# Vector

$$\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial y}\hat{k}$$

Gradient : $\phi(x, y, z) \rightarrow$ scalar function

$$\nabla\phi = \frac{\partial \phi}{\partial x}\hat{i} + \frac{\partial \phi}{\partial y}\hat{j} + \frac{\partial \phi}{\partial z}\hat{k}$$

Divergence : $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

$$\nabla\cdot\vec{a} = \frac{\partial}{\partial x}a_x + \frac{\partial}{\partial y}a_y + \frac{\partial}{\partial z}a_z$$

Laplausia (scalar operator) : $\nabla^2$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

in terms of $f$, $(f_{xx} + f_{yy} + f_{zz}) : \nabla^2 f$

curl : $\nabla \times \vec{a} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ a_x & a_y & a_z \end{vmatrix}$

$(a, b, c)$ poins given,

Area of triangle $= \frac{1}{2}|\vec{AB} \times \vec{AC}|$

# Maxima Minima of Multi variable

$f(x, y)$ = Function

$f_x = 0$, $f_y = 0$ } To find critical points

minima : (i) $f_{xx}, f_{yy} > 0$

(ii) $f_{xy} < f_{xx} \cdot f_{yy}$

maxima : (i) $f_{xx}, f_{yy} < 0$

(ii) $f_{xy} < f_{xx} \cdot f_{yy}$

saddle : (i) $f_{xx}$ and $f_{yy} \Rightarrow$ opposite signs / zero

(ii) $f_{xy} > f_{xx} \cdot f_{yy}$

# SINGLE VARIABLE

$f(x)$ = Function

$f'(x) = 0$ ⟩ To find critical points

$f''(x) > 0 \Rightarrow$ Minimum

$f''(x) < 0 \Rightarrow$ Maximum

$f''(x) = 0 \Rightarrow$ inconclusive

To find a function's concavity :-

$f''(x) > 0 \longrightarrow$ concave up

$f''(x) < 0 \longrightarrow$ concave down

Let $f''(x) = 0$ to find intervals for concavity.

## Taylor expansion for multivariable function

1st degree:

$$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

$$Q(x,y) = L(x,y) + \frac{f_{xx}(a,b)}{2!}(x-a)^2 + f_{xy}(a,b)(x-a)(y-b) + \frac{f_{yy}(a,b)}{2!}(y-b)^2$$

$-\frac{1}{2}+\frac{1}{2}$

1/10/24

$$P_n(0) = \frac{f(x)}{0!}x^0 + \frac{f'(x)}{1!}x^1 + \frac{f''(x)}{2!}x^2 + \frac{f'''(x)}{3!}x^3 + \frac{f^4(x)}{4!}x^4 + \cdots$$

Taylor : $\boxed{\text{at } x_0 \text{, for } f(x)}$

$$P_n(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \frac{f'''}{3!}(x-x_0)^3 + \cdots$$

second degree equation

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

$$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$$

① $\Delta = 0$ (Degenerate case) :

ⓐ Parallal / straigt lines :$\Rightarrow$ $h^2 - ab = 0$

ⓑ Perpendicular $\Rightarrow$ $a + b = 0$

② $\Delta \neq 0$ :

ⓐ Parabola $\Rightarrow$ $h^2 - ab = 0$

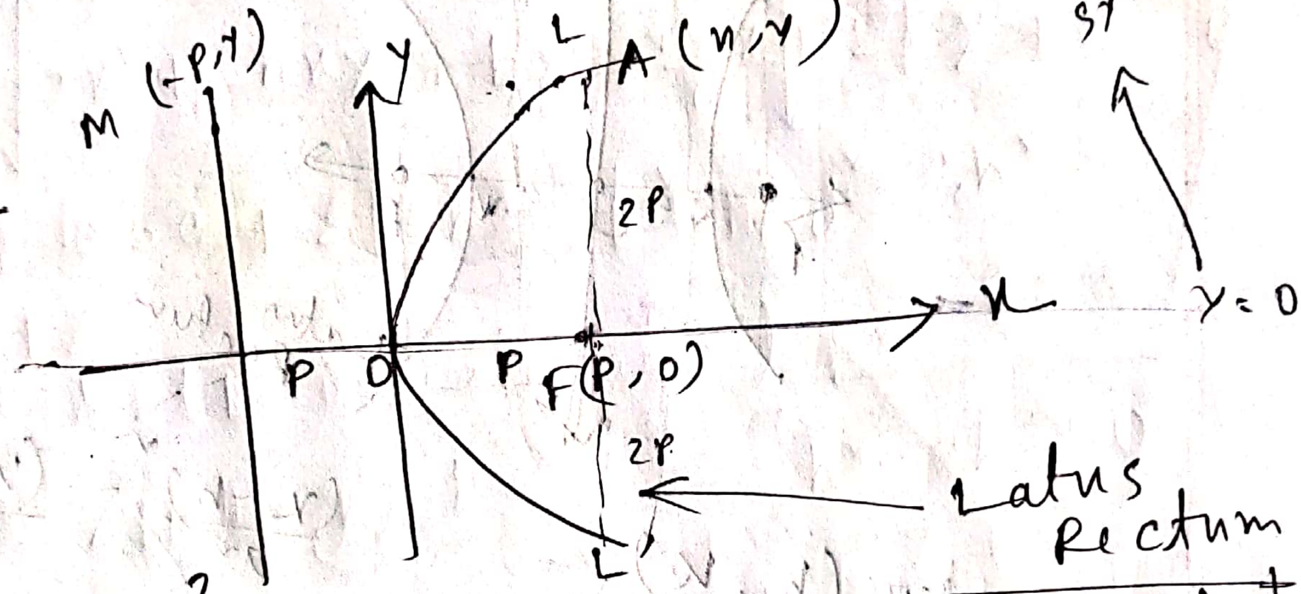
ⓑ Hyperbola $\Rightarrow$ $h^2 - ab > 0$

ⓒ Ellipse $\Rightarrow$ $h^2 - ab < 0$

ⓓ Circle $\Rightarrow$ $a = b$, $h = 0$, $h^2 - ab < 0$

# Parabola

$\boxed{P>0}$




$\boxed{y^2 = 4Px}$ $\boxed{(y-k)^2 = 4P(x-h)}$

vertex $(x,y) = (0,0)$ , $(h,k)$

Focus $\Rightarrow (P,0)$ | $(h+P,k)$

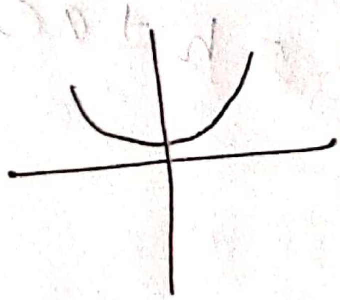
Equation directrix $\Rightarrow$ $x + P = 0$ | $x - h = -P$

$\Rightarrow \boxed{x = -P}$

Equation axis of symatry , $y = 0$

Eq of Latus Rectum , $x = P$

Length of Latus Rectum $= |4P|$



vertex $(0,0)$

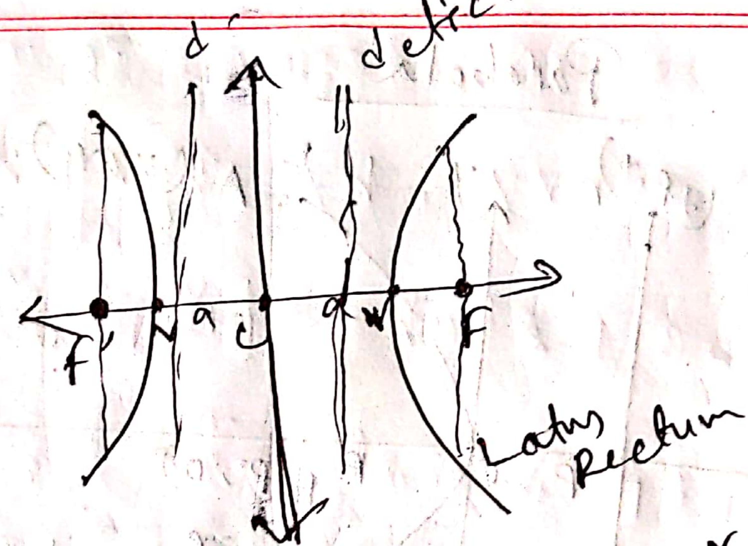
F $(0,P)$

Directrix $\Rightarrow$ $y = -P$

sym $\Rightarrow$ $x = 0$

Rec : L $|4P|$ . E : $(x = 0)$

# Hyperbola




$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

center $(h, k)$

vertices $(h \pm a, k)$

Eccentricity, $e = \sqrt{1 + \frac{b^2}{a^2}}$

Foci, F ⇒ $(h \pm c, \cancel{o}k)$ or $\boxed{(h \pm ae, k)}$

Detectrices : $x = 1 \pm \frac{a}{e}$

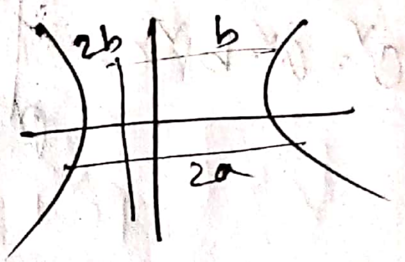
Latus Rectum, Length = $\frac{2b^2}{a}$

equation, $x = h \pm ae$

Length of transververse axis = 2a

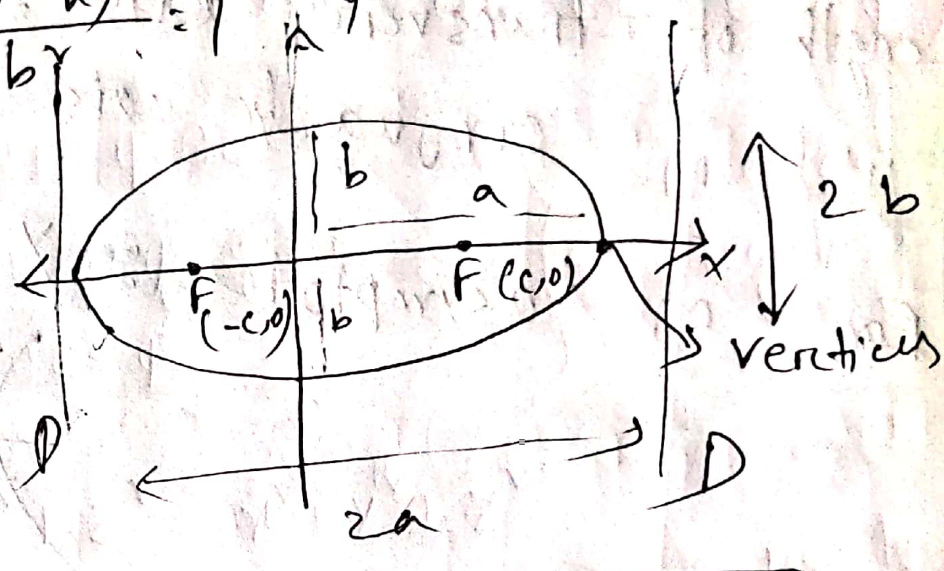
" " conjugate axis : 2b

For. of asymptotes : $y = \pm \frac{b}{a} x$



# Ellipse

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$




$c^2 = a^2 - b^2$

Eccentricity, $e = \sqrt{1 - b^2/a^2}$

center, $(h, k)$

foci : $(h \pm ae, k)$ or $(h \pm c, k)$

verticus : $(h \pm a, k)$

coverticus : $(h, k \pm b)$

Eq of major axis, $y = k$

Eq of minor axis, $x = h$

Length of major axis $= 2a$

" " minor axis $= 2b$

Directrices : $x = h \pm \frac{a}{e}$

Eq of letus rectum : $x = h \pm ae$

Length $= 2b^2/a$

## Conversion of equation:

Polar coordinate from cartesian:

$r^2 = x^2 + y^2$

$\tan\theta = \frac{y}{x}$

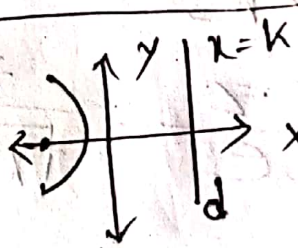
Cartesian coordinate from polar:

$x = r\cos\theta$

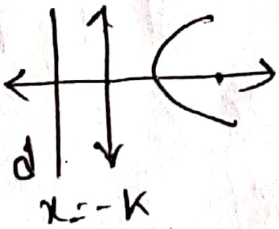
$y = r\sin\theta$

### Polar equations

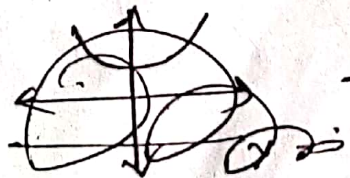
(I) $r = \frac{ke}{1+e\cos\theta}$

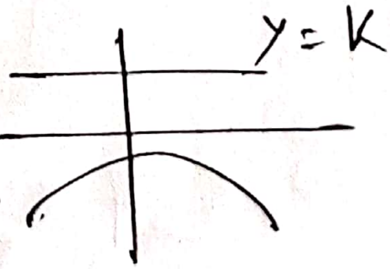



(II) $r = \frac{ke}{1-e\cos\theta}$

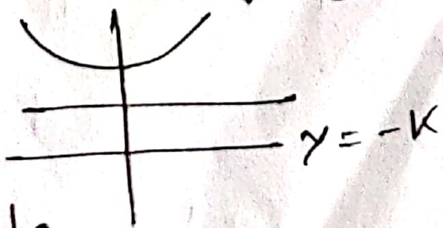



(III) $r = \frac{ke}{1+e\sin\theta}$






(IV) $r = \frac{ke}{1-e\sin\theta}$




* $e = 1$ : Parabola
* $0 < e < 1$ : ellipse
* $e > 1$ : Hyperbola