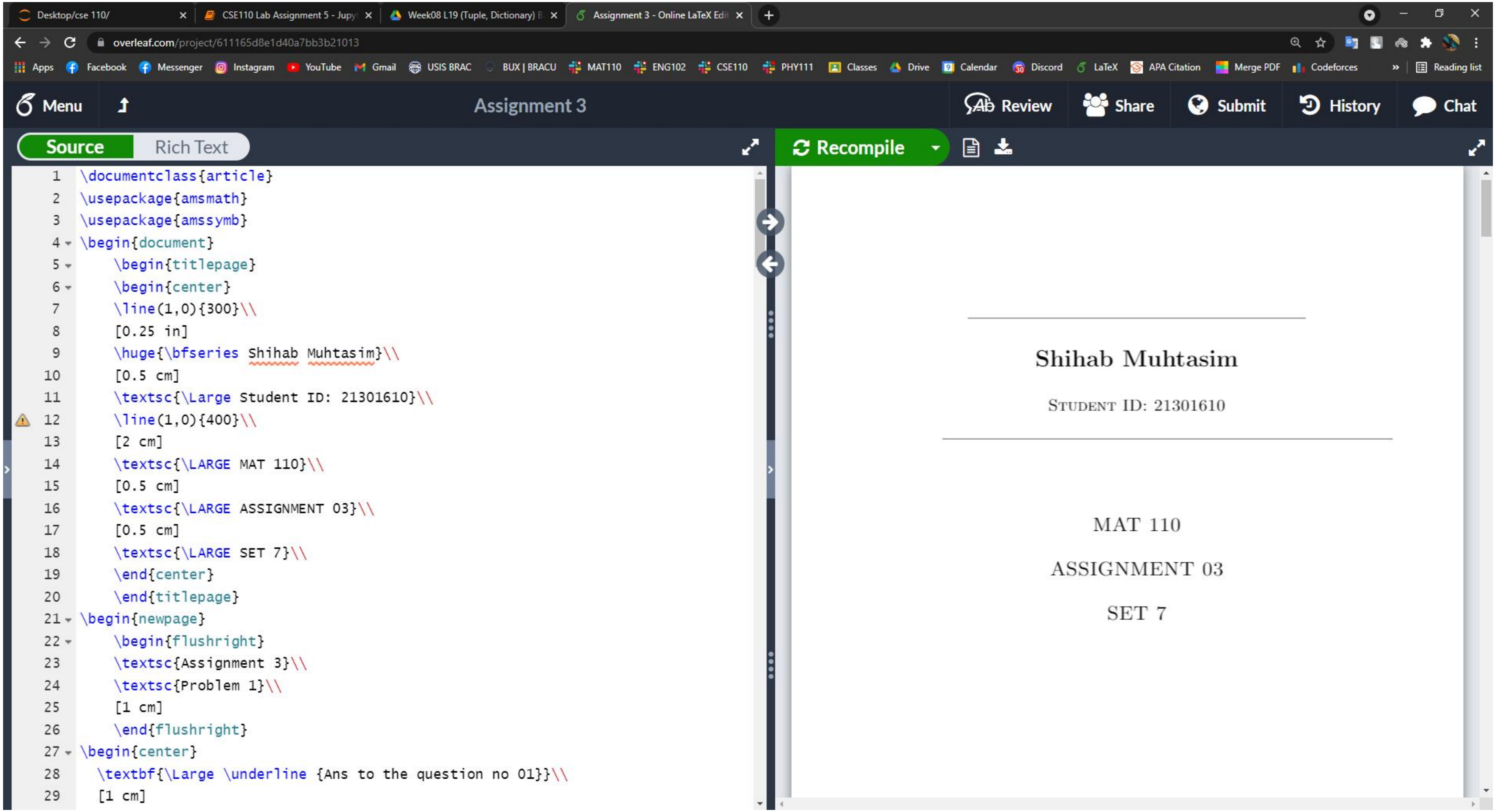
$h_{yz} = 3y^2 3z^2 \cos(x^4) = 9y^2 z^2 \cos(x^4)$

Finally partial differentiation of h_{yz} with respect to x,

$h_{yzx} = 9y^2 z^2 \{-\sin(x^4)(4x^3)\} = 36x^3 y^2 z^2 \{-\sin(x^4)\}$

evaluating the expression of h_{yzx} at point (0,1,1) we get,

$h_{yzx} = 36 \cdot 0^3 \cdot 1^2 \cdot 1^2 \{-\sin(0^4)\} = 0$


Assignment 3
\documentclass{article}
\usepackage{amsmath}
\usepackage{amssymb}
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\huge{\bfseries Shihab Muhtasim}\\
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\textsc{Assignment 3}\\
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\textbf{\Large \underline {Ans to the question no 01}}\\
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Shihab Muhtasim
STUDENT ID: 21301610
MAT 110
ASSIGNMENT 03
SET 7

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    \textsc{Assignment 3}\\
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  \textbf{\Large \underline {Ans to the question no 01}}\\
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\Large {Given function, \\[3mm]
$ f(x)=\sin{x}+2\cos{x}$\\[3mm]
By differentiating this function we get,\\[3mm]
$f'(x)=\cos{x}-2\sin{x}\\[3mm]
f''(x)=-\sin{x}-2\cos{x}\\[3mm]
f'''(x)=-\cos{x}+2\sin{x}\\[5mm]$
Now, at x=0,\\[3mm]
$f(0)=\sin{0}+2\cos{0}=2\\[3mm]
f'(0)=\cos{0}-2\sin{0}=1\\[3mm]
f''(0)=-\sin{0}-2\cos{0}=-2\\[3mm]
f'''(0)=-\cos{0}+2\sin{0}=-1\\[5mm]$
We know,
$P_{n}(0)=\frac{f^0(x)}{0!}x^0+\frac{f'(x)}{1!}x^1+\frac{f''(x)}{2!}x^2+\frac{f'''(x)}{3!}x^3+....
.$\\[3mm]
According to Taylor series around x=0,\\[3mm]
$P_{3}(0)=\frac{f^0(0)}{0!}x^0+\frac{f'(0)}{1!}x^1+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3\\[3m
m]
\Rightarrow P_{3}(0) =\frac{2}{0!}(x-0)^0+\frac{1}{1!}(x-0)^1+\frac{-2}{2!}(x-0)^2+\frac{-1}{3!}(x
-0)^3\\[3mm]
\Rightarrow P_{3}(0)=2+(x-0)-(x-0)^2-\frac{x^3}{6}\\[3mm]
\Rightarrow P_{3}(0)=2+x-x^2-\frac{1}{6}x^3\\[3mm]
\therefore P_{3}(0)=2+x-x^2-\frac{1}{6}x^3$}
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    \textsc{Assignment 3}\\
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<u>Ans to the question no 01</u>

Given function,

$f(x) = \sin x + 2\cos x$

By differentiating this function we get,

$f'(x) = \cos x - 2\sin x$

$f''(x) = -\sin x - 2\cos x$

$f'''(x) = -\cos x + 2\sin x$

Now, at x=0,

$f(0) = \sin 0 + 2\cos 0 = 2$

$f'(0) = \cos 0 - 2\sin 0 = 1$

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We know, $P_n(0) = \frac{f^0(x)}{0!}x^0 + \frac{f'(x)}{1!}x^1 + \frac{f''(x)}{2!}x^2 + \frac{f'''(x)}{3!}x^3 + ...$

According to Taylor series around x=0,

$P_3(0) = \frac{f^0(0)}{0!}x^0 + \frac{f'(0)}{1!}x^1 + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3$

$\Rightarrow P_3(0) = \frac{2}{0!}(x-0)^0 + \frac{1}{1!}(x-0)^1 + \frac{-2}{2!}(x-0)^2 + \frac{-1}{3!}(x-0)^3$

$\Rightarrow P_3(0) = 2 + (x-0) - (x-0)^2 - \frac{x^3}{6}$

$\Rightarrow P_3(0) = 2 + x - x^2 - \frac{1}{6}x^3$

$\therefore P_3(0) = 2 + x - x^2 - \frac{1}{6}x^3$

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    \textsc{Assignment 3}\\
    \textsc{Problem 2}\\
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  \textbf{\Large \underline {Ans to the question no 02}}\\
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\Large {We know,\\[3mm]
$P_{n}(0)=\frac{f^0(x)}{0!}x^0+\frac{f'(x)}{1!}x^1+\frac{f''(x)}{2!}x^2+\frac{f'''(x)}{3!}x^3+....
.$\\[3mm]
Given function, \\[2mm]
$f(x) = \ln(e^{2x} + e^{-2x})$\\[3mm]
By differentiating we get,\\[2mm]
$f'(x) = \frac{1}{e^{2x} + e^{-2x}}\cdot(2{e^{2x}}-2e^{-2x})=\frac{2(e^{4x}-1)}{e^{4x}+1}\\[3mm]
f''(x)=\frac{2\cdot4e^{4x}(e^{4x}+1)-4e^{4x}2(e^{4x}-1)}{(e^{4x}+1)^2}\\[3mm]
\Rightarrow f''(x)=\frac{8e^{4x}(e^{4x}+1)-8e^{4x}(e^{4x}-1)}{(e^{4x}+1)^2}\\[3mm]
\Rightarrow f''(x)=\frac{8e^{4x}(e^{4x}-e^{4x}+2)}{(e^{4x}+1)^2}\\[3mm]
\Rightarrow f''(x)=\frac{16e^{4x}}{(e^{4x}+1)^2}\\[3mm]$
Putting x=0 in f(x) and its derivatives we get,\\[3mm]
$f(0) = \ln (e^{2\cdot0} + e^{-2\cdot0})=\ln2\\[3mm]
f'(0) =\frac{2(e^{4\cdot0}-1)}{e^{4\cdot0}+1}=\frac{0}{2}=0\\[3mm]
f''(0)=\frac{16e^{4\cdot0}}{(e^{4\cdot0}+1)^2}=\frac{16}{4}=4\\[3mm]$
Therefore,\\[3mm]
$P_{o}=\frac{f^0(0)x^0}{0!}=\ln2 \\[3mm]
P_{1}=\frac{f^0(0)x^0}{0!}+\frac{f'(0)x^1}{1!}=\ln2+0=\ln2\\[3mm]
P_{2}=\frac{f^0(0)x^0}{0!}+\frac{f'(0)x^1}{1!}+\frac{f''(0)x^2}{2!}=\ln2+2x^2$}
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    \textsc{Assignment 3}\\
    \textsc{Problem 3}\\
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<u>Ans to the question no 02</u>

We know,

$P_n(0) = \frac{f^0(x)}{0!}x^0 + \frac{f'(x)}{1!}x^1 + \frac{f''(x)}{2!}x^2 + \frac{f'''(x)}{3!}x^3 + ...$

Given function,

$f(x) = \ln(e^{2x} + e^{-2x})$

By differentiating we get,

$f'(x) = \frac{1}{e^{2x}+e^{-2x}} \cdot (2e^{2x} - 2e^{-2x}) = \frac{2(e^{4x}-1)}{e^{4x}+1}$

$f''(x) = \frac{2\cdot4e^{4x}(e^{4x}+1)-4e^{4x}2(e^{4x}-1)}{(e^{4x}+1)^2}$

$\Rightarrow f''(x) = \frac{8e^{4x}(e^{4x}+1)-8e^{4x}(e^{4x}-1)}{(e^{4x}+1)^2}$

$\Rightarrow f''(x) = \frac{8e^{4x}(e^{4x}-e^{4x}+2)}{(e^{4x}+1)^2}$

$\Rightarrow f''(x) = \frac{16e^{4x}}{(e^{4x}+1)^2}$

Putting x=0 in f(x) and its derivatives we get,

$f(0) = \ln(e^{2\cdot0} + e^{-2\cdot0}) = \ln 2$

$f'(0) = \frac{2(e^{4\cdot0}-1)}{e^{4\cdot0}+1} = \frac{0}{2} = 0$

$f''(0) = \frac{16e^{4\cdot0}}{(e^{4\cdot0}+1)^2} = \frac{16}{4} = 4$

Therefore,

$P_o = \frac{f^0(0)x^0}{0!} = \ln 2$

$P_1 = \frac{f^0(0)x^0}{0!} + \frac{f'(0)x^1}{1!} = \ln 2 + 0 = \ln 2$

$P_2 = \frac{f^0(0)x^0}{0!} + \frac{f'(0)x^1}{1!} + \frac{f''(0)x^2}{2!} = \ln 2 + 2x^2$

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f''(0)=\frac{16e^{4\cdot0}}{(e^{4\cdot0}+1)^2}=\frac{16}{4}=4\\[3mm]$
Therefore,\\[3mm]
$P_{o}=\frac{f^0(0)x^0}{0!}=\ln2 \\[3mm]
P_{1}=\frac{f^0(0)x^0}{0!}+\frac{f'(0)x^1}{1!}=\ln2+0=\ln2\\[3mm]
P_{2}=\frac{f^0(0)x^0}{0!}+\frac{f'(0)x^1}{1!}+\frac{f''(0)x^2}{2!}=\ln2+2x^2$
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  \textbf{\Large \underline {Ans to the question no 03}}\\
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\Large {Given multi-variable function, \\[3mm]
$f(x,y) = (x^2 + 2y^2x + 3x + y^2)^2$\\[5mm]
Derivatives of f(x,y) with respect to x considering other variables constant,\\[3mm]
$f_{x}=2(x^2 + 2y^2x + 3x + y^2)(2x+2y^2+3)\\[3mm]
f_{xx}=2\{2(x^2 + 2y^2x + 3x + y^2)+(2x+2y^2+3)^2\}\\[3mm]
\Rightarrow f_{xx}=2\{(2x^2 + 4xy^2 + 6x + 2y^2)+(4x^2+4y^4+9+8xy^2+12y^2+12x)\}\\[3mm]
\Rightarrow f_{xx}=2(6x^2 + 12xy^2 + 18x + 14y^2+ 4y^2+9) \\[3mm]
\therefore f_{xx}=2(6x^2 + 12xy^2 + 18x + 14y^2+ 4y^2+9) \\[5mm]$
Similarly,\\[3mm]
$f_{y}=2(x^2 + 2y^2x + 3x + y^2)(4xy+2y)\\[3mm]
f_{yy}=2\{(4x+2)(x^2 + 2y^2x + 3x + y^2)+(4xy+2y)^2\}\\[3mm]
\Rightarrow f_{yy}=2\{(4x^3+8x^2y^2+12x^2+4xy^2+2x^2+4xy^2+6x+2y^2)+(16x^2y^2+4y^2+16xy^2)\}\\[3mm
]
\Rightarrow f_{yy}=2(4x^3+24x^2y^2+14x^2+24xy^2+6x+6y^2)\\[3mm]
\therefore f_{yy}=2(4x^3+24x^2y^2+14x^2+24xy^2+6x+6y^2)$}
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    \textsc{Assignment 3}\\
    \textsc{Problem 4}\\
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Ans to the question no 03

Given multi-variable function,

$f(x,y) = (x^2 + 2y^2x + 3x + y^2)^2$

Derivatives of f(x,y) with respect to x considering other variables constant,

$f_x = 2(x^2 + 2y^2x + 3x + y^2)(2x + 2y^2 + 3)$

$f_{xx} = 2\{2(x^2 + 2y^2x + 3x + y^2) + (2x + 2y^2 + 3)^2\}$

$\Rightarrow f_{xx} = 2\{(2x^2 + 4xy^2 + 6x + 2y^2) + (4x^2 + 4y^4 + 9 + 8xy^2 + 12y^2 + 12x)\}$

$\Rightarrow f_{xx} = 2(6x^2 + 12xy^2 + 18x + 14y^2 + 4y^2 + 9)$

$\therefore f_{xx} = 2(6x^2 + 12xy^2 + 18x + 14y^2 + 4y^2 + 9)$

Similarly,

$f_y = 2(x^2 + 2y^2x + 3x + y^2)(4xy + 2y)$

$f_{yy} = 2\{(4x + 2)(x^2 + 2y^2x + 3x + y^2) + (4xy + 2y)^2\}$

$\Rightarrow f_{yy} = 2\{(4x^3 + 8x^2y^2 + 12x^2 + 4xy^2 + 2x^2 + 4xy^2 + 6x + 2y^2) + (16x^2y^2 + 4y^2 + 16xy^2)\}$

$\Rightarrow f_{yy} = 2(4x^3 + 24x^2y^2 + 14x^2 + 24xy^2 + 6x + 6y^2)$

$\therefore f_{yy} = 2(4x^3 + 24x^2y^2 + 14x^2 + 24xy^2 + 6x + 6y^2)$

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    \textsc{Assignment 3}\\
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  \textbf{\Large \underline {Ans to the question no 04}}\\
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\Large {Given function, \\[2mm]
$f(x, y) = \ln((x^2 + 3y + xy)^2)$\\[2mm]
Where $x(t) = 2t+3, y(t) = t^2+3t $\\[2mm]
Chair rule for partial derivatives,\\[2mm]
$\frac{df}{dt}=\frac{\partial{f}}{\partial{x}}\cdot
\frac{dx}{dt}+\frac{\partial{f}}{\partial{y}}\cdot \frac{dy}{dt}\\[2mm]$
Now,\\[2mm]
$\frac{\partial{f}}{\partial{x}}=\frac{1}{(x2 + 3y + xy)^2}\cdot 2(x^2 + 3y +
xy)\cdot(2x+y)=\frac{2(2x+y)}{(x^2 + 3y + xy)}\\[2mm]
\frac{dx}{dt}=2\\[2mm]
\frac{\partial{f}}{\partial{y}}=\frac{1}{(x2 + 3y + xy)^2}\cdot 2(x^2 + 3y +
xy)\cdot(3+x)=\frac{2(x+3)}{(x^2 + 3y + xy)}\\[2mm]
\frac{dy}{dt}=2t+3\\[3mm]$
Imposing these values on the chain rule formula,\\[2mm]
$\frac{df}{dt}=\frac{2\cdot2(2x+y)}{(x^2 + 3y + xy)}+\frac{2(x+3)(2t+3)}{(x^2 + 3y + xy)}\\[2mm]$
Since we have to evaluate this expression when t=0,\\[2mm]
$\frac{4(2x+y)}{(x^2 + 3y + xy)}+\frac{2(x+3)(2\cdot0+3)}{(x^2 + 3y + xy)}\\[2mm]
=\frac{8x+4y}{(x^2 + 3y + xy)}+\frac{6x+18}{(x2 + 3y + xy)}\\[2mm]
=\frac{14x+4y+18}{(x^2 + 3y + xy)}\\[2mm]
\therefore \frac{df}{dt}=\frac{14x+4y+18}{(x^2 + 3y + xy)}$ when t=0}
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    \textsc{Assignment 3}\\
    \textsc{Problem 5}\\
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<u>Ans to the question no 04</u>

Given function,

$f(x, y) = \ln((x^2 + 3y + xy)^2)$

Where $x(t) = 2t + 3, y(t) = t^2 + 3t$

Chair rule for partial derivatives,

$\frac{df}{dt} = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dt}$

Now,

$\frac{\partial f}{\partial x} = \frac{1}{(x2+3y+xy)^2} \cdot 2(x^2 + 3y + xy) \cdot (2x + y) = \frac{2(2x+y)}{(x^2+3y+xy)}$

$\frac{dx}{dt} = 2$

$\frac{\partial f}{\partial y} = \frac{1}{(x2+3y+xy)^2} \cdot 2(x^2 + 3y + xy) \cdot (3 + x) = \frac{2(x+3)}{(x^2+3y+xy)}$

$\frac{dy}{dt} = 2t + 3$

Imposing these values on the chain rule formula,

$\frac{df}{dt} = \frac{2\cdot2(2x+y)}{(x^2+3y+xy)} + \frac{2(x+3)(2t+3)}{(x^2+3y+xy)}$

Since we have to evaluate this expression when t=0,

$\frac{4(2x+y)}{(x^2+3y+xy)} + \frac{2(x+3)(2\cdot0+3)}{(x^2+3y+xy)}$

$= \frac{8x+4y}{(x^2+3y+xy)} + \frac{6x+18}{(x2+3y+xy)}$

$= \frac{14x+4y+18}{(x^2+3y+xy)}$

$\therefore \frac{df}{dt} = \frac{14x+4y+18}{(x^2+3y+xy)}$ when t=0

Assignment 3

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\therefore \frac{df}{dt}=\frac{14x+4y+18}{(x^2 + 3y + xy)}$ when t=0}
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    \textsc{Assignment 3}\\
    \textsc{Problem 5}\\
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  \textbf{\Large \underline {Ans to the question no 05}}\\
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\Large {Given function, \\[3mm]
$y^2\sin(x^3)+ze^{3x}-\cos(z^2) = z^4-y^2+x^2$\\[3mm]
Imposing partial differentiation on both sides with respect to x ,\\[3mm]
$\frac{\partial}{\partial{x}}\{y^2\sin(x^3)+ze^{3x}-\cos(z^2)\}=\frac{\partial}{\partial{x}}(z^4-y^2+x^2)\\[3mm]
\Rightarrow y^2\cos(x^3)\cdot 3x^2+\frac{\partial{z}}{\partial{x}}e^{3x}+z\cdot3e^{3x}+\sin(z^2)2z\frac{\partial{z}}{\partial{x}}=4z^3\frac{\partial{z}}{\partial{x}}+2x \\[3mm]
\Rightarrow \frac{\partial{z}}{\partial{x}} (e^{3x}+2z\sin(z^2)-4z^3)= 2x-z\cdot3e^{3x}-3x^2y^2\cos(x^3)\\[3mm]
\Rightarrow \frac{\partial{z}}{\partial{x}}=\frac{2x-3ze^{3x}-3x^2y^2\cos(x^3)}{(e^{3x}+2z\sin(z^2)-4z^3)} \\[3mm]
\therefore \frac{\partial{z}}{\partial{x}}=\frac{2x-3ze^{3x}-3x^2y^2\cos(x^3)}{(e^{3x}+2z\sin(z^2)-4z^3)} \\[4mm]$
Similarly partial  differentiation  on  both  sides  with  respect to y,\\[3mm]
$\frac{\partial}{\partial{y}}\{y^2\sin(x^3)+ze^{3x}-\cos(z^2)\}=\frac{\partial}{\partial{y}}(z^4-y^2+x^2)\\[3mm]
\Rightarrow 2y\sin(x^3)+\frac{\partial{z}}{\partial{y}}e^{3x}+\sin(z^2)2z\frac{\partial{z}}{\partial{y}}=4z^3\frac{\partial{z}}{\partial{y}}-2y \\[3mm]
\Rightarrow \frac{\partial{z}}{\partial{y}} (e^{3x}+2z\sin(z^2)-4z^3)= -2y-2y\sin(x^3)\\[3mm]
\Rightarrow \frac{\partial{z}}{\partial{y}}=\frac{-2y(1+\sin x^3)}{(e^{3x}+2z\sin(z^2)-4z^3)}\\[4mm]
\therefore \frac{\partial{z}}{\partial{y}}=\frac{-2y(1+\sin x^3)}{(e^{3x}+2z\sin(z^2)-4z^3)} $}
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    \textsc{Assignment 3}\\
    \textsc{Problem 6}\\
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Ans to the question no 05

Given function,

$y^2\sin(x^3)+ze^{3x}-\cos(z^2) = z^4-y^2+x^2$

Imposing partial differentiation on both sides with respect to x ,

$$\frac{\partial}{\partial x}\{y^2\sin(x^3)+ze^{3x}-\cos(z^2)\}=\frac{\partial}{\partial x}(z^4-y^2+x^2)$$

$$\Rightarrow y^2\cos(x^3)\cdot 3x^2+\frac{\partial z}{\partial x}e^{3x}+z\cdot 3e^{3x}+\sin(z^2)2z\frac{\partial z}{\partial x}=4z^3\frac{\partial z}{\partial x}+2x$$

$$\Rightarrow \frac{\partial z}{\partial x}(e^{3x}+2z\sin(z^2)-4z^3)= 2x-z\cdot 3e^{3x}-3x^2y^2\cos(x^3)$$

$$\Rightarrow \frac{\partial z}{\partial x}=\frac{2x-3ze^{3x}-3x^2y^2\cos(x^3)}{(e^{3x}+2z\sin(z^2)-4z^3)}$$

$$\therefore \frac{\partial z}{\partial x}=\frac{2x-3ze^{3x}-3x^2y^2\cos(x^3)}{(e^{3x}+2z\sin(z^2)-4z^3)}$$

Similarly partial differentiation on both sides with respect to y,

$$\frac{\partial}{\partial y}\{y^2\sin(x^3)+ze^{3x}-\cos(z^2)\}=\frac{\partial}{\partial y}(z^4-y^2+x^2)$$

$$\Rightarrow 2y\sin(x^3)+\frac{\partial z}{\partial y}e^{3x}+\sin(z^2)2z\frac{\partial z}{\partial y}=4z^3\frac{\partial z}{\partial y}-2y$$

$$\Rightarrow \frac{\partial z}{\partial y}(e^{3x}+2z\sin(z^2)-4z^3)=-2y-2y\sin(x^3)$$

$$\Rightarrow \frac{\partial z}{\partial y}=\frac{-2y(1+\sin x^3)}{(e^{3x}+2z\sin(z^2)-4z^3)}$$

$$\therefore \frac{\partial z}{\partial y}=\frac{-2y(1+\sin x^3)}{(e^{3x}+2z\sin(z^2)-4z^3)}$$

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\therefore \frac{\partial{z}}{\partial{x}}=\frac{(e^{3x}+2z\sin(z^2)-4z^3)}{2x-3ze^{3x}-3x^2y^2\cos(x^3)} \\[4mm]$
Similarly partial  differentiation  on  both  sides  with  respect to y,\\[3mm]
$\frac{\partial}{\partial{y}}\{y^2\sin(x^3)+ze^{3x}-\cos(z^2)\}=\frac{\partial}{\partial{y}}(z^4-y^2+x^2)\\[3mm]
\Rightarrow 2y\sin(x^3)+\frac{\partial{z}}{\partial{y}}e^{3x}+\sin(z^2)2z\frac{\partial{z}}{\partial{y}}=4z^3\frac{\partial{z}}{\partial{y}}-2y \\[3mm]
\Rightarrow \frac{\partial{z}}{\partial{y}} (e^{3x}+2z\sin(z^2)-4z^3)= -2y-2y\sin(x^3)\\[3mm]
\Rightarrow \frac{\partial{z}}{\partial{y}}=\frac{-2y(1+\sin x^3)}{(e^{3x}+2z\sin(z^2)-4z^3)}
\\[4mm]
\therefore \frac{\partial{z}}{\partial{y}}=\frac{-2y(1+\sin x^3)}{(e^{3x}+2z\sin(z^2)-4z^3)} $}
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\begin{center}
  \textbf{\Large \underline {Ans to the question no 06}}\\
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\Large {Given function, \\[3mm]
$h(x,y,z) = y^3z^3 \cos(x^4)+x^3 \sin(z^2)$\\[3mm]
We have to use partial differentiation to find $ h_{yzx}$.\\[3mm]
Imposing partial differentiation of $h(x,y,x)$ with respect to y,\\[3mm]
$h_{y}=3y^2z^3 \cos(x^4)$\\[3mm]
Again partial differentiation of $h_{y}$ with respect to z, \\[3mm]
$h_{yz}=3y^23z^2 \cos(x^4)=9y^2z^2\cos(x^4)$\\[3mm]
Finally partial differentiation of $h_{yz}$ with respect to x, \\[3mm]
$h_{yzx}=9y^2z^2\{-\sin(x^4)(4x^3)\}=36x^3y^2z^2\{-\sin(x^4)\}$\\[3mm]
evaluating the expression of $h_{yzx}$ at point (0,1,1) we get,\\[3mm]
$h_{yzx}=36\cdot0^3\cdot1^2\cdot1^2\{-\sin(0^4)\}=0$\\[3mm]}
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Ans to the question no 06

Given function,

$h(x, y, z) = y^3z^3\cos(x^4) + x^3\sin(z^2)$

We have to use partial differentiation to find h_{yzx}.

Imposing partial differentiation of $h(x, y, x)$ with respect to y,

$h_y = 3y^2z^3\cos(x^4)$

Again partial differentiation of h_y with respect to z,

$h_{yz} = 3y^23z^2\cos(x^4) = 9y^2z^2\cos(x^4)$

Finally partial differentiation of h_{yz} with respect to x,

$h_{yzx} = 9y^2z^2\{-\sin(x^4)(4x^3)\} = 36x^3y^2z^2\{-\sin(x^4)\}$

evaluating the expression of h_{yzx} at point (0,1,1) we get,

$h_{yzx} = 36 \cdot 0^3 \cdot 1^2 \cdot 1^2\{-\sin(0^4)\} = 0$