BRAC University
Course Code: MAT 216

Home work* Sheet # 1

1. Solve the following matrix equation for a, b, c and d .

$$\begin{bmatrix} a-b & b+c \\ 3d+c & 2a-4d \end{bmatrix} = \begin{bmatrix} 8 & 1 \\ 7 & 6 \end{bmatrix}.$$

2. Consider the matrices :

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix},$$

$$D = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}, \quad E = \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix},$$

Compute the following (where possible)

(a) $D + E$ **(b)** $-7C$, **(c)** $2B - C$, **(d)** $-3(D + 2E)$, **(e)** $A - A$, **(f)** $\operatorname{tr}(D - 3E)$.

3. Using the matrices in exercise **(2)** , compute the following (where possible) :

(a) $2A^T + C$, **(b)** $(2E^T - 3D^T)^T$, **(c)** $(D - E)^T$, **(d)** $B^T + 5C^T$, **(e)** $\frac{1}{2}C^T - \frac{1}{4}A$.

4. Using the matrices in exercise **(2)** , compute the following (where possible) .

(a) AB , **(b)** BA , **(c)** $(3E)D$, **(d)** $(AB)C$, **(e)** $A(BC)$, **(f)** $(DA)^T$,

(g) $(C^T B)A^T$, **(h)** $\operatorname{tr}(DD^T)$, **(i)** $\operatorname{tr}(4E^T - D)$.

5. Using the matrices in exercise **(2)** , compute the following (where possible) :

(a) $(2D^T - E)A$, **(b)** $(BA^T - 2C)^T$.

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Home work* Sheet # 2

1. Solve each of the following systems by Gaussain elimination or Gauss - Jordan elimination:

(i) $\begin{aligned} x_1 + x_2 + 2x_3 &= 8 \\ -x_1 - 2x_2 + 3x_3 &= 1 \\ 3x_1 - 7x_2 + 4x_3 &= 10 \end{aligned}$ (ii) $\begin{aligned} 2x_1 + 2x_2 + 2x_3 &= 0 \\ -2x_1 + 5x_2 + 2x_3 &= 1 \\ 8x_1 + x_2 + 4x_3 &= -1 \end{aligned}$ (iii) $\begin{aligned} x - y + 2z - w &= -1 \\ 2x + y - 2z - 2w &= -2 \\ -x + 2y - 4z + w &= 1 \\ 3x \qquad\qquad - 3w &= -3 \end{aligned}$

2. Solve each of the following homogeneous system of linear equations by Gaussain elimination or Gauss - Jordan elimination:

(i) $\begin{aligned} 2x_1 + 2x_2 - x_3 \quad + \quad x_5 &= 0 \\ -x_1 - x_2 + 2x_3 - 3x_4 + x_5 &= 0 \\ x_1 + x_2 - 2x_3 \qquad - x_5 &= 0 \\ x_3 + x_4 + x_5 &= 0 \end{aligned}$ (ii) $\begin{aligned} 2x + 2y + 4z &= 0 \\ w \qquad - y - 3z &= 0 \\ 2w + 3x + y + z &= 0 \\ -2w + x + 3y - 2z &= 0 \end{aligned}$

3. Determine the values of parameter λ , such that the following system has (i) no solution (ii) a unique solution (iii) more than one solution:

$$\begin{aligned} x + y - z &= 1 \\ 2x + 3y + \lambda z &= 3 \\ x + \lambda y + 3z &= 2 \end{aligned} \quad .$$

4. Determine the values of parameters λ & μ, such that the following system has (i) no solution (ii) a unique solution (iii) more than one solution :

$$\begin{aligned} x + y + z &= 6 \\ x + 2y + 3z &= 10 \, . \\ x + 2y + \lambda z &= \mu \end{aligned}$$

5. Determine the values of parameter (s) such that the following system has (i) no solution (ii) a unique solution (iii) more than one solution :

(i) $\begin{aligned} \lambda x + y + z &= 1 \\ x + \lambda y + z &= 1 \\ x + y + \lambda z &= 1 \end{aligned}$ (ii) $\begin{aligned} x + y + kz &= 2 \\ 3x + 4y + 2z &= k \\ 2x + 3y - z &= 1 \end{aligned}$

(iii) $\begin{aligned} x \qquad - 3z &= -3 \\ 2x + \lambda y - z &= -2 \\ x + 2y + \lambda z &= 1 \end{aligned}$ (iv) $\begin{aligned} x + y + \lambda z &= 1 \\ x + \lambda y + z &= \lambda \\ \lambda x + y + z &= \lambda^2 \end{aligned}$

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6. Let A = $\begin{bmatrix} 1 & -2 & 3 \\ 6 & 7 & -1 \\ -3 & 1 & 4 \end{bmatrix}$,

(a) Find all the minors of A
(b) Find all the cofactors of A,
(c) Find adj (A) ,
(d) Find A^{-1} , Using $A^{-1} = \dfrac{1}{\det(A)}$ adj (A).

7. Find the inverse of the following matrices if it exists, using [**A: I]:**

(i) $\begin{bmatrix} 2 & 5 & 5 \\ -1 & -1 & 0 \\ 2 & 4 & 3 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & -3 & 5 \\ 0 & 1 & -3 \\ 0 & 0 & 2 \end{bmatrix}$ (iii) $\begin{bmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix}$ (iv) $\begin{bmatrix} 3 & 4 & -1 \\ 1 & 0 & 3 \\ 2 & 5 & -4 \end{bmatrix}$

(v) $\begin{bmatrix} 1 & 3 & 1 & 1 \\ 2 & 5 & 2 & 2 \\ 1 & 3 & 8 & 9 \\ 1 & 3 & 2 & 2 \end{bmatrix}$ (vi) $\begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ (vii) $\begin{bmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix}$ (viii) $\begin{bmatrix} 1 & -1 & 2 & 1 \\ 3 & 0 & 2 & 2 \\ 2 & 1 & -1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$

8. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$ & $B = \begin{bmatrix} 2 & 5 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$, prove that $(AB)^{-1} = B^{-1}.\, A^{-1}$

9. Solve the following system of linear equations using $\mathbf{x} = \mathbf{A}^{-1}\,\mathbf{b}$

(i) $\begin{aligned} x_1 + 3x_2 + x_3 &= 4 \\ 2x_1 + 2x_2 + x_3 &= -1 \\ 2x_1 + 3x_2 + x_3 &= 3 \end{aligned}$ (ii) $\begin{aligned} 5x_1 + 3x_2 + 2x_3 &= 4 \\ 3x_1 + 3x_2 + 2x_3 &= 2 \\ x_2 + x_3 &= 5 \end{aligned}$ (iii) $\begin{aligned} x + y + z &= 5 \\ x + y - 4z &= 10 \\ -4x + y + z &= 0 \end{aligned}$

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Home work* Sheet # 3

1. Verify whether the following sets are subspace of R^3 / R^4 or not.

(i) S = {(x, 2y, 5) : x,y ϵ R }
(ii) S = {(x, x + y, 3z) : x, y, z ϵ R }
(iii) S = {(x, y, z) ϵ R^3: x - y + z = 0 }
(iv) S = {(x, y, z, t) ϵ R^4 : 3x - 2y - 2z - t = 0 }
(v) S = {(x, y, z) ϵ R^3: x + y + z = 0 }

2. Write the vectors (1, 0, 0) and (0, 0, 1) as linear combinations of vectors {(1, 0, -1), (0, 1, 0), (1, 0, 1)}

3. Determine whether or not,

(i) the vector (1, 2, 6) is a linear combination of (2, 1, 0), (1, -1, 2) & (0, 3, -4).
(ii) the vector (1, 1, 1) is a linear combination of (2, 1, 0), (1, -1, 2) & (0, 3, -4).
(iii) the vector (3, 9, -4, -2) is a linear combination of (1,-2, 0, 3), (2, 3, 0, -1) & (2, -1, 2, 1).
(iv) the vector (2, 3, -7, 3) is a linear combination of (2, 1, 0, 3), (3, -1, 5, 2) & (-1, 0, 2, 1).

4. Determine whether or not the following set of vectors span R^3,

(i) {(1, 1, 2), (1, -1, 2), (1, 0, 1)}
(ii) {(-1, 1, 0), (-1, 0, 1), (1, 1, 1)}
(iii) {(2, 1, 2), (0, 1, -1), (4, 3, 3)}

5. Determine whether each of the following sets are linearly independent or dependent:

(i) {(2 ,1 ,2) , (0 , 1 , - 1) , (4 , 3 , 3)} .
(ii) {(3 , 0 , 1 , -1) , (2 , -1, 0 , 1) , (1 , 1 , 1 , -2)} .
(iii) {(1 , - 4 , 2) , (3 , - 5 , 1) , (2 , 7 , 8) , (- 1 ,1 , 1)} .
(iv) {(0 , 1 , 0 , 1) , (1 , 2 , 3 , -1) , (8 , 4 , 3 , 2) , (0 , 3 , 2 , 0)} .
(v) {(1 , 3 , 2) , (1 , -7 , - 8) , (2 , 1 , - 1)} .
(vi) {(3 , 0 , 4 , 1) , (6 , 2 , -1 , 2) , (-1 , 3 , 5 , 1) , (- 3 , 7 , 8 , 3)}
(vii) {(4 , -4 , 8 , 0) , (2 , 2 , 4 , 0) , (6 , 0 , 0 , 2) , (6 , 3 , -3 , 0)} .

6. Determine whether each of the following sets form a basis for R^3 / R^4 :

(i) {(1 , 2 , 0) , (0 , 5 , 7) & (-1 , 1 , 3)}.
(ii) {(2 , 0 , 1) , (1 , 1 , 1)} .
(iii) {(1 , 1 , 1 , 1) , (0 , 1 , 1 , 1) , (0 , 0 , 1 , 1) , (0 , 0 , 0 , 1)} .

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7. Find *a basis for the row space, a basis for the column space* and *the rank* of the following matrices:

(i) $A=\begin{bmatrix} 6 & 2 & 0 & 4 \\ -2 & -1 & 3 & 4 \\ -1 & -1 & 6 & 10 \end{bmatrix}$ **(ii)** $A=\begin{bmatrix} 1 & 2 & 0 & -1 \\ 3 & 4 & 1 & 2 \\ -2 & 3 & 2 & 5 \end{bmatrix}$ **(iii)** $A=\begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix}$.

(iv) $A=\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ **(v)** $A=\begin{bmatrix} 1 & 3 & 1 & -2 & -3 \\ 1 & 4 & 3 & -1 & -4 \\ 2 & 3 & -4 & -7 & -3 \\ 3 & 8 & 1 & -7 & -8 \end{bmatrix}$.

8. Find *a basis for the Null space, the rank* and *the Nullity* of the following matrices:

(i) $A=\begin{bmatrix} -1 & 2 & 0 & 4 & 5 & -3 \\ 3 & -7 & 2 & 0 & 1 & 4 \\ 2 & -5 & 2 & 4 & 6 & 1 \\ 4 & -9 & 2 & -4 & -4 & 7 \end{bmatrix}$ **(ii)** $A=\begin{bmatrix} 1 & -1 & 3 \\ 5 & -4 & -4 \\ 7 & -6 & 2 \end{bmatrix}$ **(iii)** $A=\begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}$

(iv) $A=\begin{bmatrix} 1 & -3 & 2 & 2 & 1 \\ 0 & 3 & 6 & 0 & -3 \\ 2 & -3 & -2 & 4 & 4 \\ 3 & -6 & 0 & 6 & 5 \\ -2 & 9 & 2 & -4 & -5 \end{bmatrix}$

9. Find *a basis* and *dimension* of the subspace generated by the set of vectors S = {(1 , 2 , 1) , (3 , 1 , 2) , (1 , -3 , 4)}.

10. Let **U** be the subspace of R^3 spanned (generated) by the set of vectors S = {(1 , 2 , 1) , (0 , - 1 , 0) & (2 , 0 , 2)}. Find *a basis* and *dimension* of **U .**

11. Let **W** be the subspace of R^4 generated by the set of vectors S = {(1 ,- 2 , 5 ,-3) , (2 , 3 , 1 , - 4) & (3 , 8 , - 3 , - 5)}. Find *a basis* and *dimension* of **W**.

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<u>Home work* Sheet # 4</u>

1. Determine whether each of the following Transformation is a linear transformation:

(i) $T(x, y, z) = (x - y, x - z)$ (ii) $T(x, y, z) = (3x - 2y + z, x - 3y - 2z)$

(iii) $T(x, y, z) = (x + 1, y + z)$. (iv) $T(x, y) = (x + y, 1)$

2. Let T : $R^4 \to R^3$ be the linear transformation defined by

$T(x, y, z, t) = (x - y + z + t, x + 2z - t, x + y + 3z - 3t)$.

Find a basis & dimension of the range space of (T) & the null space of (T).

3. Let T : $R^3 \to R^3$ be the linear transformation defined by

$T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$.

Find a basis & dimension of (i) Range(T) & (ii) Ker (T).

4. Let T : $R^3 \to R^3$ be the linear transformation defined by

$T(x, y, z) = (3x - y, y - z, 3x - 2y + z)$,

Find a basis & dimension of (i) Range(T) & (ii) Ker (T).

5. Let T : $R^3 \to R^3$ be the linear transformation defined by

$T(x, y, z) = (x + 2y - 3z, 2x - y + 4z, 4x + 3y - 2z)$,

Find a basis & dimension of (i) Range (T) & (ii) Ker (T).

6. Find all eigenvalues and the corresponding eigenvectors of the following matrices:

(i) $A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & -2 & 0 \\ 0 & -5 & 2 \end{bmatrix}$ **(ii)** $A = \begin{bmatrix} 2 & -2 & 1 \\ 2 & -8 & -2 \\ 1 & 2 & 2 \end{bmatrix}$ **(iii)** $A = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$

7. Find a matrix P that diagonalizes the following matrices , also find $P^{-1}AP$:

(i) $A = \begin{bmatrix} -14 & 12 \\ -20 & 17 \end{bmatrix}$ **(ii)** $A = \begin{bmatrix} -1 & -2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$ **(iii)** $A = \begin{bmatrix} 1 & 4 & 1 \\ 2 & 1 & 0 \\ -1 & 3 & 1 \end{bmatrix}$

Solve the following problems given in the book "Elementary linear algebra by Howard Anton and Chris Rorres, Application version, eigth edition."

_ Ex 5.5: 3(a,b), 6(a,b), 7(a,b),8(a, b,c), 11(a,c),12(a,b)
_ Ex 5.6: 1, 2(a,b,c)
_ Ex 8.1: 13, 16
_ Ex 8.2: 3,4, 10, 11
_ Ex 7.1: Consider the matrix given in 4(a,c,d). Find the eigenvalues and their corresponding
Eigenvectors that form bases for eigenspace. If possible, diagonalize those matrices.

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