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**Shihab Muhtasim**

STUDENT ID: 21301610

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MAT 110

ASSIGNMENT 03

SET 7

Ans to the question no 01

Given function,

$$f(x) = \sin x + 2 \cos x$$

By differentiating this function we get,

$$f'(x) = \cos x - 2 \sin x$$

$$f''(x) = -\sin x - 2 \cos x$$

$$f'''(x) = -\cos x + 2 \sin x$$

Now, at  $x=0$ ,

$$f(0) = \sin 0 + 2 \cos 0 = 2$$

$$f'(0) = \cos 0 - 2 \sin 0 = 1$$

$$f''(0) = -\sin 0 - 2 \cos 0 = -2$$

$$f'''(0) = -\cos 0 + 2 \sin 0 = -1$$

We know,  $P_n(x) = \frac{f^0(x)}{0!}x^0 + \frac{f'(x)}{1!}x^1 + \frac{f''(x)}{2!}x^2 + \frac{f'''(x)}{3!}x^3 + \dots$

According to Taylor series around  $x=0$ ,

$$P_3(x) = \frac{f^0(0)}{0!}x^0 + \frac{f'(0)}{1!}x^1 + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3$$

$$\Rightarrow P_3(x) = \frac{2}{0!}(x-0)^0 + \frac{1}{1!}(x-0)^1 + \frac{-2}{2!}(x-0)^2 + \frac{-1}{3!}(x-0)^3$$

$$\Rightarrow P_3(x) = 2 + (x-0) - (x-0)^2 - \frac{x^3}{6}$$

$$\Rightarrow P_3(x) = 2 + x - x^2 - \frac{1}{6}x^3$$

$$\therefore P_3(x) = 2 + x - x^2 - \frac{1}{6}x^3$$

Ans to the question no 02

We know,

$$P_n(0) = \frac{f^0(x)}{0!}x^0 + \frac{f'(x)}{1!}x^1 + \frac{f''(x)}{2!}x^2 + \frac{f'''(x)}{3!}x^3 + \dots$$

Given function,

$$f(x) = \ln(e^{2x} + e^{-2x})$$

By differentiating we get,

$$f'(x) = \frac{1}{e^{2x}+e^{-2x}} \cdot (2e^{2x} - 2e^{-2x}) = \frac{2(e^{4x}-1)}{e^{4x}+1}$$

$$f''(x) = \frac{2 \cdot 4e^{4x}(e^{4x}+1) - 4e^{4x}2(e^{4x}-1)}{(e^{4x}+1)^2}$$

$$\Rightarrow f''(x) = \frac{8e^{4x}(e^{4x}+1) - 8e^{4x}(e^{4x}-1)}{(e^{4x}+1)^2}$$

$$\Rightarrow f''(x) = \frac{8e^{4x}(e^{4x}-e^{4x}+2)}{(e^{4x}+1)^2}$$

$$\Rightarrow f''(x) = \frac{16e^{4x}}{(e^{4x}+1)^2}$$

Putting  $x=0$  in  $f(x)$  and its derivatives we get,

$$f(0) = \ln(e^{2 \cdot 0} + e^{-2 \cdot 0}) = \ln 2$$

$$f'(0) = \frac{2(e^{4 \cdot 0}-1)}{e^{4 \cdot 0}+1} = \frac{0}{2} = 0$$

$$f''(0) = \frac{16e^{4 \cdot 0}}{(e^{4 \cdot 0}+1)^2} = \frac{16}{4} = 4$$

Therefore,

$$P_0 = \frac{f^0(0)x^0}{0!} = \ln 2$$

$$P_1 = \frac{f^0(0)x^0}{0!} + \frac{f'(0)x^1}{1!} = \ln 2 + 0 = \ln 2$$

$$P_2 = \frac{f^0(0)x^0}{0!} + \frac{f'(0)x^1}{1!} + \frac{f''(0)x^2}{2!} = \ln 2 + 2x^2$$

Ans to the question no 03

Given multi-variable function,

$$f(x, y) = (x^2 + 2y^2x + 3x + y^2)^2$$

Derivatives of  $f(x, y)$  with respect to  $x$  considering other variables constant,

$$f_x = 2(x^2 + 2y^2x + 3x + y^2)(2x + 2y^2 + 3)$$

$$f_{xx} = 2\{2(x^2 + 2y^2x + 3x + y^2) + (2x + 2y^2 + 3)^2\}$$

$$\Rightarrow f_{xx} = 2\{(2x^2 + 4xy^2 + 6x + 2y^2) + (4x^2 + 4y^4 + 9 + 8xy^2 + 12y^2 + 12x)\}$$

$$\Rightarrow f_{xx} = 2(6x^2 + 12xy^2 + 18x + 14y^2 + 4y^4 + 9)$$

$$\therefore f_{xx} = 2(6x^2 + 12xy^2 + 18x + 14y^2 + 4y^2 + 9)$$

Similarly,

$$f_y = 2(x^2 + 2y^2x + 3x + y^2)(4xy + 2y)$$

$$f_{yy} = 2\{(4x + 2)(x^2 + 2y^2x + 3x + y^2) + (4xy + 2y)^2\}$$

$$\Rightarrow f_{yy} = 2\{(4x^3 + 8x^2y^2 + 12x^2 + 4xy^2 + 2x^2 + 4xy^2 + 6x + 2y^2) + (16x^2y^2 + 4y^2 + 16xy^2)\}$$

$$\Rightarrow f_{yy} = 2(4x^3 + 24x^2y^2 + 14x^2 + 24xy^2 + 6x + 6y^2)$$

$$\therefore f_{yy} = 2(4x^3 + 24x^2y^2 + 14x^2 + 24xy^2 + 6x + 6y^2)$$

Ans to the question no 04

Given function,

$$f(x, y) = \ln((x^2 + 3y + xy)^2)$$

$$\text{Where } x(t) = 2t + 3, y(t) = t^2 + 3t$$

Chain rule for partial derivatives,

$$\frac{df}{dt} = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dt}$$

Now,

$$\frac{\partial f}{\partial x} = \frac{1}{(x^2+3y+xy)^2} \cdot 2(x^2 + 3y + xy) \cdot (2x + y) = \frac{2(2x+y)}{(x^2+3y+xy)}$$

$$\frac{dx}{dt} = 2$$

$$\frac{\partial f}{\partial y} = \frac{1}{(x^2+3y+xy)^2} \cdot 2(x^2 + 3y + xy) \cdot (3 + x) = \frac{2(x+3)}{(x^2+3y+xy)}$$

$$\frac{dy}{dt} = 2t + 3$$

Imposing these values on the chain rule formula,

$$\frac{df}{dt} = \frac{2 \cdot 2(2x+y)}{(x^2+3y+xy)} + \frac{2(x+3)(2t+3)}{(x^2+3y+xy)}$$

Since we have to evaluate this expression when  $t=0$ ,

$$\frac{4(2x+y)}{(x^2+3y+xy)} + \frac{2(x+3)(2 \cdot 0+3)}{(x^2+3y+xy)}$$

$$= \frac{8x+4y}{(x^2+3y+xy)} + \frac{6x+18}{(x^2+3y+xy)}$$

$$= \frac{14x+4y+18}{(x^2+3y+xy)}$$

$$\therefore \frac{df}{dt} = \frac{14x+4y+18}{(x^2+3y+xy)} \text{ when } t=0$$

Ans to the question no 05

Given function,

$$y^2 \sin(x^3) + ze^{3x} - \cos(z^2) = z^4 - y^2 + x^2$$

Imposing partial differentiation on both sides with respect to x ,

$$\begin{aligned}\frac{\partial}{\partial x}\{y^2 \sin(x^3) + ze^{3x} - \cos(z^2)\} &= \frac{\partial}{\partial x}(z^4 - y^2 + x^2) \\ \Rightarrow y^2 \cos(x^3) \cdot 3x^2 + \frac{\partial z}{\partial x}e^{3x} + z \cdot 3e^{3x} + \sin(z^2)2z\frac{\partial z}{\partial x} &= 4z^3\frac{\partial z}{\partial x} + 2x \\ \Rightarrow \frac{\partial z}{\partial x}(e^{3x} + 2z \sin(z^2) - 4z^3) &= 2x - z \cdot 3e^{3x} - 3x^2y^2 \cos(x^3) \\ \Rightarrow \frac{\partial z}{\partial x} &= \frac{2x - 3ze^{3x} - 3x^2y^2 \cos(x^3)}{(e^{3x} + 2z \sin(z^2) - 4z^3)} \\ \therefore \frac{\partial z}{\partial x} &= \frac{2x - 3ze^{3x} - 3x^2y^2 \cos(x^3)}{(e^{3x} + 2z \sin(z^2) - 4z^3)}\end{aligned}$$

Similarly partial differentiation on both sides with respect to y,

$$\begin{aligned}\frac{\partial}{\partial y}\{y^2 \sin(x^3) + ze^{3x} - \cos(z^2)\} &= \frac{\partial}{\partial y}(z^4 - y^2 + x^2) \\ \Rightarrow 2y \sin(x^3) + \frac{\partial z}{\partial y}e^{3x} + \sin(z^2)2z\frac{\partial z}{\partial y} &= 4z^3\frac{\partial z}{\partial y} - 2y \\ \Rightarrow \frac{\partial z}{\partial y}(e^{3x} + 2z \sin(z^2) - 4z^3) &= -2y - 2y \sin(x^3) \\ \Rightarrow \frac{\partial z}{\partial y} &= \frac{-2y(1 + \sin x^3)}{(e^{3x} + 2z \sin(z^2) - 4z^3)} \\ \therefore \frac{\partial z}{\partial y} &= \frac{-2y(1 + \sin x^3)}{(e^{3x} + 2z \sin(z^2) - 4z^3)}\end{aligned}$$

Ans to the question no 06

Given function,

$$h(x, y, z) = y^3 z^3 \cos(x^4) + x^3 \sin(z^2)$$

We have to use partial differentiation to find  $h_{yzx}$ .

Imposing partial differentiation of  $h(x, y, x)$  with respect to y,

$$h_y = 3y^2 z^3 \cos(x^4)$$

Again partial differentiation of  $h_y$  with respect to z,

$$h_{yz} = 3y^2 3z^2 \cos(x^4) = 9y^2 z^2 \cos(x^4)$$

Finally partial differentiation of  $h_{yz}$  with respect to x,

$$h_{yzx} = 9y^2 z^2 \{-\sin(x^4)(4x^3)\} = 36x^3 y^2 z^2 \{-\sin(x^4)\}$$

evaluating the expression of  $h_{yzx}$  at point (0,1,1) we get,

$$h_{yzx} = 36 \cdot 0^3 \cdot 1^2 \cdot 1^2 \{-\sin(0^4)\} = 0$$

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1 \documentclass{article}
2 \usepackage{amsmath}
3 \usepackage{amssymb}
4 \begin{document}
5   \begin{titlepage}
6     \begin{center}
7       \line(1,0){300}\\
8       [0.25 in]
9       \huge{\bfseries Shihab Muhtasim}\\
10      [0.5 cm]
11      \textsc{\Large Student ID: 21301610}\\
12      \line(1,0){400}\\
13      [2 cm]
14      \textsc{\LARGE MAT 110}\\
15      [0.5 cm]
16      \textsc{\LARGE ASSIGNMENT 03}\\
17      [0.5 cm]
18      \textsc{\LARGE SET 7}\\
19      \end{center}
20    \end{titlepage}
21    \begin{newpage}
22      \begin{flushright}
23        \textsc{Assignment 3}\\
24        \textsc{Problem 1}\\
25        [1 cm]
26      \end{flushright}
27    \begin{center}
28      \textbf{\Large \underline{Ans to the question no 01}}\\
29      [1 cm]

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Shihab Muhtasim

STUDENT ID: 21301610

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MAT 110

ASSIGNMENT 03

SET 7



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22 \begin{flushright}
23 \textsc{Assignment 3}\\
24 \textsc{Problem 1}\\
25 [1 cm]
26 \end{flushright}
27 \begin{center}
28 \textbf{\Large \underline {Ans to the question no 01}}\\
29 [1 cm]
30 \end{center}
31 \Large {Given function, \[3mm]}
32 $f(x)=\sin x+2\cos x$ \[3mm]
33 By differentiating this function we get, \[3mm]
34 $f'(x)=\cos x-2\sin x$ \[3mm]
35 $f''(x)=-\sin x-2\cos x$ \[3mm]
36 $f'''(x)=-\cos x+2\sin x$ \[5mm]
37 Now, at x=0, \[3mm]
38 $f(0)=\sin 0+2\cos 0=2$ \[3mm]
39 $f'(0)=\cos 0-2\sin 0=1$ \[3mm]
40 $f''(0)=-\sin 0-2\cos 0=-2$ \[3mm]
41 $f'''(0)=-\cos 0+2\sin 0=-1$ \[5mm]
42 We know,
43 $P_n(0)=\frac{f^0(x)}{0!}x^0+\frac{f'(x)}{1!}x^1+\frac{f''(x)}{2!}x^2+\frac{f'''(x)}{3!}x^3+\dots$ \[3mm]
44 According to Taylor series around x=0, \[3mm]
45 $P_3(0)=\frac{f^0(0)}{0!}x^0+\frac{f'(0)}{1!}x^1+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3$ \[3mm]
46 \Rightarrow P_3(0)=\frac{2}{0!}(x-0)^0+\frac{1}{1!}(x-0)^1+\frac{-2}{2!}(x-0)^2+\frac{-1}{3!}(x-0)^3 \[3mm]
47 \Rightarrow P_3(0)=2+(x-0)-(x-0)^2-\frac{1}{6}x^3 \[3mm]
48 \Rightarrow P_3(0)=2+x-x^2-\frac{1}{6}x^3 \[3mm]
49 \therefore P_3(0)=2+x-x^2-\frac{1}{6}x^3
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51 \begin{newpage}
52 \begin{flushright}
53 \textsc{Assianment 3}

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ASSIGNMENT 3  
PROBLEM 1

### Ans to the question no 01

Given function,

$$f(x) = \sin x + 2 \cos x$$

By differentiating this function we get,

$$f'(x) = \cos x - 2 \sin x$$

$$f''(x) = -\sin x - 2 \cos x$$

$$f'''(x) = -\cos x + 2 \sin x$$

Now, at x=0,

$$f(0) = \sin 0 + 2 \cos 0 = 2$$

$$f'(0) = \cos 0 - 2 \sin 0 = 1$$

$$f''(0) = -\sin 0 - 2 \cos 0 = -2$$

$$f'''(0) = -\cos 0 + 2 \sin 0 = -1$$

We know,  $P_n(0) = \frac{f^0(x)}{0!}x^0 + \frac{f'(x)}{1!}x^1 + \frac{f''(x)}{2!}x^2 + \frac{f'''(x)}{3!}x^3 + \dots$

According to Taylor series around x=0,

$$P_3(0) = \frac{f^0(0)}{0!}x^0 + \frac{f'(0)}{1!}x^1 + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3$$

$$\Rightarrow P_3(0) = \frac{2}{0!}(x-0)^0 + \frac{1}{1!}(x-0)^1 + \frac{-2}{2!}(x-0)^2 + \frac{-1}{3!}(x-0)^3$$

$$\Rightarrow P_3(0) = 2 + (x-0) - (x-0)^2 - \frac{x^3}{6}$$

$$\Rightarrow P_3(0) = 2 + x - x^2 - \frac{1}{6}x^3$$

$$\therefore P_3(0) = 2 + x - x^2 - \frac{1}{6}x^3$$

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51 \begin{newpage}
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53 \textsc{Assignment 3}\\
54 \textsc{Problem 2}\\
55 [0.5 cm]
56 \end{flushright}
57 \begin{center}
58 \textbf{\Large \underline{Ans to the question no 02}}\\
59 [0.5 cm]
60 \end{center}
61 \Large We know,\\
62 $P_n(0)=\frac{f^{(0)}(x)}{0!}x^0+\frac{f'(x)}{1!}x^1+\frac{f''(x)}{2!}x^2+\frac{f'''(x)}{3!}x^3+...$\\
63 . $\\
64 Given function, \\
65 $f(x) = \ln(e^{2x} + e^{-2x})$\\
66 By differentiating we get,\\
67 $f'(x) = \frac{1}{e^{2x}+e^{-2x}} \cdot (2e^{2x} - 2e^{-2x}) = \frac{2(e^{4x}-1)}{e^{4x}+1}$\\
68 $f''(x) = \frac{2 \cdot 4e^{4x}(e^{4x}+1) - 4e^{4x} \cdot 2(e^{4x}-1)}{(e^{4x}+1)^2}$\\
69 $\Rightarrow f''(x) = \frac{8e^{4x}(e^{4x}+1) - 8e^{4x}(e^{4x}-1)}{(e^{4x}+1)^2}$\\
70 $\Rightarrow f''(x) = \frac{8e^{4x}(e^{4x}-e^{4x}+2)}{(e^{4x}+1)^2}$\\
71 $\Rightarrow f''(x) = \frac{16e^{4x}}{(e^{4x}+1)^2}$\\
72 Putting x=0 in f(x) and its derivatives we get,\\
73 $f(0) = \ln(e^{2 \cdot 0} + e^{-2 \cdot 0}) = \ln 2$\\
74 $f'(0) = \frac{2(e^{4 \cdot 0}-1)}{e^{4 \cdot 0}+1} = \frac{0}{2} = 0$\\
75 $f''(0) = \frac{16e^{4 \cdot 0}}{(e^{4 \cdot 0}+1)^2} = \frac{16}{4} = 4$\\
76 Therefore,\\
77 $P_o = \frac{f^{(0)}(0)x^0}{0!} = \ln 2$\\
78 $P_1 = \frac{f^{(0)}(0)x^0}{0!} + \frac{f'(0)x^1}{1!} = \ln 2 + 0 = \ln 2$\\
79 $P_2 = \frac{f^{(0)}(0)x^0}{0!} + \frac{f'(0)x^1}{1!} + \frac{f''(0)x^2}{2!} = \ln 2 + 0 + 2x^2$

```

### Ans to the question no 02

We know,

$$P_n(0) = \frac{f^{(0)}(x)}{0!}x^0 + \frac{f'(x)}{1!}x^1 + \frac{f''(x)}{2!}x^2 + \frac{f'''(x)}{3!}x^3 + \dots$$

Given function,

$$f(x) = \ln(e^{2x} + e^{-2x})$$

By differentiating we get,

$$f'(x) = \frac{1}{e^{2x}+e^{-2x}} \cdot (2e^{2x} - 2e^{-2x}) = \frac{2(e^{4x}-1)}{e^{4x}+1}$$

$$f''(x) = \frac{2 \cdot 4e^{4x}(e^{4x}+1) - 4e^{4x} \cdot 2(e^{4x}-1)}{(e^{4x}+1)^2}$$

$$\Rightarrow f''(x) = \frac{8e^{4x}(e^{4x}+1) - 8e^{4x}(e^{4x}-1)}{(e^{4x}+1)^2}$$

$$\Rightarrow f''(x) = \frac{8e^{4x}(e^{4x}-e^{4x}+2)}{(e^{4x}+1)^2}$$

$$\Rightarrow f''(x) = \frac{16e^{4x}}{(e^{4x}+1)^2}$$

Putting x=0 in f(x) and its derivatives we get,

$$f(0) = \ln(e^{2 \cdot 0} + e^{-2 \cdot 0}) = \ln 2$$

$$f'(0) = \frac{2(e^{4 \cdot 0}-1)}{e^{4 \cdot 0}+1} = \frac{0}{2} = 0$$

$$f''(0) = \frac{16e^{4 \cdot 0}}{(e^{4 \cdot 0}+1)^2} = \frac{16}{4} = 4$$

Therefore,

$$P_o = \frac{f^{(0)}(0)x^0}{0!} = \ln 2$$

$$P_1 = \frac{f^{(0)}(0)x^0}{0!} + \frac{f'(0)x^1}{1!} = \ln 2 + 0 = \ln 2$$

$$P_2 = \frac{f^{(0)}(0)x^0}{0!} + \frac{f'(0)x^1}{1!} + \frac{f''(0)x^2}{2!} = \ln 2 + 0 + 2x^2$$

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74 f''(0)=\frac{16e^{4\cdot 0}}{(e^{4\cdot 0}+1)^2}=\frac{16}{4}=4\ll[3mm]$
75 Therefore,\ll[3mm]
76 $P_o=\frac{f^{(0)}(0)x^{0!}}{0!}=\ln 2 \ll[3mm]
77 $P_1=\frac{f^{(1)}(0)x^{1!}}{1!}+\frac{f^{(2)}(0)x^{2!}}{2!}=\ln 2+0=\ln 2\ll[3mm]
78 $P_2=\frac{f^{(2)}(0)x^{2!}}{2!}+\frac{f^{(3)}(0)x^{3!}}{3!}=\ln 2+2x^2$
79 \end{newpage}
80 \begin{newpage}
81 \begin{flushright}
82 \textsc{Assignment 3}\ll
83 \textsc{Problem 3}\ll
84 [1 cm]
85 \end{flushright}
86 \begin{center}
87 \textbf{\underline{Ans to the question no 03}}\ll
88 [1 cm]
89 \end{center}
90 \Large {Given multi-variable function, \ll[3mm]
91 $f(x,y) = (x^2 + 2y^2x + 3x + y^2)^2\ll[5mm]
92 Derivatives of f(x,y) with respect to x considering other variables constant,\ll[3mm]
93 $f_x=2(x^2 + 2y^2x + 3x + y^2)(2x+2y^2+3)\ll[3mm]
94 $f_{xx}=2\{2(x^2 + 2y^2x + 3x + y^2)+(2x+2y^2+3)^2\}\ll[3mm]
95 \Rightarrow f_{xx}=2\{(2x^2 + 4xy^2 + 6x + 2y^2)+(4x^2+4y^4+9+8xy^2+12y^2+12x)\}\ll[3mm]
96 \Rightarrow f_{xx}=2(6x^2 + 12xy^2 + 18x + 14y^2 + 4y^2 + 9)
97 \therefore f_{xx}=2(6x^2 + 12xy^2 + 18x + 14y^2 + 4y^2 + 9) \ll[5mm]$
98 Similarly,\ll[3mm]
99 $f_y=2(x^2 + 2y^2x + 3x + y^2)(4xy+2y)\ll[3mm]
100 $f_{yy}=2\{(4x+2)(x^2 + 2y^2x + 3x + y^2)+(4xy+2y)^2\}\ll[3mm]
101 \Rightarrow f_{yy}=2\{(4x^3+8x^2y^2+12x^2+4xy^2+2x^2+4xy^2+6x+2y^2)+(16x^2y^2+4y^2+16xy^2)\}\ll[3mm]
102 \Rightarrow f_{yy}=2(4x^3+24x^2y^2+14x^2+24xy^2+6x+6y^2)\ll[3mm]
103 \therefore f_{yy}=2(4x^3+24x^2y^2+14x^2+24xy^2+6x+6y^2)$
104 \end{newpage}
105 \begin{newpage}
106 \begin{flushright}
107 \textsc{Assignment 3}\ll
108 \textsc{Problem 1}\ll

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ASSIGNMENT 3  
PROBLEM 3

### Ans to the question no 03

Given multi-variable function,

$$f(x,y) = (x^2 + 2y^2x + 3x + y^2)^2$$

Derivatives of f(x,y) with respect to x considering other variables constant,

$$f_x = 2(x^2 + 2y^2x + 3x + y^2)(2x + 2y^2 + 3)$$

$$f_{xx} = 2\{2(x^2 + 2y^2x + 3x + y^2) + (2x + 2y^2 + 3)^2\}$$

$$\Rightarrow f_{xx} = 2\{(2x^2 + 4xy^2 + 6x + 2y^2) + (4x^2 + 4y^4 + 9 + 8xy^2 + 12y^2 + 12x)\}$$

$$\Rightarrow f_{xx} = 2(6x^2 + 12xy^2 + 18x + 14y^2 + 4y^2 + 9)$$

$$\therefore f_{xx} = 2(6x^2 + 12xy^2 + 18x + 14y^2 + 4y^2 + 9)$$

Similarly,

$$f_y = 2(x^2 + 2y^2x + 3x + y^2)(4xy + 2y)$$

$$f_{yy} = 2\{(4x + 2)(x^2 + 2y^2x + 3x + y^2) + (4xy + 2y)^2\}$$

$$\Rightarrow f_{yy} = 2\{(4x^3 + 8x^2y^2 + 12x^2 + 4xy^2 + 2x^2 + 4xy^2 + 6x + 2y^2) + (16x^2y^2 + 4y^2 + 16xy^2)\}$$

$$\Rightarrow f_{yy} = 2(4x^3 + 24x^2y^2 + 14x^2 + 24xy^2 + 6x + 6y^2)$$

$$\therefore f_{yy} = 2(4x^3 + 24x^2y^2 + 14x^2 + 24xy^2 + 6x + 6y^2)$$



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104 \end{newpage}
105 \begin{newpage}
106 \begin{flushright}
107 \textsc{Assignment 3}\\
108 \textsc{Problem 4}\\
109 [1 cm]
110 \end{flushright}
111 \begin{center}
112 \textbf{\Large \underline{Ans to the question no 04}}\\
113 [0.5 cm]
114 \end{center}
115 \Large {Given function, \\[2mm]
116 $f(x, y) = \ln((x^2 + 3y + xy)^2)$\\[2mm]
117 where $x(t) = 2t+3, y(t) = t^2+3t$ \\[2mm]
118 Chair rule for partial derivatives,\\[2mm]
119 $\frac{df}{dt} = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dt}$\\[2mm]
120 Now, \\[2mm]
121 $\frac{\partial f}{\partial x} = \frac{1}{(x^2 + 3y + xy)^2} \cdot 2(x^2 + 3y + xy) \cdot 2x = \frac{2x}{x^2 + 3y + xy}$\\[2mm]
122 $\frac{\partial f}{\partial y} = \frac{1}{(x^2 + 3y + xy)^2} \cdot 2(x^2 + 3y + xy) \cdot (3 + x) = \frac{2(3+x)}{x^2 + 3y + xy}$\\[2mm]
123 $\frac{dx}{dt} = 2, \frac{dy}{dt} = 2t+3$\\[3mm]
124 Imposing these values on the chain rule formula,\\[2mm]
125 $\frac{df}{dt} = \frac{2x}{x^2 + 3y + xy} \cdot 2 + \frac{2(3+x)}{x^2 + 3y + xy} \cdot (2t+3)$\\[2mm]
126 Since we have to evaluate this expression when $t=0$,\\[2mm]
127 $\frac{df}{dt} = \frac{4(2x+y)}{(x^2 + 3y + xy)} + \frac{2(3+x)(2+3)}{(x^2 + 3y + xy)}$\\[2mm]
128 $= \frac{8x+4y}{(x^2 + 3y + xy)} + \frac{6x+18}{(x^2 + 3y + xy)}$\\[2mm]
129 $= \frac{14x+4y+18}{(x^2 + 3y + xy)}$\\[2mm]
130 \therefore $\frac{df}{dt} = \frac{14x+4y+18}{(x^2 + 3y + xy)}$ when $t=0$
131 \end{newpage}
132 \begin{newpage}
133 \begin{flushright}
134 \textsc{Assignment 3}\\
135 \textsc{Problem 5}

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ASSIGNMENT 3  
PROBLEM 4

### Ans to the question no 04

Given function,

$$f(x, y) = \ln((x^2 + 3y + xy)^2)$$

Where  $x(t) = 2t + 3, y(t) = t^2 + 3t$

Chair rule for partial derivatives,

$$\frac{df}{dt} = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dt}$$

Now,

$$\frac{\partial f}{\partial x} = \frac{1}{(x^2 + 3y + xy)^2} \cdot 2(x^2 + 3y + xy) \cdot (2x + y) = \frac{2(2x+y)}{(x^2 + 3y + xy)}$$

$$\frac{\partial f}{\partial y} = \frac{1}{(x^2 + 3y + xy)^2} \cdot 2(x^2 + 3y + xy) \cdot (3 + x) = \frac{2(3+x)}{(x^2 + 3y + xy)}$$

$$\frac{dx}{dt} = 2, \frac{dy}{dt} = 2t + 3$$

Imposing these values on the chain rule formula,

$$\frac{df}{dt} = \frac{2 \cdot 2(2x+y)}{(x^2 + 3y + xy)} + \frac{2(3+x)(2t+3)}{(x^2 + 3y + xy)}$$

Since we have to evaluate this expression when  $t=0$ ,

$$\frac{df}{dt} = \frac{4(2x+y)}{(x^2 + 3y + xy)} + \frac{2(3+x)(2+3)}{(x^2 + 3y + xy)}$$

$$= \frac{8x+4y}{(x^2 + 3y + xy)} + \frac{6x+18}{(x^2 + 3y + xy)}$$

$$= \frac{14x+4y+18}{(x^2 + 3y + xy)}$$

$\therefore \frac{df}{dt} = \frac{14x+4y+18}{(x^2 + 3y + xy)}$  when  $t=0$

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131 \therefore \frac{df}{dt}=\frac{14x+4y+18}{(x^2 + 3y + xy)}$ when t=0
132 \end{newpage}
133 \begin{newpage}
134 \begin{flushright}
135 \textsc{Assignment 3}\\
136 \textsc{Problem 5}\\
137 [1 cm]
138 \end{flushright}
139 \begin{center}
140 \textbf{\Large \underline{Ans to the question no 05}}\\
141 [1 cm]
142 \end{center}
143 \Large {Given function, \[3mm]}
144 $y^2\sin(x^3)+ze^{3x}-\cos(z^2) = z^4-y^2+x^2$\\[3mm]
145 Imposing partial differentiation on both sides with respect to x ,\[3mm]
146 $\frac{\partial}{\partial x}\{y^2\sin(x^3)+ze^{3x}-\cos(z^2)\}=\frac{\partial}{\partial x}\{z^4-y^2+x^2\}$\\[3mm]
147 \rightarrow y^2\cos(x^3)\cdot 3x^2+\frac{\partial}{\partial x}e^{3x}+z\cdot 3e^{3x}+\sin(z^2)2z\frac{\partial z}{\partial x}=4z^3\frac{\partial z}{\partial x}+2x
148 \rightarrow \frac{\partial z}{\partial x}(e^{3x}+2z\sin(z^2)-4z^3)=2x-z\cdot 3e^{3x}-3x^2y^2\cos(x^3)
149 \rightarrow \frac{\partial z}{\partial x}=\frac{2x-3ze^{3x}-3x^2y^2\cos(x^3)}{(e^{3x}+2z\sin(z^2)-4z^3)}
150 \therefore \frac{\partial z}{\partial x}=\frac{2x-3ze^{3x}-3x^2y^2\cos(x^3)}{(e^{3x}+2z\sin(z^2)-4z^3)}$\\[4mm]
151 similarly partial differentiation on both sides with respect to y,\[3mm]
152 $\frac{\partial}{\partial y}\{y^2\sin(x^3)+ze^{3x}-\cos(z^2)\}=\frac{\partial}{\partial y}\{z^4-y^2+x^2\}$\\[3mm]
153 \rightarrow 2y\sin(x^3)+\frac{\partial}{\partial y}e^{3x}+\sin(z^2)2z\frac{\partial z}{\partial y}=4z^3\frac{\partial z}{\partial y}-2y
154 \rightarrow \frac{\partial z}{\partial y}(e^{3x}+2z\sin(z^2)-4z^3)=-2y-2y\sin(x^3)
155 \rightarrow \frac{\partial z}{\partial y}=\frac{-2y(1+\sin(x^3))}{(e^{3x}+2z\sin(z^2)-4z^3)}$\\[4mm]
156 \therefore \frac{\partial z}{\partial y}=\frac{-2y(1+\sin(x^3))}{(e^{3x}+2z\sin(z^2)-4z^3)}$
157 \end{newpage}
158 \begin{newpage}
159 \begin{flushright}
160 \textsc{Assignment 3}\\
161 \textsc{Problem 6}\\
162 [1 cm]

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### Ans to the question no 05

Given function,

$$y^2 \sin(x^3) + ze^{3x} - \cos(z^2) = z^4 - y^2 + x^2$$

Imposing partial differentiation on both sides with respect to x ,

$$\frac{\partial}{\partial x}\{y^2 \sin(x^3) + ze^{3x} - \cos(z^2)\} = \frac{\partial}{\partial x}(z^4 - y^2 + x^2)$$

$$\Rightarrow y^2 \cos(x^3) \cdot 3x^2 + \frac{\partial}{\partial x}e^{3x} + z \cdot 3e^{3x} + \sin(z^2)2z\frac{\partial z}{\partial x} = 4z^3\frac{\partial z}{\partial x} + 2x$$

$$\Rightarrow \frac{\partial z}{\partial x}(e^{3x} + 2z \sin(z^2) - 4z^3) = 2x - z \cdot 3e^{3x} - 3x^2y^2 \cos(x^3)$$

$$\Rightarrow \frac{\partial z}{\partial x} = \frac{2x - 3ze^{3x} - 3x^2y^2 \cos(x^3)}{(e^{3x} + 2z \sin(z^2) - 4z^3)}$$

$$\therefore \frac{\partial z}{\partial x} = \frac{2x - 3ze^{3x} - 3x^2y^2 \cos(x^3)}{(e^{3x} + 2z \sin(z^2) - 4z^3)}$$

Similarly partial differentiation on both sides with respect to y,

$$\frac{\partial}{\partial y}\{y^2 \sin(x^3) + ze^{3x} - \cos(z^2)\} = \frac{\partial}{\partial y}(z^4 - y^2 + x^2)$$

$$\Rightarrow 2y \sin(x^3) + \frac{\partial}{\partial y}e^{3x} + \sin(z^2)2z\frac{\partial z}{\partial y} = 4z^3\frac{\partial z}{\partial y} - 2y$$

$$\Rightarrow \frac{\partial z}{\partial y}(e^{3x} + 2z \sin(z^2) - 4z^3) = -2y - 2y \sin(x^3)$$

$$\Rightarrow \frac{\partial z}{\partial y} = \frac{-2y(1 + \sin(x^3))}{(e^{3x} + 2z \sin(z^2) - 4z^3)}$$

$$\therefore \frac{\partial z}{\partial y} = \frac{-2y(1 + \sin(x^3))}{(e^{3x} + 2z \sin(z^2) - 4z^3)}$$

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150 \therefore \frac{\partial z}{\partial x} = \frac{(e^{3x} + 2z \sin(z^2) - 4z^3)\{2x - 3ze^{3x} - 3x^2y^2 \cos(x^3)\}}{(e^{3x} + 2z \sin(z^2) - 4z^3)^2}
151 Similarly partial differentiation on both sides with respect to y,
152 \frac{\partial}{\partial y} \{y^2 \sin(x^3) + ze^{3x} - \cos(z^2)\} = \frac{\partial}{\partial y} \{z^4 - y^2 + x^2\}
153 \Rightarrow 2y \sin(x^3) + \frac{\partial z}{\partial y} \{e^{3x} + \sin(z^2) 2z\} = 4z^3 - 2y
154 \Rightarrow \frac{\partial z}{\partial y} \{e^{3x} + 2z \sin(z^2) - 4z^3\} = -2y - 2y \sin(x^3)
155 \Rightarrow \frac{\partial z}{\partial y} = \frac{-2y(1 + \sin(x^3))}{(e^{3x} + 2z \sin(z^2) - 4z^3)}
156 \therefore \frac{\partial z}{\partial y} = \frac{-2y(1 + \sin(x^3))}{(e^{3x} + 2z \sin(z^2) - 4z^3)}
157 \end{newpage}
158 \begin{newpage}
159 \begin{flushright}
160 \textsc{Assignment 3}
161 \textsc{Problem 6}
162 [1 cm]
163 \end{flushright}
164 \begin{center}
165 \textbf{\Large \underline{Ans to the question no 06}}}
166 [1 cm]
167 \end{center}
168 \Large {Given function,}
169 $h(x,y,z) = y^3z^3 \cos(x^4) + x^3 \sin(z^2)$
170 We have to use partial differentiation to find $h_{yzx}$.
171 Imposing partial differentiation of $h(x,y,x)$ with respect to y,
172 $h_y = 3y^2z^3 \cos(x^4)$
173 Again partial differentiation of $h_y$ with respect to z,
174 $h_{yz} = 3y^2 3z^2 \cos(x^4) = 9y^2z^2 \cos(x^4)$
175 Finally partial differentiation of $h_{yz}$ with respect to x,
176 $h_{yzx} = 9y^2z^2 \{-\sin(x^4)(4x^3)\} = -36x^3y^2z^2 \sin(x^4)$
177 evaluating the expression of $h_{yzx}$ at point (0,1,1) we get,
178 $h_{yzx} = 36 \cdot 0^3 \cdot 1^2 \cdot 1^2 \{-\sin(0^4)\} = 0$
179 \end{newpage}
180 \end{document}

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ASSIGNMENT 3  
PROBLEM 6

Ans to the question no 06

Given function,

$$h(x, y, z) = y^3 z^3 \cos(x^4) + x^3 \sin(z^2)$$

We have to use partial differentiation to find  $h_{yzx}$ .

Imposing partial differentiation of  $h(x, y, x)$  with respect to y,

$$h_y = 3y^2 z^3 \cos(x^4)$$

Again partial differentiation of  $h_y$  with respect to z,

$$h_{yz} = 3y^2 3z^2 \cos(x^4) = 9y^2 z^2 \cos(x^4)$$

Finally partial differentiation of  $h_{yz}$  with respect to x,

$$h_{yzx} = 9y^2 z^2 \{-\sin(x^4)(4x^3)\} = -36x^3 y^2 z^2 \sin(x^4)$$

evaluating the expression of  $h_{yzx}$  at point (0,1,1) we get,

$$h_{yzx} = 36 \cdot 0^3 \cdot 1^2 \cdot 1^2 \{-\sin(0^4)\} = 0$$