Department of Mathematics and Natural Sciences

MAT 110

# ASSIGNMENT 1

SUMMER 2021

## SET: 15 (AII)

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*Please write your name and ID on the first page of the assignment answer script - you have to do this for both handwritten or LaTeX submission. The last date of submission is 4-7-2021, 1200 am. Solve all problems.*

*You can only submit a PDF file - image or doc files won't be accepted. Before submitting the PDF, please rename the PDF file in the format - SET_ID_SECTION.*

*Answer the questions by yourself. Plagiarism will lead to an F grade in the course.* ***Total marks is 300. Each question is worth 50 marks.*** *If you do your work using LaTeX you will get a maximum bonus of one mark which will be added as a LaTeXbonus to your course grade.*

*If you use LaTeX, you must add a screenshot of the raw code and compiled pdf side by side, in order to earn your bonus.*

*This set was prepared by and will be checked by AII. If you have any questions, please text AII on Slack.*

1. Determine whether $f(x)$ is continuous at $x = 0$, where

$$f(x) = \begin{cases} x+2; & x<0 \\ x^2-1; & 0 \le x \le 2 \\ 2x+1; & x>2. \end{cases}$$

2. Check whether $\lim_{x \to 1} f(x)$ exists or not, where

$$f(x) = \begin{cases} x^2; & x < -1 \\ x + 5; & -1 \leq x \leq 1 \\ x^3 - 4x; & x > 1. \end{cases}$$

3. Evaluate $\frac{d^2}{dx^2}\left(\ln\left(\frac{x^4+2x^3}{x^2+1}\right)\right)$.

4. Find $\frac{dy}{dx}$ from $(x - y)^2 = x + y - 1$.

5. Evaluate $\frac{d}{dx}\left(\sin^2\left(\frac{2x}{x+1}\right)\right)$.

6. Some populations initally grow exponentially but eventually level off. Equations of the form

$$P(t) = \frac{M}{1 + Ae^{-kt}}$$

where $M$, $A$, and $k$ are positive constants, are called logistic equations and are often used to model such populations. Here $M$ is called the carrying capacity and represents the maximum population size that can be supported, and $A = \frac{M-P_0}{P_0}$ where $P_0$ is the initial population.

   (a) Compute $\lim_{t \to \infty} P(t)$. Explain why your answer is to be expected.

   (b) Compute $\lim_{M \to \infty} P(t)$. (Note that $A$ is defined in terms of $M$.) What kind of function is your result ?