Department of Computer Science and Engineering (CSE)
BRAC University

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CSE250 – Circuits and Electronics

Capacitors, Inductors, First order circuits

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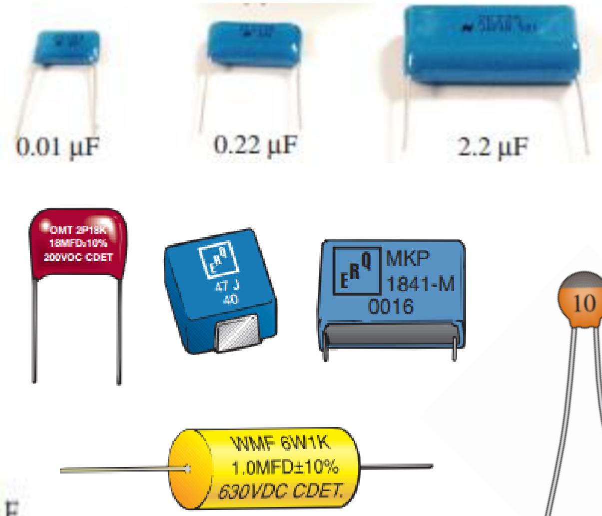
Circuit elements

- Active element
  - An active element is capable of generating energy while a passive element is not.
  - Active elements have the ability to electrically control electron flow
  - Voltage/current sources, generators, transistors, operational amplifiers.
- Passive element
  - Passive elements cannot supply energy to a circuit. They can only absorb/dissipate or store energy.
  - Resistors, capacitors, inductors, transformers

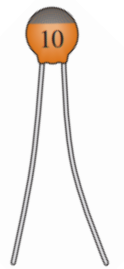
Capacitor

- A capacitor is a passive circuit element designed to store energy in its electric field.
- Unlike resistors, which dissipate energy, capacitors and inductors do not dissipate but store energy, which can be retrieved at a later time. For this reason, capacitors and inductors are called *storage* elements.

Electrolytic

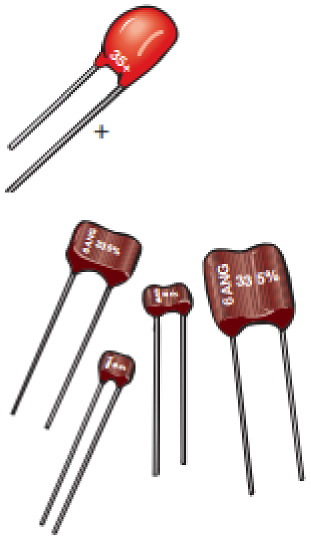


Polyester Film

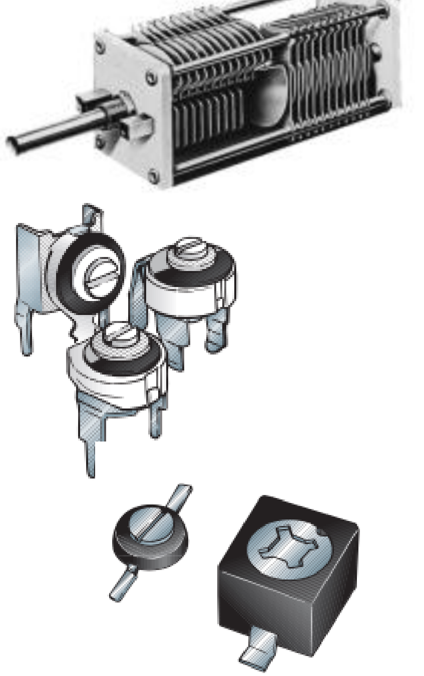


Ceramic

Mica Capacitors



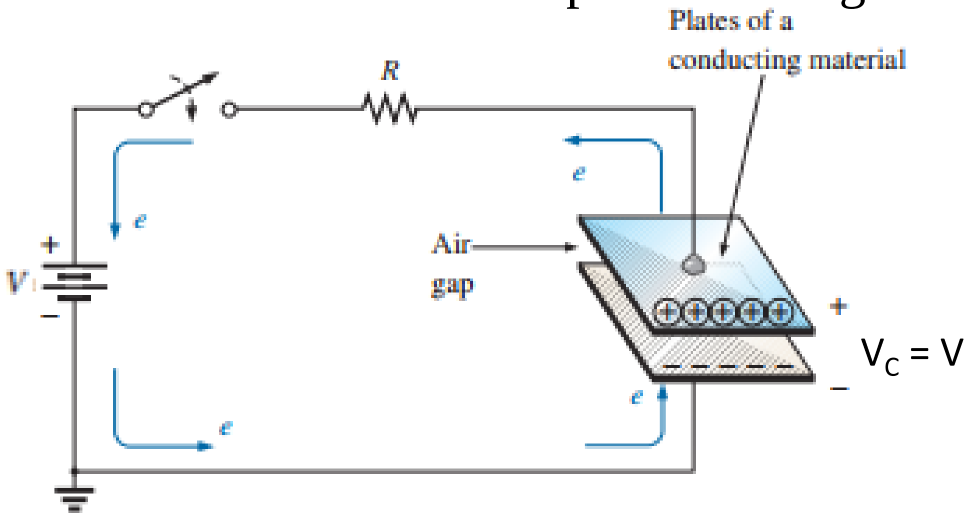
Dipped Capacitors

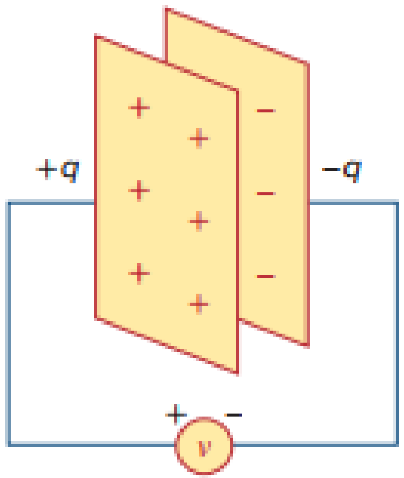


Variable Capacitors

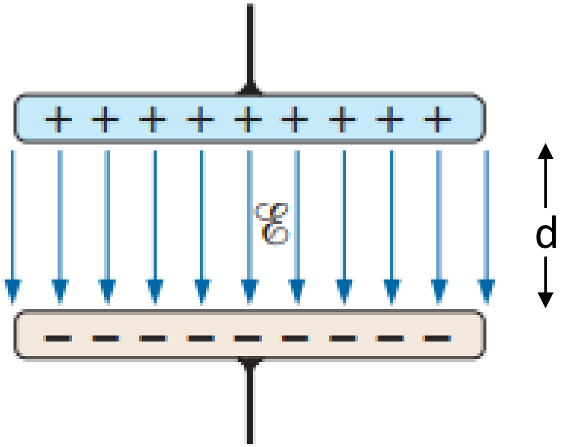
Parallel plate capacitor

- Most widely used configuration is the two conducting surfaces (aluminium mainly) separated by a dielectric (air, ceramic, paper, or mica).
- Switch open initially (no net charge)
- Closing switch causes electrons to flow from and to the upper and lower plates respectively as shown by the arrows.
- Electron flow continues until the potential difference between the plates equals the applied potential.
- The final result is a net positive charge on the top plate and a negative charge on the bottom plate.



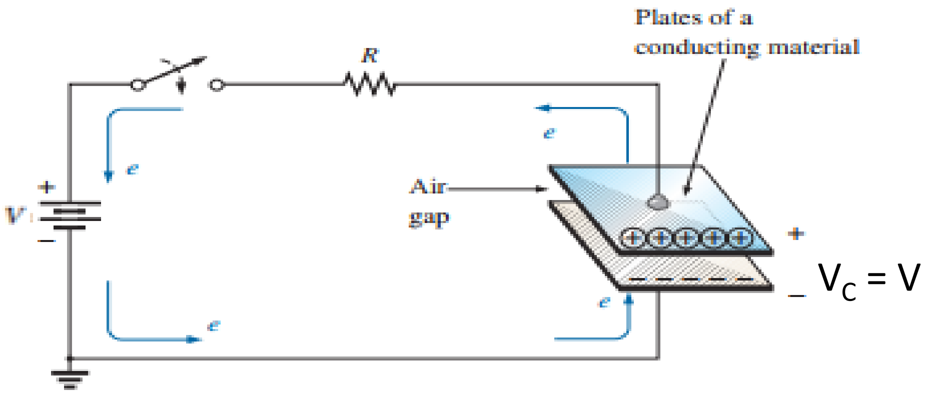


Capacitance



- Capacitance is a measure of a capacitor's ability to store charge.
- Increasing V increases E as E $\propto \frac{V}{d}$ as long as d is constant. An increase in E field causes increased charge separation i.e. increases q.
- So, q $\propto$ V

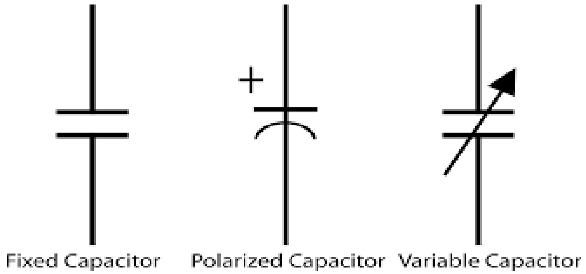
$\Rightarrow$ q = CV [C is a proportionality constant ≡ Capacitance]



$\Rightarrow$ C = $\frac{q}{V}$ (Farad, F, mF, μF)

$\Rightarrow$ For a particular capacitor ↑ V, ↑ q but $\frac{q}{V}$ = const. So, C does not depend on q or v. It depends on the physical dimension of the capacitor.

$\Rightarrow$ For the parallel plate capacitor, C = $\frac{\varepsilon A}{d}$



I-V characteristics

$\Rightarrow \quad C = \frac{q}{v}$

$\Rightarrow \quad q = Cv$

Differentiating with respect to time,

$\Rightarrow \quad \frac{dq}{dt} = C\frac{dv}{dt}$

$\Rightarrow \quad \boxed{i = C\frac{dv}{dt}}$

$\Rightarrow \quad v(t) = \frac{1}{C}\int i(t)dt$

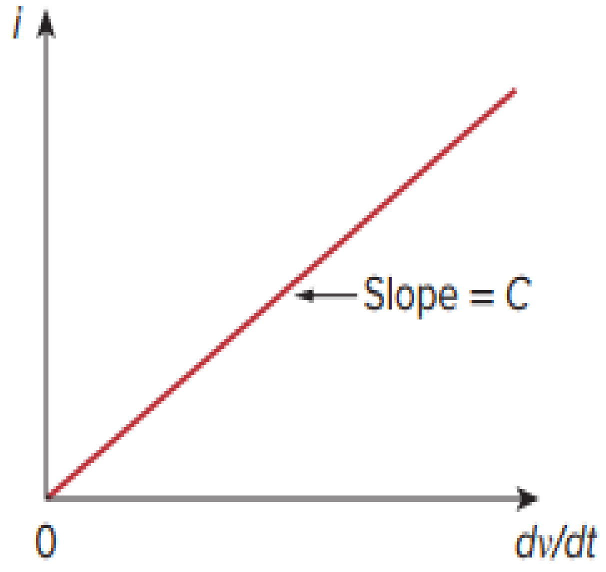
$\Rightarrow \quad v(t) = \frac{1}{C}\int_{i_0}^{t} i(t)\,dt + v(t_0)$

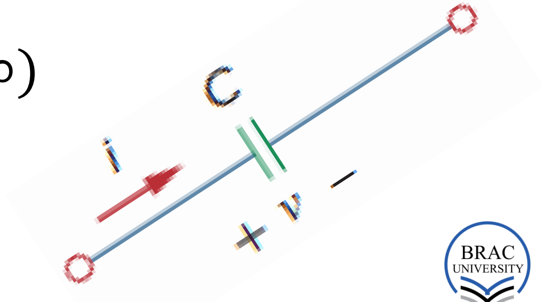
$\Rightarrow \quad$ Power, $p = vi = Cv\frac{dv}{dt}$

$\Rightarrow \quad$ Stored Energy, $w(t) = \int_{-\infty}^{t} p(t)\,dt = C\int_{-\infty}^{t} v\frac{dv}{dt}dt$

$= C\int_{v(-\infty)}^{v(t)} v\,dv = \frac{1}{2}C[v^2]_{v(-\infty)}^{v(t)}$

$\Rightarrow \quad \boxed{w(t) = \frac{1}{2}Cv2 - \frac{1}{2}Cv_0^2}$



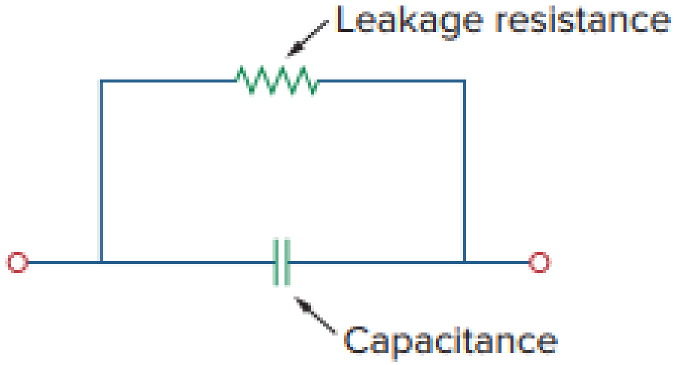


Capacitor: some important properties

1. A capacitor is an open circuit to dc.
2. The voltage on a capacitor cannot change abruptly.
3. The ideal capacitor does not dissipate energy.
4. A real, nonideal capacitor has a parallel-model leakage resistance.

1 At dc, $i_C = C\frac{dV_c}{dt} = 0$ [Open circuit]

4



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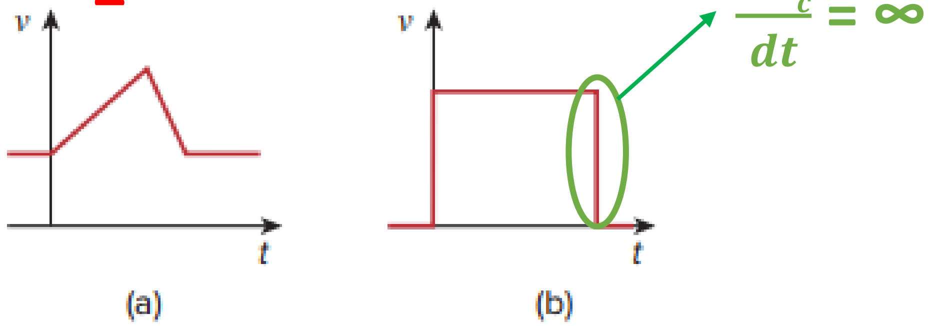
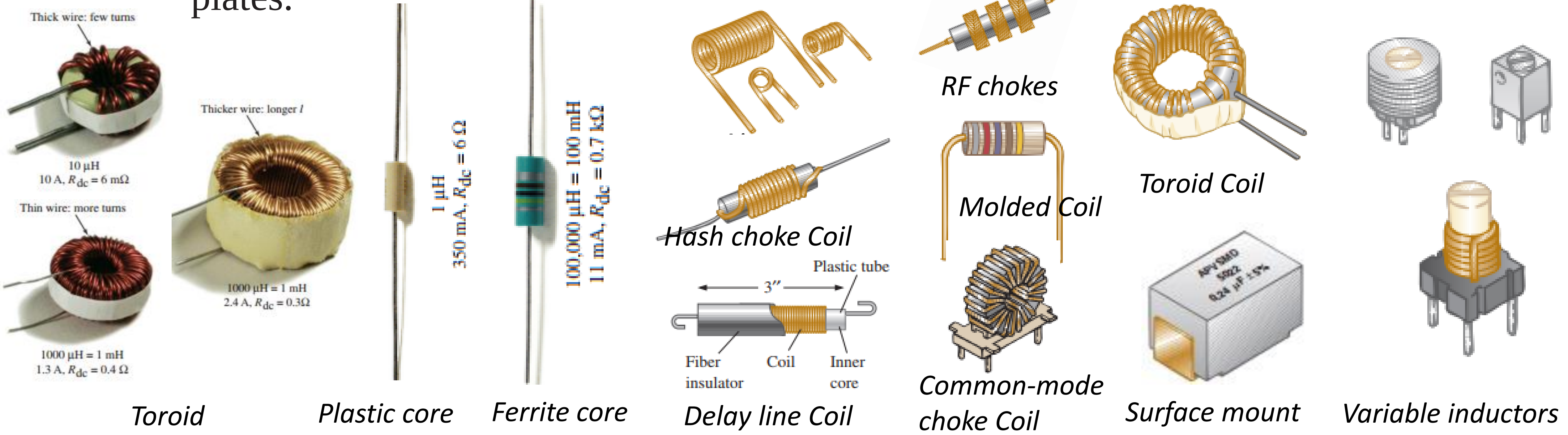


Voltage across a capacitor
(a) allowed and (b) not allowed

Inductor

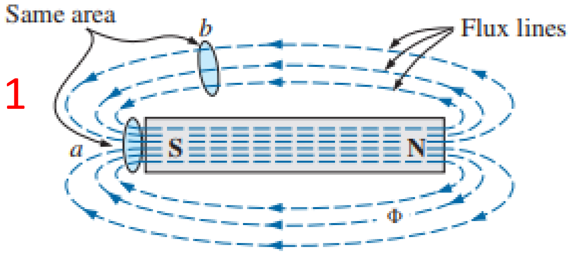
- An inductor is a passive circuit element designed to store energy in its magnetic field.
- Inductors are designed to set up a strong magnetic field linking the unit, whereas capacitors are designed to set up a strong electric field between the plates.



Magnetic field

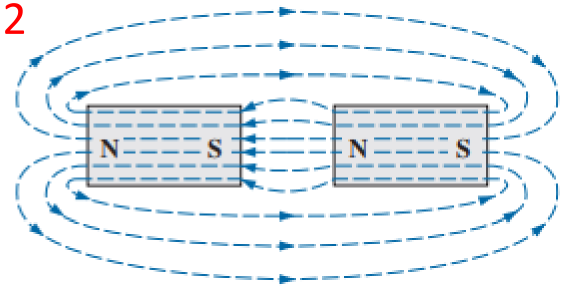
- A magnetic field exists in the region surrounding a permanent magnet, which can be represented by magnetic flux lines (imaginary) similar to electric flux lines. Magnetic flux lines, however, do not have origins or terminating points as do electric flux lines but exist in continuous loops
- In 1820, the Danish physicist Hans Christian Oersted discovered that the needle of a compass deflects if brought near a current-carrying conductor. This was the first demonstration that electricity and magnetism were related.

1



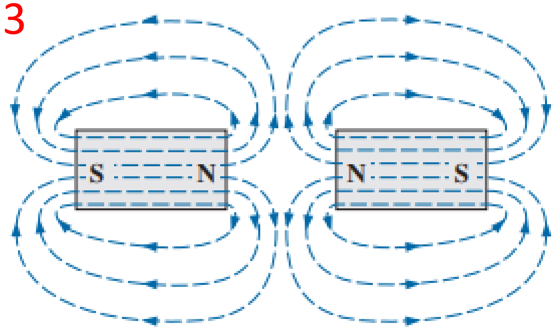
Flux distribution for a permanent magnet.

2



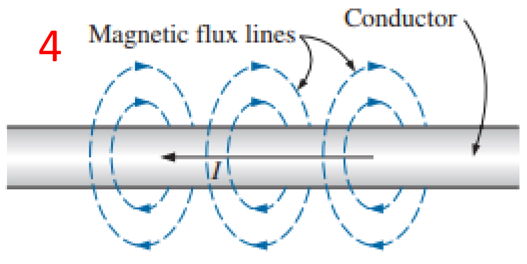
Flux distribution for two adjacent, opposite poles.

3



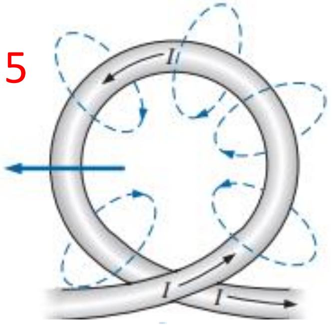
Flux distribution for two adjacent, like poles.

4



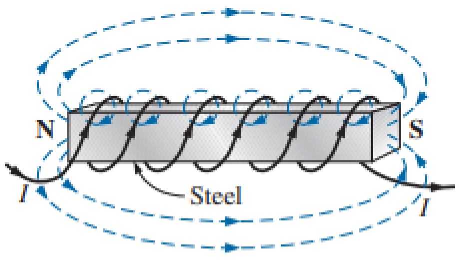
Magnetic flux lines around a current-carrying conductor.

5



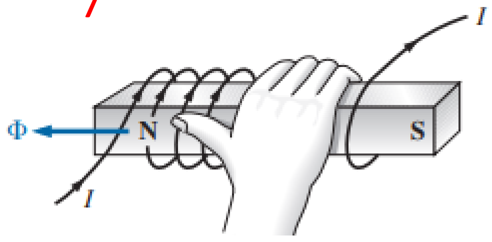
Flux distribution of a single-turn coil.

6



Electromagnet.

7



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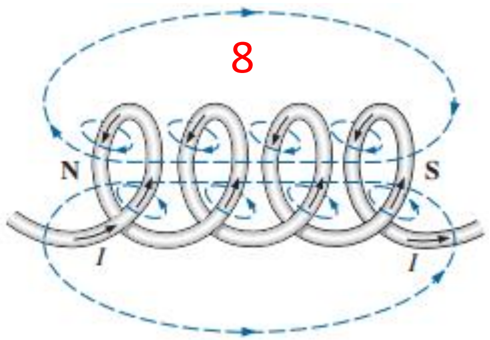
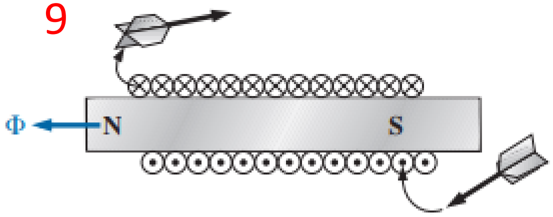


Flux distribution of a current- carrying coil.

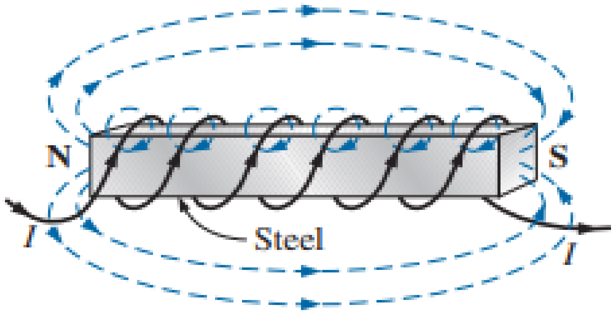
9



Flux distribution is quite similar to that of permanent magnet

Magnetic flux density

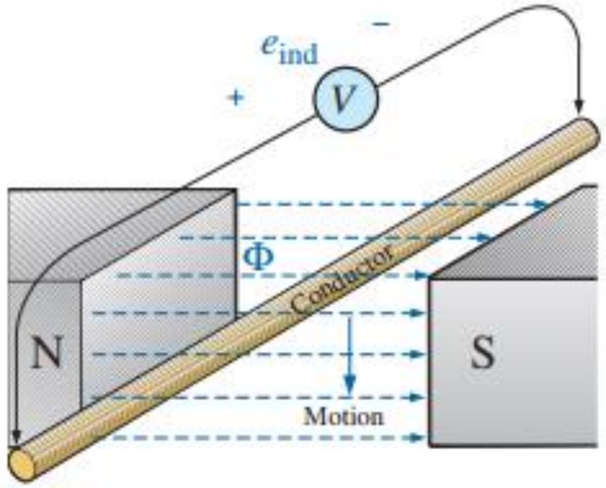
- *Magnetic flux* (ϕ) is measured in *webers (Wb)* as derived from the surname of Wilhelm Eduard Weber.
- The number of flux lines pern unit area is called *flux density (B)*. Measured in *tesla (T)*.
- In equation form,
- $B = \frac{\phi}{A}$ ($Wb/m^2 = T = 10^4$ *Gauss*)
- B of an electromagnet is directly proportional to the number of turns of, and current through, coil. Increasing either one (or both) results in increasing magnetic field.
- Another factor that affects the magnetic field strength is the type of core used. Ferromagnetic materials such as iron, cobalt, nickel, steel, alloys provide higher magnetic flux, high permeability (μ).



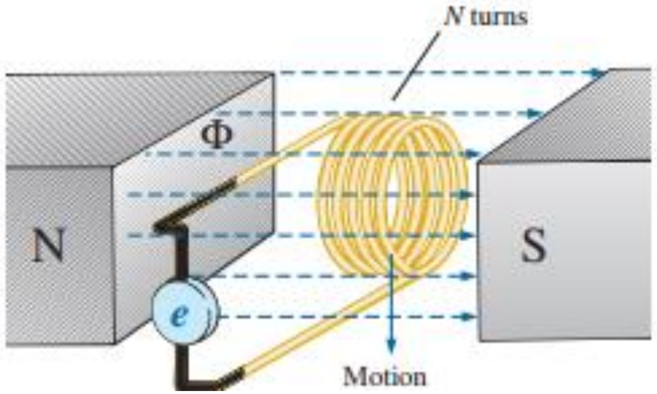
Electromagnet.

Faraday's law of electromagnetic induction

- *Faraday's law* states that, a conductor exposed to a **changing magnetic flux** will develop an induced voltage across it.
- It doesn't matter whether the changing flux is due to moving the magnetic field or moving the coil in the vicinity of a magnetic field: The only requirement is that the flux linking (passing through) the coil changes with time.
- In the form of equation,
- $e = N\dfrac{d\phi}{dt}$ (volts, V)
- This important phenomenon can now be applied to an inductor



Generating an induced voltage by moving a conductor through a magnetic field.



Demonstrating Faraday's law.

Inductance

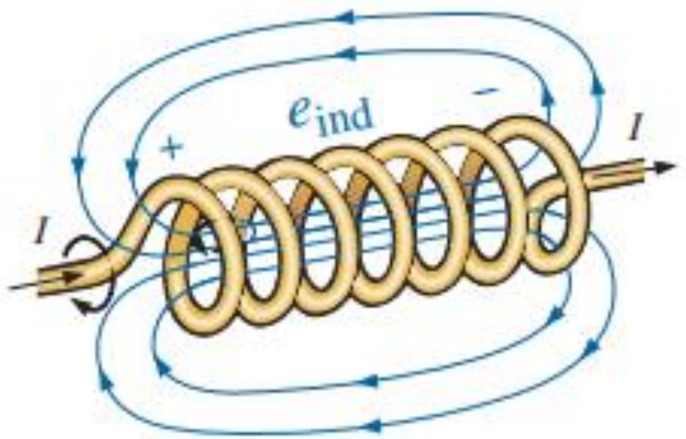
- We found that the magnetic flux linking the coil of N turns with a current I has the distribution shown in the figure.
- If the current (I) through the coil increases in magnitude, the flux (ϕ) linking the coil also increases.
- So, $\phi \propto I$

$\Rightarrow \quad \phi = LI$ [L is a proportionality constant ≡ Self inductance]

- If the loop has N number of turns,

$\Rightarrow \quad N\phi = LI$

$\Rightarrow \quad L = \frac{N\phi}{I}$ (*Henry, H, mH, μH*); $\left[L = \frac{N^2 \mu A}{l}\right.$ for a solenoid$\left.\right]$



Demonstrating the effect of Lenz's law.

- For a particular capacitor $\uparrow$ I, $\uparrow \phi$ but $\frac{\phi}{I}$ = const. So, L does not depend on ϕ or I. It depends on the physical dimension (length, # of turns, area, material) of the inductor.

Lenz's law of electromagnetic induction

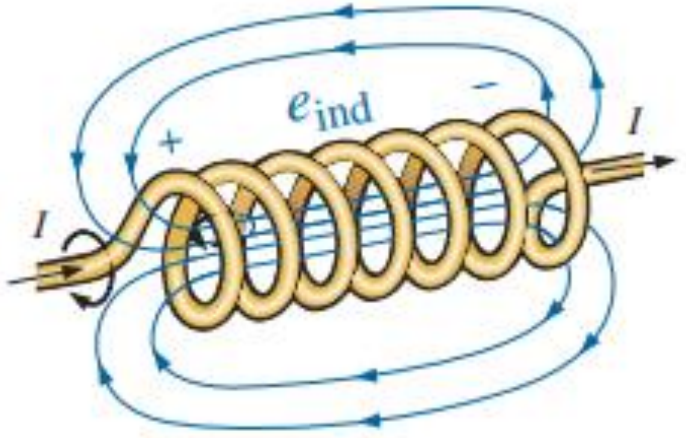
- It is very important to note in the figure that the polarity of the induced voltage across the coil is such that it opposes the increasing level of current in the coil.
- The quicker the change in current through the coil, the greater is the opposing induced voltage to squelch the attempt of the current to increase in magnitude
- Lenz's law says that an induced effect is always such as to oppose the cause that produced it.

$\Rightarrow \quad L = \dfrac{N\phi}{I}$

$\Rightarrow$ Differentiating with respect to t, $\dfrac{d\phi}{dt} = L\dfrac{dI}{dt}$

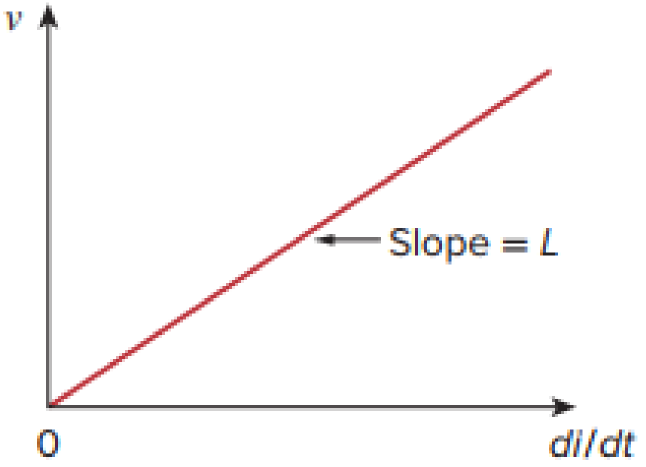
$\Rightarrow$ Substituting into Faraday's law, $e = L\dfrac{dI}{dt}$

$$L = \frac{N^2\mu A}{l}$$



Demonstrating the effect of Lenz's law.

I-V characteristics



$\Rightarrow$ In network analysis, $e = L\frac{dI}{dt}$ is expressed as,

$\Rightarrow$ $\boxed{v = L\frac{di}{dt}}$

$\Rightarrow$ di = $\frac{1}{L}vdt$

$\Rightarrow$ Power, $p = v_L i_L = L\frac{di}{dt}i$

$\Rightarrow$ Stored energy, $w(t) = \int_{-\infty}^{t} p(t)\,dt$

$$= \text{L}\int_{-\infty}^{t} i\frac{di}{dt}dt$$

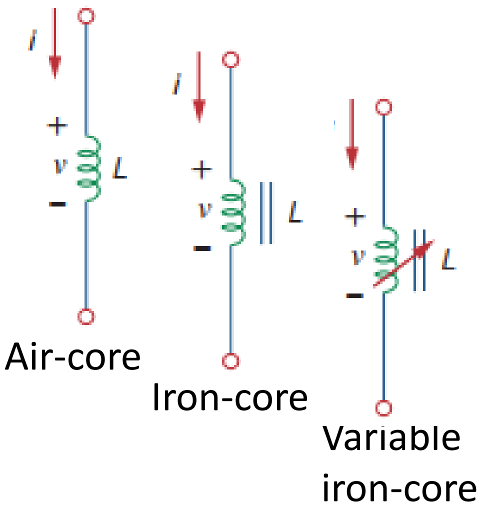
$$= \text{L}\int_{i(-\infty)}^{i(t)} i\,di$$

$$= \boxed{\frac{1}{2}Li^2}$$ [i(-∞)= 0, assumed]

$\Rightarrow$ Integrating gives,

$\Rightarrow$ i (t) = $\frac{1}{L}\int_{\infty}^{t} v(t)\,dt$

$\Rightarrow$ $\boxed{\text{i (t)} = \frac{1}{L}\int_{t_0}^{t} v(t)\,dt + v(t_0)}$

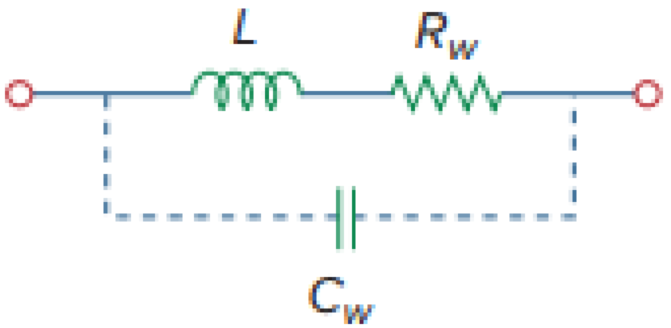


Inductor: some important properties

1. An inductor is a short circuit to dc.
2. The current on an inductor cannot change abruptly.
3. The ideal inductor does not dissipate energy.
4. A real, nonideal inductor has a series-model leakage resistance.

1 At dc, $v_L = L\frac{di}{dt} = 0$ [Short circuit]

4



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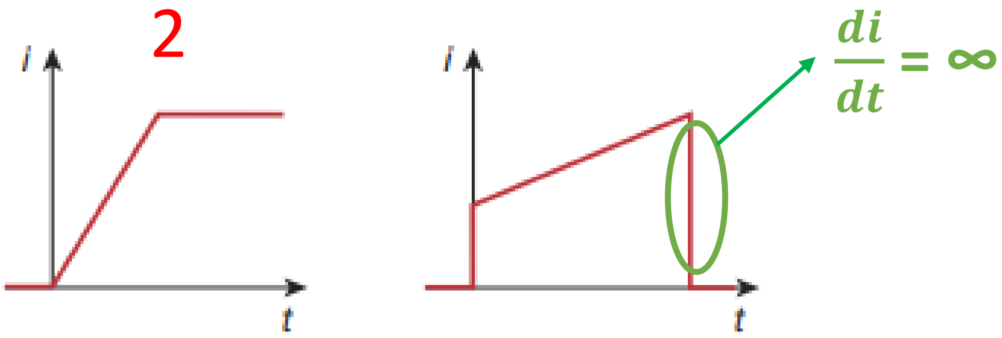


Current through an inductor
(a) allowed and (b) not allowed

First order circuits

- A first-order circuit is characterized by a first-order differential equation.
- We shall examine two types of differential circuits: circuit comprising resistors and capacitors (RC circuit) and circuit comprising resistors and inductors (RL circuit).
- Two ways to excite the circuits: (i) by initial conditions of storage elements (source free circuits) and (ii) by independent sources (DC for this course).
- The two types of first-order circuits and the two ways of exciting them add up to the four possible situations:

  (i) source free RC circuits,

  (ii) source free RL circuits,

  (iii) step response of an RC circuits, and

  (iv) step response of an RL circuits.

The source-free RC circuit

- A *source-free RC circuit* occurs when its dc source is suddenly disconnected. The energy already stored in the capacitor is released to the resistors.

$\Rightarrow$ At t = 0, $v(0) = V_0$ $(initial\ voltage\ across\ the\ capacitor)$

$\Rightarrow$ So, initially stored charge, w(0) = $\frac{1}{2}Cv_0^2$

$\Rightarrow$ From the figure, $i_C + i_R = 0$

$$\Rightarrow C\frac{dv}{dt} + \frac{v}{R} = 0$$

$$\Rightarrow \frac{dv}{dt} + \frac{v}{RC} = 0$$

$$\Rightarrow \frac{dv}{v} = -\frac{1}{RC}dt$$

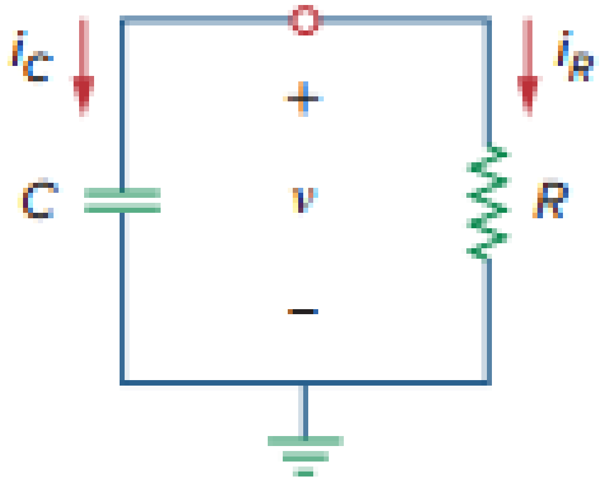
Integrating both sides,

$$\Rightarrow lnv = -\frac{t}{RC} + lnA$$

$$\Rightarrow ln\frac{v}{A} = -\frac{t}{RC}$$

$$\Rightarrow v = Ae^{-\frac{t}{RC}}$$

At t = 0, $v(0) = A = V_0$ So, $\boxed{v(t) = V_0e^{-\frac{t}{RC}}}$



Time constant for RC circuit

- $v(t) = V_0 e^{-\frac{t}{RC}}$

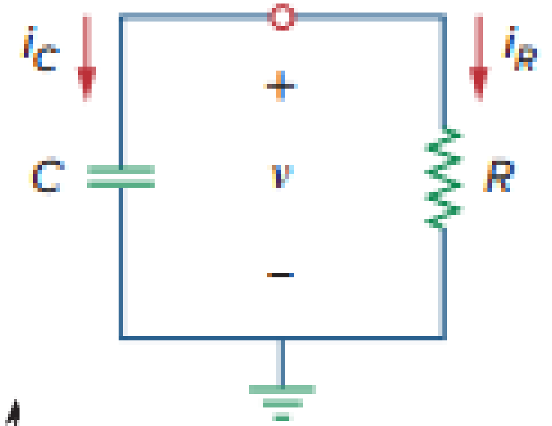
- This shows that the voltage response of the *RC* circuit is an exponential decay of the initial voltage. It is called the *natural response* of the circuit.

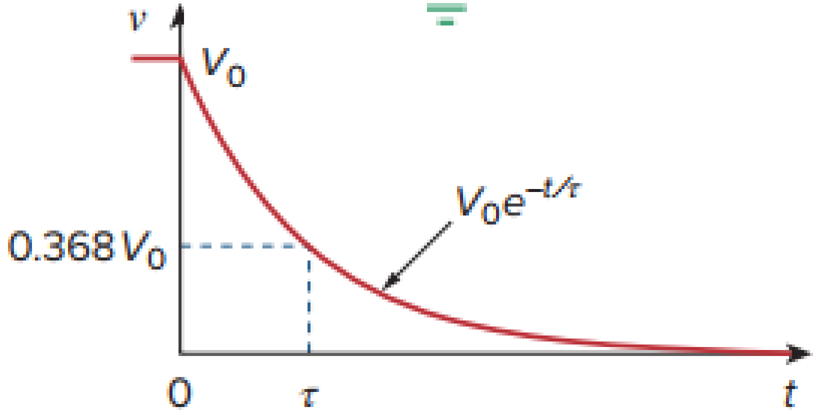
$\Rightarrow$ $\boxed{v(t) = V_0 e^{-\frac{t}{\tau}}}$, where τ = RC is the time constant (unit in sec)

R = R_{eq} as seen from the capacitor terminal

- The time constant of a circuit is the time required for the response to decay to a factor of 1/e or 36.8 percent of its initial value.

$\Rightarrow$ At t = τ, $v(t) = V_0 e^{-1} = 0.368\, V_0$



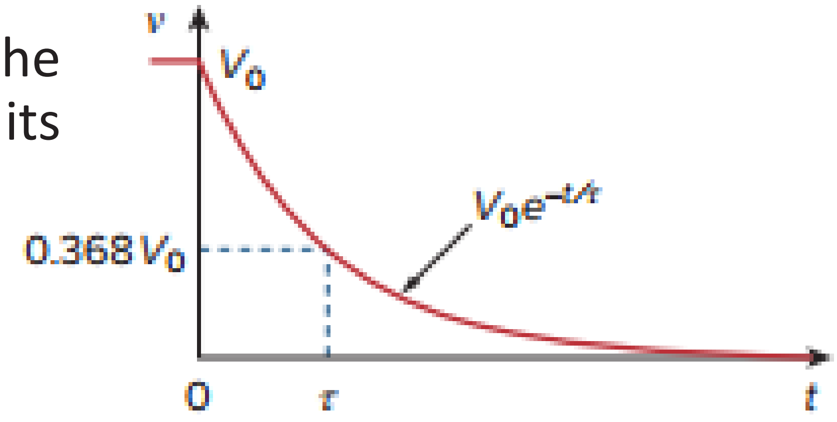


The voltage response of the *RC* circuit.

Significance of 5τ time (discharging)

- The time constant of a circuit is the time required for the response to decay to a factor of 1/e or 36.8 percent of its initial value.

$$\Rightarrow \quad v(t) = V_0 e^{-\frac{t}{\tau}}$$



- The values of $\boldsymbol{v(t)\,/\,V_0}$ for t = τ, 2τ, 3τ, 4τ, τ are calculated and listed in the table below. It is evident that $v(t)$ *is less than* 1% V_0 after 5τ time

Values of $v(t)/V_0 = e^{-t/\tau}$.

t	$v(t)/V_0$
τ	0.36788
2τ	0.13534
3τ	0.04979
4τ	0.01832
5τ	0.00674

- Thus, it is customary to assume that the capacitor is fully discharged (or charged, we will see later) after five time constants. it takes 5τ for the circuit to reach its final state or steady state when no changes take place with time.

Power & Energy (source-free RC circuit)

$\Rightarrow \; v(t) = V_0 e^{-\frac{t}{\tau}}$

$\Rightarrow \; i_R(t) = \frac{v(t)}{R} = \frac{V_0}{R} e^{-\frac{t}{\tau}}$

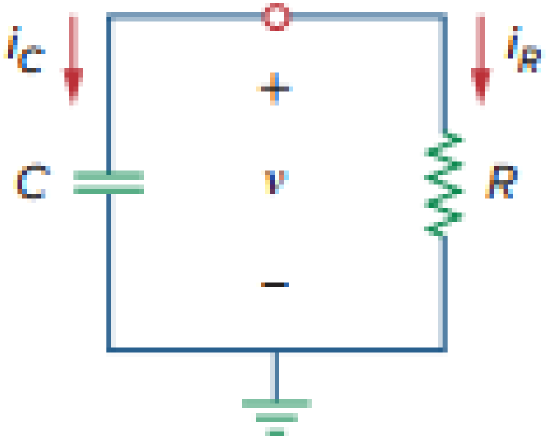
- The power dissipated in the resistor is

$\Rightarrow \; p(t) = v i_R = \frac{V_0^2}{R} e^{-2t/\tau}$

- The energy absorbed by the resistor up to time t is,

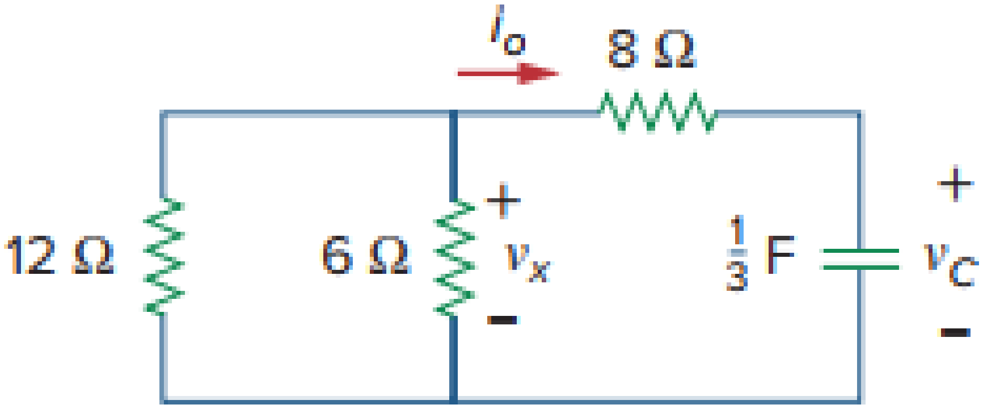
$\Rightarrow \; \mathrm{w_R}(t) = \int_0^t p(t)\,dt = \int_0^t \frac{V_0^2}{R} e^{-2t/\tau}\,dt$

$\Rightarrow \; \boxed{\mathrm{w_R}(t) = \frac{1}{2} C V_0^2 (1 - e^{-2t/\tau}),} \qquad \tau = \mathrm{RC}$



Example 1

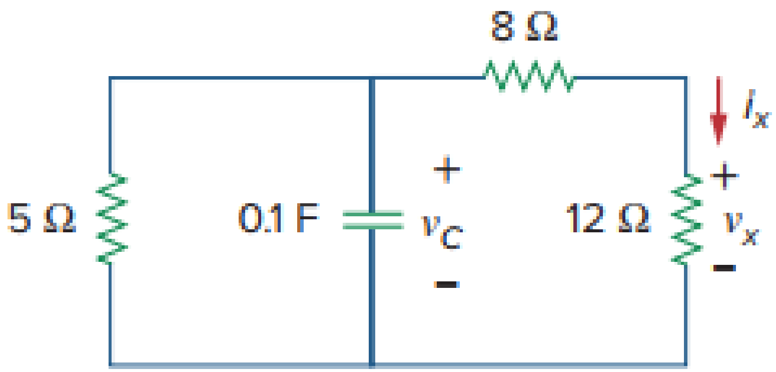
- Let V_C (0) = 60 V, Determine $\boldsymbol{V_C}$, $\boldsymbol{V_x}$, and $\boldsymbol{i_0}$ for t > 0.



Ans: $\mathbf{V_C = 60e^{-0.25t}}$ **(V);** $\mathbf{V_x = 60e^{-0.25t}}$ **(V);** $\mathbf{i_0 = -5e^{-0.25t}}$ **(A)**

Example 2

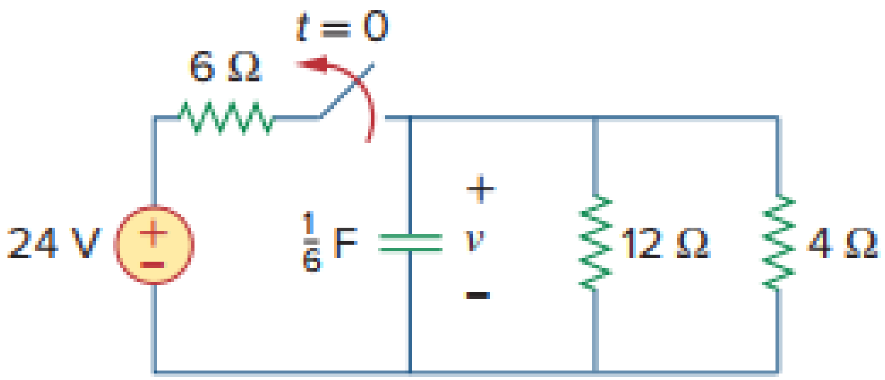
- Let $V_C(0)$ = 15 V, Find $\boldsymbol{V_C}$, $\boldsymbol{V_x}$, and $\boldsymbol{i_x}$ for t > 0.



Ans: $\mathbf{V_C = 25e^{-2.5t}\ V;\ V_x = 15e^{-2.5t}\ V;\ i_x = 1.25e^{-2.5t}\ A}$

Example 3

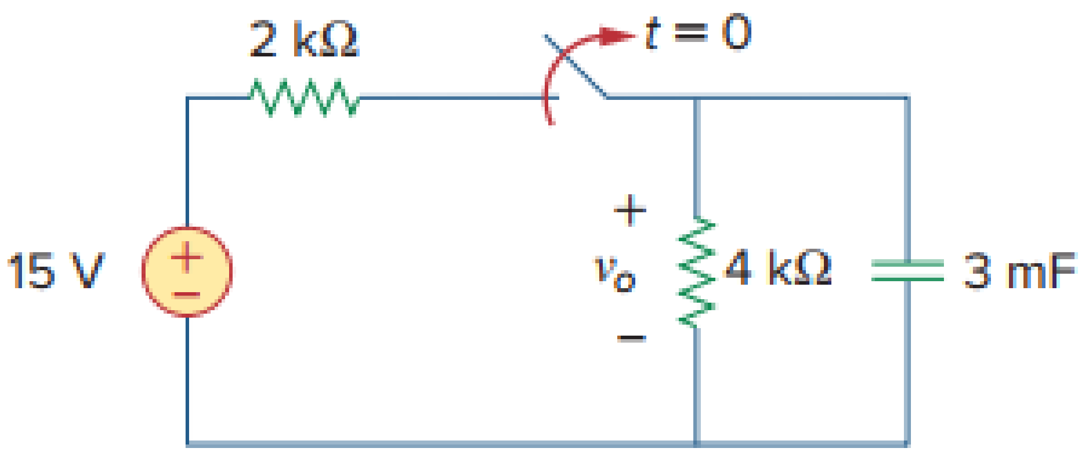
- The switch in the circuit has been closed for a long time, and it is opened at t = 0. Find ***v(t)*** for t > 0. Calculate the initial energy stored in the capacitor.



Ans: $\boldsymbol{v}\mathbf{(t) = 8e^{-2t}\ V;\ w_C(0) = 5.333\ J}$

Example 4

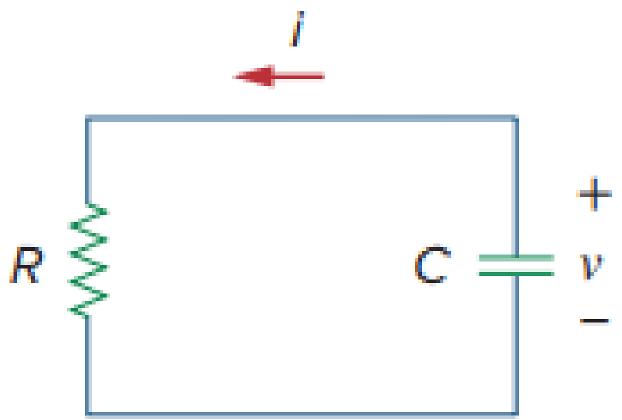
- The switch opens at t = 0. Find ***v(t)*** for t > 0.



Ans: $\boldsymbol{v}(\mathbf{t}) = \mathbf{10e^{-t/12}\ V}$

Example 5

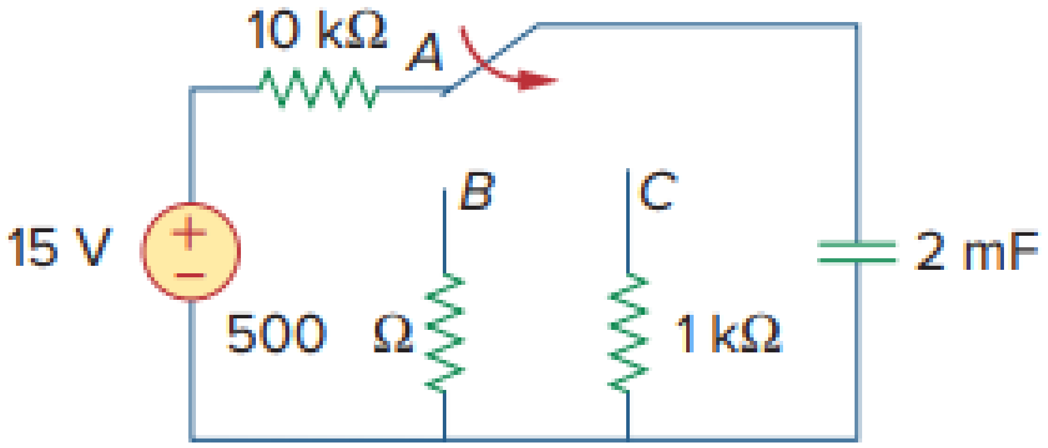
- For the circuit below, $v = 10e^{-4t}$ V and $i = 0.2e^{-4t}$ A
  - (a) Find R and C.
  - (b) Determine the time constant.
  - (c) Calculate the initial energy in the capacitor.
  - (d) Obtain the time it takes to dissipate 50% of the initial energy.



Ans: $\boldsymbol{R}$ **= 50 Ω; C = 5 mF;** $\boldsymbol{\tau}$ **= 0.25 s;** $\mathbf{w_C(0) = 0.25}$ **J; t ≈ 86 ms**

Example 6

- Assume that the switch has been in position A for a long time and is moved to position B at $t = 0$. Then at $t = 1s$, switch moves from B to C. Find ***$V_C(t)$*** for $t > 0$



Ans: $v(t) = 15e^{-t}$ V for $0 < t < 1$ sec;
$v(t) = 5.518e^{-2(t-1)}$ V for $1 < t < \infty$ sec;

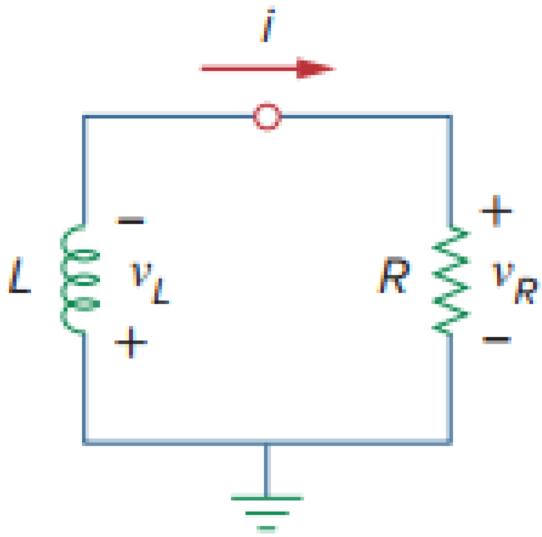
The source-free RL circuit

- A *source-free RL circuit* occurs when its dc source is suddenly disconnected. The energy already stored in the inductor is released to the resistors.

$\Rightarrow$ At t = 0, $i(0) = I_0$ $(initial\ voltage\ across\ the\ capacitor)$

$\Rightarrow$ So, initially stored charge, w(0) = $\frac{1}{2}LI_0^2$

$\Rightarrow$ From the figure, $v_L + v_R = 0$



$$\Rightarrow L\frac{di}{dt} + Ri = 0$$

$$\Rightarrow \frac{di}{dt} + \frac{R}{L}i = 0$$

$$\Rightarrow \frac{di}{i} = -\frac{R}{L}dt$$

Integrating both sides,

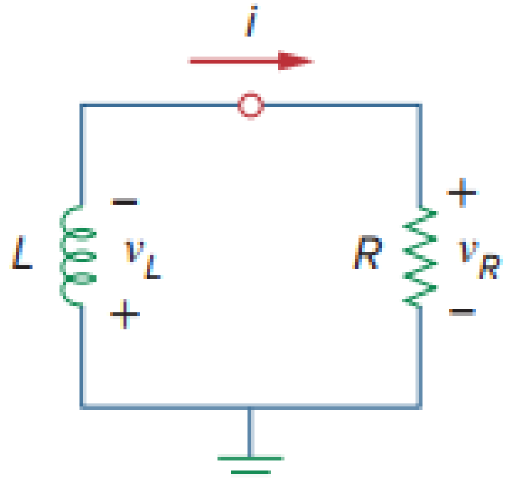
$$\Rightarrow \ln i = -\frac{R}{L}t + \ln A$$

$$\Rightarrow \ln\frac{i}{A} = -\frac{R}{L}t$$

$$\Rightarrow i = Ae^{-\frac{R}{L}t}$$

At t = 0, $i(0) = A = I_0$ So, $\boxed{i(t) = I_0e^{-\frac{R}{L}t}}$

Time constant for RL circuit



- $i(t) = I_0 e^{-\frac{R}{L}t}$

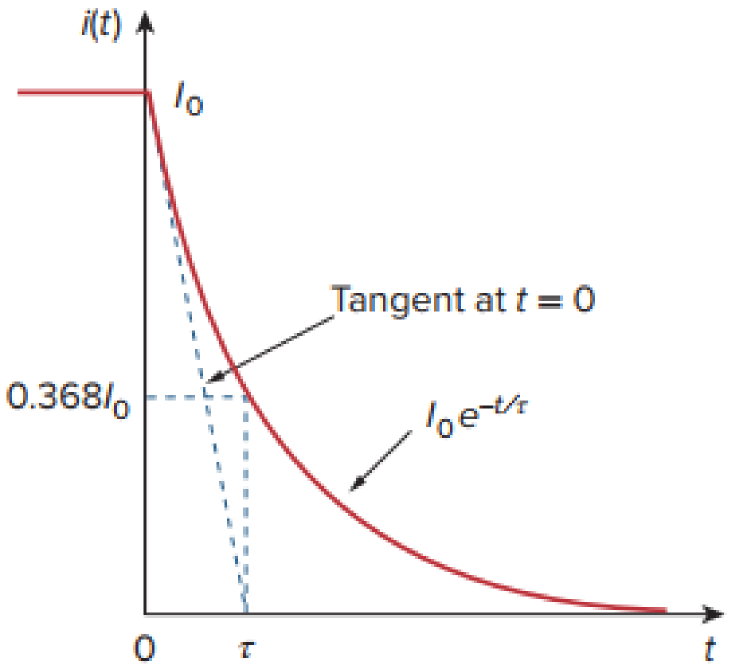
- This shows that the current response of the *RL* circuit is an exponential decay of the initial current. It is called the *natural response* of the circuit.

$\Rightarrow$ $\boxed{i(t) = I_0 e^{-\frac{t}{\tau}}}$, where $\tau = \frac{L}{R}$ is the time constant (unit in sec)

R = R_{eq} as seen from the inductor terminal

- The time constant of a circuit is the time required for the response to decay to a factor of 1/e or 36.8 percent of its initial value.

$\Rightarrow$ At t = τ, $i(t) = I_0 e^{-1} = 0.368\ I_0$



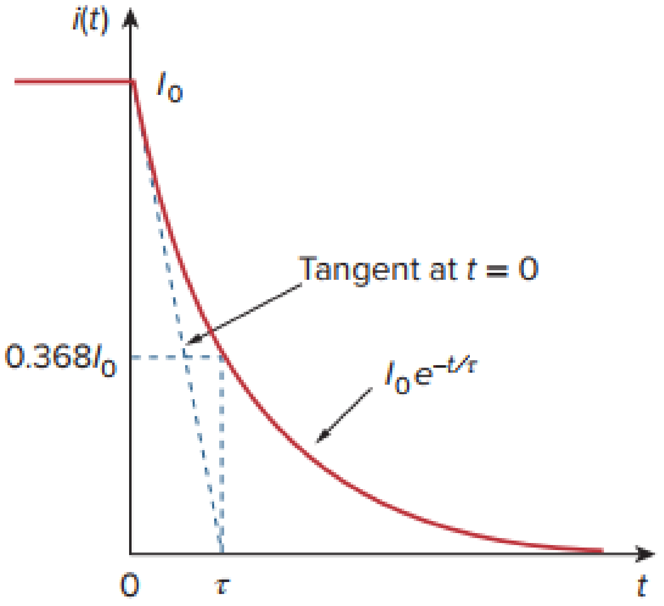
Significance of 5τ time (discharging)

- The time constant of a circuit is the time required for the response to decay to a factor of 1⁄e or 36.8 percent of its initial value.

$$\Rightarrow \; i(t) = I_0 e^{-\frac{t}{\tau}}$$

- The values of $\boldsymbol{i(t)\,/\,I_0}$ for t = τ, 2τ, 3τ, 4τ, τ are calculated and listed in the table below. It is evident that $i(t)$ *is less than* 1% I_0 after 5τ time

- Thus, it is customary to assume that the inductor is fully discharged (or charged, we will see later) after five time constants. it takes 5τ for the circuit to reach its final state or steady state when no changes take place with time.



Values of $i(t)/I_0 = e^{-t/\tau}$.

t	$i(t)/I_0$
τ	0.36788
2τ	0.13534
3τ	0.04979
4τ	0.01832
5τ	0.00674

Power & Energy (source-free RL circuit)

$\Rightarrow \; i(t) = I_0 e^{-\frac{t}{\tau}}$

$\Rightarrow \; v_R(t) = \mathrm{iR} = I_0 R e^{-\frac{t}{\tau}}$

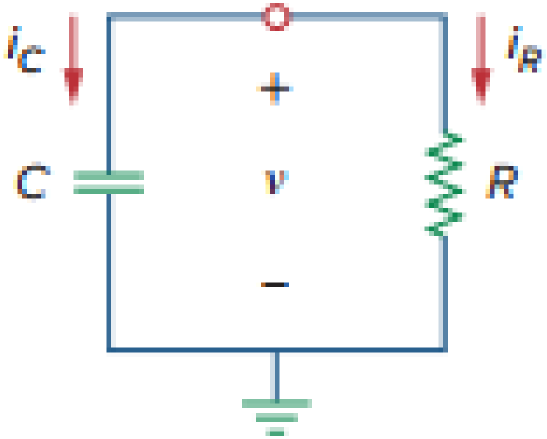
- The power dissipated in the resistor is

$\Rightarrow \; p(t) = vRi = I_0^2 R e^{-2t/\tau}$

- The energy absorbed by the resistor up to time t is,

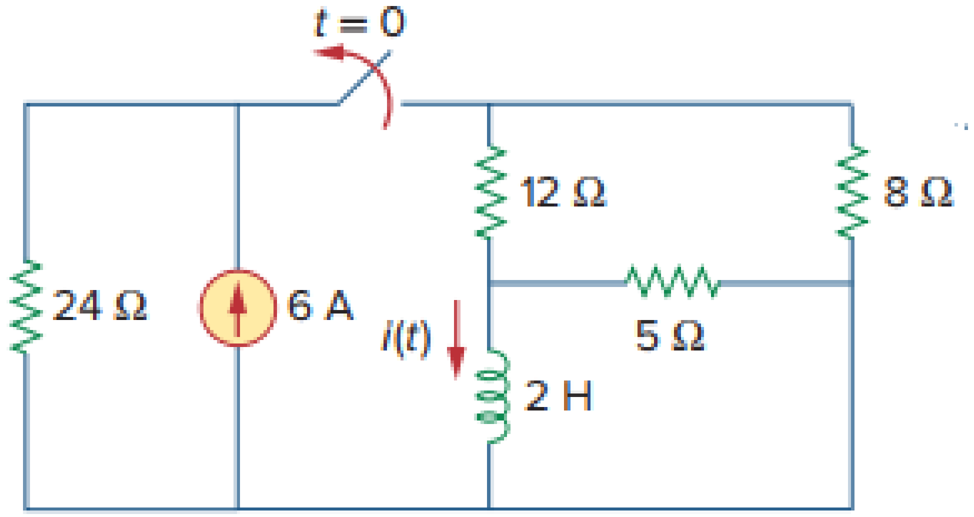
$\Rightarrow \; \mathrm{w_R}(t) = \int_0^t p(t)\, dt = \int_0^t I_0^2 R e^{-2t/\tau} dt$

$\Rightarrow \; \boxed{\mathrm{w_R}(t) = \frac{1}{2} L I_0^2 (1 - e^{-2t/\tau}),} \quad \tau = \frac{L}{R}$



Example 7

- Find ***i(t)*** for t > 0.



Ans: **i(t) = $2e^{-2t}$ (A)**

Example 8

- Find i_0, v_0, and i for all time, assuming that the switch was open for a long time.

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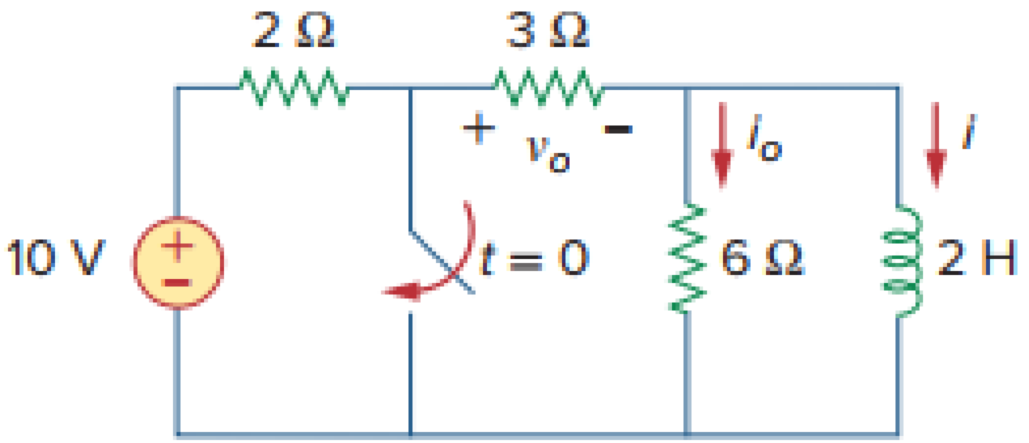

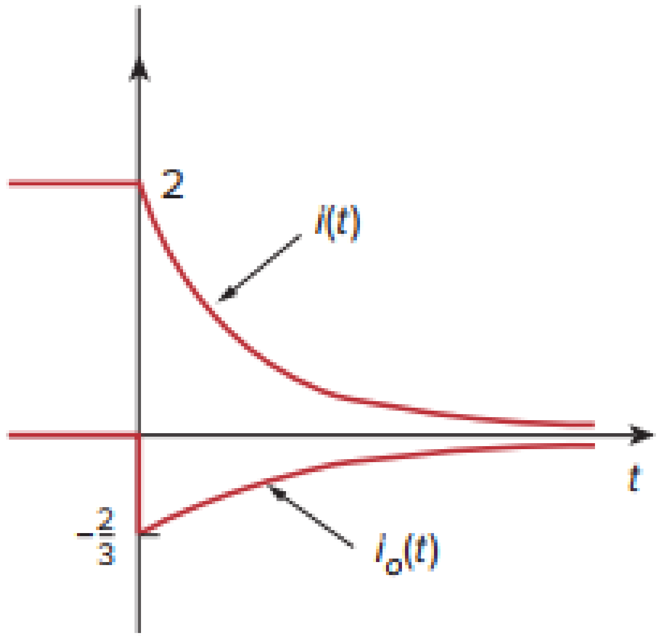

Ans:

$$i_o(t) = \begin{cases} 0 \text{ A}, & t < 0 \\ -\frac{2}{3}e^{-t} \text{ A}, & t > 0 \end{cases}, \quad v_o(t) = \begin{cases} 6 \text{ V}, & t < 0 \\ 4e^{-t} \text{ V}, & t > 0 \end{cases}$$

$$i(t) = \begin{cases} 2 \text{ A}, & t < 0 \\ 2e^{-t} \text{ A}, & t \geq 0 \end{cases}$$

Example 9

- Determine $\boldsymbol{i}$, $\boldsymbol{i_0}$, and $\boldsymbol{v_0}$ for all t in the circuit shown below. Assume that the switch was closed for a long time.

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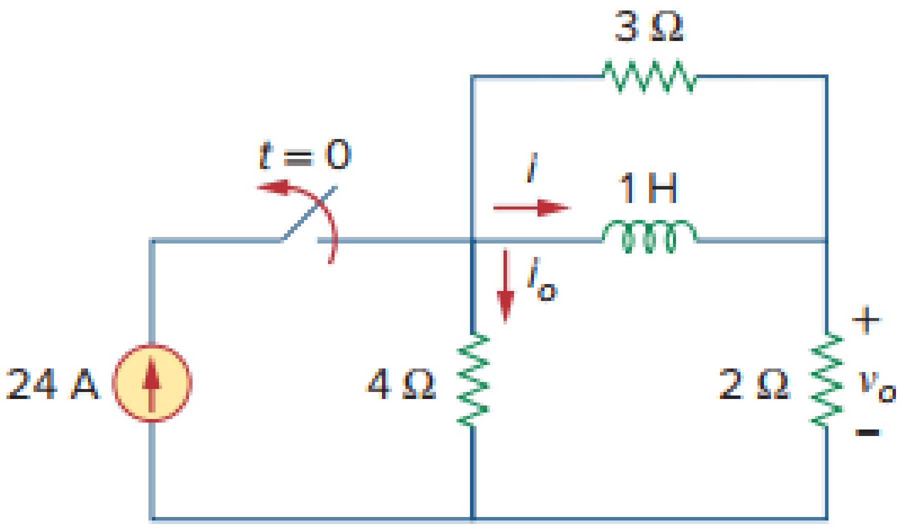


<u>Ans:</u>

$$i = \begin{cases} 16 \text{ A}, & t < 0 \\ 16e^{-2t} \text{ A}, & t \geq 0 \end{cases}, \quad i_o = \begin{cases} 8 \text{ A}, & t < 0 \\ -5.333e^{-2t} \text{ A}, & t > 0 \end{cases},$$

$$v_o = \begin{cases} 32 \text{ V}, & t < 0 \\ 10.667e^{-2t} \text{ V}, & t > 0 \end{cases}$$

Example 10

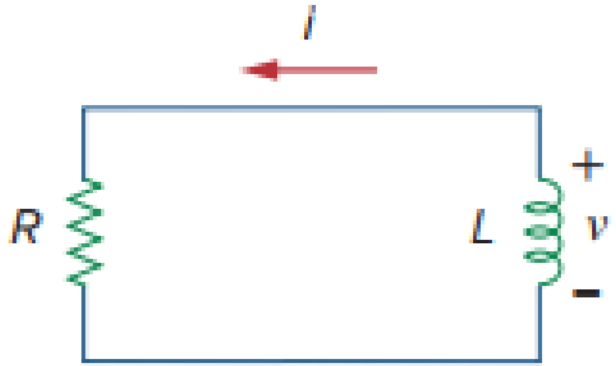
- For the circuit below, $v = 90e^{-50t}$ V and $i = 30e^{-50t}$ A for t > 0

  (a) Find L and R.

  (b) Determine the time constant.

  (c) Calculate the initial energy in the inductor.

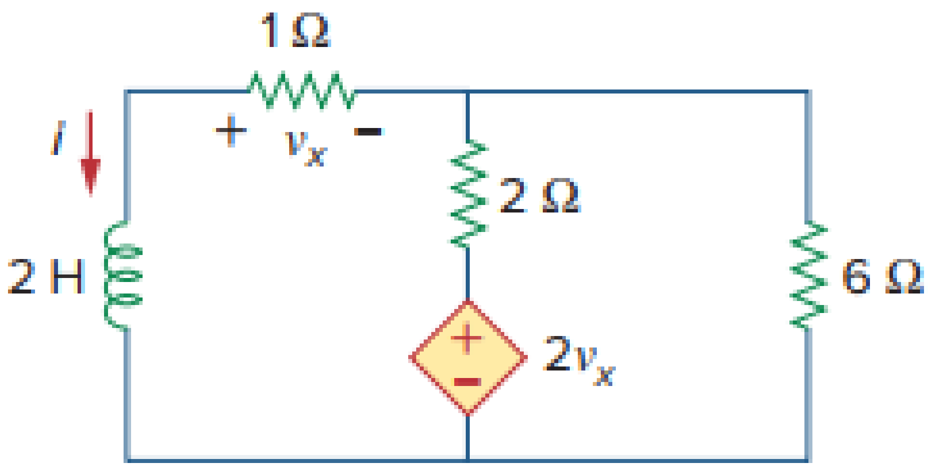
  (d) What fraction of the initial energy is dissipated in 10 ms.



Ans: $R = 3\ \Omega$; L = 60 mH; $\tau = 0.02$ s; $w_L(0) = 27$ J; % ≈ 94.7

Example 11

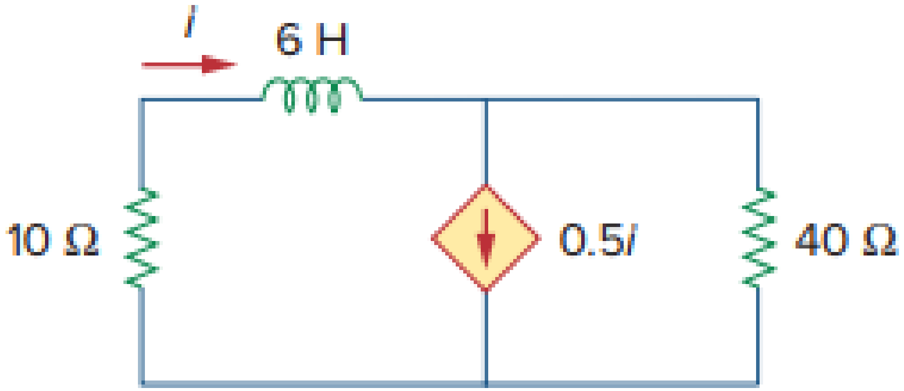
- Find ***i*** and $\boldsymbol{v_x}$ if i(0) = 7 A



Ans: $\boldsymbol{i}(t) = 7e^{-2t}$ **A;** $\boldsymbol{v_x}(t) = -\ 7e^{-2t}$ **V**

Example 12

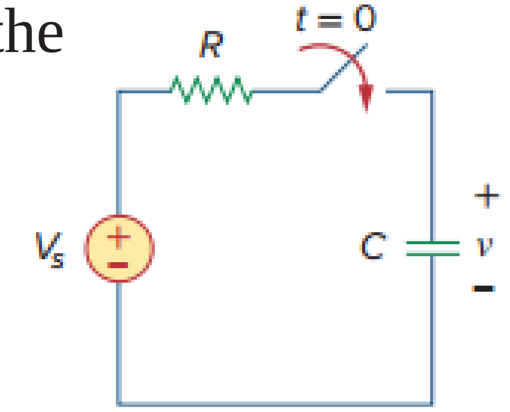
- Find ***i(t)*** for $t > 0$ if $i(0) = 5A$.



Ans: $\boldsymbol{i}(\mathbf{t}) = \mathbf{5e^{-5t}}$ **V**

Step response of a RC circuit

- The *step response* of a circuit is its behaviour under the sudden application of dc voltage or current source. We assume the circuit response to be the capacitor voltage.



⇒ Since the voltage of a capacitor cannot change instantaneously

⇒ $v(0^-) = v(0^+) = V_0$

⇒ Using KCL (for t > 0),

⇒ $C\frac{dv}{dt} + \frac{v - Vs}{R} = 0$

⇒ $\frac{dv}{dt} = -\frac{v - Vs}{RC}$

⇒ $\frac{dv}{v - Vs} = -\frac{1}{RC}dt$

Integrating both sides,

⇒ $[\ln(v - Vs)]_{V_0}^{v(t)} = -\left[\frac{t}{RC}\right]_0^t$

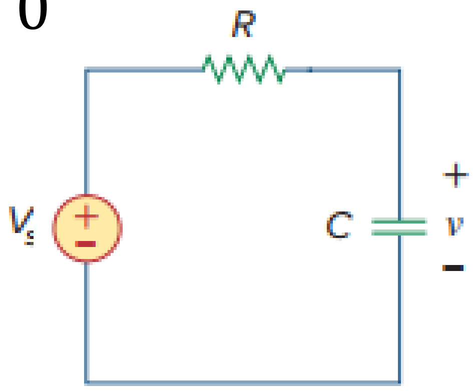
⇒ $\ln(v(t) - Vs) - \ln(V_0 - Vs) = -\frac{t}{RC} + 0$

⇒ $ln\frac{v - Vs}{V_0 - Vs} = -\frac{t}{RC}$

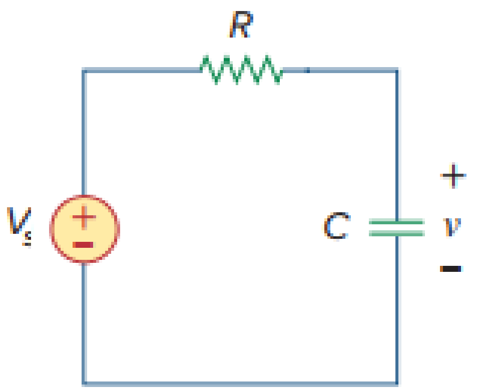
⇒ $\frac{v - Vs}{V_0 - Vs} = e^{-t/RC}$

⇒ $v - Vs = (V_0 - Vs)e^{-t/RC}$

⇒ $v(t) = Vs + (V_0 - Vs)e^{-t/RC}$



Time constant for RC circuit

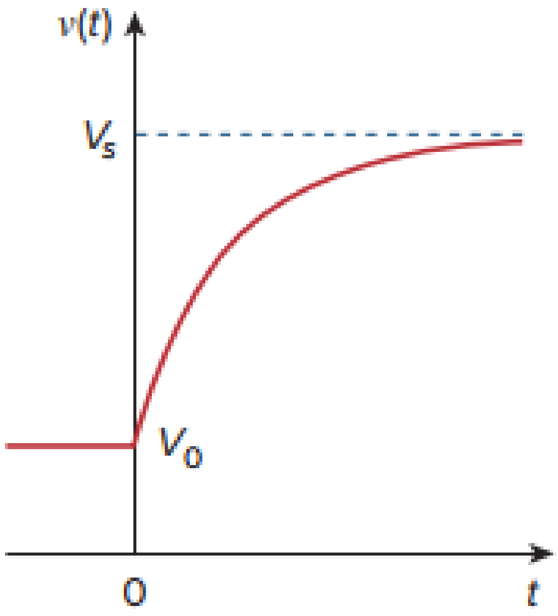


- $$v(t) = \begin{cases} V_0, & t < 0 \\ V_s + (V_0 - Vs)e^{-\frac{t}{RC}}, & t > 0 \end{cases}$$

- This is known as the complete response (or total response) of the RC circuit to a sudden application of a dc voltage source, assuming the capacitor is initially charged.

$$\Rightarrow v(t) = \begin{cases} V_0, & t < 0 \\ V_s + (V_0 - Vs)e^{-\frac{t}{RC}}, & t > 0 \end{cases}$$, where $\tau = RC$ is

the time constant (unit in sec)



- The time constant of this circuit is the time required for the response to reach to a factor of (1-1/e) or 63.2% of its final value from a full-discharged initial state.

Significance of 5τ time (charging)

- If the capacitor has an initial voltage V_0, then the charging time constant is the time required for the response to reach to a factor of (1-1/e) or 63.2% of ($V_{Final} - V_0$) from V_0.
- At t = τ,

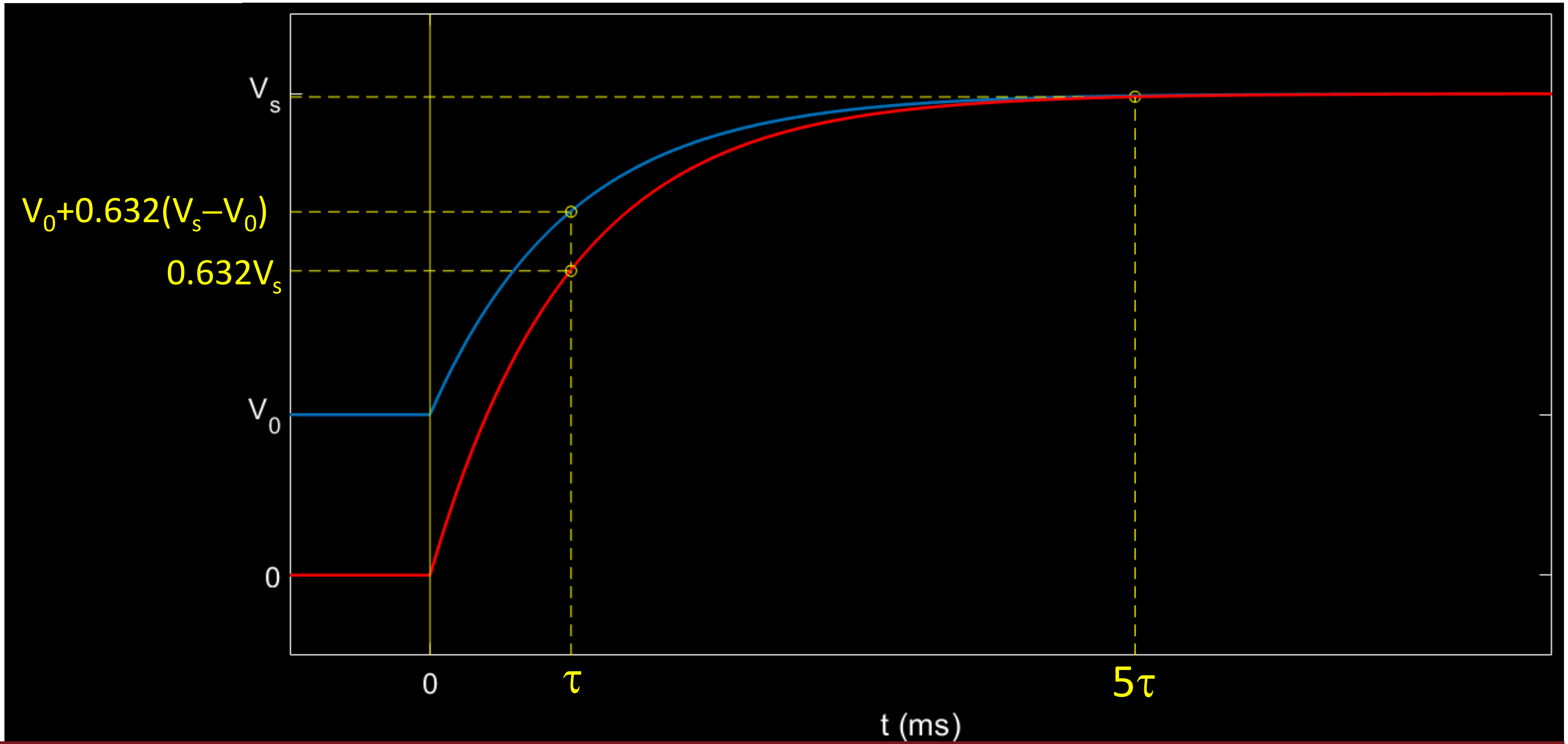
$$\Rightarrow \quad v(t) = V_s - V_s e^{-1}$$

$$= 0.632V_s \; (V_0 = 0)$$

$$\Rightarrow \quad v(t) = V_s + (V_0 - V_s)e^{-1}$$

$$= V_0 + 0.632(V_s - V_0) \; (V_0 \neq 0)$$

- The capacitor is fully charged after five time constants. It takes 5τ for the circuit to reach its final state or steady state when no changes take place with time.



Transient and steady-state response

- $$v(t) = \begin{cases} V_0, & t < 0 \\ V_s + (V_0 - V_s)e^{-\frac{t}{\tau}}, & t > 0 \end{cases}$$
- The *complete response* can be broken into two parts—one temporary and the other permanent, that is,
- $v(t) = v_{ss} + v_t,$ where, $\mathrm{v_{ss}} = \mathrm{V_s}$ and $\mathrm{v_t} = (V_0 - V_s)e^{-\frac{t}{RC}}$
- The *transient response (v_t)* is the circuit's temporary response that will die out with time.
- The *steady-state response (v_{ss})* is the behaviour of the circuit a long time after an external excitation is applied.
- The complete response can be written as,
- $\boxed{v(t) = v(\infty) + [v(0) - v(\infty)]e^{-\frac{t}{\tau}}}$

Example 13

- Find ***v(t)*** for t > 0 in the circuit shown below. Assume the switch has been open for a long time and is closed at t = 0. Calculate ***v(t)*** at t = 0.5

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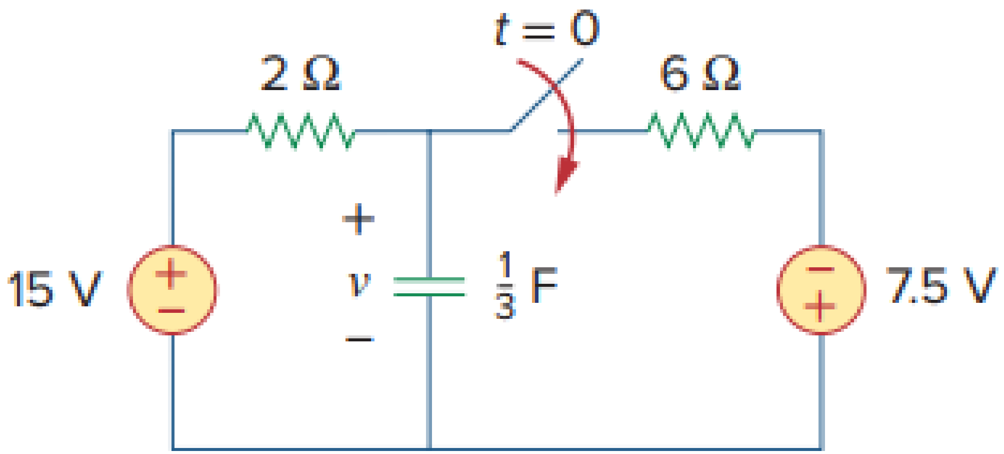
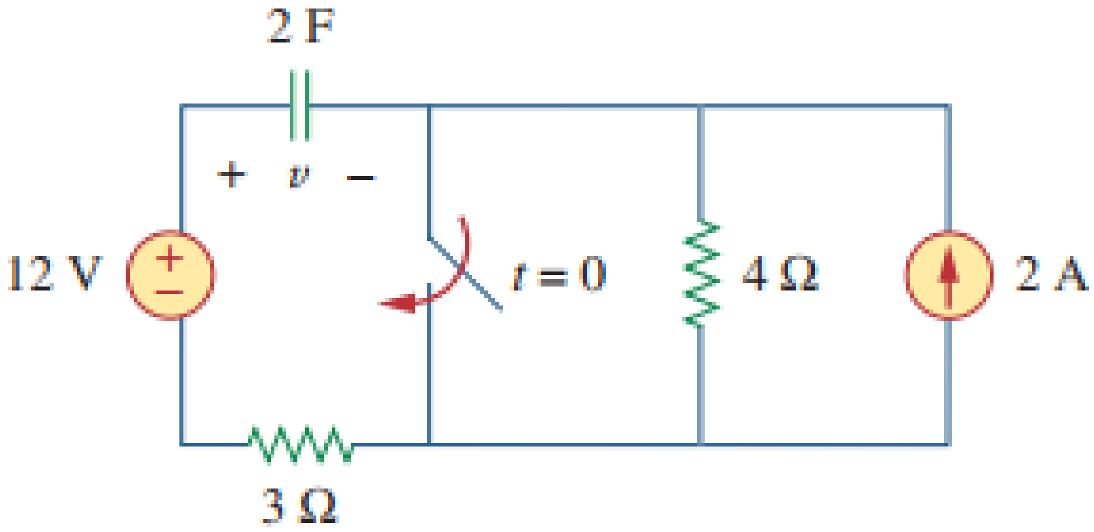


Ans: $v_c(t) = 9.375 + 5.625e^{-2t}$ V for t > 0; $v_c(0.5) = 11.444$ V

Example 14

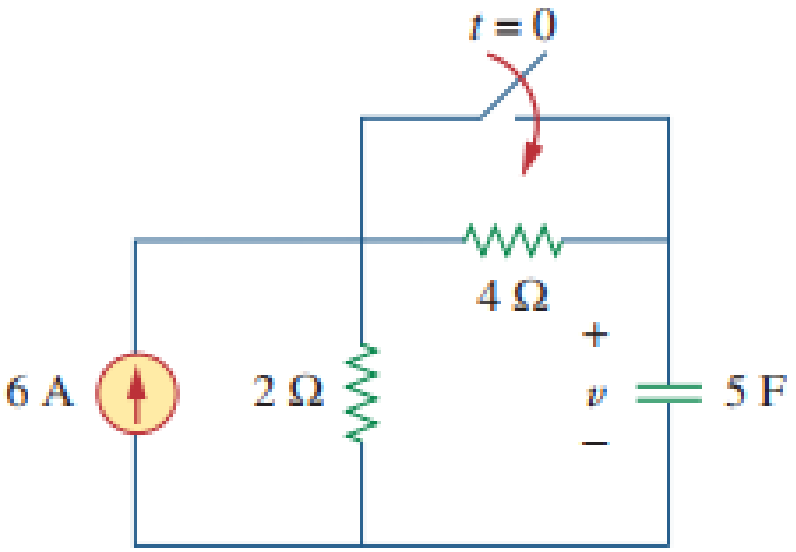
- Calculate the capacitor voltage ***v(t)*** for t < 0 and for t > 0.



Ans: $v(t) = 4$ V for $t < 0$; $v(t) = 12 - 8e^{-t/6}$ V for $t > 0$;

Example 15

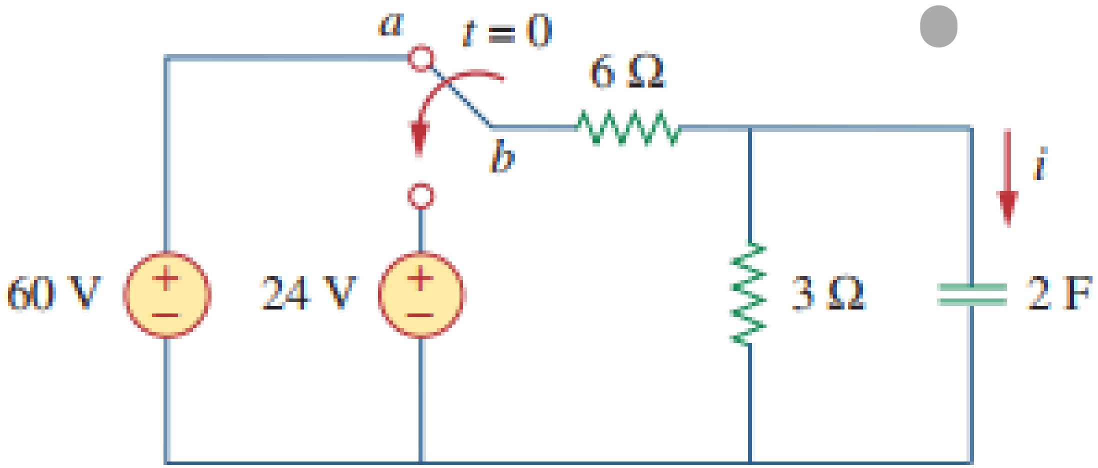
- Calculate the capacitor voltage for t < 0 and for t > 0.



Ans: $v(t) = 12$ V for $t < 0$; $v(t) = 12$ V for $t > 0$

Example 16

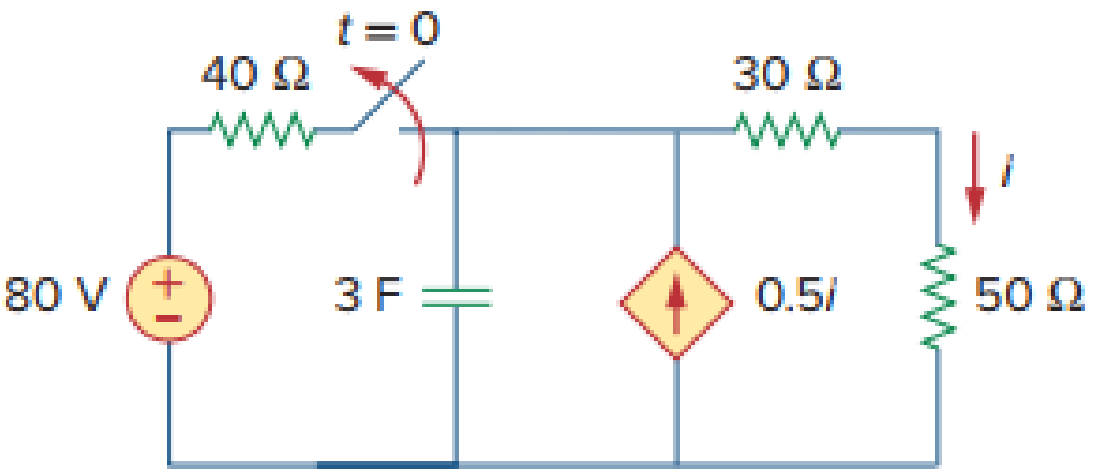
- The switch in has been in position a for a long time. At t = 0 it moves to position b. Calculate ***i(t)*** for all t > 0



Ans: $\boldsymbol{i}(t) = -6e^{-0.25t}$ **A for t > 0;**

Example 17

- Consider the circuit shown below. Find ***i(t)*** for t < 0 and t > 0.



Ans: $i(t) = 0.8$ A for t < 0; $i(t) = -0.8e^{-t/480}$ A for t > 0

Example 18

- The figure below shows the voltage response of an RC circuit to a sudden DC voltage applied through an equivalent resistance of 4 kΩ.

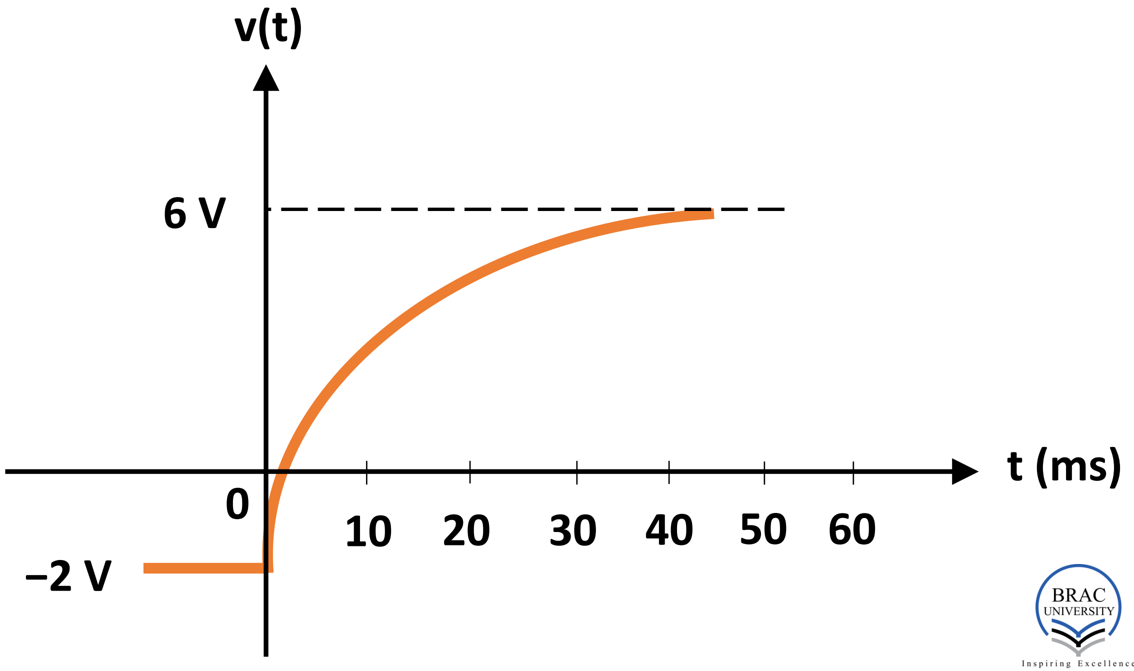
I. Define time constant.

II. Determine the approximate time constant from the figure.

III. Find the mathematical expression of v(t) for t > 0.

IV. What is the initial energy stored in the capacitor?

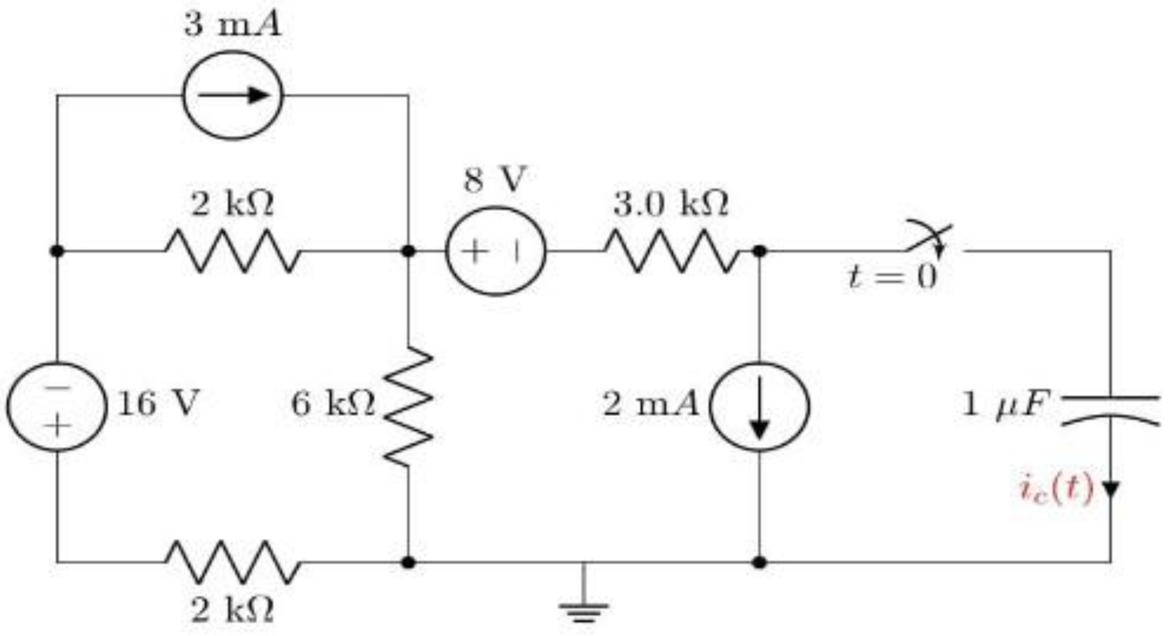
V. Draw the circuit diagram.



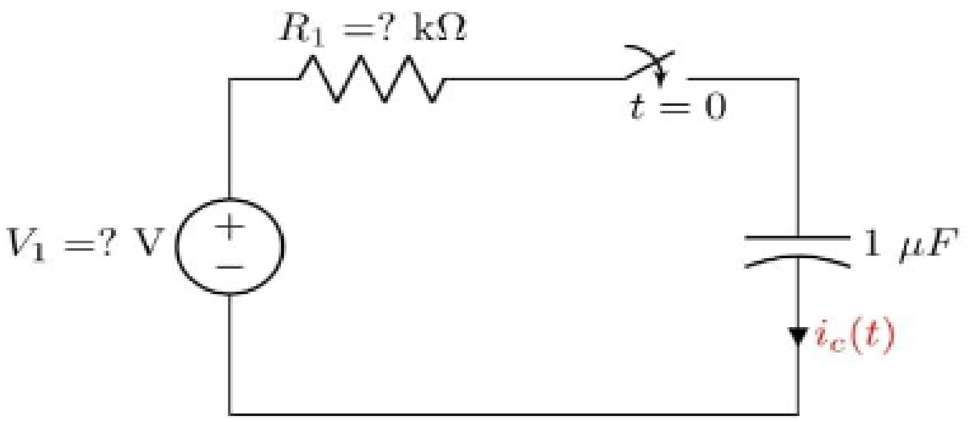
<u>Ans</u>: (ii) **τ = 9 ms;** (iii) **$v(t) = 6 - 8e^{-1000t/9}$ V for t > 0;**
(iv) **$w = 4.5 \times 10^{-6}$ J**

Example 19

- Simplify the circuit 1 below so that it takes the form of the circuit 2. Determine the values of $\boldsymbol{V_1}$ and $\boldsymbol{R_1}$.
- Perform transient analysis to determine $\boldsymbol{i_c(t)}$ through the capacitor for t > 0.



Circuit 1

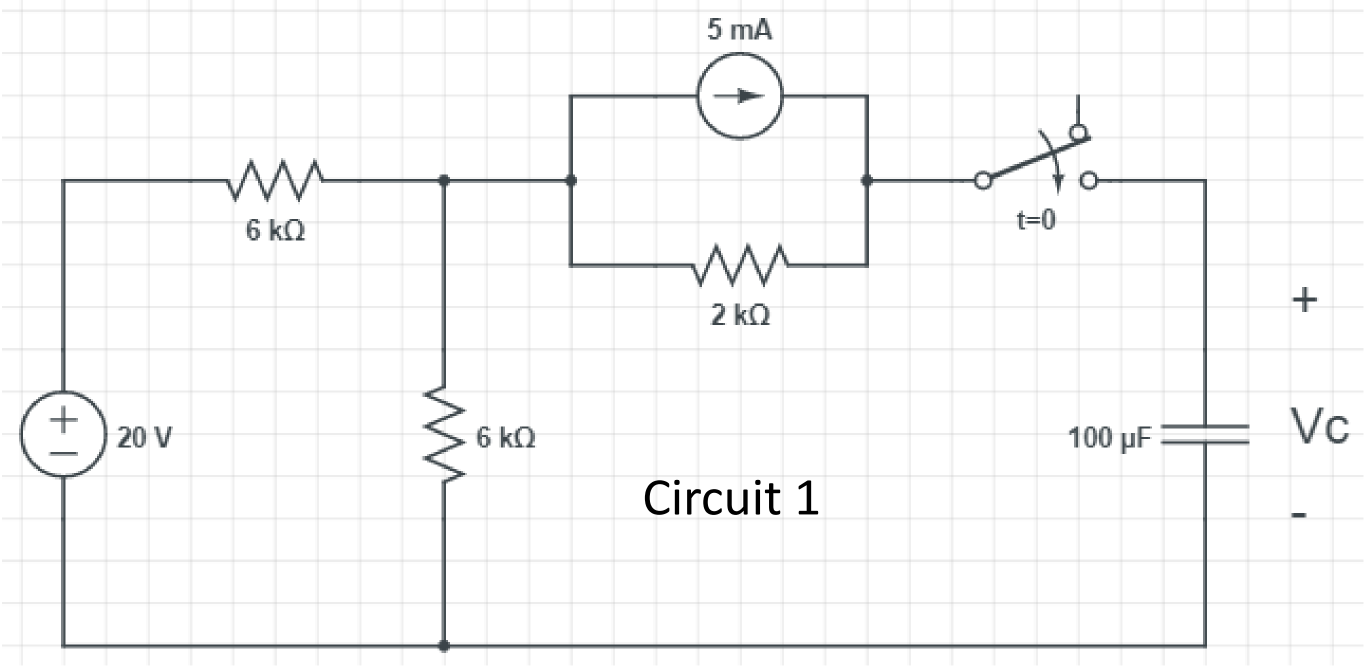


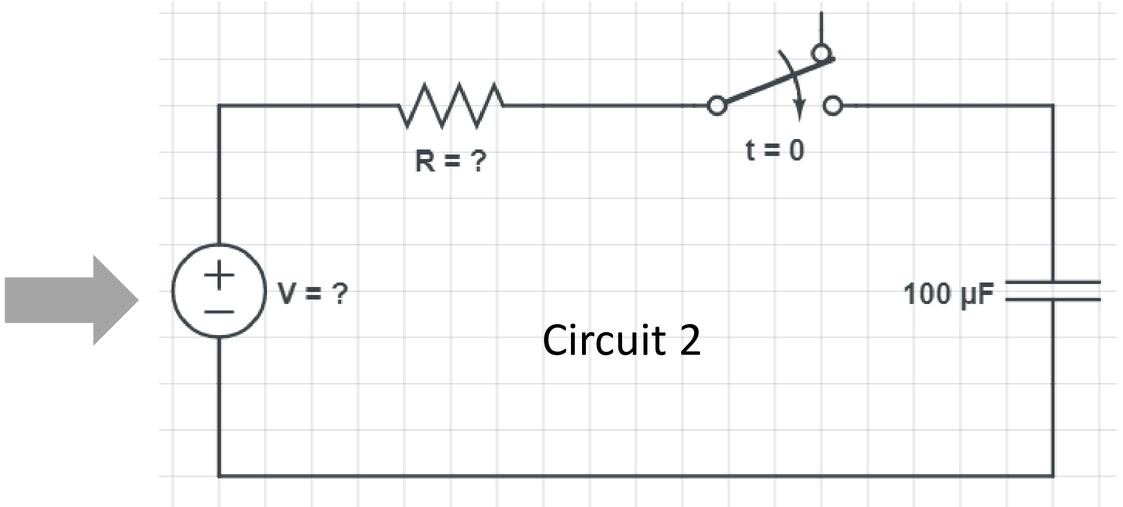
Circuit 2

<u>Ans</u>: $\mathbf{V_1 = -37.2\ V; R1 = 8.1\ k\Omega;}$ $\boldsymbol{i}\mathbf{(t) = -4.59e^{-1000t/8.1}\ A;}$

Example 20

- Simplify the circuit 1 below so that it takes the form of the circuit 2. Determine the values of ***V*** and ***R***.
- Perform transient analysis to determine $\boldsymbol{v_c(t)}$ across the capacitor for t > 0.

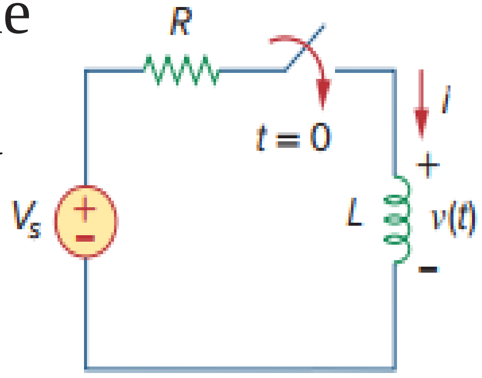




<u>Ans</u>: $\mathbf{V = 20\ V; R = 5\ k\Omega;}\ \boldsymbol{vc}\mathbf{(t) = 20e^{-2t}\ V;}$

Step response of a RL circuit

- The *step response* of a circuit is its behaviour under the sudden application of dc voltage or current source. We assume the circuit response to be the inductor current.



$\Rightarrow$ Since the current through an inductor cannot change instantaneously

$\Rightarrow$ $i(0^-) = i(0^+) = I_0$

$\Rightarrow$ Using KVL (for t > 0),

$\Rightarrow$ $V_L + iR = V_s$

$\Rightarrow$ $L\dfrac{di}{dt} + iR = V_s$

$\Rightarrow$ $\dfrac{di}{dt} + \dfrac{R}{L}i = \dfrac{V_s}{L}$

Multiplying both sides by $e^{\frac{R}{L}t}$

$\Rightarrow$ $e^{\frac{R}{L}t}\dfrac{di}{dt} + e^{\frac{R}{L}t}\dfrac{R}{L}i = \dfrac{V_s}{L}e^{\frac{R}{L}t}$

$\Rightarrow$ $\dfrac{d}{dt}\left[e^{\frac{R}{L}t}.\,i\right] = \dfrac{V_s}{L}e^{\frac{R}{L}t}$

Integrating with respect to t

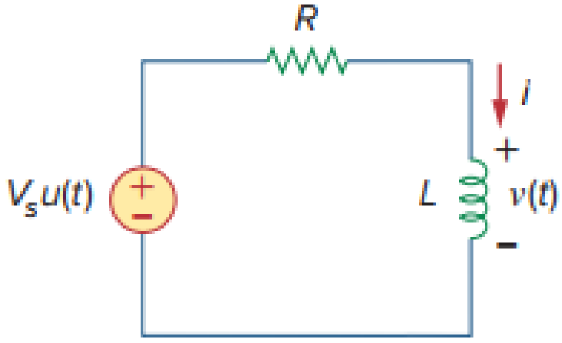
$\Rightarrow$ $e^{\frac{R}{L}t}.\,i = \dfrac{V_s}{R}e^{\frac{R}{L}t} + C$

$\Rightarrow$ $i = \dfrac{V_s}{R} + Ce^{-\frac{R}{L}t}$

$At\ t = 0, i = I_0.$ $\qquad So, C = i - \dfrac{V_s}{R}$

Substituting,

$\Rightarrow$ $i = \dfrac{V_s}{R} + \left(I_0 - \dfrac{V_s}{R}\right)e^{-\frac{R}{L}t}$



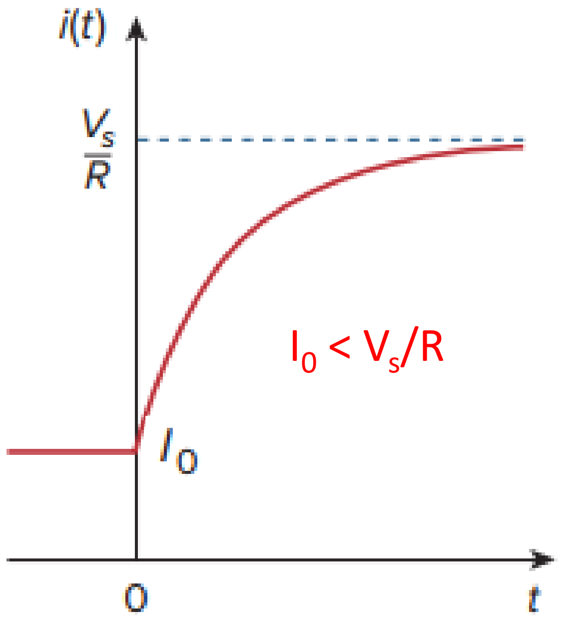
Time constant for RL circuit

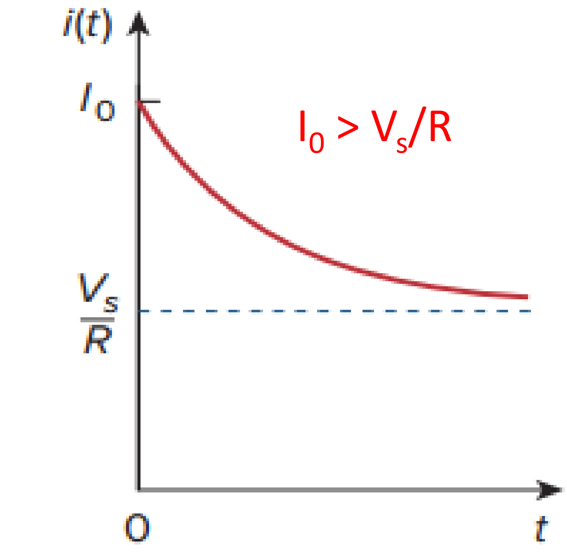
- $$i(t) = \begin{cases} I_0, & t < 0 \\ \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right) e^{-\frac{R}{L}t}, & t > 0 \end{cases}$$

- This is known as the complete response (or total response) of the RL circuit to a sudden application of a dc voltage source, assuming the inductor is initially charged.

$$\Rightarrow \quad i(t) = \begin{cases} I_0, & t < 0 \\ \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right) e^{-t/\tau}, & t > 0\text{'} \end{cases}$$ where $\tau = \frac{L}{R}$ is the time constant (unit in sec)

- The time constant of this circuit is the time required for the response to reach to a factor of (1-1⁄e) or 63.2% of its final value from a full-discharged initial state.





Significance of 5τ time (charging)

- If the inductor has an initial current I_0, then the charging time constant is the time required for the response to reach to a factor of (1-1/e) or 63.2% of (I_{final}–I_0) from I_0.

- At t = τ,

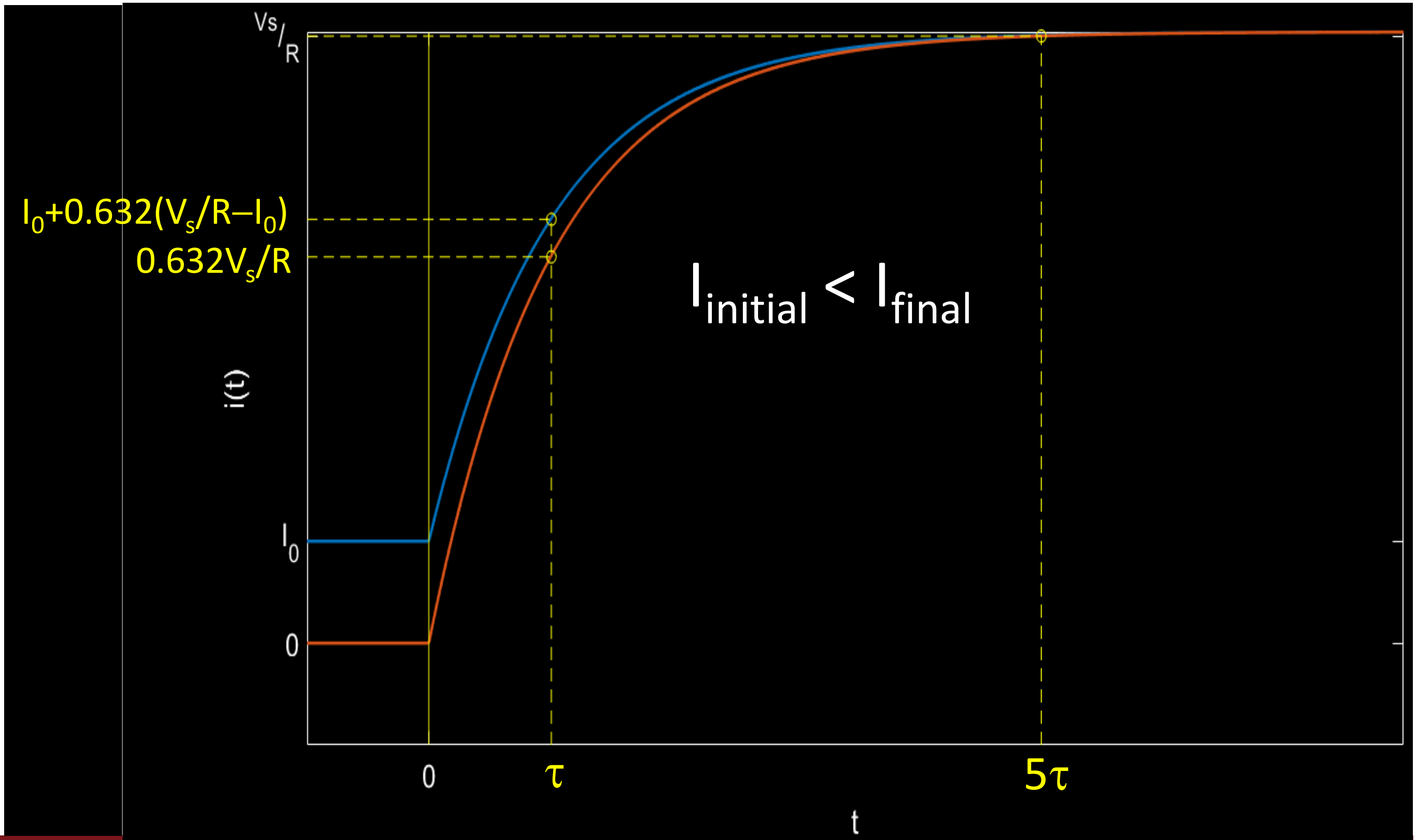
$$\Rightarrow \quad i(t) = \frac{V_s}{R} - \frac{V_s}{R} e^{-1}$$

$$= 0.632 \frac{V_s}{R} (I_0 = 0)$$

$$\Rightarrow \quad i(t) = \frac{V_s}{R} + (I_0 - \frac{V_s}{R}) e^{-1}$$

$$= I_0 + 0.632 \left(\frac{V_s}{R} - I_0\right) (I_0 \neq 0)\ (I_0 < \frac{V_s}{R})$$

- The inductor is fully charged after five time constants. It takes 5τ for the circuit to reach its final state or steady state when no changes take place with time.



Significance of 5τ time (discharging)

- If the inductor has an initial current I_0, then the discharging time constant is the time required for the response to reach to a factor of (1-1/e) or 63.2% of ($I_0 - I_{final}$) from I_0.
- At $t = \tau$,

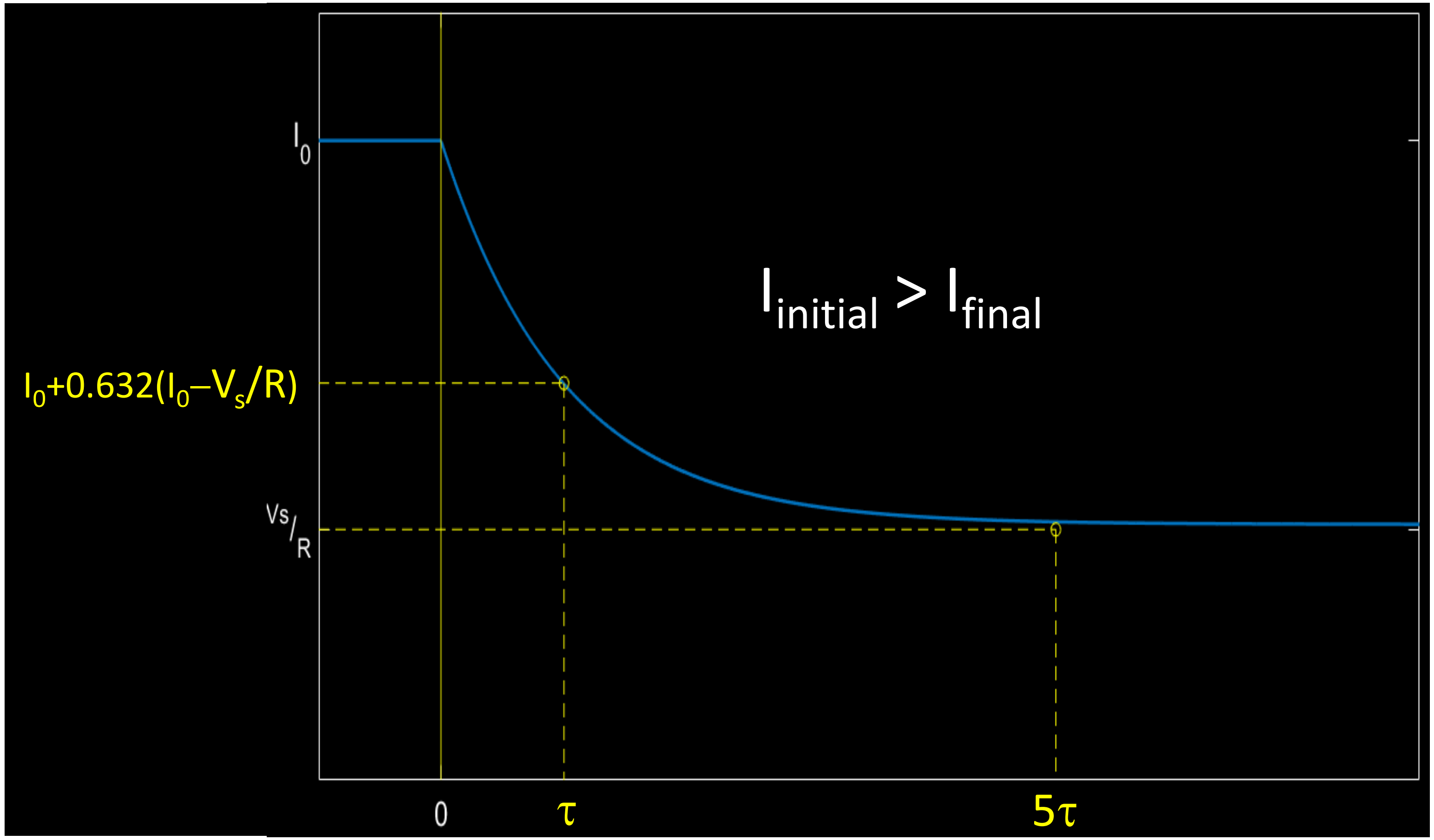
$$\Rightarrow \quad i(t) = \frac{V_s}{R} - \frac{V_s}{R} e^{-1}$$

$$= 0.632 \frac{V_s}{R} \, (I_0 = 0)$$

$$\Rightarrow \quad i(t) = \frac{V_s}{R} + (I_0 - \frac{V_s}{R}) e^{-1}$$

$$= I_0 + 0.632 \left(\frac{V_s}{R} - I_0\right) (I_0 \neq 0) \; (I_0 > \frac{V_s}{R})$$

- The inductor is fully charged after five time constants. It takes 5τ for the circuit to reach its final state or steady state when no changes take place with time.



Transient and steady-state response

- $$i(t) = \begin{cases} I_0, & t < 0 \\ \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right) e^{-\frac{R}{L}t}, & t > 0 \end{cases}$$
- The *complete response* can be broken into two parts—one temporary and the other permanent, that is,
- $i(t) = iss + it,$ where, $\text{iss} = \frac{V_s}{R}$ and $\text{it} = \left(I_0 - \frac{V_s}{R}\right) e^{-\frac{R}{L}t}$
- The *transient response (i_t)* is the circuit's temporary response that will die out with time.
- The *steady-state response (i_{ss})* is the behaviour of the circuit a long time after an external excitation is applied.
- The complete response can be written as,
- $\boxed{i(t) = i(\infty) + [i(0) - i(\infty)]e^{-\frac{t}{\tau}}}$

Example 21

- The switch has been closed for a long time. It opens at t = 0. Find ***i(t)*** for t > 0.

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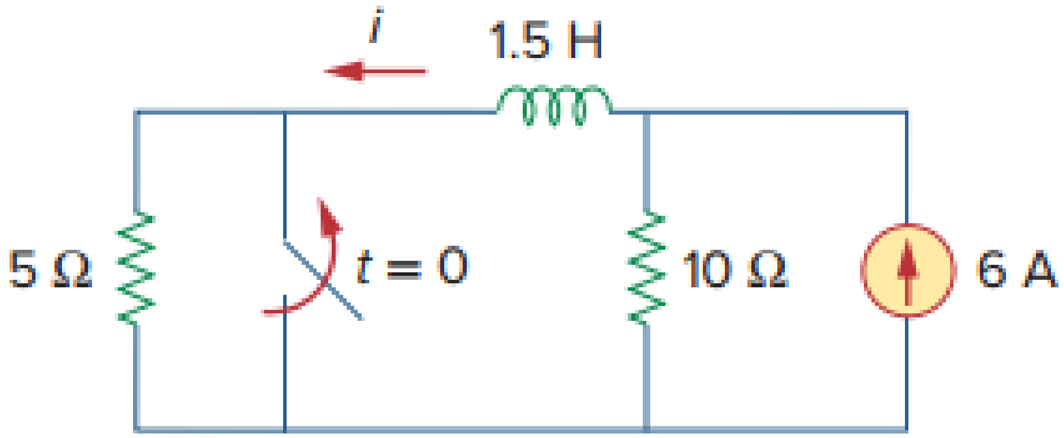
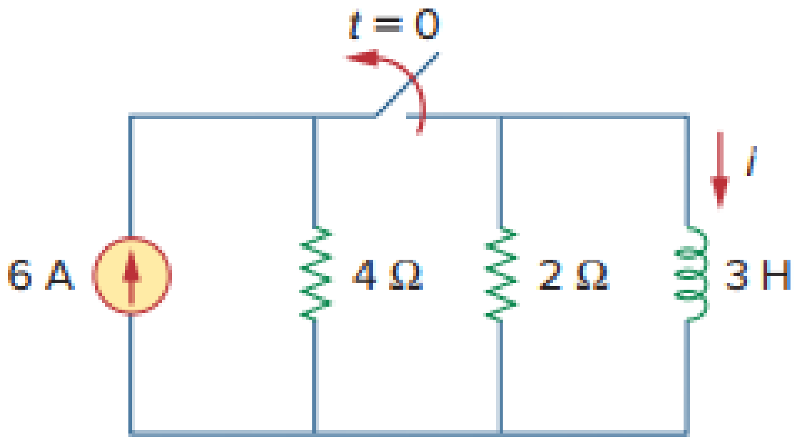


Ans: $\boldsymbol{i}(t) = 4+2e^{-10t}$ **A for t > 0;**

Example 22

- Determine the inductor current ***i(t)*** for both t < 0 and t > 0.



Ans: $i(t) = 6$ A for $t < 0$; $i(t) = 6e^{-2t/3}$ A for $t > 0$

Example 23

- Obtain the inductor current ***i(t)*** for and t > 0.

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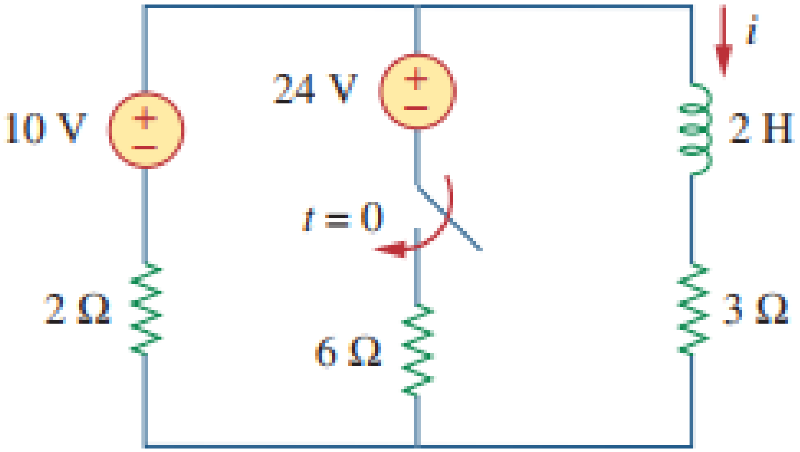
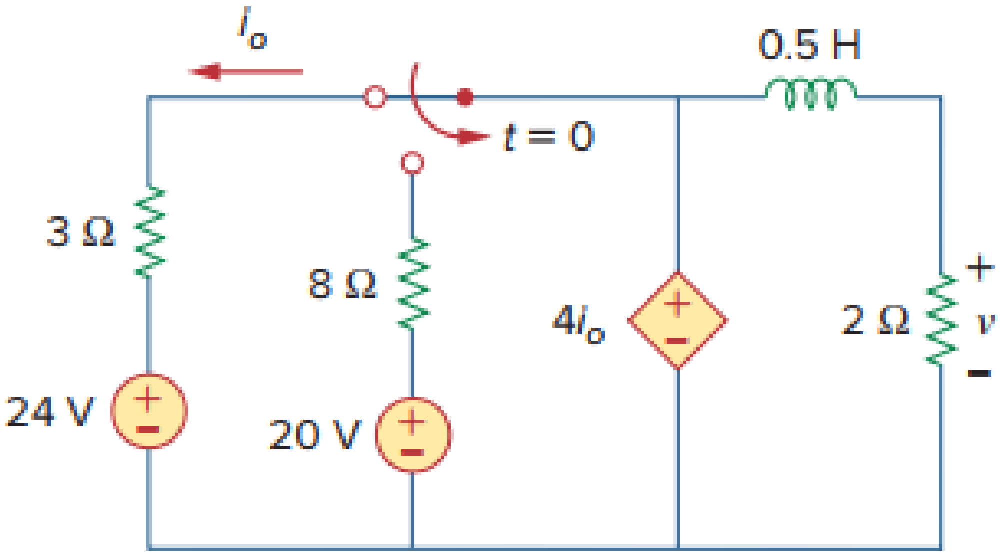


<u>Ans</u>: $\boldsymbol{i}\textbf{(t)} = \mathbf{3 - e^{-9t/4}}$ **A for t > 0**

Example 24

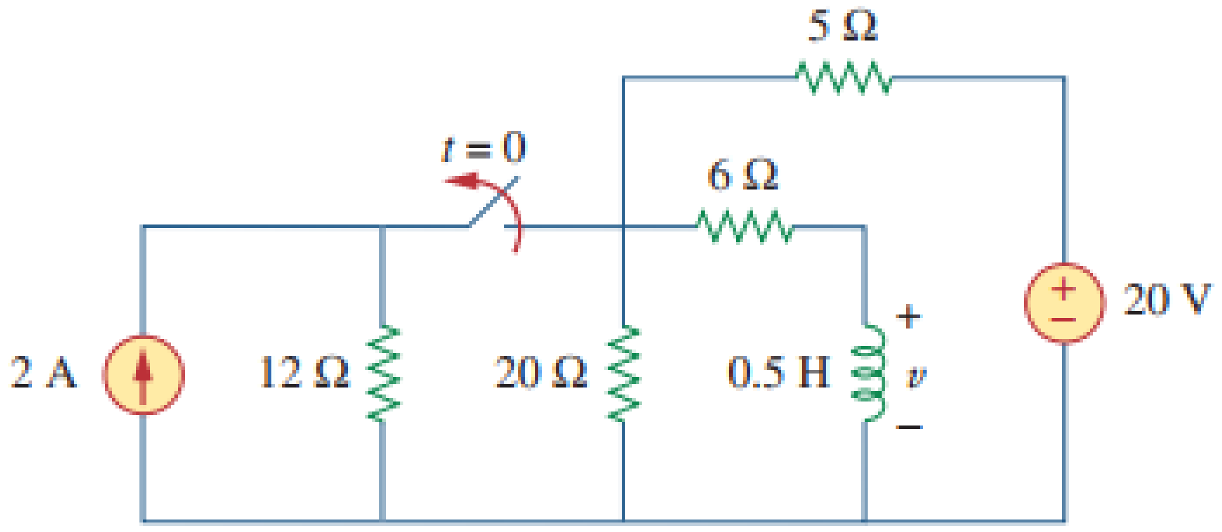
- Find ***v(t)*** for both t < 0 and t > 0.



Ans: $v(t) = 96$ V for $t < 0$; $v(t) = 96e^{-4t}$ V for $t > 0$

Example 25

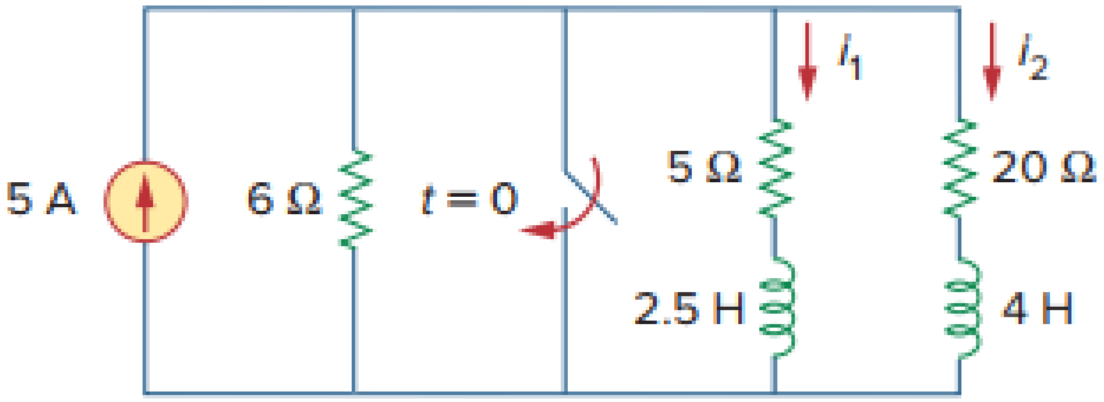
- Find ***v(t)*** for t > 0.



<u>Ans</u>: $\boldsymbol{v}$**(t) = – 4e^{-20t} V for t > 0**

Example 26

- Find $\boldsymbol{i_1(t)}$ and $\boldsymbol{i_2(t)}$ for t > 0.



Ans: $\boldsymbol{i_1(t) = 2.4e^{-2t}}$ **A for t > 0;** $\boldsymbol{i_2(t) = 0.6e^{-5t}}$ **A for t > 0**

Thank you for your attention