

**Final Examination : Questions for CSE330. All Sections.**

Department of Computer Science & Engineering

BRAC University

Spring Semester

Date : May 07, 2022

Time : One hour 30 minutes

Faculty Name (Initial) : _____

Student ID# : _____

Section#: _____

Instructions:

- There are six question. **Answer any four questions.** Total marks 40.
- Use pencil for your answers. No break for bathroom/freshroom is allowed. **Must use your own calculator.** Cell phones must be turned off (Not in vibration mode). We assume that you know how to use scientific calculator of model CASIO fx-991 ES or equivalent.
- Return this question along with your answer script.
- All examinees must abide by the 'Regulations of Students Conduct' of Brac university.

Read carefully the questions below and answer properly:

- (a) (5 marks) Consider the function $f(x) = e^x$ and a point $x_0 = 2$. Find the truncation error using the backward difference method for $h = 1, h = 0.1, h = 0.01, h = 0.001$ and figure out the relationship of the error with the order of h . Keep up to six decimal places.
- (b) (5 marks) A rocket has been launched, and its velocities at different times are collected. From these data, the acceleration of the rocket, $a(t)$, at $t = 16$ sec is calculated numerically by using different methods ($h = 1$) as shown in the table below (up to six decimal places):

Difference Method	Forward	Backward	Central
$a(t = 16)$	33.880085	32.898426	33.389255

Now, the velocity of a rocket as function of time obeys the equation

$$v(t) = 1900 \ln \left(\frac{12 \times 10^4}{12 \times 10^4 - 2000t} \right) - 9.8t ,$$

where v is in m/s and t is in seconds. Find the truncation errors for the acceleration at $t = 16$ sec for Forward, Backward and Central Difference methods. Note that acceleration is the first derivative of velocity.

- Consider two functions given as

$$f(x) = 2x - \cos x \quad \text{for } x \in [-1, 1] \quad (1)$$

$$f(x) = 2x^3 - 2x^2 - 3x + 3 . \quad (2)$$

- (2 marks) Is it possible to find root of function in Eq.(1)? [Hint: You can draw a graph for better understanding].
- (4 marks) Construct two different fixed point functions for the function given in Eq.(2) such that $f(x) = 0$.
- (4 marks) Compute the convergence rate of each fixed point function obtained in the previous part, and state if the root is converging linearly, superlinearly or it is diverging.

3. Consider the function $f(x) = 7 - e^{2x-3}$. Find the solution of the function, $f(x) = 0$, up to 8 iterations, where $x_0 = 5$ by using the following methods
- (a) (6 marks) Newton's method.
 - (b) (4 marks) Also by applying Aitken acceleration.
4. A linear system is described by the following equations:

$$x_1 + x_2 + x_3 = 6$$

$$2x_1 + 3x_2 + 4x_3 = 20$$

$$3x_1 + 4x_2 + 2x_3 = 17.$$

- (a) (4 marks) Construct the Frobenius matrices $F^{(1)}$ and $F^{(2)}$ for this system.
 - (b) (2 marks) Find the unit lower triangular matrix L .
 - (c) (4 marks) Now find the solution of this linear system using LU decomposition. Use the lower triangular matrix, L , obtained in the previous part.
5. "Best Quality Vacuum" is a store that sells vacuum cleaners of various models. Recently, they have put up a website for their business, and they are tracking the number of website traffic vs the total number of vacuum cleaners sold. The following table shows the record for the first three months. x represents the total number of their website traffic, in thousands, for a particular month. y represents the total number of vacuum cleaners sold:

x	100	220	430
y	70	180	300

Your task is to find the equation of a best-fit straight line that gives the least error while fitting the data above, by answering the following:

- (a) (2 marks) Represent the above information of the over-determined system using matrices.
 - (b) (7 marks) Use the QR -decomposition method to find the equation of the best-fit straight line that gives the least error while fitting the data above.
 - (c) (1 mark) To boost the sale, "Best Quality Vacuum" launched a TV commercial expecting a huge surge in the number of internet traffic. Estimate the total number of vacuum cleaners that needs to be in stock if their website traffic increases to 1,500,000.
6. Consider a function $f(x) = \ln x$ which is continuous on the interval $[1, 4]$. Use this function to answer the following:
- (a) (2 marks) Find the actual integral value for this function.
 - (b) (6 marks) Use the closed Composite Newton-cotes formula to find the numerical integration for $m = 4$.
 - (c) (2 marks) Compute the error in percentage between the results obtained in the previous two parts. How can we decrease the error more?