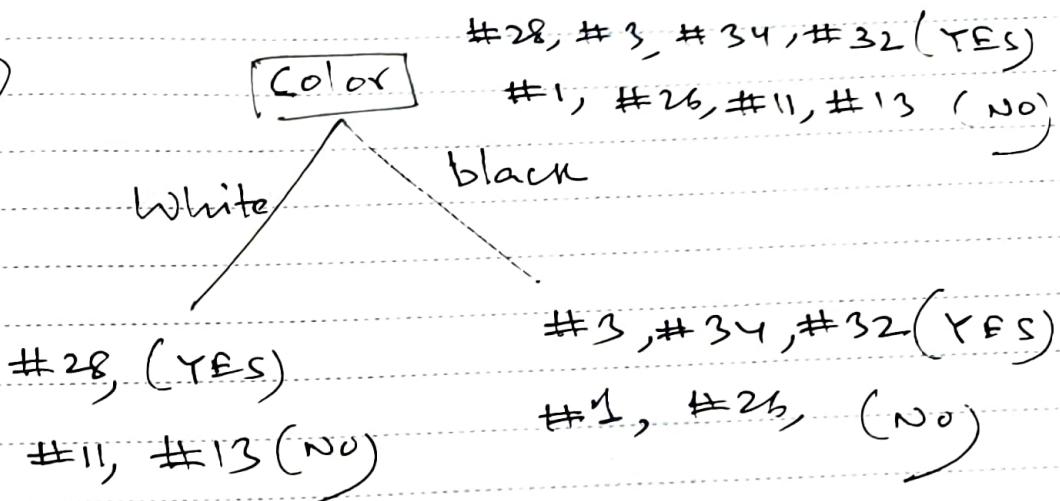




1.

(a)

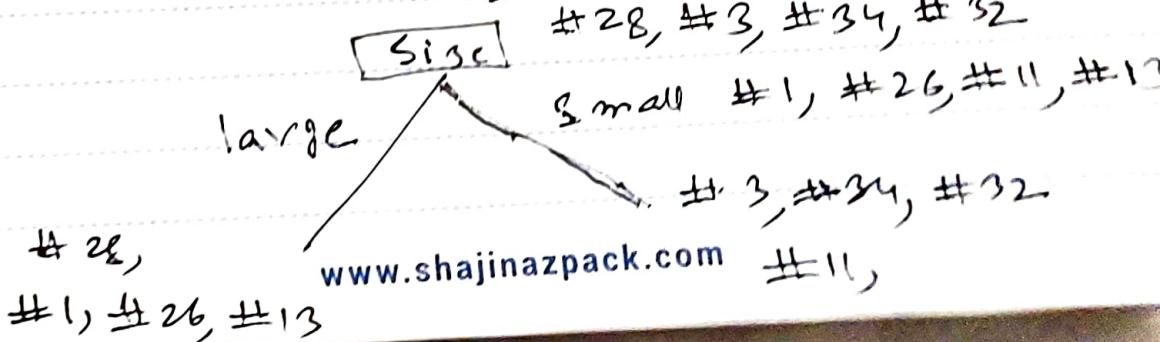


$$\text{Info}(D) = -\frac{4}{8} \log_2 \frac{4}{8} - \frac{4}{8} \log_2 \frac{4}{8} = 1$$

$$\begin{aligned} \text{Info}(\text{color}) &= \frac{3}{8} \left( -\frac{1}{3} \log_2 \frac{1}{3} - \frac{2}{3} \log_2 \frac{2}{3} \right) \\ &\quad + \frac{5}{8} \left( -\frac{3}{5} \log_2 \frac{3}{5} - \frac{2}{5} \log_2 \frac{2}{5} \right) \end{aligned}$$

$$= 0.3431 + 0.6068 = 0.95$$

$$\text{Information gain}(\text{color}) = 1 - 0.95 = 0.05$$



$$\text{Info. (Size)} = \frac{4}{8} \left( -\frac{1}{4} \log_2 \frac{1}{4} - \frac{3}{4} \log_2 \frac{3}{4} \right) + \frac{4}{8} \left( -\frac{3}{4} \log_2 \frac{3}{4} - \frac{1}{4} \log_2 \frac{1}{4} \right)$$

$$= 0.56$$

$$\text{Information Gain (size)} = 1 - 0.56$$

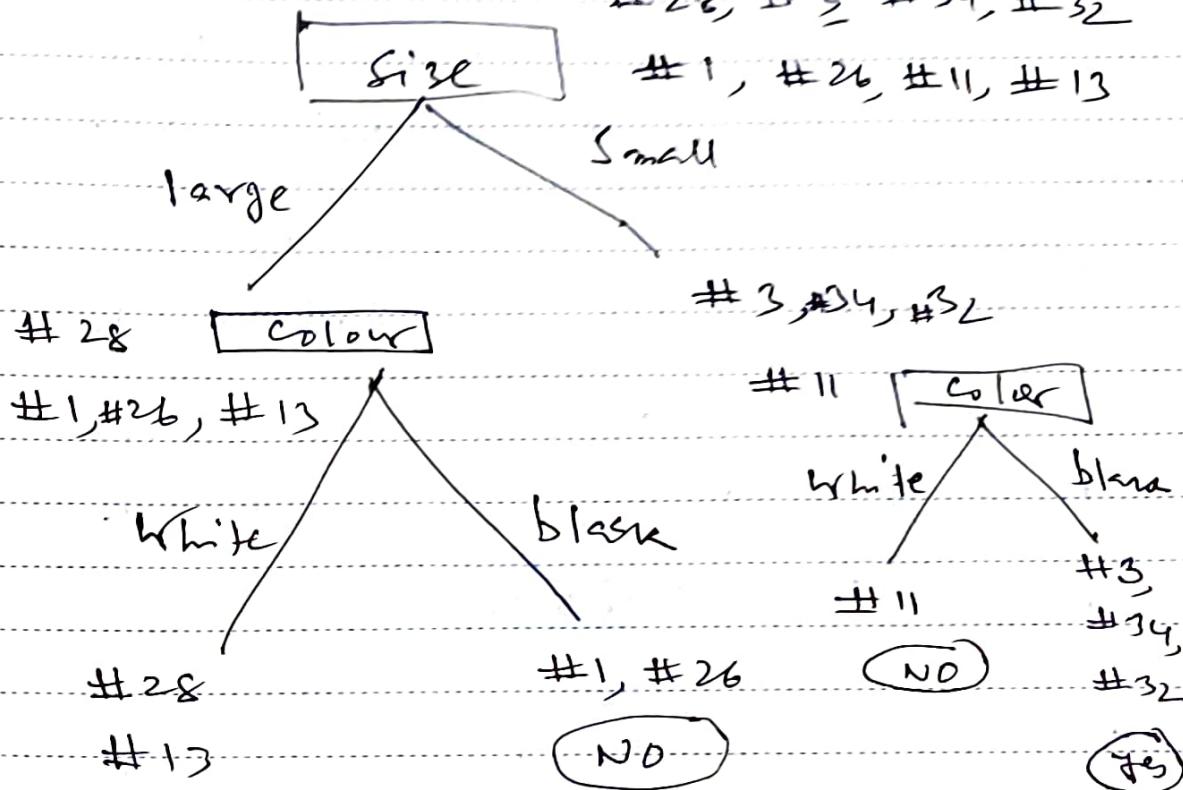
$$= 0.44$$

Since information gain of size is larger than that of colour, size should be used as the root of the tree. Now we have to construct a decision tree w.r.t size and colour attribute. We should use size as the root.

Date:

#28, #3, #34, #32

#1, #26, #11, #13



b.

unique-id will have 8 branches and each branch will contain a single yes or no example. This feature will have the highest information gain because the examples have been separated. This unique-id need to be selected as the root of the tree. But this tree will have no predictive power because

it does has no correlation with unique-id. so this type of attribute should be avoided in constructing the decision tree. this type of attribute is automatically avoided if gain ratio is used instead of information gain to select the attributes.

- ① tail-length is a numeric attribute. We can discretize it before we start constructing the tree. We can convert tail-length values into two categories, one is large and the other is small. If we choose, for example, 2.5 is the cut-off value, then 5.6, 3.8, 4.2, 3.5 will be converted to large and the others will be converted to small.

(d)

$X = \text{Single event each with probability } \frac{1}{6}$

$Y = \text{two events each with probability } \frac{1}{2}$

$$\begin{aligned} \text{Entropy}(X) &= -\frac{1}{6} \log_2 \frac{1}{6} - \frac{1}{6} \log_2 \frac{1}{6} \\ &\quad - \frac{1}{6} \log_2 \frac{1}{6} - \frac{1}{6} \log_2 \frac{1}{6} - \frac{1}{6} \log_2 \frac{1}{6} \\ &\quad - \frac{1}{6} \log_2 \frac{1}{6} \end{aligned}$$

$$\begin{aligned} H(X) &= \log_2 \frac{1}{6} = -\frac{1}{6} \log_2 \frac{1}{6} - \frac{5}{6} \log_2 \frac{5}{6} \\ &= 0.65 \end{aligned}$$

$$\begin{aligned} \text{Entropy}(Y) &= -\frac{1}{2} \log_2 \frac{1}{2} - \frac{1}{2} \log_2 \frac{1}{2} \\ &= 1 \end{aligned}$$

$\text{Entropy}(Y) > \text{Entropy}(X)$

2.

$$x_1 = 1 \quad x_2 = a \quad x_3 = v$$

$$P(Y=0 \mid x_1=1, x_2=a, x_3=v)$$

$$= P(x_1=1, x_2=a, x_3=v \mid Y=0) \times P(Y=0)$$

$$= P(x_1=1 \mid Y=0) \times P(x_2=a \mid Y=0)$$

$$\times P(x_3=v \mid Y=0) \times P(Y=0)$$

$$= \frac{1}{4} \times \frac{2}{4} \times \frac{1}{4} \times \frac{4}{9} = \frac{1}{72}$$

$$P(Y=1 \mid x_1=1, x_2=a, x_3=v)$$

$$= P(x_1=1 \mid Y=1) \quad P(x_2=a \mid Y=1) \quad P(x_3=v \mid Y=1)$$

$$P(Y=1)$$

$$= \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{8}{9} = \frac{1}{27}$$

Since

∴ most likely value of  $\gamma = 0$

B.

(a)  $X = \{A, B, C\}$

$$P(A) = 0.5 \quad P(B) = 0.3$$

$$\therefore P(C) = 1 - 0.5 - 0.3 = 0.2$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.5 + 0.3 - 0.5 \times 0.3 = 0.8 - 0.15$$

$$= 0.65$$

(b)  $P(A) = H, H \text{ or } H, T$   
 out of HH, HT, TH and TT

$$= \frac{2}{4} = \frac{1}{2}$$

$$P(B) \rightarrow XH \text{ or } TT = \frac{2}{4} = \frac{1}{2}$$

(i)  $P(A|A) = 0.5$

$$P(A) = 0.1 + x + 0.2 + 0.1 = 0.4 + x$$

As absolute independence

$$P(B|A) = P(B)$$

(ii)  $0.4 + x = 0.5$

$$\therefore x = 0.1$$

(ii)  $P(B) = 0.5 = P(B|A)$

$$\therefore P(B') = 1 - 0.5 = 0.5$$

$$P(B') = 0.2 + 0.1 + x + 0.1 = 0.5$$

$$\therefore x = 0.1$$

(iii)

$$P(A \mid B \wedge C)$$

$$\begin{aligned}
 P(A \wedge B \wedge C) &= \frac{0.1}{P(B \wedge C)} = \frac{0.1}{0.1 + 0.2} = \frac{0.1}{0.3} \\
 &= 0.33
 \end{aligned}$$

4.

(i) Entropy (D)

$$= \text{Entropy } (9P, 7N)$$

$$= -\frac{9}{16} \log_2 \frac{9}{16} - \frac{7}{16} \log_2 \frac{7}{16}$$

(ii) As usual calculate

(iii) Based on information gain, if information gain of color is greater than that of color size, then

Color is better than white size 5  
better

$$(i) P(\text{Pandemic} \wedge \text{Public Univ} \mid \text{online class})$$

$$= P(\text{Pandemic} \mid \text{online class}) \times$$

$$P(\text{Public Univ} \mid \text{online class})$$

- if conditional independence holds.

$$P(\text{Pandemic} \wedge \text{Public Univ} \mid \text{online class})$$

$$P(\text{Pandemic} \wedge \text{Public Univ} \wedge \text{online class})$$

$$= \frac{P(\text{online class})}{P(\text{online class})}$$

$$0.142$$

$$= \frac{0.142 + 0.103 + 0.165 + 0.217}{0.627} = \frac{0.142}{0.627} = 0.23$$

$P(\text{Pandemic} \mid \text{online class})$

$P(\text{Pandemic} \wedge \text{online class})$

=

$P(\text{online class})$

$$= 0.142 + 0.103$$

$$0.245$$

$$= \frac{0.142 + 0.103 + 0.165 + 0.217}{0.627} = 0.39$$

$$= 0.39$$

$P(\text{Public mix} \mid \text{online class})$

$P(\text{Public mix} \wedge \text{online class})$

$= \frac{P(\text{online class})}{0.142 + 0.165}$

$$0.142 + 0.165$$

$$= \frac{0.307}{0.627} = 0.49$$

Since  $0.23 \neq 0.39 \times 0.49$ , the statement is false.

(ii)

$$P(\text{Priv mix} \mid \text{online class})$$

$= P(\text{Priv mix})$  if they are independent.

~~$P(\text{Priv mix} \mid \text{online class})$~~

~~$P(\text{Priv mix}) \wedge P(\text{online class})$~~

~~$P(\text{online class})$~~

5

$$(0.103 + 0.146 + 0.217 + 0.118) (0.142 + 0.103 + 0.115 + 0.212)$$

$0.627$  (calculated earlier)

$$0.584 \times 0.627$$

$P(\text{priv viv} | \text{online class})$

$P(\text{priv viv} \wedge \text{online class})$

$$= \frac{P(\text{online class})}{P(\text{priv viv})}$$

$$= \frac{0.103 + 0.217}{0.627} = \cancel{0.45} 0.51$$

$$\begin{aligned} P(\text{priv viv}) &= 0.103 + 0.146 + 0.217 + 0.118 \\ &= 0.584 \end{aligned}$$

Statement is false



(iii)  $P(\text{offline class})$

$$= 0.037 + 0.166 + 0.072 + 0.118$$

$$= 0.373$$

(iv)  $P(\text{privilege} \wedge \text{offline class} \mid \text{No Pandemic})$

$P(\text{privilege} \wedge \text{offline class} \wedge \text{No Pandemic})$

$$= \frac{P(\text{No Pandemic})}{0.118}$$

$$= \frac{0.118}{0.165 + 0.072 + 0.217 + 0.118} = 0.21$$

v) Naive Bayes is called Naive Bayes because adopting the strong assumption that attributes are independent. It outperforms bayes theory because it avoids calculating the denominators of the bayes rule.

$$(\text{v}^{\prime }) \quad P(\text{yes} | x) = \frac{P(x | \text{yes}) \cdot P(\text{yes})}{P(x)} .$$

$$P(\text{No} | x) = \frac{P(x | \text{No}) \cdot P(\text{No})}{P(x)}$$

Since we need to compare  $P(\text{yes}|x)$  and  $P(\text{No}|x)$ , we can omit  $P(x)$  from the right hand side of both equations.

Weight value can be replaced by

12 (heavy) 8 (light) 13 (heavy)

15 (heavy) 4 (light) 5 (light)

18 (heavy) 11 (heavy) 9 (light)

6 (light)

The numeric attribute weight is converted to discrete attribute having only two vals heavy and light.

② ~~P{dog a heavy f}~~

P {dog a heavy | blacky}

If dog is conditionally independent  
of heavy given black then the  
following condition holds:

$$P(\text{dog} \cap \text{heavy} | \text{color} = \text{black})$$

$$= P(\text{dog} | \text{black}) \cdot P(\text{heavy} | \text{black})$$

$$\text{Now } P(\text{dog} \cap \text{heavy} | \text{color} = \text{black})$$

$$= \frac{2}{3}$$

[ 3 black animals  
out of which  
2 are dogs and  
heavy ]



$$P(\text{dog} \mid \text{black}) = \frac{2}{3} \quad (\text{2 are dogs out of 3 black animals})$$

$$P(\text{heavy} \mid \text{black})$$

$$= \frac{2}{3} \quad (\text{2 are heavy out of 3 black animals})$$

Ans

$$\frac{2}{3} \neq \frac{2}{3} \times \frac{2}{3}$$

So the statement is false.

(b)  $P(\text{pet} \mid \text{orange a cat})$   
 $= \frac{1}{2}$

$$= P(\text{orange a cat} \mid \text{pet}) \times P(\text{pet})$$
$$= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$= P(\text{orange} \mid \text{Pet}) \times P(\text{cat} \mid \text{Pet} = \text{cat}) \times P(\text{Pet} = \text{cat})$$

$$= \frac{2}{8} \times \frac{2}{4} \times \frac{6}{10} = \frac{1}{15}$$

$$P(\text{Pet} = \text{No} \mid \text{orange a cat})$$

$$= P(\text{orange} \mid \text{Pet} = \text{No}) \times P(\text{cat} \mid \text{Pet} = \text{No}) \times P(\text{Pet} = \text{No})$$

$$= \frac{1}{4} \times \frac{1}{4} \times \frac{4}{10} = \frac{1}{40}$$

∴ It is more likely that  
an orange cat is a Pet animal.

7.

(a) Find information gain of Alpha and Beta and decide the better attribute is the one which has more information. (Do it

yourself)

(b)

A H T T T

B H T T - -

The fourth and fifth outcome could be HH HT TH or TT

In every case we calculate the entropy and take the combination which has maximum entropy.

B can be as follows

$$H\bar{T}\bar{T}H\bar{H} = -\frac{3}{5} \log_2 \frac{3}{5} - \frac{2}{5} \log_2 \frac{2}{3}$$

$$= 0.971$$

$$H\bar{T}\bar{T}HT = -\frac{2}{5} \log_2 \frac{2}{5} - \frac{3}{5} \log_2 \frac{3}{5}$$

$$= 0.971$$

$$H\bar{T}\bar{T}TH = -\frac{2}{5} \log_2 \frac{2}{5} - \frac{3}{5} \log_2 \frac{3}{5}$$

$$= 0.971$$

$$H\bar{T}\bar{T}\bar{T}T = -\frac{1}{5} \log_2 \frac{1}{5} - \frac{4}{5} \log_2 \frac{4}{5}$$

$$= 0.721$$

For A

$$A \cdot H\bar{T}\bar{T}\bar{T}T = -\frac{1}{5} \log_2 \frac{1}{5} - \frac{4}{5} \log_2 \frac{4}{5}$$

$$= 0.721$$

So  $H\bar{T}\bar{T}H\bar{H}$ ,  $H\bar{T}\bar{T}HT$  and  $H\bar{T}\bar{T}TH$  (B)  
have high quality.

④ If variable is continuous we have to discretize it before constructing the deviation tree.

$$8. P(\text{Positive} | \text{test} = \text{Positive}) = 0.85$$

$$P(\text{Positive} | \text{test} = \text{Negative}) = 0.10$$

$$\textcircled{1} \quad P(\text{Positive}) = 0.7 \quad \therefore P(\text{negative})$$

$$P(\text{test} = \text{Positive}) = 0.3$$

$$= P(\text{test} = \text{Positive} \wedge \text{Positive})$$

$$+ P(\text{test} = \text{Positive} \wedge \text{Negative})$$

$$= P(\text{Positive} \mid \text{test = Positive}) \cdot P(\text{test = Positive})$$

$$+ P(\text{Negative} \mid \text{test = Positive}) \cdot P(\text{test} = \text{Positive})$$

~~$$= 0.85 \times$$~~

$$= P(\text{test = Positive} \mid \text{Positive}) \cdot P(\text{Positive})$$

$$+ P(\text{test = Positive} \mid \text{Negative}) \cdot P(\text{Negative})$$

$$= P(\text{Positive} \mid \text{test = positive}) \cdot P(\text{Positive}) \cdot P(\text{Positive})$$

$$+ P(\text{negative} \mid \text{test = positive}) \cdot P($$

8.

$$P(\text{covid} \mid \text{test} = \text{positive}) = 0.85$$

$$P(\text{covid} \mid \text{test} = \text{positive}) = 0.15$$

$$P(\text{covid} \mid \text{test} = \text{negative}) = 0.10$$

$$P(\text{covid} \mid \text{test} = \text{negative}) = 0.90$$

$$P(\text{covid}) = 0.70 \quad P(\text{test} = \text{Positive}) = 0.70$$

$$P(\text{test} = \text{positive} \mid \text{covid})$$

$$\frac{P(\text{covid} \mid \text{test} = \text{positive}) \times P(\text{test} = \text{positive})}{P(\text{covid})}$$

$$P(\text{covid})$$

$$P(A \mid B) = \frac{P(B \mid A) \cdot P(A)}{P(B)}$$

Sat Sun Mon Tue Wed Thu Fri

Date:

$$= \frac{0.85 \times 0.70}{P(\text{covid})} = \frac{0.6}{\alpha} = \frac{0.6}{0.6 + 0.03}$$

$$= 0.95$$

$$P(\text{test} = \text{negative} \mid \text{covid})$$

$$= \frac{P(\text{covid} \mid \text{test} = \text{negative}) \cdot P(\text{test} = \text{negative})}{P(\text{covid})}$$

$$0.10 \times 0.30$$

$$= \frac{0.03}{P(\text{covid})} = \frac{0.03}{\alpha} = \frac{0.03}{0.6 + 0.03}$$

$$= 0.05$$

test = Positive more likely

(A)

$$P(\text{covid} \mid \text{test} = \text{positive}) = 0.85$$

$$P(\neg \text{covid} \mid \text{test} = \text{+ve}) = 0.15$$

$$P(\text{covid} \mid \text{test} = \text{-ve}) = 0.10$$

$$P(\neg \text{covid} \mid \text{test} = \text{-ve}) = 0.90$$

$$P(\text{covid}) = 0.70$$

$$P(\text{test} = \text{+ve}) = ?$$

$$P(\text{test} = \text{+ve}) = P(\text{test} = \text{+ve} \wedge \text{covid})$$

$$+ P(\text{test} = \text{+ve} \wedge \neg \text{covid})$$

$$= P(\text{covid} \mid \text{test} = +ve) \cdot p$$

$$= P(\text{test} = +ve \mid \text{covid}) \cdot P(\text{covid})$$

$$+ P(\text{test} = +ve \mid \neg \text{covid}) P(\neg \text{covid})$$

$$= 0.7 \cdot P(\text{test} = +ve \mid \text{covid})$$

$$+ 0.3 \cdot P(\text{test} = +ve \mid \neg \text{covid})$$



Ref. No. MAI 2021-2022

Date: \_\_\_\_\_

9.

$$\textcircled{a} \quad P(\text{Rugby} | \text{male}) = \frac{P(\text{Rugby} \cap \text{male})}{P(\text{male})}$$

$$= \frac{0.2}{0.54}$$

$$\textcircled{b} \quad P(\text{female} | \text{football}) = \frac{P(\text{female} \cap \text{football})}{P(\text{football})}$$

$$= \frac{0.15}{0.39}$$

$$\textcircled{c} \quad P(\text{Football} \vee \text{others})$$

$$= P(\text{Football}) + P(\text{others}) - P(\text{Football} \cap \text{others})$$

$$= 0.39 + 0.36 - 0.39 \times 0.36 \quad (x = \text{others})$$

$$= 0.39 \times 0.36$$

Q)

$$P(\text{Rugby} | \text{Female})$$

$$P(\text{Rugby} \wedge \text{Female})$$

$$\underline{P(\text{Female})}$$

$$= \frac{0.05}{0.46} = 0.11$$

$$P(\text{Rugby}) = 0.25$$

$$\text{So } P(\text{Rugby} | \text{Female}) \neq P(\text{Rugby})$$

Therefore Playing Rugby depends on females.

10.

(A) Find information gain of  $\text{P}_1, \text{P}_2$  and  $\text{P}_3$  and select that one which has the maximum information gain.

(B) No, ID3 will not be suitable for this classification task. we have to convert continuous attributes into discrete attribute before we start building the tree.

(C) 10 male  $\rightarrow$  Smoker/Non-smoker = 70 : 30

5 Female  $\rightarrow$  Smoker/Non-smoker = 20 : 80

~~Entropy~~<sup>P</sup> (smoking female)

$$= \frac{10}{15} \times \frac{70}{100} = \frac{7}{15} = 0.47$$

Entropy (smoking male)

$$= - r \log_2 r$$

$$= - 0.47 \log_2 0.47 = 0.52$$

ii.

②  $P(c=1, m=0, Y=1)$

$$= P(c=1, m=0 | Y=1) \cdot P(Y=1)$$

$$= P(c=1 | y=1) \cdot P(m=0 | Y=1) \cdot P(Y=1)$$

$$= P_2 \times (1-P_4) \cdot (1-q)$$

③  $P(c=1, m=0, Y=0)$

$$= P(c=1, m=0 | Y=0) \cdot P(Y=0)$$

$$= P(C=1 \mid Y=0) \cdot P(M=0 \mid Y=0) \cdot P(Y=0)$$

$$= (1-P_1) \cdot (1-P_3) \cdot q$$

(c)  $P(Y=1 \mid C=1, M=0)$

$$P(Y=1, C=1, M=0)$$

$$= \frac{P(C=1, M=0)}{P(C=1, M=0)}$$

$$= \frac{P_2 \times (1-P_4) \times (1-q)}{P(C=1, M=0, Y=1) + P(C=1, M=0, Y=0)} \quad [ \text{From (1)} ]$$

$$= \frac{P_2 \times (1-P_4) \times (1-q)}{P(C=1, M=0, Y=1) + P(C=1, M=0, Y=0)}$$

$$= \frac{P_2 \times (1-P_4) \times (1-q)}{P_2 \times (1-P_4) \times (1-q) + (1-P_1) \times (1-P_3) \times q}$$

$$= \frac{P_2 \times (1-P_4) \times (1-q)}{P_2 \times (1-P_4) \times (1-q) + (1-P_1) \times (1-P_3) \times q}$$

[ from (2)

and (1)

and marginilizati<sup>n</sup>]

③ Find the table

$$q = P(Y=0) = \frac{3}{10} = 0.3$$

$$P_1 = P(c=0 | Y=0) = \frac{2}{3}$$

$$P_2 = P(c=1 | Y=1) = \frac{4}{7}$$

$$P_3 = P(m=1 | Y=0) = \frac{1}{3}$$

$$P_4 = P(m=1 | Y=1) = \frac{2}{7}$$

$$④ P(Y=1 | c=1, m=0)$$

$$= P(m=0 | Y=1) \cdot P(c=1 | Y=1) \cdot P(Y=1)$$

$$= \left(1 - \frac{2}{7}\right) \left(1 - \frac{4}{7}\right) (1 - 0.3) = 0.214$$

$$P(Y=0 \mid C=1, M=0)$$

$$= P(C=1 \mid Y=0) \cdot P(M=0 \mid Y=0) \cdot P(Y=0)$$

$$= \left(1 - \frac{2}{5}\right) \left(1 - \frac{1}{3}\right) \times 0.3$$

$$= \frac{1}{3} \times \frac{2}{5} \times 0.3 = 0.07$$

The minister is more likely to be in the office.