Example

Calculate the mean for the following frequency distribution for n=100.

Class Interval	Frequency
0-10	10
10-20	20
20-30	40
30-40	20
40-50	10

Solution

Class Interval	Frequency(f_i)	Mid values (x_i)	(f_i)*(x_i)
0-10	10	5	50
10-20	20	15	300
20-30	40	25	1000
30-40	20	35	700
40-50	10	45	450
Total	100		2500

Arithmetic mean,

$$\overline{x} = \frac{f_1x_1 + f_2x_2 + f_3x_3 + f_4x_4 + f_5x_5}{n}$$

$$= \frac{50+300+1000+}{100} = 25$$

Test Yourself

The following data represent the distribution of the age of employees within two different divisions of publishing company. Determine which company have relatively aged group of employees.

Age of employees	Number of employees of division	
	X	Y
20-30	6	13
30-40	19	30
40-50	9	24
50-60	10	0
60-70	2	4

Solution

Age of employees	Mid values(x_i)	Frequency(f_{xi})	Frequency(f_{yi})	(f_{xi})*(x_i)	(f_{yi})*(x_i)
20-30	25	6	13	150	325
30-40	35	19	30	665	1050
40-50	45	9	24	405	1080
50-60	55	10	0	550	0
60-70	65	2	4	130	260
Total		46	71	1900	2715

Arithmetic mean age of employee division X $= \frac{\sum_{i=1}^{5} f_{xi} * x_i}{\sum_{i=1}^{5} f_{xi}} = \frac{1900}{46} = 41.3$

Arithmetic mean age of employee division Y $= \frac{\sum_{i=1}^{5} f_{yi} * x_i}{\sum_{i=1}^{5} f_{yi}} = \frac{2715}{71} = 38.2$

Since, A.M of X group of employees > A.M of Y group of employees, X group of employees are relatively aged more.

Example: n is odd

The ages of a family of seven members are given as 12, 7, 2, 34, 17, 21 and 19. Find the median age.

Step 1	Count the total number of elements, n=? Here n= 7, an odd number
Step 2	Arrange the values in ascending order : 2, 7, 12, 17, 19, 21, 34
Step 3	Median: Me = Value of $\frac{n+1}{2}$ th observation = Value of $\frac{7+1}{2}$ th observation =value of 4^{th} observation = 17
Step 4	Median Age of the family is 17 years

Example: n is even

The ages of a family of eight members are given as 12, 7, 2, 34, 17, 40, 21 and 19. Find the median age.

Step 1	Count the total number of elements, n=? Here n= 8, an even number
Step 2	Arrange the values in ascending order : 2, 7, 12, 17, 19, 21, 34, 40
Step 3	Median: Me = AM of the values of $\frac{n}{2}th$ and $(\frac{n}{2}+1)th$ observation = AM of the values of $4th$ and $5th$ observation = $\frac{17}{2}$ = 18
Step 4	Median Age of the family is 18 years

Test Yourself

The following data represents the amount (in thousands taka) of loan requirements of the people of two different upazilla. Using median, comment on which upazilla has the greater average demand of loans.

Upazilla 1	42	12	26	18	9	35	28	39	8
Upazilla 2	8	15	10	18	22	20	26	42	35

Solution

Here, n = 9 (odd)

Arranging Upazilla 1 observations in ascending order:

8, 9, 12, 18, 26, 28, 35, 39, 42

Therefore, median of Upazilla 1 = $\frac{9+1}{2}$ th observation = 26

Arranging Upazilla 2 observations in ascending order:

8, 10, 15, 18, 20, 22, 26, 35, 42

Therefore, median of Upazilla 2 = $\frac{9+1}{2}$ th observation = 20

Since, median of Upazilla 1 > median of Upazilla 2, Upazilla 1 has the greater average demand of loans.

Test Yourself

The following table gives the data pertaining to kilowatt hours of electricity consumed by 100 randomly selected flat owners of Japan garden city.

Consumption (in K-watt hours)	0-100	100-200	200-300	300-400	400-500
No. of users	6	25	36	20	13

Calculate

i. Mean consumption of electricity

ii. Median use of electricity

Solution

Consumption (in K-watt hours)	Mid Value(x_i)	No. of users(f_i)	(f_i)*(x_i)	Cumulative Frequency
0-100	50	6	300	6
100-200	150	25	3750	31
200-300	250	36	9000	67
300-400	350	20	7000	87
400-500	450	13	5850	100
Total		100	25900	

i) Mean consumption of electricity = $\frac{\sum_{i=1}^{5} f_i * x_i}{\sum_{i=1}^{5} f_i} = \frac{25900}{100} = 259$

ii) Median = $\frac{100}{2} = 50th$ Observation

Median class = (200-300)

Lower Limit of the median class (L_0) = 200

Sum of the frequencies of all classes prior the median class (F_{-Me}) = 31

Frequency of median class (f_{Me}) = 36

Width of the median class (W_{Me}) = 300-200 = 100

$$\text{Median, } Me = L_0 + \frac{(\frac{n}{2} - F_{-Me})}{f_{Me}} * W_{Me}$$

$$= 200 + \frac{\frac{100}{2} - 31}{36} * 100$$

$$= 252.78 \text{ (Answer)}$$

Mode: Ungrouped Data

For the data sets:

i. 7, 8, 6, 7, 9, 7, and 4: Here ‘7’ appears highest 3 times, hence mode is ‘7’and the data is unimodal.

ii. 6, 4, 8, 5, 8, 1, 2, 5, 4, 7, 5, 2, 4, and 3: here ‘5’ and ‘4’ both occur highest 3 times hence the mode ‘5’ and ‘4’ and the data is bimodal.

iii. 1, 5, 7, 2, 6, 9, and 4: there is no mode.

iv. Consider the following table representing the frequency distribution of religion

Religion	Muslim	Hindu	Buddhist	Christian	Others
Frequency	18	75	12	4	2

- Here the highest frequency ‘75’ occurs for the category ‘Hindu’. Hence mode for the given data is Hindu.

Test Yourself

The Population of Saint Martin's Island changed by the following percentages each year for the last decade ("+" means increase, "-" means decrease):

Year	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019
Change	+2%	+3%	-1%	+5%	+7%	+5%	+2%	-1%	+2%	+3%

What is the average population growth per year for the last decade?

Solution

Number of observations, n = 10

Change in Year 2010 = $(1 + \frac{2}{100}) = 1.02$

Change in Year 2011 = $(1 + \frac{3}{100}) = 1.03$

Change in Year 2012 = $(1 - \frac{1}{100}) = 0.99$

Change in Year 2013 = $(1 + \frac{5}{100}) = 1.05$

Change in Year 2014 = $(1 + \frac{7}{100}) = 1.07$

Change in Year 2015 = $(1 + \frac{5}{100}) = 1.05$

Change in Year 2016 = $(1 + \frac{2}{100}) = 1.02$

Change in Year 2017 = $(1 - \frac{1}{100}) = 0.99$

Change in Year 2018 = $(1 + \frac{2}{100}) = 1.02$

Change in Year 2019 = $(1 + \frac{3}{100}) = 1.03$

Geometric Mean = $\sqrt[10]{1.02 * 1.03 * 0.99 * 1.05 * 1.07 * 1.05 * 1.02 * 0.99 * 1.02 * 1.03}$

Geometric Mean = 1.0267

Therefore, the average population growth per year for the last decade is 1.0267

Test Yourself

The price of Toyota Camry has fluctuated for last few years. Given below is the change in value for each year. (“+” means increase, “-” means decrease.)

Year	2012	2013	2014	2015	2016	2017	2018	2019
Change	+6%	+3%	+5%	+1%	-2%	-3%	+1%	-4%

What is the average annual change in the price of Toyota Camry within this period? Answer in percentage.

Solution

Number of observations, n = 8

Change in Year 2012 = $(1 + \frac{6}{100}) = 1.06$

Change in Year 2013 = $(1 + \frac{3}{100}) = 1.03$

Change in Year 2014 = $(1 + \frac{5}{100}) = 1.05$

Change in Year 2015 = $(1 + \frac{1}{100}) = 1.01$

Change in Year 2016 = $(1 - \frac{2}{100}) = 0.98$

Change in Year 2017 = $(1 - \frac{3}{100}) = 0.97$

Change in Year 2018 = $(1 + \frac{1}{100}) = 1.01$

Change in Year 2019 = $(1 - \frac{4}{100}) = 0.96$

Geometric Mean = $\sqrt[8]{1.06 * 1.03 * 1.05 * 1.01 * 0.98 * 0.97 * 1.01 * 0.96}$

= 1.0082

Therefore, the average annual change in the price of Toyota Camry in percentage is 0.82%

Test Yourself

Evan was driving up a mountainous road, with some flat stretches among the steep climbs. As a result, his speed varied for each 10 kilometers, as seen below:

Distance Travelled (km)	10	10	10	10	10
Speed (km/h)	10	25	6	15	8

What was his average speed throughout the drive?

Solution

Harmonic Mean = $\dfrac{n}{\frac{1}{x_1}+\frac{1}{x_2}+\cdots+\cdots+\frac{1}{x_n}}$

Number of observations, n = 5

Harmonic Mean = $\dfrac{5}{\frac{1}{10}+\frac{1}{25}+\frac{1}{6}+\frac{1}{15}+\frac{1}{8}}$ = 10.03

Therefore, the average speed throughout the drive was 10.03 km/h

Test Yourself

Selim rides his bicycle to deliver food. Given below is his speed for each kilometer he travelled today.

Kilometer	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}
Speed (km/h)	3	2	6	4	4	5

What was his average speed on the 6km ride?

Solution

$$\text{Harmonic Mean} = \frac{n}{\frac{1}{x_1}+\frac{1}{x_2}+\cdots+\cdots+\frac{1}{x_n}}$$

$$\text{Harmonic Mean} = \frac{6}{\frac{1}{3}+\frac{1}{2}+\frac{1}{6}+\frac{1}{4}+\frac{1}{4}+\frac{1}{5}} = 3.53$$

Therefore, his average speed on the 6km ride is 3.53 km/h

Test Yourself

The postal service handles five basic types of letters and cards: 1st class, airmail, special delivery, registered and certified. The mail volume during a given year is given in the following table.

Types of Mailing	gm delivered (in millions)	Price per gm
1st class	77600	0.13
AIR mail	19000	0.17
Special delivery	1300	0.35
Registered mail	750	0.40
Certified mail	800	0.45

What was the average revenue per gm for these services during the year?

Solution

Types of Mailing	gm delivered (in millions)	Price per gm	Total price of each type (price * gm delivered)
1st class	77600	0.13	10088
AIR mail	19000	0.17	3230
Special delivery	1300	0.35	455
Registered mail	750	0.40	300
Certified mail	800	0.45	360

Total gm delivered (in millions) = 77600 + 19000 + 1300 + 750 + 800 = 99450

Total price of all the types of Mailing = 10088 + 3230 + 455 + 300 + 360 = 14433

$$\text{Average revenue of all types of services per gm} = \frac{\text{Total price of all the types of Mailing}}{\text{Total gm delivered (in millions)}}$$

$$= \frac{14433}{99450} = 0.15$$