Assignment 4

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sec : 3

course : CSE 330

Ans to qr no 1(a)

interval $[-10, -1.5] = [a, b]$

$\epsilon = 1\times10^{-2}$

We know,

$$n \geq \frac{\log(|b-a|) - \log(\epsilon)}{\log(2)} - 1$$

$$n \geq \frac{\log(8.5) - \log(10^{-2})}{\log(2)} - 1$$

$$n \geq 8.731$$

$$n = 9$$

$\therefore$ Minimum numbers of iterations required $= 9$

Ans to Qr 1 (b)

$f(x) = x^3 + x^2 - 4x + 4$, $[-10, -1.5] = [a, b]$

k	a_k	b_k	m_k	$f(a_k)$	$f(b_k)$	$f(m_k)$	$[a, b]$
0	-10	-1.5	-5.75	-864	0.875	-138.04	[-5.75, -1.5]
1	-5.75	-1.5	-3.625	-138.04	0.875	-23.99	[-3.625, -1.5]
2	-3.625	-1.5	-2.5625	-23.99	0.875	-4.01	[-2.5625, -1.5]
3	-2.5625	-1.5	-2.03125	-4.01	0.875	-0.129	[-2.03125, -1.5]
4	-2.03125	-1.5	-1.765625	-0.129	0.875	0.6757	[-2.03125, -1.765625]

$\therefore x_* = -1.765625$

Ans to Q 1 (c)

$f(x) = x^3 + x^2 - 4x - 4$

roots, $x_1 = 2$, $x_2 = -1$, $x_3 = -2$

Let, $x^3 + x^2 - 4x - 4 = 0$

$$\Rightarrow x = \frac{x^3 + x^2 - 4}{4} \quad \ldots\ldots (i)$$

Again,

$$x^2 = 4x + 4 - x^3$$

$$\Rightarrow x = \pm\sqrt{4x + 4 - x^3}$$

$$\therefore x = \sqrt{4x + 4 - x^3} \quad \text{—— (ii)}$$

Ans to Q 1 (d)

(1) $g(x) = \dfrac{x^3 + x^2 - 4}{4}$

For $x_1 = 2$,

$g'(2) = 4$

since $4 \nless 1$, this root is diverging for this $g(x)$

for $x_2 = -1$,

$g'(-1) = 0.25$

$\therefore$ Since $0.25 < 1$, this root is linearly converging for this $g(x)$

for $x_3 = -2$,

$g'(-2) = 2$,

Since $2 \nless 1$, this is diverging.

Again, $g(x) = \sqrt{4x+4-x^3}$

for $x_1 = 2$,

$g'(2) = |-2| = 2$

since $2 \nless 1$, this root is diverging for the $g(x)$

for $x_2 = -1$,

$g'(-1) = 0.5$

$\therefore$ since $0.5 < 1$, this is converging

for $x_3 = -2$,

$g'(-2) = |-2| = 2$

since $2 \nless 1$, this is diverging.

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Ans to qr 2 (a)

We know,

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

Given, $f(x) = xe^x - 1$ $\quad x_0 = 1.5$

$$f'(x) = xe^x + e^x$$

k	x_k
0	$x_0 = 1.5$
1	$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0.9893$
2	$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0.6789$
3	$x_3 = 0.5766$
4	$x_4 = 0.5672$

$\therefore x_* = 0.5672$

Ans to qr 2 (b)

Given,

$$g(x) = \frac{2x+1}{\sqrt{x+1}}$$

$$g'(x) = \frac{2x+3}{2(x+1)\sqrt{x+1}}$$

For, $x_* = -3/2$

$$g'(-3/2) = \frac{-3+3}{2(-3/2+1)\sqrt{-3/2+1}}$$

$$= 0$$

$\therefore$ To be superlinearly convergent the root must satisfy $x_* = -3/2$

<u>Ans to or 3 (a)</u>

$f(x) = x^2 - x + 1$

We know, $x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$

$x_0 = 0$

k	x_k
0	1
1	$x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{1}{1} = 0$
2	$0 - \frac{1}{-1} = 1$
3	$1 - \frac{1}{1} = 0$
4	$0 - \frac{1}{-1} = 1$
5	$1 - \frac{1}{1} = 0$
6	$0 - \frac{1}{-1} = 1$

Actual solution -

$x^2 - x + 1 = 0$

$x = \frac{1}{2} \pm \frac{\sqrt{3}}{2} i$

We can see that this function from x_0 in Newton's method fails to approach it's root as it keeps getting same value.

Ans to 3(b)

$f(x) = \cos(2x) - \sin x$

$\epsilon = 1 \times 10^{-3}$

let, $f(x) = \cos(2x) - \sin x = 0$

Given $x_0 = 0$, $x_{k+1} = x_k - \dfrac{f(x_k)}{f'(x_k)}$

Atiken Acceleration,

$$\hat{x}_{k+2} = x_k - \frac{x(k+1) - x_k}{x_{k+2} - 2x(k+1) + x_k}$$

k	x_k	$f(x_k)$
0	$x_0 = 0$	1
1	$x_1 = 1$	-1.25761
2	$x_2 = 0.46686$	0.14476
2	$\hat{x}_2 = \dfrac{-1}{0.46686 - 2} = 0.652256$	-0.3438
3	$x_3 = 0.52604$	-6.346×10^{-3}
	$x_4 = 0.523710$	-2.8897×10^{-4}

$\therefore$ Solution, $x = 0.52371$