Differentiation Formulas

The following table provides the differentiation formulas for common functions. The first six rows correspond to general rules (such as the addition rule or the product rule) whereas the remaining rows contain the formulas for specific functions.

	$F(x)$	$F'(x)$
Addition	$f(x) \pm g(x)$	$f'(x) \pm g'(x)$
Linearity	$af(x)$	$af'(x)$
Product Rule	$f(x)g(x)$	$f'(x)g(x) + f(x)g'(x)$
Quotient Rule	$\frac{f(x)}{g(x)}$	$\frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$
Chain Rule	$f(g(x))$	$f'(g(x)) \cdot g'(x)$
	$f^{-1}(x)$	$\frac{1}{f'(f^{-1}(x))}$
Basic functions	x^n for any real n	nx^{n-1}
	e^x	e^x
	$a^x \quad (a > 0)$	$(\ln a)a^x$
	$\ln x$	$\frac{1}{x}$
Trig functions	$\sin x$	$\cos x$
	$\cos x$	$-\sin x$
	$\tan x$	$\frac{1}{\cos^2 x} = 1 + \tan^2 x$
	$\arctan x = \tan^{-1} x$	$\frac{1}{1+x^2}$
	$\arcsin x = \sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
Hyperbolic Trig	$\sinh x$	$\cosh x$
	$\cosh x$	$\sinh x$
	$\tanh x$	$\frac{1}{\cosh^2 x}$
	$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
	$\tanh^{-1} x$	$\frac{1}{1-x^2}$

TRIGONOMETRIC IDENTITIES

The six trigonometric functions:

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{y}{r} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{r}{y} = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{x}{r} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{r}{x} = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{y}{x} = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{x}{y} = \frac{1}{\tan\theta}$$

Sum or difference of two angles:

$$\sin(a \pm b) = \sin a\cos b \pm \cos a\sin b$$

$$\cos(a \pm b) = \cos a\cos b \mp \sin a\sin b$$

$$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a\tan b}$$

Double angle formulas:

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$\sin 2\theta = 2\sin\theta\cos\theta \qquad \cos 2\theta = 2\cos^2\theta - 1$$

$$\cos 2\theta = 1 - 2\sin^2\theta \qquad \cos 2\theta = \cos^2\theta - \sin^2\theta$$

Pythagorean Identities:

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta \qquad \cot^2\theta + 1 = \csc^2\theta$$

Half angle formulas:

$$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta) \qquad \cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$$

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}} \qquad \cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \frac{\sin\theta}{1+\cos\theta} = \frac{1-\cos\theta}{\sin\theta}$$

Sum and product formulas:

$$\sin a\cos b = \tfrac{1}{2}[\sin(a+b)+\sin(a-b)]$$

$$\cos a\sin b = \tfrac{1}{2}[\sin(a+b)-\sin(a-b)]$$

$$\cos a\cos b = \tfrac{1}{2}[\cos(a+b)+\cos(a-b)]$$

$$\sin a\sin b = \tfrac{1}{2}[\cos(a-b)-\cos(a+b)]$$

$$\sin a + \sin b = 2\sin\left(\tfrac{a+b}{2}\right)\cos\left(\tfrac{a-b}{2}\right)$$

$$\sin a - \sin b = 2\cos\left(\tfrac{a+b}{2}\right)\sin\left(\tfrac{a-b}{2}\right)$$

$$\cos a + \cos b = 2\cos\left(\tfrac{a+b}{2}\right)\cos\left(\tfrac{a-b}{2}\right)$$

$$\cos a - \cos b = -2\sin\left(\tfrac{a+b}{2}\right)\sin\left(\tfrac{a-b}{2}\right)$$

Law of cosines: $a^2 = b^2 + c^2 - 2bc\cos A$

where A is the angle of a scalene triangle opposite side a.

Radian measure: 8.1 p420

$$1^\circ = \frac{\pi}{180}\text{ radians}$$

$$1\text{ radian} = \frac{180^\circ}{\pi}$$

Reduction formulas:

$$\sin(-\theta) = -\sin\theta \qquad \cos(-\theta) = \cos\theta$$

$$\sin(\theta) = -\sin(\theta-\pi) \qquad \cos(\theta) = -\cos(\theta-\pi)$$

$$\tan(-\theta) = -\tan\theta \qquad \tan(\theta) = \tan(\theta-\pi)$$

$$\mp\sin x = \cos\left(x \pm \tfrac{\pi}{2}\right) \qquad \pm\cos x = \sin\left(x \pm \tfrac{\pi}{2}\right)$$

Complex Numbers:

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$\cos\theta = \tfrac{1}{2}(e^{j\theta} + e^{-j\theta}) \qquad \sin\theta = \tfrac{1}{j2}(e^{j\theta} - e^{-j\theta})$$

TRIGONOMETRIC VALUES FOR COMMON ANGLES

Degrees	Radians	sin θ	cos θ	tan θ	cot θ	sec θ	csc θ
0°	0	0	1	0	Undefined	1	Undefined
30°	π/6	1/2	√3 / 2	√3 / 3	√3	2√3 / 3	2
45°	π/4	√2 / 2	√2 / 2	1	1	√2	√2
60°	π/3	√3 / 2	1/2	√3	√3 / 3	2	2√3 / 3
90°	π/2	1	0	Undefined	0	Undefined	1
120°	2π/3	√3 / 2	-1/2	-√3	-√3 / 3	-2	2√3 / 3
135°	3π/4	√2 / 2	-√2 / 2	-1	-1	-√2	√2
150°	5π/6	1/2	-√3 / 2	-√3 / 3	-√3	-2√3 / 3	2
180°	π	0	-1	0	Undefined	-1	Undefined
210°	7π/6	-1/2	-√3 / 2	√3 / 3	√3	-2√3 / 3	-2
225°	5π/4	-√2 / 2	-√2 / 2	1	1	-√2	√2
240°	4π/3	-√3 / 2	-1/2	√3	√3 / 3	-2	2√3 / 3
270°	3π/2	-1	0	Undefined	0	Undefined	-1
300°	5π/3	-√3 / 2	1/2	-√3	-√3	2	-2√3 / 3
315°	7π/4	-√2 / 2	√2 / 2	-1	-1	√2	-√2
330°	11π/6	-1/2	√3 / 2	-√3 / 3	-√3	2√3 / 3	-2
360°	2π	0	1	0	Undefined	1	Undefined

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Trigonometric Formula

1. $\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1$
2. $\sin 2A = 2\sin A \cos A$
3. $\cos(A + B) = \cos A \cos B - \sin A \sin B$
4. $\cos(A - B) = \cos A \cos B + \sin A \sin B$
5. $\sin(A + B) = \sin A \cos B + \cos A \sin B$
6. $\sin(A - B) = \sin A \cos B - \cos A \sin B$

7. $\cos A = 1 - \frac{A^2}{2!} + \frac{A^4}{4!} - \frac{A^6}{6!} + \cdots\cdots\cdots$
8. $\sin A = A - \frac{A^3}{3!} + \frac{A^5}{5!} - \frac{A^7}{7!} + \cdots\cdots\cdots$
9. $\tan A = A + \frac{A^3}{3} + \frac{A^5}{5} + \frac{A^7}{7} + \cdots\cdots\cdots$
10. $e^A = 1 + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \frac{A^4}{4!} + \cdots\cdots$
11. $e^{-A} = 1 - A + \frac{A^2}{2!} - \frac{A^3}{3!} + \frac{A^4}{4!} - \cdots\cdots$

12. $(1 + A)^{-1} = 1 - A + A^2 - A^3 + \cdots\cdots$
13. $(1 - A)^{-1} = 1 + A + A^2 + A^3 + \cdots\cdots$
14. $(A + B)^n = A^n + {}^nC_1 A^{n-1} B + {}^nC_2 A^{n-2} B^2 + \cdots\cdots + B^n$
15. $\ln(1 + A) = A - \frac{A^2}{2} + \frac{A^3}{3} - \frac{A^4}{4} + \cdots\cdots$
16. $\ln(1 - A) = -A - \frac{A^2}{2} - \frac{A^3}{3} - \frac{A^4}{4} - \cdots\cdots$

Integration Formulas

The following list provides some of the rules for finding integrals and a few of the common antiderivatives of functions.

Linearity	$\int af(x) + bg(x)\,dx = a\int f(x)\,dx + b\int g(x)\,dx$
Substitution	$\int f(w(x))w'(x)\,dx = \int f(w)\,dw$
Integration by parts	$\int u(x)v'(x)\,dx = u(x)v(x) - \int u'(x)v(x)\,dx$

Basic Functions

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \qquad \int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int e^{ax}\,dx = \frac{1}{a}e^x + C \qquad \int a^x\,dx = \frac{a^x}{\ln a} + C$$

Trigonometric functions

$$\int \sin x\,dx = -\cos x + C \qquad \int \cos x\,dx = \sin x + C$$

$$\int \frac{1}{\cos^2 x}\,dx = \tan x + C \qquad \int \tan x\,dx = -\ln|\cos x| + C$$

$$\int \cot x\,dx = \ln|\sin x| + C$$

Hyperbolic Trig functions

$$\int \sinh x\,dx = \cosh x + C \qquad \int \cosh x\,dx = \sinh x + C$$

$$\int \tanh x\,dx = \ln(\cosh x) + C \qquad \int \coth x\,dx = \ln|\sinh x| + C$$

Functions with $a^2 \pm x^2$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C \qquad \int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\ln\left|\frac{x+a}{x-a}\right| + C$$

$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1}\left(\frac{x}{a}\right) + C \qquad \int \frac{dx}{\sqrt{x^2 + a^2}} = \sinh^{-1}\left(\frac{x}{a}\right) + C$$

Inverse Functions

$$\int \ln x\,dx = x\ln x - x + C \qquad \int \arcsin x\,dx = x\arcsin x + \sqrt{1 - x^2} + C$$

$$\int \arctan x = x\arctan x - \frac{1}{2}\ln(1 + x^2) + C$$

Expansions for sine, cosine, tangent, cotangent:

$$\sin y = y - \frac{y^3}{6} + \frac{y^5}{5!} - \frac{y^7}{7!} + \cdots$$

$$\cos y = 1 - \frac{y^2}{2} + \frac{y^4}{4!} - \frac{y^6}{6!} + \cdots$$

$$\tan y = y + \frac{y^3}{3} + \frac{2y^5}{15} + \cdots$$

$$\cot y = \frac{1}{y} - \frac{y}{3} - \frac{y^3}{45} - \frac{2y^5}{945} - \cdots$$

Hyperbolic functions:

$$\sinh y = \frac{1}{2}\left(e^{y} - e^{-y}\right) \qquad \sinh jy = j\sin y$$

$$\cosh y = \frac{1}{2}\left(e^{y} + e^{-y}\right) \qquad \cosh jy = j\cos y$$

$$\tanh jy = j\tan y$$

Expansions for hyperbolic functions:

$$\sinh y = y + \frac{y^3}{6} + \cdots$$

$$\cosh y = 1 + \frac{y^2}{2} + \cdots$$

$$\operatorname{sech} y = 1 - \frac{y^2}{2} + \frac{5y^4}{24} - \cdots$$

$$\operatorname{ctnh} y = \frac{1}{y} + \frac{y}{3} - \frac{y^3}{45} + \cdots$$

$$\operatorname{csch} y = \frac{1}{y} - \frac{y}{6} + \frac{7y^3}{360} - \cdots$$

$$\tanh y = y - \frac{y^3}{3} + \frac{2y^5}{15} - \cdots$$

(3) $\int x^n\,dx$ নির্ণয় কর

আমরা জানি, $\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = (n+1)\frac{x^n}{n+1} = x^n \quad \therefore \int x^n\,dx = \frac{x^{n+1}}{n+1} + c$. [যখন $n \neq -1$]

কতিপয় ফাংশনের অন্তরজ ও প্রতিঅন্তরজ নিচে প্রদত্ত হলো :

$\frac{d}{dx}(x^n) = nx^{n-1}$

$\frac{d}{dx}(x) = 1, \frac{d}{dx}(c) = 0.$

$\frac{d}{dx}(ln\ x) = \frac{1}{x}, (x > 0)$

$\frac{d}{dx}(e^{mx}) = me^{mx}$

$\frac{d}{dx}(e^x) = e^x$

$\frac{d}{dx}(a^x) = a^x.\ ln\ a$

$\frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\sin mx) = m\cos mx$

$\frac{d}{dx}(\cos mx) = -m\sin mx$

$\frac{d}{dx}(\tan x) = \sec^2 x$

$\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

$\frac{d}{dx}(\sec x) = \sec x\tan x$

$\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x\cot x,$

$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$

$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$

$\frac{d}{dx}(\operatorname{cosec}^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$

$\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$È

$\Rightarrow \int x^n\,dx = \frac{x^{n+1}}{n+1} + c$ [যখন $n \neq -1$]

$\Rightarrow \int dx = x + c$

$\Rightarrow \int \frac{dx}{x} = ln\ x + c$

$\Rightarrow \int e^{mx}\,dx = \frac{1}{m}e^{mx} + c$

$\Rightarrow \int e^x\,dx = e^x + c$

$\Rightarrow \int a^x dx = a^x/ln\ a + c$

$\Rightarrow \int \cos x\,dx = \sin x + c$

$\Rightarrow \int \sin x\,dx = -\cos x + c$

$\Rightarrow \int \cos mx\,dx = \frac{1}{m}\sin mx + c$

$\Rightarrow \int \sin mx\,dx = -\frac{1}{m}\cos mx + c$

$\Rightarrow \int \sec^2 x\,dx = \tan x + c$

$\Rightarrow \int \operatorname{cosec}^2 x\,dx = -\cot x + c$

$\Rightarrow \int \sec x\tan x\,dx = \sec x + c$

$\Rightarrow \int \operatorname{cosec} x\cot x\,dx = -\operatorname{cosec} x + c$

$\Rightarrow \int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + c$

$\Rightarrow \int \frac{dx}{1+x^2} = \tan^{-1} x + c$

$\Rightarrow \int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + c$

$\Rightarrow \int \frac{1}{\sqrt{1-x^2}} = -\cos^{-1} x + c$

$\Rightarrow \int \frac{1}{1+x^2} = -\cot^{-1} x + c$

$\Rightarrow \int \frac{1}{x\sqrt{x^2-1}} = -\operatorname{cosec}^{-1} x + c$

$\Rightarrow \int \frac{dx}{\sqrt{x}} = 2\sqrt{x} + c$

Differentiation Formulas

$$\frac{d}{dx}k = 0 \tag{1}$$

$$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x) \tag{2}$$

$$\frac{d}{dx}[k \cdot f(x)] = k \cdot f'(x) \tag{3}$$

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x) \tag{4}$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2} \tag{5}$$

$$\frac{d}{dx}f(g(x)) = f'(g(x)) \cdot g'(x) \tag{6}$$

$$\frac{d}{dx}x^n = nx^{n-1} \tag{7}$$

$$\frac{d}{dx}\sin x = \cos x \tag{8}$$

$$\frac{d}{dx}\cos x = -\sin x \tag{9}$$

$$\frac{d}{dx}\tan x = \sec^2 x \tag{10}$$

$$\frac{d}{dx}\cot x = -\csc^2 x \tag{11}$$

$$\frac{d}{dx}\sec x = \sec x \tan x \tag{12}$$

$$\frac{d}{dx}\csc x = -\csc x \cot x \tag{13}$$

$$\frac{d}{dx}e^x = e^x \tag{14}$$

$$\frac{d}{dx}a^x = a^x \ln a \tag{15}$$

$$\frac{d}{dx}\ln|x| = \frac{1}{x} \tag{16}$$

$$\cdot \sin^{-1} x = \frac{1}{\sqrt{1-x^2}} \tag{17}$$

$$\cos^{-1} x = \frac{-1}{\sqrt{1-x^2}} \tag{18}$$

$$\text{an}^{-1} x = \frac{1}{x^2+1} \tag{19}$$

$$\text{ot}^{-1} x = \frac{-1}{x^2+1} \tag{20}$$

$$^{-1} x = \frac{1}{|x|\sqrt{x^2-1}} \tag{21}$$

$$^{-1} x = \frac{-1}{|x|\sqrt{x^2-1}} \tag{22}$$

Integration Formulas

$$\int dx = x + C \tag{1}$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \tag{2}$$

$$\int \frac{dx}{x} = \ln|x| + C \tag{3}$$

$$\int e^x\,dx = e^x + C \tag{4}$$

$$\int a^x\,dx = \frac{1}{\ln a}a^x + C \tag{5}$$

$$\int \ln x\,dx = x\ln x - x + C \tag{6}$$

$$\int \sin x\,dx = -\cos x + C \tag{7}$$

$$\int \cos x\,dx = \sin x + C \tag{8}$$

$$\int \tan x\,dx = -\ln|\cos x| + C \tag{9}$$

$$\int \cot x\,dx = \ln|\sin x| + C \tag{10}$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C \tag{11}$$

$$\int \csc x\,dx = -\ln|\csc x + \cot x| + C \tag{12}$$

$$\int \sec^2 x\,dx = \tan x + C \tag{13}$$

$$\int \csc^2 x\,dx = -\cot x + C \tag{14}$$

$$\int \sec x\tan x\,dx = \sec x + C \tag{15}$$

$$\int \csc x\cot x\,dx = -\csc x + C \tag{16}$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + C \tag{17}$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C \tag{18}$$

$$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\frac{|x|}{a} + C \tag{19}$$