

There are a total of five problems. You have to solve **all** of them.

Problem 1 (CO2): Designing Context-Free Grammar (10 points)

Let $\Sigma = \{0, 1, 2\}$. Consider the following languages over Σ .

$$L_1 = \{1^i 0 2^j 1^k \mid i, j, k \geq 0, i = k\}$$

$$L_2 = \{1^i 0 2^j 1^k \mid i, j, k \geq 0, k = i + 2j\}$$

Now solve the following problems.

- (a) **Give** a context-free grammar for the language L_1 . (5 points)
- (b) **Give** a context-free grammar for the language L_2 . (5 points)

Problem 2 (CO2): Derivations, Parse Trees and Ambiguity (10 points)

Take a look at the grammar below and solve the following problems.

$$S \rightarrow aaS \mid abS \mid baS \mid bbS \mid X$$

$$X \rightarrow aaY \mid baY$$

$$Y \rightarrow aY \mid bY \mid \varepsilon$$

- (a) **Give** a leftmost derivation for the string **abbabbaa**. (3 points)
- (b) **Sketch** the parse tree corresponding to the derivation you gave in (a). (2 points)
- (c) **Demonstrate** that the given grammar is ambiguous by showing one more parse tree (apart from the one you already found in (b)) for the same string. (4 points)
- (d) **Find** a string w of length nine such that w has exactly one parse tree in the grammar above. (1 point)

Problem 3 (CO2): Chomsky Normal Form (10 points)

- (a) **List** the rules that violate the conditions of Chomsky Normal form in the following grammar. Here **a**, **b**, and **c** are terminals and the rest are variables.

$$A \rightarrow BC \mid bB \mid a$$

$$B \rightarrow bb \mid Cb \mid b \mid C$$

$$C \rightarrow c$$

- (b) **Write** down the additional rules that need to be added to the following grammar if the production, $B \rightarrow \varepsilon$ is removed. Here 0 and 1 are terminals and the rest are variables.

$$S \rightarrow AB \mid 1$$

$$A \rightarrow BAB \mid ABA \mid B \mid 11$$

$$B \rightarrow 00 \mid \varepsilon$$

- (c) **Write** down the additional rules that need to be added to the following grammar if the unit productions are removed. Here 0 and 1 are terminals and the rest are variables.

$$S \rightarrow XYX \mid YX \mid X \mid Y$$

$$Y \rightarrow XY \mid X0 \mid 0$$

$$X \rightarrow 1 \mid Y$$

Problem 4 (CO4): The CYK Algorithm (10 points)

Apply the CYK algorithm to determine whether the string **baaab** can be derived in the following grammar. You must show the entire CYK table. Here **a** and **b** are terminals and the rest are variables.

$$S \rightarrow CA \mid DB \mid a \mid b$$

$$Z \rightarrow CA \mid DB \mid a \mid b$$

$$C \rightarrow AZ$$

$$D \rightarrow BZ$$

$$A \rightarrow a$$

$$B \rightarrow b$$

Problem 5 (CO2): Constructing Pushdown Automata (10 points)

Let $\Sigma = \{0, 1\}$. Consider the following pair of languages over Σ .

$$L_1 = \{w \mid \text{the length of } w \text{ is divisible by four}\}$$

$$L_2 = \{w \mid w = 0^{n+2}1^{2n}, n \geq 0\}$$

Now solve the following problems.

- (a) **Construct** a pushdown automaton that recognizes L_1 . (4 points)
- (b) **Construct** a pushdown automaton that recognizes L_2 . (6 points)

Problem 6: (Bonus) Closed Under Intersection? (5 points)

(Note that this is a bonus problem. Attempt it only after you are done with everything else. Even if you do not attempt it, you can get a perfect score. So, do not worry if you find it too hard!)

Consider the language $L = L_1 \cap L_2$ where L_1 and L_2 were defined in Problem 5. **Construct** a pushdown automaton that recognizes L . Describe what your automaton is doing in two or three sentences.

After you are done with the test, please indicate where you stand on the smiley face spectrum.

