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MAT 110

ASSIGNMENT 05

SET 6

Ans to the question no 01

Given,

$-15 + 12x - 6y + y^2 = 0$

$\Rightarrow y^2 - 6y + 3^2 = 15 - 12x + 9$

$\Rightarrow (y-3)^2 = -12x + 24$

$\Rightarrow (y-3)^2 = 4(-3)(x-2)$

$\therefore$ Equation into the standard form of the equation of $parabola : (y-3)^2 = 4(-3)(x-2)$

Comparing this equation with $Y^2 = 4pX$ we get,

$Y = y - 3,$

$4p = 12 \Rightarrow p = -3,$

$X = 2 - x$

Vertex:

$Y = 0$

$\Rightarrow y - 3 = 0$

$\Rightarrow y = 3$

Again,

$X = 0$

$\Rightarrow x - 2 = 0$

$\Rightarrow x = 2$

$\therefore$ Vertex$(x, y) = (2, 3)$

Focus:

The focus of the parabola's following $Y^2 = 4pX$ will be on x axis

$\therefore Y = 0$

$\Rightarrow y - 3 = 0$

$\Rightarrow y = 3$

$And,\ X = p \Rightarrow x - 2 = -3 \Rightarrow x = -1$

$\therefore$ Focus$= (-1, 3)$

Equation of directrix:

$X + p = 0$

$\Rightarrow x - 2 - 3 = 0$

$\Rightarrow x = 5$

Equation of directrix is $x = 5$

Ans to the question no 02

Given,

$256 + 9x^2 - 160y + 16y^2 = 0$

$\Rightarrow 9x^2 + 16y^2 - 160y + 256 = 0$

$\Rightarrow 9x^2 + 16(y^2 - 10y + 25) + 256 - 25 \cdot 16 = 0$

$\Rightarrow 9x^2 + 16(y - 5)^2 + 144 = 0$

$\Rightarrow \frac{9x^2}{144} + \frac{16(y-5)^2}{144} = 1$

$\Rightarrow \frac{x^2}{4^2} + \frac{(y-5)^2}{3^2} = 1$

$\therefore$ Equation into the standard form of the equation of $ellipse : \frac{x^2}{4^2} + \frac{(y-5)^2}{3^2} = 1$

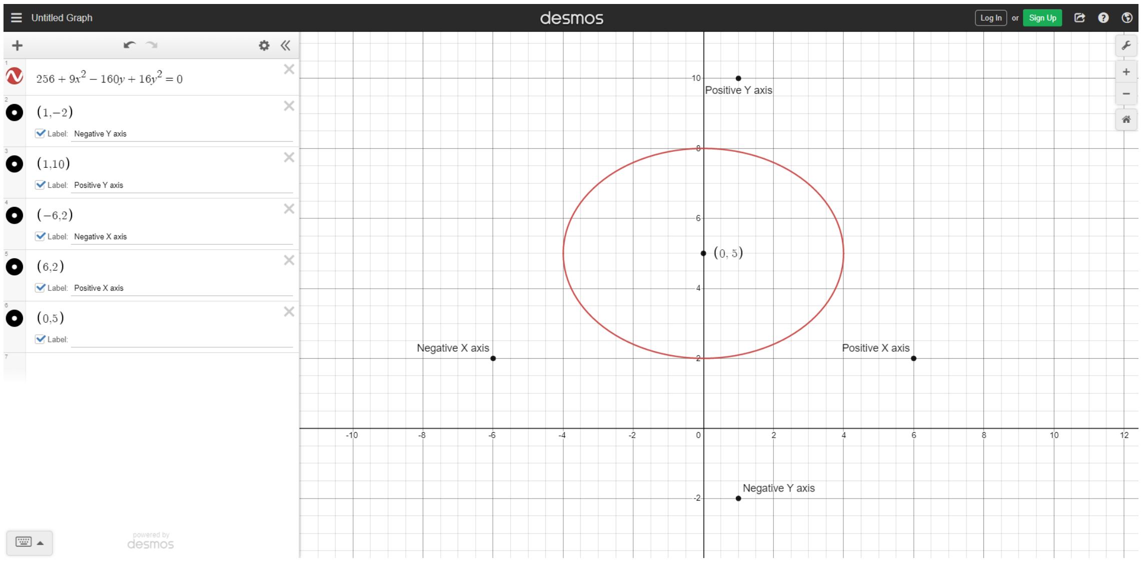
Comparing this equation with the standard form of equation of ellipse $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

vertex:

$(h, k) = (0, 5)$

$a = 4, b = 3$

sketch of the ellipse:



Ans to the question no 03

Given,

$-24 - 24x + 12x^2 - 3y^2 = 0$

$\Rightarrow 12(x^2 - 2x + 1) - 3y^2 - 24 - 12 = 0$

$\Rightarrow 12(x-1)^2 - 3y^2 = 36$

$\Rightarrow \frac{(x-1)^2}{(\sqrt{3})^2} - \frac{y^2}{(2\sqrt{3})^2} = 1$

$\therefore$ Equation into the standard form of the equation of hyperbola:$\frac{(x-1)^2}{(\sqrt{3})^2} - \frac{y^2}{(2\sqrt{3})^2} = 1$

Comparing this equation with the standard form of equation of hyperbola $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$,

$a = \sqrt{3}, b = 2\sqrt{3}$

Center:$(h, k) = (1, 0)$

vertices:

The vertices of this hyperbola are on x axis

$\therefore y = k = 0$

And, $x = h \pm a = 1 \pm \sqrt{3}$

$\therefore$ Vertices$= (1 + \sqrt{3}, 0), (1 - \sqrt{3}, 0)$

Eccentricity:

$e = \sqrt{1 + \frac{b^2}{a^2}}$

$= \sqrt{1 + \frac{2\sqrt{3})^2}{(\sqrt{3})^2}}$

$= \sqrt{5}$

Foci:

$(h \pm ae, k) = (1 \pm \sqrt{3} \cdot \sqrt{5}, 0) = (1 \pm \sqrt{15}, 0)$

$\therefore$ Foci=$(1 + \sqrt{15}, 0), (1 - \sqrt{15}, 0)$

Equation of directrices:

$x - h = \pm\frac{a}{e}$

$\Rightarrow x - 1 = \pm\frac{\sqrt{3}}{\sqrt{5}}$

$\Rightarrow x = 1 \pm \frac{\sqrt{3}}{\sqrt{5}}$

$\Rightarrow x = \frac{5 \pm \sqrt{15}}{5}$

$\therefore$ Equation of directrices: $x = \frac{5+\sqrt{15}}{5}, x = \frac{5+\sqrt{15}}{5}$

Ans to the question no 04

Given,

$r = \frac{9}{6+2cos\theta}$

$\Rightarrow r = \frac{\frac{9}{6}}{1+\frac{cos\theta}{3}}$

$\Rightarrow r = \frac{\frac{3}{2}}{1+\frac{1}{3}cos\theta}$

Comparing this equation with $r = \frac{ke}{1+ecos\theta}$,

(a)Eccentricity: $e = \frac{1}{3}$

(b) As we know, for ellipse the eccentricity value is $0 < e < 1$,

$Here, 0 < e = \frac{1}{3} < 1$

$\therefore$ The conic is an ellipse

(c)Equation of directrix:

Here,

$ke = \frac{3}{2}$

$\Rightarrow k = \frac{3}{2} \cdot \frac{3}{1} = \frac{9}{2}$

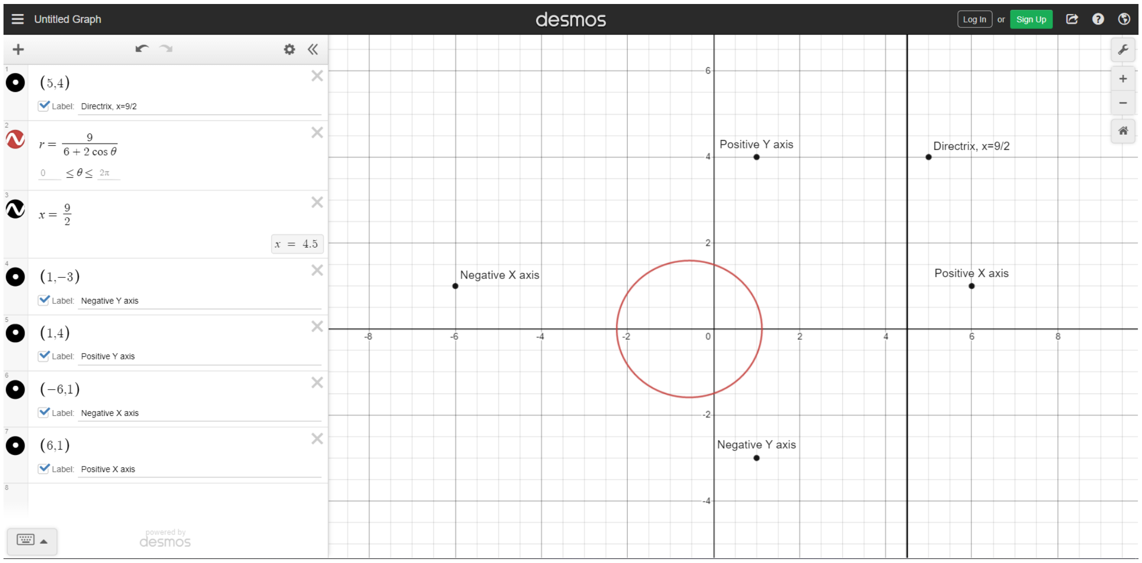
Since we have a positive value the directrix will be,

$x = k$

$\Rightarrow x = \frac{9}{2}$

$\therefore$(c)Equation of directrix:$x = \frac{9}{2}$

(d)Sketch of the conic:



Ans to the question no 05

Given,

The cylindrical coordinates$(r, \theta, z) = (\pi, \frac{\pi}{2}, -2)$

We know in terms of rectangular coordinates,

$x = rcos\theta$

$y = rsin\theta$

$z = z$

Here,

$x = \pi cos\frac{\pi}{2}$

$\Rightarrow x = \pi \cdot 0 = 0$

Again,

$y = \pi sin\frac{\pi}{2}$

$\Rightarrow y = \pi \cdot 1$

$\Rightarrow y = \pi = 3.1416$

And, $z = -2$

$\therefore$ Rectangular coordinates $(x, y, z) = (0, 3.1416, -2)$

Ans to the question no 06

Given,

The spherical coordinates$(2, \theta, \phi) = (\frac{5\pi}{6}, \frac{\pi}{2}, \pi)$

We know in terms of rectangular coordinates,

$x = e \cdot sin\phi cos\theta$

$y = e \cdot sin\phi sin\theta$

$z = e \cdot cos\phi$

Here,

$x = \frac{5\pi}{6} sin\pi cos\frac{\pi}{2}$

$\Rightarrow x = \frac{5\pi}{6} \cdot 0 \cdot 0 = 0$

Again,

$y = \frac{5\pi}{6} sin\pi sin\frac{\pi}{2}$

$\Rightarrow y = \frac{5\pi}{\cdot} 0 \cdot 1 = 0$

$And, z = \frac{5\pi}{6} cos\pi$

$\Rightarrow z = -\frac{5\pi}{6}$

$\Rightarrow z = -\frac{5 \cdot 3.1416}{6}$

$\Rightarrow z = -2.618$

$\therefore$ Rectangular coordinates $(x, y, z) = (0, 0, -2.618)$