Shihab Muhtasim

STUDENT ID: 21301610

MAT 110
ASSIGNMENT 05

SET 6

Given,

$$-15 + 12x - 6y + y^{2} = 0$$

$$\Rightarrow y^{2} - 6y + 3^{2} = 15 - 12x + 9$$

$$\Rightarrow (y - 3)^{2} = -12x + 24$$

$$\Rightarrow (y - 3)^{2} = 4(-3)(x - 2)$$

... Equation into the standard form of the equation of $parabola: (y-3)^2 = 4(-3)(x-2)$

Comparing this equation with $Y^2 = 4pX$ we get,

$$Y = y - 3,$$

$$4p = 12 \Rightarrow p = -3$$

$$X = 2 - x$$

Vertex:

$$Y = 0$$

$$\Rightarrow y - 3 = 0$$

$$\Rightarrow y = 3$$

Again,

$$X = 0$$

$$\Rightarrow x - 2 = 0$$

$$\Rightarrow x = 2$$

$$\therefore Vertex(x,y) = (2,3)$$

Focus:

The focus of the parabola's following $Y^2=4pX$ will be on x axis

$$\therefore Y = 0$$

$$\Rightarrow y - 3 = 0$$

$$\Rightarrow y = 3$$

$$And,\,X=p\Rightarrow x-2=-3\Rightarrow x=-1$$

$$\therefore$$
 Focus= $(-1,3)$

Equation of directrix:

$$X + p = 0$$

$$\Rightarrow x - 2 - 3 = 0$$

$$\Rightarrow x = 5$$

Equation of directrix is x = 5

Given,

$$256 + 9x^{2} - 160y + 16y^{2} = 0$$

$$\Rightarrow 9x^{2} + 16y^{2} - 160y + 256 = 0$$

$$\Rightarrow 9x^{2} + 16(y^{2} - 10y + 25) + 256 - 25 \cdot 16 = 0$$

$$\Rightarrow 9x^{2} + 16(y - 5)^{2} + 144 = 0$$

$$\Rightarrow \frac{9x^{2}}{144} + \frac{16(y - 5)^{2}}{144} = 1$$

$$\Rightarrow \frac{x^{2}}{4^{2}} + \frac{(y - 5)^{2}}{3^{2}} = 1$$

... Equation into the standard form of the equation of $ellipse: \frac{x^2}{4^2} + \frac{(y-5)^2}{3^2} = 1$

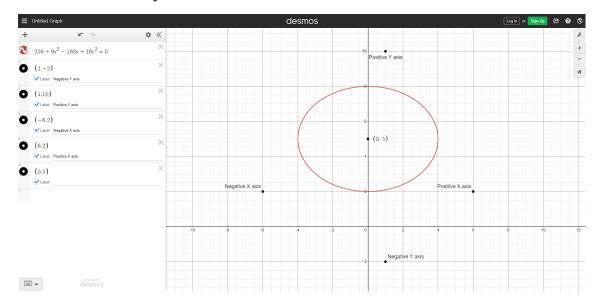
Comparing this equation with the standard form of equation of ellipse $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$

vertex:

$$(h,k) = (0,5)$$

$$a = 4, b = 3$$

sketch of the ellipse:



Given,

$$-24 - 24x + 12x^{2} - 3y^{2} = 0$$

$$\Rightarrow 12(x^{2} - 2x + 1) - 3y^{2} - 24 - 12 = 0$$

$$\Rightarrow 12(x - 1)^{2} - 3y^{2} = 36$$

$$\Rightarrow \frac{(x - 1)^{2}}{(\sqrt{3})^{2}} - \frac{y^{2}}{(2\sqrt{3})^{2}} = 1$$

... Equation into the standard form of the equation of hyperbola: $\frac{(x-1)^2}{(\sqrt{3})^2} - \frac{y^2}{(2\sqrt{3})^2} = 1$

Comparing this equation with the standard form of equation of hyperbola $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$,

$$a = \sqrt{3}, b = 2\sqrt{3}$$

Center:
$$(h, k) = (1, 0)$$

vertices:

The vertices of this hyperbola are on x axis

$$\therefore y = k = 0$$

And,
$$x = h \pm a = 1 \pm \sqrt{3}$$

:. Vertices =
$$(1 + \sqrt{3}, 0), (1 - \sqrt{3}, 0)$$

Eccentricity:

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

$$= \sqrt{1 + \frac{2\sqrt{3})^2}{(\sqrt{3})^2}}$$

$$=\sqrt{5}$$

Foci:

$$(h \pm ae, k) = (1 \pm \sqrt{3} \cdot \sqrt{5}, 0) = (1 \pm \sqrt{15}, 0)$$

:. Foci=
$$(1 + \sqrt{15}, 0), (1 - \sqrt{15}, 0)$$

Equation of directrices:

$$x - h = \pm \frac{a}{e}$$

$$\Rightarrow x - 1 = \pm \frac{\sqrt{3}}{\sqrt{5}}$$

$$\Rightarrow x = 1 \pm \frac{\sqrt{3}}{\sqrt{5}}$$

$$\Rightarrow x = \frac{5 \pm \sqrt{15}}{5}$$

.: Equation of directrices:
$$x = \frac{5+\sqrt{15}}{5}, x = \frac{5+\sqrt{15}}{5}$$

Given,

$$r = \frac{9}{6 + 2cos\theta}$$

$$\Rightarrow r = \frac{\frac{9}{6}}{1 + \frac{\cos\theta}{3}}$$

$$\Rightarrow r = \frac{\frac{3}{2}}{1 + \frac{1}{3}cos\theta}$$

Comparing this equation with $r = \frac{ke}{1 + e\cos\theta}$,

- (a) Eccentricity: $e = \frac{1}{3}$
- (b) As we know, for ellipse the eccentricity value is 0 < e < 1,

$$Here, 0 < e = \frac{1}{3} < 1$$

- ... The conic is an ellipse
- (c)Equation of directrix:

Here,

$$ke = \frac{3}{2}$$

$$\Rightarrow k = \frac{3}{2} \cdot \frac{3}{1} = \frac{9}{2}$$

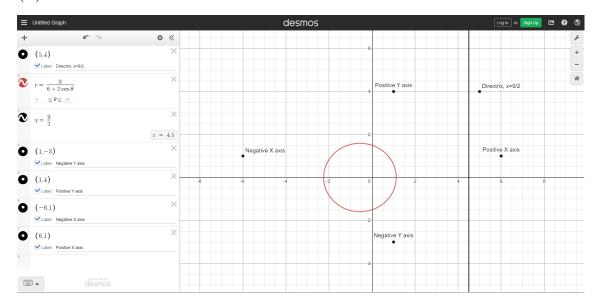
Since we have a positive value the directrix will be,

$$x = k$$

$$\Rightarrow x = \frac{9}{2}$$

∴(c)Equation of directrix: $x = \frac{9}{2}$

(d)Sketch of the conic:



Given,

The cylindrical coordinates $(r, \theta, z) = (\pi, \frac{\pi}{2}, -2)$

We know in terms of rectangular coordinates,

$$x = rcos\theta$$

$$y = rsin\theta$$

$$z = z$$

Here,

$$x = \pi \cos \frac{\pi}{2}$$

$$\Rightarrow x = \pi \cdot 0 = 0$$

Again,

$$y = \pi sin\frac{\pi}{2}$$

$$\Rightarrow y = \pi \cdot 1$$

$$\Rightarrow y = \pi = 3.1416$$

And,
$$z = -2$$

.:. Rectangular coordinates (x, y, z) = (0, 3.1416, -2)

Given,

The spherical coordinates $(2, \theta, \phi) = (\frac{5\pi}{6}, \frac{\pi}{2}, \pi)$

We know in terms of rectangular coordinates,

$$x = e \cdot sin\phi cos\theta$$

$$y = e \cdot \sin\phi\sin\theta$$

$$z = e \cdot cos\phi$$

Here,

$$x = \frac{5\pi}{6} sin\pi cos\frac{\pi}{2}$$

$$\Rightarrow x = \frac{5\pi}{6} \cdot 0 \cdot 0 = 0$$

Again,

$$y = \frac{5\pi}{6} sin\pi sin\frac{\pi}{2}$$

$$\Rightarrow y = \frac{5\pi}{\cdot} 0 \cdot 1 = 0$$

$$And, z = \frac{5\pi}{6}cos\pi$$

$$\Rightarrow z = -\frac{5\pi}{6}$$

$$\Rightarrow z = -\frac{5 \cdot 3.1416}{6}$$

$$\Rightarrow z = -2.618$$

 \therefore Rectangular coordinates (x, y, z) = (0, 0, -2.618)

