Assignment 3

CSE330

Name: Shihab Muhtasim

ID: 21301610

sec: 3

Ans to qr no 1

(a)

Given, $f(x) = 2x - e^{-6x}$

$x_0 = 0.5$, $h = 0.2$

forward diffrenciation:

$$f'(x) = \frac{f(x_0+h) - f(x_0)}{h}$$

$$= \frac{f(0.7) - f(0.5)}{0.2}$$

$$= \frac{2\times0.7 - e^{-6\times0.7} - \left[2\times0.5 - e^{-6\times0.5}\right]}{0.2}$$

$$= 2.1739$$

b

$x_0 = 0.5, \; h = 0.2$

Central diffrenciation :

$$f'(x) = \frac{f(x_0+h) - f(x_0-h)}{2h}$$

$$= \frac{f(0.5+0.2) - f(0.5-0.2)}{0.2 \times 2}$$

$$= \frac{f(0.7) - f(0.3)}{0.4}$$

$$= \frac{2\times0.7 - e^{-6\times0.7} - \left[2\times0.3 - e^{-6\times0.3}\right]}{0.4}$$

$$= \frac{1.3850 - 0.43470}{0.4}$$

$$= \cancel{0.975148}$$

$$= 2.37575$$

Ans to 1(C)

$f(x) = 2x - e^{-6x}$ $\quad$ $h = 0.1$

$[2.4, 2.7]$

$f'(x) = 2 + 6e^{-6x}$

$f''(x) = -36e^{-6x}$

$f'''(x) = 216e^{-6x}$

(i) upper bound Truncation error in forward diffrenciation:

$f''(2.7) = -36e^{-6\times2.7} = -3.316\times10^{-6}$

$f''(2.4) = -36e^{-6\times2.4} = -2.006\times10^{-5}$

Here, $f''(2.7)$ is maximum.

$\therefore$ upper bound of trancation error

$$= \left|\frac{f''(\xi)}{2!}\times(-h)\right| = \left|\frac{f''(2.7)}{2!}\times(-0.1)\right|$$

$$= \left|\frac{-3.316\times10^{-6}}{2!}\times(-0.1)\right| = 1.658\times10^{-7}$$

CamScanner

~~Ans to 1(d)~~

Using central diffrence :

$f'''(2.7) = 216 \times e^{-6\times2.7} = 1.990\times10^{-5}$

$f'''(2.4) = 216 \times e^{-6\times2.4} = 1.2039\times10^{-4}$

$\therefore f'''(2.4)$ is max. $f^3(\xi) = f^3(2.4)$

Now, uppen bound traction ennon =

$$\left|\frac{f^3(\xi)}{3!} \times (-h^2)\right| = \left|\frac{1.2039\times10^{-4}}{3!} \times (-0.1)^2\right|$$

$= 2.0065\times10^{-7}$

Ans to 1(d)

$f(x) = 2x - e^{-6x}$, $x_0 = 0.2$

$$D_h = \frac{f(x_0+h) - f(x_0-h)}{2h}$$

$$D_{0.5} = \frac{f(0.2+0.5) - f(0.2-0.5)}{2 \times 0.5}$$

$$= \frac{f(0.7) - f(-0.3)}{1}$$

$$= 2 \times 0.7 - e^{-6 \times 0.7} - \left[2 \times (-0.3) - e^{-6 \times (-0.3)}\right]$$

$$= 1.3850 + 6.64964$$

$$= 8.03464$$

$D_{h/2}$:

$$D_{0.25} = \frac{f(0.2+0.25) - f(0.2-0.25)}{2 \times 0.25}$$

$$= \frac{f(0.45) - f(-0.05)}{0.5}$$

$$= \frac{2\times 0.45 - e^{-6\times 0.45} - [2\times(-0.05) - \ldots}{0.5}$$

$$= 0.832794 + 1.449858 / 0.5$$

$$= 2.282652 / 0.5 = 4.565304$$

$$D_n^{(1)} = \frac{4D_{n/2} - D_n}{3}$$

$$D_{0.5}^{(1)} = \frac{4D_{0.25} - D_{0.5}}{3}$$

$$= \frac{4\times 4.565304 - 8.03464}{3}$$

$$= 3.408858$$

$$f'(x) = 2 + 6e^{-6x}$$

$$f'(0.2) = 2 + 6e^{-6\times 0.2} = 3.807165$$

$\therefore$ Truncation error $= f'(0.2) - D_{0.5}^{(1)}$

$$= 3.807165 - 3.408858$$

$$= 0.398306$$

CamScanner

Ans to Q 2

(a)

Given,

$$D_h^{(1)} = f'(x_0) - \frac{h^4}{480} f^5(x_0) + O(h^6)$$

$$D_{h/2}^{(1)} = f'(x_0) - \frac{f^5(x_0)}{480} \times \frac{h^4}{16} + O(h^6)$$

$$16 D_{h/2}^{(1)} - D_h^{(1)} = 15 f'(x_0) + 0 + O(h^6)$$

$$\frac{16 D_{h/2}^{(1)} - D_h^{(1)}}{15} = f'(x_0) + O(h^6)$$

Expression for $D_h^{(2)} = \frac{16 D_{h/2}^{(1)} - D_h^{(1)}}{15} = f'(x_0) + O(h^6)$

Ans to Q 2(b)

$$D_h = \frac{f(x+h) - f(x-h)}{2h}$$

$$f(x+h) = f(x) + f'(x)h + \frac{f^2(x)}{2!}h^2 + \frac{f^3(x)}{3!}h^3 + \frac{f^4(x)}{4!}h^4 + \frac{f^5(x)}{5!}h^5 + O(h^6)$$

$$f(x-h) = f(x) - f'(x)h + \frac{f^2(x)}{2!}h^2 - \frac{f^3(x)}{3!}h^3 + \frac{f^4(x)}{4}h^4 - \frac{f^5(x)}{5!}h^5 + O(h^6)$$

$$\therefore f(x+h) - f(x-h) = 2f'(x)h + \frac{2f^3(x)h^3}{3!} + \frac{2f^5(x)}{5!}h^5 + O(h7)$$

$$D_h = f'(x) + \frac{f^3(x)}{3!}h^2 + \frac{f^5(x)h^4}{5!} + O(h^6) \quad \text{——(1)}$$

$$D_{h/3} = f'(x) + \frac{f^3(x)}{3!} \times \frac{h^2}{9} + \frac{f^5(x)}{5!} \times \frac{h^4}{81} + O(h^6) \quad \text{——(2)}$$

$$9D_{h/3} - D_h = 8f'(x) - \frac{8}{9}\frac{f^5(x)}{5!} \times h^4 + O(h^6)$$

$$\frac{9D_{h/3} - D_h}{8} = f'(x) - \frac{1}{9}\frac{f^5(x)}{5!} h^4 + O(h^6)$$

$$\therefore D_h^1 = f'(x) - \frac{1}{9}\frac{f^5(x)}{5!} h^4 + O(h^6)$$

Ans to Q 2(c)

Error part in $D_h^{(1)} = -\frac{1}{9}\frac{f^5(x)}{5!} h^4 + O(h^6)$

Error bound of $D_h^{(1)} = -\frac{1}{9}\frac{f^5(x)}{5!} h^4$

Ans to Qr 2(d)

If $f(x) = \ln x$, $x_0 = 1$, $h = 0.1$

$f'(x) = \frac{1}{x}$ $f'(1) = 1$

$f''(x) = \frac{-1}{x^2}$; $f^3(x) = \frac{2}{x^3}$; $f^4(x) = -\frac{6}{x^4}$

$f^5(x) = \frac{24}{x^5}$ $f^5(1) = 24$

Given

$$D_h^{(1)} = f'(x_0) - \frac{h^4}{480} f^5(x_0) + O(h^6)$$

upper bound error $= \left| -\frac{h^4}{480} \times f^5(x_0) \right|$

$$= \left| -\frac{(0.1)^4}{480} \times 24 \right|$$

$$= 5 \times 10^{-6}$$