Differential Calculus

*Home work sheet #1

Domain & Range

1. Sketch the following functions and find the domain and range:

   i. $f(x)=\dfrac{1}{x-2}$

   ii. $g(x)=\dfrac{|x|}{x}$

   iii. $h(x)=x^2-2$

   iv. $k(x)=\sqrt{4-x^2}$

Find out the domain and range of the following functions:

1. $f(x)=\dfrac{1}{x-3}$

2. $f(x)=\sqrt{x^2-9}$

3. $f(x)=\sqrt{9-x^2}$

4. $f(x)=\begin{cases}2x+6 & ,-3\le x\le 0\\ 6 & ,0<x<2\\ 2x-6 & ,2\le x\le 5\end{cases}$

5. $f(x)=\dfrac{x}{|x|}$

6. $f(x)=x^3+2$

7. $f(x)=\begin{cases}x+2 & ,x\le -1\\ x^3 & ,|x|<1\\ -x+3 & ,x\ge 1\end{cases}$

8. $f(x)=e^x$

10. $f(x)=\sqrt{2x+4}$

11. $f(x)=\dfrac{1}{5x+7}$

12. $f(x)=-\sqrt{x^2-7x+10}$

13. $f(x)=\begin{cases}x^2 & ,x<0\\ x & ,0\le x\le 1\\ \dfrac{1}{x} & ,x>1\end{cases}$

14. $f(x)=\sqrt{x^2-5x+6}$

15. $f(x)=Sin^2x$

16. $f(x)=\begin{cases}\dfrac{x^2-1}{x-1} & ,x\ne 1\\ 2 & ,x=1\end{cases}$

17. $f(x)=\log x$

**These problems are for the students only as home work. Search the reference books for more examples.*

Differential Calculus

9. $f(x) = 3Sinx$

18. $f(x) = \ln(x^2 + 1)$

Sketch the above functions except 16 and 18.

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Differential Calculus

*Home work sheet #2

Limit

Find the limit of the following functions:

1. $\lim_{x\to 0}\dfrac{x}{\sqrt{x+1}-1}$

2. $\lim_{x\to 2}\dfrac{2x^2-5x+2}{5x^2-7x-6}$

3. $f(x)=\begin{cases}x^2+1 & ,x>0\\ 1 & ,x=0\\ 1+x & ,x<0\end{cases}$

   Find $\lim_{x\to 0} f(x)$

4. $f(x)=\begin{cases}3x-1 & ,x<1\\ 3-x & ,x>1\end{cases}$

   Find $\lim_{x\to 1} f(x)$

5. $\lim_{x\to 0}\dfrac{x}{|x|}$

6. $\lim_{x\to\infty}\dfrac{3x+5}{6x-8}$

7. $f(x)=\begin{cases}2-x & ,x<1\\ x^2+1 & ,x>1\end{cases}$

   Find $\lim_{x\to 1} f(x)$

8. $f(x)=\begin{cases}e^{\frac{-|x|}{2}} & ,-1<x<0\\ x^2 & ,0<x<2\end{cases}$

   Find $\lim_{x\to 0} f(x)$

9. $f(x)=\begin{cases}\dfrac{1}{x+2} & ,x<-2\\ x^2-5 & ,-2<x<3\\ \sqrt{x+13} & ,x>3\end{cases}$

   Find $\lim_{x\to -2} f(x)$ and $\lim_{x\to 3} f(x)$

10. $f(x)=\begin{cases}x^2 & ,x<1\\ 2.4 & ,x=1\\ x^2+1 & ,x>1\end{cases}$

    Does $\lim_{x\to 1} f(x)$ exist?

11. $\lim_{x\to\infty}\left(\sqrt{x^6+5x^3}-x^3\right)$

12. Prove that $\lim_{x\to\infty}\left(1+\dfrac{1}{x}\right)^x=e$

13. $\lim_{x\to\infty}\sqrt[3]{\dfrac{3x+5}{6x-8}}$

14. $\lim_{x\to-\infty}\dfrac{4x^2-x}{2x^3-5}$

15. Let $f(x)=\begin{cases}2x+1 & ,x<1\\ 3-x & ,x>1\end{cases}$, find $\lim_{x\to 1} f(x)$.

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Differential Calculus

*Home work sheet # 3

Continuity and Differentiability

a) Test the continuity of the following functions:

1. $f(x)=\begin{cases}\cos x, & x\geq 0\\ -\cos x, & x<0\end{cases}$ at $x=0$.

2. $f(x)=\begin{cases}x\cos(1/x), & x\neq 0\\ 0, & x=0\end{cases}$ at $x=0.$

3. $f(x)=\begin{cases}e^{1/x}, & x\neq 0\\ 1, & x=0\end{cases}$ at $x=0.$

4. $f(x)=\begin{cases}e^{\frac{-|x|}{2}}, & -1<x<0\\ x^2, & 0\leq x<2\end{cases}$ at $x=0.$

5. $f(x)=\begin{cases}(x-a)\sin\left(\dfrac{1}{x-a}\right), & x\neq a\\ 0, & x=a\end{cases}$ at $x=a.$

6. $f(x)=\begin{cases}1, & x<0\\ 1+\sin x, & 0\leq x<\pi/2\\ 2+(x-\pi/2)^2, & x\geq \pi/2\end{cases}$ at $x=0$ and $x=\pi/2.$

7. $f(x)=\begin{cases}x\sin(1/x), & x\neq 0\\ 0, & x=0\end{cases}$ at $x=0.$

8. $f(x)=\begin{cases}\dfrac{|x-3|}{x-3}, & x\neq 3\\ 0, & x=3\end{cases}$ at $x=3.$

9. $f(x)=|x|+|x-1|$ at $x=0$ and $x=1.$

10. $f(x)=\begin{cases}(1+x)^{1/x}, & x\neq 0\\ 1, & x=0\end{cases}$ at $x=0.$

b) Test the differentiability of the following functions:

2. $f(x)=\begin{cases}x^2\cos(1/x), & x\neq 0\\ 0, & x=0\end{cases}$ at $x=0.$

3. $f(x)=|x|$ at $x=0.$

2. $f(x)=\begin{cases}x^2\sin(1/x), & x\neq 0\\ 0, & x=0\end{cases}$ at $x=0.$

(*c*) Let $f(x)=\begin{cases}x^2-16x, & x<9\\ 12\sqrt{x}, & x\geq 9\end{cases}$. Is *f*(*x*) continuous at $x=9$? Determine whether $f(x)$ is differentiable at $x=9$.

(*d*) Let $f(x)=\begin{cases}x^2, & x\leq 1\\ \sqrt{x}, & x>1\end{cases}$. Is $f(x)$ continuous at $x=1$? Determine whether $f(x)$ is

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differentiable at $x=1$.

(*e*) Show that $f(x)=\begin{cases} x^2+1 & ,x\le 1 \\ x & ,x>1 \end{cases}$ is neither continuous nor differentiable at $x=1$. Also sketch the graph of $f(x)$.

*Home work sheet # 4

Techniques of Differentiation

1. Find the differential coefficients of the following functions with respect to *x*. (i.e. $\frac{dy}{dx}$).

$(i)y=\sin x\sin 2x\sin 3x,\ (ii)y=\cos ec^3 x,\ (iii)y=\cos 2x\cos 3x,\ (iv)y=\sin^{-1}(x^2),$

$(v)y=\tan(\sin^{-1}x),\quad (vi)\cot^{-1}\left(\frac{1+x}{1-x}\right),\quad (vii)\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right),\quad (viii)\sin^{-1}\left(\frac{2x}{1+x^2}\right),$

$(ix)\tan^{-1}\left(\frac{2x}{1-x^2}\right),\quad (x)\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right),\quad (xi)\sin\left(2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right),\quad (xii)\ln\sqrt{\frac{1-\cos x}{1+\cos x}}.$

2. Find the differential coefficients of:

$(i)(\sin x)^{\ln x},\qquad (ii)(\sin x)^{\cos x}+(\cos x)^{\sin x}$.

3. Find $\frac{dy}{dx}$ in the following cases:

$(i)3x^4-x^2y+2y^3=0,\quad (ii)x^3+y^3+4x^2y-25=0,\quad (iii)x^y=y^x.$

4. Find $\frac{dy}{dx}$ when

$(i)x=a\cos^3\theta,\ y=a\sin^3\theta,\quad (ii)x=\sin^2\theta, y=\tan\theta,\quad (iii)x=a\sec^2\theta, y=a\tan^2\theta.$

5. Differentiate the left-side functions with respect to the right-side ones:

$(i)\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ with respect to $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ $(ii)x^{\sin^{-1}(x)}$ with respect to $\sin^{-1}x$.

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Differential Calculus

**These problems are for the students only as home work. Search the reference books for more examples.*

Differential Calculus

*Home work sheet #5

Maxima and minima

1. Find (*a*) the open intervals on which f is increasing , (*b*) the open intervals on which f is decreasing, (*c*) the open intervals on which f is concave up, (*d*)) the open intervals on which f is concave down and (*e*) the x- coordinate of all inflection points.

$(i)\ f(x) = x^2 - 5x + 6 \qquad (ii)\ f(x) = 5 + 12x - x^3$

$(iii)\ f(x) = x^4 - 8x^2 + 16 \qquad (iv)\ f(x) = \dfrac{x^2}{x^2 + 2} \qquad (v)\ f(x) = \sqrt[3]{x+2}.$

2. Locate the critical numbers and identify which critical numbers correspond to stationary points.

$(i)\ f(x) = x^3 + 3x^2 - 9x + 1 \quad (ii)\ f(x) = x^4 - 6x^2 - 3$

$(iii)\ f(x) = \dfrac{x}{x^2 + 2} \qquad (iv)\ f(x) = x^{2/3}$

$(v)\ f(x) = x^{1/3}(x+4) \qquad (vi)\ f(x) = \cos 3x.$

3. Find the relative extrema (maxima/ minima) using both the first and second derivative tests.

$(i)\ f(x) = 2x^3 - 9x^2 + 12x \qquad (ii)\ f(x) = \dfrac{x}{2} - \sin x, \quad 0 < x < 2\pi.$

4. Use the given derivative to find all critical numbers of $f$and determine whether a relative maximum, relative minimum, or neither occurs there.

$(i)\ f'(x) = x^3(x^2 - 5) \qquad (ii)\ f'(x) = \dfrac{x^2 - 1}{x^2 + 1}.$

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Differential Calculus

*Home work sheet # 6

Successive Differentiation

*a.*Find the *n*th derivative of the following functions:

1. $y = x^n$
2. $y = (ax + b)^n$
3. $y = \ln(ax + b)$
4. $y = \dfrac{1}{x + a}$
5. $y = e^{ax}$
6. $y = \sin(ax + b)$
7. $y = \cos(ax + b)$

b. If $y = e^{ax} \sin bx,$ then show that $y_2 - 2ay_1 + (a^2 + b^2)y = 0.$

c. If $y = e^x \sin x,$ then show that $y_4 + 4y = 0.$

Rolle's and Mean Value Theorem

a. Verify the hypothesis of Rolle's Theorem for the following functions:

1. $f(x) = x^2 - 6x + 8; \quad [2, 4]$
2. $f(x) = \cos x; \quad [\pi / 2, 3\pi / 2]$
3. $f(x) = \dfrac{x}{2} - \sqrt{x}; \quad [0, 4].$

b. Verify the hypothesis of Mean Value Theorem for the following functions:

1. $f(x) = x^3 + x - 4; \quad [-1, 2]$
2. $f(x) = \sqrt{x + 1}; \quad [0, 3]$
3. $f(x) = \sqrt{25 - x^2}; \quad [0, 5].$

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Differential Calculus

Maclaurin and Taylor Series

1. Find the Taylor series for the following functions:

   $(i)\ \sin x$, at $x_0 = \frac{\pi}{2}$. $(ii)\ \ln x$, at $x_0 = 2$.

2. Expand $y = \ln x$ in the power of $x-2$ and $y = e^{ax}$ in the power of $x-1$.

3. Find the Maclaurin series for the function e^{ax} and $\cos x$.

4. Find the Maclaurin polynomial p_0, p_1, p_2, p_3 for $e^x \cos x$.

5. Expand $y = \ln(x+1)$, $y = \sin x$, $y = \cos x$ in the power of x.

*Home work sheet # 7

Leibnitz's Theorem

1. If $y = \tan^{-1} x$, then show that $(1+x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0.$
2. If $y = \cot^{-1} x$, then show that $(1+x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0.$
3. If $y\sqrt{1-x^2} = \sin^{-1} x$, then show that $(1-x^2)y_{n+1} - (2n+1)xy_n - n^2 y_{n-1} = 0.$
4. If $y = e^{\tan^{-1} x}$, then show that $(1+x^2)y_{n+2} + (2nx+2x-1)y_{n+1} + n(n+1)y_n = 0.$
5. If $y = e^{m\sin^{-1} x}$, then show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0.$
6. If $y = (\sin^{-1} x)^2$, then show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0.$
7. If $\log_e y = a\sin^{-1} x$ then show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0.$

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Differential Calculus

8. If $y = e^{m\cos^{-1}x}$ then show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0.$

9. If $\log_e y = \tan^{-1}x$ then show that $(1+x^2)y_{n+2} + (2nx+2x-1)y_{n+1} + n(n+1)y_n = 0.$

10. If $y = \left(\cos^{-1}x\right)^2$, then show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0.$

11. If $\ln y = m\cos^{-1}x,$ then show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0.$

12. If $x = \tan(\ln y),$ then show that $(1+x^2)y_{n+2} + (2nx+2x-1)y_{n+1} + n(n+1)y_n = 0.$

Indeterminate Forms

Find the limit using L' Hospital's rule:

1. $\lim_{x\to 1}\frac{\ln x}{x-1}$, 2. $\lim_{x\to 3}\frac{x-3}{3x^2-13x+12}$, 3. $\lim_{x\to 0}\left(\frac{1}{x^2}-\frac{\cos 3x}{x^2}\right)$, 4. $\lim_{x\to\pi}\frac{\sin x}{x-\pi}$

5. $\lim_{x\to 0}\frac{x-\tan^{-1}x}{x^3}$, 6. $\lim_{x\to+\infty}\frac{e^{3x}}{x^2}$, 7. $\lim_{x\to 0}\frac{a^x-1-x\log a}{x^2}$, 8. $\lim_{x\to 0}(e^x+x)^{\frac{1}{x}}$

9. $\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{\sin x}\right)$, 10. $\lim_{x\to\pi}(x-\pi)\cot x$, 11. $\lim_{x\to 0}\frac{\ln(\sin x)}{\ln(\tan x)}$, 12. $\lim_{x\to\infty}xe^{-x}$

13. $\lim_{x\to 0}\frac{\sin 2x}{x}$, 14. $\lim_{x\to 0}\frac{\sin x}{x^2}$, 15. $\lim_{x\to\infty}\frac{x}{e^x}$, 16. $\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{xe^x}\right)$, 17. $\lim_{x\to 0}\frac{\sin 2x}{\sin 5x}$.

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Differential Calculus

*Home work sheet # 8

Partial derivative

1. Let $f(x,y)=3x^3y^2$. Find

(a) $f_x(x,y)$ (b) $f_y(x,y)$ (c) $f_x(x,1)$ (d) $f_y(1,y)$ (f) $f_x(1,2)$ (g) $f_y(1,2)$.

2.Let $f(x,y)=xe^{-y}+5y$, $f(x,y)=\sqrt{3x+2y}$

(a) Find the Slope of the surface $f(x,y)$ in the x- direction at the point (3,0) and (4,2)

(b) Find the slope of the surface $f(x,y)$ in the y direction of the point (3,0) and (4,2)

3. Let $f(x,y)=4x^2-2y+7x^4y^5$, Find (a) f_{xx} (b) f_{yy} (c) f_{xy} (d) f_{yx}.

4. Let $z=Sin(y^2-4x)$ *and* $z=(x+y)^{-1}$

(a) Find the rate of Change of z w. r. to x at the point (2,1) and (-2, 4) with y kept fixed.
(b) Find the rate of Change of z w. r. to y at the point (2,1) and (-2, 4) with x kept fixed.

5. Let $f(x,y,z)=x^3y^5z^7+xy^2+y^3z$, find

(a) f_{xy} (b) f_{yz} (c) f_{xz} (d) f_{zz} (e) f_{zyy} (f) f_{zxy} (g) f_{zyx} (h) f_{xxyz}.

6. Let $f(x,y,z)=\sqrt{xy}+\ln\left(x^2z^3\right)-x\tan(z)$. Compute f_x, f_z, f_{xy}, f_{xyz}

Maxima Minima of function of several variable

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9–20 Locate all relative maxima, relative minima, and saddle points, if any.

9. $f(x,y)=y^2+xy+3y+2x+3$
10. $f(x,y)=x^2+xy-2y-2x+1$
11. $f(x,y)=x^2+xy+y^2-3x$
12. $f(x,y)=xy-x^3-y^2$
13. $f(x,y)=x^2+y^2+\dfrac{2}{xy}$
14. $f(x,y)=xe^y$
15. $f(x,y)=x^2+y-e^y$
16. $f(x,y)=xy+\dfrac{2}{x}+\dfrac{4}{y}$
17. $f(x,y)=e^x\sin y$
18. $f(x,y)=y\sin x$
19. $f(x,y)=e^{-(x^2+y^2+2x)}$
20. $f(x,y)=xy+\dfrac{a^3}{x}+\dfrac{b^3}{y}$ $(a\neq 0, b\neq 0)$

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Differential Calculus

Lagrange Multiplier

5–12 Use Lagrange multipliers to find the maximum and m
imum values of f subject to the given constraint. Also, find
points at which these extreme values occur. ■

5. $f(x, y) = xy;\ 4x^2 + 8y^2 = 16$
6. $f(x, y) = x^2 - y^2;\ x^2 + y^2 = 25$
7. $f(x, y) = 4x^3 + y^2;\ 2x^2 + y^2 = 1$
8. $f(x, y) = x - 3y - 1;\ x^2 + 3y^2 = 16$
9. $f(x, y, z) = 2x + y - 2z;\ x^2 + y^2 + z^2 = 4$
10. $f(x, y, z) = 3x + 6y + 2z;\ 2x^2 + 4y^2 + z^2 = 70$
11. $f(x, y, z) = xyz;\ x^2 + y^2 + z^2 = 1$
12. $f(x, y, z) = x^4 + y^4 + z^4;\ x^2 + y^2 + z^2 = 1$

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Differential Calculus

*Home work sheet # 9

Transformation of Co-ordinates

1. Find the polar co-ordinates of the points $(2\sqrt{3},-2),\ (0,-2),\ (1,1)$.
2. Find the rectangular co-ordinates of the points $(7,2\pi/3),\ (8,9\pi/4),\ (0,\pi)$.
3. Change to Cartesian coordinates the equations $(i)\, r = a\sin\theta,\ (ii)\sqrt{r} = \sqrt{a}\cos\left(\frac{\theta}{2}\right).$
4. Transform to polar coordinates the equations $(i)9x^2 + 4y^2 = 36,\ (ii)x^3 = y^2(2a - x).$
5. Transform to parallel axes through the new origin (1,-2) of the equation $2x^2 + y^2 - 4x + 4y = 0$.
6. Transform the equation $x^2 + y^2 - 8x + 14y + 5 = 0$ to parallel axes through (4,-7).
7. Transform the equation $7x^2 - 2xy + y^2 + 1 = 0$ to axes turned through the angle $\tan^{-1}\left(\frac{1}{2}\right)$.
8. Transform the equation $11x^2 + 24xy + 4y^2 - 20x - 40y - 5 = 0$ to rectangular axes through the point (2,-1) and inclined at an angle $\tan^{-1}\left(\frac{4}{3}\right)$.
9. Transform the equation $9x^2 + 15xy + y^2 + 12x - 11y - 5 = 0$, so as to remove the terms x and y.
10. Transform the equation $11x^2 + 3xy + 7y^2 + 19 = 0$, so as to remove the term xy.
11. Determine the equation of the curve $2x^2 + 4xy + 5y^2 - 4x - 22y + 7 = 0$ when the origin is transferred to the point $(-2,3)$.

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Differential Calculus

12. Remove the xy term from the equation $9x^2 + 24xy + 2y^2 + 54 = 0.$

13. Determine the equation $x^2 + 2\sqrt{3}xy - y^2 = 2a^2$ after rotating of axes through 30°.

14. Transform the equation $9x^2 + 24xy + 2y^2 - 6x + 20y + 41 = 0$ so as to remove the terms in x and y.

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Differential Calculus

*Home work sheet # 10

Pair of straight lines

1. Show that the following equations represents a pair of straight lines; find also their point of intersection and the angle between them:

$(i)\ 2y^2 - xy - x^2 + y + 2x - 1 = 0, \quad (ii) 2x^2 - 2xy + x + 2y - 3 = 0,$
$(iii)\ x^2 + 3xy + 2y^2 + \frac{1}{8}x - \frac{1}{32} = 0, \quad (iv) 21x^2 + 40xy - 21y^2 + 44x + 122y - 17 = 0.$

2. Find the value of λ or k so that the following equations may represent pairs of straight lines:

$(i)\ 2\lambda xy - y^2 + 4x + 2y + 8 = 0, \qquad (ii) 2x^2 + xy - y^2 - 2x - 5y + k = 0$
$(iii)\ x^2 - \lambda xy + 2y^2 + 3x - 5y + 2 = 0, \qquad (iv) 12x^2 - 10xy + 2y^2 + 11x - 5y + \lambda = 0.$

3. Find the equations of the bisectors of the angles between the following pairs of straight lines:

$(i) x^2 + xy - 6y^2 - x - 8y - 2 = 0, \quad (ii) 8x^2 - 14xy + 6y^2 + 2x - y - 1 = 0,$
$(iii) 2x^2 + xy - y^2 - 3x + 6y - 9 = 0, \quad (iv) 2x^2 + 7xy + 6y^2 + 13x + 22y + 20 = 0$

Circle

1. Find the equation of the circle with
   (i) centre (-2, -1) and radius 4, (ii) centre (9, 0) and radius 1,
   (iii) centre (0, 0) and radius 5.
2. Find the centre and radius of the following circles:

$(i) 5x^2 + 5y^2 - 11x - 9y - 12 = 0, \quad (ii) x^2 + y^2 + 2x + 2y + 1 = 0$
$(iii) x^2 + y^2 + 2x - 4y - 8 = 0.$

3. Find the equation of the circle passing through the points:

$(i)\ (1, 3), (2, -1), (-1, 1), \quad (ii)\ (-4, -3), (-1, -7), (0, 0), \quad (iii)\ (3, 1), (4, -3), (1, -1).$

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Differential Calculus

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Differential Calculus

*Home work sheet # 11

Tangent and Normal of Circle

1. Show that the circles $x^2+y^2-2x+4y+3=0$ and $x^2+y^2-8x-2y+9=0$ touch one another at (2, -1).
2. Find the equation of the circle through the intersection of the circles $x^2+y^2-9x+14y-7=0$ and $x^2+y^2+15x+14=0$ and passes through the point (2, 5).
3. Find the equation of the circle which passes through the intersection of the circles $x^2+y^2=1$ and $x^2+y^2+2x+4y+1=0$ and touches the straight line $x+2y+5=0$.
4. Find the radical centre of the three circles $x^2+y^2+x+2y+3=0$, $x^2+y^2+2x+4y+5=0$ and $x^2+y^2-7x-8y-9=0$.

Conic Section

1. Reduce the following equations to their standard forms:

$(i)\ \ x^2-6xy+9y^2-2x-3y+1=0$
$(ii)\ x^2-4xy+y^2+8x+2y-5=0$
$(iii)\ 4x^2-24xy-6y^2+4x-12y+1=0$
$(iv)\ 9x^2-4xy+6y^2-10x-7=0$
$(v)\ \ x^2-4xy-2y^2+10x+4y=0$
$(vi)\ \ x^2+4y^2-2x-16y+1=0$
$(vii)\ 9x^2+24xy+16y^2+22x+46y+9=0$
$(viii)\ 3x^2+2xy+3y^2+2x-6y+\frac{25}{2}=0.$

2. Find the centre of the following conics:

**These problems are for the students only as home work. Search the reference books for more examples.*

Differential Calculus

(*i*) $x^2 - 4xy + y^2 + 8x + 2y - 5 = 0$

(*ii*) $x^2 - 2xy + 2y^2 - 3x + 7y - 1 = 0$

(*iii*) $3x^2 - 7xy - 6y^2 + 3x - 9y + 5 = 0.$

**These problems are for the students only as home work. Search the reference books for more examples.*