Ans to the qr no 1(a)

Given,

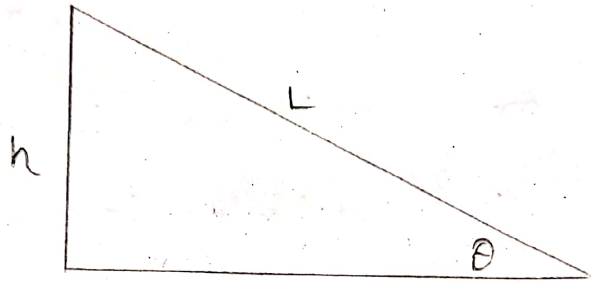
Mass of cylinder = m
Radius of cylinder = r
Mass of sphere = M
Radius of sphere = R

Now,

Moment of inertia of the cylinder is $\frac{1}{2} mr^2$

Moment of inertia of the sphere is $\frac{2}{5} MR^2$

Ans to the qr no 1 (b)



Here,

$\sin\theta = h/L$

$\Rightarrow h = L\sin\theta$

$\therefore$ The potential energy of the cylinder at the top, $PE_{cylinder} = mgh = mgL\sin\theta$

$\therefore$ The potential energy of the sphere at the top, $PE_{sphere} = Mgh = MgL\sin\theta$

Ans to the qr no 1(c)

After letting go the cylinder down the inclined plane according to conservation of energy,

$$\frac{1}{2} m v_1^2 + \frac{1}{2} I_1 \omega_1^2 = mgL\sin\theta$$

$$\Rightarrow \frac{1}{2} m v_1^2 + \frac{1}{2}\left(\frac{1}{2} mr^2\right)\frac{v_1^2}{r^2} = mgL\sin\theta$$

$$\Rightarrow \cancel{\frac{1}{2}} m\left(\frac{v_1^2}{2} + \frac{v_1^2}{4}\right) = mgL\sin\theta$$

$$\Rightarrow \frac{3v_1^2}{4} = gL\sin\theta$$

$$\Rightarrow v_1 = \sqrt{\frac{4gL\sin\theta}{3}} = 1.1547\sqrt{L\sin\theta}$$

Similarly. In case of sphere,

$$\frac{1}{2} M v_2^2 + \frac{1}{2} I_2 W_2^2 = MgL\sin\theta$$

$$\Rightarrow \frac{1}{2} M v_2^2 + \frac{1}{2}\cdot\left(\frac{2}{5} MR^2\right)\frac{v_2^2}{R^2} = MgL\sin\theta$$

$$\Rightarrow M\left(\frac{1}{2}v_2^2 + \frac{v_2^2}{5}\right) = MgL\sin\theta$$

$$\Rightarrow \frac{7v_2^2}{10} = gL\sin\theta$$

$$\Rightarrow v_2 = \sqrt{\frac{10gL\sin\theta}{7}}$$

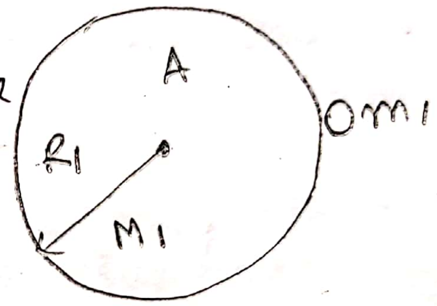
$$\Rightarrow v_2 = 1.195\sqrt{L\sin\theta}$$

$\because v_2 > v_1$.

The solid sphere has greater linear velocity than the solid cylinder.

Ans to the qr no 2(a)

Let,



Mass of an object on the surface of A $= m_1$

Mass of A $= M_1$

Radius of A $= R_1$

We know, $F = \dfrac{G M_1 m_1}{R_1^2}$ ... (i)

Again, $F = m_1 g_A$ ... (ii)

From (i) and (ii),

$$m g_A = \frac{G M_1 m_1}{R_1^2}$$

$$\Rightarrow g_A = \frac{G M_1}{R_1^2}$$

$\therefore$ Gravitational acceleration, $g_A = \dfrac{G M_1}{R_1^2}$

Ans to the Q no 2 (b)

Let,

Mass of B = M_2

Radius of B = R_2

Mass of an object on surface of B = m_2

Now, kinetic energy = $\frac{1}{2} m_2 v_2^2$

Potential energy = $\frac{G M_2 m_2}{R_2}$

According to conservation of energy,

potential energy = kinetic energy

$$\Rightarrow \frac{G M_2 m_2}{R_2} = \frac{1}{2} m_2 v_2^2$$

$$\Rightarrow v_2 = \sqrt{\frac{G M_2}{R_2}}$$

$\therefore$ Escape velocity of B is $\sqrt{\frac{G M_2}{R_2}}$

Ans to the q no 2 (c)

A and B starting to move towards each other is a two body problem

where,

Distance between the spherical objects $= d$

Radius of the circular motion, $R = \frac{d}{2}$

The reduced mass, $\mu = \frac{M_1 . M_2}{M_1 + M_2}$

$\therefore$ centripetal force, $\mu R \omega^2 = \frac{G M_1 M_2}{R^2}$

$$\Rightarrow \frac{M_1 M_2}{M_1 + M_2} R \omega^2 = \frac{G M_1 M_2}{R^2}$$

$$\Rightarrow \omega^2 = \frac{G (M_1 + M_2)}{R^3}$$

$$\Rightarrow \frac{4\pi^2}{T^2} = \frac{G (M_1 + M_2)}{R^3}$$

$$\Rightarrow \quad T = \sqrt{\frac{4\pi^2 R^3}{G(M_1 + M_2)}}$$

$$\Rightarrow \quad T = \frac{\pi}{\sqrt{2}} \cdot \sqrt{\frac{d^3}{G(M_1 + M_2)}}$$

Since for colliding at center of mass,

$$t = \frac{T}{2}$$

$$\Rightarrow \quad t = \frac{\pi}{2\sqrt{2}} \cdot \sqrt{\frac{d^3}{G(M_1 + M_2)}}$$

$\therefore$ Time taken when A and B collide with each other is $\frac{\pi}{2\sqrt{2}} \sqrt{d^3 / G(M_1 + M_2)}$

Ans to the qr no 3 (a)

Given,

Mass of the person, $m = 65\ kg$

Net stretched, $x = 1.1\ m$

Height, $h = 18\ m$

Assuming the net as a spring,

spring constant $= k$

When the person is falling down,

kinetic energy = potential energy

$$\Rightarrow \frac{1}{2} m v^2 = \frac{1}{2} k x^2 \quad \text{--- (1)}$$

We know,

$$v^2 = 2g(h+x) = 2 \times 9.8 \times (18 + 1.1)$$
$$= 374.36$$

Now, From (1) we get,

$$m.v^2 = k x^2$$

$$\Rightarrow \quad k = \frac{mv^2}{x^2}$$

$$\Rightarrow \quad k = \frac{65 \times 374.36}{(1.1)^2}$$

$$\Rightarrow \quad k = 20110.247 \ N m^{-1}$$

Now, If the person was lying in it,

$$F = kx$$

$$\Rightarrow \quad x = \frac{F}{k}$$

$$\Rightarrow \quad x = \frac{mg}{k}$$

$$\Rightarrow \quad x = \frac{65 \times 9.8}{20110.247}$$

$$\Rightarrow \quad x = 0.03167 \ m$$

$\therefore$ I would stretch 0.03167 m if the person of 65 kg was lying in it.

Ans to the Q. no 3(b)

Here,

New height, $h = 35\,m$

from (a) we get,

spring constant, $k = 20110.247\ Nm^{-1}$

If the person jumps from the new height

velocity of the person, $v = \sqrt{2g(h+x)}$

$$\Rightarrow v^2 = 2g(35+x)$$

According to conservation of energy,

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

$$\Rightarrow m\,2g(35+x) = kx^2$$

$$\Rightarrow kx^2 - 2mgx - 2mg \cdot 35 = 0$$

$$\Rightarrow 20110.247x^2 - 2\times65\times9.8 \cdot x - 2\times65\times9.8\times35 = 0$$

$$\Rightarrow 20110.247x^2 - 1274x - 44590 = 0$$

$$\Rightarrow 20110.247x^2$$

$$\Rightarrow x = 1.521$$

$\therefore$ It would stretch $1.521\,m$ if the person jumped from $35\,m$.