Ass 2

Friday, February 17, 2023 11:37 AM

1(a)

$f(x) = \tan x_0 = 0$

$f'(x_0) = \sec^2 x_0 = 1$

$\tan 0 = 0$

$\sec 0 = 1$

$f''(x) = 2\sec x_0 \cdot \frac{d}{dx_0} \sec x_0 = 0$

$= 2\sec^2 x_0 \tan x_0$

$f^3(x) = 2\left[\sec^2 x_0 \cdot \sec^2 x_0 + \tan x_0 \cdot 2\sec^2 x_0 \tan x_0\right] = 2$

$= 2\sec^4 x_0 + 4\sec^2 x_0 \tan^2 x_0$

$$f(x) = \tan x_0 + \sec^2 x_0 (x - x_0) + \frac{2\sec^2 x_0 \tan x_0 (x - x_0)^2}{2!} + \frac{(2\sec^4 x_0 + 4\sec^2 x_0 \tan^2 x_0)(x - x_0)^3}{3!}$$

$$= 0 + x - 0 + 0 + \frac{2x^3}{3!}$$

$$= x + \frac{2x^3}{3!}$$

$$= \frac{\pi}{4} + \frac{2\left(\frac{\pi}{4}\right)^3}{3!}$$

$$= 0.946$$

C

$\tan \frac{\pi}{4} = 1$

$P\left(\frac{\pi}{4}\right)$

$1 - 0.946 = 0.054$

$= 5.4\%$

Taylor expansion:

$$P_n(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)(x - x_0)^2}{2!} + \frac{f^3(x_0)(x - x_0)^3}{3!} \cdots$$

$$|f(x) - P(x)| \leq \frac{f^{n+1}(\xi)}{(n+1)!} \times (x - x_0)^{n+1}$$

Vandan

$$V \cdot a = f$$

$$V = \begin{bmatrix} 1 & x_0^1 & x_0^2 & \cdots & x_0^n \\ 1 & x_1^1 & x_1^2 & \cdots & x_1^n \\ \vdots & & & & \\ 1 & x_n^1 & x_n^2 & \cdots & x_n^n \end{bmatrix} \quad a = \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_n \end{pmatrix} \quad f = \begin{bmatrix} f(x_0) \\ f(x_1) \\ \vdots \\ f(x_n) \end{bmatrix}$$

$$P_n(x) = a_0 x^0 + a_1 x^1 + a_2 x^2 \cdots a_n x^n$$

Lagrange ò

Lagrange

$$P_n(x) = \sum f(x_i) l_i(x)$$

$$l_0(x) = \frac{x-x_0}{x_0-x_0} \times \frac{x-x_1}{x_0-x_1} \times \frac{x-x_2}{x_0-x_2}$$

Newton:

$$P_n(x) = a_0 n_0(x) + a_1 n_1(x) + a_2 n_2(x) \ldots\ldots$$

$a_0 = f[x_0]$, $a_1 = f[x_0, x_1]$

$a_n = f[x_0, x_1, \ldots x_n]$

$n_0(x) = 1$

$n_1(x) = x - x_0$

$n_2(x) = (x-x_0)(x-x_1)$

$n_3(x) = (x-x_0)(x-x_1)(x-x_2)$

$f[x_0]$ $\quad \frac{f(x_1)-f(x_0)}{x_1-x_0} = f[x_0, x_1]$

$f[x_0, x_1, x_2] = \frac{f(x_1, x_2) - f(x_0, x_1)}{x_2 - x_0}$

$f[x_1]$ $\quad f x_2 - f x_1 = f[x_1, x_2]$

$f(x_1)$ $\frac{f x_2 - f x_1}{x_2 - x_1}$ $f(x_1, x_2)$

$f(x_2)$

$f(x) = \ln x$

$f'(x) = \frac{1}{x} = x^{-1}$

$f''(x) = -x^{-2} = -\frac{1}{x^2}$

$\int \tan x \rightarrow \sec^2 x$

$\sec \rightarrow \sec x \tan x$

$\cot \quad -\operatorname{cosec}^2 x$

$\cos x - r - \operatorname{cosec} x \tan x$

$$a_0 x^0 + a_1 x^1 + a_2$$