(3 marks) Consider the quadratic equation, $x^2 - 60x + 1 = 0$. Working to 6 significant figures, compute the roots of the quadratic equation and check that there is a loss of significance. Find the correct roots such that loss of significance does not occur.

**Solution**: The discriminant of the given quadratic equation is

$$\text{Discriminant} = \frac{-(-60) \pm \sqrt{(-60)^2 - 4 \times 1 \times 1}}{2 \times 1} = 30 \pm 29.9833 = 59.9833 \quad \text{and} \quad 0.0167000 \quad \text{(upto 6 sig. fig.)}$$

But the product of the roots must be equal to 1, but here we get, $59.9833 \times 0.0167000 = 1.00172 \neq 1$. Since the product is not equal to one there is loss of significance due to the subtraction of two close numbers 30 and 29.9833.

One root is $x_1 = 59.9833$. To find the second root, we use, $x_2 = 1/x_1 = 1/(59.9833) = 0.0166713$ up to 6 sig. fig. Now, check that $x_1 + x_2 = 59.9833 + 0.0166713 = 59.99997 = 60.0000$ up tp 6 sig. fig. This is the correct solution that does not have any loss of significance.

Consider the quadratic equation $x^2 - 16x + 3 = 0$. Explain how the loss of significance occurs in finding the roots of the quadratic equation if we restrict to 4 significant figures. Discuss how to avoid this and find the roots.

Answer: $x^2 - 16x + 3 = 0$ =>

$x = \frac{16 \pm \sqrt{16^2 - 12}}{2} = 8 \pm \sqrt{61} = 8 + 7.810;\ 8 - 7.810 = 15.81;\ 0.19$

But $15.81$ * $0.19$ = 3.004 $\neq$ 3

So, 2nd root = $\frac{3}{15.81} = 0.1898$ (4 s.f.)

Consider the quadratic equation $x^2 - 12x + 5 = 0$. Explain how the loss of significance occurs in finding the roots of the quadratic equation if we restrict to 4 significant figures. Discuss how to avoid this and find the roots.

Answer: $x^2 - 12x + 5 = 0$ =>

$x = \frac{12 \pm \sqrt{12^2 - 20}}{2} = 6 \pm \sqrt{31} = 6 + 5.568;\ 6 - 5.568 = 11.57;\ 0.432$

But 11.57 * 0.432 = 4.998 $\neq$ 5

So, 2nd root = $\frac{5}{11.57} = 0.4321$ (4 s.f.)