BRAC UNIVERSITY

Inspiring Excellence

Department of Mathematics and Natural Sciences

MAT 110

MID ASSIGNMENT

SUMMER 2021

SET: 12(SADT)

Please write your name and ID on the first page of the assignment answer script. The deadline is 30th july, 9.00 am to 10.30 am. Solve all problems.

You can only submit a PDF file - image or doc files won't be accepted. Before submitting the PDF, please rename the PDF file in the format - SET_ID_SECTION.

Answer the questions by yourself. Plagiarism will lead to an F grade in the course. ***Total marks is 250.*** *It will be converted to 20. If you have issues with the questions, please contact SADT on Slack.*

1. Calculate the following limit:

$$\lim_{x\to 0} x \left\lceil \frac{1}{x} \right\rceil .$$

   Here, the symbols around $\frac{1}{x}$ indicate the least integer greater that $\frac{1}{x}$, i.e. the ceiling function of $\frac{1}{x}$. [*Hint*: Use the squeeze theorem.]

2. Without using L'Hospital's rule, evaluate

$$\lim_{x\to 0} \frac{\tan(4x)}{x}.$$

   [*Hint*: Use the fact that $\lim_{x\to 0} \frac{\sin x}{x} = 1$.]

3. By using the first four terms of the Taylor expansion of $f(x) = \arctan(x)$ about $x = 0$, find an estimate for π to 1 decimal place.

4. You are constructing a box for your cat to sleep in. The material for the square bottom of the box costs \$5/ft^2 and the material for the sides costs \$2/ft^2. You need a box with volume \$4ft^2. Find the dimensions of the box that minimize cost and show that the dimensions that you get actually give you minimum cost.

5. Consider the following function f:

$$f(x) = \begin{cases} 3ax & x < 2 \\ ax^2 + bx + 1 & x \geq 2. \end{cases}$$

What are the values of a and b for which f is differentiable at $x = 2$?