Shihab Muhtasim

Student ID: 21301610

MAT 110

ASSIGNMENT 04

SET 7

Ans to the question no 01

Given function,

$f(x, y) = y \cdot e^{(xy+3x+2y^2)}$

Now,

$f_x = y \cdot e^{(xy+3x+2y^2)} \cdot (y + 3) = (y^2 + 3y) \cdot e^{(xy+3x+2y^2)}$

$f_y = e^{(xy+3x+2y^2)} + y \cdot e^{(xy+3x+2y^2)} \cdot (x + 4y)$

$\Rightarrow f_y = e^{(xy+3x+2y^2)}(1 + xy + 4y^2)$

$f_{xx} = e^{(xy+3x+2y^2)} \cdot (y+3) \cdot y(y+3) = e^{(xy+3x+2y^2)} \cdot y(y+3)^2$

$f_{yy} = e^{(xy+3x+2y^2)} \cdot (x + 4y) \cdot (1 + xy + 4y^2) + e^{(xy+3x+2y^2)} \cdot (x + 8y)$

$\Rightarrow f_{yy} = e^{(xy+3x+2y^2)} \cdot \{(x + 4y)(1 + xy + 4y^2) + (x + 8y)\}$

$f_{xy} = (2y+3) \cdot e^{(xy+3x+2y^2)} + (y^2+3y) \cdot e^{(xy+3x+2y^2)} \cdot (x+4y)$

$\Rightarrow f_{xy} = e^{(xy+3x+2y^2)}\{(2y + 3) + (y^2 + 3y)(x + 4y)\}$

Now, $f(0, 0) = 0$

$f_x(0, 0) = 0$

$f_y(0, 0) = 1$

$f_{xx}(0, 0) = 0$

$f_{yy}(0, 0) = 0$

$f_{xy}(0, 0) = 3$

The first degree Maclaurin polynomial approximation,

$\therefore L(x,y) = f(0,0) + f_x(0,0)(x-0) + f_y(0,0)(y-0)$

$\Rightarrow L(x,y) = 0 + 0 + 1 \cdot y = y$

$\therefore L(x,y) = y$

The second degree Maclaurin polynomial approximation,

$Q(x,y) = L(x,y) + \frac{f_{xx}(0,0)}{2!} \cdot (x-0)^2 + \frac{f_{yy}(0,0)}{2!} \cdot (y-0)^2 + \frac{f_{xy}(0,0)}{1!} \cdot (x-0)(y-0)$

$\Rightarrow Q(x,y) = y + 0 + 0 + 3xy = y(3x+1)$

$\therefore Q(x,y) = y(3x+1)$

<u>Ans to the question no 02</u>

Given function,

$f(x, y) = f(x, y) = 4xye^{-x^2-y^2}$

Now,

$f_x = 4y \cdot \{e^{-x^2-y^2} + xe^{-x^2-y^2}(-2x)\}$

$\Rightarrow f_x = 4y \cdot e^{-x^2-y^2}(1 - 2x^2)$

$f_y = 4x \cdot \{e^{-x^2-y^2} + ye^{-x^2-y^2}(-2x)\}$

$\Rightarrow f_y = 4x \cdot e^{-x^2-y^2}(1 - 2y^2)$

For extreme values,

$f_x = 0$

$\Rightarrow 4y \cdot e^{-x^2-y^2}(1 - 2x^2) = 0$

$e^{-x^2-y^2}$ cannot be zero for any finite values of x or y

$\therefore y = 0$ or $x = \pm\frac{1}{\sqrt{2}}$

Similarly,

$f_y = 0$

$\Rightarrow 4x \cdot e^{-x^2-y^2}(1 - 2y^2) = 0$

$e^{-x^2-y^2}$ cannot be zero for any finite values of x or y

$\therefore x = 0$ or $y = \pm\frac{1}{\sqrt{2}}$

The critical points are: $(0, 0), (0, \frac{1}{\sqrt{2}}), (0, -\frac{1}{\sqrt{2}}), (\frac{1}{\sqrt{2}}, 0)$

$(-\frac{1}{\sqrt{2}},0),(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}),(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}),(-\frac{1}{\sqrt{2}},+\frac{1}{\sqrt{2}}),(-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})$

Now,

$f_{xx} = 4y\{e^{-x^2-y^2}(-4x) + (1-2x^2)e^{-x^2-y^2}(-2x)\}$

$\Rightarrow f_{xx} = 4y\{2xe^{-x^2-y^2}(2x^2-3)\}$

$\Rightarrow f_{xx} = 8xy(2x^2-3)e^{-x^2-y^2}$

$f_{yy} = 4x\{e^{-x^2-y^2}(-4y) + (1-2y^2)e^{-x^2-y^2}(-2y)\}$

$\Rightarrow f_{xx} = 4x\{2xe^{-x^2-y^2}(2y^2-3)\}$

$\Rightarrow f_{xx} = 8xy(2y^2-3)e^{-x^2-y^2}$

$f_{xy} = 4\{e^{-x^2-y^2}(1-2x^2) + ye^{-x^2-y^2}(-2y)(1-2x^2)\}$

$\Rightarrow f_{xy} = 4e^{-x^2-y^2}(1-2x^2)(1-2y^2)$

For $(0,0)$,

$A = f_{xx}(0,0) = 0$

$B = f_{xy}(0,0) = 4$

$C = f_{yy}(0,0) = 0$

$D = AC - B^2 = 0 - 4^2 = -16$

Since $D < 0$, f has a saddle point at (0,0)

For $(0,\frac{1}{\sqrt{2}})$,

$A = f_{xx}(0,\frac{1}{\sqrt{2}}) = 0$

$B = f_{xy}(0,\frac{1}{\sqrt{2}}) = 0$

$C = f_{yy}(0,\frac{1}{\sqrt{2}}) = 0$

$D = AC - B^2 = 0$

Since $D = 0$, No conclusion can be done

For $(0, -\frac{1}{\sqrt{2}})$,

$A = f_{xx}(0, -\frac{1}{\sqrt{2}}) = 0$

$B = f_{xy}(0, -\frac{1}{\sqrt{2}}) = 0$

$C = f_{yy}(0, -\frac{1}{\sqrt{2}}) = 0$

$D = AC - B^2 = 0$

Since $D = 0$, No conclusion can be done

For $(\frac{1}{\sqrt{2}}, 0)$,

$A = f_{xx}(\frac{1}{\sqrt{2}}, 0) = 0$

$B = f_{xy}(\frac{1}{\sqrt{2}}, 0) = 0$

$C = f_{yy}(\frac{1}{\sqrt{2}}, 0) = 0$

$D = AC - B^2 = 0$

Since $D = 0$, No conclusion can be done

For $(-\frac{1}{\sqrt{2}}, 0)$,

$A = f_{xx}(-\frac{1}{\sqrt{2}}, 0) = 0$

$B = f_{xy}(-\frac{1}{\sqrt{2}}, 0) = 0$

$C = f_{yy}(-\frac{1}{\sqrt{2}}, 0) = 0$

$D = AC - B^2 = 0$

Since $D = 0$, No conclusion can be done

For $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$,

$A = f_{xx}(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) = -8e^{-1} = -2.943$

$B = f_{xy}(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) = 0$

$C = f_{yy}(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) = -8e^{-1} = -2.943$

$D = AC - B^2 = 8.66$
Since $D > 0$ and $A < 0$, f has a relative maximum point at $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

For $(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$,

$A = f_{xx}(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}) = 8e^{-1} = 2.943$

$B = f_{xy}(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}) = 0$

$C = f_{yy}(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}) = 8e^{-1} = 2.943$

$D = AC - B^2 = 8.66$
Since $D > 0$ and $A > 0$, f has a relative minimum point at $(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$

For $(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$,

$A = f_{xx}(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) = 8e^{-1} = 2.943$

$B = f_{xy}(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) = 0$

$C = f_{yy}(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) = 8e^{-1} = 2.943$

$D = AC - B^2 = 8.66$
Since $D > 0$ and $A > 0$, f has a relative minimum point at $(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

For $(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$,

$A = f_{xx}(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}) = -8e^{-1} = -2.943$

$B = f_{xy}(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}) = 0$

$C = f_{yy}(-\frac{1}{-\sqrt{2}}, -\frac{1}{\sqrt{2}}) = -8e^{-1} = -2.943$

$D = AC - B^2 = 8.66$

Since $D > 0$ and $A < 0$, f has a relative maximum point at $(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$

$\therefore$ Saddle point:(0,0)

Maxima:$(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$,$(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

Minima:$(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$,$(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

Ans to the question no 03

Given function,

$x - 2y + z = 16 \Rightarrow z = 16 - x + 2y$

At origin (0,0),

$D = (Distance)^2 = (x-0)^2 + (y-0)^2 + (z-0)^2$

$= x^2 + y^2 + z^2$

$= x^2 + y^2 + (16 - x + 2y)^2$

Here,

$D_x = 2x + 2(16 - x + 2y)(-1)$

$= 2x - 32 + 2x - 4y$

$= 4x - 4y - 32$

When $D_x = 0$,

$4x - 4y - 32 = 0$

$\Rightarrow x - y - 8 = 0$

$\Rightarrow y = x - 8$

Again, $D_y = 2y + 2(16 - x + 2y)(2)$

$= 2y + 64 - 4x + 8y$

$= 10y - 4x + 64$

When $D_y = 0$

$10y - 4x + 64 = 0$

$\Rightarrow 5(x - 8) - 4x + 64 = 0$

$\Rightarrow 3x - 8 = 0$

$\Rightarrow x = \frac{8}{3}$

Substituting the value of x in the equation $y = x - 8$,

$y = \frac{-16}{3}$

$\therefore (x, y) = (\frac{8}{3}, \frac{-16}{3})$

Now, $A = D_{xx} = 4$

$C = D_{yy} = 10$

$B = D_{xy} = -4$

$D = AC - B^2 = 24$

Since $D > 0$ and $A > 0$, D has a minimum at point $(\frac{8}{3}, \frac{-16}{3})$

Now,

$D(\frac{8}{3}, \frac{-16}{3}) = (\frac{8}{3})^2 + (\frac{-16}{3})^2 + (16 - \frac{8}{3} - \frac{2 \cdot 16}{3})^2$

$= \frac{64}{9} + \frac{256}{9} + (\frac{40}{3} - \frac{32}{3})^2$

$= \frac{64}{9} + \frac{256}{9} + (\frac{8}{3})^2$

$= \frac{128}{9} + \frac{256}{9}$

$= \frac{128}{3}$

$\therefore Distance = \sqrt{\frac{128}{3}} = \frac{8\sqrt{6}}{3}$

$Since, x = \frac{8}{3}, y = \frac{-16}{3},$

$z = 16 - \frac{8}{3} + \frac{2 \cdot -16}{3} = \frac{8}{3}$

the point in the given plane that is closest to the origin is$(\frac{8}{3}, \frac{-16}{3}, \frac{8}{3})$

Ans to the question no 04

Given function,

$\vec{F} = -x^2(y-z)\hat{i} - \left(x^2 + y^4\right)\hat{j} + \left(\frac{4z^2}{y^2}\right)\hat{k}$

$\Rightarrow \vec{F} = (-x^2y + -x^2z)\hat{i} + \left(-x^2 - y^4\right)\hat{j} + \left(4z^2y^{-2}\right)\hat{k}$

$\text{Curl } \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -x^2y + x^2z & -x^2 - y^4 & 4z^2y^{-2} \end{vmatrix}$

$= \hat{i}\{\frac{\partial}{\partial y}(4z^2y^{-2}) - \frac{\partial}{\partial z}(-x^2 - y^4)\} - \hat{j}\{\frac{\partial}{\partial x}(4z^2y^{-2}) - \frac{\partial}{\partial z}(-x^2y + x^2z) + \hat{k}\{\frac{\partial}{\partial x}(-x^2 - y^4) - \frac{\partial}{\partial y}(-x^2y + x^2z)\}$

$= (-8z^2y^{-3})\hat{i} - (0 - 0 + x^2)\hat{j} + (-2x + x^2)\hat{k}$

$= -8z^2y^{-3}\hat{i} - x^2\hat{j} - (2x + x^2)\hat{k}$

$\text{Div } \vec{F} = \Delta F$

$= \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$

$= \frac{\partial}{\partial x}(-x^2y + -x^2z) + \frac{\partial}{\partial y}(-x^2 - y^4) + \frac{\partial}{\partial z}(4z^2y^{-2})$

$= -2x(y-z) - 4y^3 + \frac{8z}{y^2}$

$\therefore Curl\vec{F} = -8z^2y^{-3}\hat{i} - x^2\hat{j} - (2x + x^2)\hat{k}$

$\therefore \Delta F = -2x(y-z) - 4y^3 + \frac{8z}{y^2}$

Ans to the question no 05

Given function,

$\vec{F} = 2yz\hat{i} - x^2y\hat{j} + xz^2\hat{k}$

$\phi = 2x^2yz^3$

$\nabla\phi = (\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}) \cdot (2x^2yz^3)$

$= \frac{\partial}{\partial x}(2x^2yz^3)\hat{i} + \frac{\partial}{\partial y}(2x^2yz^3)\hat{j} + \frac{\partial}{\partial z}(2x^2yz^3)\hat{k}$

$= 4xyz^3\hat{i} + 2x^2z^3\hat{j} + 6x^2yz^2\hat{j}$

Now,

$\vec{F} \cdot (\nabla\phi) = (2yz\hat{i} - x^2y\hat{j} + xz^2\hat{k}) \cdot (4xyz^3\hat{i} + 2x^2z^3\hat{j} + 6x^2yz^2\hat{j})$

$= 8xy^2z^4 - 2x^4yz^3 + 6x^3yz^4$

$\therefore \vec{F} \cdot (\nabla\phi) = 8xy^2z^4 - 2x^4yz^3 + 6x^3yz^4$

Ans to the question no 06

Given function,

$f(x,y,x) = x^2y + y^2z + xz^2$

$\nabla \cdot f(x,y,x) = (\frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k})$

$= \frac{\partial}{\partial x}(x^2y+y^2z+xz^2)\hat{i} + \frac{\partial f}{\partial y}(x^2y+y^2z+xz^2)\hat{j} + \frac{\partial f}{\partial z}(x^2y + y^2z + xz^2)\hat{k})$

$= (2xy + z^2)\hat{i} + (x^2 + 2yz)\hat{j} + (y^2 + 2zx)\hat{k}$

At the point (2,4,5),

$\nabla \cdot f(2,4,5) = (2\cdot2\cdot4+5^2)\hat{i}+(2^2+2\cdot4\cdot5)\hat{j}+(4^2+2\cdot5\cdot2)\hat{k}$

$= 41\hat{i} + 44\hat{j} + 36\hat{k}$

unit vector along (1,-1,3),

$\hat{u} = \frac{\hat{i}-\hat{j}+3\hat{k}}{\sqrt{1^2+1^2+3^2}}$

$= \frac{\hat{i}-\hat{j}+3\hat{k}}{\sqrt{11}}$

Directional derivative of the function f(x,y,z) at the point (2,4,5) in the direction of the point (1,-1,3),

$(\nabla \cdot f(2,4,5)) \cdot \hat{u} = \frac{41-44+108}{\sqrt{11}}$

$= \frac{105}{\sqrt{11}} = 31.66$

assignment 4

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\documentclass{article}
\usepackage{amsmath}
\usepackage{amssymb}
\begin{document}
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    \begin{center}
    \line(1,0){300}\\
    [0.25 in]
    \huge{\bfseries Shihab Muhtasim}\\
    [0.5 cm]
    \textsc{\Large Student ID: 21301610}\\
    \line(1,0){400}\\
    [2 cm]
    \textsc{\LARGE MAT 110}\\
    [0.5 cm]
    \textsc{\LARGE ASSIGNMENT 04}\\
    [0.5 cm]
    \textsc{\LARGE SET 7}\\
    \end{center}
    \end{titlepage}
\begin{newpage}
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    \textsc{Assignment 4}\\
    \textsc{Problem 1}\\
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\begin{center}
  \textbf{\Large \underline {Ans to the question no 01}}\\
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\end{center}
\Large {Given function, \\[3mm]
$ f(x,y)=y\cdot e^{(xy+3x+2y^2)}$\\[3mm]
Now,\\[3mm]
$f_x=y\cdot e^{(xy+3x+2y^2)}\cdot(y+3)=(y^2+3y)\cdot e^{(xy+3x+2y^2)}\\[3mm]
f_y=e^{(xy+3x+2y^2)}+y\cdot e^{(xy+3x+2y^2)}\cdot (x+4y)\\[3mm]
\Rightarrow f_y= e^{(xy+3x+2y^2)}(1+xy+4y^2)\\[3mm]
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Shihab Muhtasim

STUDENT ID: 21301610

MAT 110

ASSIGNMENT 04

SET 7

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\begin{newpage}
    \begin{flushright}
    \textsc{Assignment 4}\\
    \textsc{Problem 1}\\
    [1 cm]
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\begin{center}
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\end{center}
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\Rightarrow f_y= e^{(xy+3x+2y^2)}(1+xy+4y^2)\\[3mm]
f_{xx}=e^{(xy+3x+2y^2)} \cdot (y+3)\cdot y(y+3)=e^{(xy+3x+2y^2)} \cdot y(y+3)^2\\[3mm]
f_{yy}=e^{(xy+3x+2y^2)}\cdot (x+4y)\cdot (1+xy+4y^2)+e^{(xy+3x+2y^2)}\cdot (x+8y)\\[3mm]
\Rightarrow f_{yy}=e^{(xy+3x+2y^2)}\cdot \{(x+4y)(1+xy+4y^2)+(x+8y)\}\\[3mm]
f_{xy}=(2y+3)\cdot e^{(xy+3x+2y^2)} +(y^2+3y)\cdot e^{(xy+3x+2y^2)}\cdot (x+4y)\\[3mm]
\Rightarrow f_{xy}= e^{(xy+3x+2y^2)}\{(2y+3)+(y^2+3y)(x+4y)\}\\[3mm]$
Now,
$f(0,0)=0\\[3mm]
f_x(0,0)=0\\[3mm]
f_y(0,0)=1\\[3mm]
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```


Ans to the question no 01

Given function,

$f(x,y) = y \cdot e^{(xy+3x+2y^2)}$

Now,

$f_x = y \cdot e^{(xy+3x+2y^2)} \cdot (y+3) = (y^2+3y) \cdot e^{(xy+3x+2y^2)}$

$f_y = e^{(xy+3x+2y^2)} + y \cdot e^{(xy+3x+2y^2)} \cdot (x+4y)$

$\Rightarrow f_y = e^{(xy+3x+2y^2)}(1+xy+4y^2)$

$f_{xx} = e^{(xy+3x+2y^2)} \cdot (y+3) \cdot y(y+3) = e^{(xy+3x+2y^2)} \cdot y(y+3)^2$

$f_{yy} = e^{(xy+3x+2y^2)} \cdot (x+4y) \cdot (1+xy+4y^2) + e^{(xy+3x+2y^2)} \cdot (x+8y)$

$\Rightarrow f_{yy} = e^{(xy+3x+2y^2)} \cdot \{(x+4y)(1+xy+4y^2)+(x+8y)\}$

$f_{xy} = (2y+3) \cdot e^{(xy+3x+2y^2)} + (y^2+3y) \cdot e^{(xy+3x+2y^2)} \cdot (x+4y)$

$\Rightarrow f_{xy} = e^{(xy+3x+2y^2)}\{(2y+3)+(y^2+3y)(x+4y)\}$

Now, $f(0,0) = 0$

$f_x(0,0) = 0$

$f_y(0,0) = 1$

$f_{xx}(0,0) = 0$

$f_{yy}(0,0) = 0$

$f_{xy}(0,0) = 3$

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```
f_y(0,0)=1\\[3mm]
f_{xx}(0,0)=0\\[3mm]
f_{yy}(0,0)=0\\[3mm]
f_{xy}(0,0)=3$\\[10mm]
The first degree Maclaurin polynomial approximation,\\[3mm]
$\therefore L(x,y)=f(0,0)+f_x(0,0)(x-0)+f_y(0,0)(y-0)\\[3mm]
\Rightarrow L(x,y)=0+0+1\cdot y=y\\[3mm]
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(x-0)^2+\frac{f_{yy}(0,0)}{2!}\cdot(y-0)^2+\frac{f_{xy}(0,0)}{1!}\cdot(x-0)(y-0)\\[3mm]
\Rightarrow Q(x,y)=y+0+0+3xy=y(3x+1)\\[3mm]
\therefore Q(x,y)=y(3x+1)\\[3mm]$}
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    \begin{flushright}
    \textsc{Assignment 4}\\
    \textsc{Problem 2}\\
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\begin{center}
  \textbf{\Large \underline {Ans to the question no 02}}\\
  [0.5 cm]
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\Large {Given function, \\[3mm]
$ f(x,y)=f(x,y) = 4xye^{-x^2-y^2}$\\[3mm]
Now,\\[2mm]
$f_x=4y \cdot \{e^{-x^2-y^2}+xe^{-x^2-y^2}(-2x)\}\\[3mm]
\Rightarrow f_x=4y\cdot e^{-x^2-y^2}(1-2x^2)\\[3mm]
f_y=4x \cdot \{e^{-x^2-y^2}+ye^{-x^2-y^2}(-2x)\}\\[3mm]
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For extreme values,\\[3mm]
$f_x=0\\[3mm]
\Rightarrow 4y\cdot e^{-x^2-y^2}(1-2x^2) =0 \\[3mm]
e^{-x^2-y^2}$ cannot be zero for any finite values of x or y\\[3mm]
$\therefore y=0$ or $x=\pm \frac{1}{\sqrt{2}}$\\[3mm]
```

The first degree Maclaurin polynomial approximation,

$\therefore L(x,y)=f(0,0)+f_x(0,0)(x-0)+f_y(0,0)(y-0)$

$\Rightarrow L(x,y)=0+0+1\cdot y=y$

$\therefore L(x,y)=y$

The second degree Maclaurin polynomial approximation,

$Q(x,y)=L(x,y)+\frac{f_{xx}(0,0)}{2!}\cdot(x-0)^2+\frac{f_{yy}(0,0)}{2!}\cdot(y-0)^2+\frac{f_{xy}(0,0)}{1!}\cdot(x-0)(y-0)$

$\Rightarrow Q(x,y)=y+0+0+3xy=y(3x+1)$

$\therefore Q(x,y)=y(3x+1)$

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\end{newpage}
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    \begin{flushright}
    \textsc{Assignment 4}\\
    \textsc{Problem 2}\\
    [1 cm]
    \end{flushright}
\begin{center}
  \textbf{\Large \underline {Ans to the question no 02}}\\
  [0.5 cm]
\end{center}
\Large {Given function, \\[3mm]
$ f(x,y)=f(x,y) = 4xye^{-x^2-y^2}$\\[3mm]
Now,\\[2mm]
$f_x=4y \cdot \{e^{-x^2-y^2}+xe^{-x^2-y^2}(-2x)\}\\[3mm]
\Rightarrow f_x=4y\cdot e^{-x^2-y^2}(1-2x^2)\\[3mm]
f_y=4x \cdot \{e^{-x^2-y^2}+ye^{-x^2-y^2}(-2x)\}\\[3mm]
\Rightarrow f_y=4x\cdot e^{-x^2-y^2}(1-2y^2)\\[3mm]$
For extreme values,\\[3mm]
$f_x=0\\[3mm]
\Rightarrow 4y\cdot e^{-x^2-y^2}(1-2x^2) =0 \\[3mm]
e^{-x^2-y^2}$ cannot be zero for any finite values of x or y\\[3mm]
$\therefore y=0$ or $x=\pm \frac{1}{\sqrt{2}}$\\[3mm]
Similarly,\\[3mm]
$f_y=0\\[3mm]
\Rightarrow 4x\cdot e^{-x^2-y^2}(1-2y^2) =0 \\[3mm]
e^{-x^2-y^2}$ cannot be zero for any finite values of x or y\\[3mm]
$\therefore x=0$ or $y=\pm \frac{1}{\sqrt{2}}$\\[3mm]
The critical points are: $(0,0),(0,\frac{1}{\sqrt{2}}),(0,-\frac{1}{\sqrt{2}}),(\frac{1}{\sqrt{2}},0)\\[3mm]
(-\frac{1}{\sqrt{2}},0),(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}),(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}),(-\frac{1}{\sqrt{2}},+\frac{1}{\sqrt{2}}),(-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})$\\[3mm]
Now,\\[3mm]
$f_{xx}=4y\{e^{-x^2-y^2}(-4x)+(1-2x^2)e^{-x^2-y^2}(-2x)\} \\[3mm]
\Rightarrow f_{xx}=4y\{2xe^{-x^2-y^2}(2x^2-3)\}\\[3mm]
\Rightarrow f_{xx}=8xy(2x^2-3)e^{-x^2-y^2}\\[3mm]
```

Assignment 4
Problem 2

Ans to the question no 02

Given function,

$f(x,y) = f(x,y) = 4xye^{-x^2-y^2}$

Now,

$f_x = 4y \cdot \{e^{-x^2-y^2} + xe^{-x^2-y^2}(-2x)\}$

$\Rightarrow f_x = 4y \cdot e^{-x^2-y^2}(1-2x^2)$

$f_y = 4x \cdot \{e^{-x^2-y^2} + ye^{-x^2-y^2}(-2x)\}$

$\Rightarrow f_y = 4x \cdot e^{-x^2-y^2}(1-2y^2)$

For extreme values,

$f_x = 0$

$\Rightarrow 4y \cdot e^{-x^2-y^2}(1-2x^2) = 0$

$e^{-x^2-y^2}$ cannot be zero for any finite values of x or y

$\therefore y = 0$ or $x = \pm\frac{1}{\sqrt{2}}$

Similarly,

$f_y = 0$

$\Rightarrow 4x \cdot e^{-x^2-y^2}(1-2y^2) = 0$

$e^{-x^2-y^2}$ cannot be zero for any finite values of x or y

$\therefore x = 0$ or $y = \pm\frac{1}{\sqrt{2}}$

The critical points are: $(0,0), (0,\frac{1}{\sqrt{2}}), (0,-\frac{1}{\sqrt{2}}), (\frac{1}{\sqrt{2}},0)$

3

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```
\Rightarrow 4x\cdot e^{-x^2-y^2}(1-2y^2) =0 \\[3mm]
e^{-x^2-y^2}$ cannot be zero for any finite values of x or y\\[3mm]
$\therefore x=0$ or $y=\pm \frac{1}{\sqrt{2}}$\\[3mm]
The critical points are: $(0,0),(0,\frac{1}{\sqrt{2}}),(0,-\frac{1}{\sqrt{2}}),(\frac{1}{\sqrt{2}},0)\\[3mm]
(-\frac{1}{\sqrt{2}},0),(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}),(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}),(-\frac{1}{\sqrt{2}},+\frac{1}{\sqrt{2}}),(-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})$\\[3mm]
Now,\\[3mm]
$f_{xx}=4y\{e^{-x^2-y^2}(-4x)+(1-2x^2)e^{-x^2-y^2}(-2x)\} \\[3mm]
\Rightarrow f_{xx}=4y\{2xe^{-x^2-y^2}(2x^2-3)\}\\[3mm]
\Rightarrow f_{xx}=8xy(2x^2-3)e^{-x^2-y^2}\\[3mm]
f_{yy}=4x\{e^{-x^2-y^2}(-4y)+(1-2y^2)e^{-x^2-y^2}(-2y)\} \\[3mm]
\Rightarrow f_{xx}=4x\{2xe^{-x^2-y^2}(2y^2-3)\}\\[3mm]
\Rightarrow f_{xx}=8xy(2y^2-3)e^{-x^2-y^2}\\[3mm]
f_{xy}=4\{e^{-x^2-y^2}(1-2x^2)+ye^{-x^2-y^2}(-2y)(1-2x^2)\}\\[3mm]
\Rightarrow f_{xy}=4e^{-x^2-y^2}(1-2x^2)(1-2y^2)$\\[3mm]
For $(0,0),\\[3mm]
A=f_{xx}(0,0)=0\\[3mm]
B=f_{xy}(0,0)=4\\[3mm]
C=f_{yy}(0,0)=0\\[3mm]
D=AC-B^2=0-4^2=-16\\[3mm]$
Since $D<0$, f has a saddle point at (0,0)\\[3mm]
For $(0,\frac{1}{\sqrt{2}}),\\[3mm]
A=f_{xx}(0,\frac{1}{\sqrt{2}})=0\\[3mm]
B=f_{xy}(0,\frac{1}{\sqrt{2}})=0\\[3mm]
C=f_{yy}(0,\frac{1}{\sqrt{2}})=0\\[3mm]
D=AC-B^2=0\\[3mm]$
Since $D=0$, No conclusion can be done\\[3mm]
For $(0,-\frac{1}{\sqrt{2}}),\\[3mm]
A=f_{xx}(0,-\frac{1}{\sqrt{2}})=0\\[3mm]
B=f_{xy}(0,-\frac{1}{\sqrt{2}})=0\\[3mm]
C=f_{yy}(0,-\frac{1}{\sqrt{2}})=0\\[3mm]
D=AC-B^2=0\\[3mm]$
Since $D=0$, No conclusion can be done\\[3mm]
For $(\frac{1}{\sqrt{2}},0),\\[3mm]
A=f_{xx}(\frac{1}{\sqrt{2}},0)=0\\[3mm]
```

$(-\frac{1}{\sqrt{2}},0),(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}),(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}),(-\frac{1}{\sqrt{2}},+\frac{1}{\sqrt{2}}),(-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})$

Now,

$f_{xx}=4y\{e^{-x^2-y^2}(-4x)+(1-2x^2)e^{-x^2-y^2}(-2x)\}$

$\Rightarrow f_{xx}=4y\{2xe^{-x^2-y^2}(2x^2-3)\}$

$\Rightarrow f_{xx}=8xy(2x^2-3)e^{-x^2-y^2}$

$f_{yy}=4x\{e^{-x^2-y^2}(-4y)+(1-2y^2)e^{-x^2-y^2}(-2y)\}$

$\Rightarrow f_{xx}=4x\{2xe^{-x^2-y^2}(2y^2-3)\}$

$\Rightarrow f_{xx}=8xy(2y^2-3)e^{-x^2-y^2}$

$f_{xy}=4\{e^{-x^2-y^2}(1-2x^2)+ye^{-x^2-y^2}(-2y)(1-2x^2)\}$

$\Rightarrow f_{xy}=4e^{-x^2-y^2}(1-2x^2)(1-2y^2)$

For $(0,0)$,

$A=f_{xx}(0,0)=0$

$B=f_{xy}(0,0)=4$

$C=f_{yy}(0,0)=0$

$D=AC-B^2=0-4^2=-16$

Since $D<0$, f has a saddle point at (0,0)

For $(0,\frac{1}{\sqrt{2}})$,

$A=f_{xx}(0,\frac{1}{\sqrt{2}})=0$

$B=f_{xy}(0,\frac{1}{\sqrt{2}})=0$

$C=f_{yy}(0,\frac{1}{\sqrt{2}})=0$

$D=AC-B^2=0$

```
For $(0,\frac{1}{\sqrt{2}}),\\[3mm]
A=f_{xx}(0,\frac{1}{\sqrt{2}})=0\\[3mm]
B=f_{xy}(0,\frac{1}{\sqrt{2}})=0\\[3mm]
C=f_{yy}(0,\frac{1}{\sqrt{2}})=0\\[3mm]
D=AC-B^2=0\\[3mm]$
Since $D=0$, No conclusion can be done\\[3mm]
For $(0,-\frac{1}{\sqrt{2}}),\\[3mm]
A=f_{xx}(0,-\frac{1}{\sqrt{2}})=0\\[3mm]
B=f_{xy}(0,-\frac{1}{\sqrt{2}})=0\\[3mm]
C=f_{yy}(0,-\frac{1}{\sqrt{2}})=0\\[3mm]
D=AC-B^2=0\\[3mm]$
Since $D=0$, No conclusion can be done\\[3mm]
For $(\frac{1}{\sqrt{2}},0),\\[3mm]
A=f_{xx}(\frac{1}{\sqrt{2}},0)=0\\[3mm]
B=f_{xy}(\frac{1}{\sqrt{2}},0)=0\\[3mm]
C=f_{yy}(\frac{1}{\sqrt{2}},0)=0\\[3mm]
D=AC-B^2=0\\[3mm]$
Since $D=0$, No conclusion can be done\\[3mm]
For $(-\frac{1}{\sqrt{2}},0),\\[3mm]
A=f_{xx}(-\frac{1}{\sqrt{2}},0)=0\\[3mm]
B=f_{xy}(-\frac{1}{\sqrt{2}},0)=0\\[3mm]
C=f_{yy}(-\frac{1}{\sqrt{2}},0)=0\\[3mm]
D=AC-B^2=0\\[3mm]$
Since $D=0$, No conclusion can be done\\[3mm]
For $(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}),\\[3mm]
A=f_{xx}(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})=-8e^{-1}=-2.943\\[3mm]
B=f_{xy}(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})=0\\[3mm]
C=f_{yy}(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})=-8e^{-1}=-2.943\\[3mm]
D=AC-B^2=8.66\\[3mm]$
Since $D>0$ and $A<0$, f has a relative maximum point at
$(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})$\\[3mm]
For $(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}),\\[3mm]
A=f_{xx}(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})=8e^{-1}=2.943\\[3mm]
B=f_{xy}(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})=0\\[3mm]
C=f_{yy}(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})=8e^{-1}=2.943\\[3mm]
D=AC-B^2=8.66\\[3mm]$
```

Since $D=0$, No conclusion can be done

For $(0,-\frac{1}{\sqrt{2}})$,

$A=f_{xx}(0,-\frac{1}{\sqrt{2}})=0$

$B=f_{xy}(0,-\frac{1}{\sqrt{2}})=0$

$C=f_{yy}(0,-\frac{1}{\sqrt{2}})=0$

$D=AC-B^2=0$

Since $D=0$, No conclusion can be done

For $(\frac{1}{\sqrt{2}},0)$,

$A=f_{xx}(\frac{1}{\sqrt{2}},0)=0$

$B=f_{xy}(\frac{1}{\sqrt{2}},0)=0$

$C=f_{yy}(\frac{1}{\sqrt{2}},0)=0$

$D=AC-B^2=0$

Since $D=0$, No conclusion can be done

For $(-\frac{1}{\sqrt{2}},0)$,

$A=f_{xx}(-\frac{1}{\sqrt{2}},0)=0$

$B=f_{xy}(-\frac{1}{\sqrt{2}},0)=0$

$C=f_{yy}(-\frac{1}{\sqrt{2}},0)=0$

$D=AC-B^2=0$

Since $D=0$, No conclusion can be done

For $(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})$,

```
A=f_{xx}(-\frac{1}{\sqrt{2}},0)=0\\[3mm]
B=f_{xy}(-\frac{1}{\sqrt{2}},0)=0\\[3mm]
C=f_{yy}(-\frac{1}{\sqrt{2}},0)=0\\[3mm]
D=AC-B^2=0\\[3mm]$
Since $D=0$, No conclusion can be done\\[3mm]
For $(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}),\\[3mm]
A=f_{xx}(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})=-8e^{-1}=-2.943\\[3mm]
B=f_{xy}(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})=0\\[3mm]
C=f_{yy}(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})=-8e^{-1}=-2.943\\[3mm]
D=AC-B^2=8.66\\[3mm]$
Since $D>0$ and $A<0$, f has a relative maximum point at
$(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})$\\[3mm]
For $(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}),\\[3mm]
A=f_{xx}(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})=8e^{-1}=2.943\\[3mm]
B=f_{xy}(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})=0\\[3mm]
C=f_{yy}(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})=8e^{-1}=2.943\\[3mm]
D=AC-B^2=8.66\\[3mm]$
Since $D>0$ and $A>0$, f has a relative minimum point at
$(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})$\\[3mm]
For $(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}),\\[3mm]
A=f_{xx}(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})=8e^{-1}=2.943\\[3mm]
B=f_{xy}(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})=0\\[3mm]
C=f_{yy}(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})=8e^{-1}=2.943\\[3mm]
D=AC-B^2=8.66\\[3mm]$
Since $D>0$ and $A>0$, f has a relative minimum point at
$(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})$\\[3mm]
For $(-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}),\\[3mm]
A=f_{xx}(-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})=-8e^{-1}=-2.943\\[3mm]
B=f_{xy}(-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})=0\\[3mm]
C=f_{yy}(-\frac{1}{-\sqrt{2}},-\frac{1}{\sqrt{2}})=-8e^{-1}=-2.943\\[3mm]
D=AC-B^2=8.66\\[3mm]$
Since $D>0$ and $A<0$, f has a relative maximum point at
$(-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})$\\[3mm]
$\therefore$ Saddle point:(0,0)\\[3mm]
Maxima:$(-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})$,$(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})$\\[3mm]
Minima:$(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})$,$(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})$ \\[3mm]}
```

$A = f_{xx}(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}) = -8e^{-1} = -2.943$

$B = f_{xy}(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}) = 0$

$C = f_{yy}(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}) = -8e^{-1} = -2.943$

$D = AC - B^2 = 8.66$

Since $D > 0$ and $A < 0$, f has a relative maximum point at $(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})$

For $(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})$,

$A = f_{xx}(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}) = 8e^{-1} = 2.943$

$B = f_{xy}(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}) = 0$

$C = f_{yy}(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}) = 8e^{-1} = 2.943$

$D = AC - B^2 = 8.66$

Since $D > 0$ and $A > 0$, f has a relative minimum point at $(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})$

For $(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})$,

$A = f_{xx}(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}) = 8e^{-1} = 2.943$

$B = f_{xy}(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}) = 0$

$C = f_{yy}(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}) = 8e^{-1} = 2.943$

$D = AC - B^2 = 8.66$

Since $D > 0$ and $A > 0$, f has a relative minimum point at $(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})$

For $(-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})$,

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```
A=f_{xx}(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})=8e^{-1}=2.943\\[3mm]
B=f_{xy}(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})=0\\[3mm]
C=f_{yy}(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})=8e^{-1}=2.943\\[3mm]
D=AC-B^2=8.66\\[3mm]$
Since $D>0$ and $A>0$, f has a relative minimum point at
$(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})$\\[3mm]
For $(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}),\\[3mm]
A=f_{xx}(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})=8e^{-1}=2.943\\[3mm]
B=f_{xy}(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})=0\\[3mm]
C=f_{yy}(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})=8e^{-1}=2.943\\[3mm]
D=AC-B^2=8.66\\[3mm]$
Since $D>0$ and $A>0$, f has a relative minimum point at
$(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})$\\[3mm]
For $(-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}),\\[3mm]
A=f_{xx}(-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})=-8e^{-1}=-2.943\\[3mm]
B=f_{xy}(-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})=0\\[3mm]
C=f_{yy}(-\frac{1}{-\sqrt{2}},-\frac{1}{\sqrt{2}})=-8e^{-1}=-2.943\\[3mm]
D=AC-B^2=8.66\\[3mm]$
Since $D>0$ and $A<0$, f has a relative maximum point at
$(-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})$\\[3mm]
$\therefore$ Saddle point:(0,0)\\[3mm]
Maxima:$(-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})$,$(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})$\\[3mm]
Minima:$(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})$,$(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})$ \\[3mm]}
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    \begin{flushright}
    \textsc{Assignment 4}\\
    \textsc{Problem 3}\\
    [1 cm]
    \end{flushright}
\begin{center}
  \textbf{\Large \underline {Ans to the question no 03}}\\
  [0.5 cm]
\end{center}
\Large {Given function, \\[3mm]
```

$A = f_{xx}(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}) = -8e^{-1} = -2.943$

$B = f_{xy}(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}) = 0$

$C = f_{yy}(-\frac{1}{-\sqrt{2}}, -\frac{1}{\sqrt{2}}) = -8e^{-1} = -2.943$

$D = AC - B^2 = 8.66$

Since $D > 0$ and $A < 0$, f has a relative maximum point at $(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$

$\therefore$ Saddle point:(0,0)

Maxima:$(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$,$(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

Minima:$(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$,$(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

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Minima:$(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})$,$(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})$ \\[3mm]}
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    \begin{flushright}
    \textsc{Assignment 4}\\
    \textsc{Problem 3}\\
    [1 cm]
    \end{flushright}
\begin{center}
  \textbf{\Large \underline {Ans to the question no 03}}\\
  [0.5 cm]
\end{center}
\Large {Given function, \\[3mm]
$ x-2y + z = 16
\Rightarrow z=16-x+2y$\\[3mm]
At origin (0,0),\\[3mm]
$D=(Distance)^2=(x-0)^2+(y-0)^2+(z-0)^2\\[3mm]
=x^2+y^2+z^2\\[3mm]
=x^2+y^2+(16-x+2y)^2$\\[3mm]
Here,\\[3mm]
$D_x=2x+2(16-x+2y)(-1)\\[3mm]
=2x-32+2x-4y\\[3mm]
=4x-4y-32\\[3mm]$
When $D_x=0,\\[3mm]
4x-4y-32=0\\[3mm]
\Rightarrow x-y-8=0\\[3mm]
\Rightarrow y=x-8\\[3mm]$
Again,
$D_y=2y+2(16-x+2y)(2)\\[3mm]
=2y+64-4x+8y\\[3mm]
=10y-4x+64\\[3mm]$
When $D_y=0\\[3mm]
10y-4x+64=0\\[3mm]
\Rightarrow 5(x-8)-4x+64=0\\[3mm]
\Rightarrow 3x-8=0 \\[3mm]
\Rightarrow x=\frac{8}{3}\\[3mm]$
```


<u>Ans to the question no 03</u>

Given function,

$x - 2y + z = 16 \Rightarrow z = 16 - x + 2y$

At origin (0,0),

$D = (Distance)^2 = (x-0)^2 + (y-0)^2 + (z-0)^2$

$= x^2 + y^2 + z^2$

$= x^2 + y^2 + (16 - x + 2y)^2$

Here,

$D_x = 2x + 2(16 - x + 2y)(-1)$

$= 2x - 32 + 2x - 4y$

$= 4x - 4y - 32$

When $D_x = 0$,

$4x - 4y - 32 = 0$

$\Rightarrow x - y - 8 = 0$

$\Rightarrow y = x - 8$

Again, $D_y = 2y + 2(16 - x + 2y)(2)$

$= 2y + 64 - 4x + 8y$

$= 10y - 4x + 64$

When $D_y = 0$

$10y - 4x + 64 = 0$

$\Rightarrow 5(x-8) - 4x + 64 = 0$

$\Rightarrow 3x - 8 = 0$

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```
$D_y=2y+2(16-x+2y)(2)\\[3mm]
=2y+64-4x+8y\\[3mm]
=10y-4x+64\\[3mm]$
when $D_y=0\\[3mm]
10y-4x+64=0\\[3mm]
\Rightarrow 5(x-8)-4x+64=0\\[3mm]
\Rightarrow 3x-8=0 \\[3mm]
\Rightarrow x=\frac{8}{3}\\[3mm]$
Substituting the value of x in the equation $ y=x-8,\\[3mm]
y=\frac{-16}{3}\\[3mm]
\therefore (x,y)=(\frac{8}{3},\frac{-16}{3})\\[3mm]$
Now,
$A=D_{xx}=4\\[3mm]
C=D_{yy}=10\\[3mm]
B=D_{xy}=-4\\[3mm]
D=AC-B^2=24\\[3mm]$
Since $D>0$ and $A>0$, D has a minimum at point $(\frac{8}{3},\frac{-16}{3})$\\[3mm]
Now, \\[3mm]
$D(\frac{8}{3},\frac{-16}{3})=(\frac{8}{3})^2+(\frac{-16}{3})^2+(16-\frac{8}{3}-\frac{2\cdot
16}{3})^2\\[3mm]
=\frac{64}{9}+\frac{256}{9}+(\frac{40}{3}-\frac{32}{3})^2\\[3mm]
=\frac{64}{9}+\frac{256}{9}+(\frac{8}{3})^2\\[3mm]
=\frac{128}{9}+\frac{256}{9}\\[3mm]
=\frac{128}{3}\\[3mm]
\therefore Distance= \sqrt{\frac{128}{3}}=\frac{8\sqrt{6}}{3}\\[3mm]
Since, x=\frac{8}{3}, y=\frac{-16}{3},\\[3mm]
z=16-\frac{8}{3}+\frac{2\cdot-16}{3}=\frac{8}{3}\\[3mm]$
the point in the given plane that is closest to the origin
is$(\frac{8}{3},\frac{-16}{3},\frac{8}{3})\\[3mm]$}
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$\Rightarrow x=\frac{8}{3}$

Substituting the value of x in the equation $y=x-8$,

$y=\frac{-16}{3}$

$\therefore (x,y)=(\frac{8}{3},\frac{-16}{3})$

Now, $A=D_{xx}=4$

$C=D_{yy}=10$

$B=D_{xy}=-4$

$D=AC-B^2=24$

Since $D>0$ and $A>0$, D has a minimum at point $(\frac{8}{3},\frac{-16}{3})$

Now,

$D(\frac{8}{3},\frac{-16}{3})=(\frac{8}{3})^2+(\frac{-16}{3})^2+(16-\frac{8}{3}-\frac{2\cdot 16}{3})^2$

$=\frac{64}{9}+\frac{256}{9}+(\frac{40}{3}-\frac{32}{3})^2$

$=\frac{64}{9}+\frac{256}{9}+(\frac{8}{3})^2$

$=\frac{128}{9}+\frac{256}{9}$

$=\frac{128}{3}$

$\therefore Distance=\sqrt{\frac{128}{3}}=\frac{8\sqrt{6}}{3}$

$Since, x=\frac{8}{3}, y=\frac{-16}{3},$

$z=16-\frac{8}{3}+\frac{2\cdot-16}{3}=\frac{8}{3}$

the point in the given plane that is closest to the origin is$(\frac{8}{3},\frac{-16}{3},\frac{8}{3})$

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\begin{center}
  \textbf{\Large \underline {Ans to the question no 04}}\\
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\Large {Given function, \\[3mm]
$ \vec{F}=-x^{2}(y-z) \hat{i}-\left(x^{2}+y^{4}\right) \hat{j}+\left(\frac{4 z^{2}}{y^{2}}\right)\hat{k}\\[3mm]
\Rightarrow \vec{F}=(-x^{2}y+-x^{2}z) \hat{i}+\left(-x^{2}-y^{4}\right) \hat{j}+\left(4 z^{2}y^{-2}\right)\hat{k}\\[3mm]$
Curl $\vec{F}=\left|\begin{array}{ccc}\hat{i} & \hat{j} & \hat{k}\\
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
-x^{2}y+x^{2}z & -x^{2}-y^{4} & 4 z^{2} y^{-2}
\end{array}\right|\\[3mm]
=\hat{i}\{\frac{\partial}{\partial y}(4 z^{2} y^{-2})-\frac{\partial}{\partial z}(-x^{2}-y^{4})\}-\hat{j}\{\frac{\partial}{\partial x}(4 z^{2} y^{-2})-{\frac{\partial}{\partial z}}(-x^{2}y+x^{2}z)+\hat{k}\{\frac{\partial}{\partial x}(-x^{2}-y^{4})-\frac{\partial}{\partial y}(-x^{2}y+x^{2}z)\}\\[3mm]
=(-8z^2y^{-3})\hat{i}-(0-0+x^2)\hat{j}+(-2x+x^2)\hat{k}\\[3mm]
=-8z^2y^{-3}\hat{i}-x^2\hat{j}-(2x+x^2)\hat{k}$\\[3mm]
Div $\vec{F}=\Delta F\\[3mm]
=\frac{\partial F_x}{\partial x}+\frac{\partial F_y}{\partial y}+\frac{\partial F_z}{\partial z}\\[3mm]
=\frac{\partial }{\partial x}(-x^{2}y+-x^{2}z)+\frac{\partial}{\partial y}(-x^{2}-y^{4})+\frac{\partial}{\partial z}(4 z^{2}y^{-2})\\[3mm]
=-2x(y-z)-4y^3+\frac{8z}{y^2}\\[3mm]
\therefore Curl \vec{F}=-8z^2y^{-3}\hat{i}-x^2\hat{j}-(2x+x^2)\hat{k}\\[3mm]
\therefore \Delta F=-2x(y-z)-4y^3+\frac{8z}{y^2}$}
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Ans to the question no 04

Given function,

$\vec{F}=-x^{2}(y-z) \hat{i}-\left(x^{2}+y^{4}\right) \hat{j}+\left(\frac{4 z^{2}}{y^{2}}\right)\hat{k}$

$\Rightarrow \vec{F}=(-x^{2}y+-x^{2}z) \hat{i}+\left(-x^{2}-y^{4}\right) \hat{j}+\left(4 z^{2}y^{-2}\right)\hat{k}$

Curl $\vec{F}=\left|\begin{array}{ccc}\hat{i} & \hat{j} & \hat{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -x^{2}y+x^{2}z & -x^{2}-y^{4} & 4 z^{2} y^{-2} \end{array}\right|$

$=\hat{i}\{\frac{\partial}{\partial y}(4 z^{2} y^{-2})-\frac{\partial}{\partial z}(-x^{2}-y^{4})\}-\hat{j}\{\frac{\partial}{\partial x}(4 z^{2} y^{-2})-\frac{\partial}{\partial z}(-x^{2}y+x^{2}z)+\hat{k}\{\frac{\partial}{\partial x}(-x^{2}-y^{4})-\frac{\partial}{\partial y}(-x^{2}y+x^{2}z)\}$

$=(-8z^2y^{-3})\hat{i}-(0-0+x^2)\hat{j}+(-2x+x^2)\hat{k}$

$=-8z^2y^{-3}\hat{i}-x^2\hat{j}-(2x+x^2)\hat{k}$

Div $\vec{F}=\Delta F$

$=\frac{\partial F_x}{\partial x}+\frac{\partial F_y}{\partial y}+\frac{\partial F_z}{\partial z}$

$=\frac{\partial }{\partial x}(-x^{2}y+-x^{2}z)+\frac{\partial}{\partial y}(-x^{2}-y^{4})+\frac{\partial}{\partial z}(4 z^{2}y^{-2})$

$=-2x(y-z)-4y^3+\frac{8z}{y^2}$

$\therefore Curl \vec{F}=-8z^2y^{-3}\hat{i}-x^2\hat{j}-(2x+x^2)\hat{k}$

$\therefore \Delta F=-2x(y-z)-4y^3+\frac{8z}{y^2}$

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=\frac{\partial }{\partial x}(-x^{2}y+-x^{2}z)+\frac{\partial}{\partial y}(-x^{2}-y^{4})+\frac{\partial}{\partial z}(4 z^{2}y^{-2})\\[3mm]
=-2x(y-z)-4y^3+\frac{8z}{y^2}\\[3mm]
\therefore Curl \vec{F}=-8z^2y^{-3}\hat{i}-x^2\hat{j}-(2x+x^2)\hat{k}\\[3mm]
\therefore \Delta F=-2x(y-z)-4y^3+\frac{8z}{y^2}$}
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  \textbf{\Large \underline {Ans to the question no 05}}\\
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\Large {Given function, \\[3mm]
$ \vec{F} = 2yz\hat{i}-x^2y\hat{j} + xz^2\hat{k}\\[3mm]
\phi =2x^2yz^3\\[3mm]
\nabla \phi = (\frac{\partial}{\partial x}\hat{i}+\frac{\partial }{\partial y}\hat{j}+\frac{\partial }{\partial z}\hat{k}) \cdot (2x^2yz^3)\\[3mm]
=\frac{\partial}{\partial x}(2x^2yz^3)\hat{i}+\frac{\partial}{\partial y}(2x^2yz^3)\hat{j}+\frac{\partial}{\partial z}(2x^2yz^3)\hat{k}\\[3mm]
=4xyz^3\hat{i}+2x^2z^3\hat{j}+6x^2yz^2\hat{j}$\\[3mm]
Now,\\[3mm]
$\vec{F}\cdot (\nabla \phi)=(2yz\hat{i}-x^2y\hat{j} + xz^2\hat{k}) \cdot (4xyz^3\hat{i}+2x^2z^3\hat{j}+6x^2yz^2\hat{j})\\[3mm]
=8xy^2z^4-2x^4yz^3+6x^3yz^4\\[5mm]
\therefore \vec{F}\cdot (\nabla \phi)=8xy^2z^4-2x^4yz^3+6x^3yz^4$}
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Ans to the question no 05

Given function,

$\vec{F} = 2yz\hat{i} - x^2y\hat{j} + xz^2\hat{k}$

$\phi = 2x^2yz^3$

$\nabla\phi = (\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}) \cdot (2x^2yz^3)$

$= \frac{\partial}{\partial x}(2x^2yz^3)\hat{i} + \frac{\partial}{\partial y}(2x^2yz^3)\hat{j} + \frac{\partial}{\partial z}(2x^2yz^3)\hat{k}$

$= 4xyz^3\hat{i} + 2x^2z^3\hat{j} + 6x^2yz^2\hat{j}$

Now,

$\vec{F} \cdot (\nabla\phi) = (2yz\hat{i} - x^2y\hat{j} + xz^2\hat{k}) \cdot (4xyz^3\hat{i} + 2x^2z^3\hat{j} + 6x^2yz^2\hat{j})$

$= 8xy^2z^4 - 2x^4yz^3 + 6x^3yz^4$

$\therefore \vec{F} \cdot (\nabla\phi) = 8xy^2z^4 - 2x^4yz^3 + 6x^3yz^4$

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$\vec{F}\cdot (\nabla \phi)=(2yz\hat{i}-x^2y\hat{j} + xz^2\hat{k}) \cdot
(4xyz^3\hat{i}+2x^2z^3\hat{j}+6x^2yz^2\hat{j})\\[3mm]
=8xy^2z^4-2x^4yz^3+6x^3yz^4\\[5mm]
\therefore \vec{F}\cdot (\nabla \phi)=8xy^2z^4-2x^4yz^3+6x^3yz^4$}
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  \textbf{\Large \underline {Ans to the question no 06}}\\
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\Large {Given function, \\[3mm]
$f(x,y,x)=x^2y + y^2z + xz^2\\[3mm]
\nabla \cdot f(x,y,x)=(\frac{\partial f}{\partial x}\hat{i}+\frac{\partial f}{\partial y}\hat{j}+\frac{\partial f}{\partial z}\hat{k})\\[3mm]
=\frac{\partial}{\partial x}(x^2y + y^2z + xz^2)\hat{i}+\frac{\partial f}{\partial y}(x^2y + y^2z + xz^2)\hat{j}+\frac{\partial f}{\partial z}(x^2y + y^2z + xz^2)\hat{k})\\[2mm]
=(2xy+z^2)\hat{i}+(x^2+2yz)\hat{j}+(y^2+2zx)\hat{k}\\[3mm]$
At the point (2,4,5),\\[3mm]
$\nabla \cdot f(2,4,5)=(2\cdot 2\cdot 4+5^2)\hat{i}+(2^2+2\cdot 4\cdot 5)\hat{j}+(4^2+2\cdot 5\cdot 2)\hat{k}\\[3mm]
=41\hat{i}+44\hat{j}+36\hat{k}\\[3mm]$
unit vector along (1,-1,3),\\[3mm]
$\hat{u}=\frac{\hat{i}-\hat{j}+3\hat{k}}{\sqrt{1^2+1^2+3^2}}\\[3mm]
=\frac{\hat{i}-\hat{j}+3\hat{k}}{\sqrt{11}}\\[3mm]$
Directional derivative of the function f(x,y,z) at the point (2,4,5) in the direction of the point (1,-1,3),\\[3mm]
$ (\nabla \cdot f(2,4,5))\cdot \hat{u}= \frac{41-44+108}{\sqrt{11}}\\[3mm]
=\frac{105}{\sqrt{11}}=31.66$}
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Ans to the question no 06

Given function,

$f(x, y, x) = x^2y + y^2z + xz^2$

$\nabla \cdot f(x, y, x) = (\frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k})$

$= \frac{\partial}{\partial x}(x^2y+y^2z+xz^2)\hat{i} + \frac{\partial f}{\partial y}(x^2y+y^2z+xz^2)\hat{j} + \frac{\partial f}{\partial z}(x^2y + y^2z + xz^2)\hat{k})$

$= (2xy + z^2)\hat{i} + (x^2 + 2yz)\hat{j} + (y^2 + 2zx)\hat{k}$

At the point (2,4,5),

$\nabla \cdot f(2, 4, 5) = (2\cdot 2\cdot 4+5^2)\hat{i}+(2^2+2\cdot 4\cdot 5)\hat{j}+(4^2+2\cdot 5\cdot 2)\hat{k}$

$= 41\hat{i} + 44\hat{j} + 36\hat{k}$

unit vector along (1,-1,3),

$\hat{u} = \frac{\hat{i}-\hat{j}+3\hat{k}}{\sqrt{1^2+1^2+3^2}}$

$= \frac{\hat{i}-\hat{j}+3\hat{k}}{\sqrt{11}}$

Directional derivative of the function f(x,y,z) at the point (2,4,5) in the direction of the point (1,-1,3),

$(\nabla \cdot f(2, 4, 5)) \cdot \hat{u} = \frac{41-44+108}{\sqrt{11}}$

$= \frac{105}{\sqrt{11}} = 31.66$