
Shihab Muhtasim

STUDENT ID: 21301610

MAT 110

ASSIGNMENT 04

SET 7

Ans to the question no 01

Given function,

$$f(x, y) = y \cdot e^{(xy+3x+2y^2)}$$

Now,

$$f_x = y \cdot e^{(xy+3x+2y^2)} \cdot (y + 3) = (y^2 + 3y) \cdot e^{(xy+3x+2y^2)}$$

$$f_y = e^{(xy+3x+2y^2)} + y \cdot e^{(xy+3x+2y^2)} \cdot (x + 4y)$$

$$\Rightarrow f_y = e^{(xy+3x+2y^2)}(1 + xy + 4y^2)$$

$$f_{xx} = e^{(xy+3x+2y^2)} \cdot (y+3) \cdot y(y+3) = e^{(xy+3x+2y^2)} \cdot y(y+3)^2$$

$$f_{yy} = e^{(xy+3x+2y^2)} \cdot (x + 4y) \cdot (1 + xy + 4y^2) + e^{(xy+3x+2y^2)} \cdot (x + 8y)$$

$$\Rightarrow f_{yy} = e^{(xy+3x+2y^2)} \cdot \{(x + 4y)(1 + xy + 4y^2) + (x + 8y)\}$$

$$f_{xy} = (2y + 3) \cdot e^{(xy+3x+2y^2)} + (y^2 + 3y) \cdot e^{(xy+3x+2y^2)} \cdot (x + 4y)$$

$$\Rightarrow f_{xy} = e^{(xy+3x+2y^2)} \{(2y + 3) + (y^2 + 3y)(x + 4y)\}$$

$$\text{Now, } f(0, 0) = 0$$

$$f_x(0, 0) = 0$$

$$f_y(0, 0) = 1$$

$$f_{xx}(0, 0) = 0$$

$$f_{yy}(0, 0) = 0$$

$$f_{xy}(0, 0) = 3$$

The first degree Maclaurin polynomial approximation,

$$\therefore L(x, y) = f(0, 0) + f_x(0, 0)(x - 0) + f_y(0, 0)(y - 0)$$

$$\Rightarrow L(x, y) = 0 + 0 + 1 \cdot y = y$$

$$\therefore L(x, y) = y$$

The second degree Maclaurin polynomial approximation,

$$Q(x, y) = L(x, y) + \frac{f_{xx}(0,0)}{2!} \cdot (x - 0)^2 + \frac{f_{yy}(0,0)}{2!} \cdot (y - 0)^2 + \frac{f_{xy}(0,0)}{1!} \cdot (x - 0)(y - 0)$$

$$\Rightarrow Q(x, y) = y + 0 + 0 + 3xy = y(3x + 1)$$

$$\therefore Q(x, y) = y(3x + 1)$$

Ans to the question no 02

Given function,

$$f(x, y) = f(x, y) = 4xye^{-x^2-y^2}$$

Now,

$$f_x = 4y \cdot \{e^{-x^2-y^2} + xe^{-x^2-y^2}(-2x)\}$$

$$\Rightarrow f_x = 4y \cdot e^{-x^2-y^2}(1 - 2x^2)$$

$$f_y = 4x \cdot \{e^{-x^2-y^2} + ye^{-x^2-y^2}(-2y)\}$$

$$\Rightarrow f_y = 4x \cdot e^{-x^2-y^2}(1 - 2y^2)$$

For extreme values,

$$f_x = 0$$

$$\Rightarrow 4y \cdot e^{-x^2-y^2}(1 - 2x^2) = 0$$

$e^{-x^2-y^2}$ cannot be zero for any finite values of x or y

$$\therefore y = 0 \text{ or } x = \pm \frac{1}{\sqrt{2}}$$

Similarly,

$$f_y = 0$$

$$\Rightarrow 4x \cdot e^{-x^2-y^2}(1 - 2y^2) = 0$$

$e^{-x^2-y^2}$ cannot be zero for any finite values of x or y

$$\therefore x = 0 \text{ or } y = \pm \frac{1}{\sqrt{2}}$$

The critical points are: $(0, 0), (0, \frac{1}{\sqrt{2}}), (0, -\frac{1}{\sqrt{2}}), (\frac{1}{\sqrt{2}}, 0)$

$$(-\frac{1}{\sqrt{2}}, 0), (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}), (\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}), (-\frac{1}{\sqrt{2}}, +\frac{1}{\sqrt{2}}), (-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$$

Now,

$$f_{xx} = 4y\{e^{-x^2-y^2}(-4x) + (1 - 2x^2)e^{-x^2-y^2}(-2x)\}$$

$$\Rightarrow f_{xx} = 4y\{2xe^{-x^2-y^2}(2x^2 - 3)\}$$

$$\Rightarrow f_{xx} = 8xy(2x^2 - 3)e^{-x^2-y^2}$$

$$f_{yy} = 4x\{e^{-x^2-y^2}(-4y) + (1 - 2y^2)e^{-x^2-y^2}(-2y)\}$$

$$\Rightarrow f_{yy} = 4x\{2ye^{-x^2-y^2}(2y^2 - 3)\}$$

$$\Rightarrow f_{yy} = 8xy(2y^2 - 3)e^{-x^2-y^2}$$

$$f_{xy} = 4\{e^{-x^2-y^2}(1 - 2x^2) + ye^{-x^2-y^2}(-2y)(1 - 2x^2)\}$$

$$\Rightarrow f_{xy} = 4e^{-x^2-y^2}(1 - 2x^2)(1 - 2y^2)$$

For $(0, 0)$,

$$A = f_{xx}(0, 0) = 0$$

$$B = f_{xy}(0, 0) = 4$$

$$C = f_{yy}(0, 0) = 0$$

$$D = AC - B^2 = 0 - 4^2 = -16$$

Since $D < 0$, f has a saddle point at $(0, 0)$

For $(0, \frac{1}{\sqrt{2}})$,

$$A = f_{xx}(0, \frac{1}{\sqrt{2}}) = 0$$

$$B = f_{xy}(0, \frac{1}{\sqrt{2}}) = 0$$

$$C = f_{yy}(0, \frac{1}{\sqrt{2}}) = 0$$

$$D = AC - B^2 = 0$$

Since $D = 0$, No conclusion can be done

For $(0, -\frac{1}{\sqrt{2}})$,

$$A = f_{xx}(0, -\frac{1}{\sqrt{2}}) = 0$$

$$B = f_{xy}(0, -\frac{1}{\sqrt{2}}) = 0$$

$$C = f_{yy}(0, -\frac{1}{\sqrt{2}}) = 0$$

$$D = AC - B^2 = 0$$

Since $D = 0$, No conclusion can be done

For $(\frac{1}{\sqrt{2}}, 0)$,

$$A = f_{xx}(\frac{1}{\sqrt{2}}, 0) = 0$$

$$B = f_{xy}(\frac{1}{\sqrt{2}}, 0) = 0$$

$$C = f_{yy}(\frac{1}{\sqrt{2}}, 0) = 0$$

$$D = AC - B^2 = 0$$

Since $D = 0$, No conclusion can be done

For $(-\frac{1}{\sqrt{2}}, 0)$,

$$A = f_{xx}(-\frac{1}{\sqrt{2}}, 0) = 0$$

$$B = f_{xy}(-\frac{1}{\sqrt{2}}, 0) = 0$$

$$C = f_{yy}(-\frac{1}{\sqrt{2}}, 0) = 0$$

$$D = AC - B^2 = 0$$

Since $D = 0$, No conclusion can be done

For $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$,

$$A = f_{xx}\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = -8e^{-1} = -2.943$$

$$B = f_{xy}\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = 0$$

$$C = f_{yy}\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = -8e^{-1} = -2.943$$

$$D = AC - B^2 = 8.66$$

Since $D > 0$ and $A < 0$, f has a relative maximum point at $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

For $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$,

$$A = f_{xx}\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) = 8e^{-1} = 2.943$$

$$B = f_{xy}\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) = 0$$

$$C = f_{yy}\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) = 8e^{-1} = 2.943$$

$$D = AC - B^2 = 8.66$$

Since $D > 0$ and $A > 0$, f has a relative minimum point at $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

For $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$,

$$A = f_{xx}\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = 8e^{-1} = 2.943$$

$$B = f_{xy}\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = 0$$

$$C = f_{yy}\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = 8e^{-1} = 2.943$$

$$D = AC - B^2 = 8.66$$

Since $D > 0$ and $A > 0$, f has a relative minimum point at $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

For $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$,

$$A = f_{xx}\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) = -8e^{-1} = -2.943$$

$$B = f_{xy}\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) = 0$$

$$C = f_{yy}\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) = -8e^{-1} = -2.943$$

$$D = AC - B^2 = 8.66$$

Since $D > 0$ and $A < 0$, f has a relative maximum point at $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

\therefore Saddle point: $(0,0)$

Maxima: $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right), \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

Minima: $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right), \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

Ans to the question no 03

Given function,

$$x - 2y + z = 16 \Rightarrow z = 16 - x + 2y$$

At origin (0,0),

$$D = (Distance)^2 = (x - 0)^2 + (y - 0)^2 + (z - 0)^2$$

$$= x^2 + y^2 + z^2$$

$$= x^2 + y^2 + (16 - x + 2y)^2$$

Here,

$$D_x = 2x + 2(16 - x + 2y)(-1)$$

$$= 2x - 32 + 2x - 4y$$

$$= 4x - 4y - 32$$

When $D_x = 0$,

$$4x - 4y - 32 = 0$$

$$\Rightarrow x - y - 8 = 0$$

$$\Rightarrow y = x - 8$$

$$\text{Again, } D_y = 2y + 2(16 - x + 2y)(2)$$

$$= 2y + 64 - 4x + 8y$$

$$= 10y - 4x + 64$$

When $D_y = 0$

$$10y - 4x + 64 = 0$$

$$\Rightarrow 5(x - 8) - 4x + 64 = 0$$

$$\Rightarrow 3x - 8 = 0$$

$$\Rightarrow x = \frac{8}{3}$$

Substituting the value of x in the equation $y = x - 8$,

$$y = \frac{-16}{3}$$

$$\therefore (x, y) = (\frac{8}{3}, \frac{-16}{3})$$

$$\text{Now, } A = D_{xx} = 4$$

$$C = D_{yy} = 10$$

$$B = D_{xy} = -4$$

$$D = AC - B^2 = 24$$

Since $D > 0$ and $A > 0$, D has a minimum at point $(\frac{8}{3}, \frac{-16}{3})$

Now,

$$D(\frac{8}{3}, \frac{-16}{3}) = (\frac{8}{3})^2 + (\frac{-16}{3})^2 + (16 - \frac{8}{3} - \frac{2 \cdot 16}{3})^2$$

$$= \frac{64}{9} + \frac{256}{9} + (\frac{40}{3} - \frac{32}{3})^2$$

$$= \frac{64}{9} + \frac{256}{9} + (\frac{8}{3})^2$$

$$= \frac{128}{9} + \frac{256}{9}$$

$$= \frac{128}{3}$$

$$\therefore \text{Distance} = \sqrt{\frac{128}{3}} = \frac{8\sqrt{6}}{3}$$

$$\text{Since, } x = \frac{8}{3}, y = \frac{-16}{3},$$

$$z = 16 - \frac{8}{3} + \frac{2 \cdot -16}{3} = \frac{8}{3}$$

the point in the given plane that is closest to the origin is $(\frac{8}{3}, \frac{-16}{3}, \frac{8}{3})$

Ans to the question no 04

Given function,

$$\vec{F} = -x^2(y - z)\hat{i} - (x^2 + y^4)\hat{j} + \left(\frac{4z^2}{y^2}\right)\hat{k}$$

$$\Rightarrow \vec{F} = (-x^2y + x^2z)\hat{i} + (-x^2 - y^4)\hat{j} + (4z^2y^{-2})\hat{k}$$

$$\begin{aligned}\text{Curl } \vec{F} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -x^2y + x^2z & -x^2 - y^4 & 4z^2y^{-2} \end{vmatrix} \\ &= \hat{i}\left\{\frac{\partial}{\partial y}(4z^2y^{-2}) - \frac{\partial}{\partial z}(-x^2 - y^4)\right\} - \hat{j}\left\{\frac{\partial}{\partial x}(4z^2y^{-2}) - \frac{\partial}{\partial z}(-x^2y + x^2z)\right\} \\ &\quad + \hat{k}\left\{\frac{\partial}{\partial x}(-x^2 - y^4) - \frac{\partial}{\partial y}(-x^2y + x^2z)\right\} \\ &= (-8z^2y^{-3})\hat{i} - (0 - 0 + x^2)\hat{j} + (-2x + x^2)\hat{k} \\ &= -8z^2y^{-3}\hat{i} - x^2\hat{j} - (2x + x^2)\hat{k}\end{aligned}$$

$$\text{Div } \vec{F} = \Delta F$$

$$\begin{aligned}&= \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \\ &= \frac{\partial}{\partial x}(-x^2y + x^2z) + \frac{\partial}{\partial y}(-x^2 - y^4) + \frac{\partial}{\partial z}(4z^2y^{-2}) \\ &= -2x(y - z) - 4y^3 + \frac{8z}{y^2}\end{aligned}$$

$$\therefore \text{Curl } \vec{F} = -8z^2y^{-3}\hat{i} - x^2\hat{j} - (2x + x^2)\hat{k}$$

$$\therefore \Delta F = -2x(y - z) - 4y^3 + \frac{8z}{y^2}$$

Ans to the question no 05

Given function,

$$\vec{F} = 2yz\hat{i} - x^2y\hat{j} + xz^2\hat{k}$$

$$\phi = 2x^2yz^3$$

$$\nabla\phi = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \cdot (2x^2yz^3)$$

$$= \frac{\partial}{\partial x}(2x^2yz^3)\hat{i} + \frac{\partial}{\partial y}(2x^2yz^3)\hat{j} + \frac{\partial}{\partial z}(2x^2yz^3)\hat{k}$$

$$= 4xyz^3\hat{i} + 2x^2z^3\hat{j} + 6x^2yz^2\hat{k}$$

Now,

$$\vec{F} \cdot (\nabla\phi) = (2yz\hat{i} - x^2y\hat{j} + xz^2\hat{k}) \cdot (4xyz^3\hat{i} + 2x^2z^3\hat{j} + 6x^2yz^2\hat{k})$$

$$= 8xy^2z^4 - 2x^4yz^3 + 6x^3yz^4$$

$$\therefore \vec{F} \cdot (\nabla\phi) = 8xy^2z^4 - 2x^4yz^3 + 6x^3yz^4$$

Ans to the question no 06

Given function,

$$f(x, y, z) = x^2y + y^2z + xz^2$$

$$\nabla \cdot f(x, y, z) = \left(\frac{\partial f}{\partial x} \hat{i} + \frac{\partial f}{\partial y} \hat{j} + \frac{\partial f}{\partial z} \hat{k} \right)$$

$$= \frac{\partial}{\partial x}(x^2y + y^2z + xz^2) \hat{i} + \frac{\partial}{\partial y}(x^2y + y^2z + xz^2) \hat{j} + \frac{\partial}{\partial z}(x^2y + y^2z + xz^2) \hat{k}$$

$$= (2xy + z^2) \hat{i} + (x^2 + 2yz) \hat{j} + (y^2 + 2zx) \hat{k}$$

At the point (2,4,5),

$$\nabla \cdot f(2, 4, 5) = (2 \cdot 2 \cdot 4 + 5^2) \hat{i} + (2^2 + 2 \cdot 4 \cdot 5) \hat{j} + (4^2 + 2 \cdot 5 \cdot 2) \hat{k}$$

$$= 41 \hat{i} + 44 \hat{j} + 36 \hat{k}$$

unit vector along (1,-1,3),

$$\hat{u} = \frac{\hat{i} - \hat{j} + 3\hat{k}}{\sqrt{1^2 + 1^2 + 3^2}}$$

$$= \frac{\hat{i} - \hat{j} + 3\hat{k}}{\sqrt{11}}$$

Directional derivative of the function $f(x,y,z)$ at the point (2,4,5) in the direction of the point (1,-1,3),

$$(\nabla \cdot f(2, 4, 5)) \cdot \hat{u} = \frac{41 - 44 + 108}{\sqrt{11}}$$

$$= \frac{105}{\sqrt{11}} = 31.66$$

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1 \documentclass{article}
2 \usepackage{amsmath}
3 \usepackage{amssymb}
4 \begin{document}
5   \begin{titlepage}
6     \begin{center}
7       \line(1,0){300}\\
8       [0.25 in]
9       \huge{\bfseries Shihab Muhtasim}\\
10      [0.5 cm]
11      \textsc{\Large Student ID: 21301610}\\
12      \line(1,0){400}\\
13      [2 cm]
14      \textsc{\LARGE MAT 110}\\
15      [0.5 cm]
16      \textsc{\LARGE ASSIGNMENT 04}\\
17      [0.5 cm]
18      \textsc{\LARGE SET 7}\\
19      \end{center}
20      \end{titlepage}
21 \begin{newpage}
22   \begin{flushright}
23     \textsc{Assignment 4}\\
24     \textsc{Problem 1}\\
25     [1 cm]
26   \end{flushright}
27   \begin{center}
28     \textbf{\Large \underline{Ans to the question no 01}}\\
29     [0.5 cm]
30   \end{center}
31   \Large {Given function, }
32   $ f(x,y)=y\cdot e^{(xy+3x+2y^2)} $
33   Now,
34   $ f_x=y\cdot e^{(xy+3x+2y^2)}\cdot (y+3)=(y^2+3y)\cdot e^{(xy+3x+2y^2)} $
35   $ f_y=e^{(xy+3x+2y^2)}+y\cdot e^{(xy+3x+2y^2)}\cdot (x+4y) $
36   \Rrightarrow f_v = e^{(xv+3x+2v^2)}\cdot (1+xv+4v^2)

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Shihab Muhtasim

STUDENT ID: 21301610

MAT 110

ASSIGNMENT 04

SET 7

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20 \end{titlepage}
21 \begin{newpage}
22 \begin{flushright}
23 \textsc{Assignment 4}\\
24 \textsc{Problem 1}\\
25 [1 cm]
26 \end{flushright}
27 \begin{center}
28 \textbf{\Large \underline{Ans to the question no 01}}\\
29 [0.5 cm]
30 \end{center}
31 \Large {Given function, \\[3mm]
32 $f(x,y)=y\cdot e^{(xy+3x+2y^2)}$\\[3mm]
33 Now, \\[3mm]
34 $f_x=y\cdot e^{(xy+3x+2y^2)}\cdot (y+3)=y^2+3y\cdot e^{(xy+3x+2y^2)}$\\[3mm]
35 $f_y=e^{(xy+3x+2y^2)}+y\cdot e^{(xy+3x+2y^2)}\cdot (x+4y)$\\[3mm]
36 \Rightarrow f_y= e^{(xy+3x+2y^2)}(1+xy+4y^2)\\[3mm]
37 $f_{xx}=e^{(xy+3x+2y^2)}\cdot (y+3)\cdot y(y+3)=e^{(xy+3x+2y^2)}\cdot y(y+3)^2$\\[3mm]
38 $f_{yy}=e^{(xy+3x+2y^2)}\cdot (x+4y)\cdot (1+xy+4y^2)+e^{(xy+3x+2y^2)}\cdot (x+8y)$\\[3mm]
39 \Rightarrow f_{yy}=e^{(xy+3x+2y^2)}\cdot \{(x+4y)(1+xy+4y^2)+(x+8y)\}$\\[3mm]
40 $f_{xy}=(2y+3)\cdot e^{(xy+3x+2y^2)}+(y^2+3y)\cdot e^{(xy+3x+2y^2)}\cdot (x+4y)$\\[3mm]
41 \Rightarrow f_{xy}= e^{(xy+3x+2y^2)}\cdot \{(2y+3)+(y^2+3y)(x+4y)\}$\\[3mm]
42 Now,
43 $f(0,0)=0$\\[3mm]
44 $f_x(0,0)=0$\\[3mm]
45 $f_y(0,0)=1$\\[3mm]
46 $f_{xx}(0,0)=0$\\[3mm]
47 $f_{yy}(0,0)=0$\\[3mm]
48 $f_{xy}(0,0)=3$\\[10mm]
49 The first degree Maclaurin polynomial approximation,\\[3mm]
50 $\therefore L(x,y)=f(0,0)+f_x(0,0)(x-0)+f_y(0,0)(y-0)$\\[3mm]
51 $\Rightarrow L(x,y)=0+0\cdot x+1\cdot y=y$\\[3mm]
52 $\therefore L(x,y)=y$\\[3mm]
53 The second degree Maclaurin polynomial approximation,\\[3mm]
54 $Q(x,y)=L(x,y)+\frac{f_{xx}(0,0)}{2!}x^2+\frac{f_{yy}(0,0)}{2!}y^2+\frac{f_{xy}(0,0)}{1!1!}xy$\\[3mm]

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ASSIGNMENT 4
PROBLEM 1

Ans to the question no 01

Given function,

$$f(x,y) = y \cdot e^{(xy+3x+2y^2)}$$

Now,

$$f_x = y \cdot e^{(xy+3x+2y^2)} \cdot (y+3) = (y^2+3y) \cdot e^{(xy+3x+2y^2)}$$

$$f_y = e^{(xy+3x+2y^2)} + y \cdot e^{(xy+3x+2y^2)} \cdot (x+4y)$$

$$\Rightarrow f_y = e^{(xy+3x+2y^2)}(1+xy+4y^2)$$

$$f_{xx} = e^{(xy+3x+2y^2)} \cdot (y+3) \cdot y(y+3) = e^{(xy+3x+2y^2)} \cdot y(y+3)^2$$

$$f_{yy} = e^{(xy+3x+2y^2)} \cdot (x+4y) \cdot (1+xy+4y^2) + e^{(xy+3x+2y^2)} \cdot (x+8y)$$

$$\Rightarrow f_{yy} = e^{(xy+3x+2y^2)} \cdot \{(x+4y)(1+xy+4y^2) + (x+8y)\}$$

$$f_{xy} = (2y+3) \cdot e^{(xy+3x+2y^2)} + (y^2+3y) \cdot e^{(xy+3x+2y^2)} \cdot (x+4y)$$

$$\Rightarrow f_{xy} = e^{(xy+3x+2y^2)} \cdot \{(2y+3) + (y^2+3y)(x+4y)\}$$

Now, $f(0,0) = 0$

$$f_x(0,0) = 0$$

$$f_y(0,0) = 1$$

$$f_{xx}(0,0) = 0$$

$$f_{yy}(0,0) = 0$$

$$f_{xy}(0,0) = 3$$

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45 f_y(0,0)=1\\[3mm]
46 f_{xx}(0,0)=0\\[3mm]
47 f_{yy}(0,0)=0\\[3mm]
48 f_{xy}(0,0)=3$\\[10mm]
49 The first degree Maclaurin polynomial approximation,\\[3mm]
50 $\\therefore L(x,y)=f(0,0)+f_x(0,0)(x-0)+f_y(0,0)(y-0)\\[3mm]
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55 \\rightarrow Q(x,y)=y+0+0+3xy=y(3x+1)\\[3mm]
56 \\therefore Q(x,y)=y(3x+1)$\\[3mm]
57 \\end{newpage}
58 \\begin{newpage}
59 \\begin{flushright}
60 \\textsc{Assignment 4}\\[1 cm]
61 \\textsc{Problem 2}\\[1 cm]
62 \\end{flushright}
63 \\begin{center}
64 \\textbf{\\Large \\underline{Ans to the question no 02}}\\[0.5 cm]
65 \\end{center}
66 \\Large {Given function, \\[3mm]
67 $ f(x,y)=f(x,y) = 4xye^{-x^2-y^2}$\\[3mm]
68 Now, \\[2mm]
69 $f_x=4y \\cdot e^{-x^2-y^2}+xe^{-x^2-y^2}(-2x)$\\[3mm]
70 \\rightarrow f_x=4y \\cdot e^{-x^2-y^2}(1-2x^2)\\[3mm]
71 $f_y=4x \\cdot e^{-x^2-y^2}+ye^{-x^2-y^2}(-2y)$\\[3mm]
72 \\rightarrow f_y=4x \\cdot e^{-x^2-y^2}(1-2y^2)\\[3mm]
73 For extreme values,\\[3mm]
74 $f_x=0$\\[3mm]
75 \\rightarrow 4y \\cdot e^{-x^2-y^2}(1-2x^2) =0 \\[3mm]
76 $e^{-x^2-y^2}$ cannot be zero for any finite values of x or y\\[3mm]
77 $\\therefore y=0$ or $x=\\pm \\frac{1}{\\sqrt{2}}$\\[3mm]

```

The first degree Maclaurin polynomial approximation,

$$\therefore L(x, y) = f(0, 0) + f_x(0, 0)(x - 0) + f_y(0, 0)(y - 0)$$

$$\Rightarrow L(x, y) = 0 + 0 + 1 \cdot y = y$$

$$\therefore L(x, y) = y$$

The second degree Maclaurin polynomial approximation,

$$Q(x, y) = L(x, y) + \frac{f_{xx}(0,0)}{2!} \cdot (x - 0)^2 + \frac{f_{yy}(0,0)}{2!} \cdot (y - 0)^2 + \frac{f_{xy}(0,0)}{1!} \cdot (x - 0)(y - 0)$$

$$\Rightarrow Q(x, y) = y + 0 + 0 + 3xy = y(3x + 1)$$

$$\therefore Q(x, y) = y(3x + 1)$$

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57 \end{newpage}
58 \begin{newpage}
59 \begin{flushright}
60 \textsc{Assignment 4}\\
61 \textsc{Problem 2}\\
62 [1 cm]
63 \end{flushright}
64 \begin{center}
65 \textbf{\Large \underline {Ans to the question no 02}}\\
66 [0.5 cm]
67 \end{center}
68 \Large {Given function, \\[3mm]
69 $f(x,y)=f(x,y) = 4xye^{-x^2-y^2}$\\[3mm]
70 Now, \\[2mm]
71 $f_x=4y \cdot e^{-x^2-y^2} + xe^{-x^2-y^2}(-2x)$\\[3mm]
72 \Rightarrow f_x=4y \cdot e^{-x^2-y^2}(1-2x^2)$\\[3mm]
73 $f_y=4x \cdot e^{-x^2-y^2} + ye^{-x^2-y^2}(-2x)$\\[3mm]
74 \Rightarrow f_y=4x \cdot e^{-x^2-y^2}(1-2y^2)$\\[3mm]
75 For extreme values, \\[3mm]
76 $f_x=0$\\[3mm]
77 \Rightarrow 4y \cdot e^{-x^2-y^2}(1-2x^2) = 0 \\[3mm]
78 $e^{-x^2-y^2}$ cannot be zero for any finite values of x or y\\[3mm]
79 $\therefore y=0$ or $x=\pm \frac{1}{\sqrt{2}}$\\[3mm]
80 Similarly, \\[3mm]
81 $f_y=0$\\[3mm]
82 \Rightarrow 4x \cdot e^{-x^2-y^2}(1-2y^2) = 0 \\[3mm]
83 $e^{-x^2-y^2}$ cannot be zero for any finite values of x or y\\[3mm]
84 $\therefore x=0$ or $y=\pm \frac{1}{\sqrt{2}}$\\[3mm]
85 The critical points are: $(0,0), (0, \frac{1}{\sqrt{2}}), (0, -\frac{1}{\sqrt{2}}), (\frac{1}{\sqrt{2}}, 0)$, \\[3mm]
86 $(-\frac{1}{\sqrt{2}}, 0), (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}), (\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}), (-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}), (-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$\\[3mm]
87 Now, \\[3mm]
88 $f_{xx}=4y \cdot e^{-x^2-y^2}(-4x) + (1-2x^2)e^{-x^2-y^2}(-2x)$\\[3mm]
89 \Rightarrow f_{xx}=4y \cdot e^{-x^2-y^2}(2x^2-3)$\\[3mm]
90 \Rightarrow f_{xx}=8xy \cdot e^{-x^2-y^2} - 6x \cdot e^{-x^2-y^2}$\\[3mm]

```

ASSIGNMENT 4
PROBLEM 2

Ans to the question no 02

Given function,

$$f(x,y) = f(x,y) = 4xye^{-x^2-y^2}$$

Now,

$$f_x = 4y \cdot \{e^{-x^2-y^2} + xe^{-x^2-y^2}(-2x)\}$$

$$\Rightarrow f_x = 4y \cdot e^{-x^2-y^2}(1-2x^2)$$

$$f_y = 4x \cdot \{e^{-x^2-y^2} + ye^{-x^2-y^2}(-2x)\}$$

$$\Rightarrow f_y = 4x \cdot e^{-x^2-y^2}(1-2y^2)$$

For extreme values,

$$f_x = 0$$

$$\Rightarrow 4y \cdot e^{-x^2-y^2}(1-2x^2) = 0$$

$$e^{-x^2-y^2} \text{ cannot be zero for any finite values of x or y}$$

$$\therefore y = 0 \text{ or } x = \pm \frac{1}{\sqrt{2}}$$

Similarly,

$$f_y = 0$$

$$\Rightarrow 4x \cdot e^{-x^2-y^2}(1-2y^2) = 0$$

$$e^{-x^2-y^2} \text{ cannot be zero for any finite values of x or y}$$

$$\therefore x = 0 \text{ or } y = \pm \frac{1}{\sqrt{2}}$$

The critical points are: $(0,0), (0, \frac{1}{\sqrt{2}}), (0, -\frac{1}{\sqrt{2}}), (\frac{1}{\sqrt{2}}, 0), (-\frac{1}{\sqrt{2}}, 0), (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}), (\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}), (-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}), (-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$

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82 \rightarrow 4x \cdot e^{-x^2-y^2}(1-2y^2) = 0 \\[3mm]
83 e^{-x^2-y^2} cannot be zero for any finite values of x or y \\[3mm]
84 \therefore x=0 \text{ or } y=\pm \frac{1}{\sqrt{2}} \\[3mm]
85 The critical points are: $(0,0), (0, \frac{1}{\sqrt{2}}), (0, -\frac{1}{\sqrt{2}}), (\frac{1}{\sqrt{2}}, 0) \\[3mm]
86 (-\frac{1}{\sqrt{2}}, 0), (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}), (\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}), (-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}), (-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$ \\[3mm]
87 Now, \\[3mm]
88 $f_{xx}=4y\{e^{-x^2-y^2}(-4x)+(1-2x^2)e^{-x^2-y^2}(-2x)\}$ \\[3mm]
89 \rightarrow f_{xx}=4y\{2xe^{-x^2-y^2}(2x^2-3)\} \\[3mm]
90 \rightarrow f_{xx}=8xy(2x^2-3)e^{-x^2-y^2} \\[3mm]
91 $f_{yy}=4x\{e^{-x^2-y^2}(-4y)+(1-2y^2)e^{-x^2-y^2}(-2y)\}$ \\[3mm]
92 \rightarrow f_{yy}=4x\{2ye^{-x^2-y^2}(2y^2-3)\} \\[3mm]
93 \rightarrow f_{yy}=8xy(2y^2-3)e^{-x^2-y^2} \\[3mm]
94 $f_{xy}=4\{e^{-x^2-y^2}(1-2x^2)+ye^{-x^2-y^2}(-2y)(1-2x^2)\}$ \\[3mm]
95 \rightarrow f_{xy}=4e^{-x^2-y^2}(1-2x^2)(1-2y^2)$ \\[3mm]
96 For $(0,0)$, \\[3mm]
97 $A=f_{xx}(0,0)=0$ \\[3mm]
98 $B=f_{xy}(0,0)=4$ \\[3mm]
99 $C=f_{yy}(0,0)=0$ \\[3mm]
100 $D=AC-B^2=0-4^2=-16$ \\[3mm]
101 Since $D<0$, $f$ has a saddle point at $(0,0)$ \\[3mm]
102 For $(0, \frac{1}{\sqrt{2}})$, \\[3mm]
103 $A=f_{xx}(0, \frac{1}{\sqrt{2}})=0$ \\[3mm]
104 $B=f_{xy}(0, \frac{1}{\sqrt{2}})=0$ \\[3mm]
105 $C=f_{yy}(0, \frac{1}{\sqrt{2}})=0$ \\[3mm]
106 $D=AC-B^2=0$ \\[3mm]
107 Since $D=0$, No conclusion can be done \\[3mm]
108 For $(0, -\frac{1}{\sqrt{2}})$, \\[3mm]
109 $A=f_{xx}(0, -\frac{1}{\sqrt{2}})=0$ \\[3mm]
110 $B=f_{xy}(0, -\frac{1}{\sqrt{2}})=0$ \\[3mm]
111 $C=f_{yy}(0, -\frac{1}{\sqrt{2}})=0$ \\[3mm]
112 $D=AC-B^2=0$ \\[3mm]
113 Since $D=0$, No conclusion can be done \\[3mm]
114 For $(\frac{1}{\sqrt{2}}, 0)$, \\[3mm]
115 $A=f_{xx}(\frac{1}{\sqrt{2}}, 0)=0$ \\[3mm]

```

$$(-\frac{1}{\sqrt{2}}, 0), (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}), (\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}), (-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}), (-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$$

Now,

$$f_{xx} = 4y\{e^{-x^2-y^2}(-4x) + (1-2x^2)e^{-x^2-y^2}(-2x)\}$$

$$\Rightarrow f_{xx} = 4y\{2xe^{-x^2-y^2}(2x^2-3)\}$$

$$\Rightarrow f_{xx} = 8xy(2x^2-3)e^{-x^2-y^2}$$

$$f_{yy} = 4x\{e^{-x^2-y^2}(-4y) + (1-2y^2)e^{-x^2-y^2}(-2y)\}$$

$$\Rightarrow f_{yy} = 4x\{2ye^{-x^2-y^2}(2y^2-3)\}$$

$$\Rightarrow f_{yy} = 8xy(2y^2-3)e^{-x^2-y^2}$$

$$f_{xy} = 4\{e^{-x^2-y^2}(1-2x^2) + ye^{-x^2-y^2}(-2y)(1-2x^2)\}$$

$$\Rightarrow f_{xy} = 4e^{-x^2-y^2}(1-2x^2)(1-2y^2)$$

For \$(0,0)\$,

$$A = f_{xx}(0,0) = 0$$

$$B = f_{xy}(0,0) = 4$$

$$C = f_{yy}(0,0) = 0$$

$$D = AC - B^2 = 0 - 4^2 = -16$$

Since \$D < 0\$, \$f\$ has a saddle point at \$(0,0)\$

For \$(0, \frac{1}{\sqrt{2}})\$,

$$A = f_{xx}(0, \frac{1}{\sqrt{2}}) = 0$$

$$B = f_{xy}(0, \frac{1}{\sqrt{2}}) = 0$$

$$C = f_{yy}(0, \frac{1}{\sqrt{2}}) = 0$$

$$D = AC - B^2 = 0$$

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102 For  $(0, \frac{1}{\sqrt{2}})$ ,
103  $A = f_{xx}(0, \frac{1}{\sqrt{2}}) = 0$ 
104  $B = f_{xy}(0, \frac{1}{\sqrt{2}}) = 0$ 
105  $C = f_{yy}(0, \frac{1}{\sqrt{2}}) = 0$ 
106  $D = AC - B^2 = 0$ 
107 Since  $D = 0$ , No conclusion can be done
108 For  $(0, -\frac{1}{\sqrt{2}})$ ,
109  $A = f_{xx}(0, -\frac{1}{\sqrt{2}}) = 0$ 
110  $B = f_{xy}(0, -\frac{1}{\sqrt{2}}) = 0$ 
111  $C = f_{yy}(0, -\frac{1}{\sqrt{2}}) = 0$ 
112  $D = AC - B^2 = 0$ 
113 Since  $D = 0$ , No conclusion can be done
114 For  $(\frac{1}{\sqrt{2}}, 0)$ ,
115  $A = f_{xx}(\frac{1}{\sqrt{2}}, 0) = 0$ 
116  $B = f_{xy}(\frac{1}{\sqrt{2}}, 0) = 0$ 
117  $C = f_{yy}(\frac{1}{\sqrt{2}}, 0) = 0$ 
118  $D = AC - B^2 = 0$ 
119 Since  $D = 0$ , No conclusion can be done
120 For  $(-\frac{1}{\sqrt{2}}, 0)$ ,
121  $A = f_{xx}(-\frac{1}{\sqrt{2}}, 0) = 0$ 
122  $B = f_{xy}(-\frac{1}{\sqrt{2}}, 0) = 0$ 
123  $C = f_{yy}(-\frac{1}{\sqrt{2}}, 0) = 0$ 
124  $D = AC - B^2 = 0$ 
125 Since  $D = 0$ , No conclusion can be done
126 For  $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ ,
127  $A = f_{xx}(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) = -8e^{-1} = -2.943$ 
128  $B = f_{xy}(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) = 0$ 
129  $C = f_{yy}(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) = -8e^{-1} = -2.943$ 
130  $D = AC - B^2 = 8.66$ 
131 Since  $D > 0$  and  $A < 0$ , f has a relative maximum point at
132  $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ 
133 For  $(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$ ,
134  $A = f_{xx}(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}) = 8e^{-1} = 2.943$ 
135  $B = f_{xy}(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}) = 0$ 
136  $C = f_{yy}(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}) = 8e^{-1} = 2.943$ 
137  $D = AC - B^2 = 8.66$ 

```

Since $D = 0$, No conclusion can be done
For $(0, -\frac{1}{\sqrt{2}})$,
 $A = f_{xx}(0, -\frac{1}{\sqrt{2}}) = 0$
 $B = f_{xy}(0, -\frac{1}{\sqrt{2}}) = 0$
 $C = f_{yy}(0, -\frac{1}{\sqrt{2}}) = 0$
 $D = AC - B^2 = 0$
Since $D = 0$, No conclusion can be done
For $(\frac{1}{\sqrt{2}}, 0)$,
 $A = f_{xx}(\frac{1}{\sqrt{2}}, 0) = 0$
 $B = f_{xy}(\frac{1}{\sqrt{2}}, 0) = 0$
 $C = f_{yy}(\frac{1}{\sqrt{2}}, 0) = 0$
 $D = AC - B^2 = 0$
Since $D = 0$, No conclusion can be done
For $(-\frac{1}{\sqrt{2}}, 0)$,
 $A = f_{xx}(-\frac{1}{\sqrt{2}}, 0) = 0$
 $B = f_{xy}(-\frac{1}{\sqrt{2}}, 0) = 0$
 $C = f_{yy}(-\frac{1}{\sqrt{2}}, 0) = 0$
 $D = AC - B^2 = 0$
Since $D = 0$, No conclusion can be done
For $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$,
 $A = f_{xx}(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) = -8e^{-1} = -2.943$
 $B = f_{xy}(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) = 0$
 $C = f_{yy}(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) = -8e^{-1} = -2.943$
 $D = AC - B^2 = 8.66$
Since $D > 0$ and $A < 0$, f has a relative maximum point at
 $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$
For $(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$,
 $A = f_{xx}(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}) = 8e^{-1} = 2.943$
 $B = f_{xy}(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}) = 0$
 $C = f_{yy}(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}) = 8e^{-1} = 2.943$
 $D = AC - B^2 = 8.66$

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121 A=f_{xx}(-\frac{1}{\sqrt{2}},0)=0\[[3mm]
122 B=f_{xy}(-\frac{1}{\sqrt{2}},0)=0\[[3mm]
123 C=f_{yy}(-\frac{1}{\sqrt{2}},0)=0\[[3mm]
124 D=AC-B^2=0\[[3mm]$
125 Since $D=0$, No conclusion can be done\[[3mm]
126 For $(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}),\frac{1}{\sqrt{2}}\$, \[[3mm]
127 A=f_{xx}(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})=-8e^{-1}=-2.943\[[3mm]
128 B=f_{xy}(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})=0\[[3mm]
129 C=f_{yy}(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})=-8e^{-1}=-2.943\[[3mm]
130 D=AC-B^2=8.66\[[3mm]$
131 Since $D>0$ and $A<0$, f has a relative maximum point at
    $(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})\$ \[[3mm]
132 For $(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}),-\frac{1}{\sqrt{2}}\$, \[[3mm]
133 A=f_{xx}(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})=8e^{-1}=2.943\[[3mm]
134 B=f_{xy}(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})=0\[[3mm]
135 C=f_{yy}(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})=8e^{-1}=2.943\[[3mm]
136 D=AC-B^2=8.66\[[3mm]$
137 Since $D>0$ and $A>0$, f has a relative minimum point at
    $(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})\$ \[[3mm]
138 For $(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}),\frac{1}{\sqrt{2}}\$, \[[3mm]
139 A=f_{xx}(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})=8e^{-1}=2.943\[[3mm]
140 B=f_{xy}(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})=0\[[3mm]
141 C=f_{yy}(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})=8e^{-1}=2.943\[[3mm]
142 D=AC-B^2=8.66\[[3mm]$
143 Since $D>0$ and $A>0$, f has a relative minimum point at
    $(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})\$ \[[3mm]
144 For $(-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}),-\frac{1}{\sqrt{2}}\$, \[[3mm]
145 A=f_{xx}(-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})=-8e^{-1}=-2.943\[[3mm]
146 B=f_{xy}(-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})=0\[[3mm]
147 C=f_{yy}(-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})=-8e^{-1}=-2.943\[[3mm]
148 D=AC-B^2=8.66\[[3mm]$
149 Since $D>0$ and $A<0$, f has a relative maximum point at
    $(-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})\$ \[[3mm]
150 $\therefore$ Saddle point:(0,0)\[[3mm]
151 Maxima:$(-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})\$, $(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})\$ \[[3mm]
152 Minima:$(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})\$, $(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})\$ \[[3mm]

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$$A = f_{xx}\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = -8e^{-1} = -2.943$$

$$B = f_{xy}\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = 0$$

$$C = f_{yy}\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = -8e^{-1} = -2.943$$

$$D = AC - B^2 = 8.66$$

Since $D > 0$ and $A < 0$, f has a relative maximum point at $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

For $(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$,

$$A = f_{xx}\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) = 8e^{-1} = 2.943$$

$$B = f_{xy}\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) = 0$$

$$C = f_{yy}\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) = 8e^{-1} = 2.943$$

$$D = AC - B^2 = 8.66$$

Since $D > 0$ and $A > 0$, f has a relative minimum point at $(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$

For $(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$,

$$A = f_{xx}\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = 8e^{-1} = 2.943$$

$$B = f_{xy}\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = 0$$

$$C = f_{yy}\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = 8e^{-1} = 2.943$$

$$D = AC - B^2 = 8.66$$

Since $D > 0$ and $A > 0$, f has a relative minimum point at $(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

For $(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$,

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133 A=f_{xx}(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})=8e^{-1}=2.943\text{\\[3mm]}
134 B=f_{xy}(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})=0\text{\\[3mm]}
135 C=f_{yy}(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})=8e^{-1}=2.943\text{\\[3mm]}
136 D=AC-B^2=8.66\text{\\[3mm]}$
137 Since $D>0$ and $A>0$, f has a relative minimum point at
138 $(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})\text{\\[3mm]}$
139 For $(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})$, $\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})=8e^{-1}=2.943\text{\\[3mm]}$
140 $B=f_{xy}(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})=0\text{\\[3mm]}$
141 $C=f_{yy}(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})=8e^{-1}=2.943\text{\\[3mm]}$
142 $D=AC-B^2=8.66\text{\\[3mm]}$
143 Since $D>0$ and $A>0$, f has a relative minimum point at
144 $(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})\text{\\[3mm]}$
145 $A=f_{xx}(-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})=-8e^{-1}=-2.943\text{\\[3mm]}$
146 $B=f_{xy}(-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})=0\text{\\[3mm]}$
147 $C=f_{yy}(-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})=-8e^{-1}=-2.943\text{\\[3mm]}$
148 $D=AC-B^2=8.66\text{\\[3mm]}$
149 Since $D>0$ and $A<0$, f has a relative maximum point at
150 $(-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})\text{\\[3mm]}$
151 $\therefore$ Saddle point: $(0,0)\text{\\[3mm]}$
152 Maxima: $(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})$, $(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})\text{\\[3mm]}$
153 Minima: $(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})$, $(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})\text{\\[3mm]}$
154 \end{newpage}
155 \begin{newpage}
156 \begin{flushright}
157 \textsc{Assignment 4}\\
158 \textsc{Problem 3}\\
159 [1 cm]
160 \end{flushright}
161 \begin{center}
162 \textbf{\Large \underline{Ans to the question no 03}}\\
163 [0.5 cm]
164 \end{center}
165 \Large {Given function, \text{\\[3mm]}

```

$$A = f_{xx}\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) = -8e^{-1} = -2.943$$

$$B = f_{xy}\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) = 0$$

$$C = f_{yy}\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) = -8e^{-1} = -2.943$$

$$D = AC - B^2 = 8.66$$

Since $D > 0$ and $A < 0$, f has a relative maximum point at $(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$

\therefore Saddle point: $(0,0)$

Maxima: $(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}), (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

Minima: $(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}), (-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

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152 Minima: $\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ \\[3mm]
153 \end{newpage}
154 \begin{newpage}
155 \begin{flushright}
156 \textsc{Assignment 4}\\
157 \textsc{Problem 3}\\
158 [1 cm]
159 \end{flushright}
160 \begin{center}
161 \textbf{\Large \underline{Ans to the question no 03}}\\
162 [0.5 cm]
163 \end{center}
164 \Large {Given function, \\[3mm]
165 $ x-2y+z=16$ \\[3mm]
166 \rightarrow z=16-x+2y \\[3mm]
167 At origin (0,0), \\[3mm]
168 $D=(Distance)^2=(x-0)^2+(y-0)^2+(z-0)^2$ \\[3mm]
169 $=x^2+y^2+z^2$ \\[3mm]
170 $=x^2+y^2+(16-x+2y)^2$ \\[3mm]
171 Here, \\[3mm]
172 $D_x=2x+2(16-x+2y)(-1)$ \\[3mm]
173 $=2x-32+2x-4y$ \\[3mm]
174 $=4x-4y-32$ \\[3mm]
175 When $D_x=0$, \\[3mm]
176 $4x-4y-32=0$ \\[3mm]
177 \rightarrow x-y-8=0 \\[3mm]
178 \rightarrow y=x-8 \\[3mm]
179 Again,
180 $D_y=2y+2(16-x+2y)(2)$ \\[3mm]
181 $=2y+64-4x+8y$ \\[3mm]
182 $=10y-4x+64$ \\[3mm]
183 When $D_y=0$, \\[3mm]
184 $10y-4x+64=0$ \\[3mm]
185 \rightarrow 5(x-8)-4x+64=0 \\[3mm]
186 \rightarrow 3x-8=0 \\[3mm]
187 \rightarrow x=\frac{8}{3} \\[3mm]

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ASSIGNMENT 4
PROBLEM 3

Ans to the question no 03

Given function,

$$x - 2y + z = 16 \Rightarrow z = 16 - x + 2y$$

At origin (0,0),

$$D = (Distance)^2 = (x - 0)^2 + (y - 0)^2 + (z - 0)^2$$

$$= x^2 + y^2 + z^2$$

$$= x^2 + y^2 + (16 - x + 2y)^2$$

Here,

$$D_x = 2x + 2(16 - x + 2y)(-1)$$

$$= 2x - 32 + 2x - 4y$$

$$= 4x - 4y - 32$$

When $D_x = 0$,

$$4x - 4y - 32 = 0$$

$$\Rightarrow x - y - 8 = 0$$

$$\Rightarrow y = x - 8$$

Again, $D_y = 2y + 2(16 - x + 2y)(2)$

$$= 2y + 64 - 4x + 8y$$

$$= 10y - 4x + 64$$

When $D_y = 0$

$$10y - 4x + 64 = 0$$

$$\Rightarrow 5(x - 8) - 4x + 64 = 0$$

$$\Rightarrow 3x - 8 = 0$$

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180 $D_y=2y+2(16-x+2y)(2)\\[3mm]
181 =2y+64-4x+8y\\[3mm]
182 =10y-4x+64\\[3mm]$
183 When $D_y=0\\[3mm]
184 10y-4x+64=0\\[3mm]
185 \\rightarrow 5(x-8)-4x+64=0\\[3mm]
186 \\rightarrow 3x-8=0 \\[3mm]
187 \\rightarrow x=\\frac{8}{3}\\[3mm]$
188 Substituting the value of x in the equation $y=x-8,\\[3mm]
189 y=\\frac{-16}{3}\\[3mm]
190 \\therefore (x,y)=(\\frac{8}{3},\\frac{-16}{3})\\[3mm]$
191 Now,
192 $A=D_{xx}=4\\[3mm]
193 C=D_{yy}=10\\[3mm]
194 B=D_{xy}=-4\\[3mm]
195 D=AC-B^2=24\\[3mm]$
196 Since $D>0$ and $A>0$, D has a minimum at point $(\\frac{8}{3},\\frac{-16}{3})$\\[3mm]
197 Now, \\[3mm]
198 $D(\\frac{8}{3},\\frac{-16}{3})=(\\frac{8}{3})^2+(\\frac{-16}{3})^2+(16-\\frac{8}{3}-\\frac{2\\cdot 16}{3})^2
199 =\\frac{64}{9}+\\frac{256}{9}+(\\frac{40}{3}-\\frac{32}{3})^2\\[3mm]
200 =\\frac{64}{9}+\\frac{256}{9}+(\\frac{8}{3})^2\\[3mm]
201 =\\frac{128}{9}+\\frac{256}{9}\\[3mm]
202 =\\frac{128}{3}\\[3mm]
203 \\therefore Distance=\\sqrt{\\frac{128}{3}}=\\frac{8\\sqrt{6}}{3}\\[3mm]
204 Since, $x=\\frac{8}{3}$, $y=\\frac{-16}{3}$,\\[3mm]
205 $z=16-\\frac{8}{3}+\\frac{2\\cdot -16}{3}=\\frac{8}{3}\\[3mm]$
206 the point in the given plane that is closest to the origin
207 is$(\\frac{8}{3},\\frac{-16}{3},\\frac{8}{3})$\\[3mm]$
207 \\end{newpage}
208 \\begin{newpage}
209 \\begin{flushright}
210 \\textsc{Assignment 4}\\[3mm]
211 \\textsc{Problem 4}\\[3mm]
212 [1 cm]
213 \\end{flushright}

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$$\Rightarrow x = \frac{8}{3}$$

Substituting the value of x in the equation $y = x - 8$,

$$y = \frac{-16}{3}$$

$$\therefore (x, y) = \left(\frac{8}{3}, \frac{-16}{3}\right)$$

Now, $A = D_{xx} = 4$

$$C = D_{yy} = 10$$

$$B = D_{xy} = -4$$

$$D = AC - B^2 = 24$$

Since $D > 0$ and $A > 0$, D has a minimum at point $\left(\frac{8}{3}, \frac{-16}{3}\right)$

Now,

$$D\left(\frac{8}{3}, \frac{-16}{3}\right) = \left(\frac{8}{3}\right)^2 + \left(\frac{-16}{3}\right)^2 + \left(16 - \frac{8}{3} - \frac{2 \cdot 16}{3}\right)^2$$

$$= \frac{64}{9} + \frac{256}{9} + \left(\frac{40}{3} - \frac{32}{3}\right)^2$$

$$= \frac{64}{9} + \frac{256}{9} + \left(\frac{8}{3}\right)^2$$

$$= \frac{128}{9} + \frac{256}{9}$$

$$= \frac{128}{3}$$

$$\therefore \text{Distance} = \sqrt{\frac{128}{3}} = \frac{8\sqrt{6}}{3}$$

Since, $x = \frac{8}{3}$, $y = \frac{-16}{3}$,

$$z = 16 - \frac{8}{3} + \frac{2 \cdot -16}{3} = \frac{8}{3}$$

the point in the given plane that is closest to the origin is $\left(\frac{8}{3}, \frac{-16}{3}, \frac{8}{3}\right)$

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207 \end{newpage}
208 \begin{newpage}
209 \begin{flushright}
210 \textsc{Assignment 4}\\
211 \textsc{Problem 4}\\
212 [1 cm]
213 \end{flushright}
214 \begin{center}
215 \textbf{\Large \underline {Ans to the question no 04}}\\
216 [0.5 cm]
217 \end{center}
218 \Large {Given function, \\\[3mm]
219 $\vec{F}=-x^2(y-z)\hat{i}-\left(x^2+y^4\right)\hat{j}+\left(\frac{4}{y^2}z^2\right)\hat{k}$\\
220 $\rightarrow \vec{F}=(-x^2y-x^2z)\hat{i}+(-x^2-y^4)\hat{j}+(4z^2y^{-2})\hat{k}$\\
221 $\text{Curl } \vec{F}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -x^2y+x^2z & -x^2-y^4 & 4z^2y^{-2} \end{vmatrix}$\\
222 $\frac{\partial}{\partial y}(-x^2y+x^2z)-\frac{\partial}{\partial z}(-x^2-y^4)\hat{i}-\left(\frac{\partial}{\partial x}(4z^2y^{-2})-\frac{\partial}{\partial z}(-x^2y+x^2z)\right)\hat{j}+\left(\frac{\partial}{\partial x}(-x^2-y^4)-\frac{\partial}{\partial y}(4z^2y^{-2})\right)\hat{k}$\\
223 $=(-8z^2y^{-3})\hat{i}-(0-0+x^2)\hat{j}+(-2x+x^2)\hat{k}$\\
224 $=-8z^2y^{-3}\hat{i}-x^2\hat{j}-(2x+x^2)\hat{k}$\\
225 $\text{Div } \vec{F}=\Delta F=\frac{\partial}{\partial x}(-x^2y+x^2z)+\frac{\partial}{\partial y}(-x^2-y^4)+\frac{\partial}{\partial z}(4z^2y^{-2})$\\
226 $=-2x(y-z)-4y^3+\frac{8z}{y^2}$\\
227 $\therefore \text{Curl } \vec{F}=-8z^2y^{-3}\hat{i}-x^2\hat{j}-(2x+x^2)\hat{k}$\\
228 $\therefore \Delta F=-2x(y-z)-4y^3+\frac{8z}{y^2}$
229 \end{newpage}
230 \begin{newpage}

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ASSIGNMENT 4
PROBLEM 4

Ans to the question no 04

Given function,

$$\vec{F} = -x^2(y-z)\hat{i} - (x^2+y^4)\hat{j} + \left(\frac{4z^2}{y^2}\right)\hat{k}$$

$$\Rightarrow \vec{F} = (-x^2y - x^2z)\hat{i} + (-x^2 - y^4)\hat{j} + (4z^2y^{-2})\hat{k}$$

$$\text{Curl } \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -x^2y+x^2z & -x^2-y^4 & 4z^2y^{-2} \end{vmatrix}$$

$$= \hat{i}\left\{\frac{\partial}{\partial y}(-x^2y+x^2z)-\frac{\partial}{\partial z}(-x^2-y^4)\right\} - \hat{j}\left\{\frac{\partial}{\partial x}(4z^2y^{-2})-\frac{\partial}{\partial z}(-x^2y+x^2z)\right\} + \hat{k}\left\{\frac{\partial}{\partial x}(-x^2-y^4)-\frac{\partial}{\partial y}(4z^2y^{-2})\right\}$$

$$= (-8z^2y^{-3})\hat{i} - (0-0+x^2)\hat{j} + (-2x+x^2)\hat{k}$$

$$= -8z^2y^{-3}\hat{i} - x^2\hat{j} - (2x+x^2)\hat{k}$$

$\text{Div } \vec{F} = \Delta F$

$$= \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

$$= \frac{\partial}{\partial x}(-x^2y - x^2z) + \frac{\partial}{\partial y}(-x^2 - y^4) + \frac{\partial}{\partial z}(4z^2y^{-2})$$

$$= -2x(y-z) - 4y^3 + \frac{8z}{y^2}$$

$\therefore \text{Curl } \vec{F} = -8z^2y^{-3}\hat{i} - x^2\hat{j} - (2x+x^2)\hat{k}$

$\therefore \Delta F = -2x(y-z) - 4y^3 + \frac{8z}{y^2}$

$$\therefore \vec{F} \cdot (\nabla \phi) = 8xy^2z^4 - 2x^4yz^3 + 6x^3yz^4$$

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252 
$$\vec{F} = (4xyz^3\hat{i} + 2x^2z^3\hat{j} + 6x^2yz^2\hat{k})$$

253 
$$= 8xy^2z^4 - 2x^4yz^3 + 6x^3yz^4$$

254 
$$\therefore \vec{F} \cdot \nabla \phi = 8xy^2z^4 - 2x^4yz^3 + 6x^3yz^4$$

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256 \begin{newpage}
257 \begin{flushright}
258 \textsc{Assignment 4}
259 \textsc{Problem 6}
260 [1 cm]
261 \end{flushright}
262 \begin{center}
263 \textbf{\Large \underline{Ans to the question no 06}}
264 [0.5 cm]
265 \end{center}
266 \Large {Given function, }
267 
$$f(x,y,z) = x^2y + y^2z + xz^2$$

268 
$$\nabla \cdot f(x,y,z) = \frac{\partial f}{\partial x} \hat{i} + \frac{\partial f}{\partial y} \hat{j} + \frac{\partial f}{\partial z} \hat{k}$$

269 
$$= \frac{\partial}{\partial x}(x^2y + y^2z + xz^2) \hat{i} + \frac{\partial}{\partial y}(x^2y + y^2z + xz^2) \hat{j} + \frac{\partial}{\partial z}(x^2y + y^2z + xz^2) \hat{k}$$

270 
$$= (2xy + z^2) \hat{i} + (x^2 + 2yz) \hat{j} + (y^2 + 2zx) \hat{k}$$

271 At the point (2,4,5),
272 
$$\nabla \cdot f(2,4,5) = (2 \cdot 4 + 5^2) \hat{i} + (2^2 + 2 \cdot 4 \cdot 5) \hat{j} + (4^2 + 2 \cdot 5 \cdot 2) \hat{k}$$

273 
$$= 41 \hat{i} + 44 \hat{j} + 36 \hat{k}$$

274 unit vector along (1,-1,3),
275 
$$\hat{u} = \frac{\hat{i} - \hat{j} + 3\hat{k}}{\sqrt{1^2 + 1^2 + 3^2}}$$

276 
$$= \frac{\hat{i} - \hat{j} + 3\hat{k}}{\sqrt{11}}$$

277 Directional derivative of the function f(x,y,z) at the point (2,4,5) in the direction of the point (1,-1,3),
278 
$$\nabla \cdot f(2,4,5) \cdot \hat{u} = \frac{41 - 44 + 108}{\sqrt{11}}$$

279 
$$= \frac{105}{\sqrt{11}} = 31.66$$

280 \end{newpage}
281 \end{document}

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ASSIGNMENT 4
PROBLEM 6

Ans to the question no 06

Given function,

$$f(x,y,z) = x^2y + y^2z + xz^2$$

$$\nabla \cdot f(x,y,z) = \left(\frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}\right)$$

$$= \frac{\partial}{\partial x}(x^2y + y^2z + xz^2)\hat{i} + \frac{\partial}{\partial y}(x^2y + y^2z + xz^2)\hat{j} + \frac{\partial}{\partial z}(x^2y + y^2z + xz^2)\hat{k}$$

$$= (2xy + z^2)\hat{i} + (x^2 + 2yz)\hat{j} + (y^2 + 2zx)\hat{k}$$

At the point (2,4,5),

$$\nabla \cdot f(2,4,5) = (2 \cdot 4 + 5^2)\hat{i} + (2^2 + 2 \cdot 4 \cdot 5)\hat{j} + (4^2 + 2 \cdot 5 \cdot 2)\hat{k}$$

$$= 41\hat{i} + 44\hat{j} + 36\hat{k}$$

unit vector along (1,-1,3),

$$\hat{u} = \frac{\hat{i} - \hat{j} + 3\hat{k}}{\sqrt{1^2 + 1^2 + 3^2}}$$

$$= \frac{\hat{i} - \hat{j} + 3\hat{k}}{\sqrt{11}}$$

Directional derivative of the function f(x,y,z) at the point (2,4,5) in the direction of the point (1,-1,3),

$$(\nabla \cdot f(2,4,5)) \cdot \hat{u} = \frac{41 - 44 + 108}{\sqrt{11}}$$

$$= \frac{105}{\sqrt{11}} = 31.66$$

12