Individual and Aggregate Mismatch in Higher Education

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Abstract

In many economies, many graduates are observed to work in non-graduate occupations. Does this mean that the level of education is inefficiently high? I propose a model that features uncertainty about the causal effect of HE and incorporates general equilibrium wage effects in a matching model of the labour market. My model highlights that there are two distinct notions of overeducation. Workers could be individually mismatched in the sense that they would be better off if they made alternative education choices. There could also be aggregate mismatch if average welfare would be higher if the average level of HE attendance changed. I find that uncertainty about returns generates the former, while endogenous education choice in a matching market generates the latter. Structurally estimating my model on UK data, I calculate that 32.9% of the population individually mismatched. I find in policy simulations that policy-makers can improve aggregate welfare by reducing HE share by 1.6 percentage points. Reducing uncertainty about returns is unambiguously good for workers but increases income inequality and worsens firms' outcomes by making labour more expensive.

JEL Codes: D81, I23, I26, J24
Keywords: Overeducation, efficient human capital investment, labour market matching

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1 Introduction

In many developed countries, a large and increasing share of young people attend higher education (HE), completing undergraduate degrees. HE attendance is often subsidised by governments, either directly through tuition fee subsidies (e.g. Germany, France) or the provision of below market-rate loans (e.g. United Kingdom, Australia). The large numbers of HE students and the high cost of such HE subsidies to the state have often prompted concerns that there are too many students who select into higher education. These concerns are exacerbated by the observation that a significant share of graduates end up working in occupations which typically do not require a college degree, a phenomenon which is known as overeducation or underemployment. These underemployed graduates typically earn less than their peers (Leuven & Oosterbeek (2011)), and are persistently employed in low-skilled employment (Barnichon & Zylberberg (2019)).

In this paper, I study the determinants of overeducation and analyse the role of policy in addressing the phenomenon. This paper makes two main contributions to the literature on these questions[1]. First, I emphasise the distinction between individual mismatch and aggregate education mismatch, and argue that the former may not be grounds for policy intervention if the individual mismatch is due to information limitations. Second, I provide evidence about the role of selection on ability in overeducation[2], and develop a matching model that accounts for selection into education on the basis of wage returns. I provide a stylised fact to show that the probability of underemployment is decreasing in graduate's ability (as measured by cognitive tests), and argue that this suggests that individual heterogeneity is important in the phenomenon of overeducation. I then develop a matching model of the labour market with an initial education decision that features Roy-type selection into education based on returns. To my knowledge, my paper is one of the first to incorporate selection into education on uncertain returns in the context of a matching framework[3].

First, this paper argues that popular discussions of over-education use the term in a different sense to that in the macroeconomic literature on underemployment. In those models, there is overeducation or undereducation in the sense that aggregate utility in the economy, defined in some specific way, would increase if the profile of HE attendance in the economy changed. This is the sense in which one may argue that there is credentialism in the labour market or inefficient competition in HE attendance, and that average worker utility may increase if fewer students chose to engage in HE. I describe this definition of overeducation as aggregate overeducation. Another way in which people use the term is to argue that an individual is over-educated when a graduate

[1] Much of the literature rationalising overeducating or underemployment has been in the macroeconomics tradition. A popular approach, following Albrecht & Vroman (2002), has focused on analysing models with two types of jobs with varying skill requirements, and workers competing for jobs either in partially segmented labour markets (Dolado et al. (2009); Barnichon & Zylberberg (2019)). The main insight of these models is that underemployment occurs because skilled workers may accept low-skilled jobs either due to search frictions or because they wish to escape competition from other skilled workers. Jackson (2021) extends these models with an endogenous education choice and show that underemployment can be efficient in the aggregate sense if expanding education leads to benefits e.g. for firms or more encouraging more productive firms to enter the market.

[2] Chevalier (2003) is an early empirical paper that makes a similar point that apparent mismatch in the labour market may not be actual mismatch if the graduate in question is of low ability. My paper expands this point by showing how ex-post mismatches can occur if students do not have perfect information about their ability, and analyses how this mechanism interacts with a matching model.

[3] These models typically assume some other form of individual heterogeneity to account for workers making different education decisions. Chade & Lindenlaub (2021) assume heterogeneity in the cost of education while Jackson (2021) assumes heterogeneity in endowments.

is working in an occupation which does not 'require graduate skills'. A possible definition based on this usage is as follows: individual education mismatch occurs when a worker selects into HE despite not benefiting from it, or if a worker does not select into HE despite benefiting from it. This usage clearly is similar to the notion of potential outcomes in causal analysis; a worker is individually mismatched if they make a choice that does not lead to the highest potential outcome. Such individual mismatch could occur ex-post when workers make ex-ante rational decision based on imperfect information (Cunha et al. (2005), Athey & Wager (2021)).

I propose a framework that rationalises both individual mismatch and possible aggregate mismatch that arises from general equilibrium effects. To this end, I develop a model of education investment under uncertainty set in a labour market characterised by matching between workers and firms. In the model, the return to HE in terms of human capital is heterogeneous across workers, varying by an index which I call ability. This return is imperfectly observed at the point of choice; workers can only observe their grades in school, which I interpret as noisy signals of their underlying ability. As such, workers make an education decision based on an ex-ante expectation of their likely returns to education, but ex-post may experience regret if they are 'unlucky', i.e. they received a signal which convinced them that they were higher ability than they actually are. Post HE, workers match to jobs in a frictionless matching market based on their post-education human capital. This matching labour market generates general equilibrium price effects; when the share of workers in HE increases, workers have less bargaining power as there are more skilled workers in the economy and their wages conditional on their human capital fall as a result.

In my framework, the main determinant of the extent of individual mismatch is the amount of uncertainty that workers face about their underlying return to HE. This suggests that an important function of testing is to enable workers to make more accurate choices about their human capital investments. However, this uncertainty does not generate aggregate mismatch by itself, as workers make individually optimal decisions ex-ante, which the government cannot improve upon unless they have more information than firms. Instead, I find that there are two forces that generate aggregate mismatch in my model. First, there is a hold-up externality where workers do not internalise the effect of their education on firms' profits because they cannot appropriate this surplus. Thus, they will under-invest in higher education. Second, because workers' wages depend on their position on the overall skill distribution in the economy, they exert positional externalities on others when they invest in education. Workers do not consider the impact of their education choice on the bargaining power of other workers in the economy in equilibrium. This leads to over-investment in higher education relative to the social optimum. Because these externalities go in opposite directions, it is ambiguous whether the level of education is too high in the economy. In general, workers and firms have differing policy preferences, so policy-makers have to decide how to weight the preferences of workers and firms in deciding aggregate policy.

I contribute empirical estimates of the extent and welfare costs of education mismatch by estimating a parametric version of my model on data from the UK using the Simulated Method of Moments (SMM). I find that students face substantial uncertainty about their eventual returns to university; the estimated correlation between unobserved ability and the signal is only 0.324. I estimate that 32.9% of workers would have been better off if they made a different education decision to the one they actually did. 18.2% of workers or 39.1% of graduates did not benefit from university despite going, and 14.7% of all workers or 27.6% of non-graduates would have benefited from university despite not going. The costs in utility terms are small on average but non-negligible. The average utility loss to going to university for overeducated workers is equivalent to earnings increases of £0.183 per hour on average, while the average utility loss to missing out on university for undereducated workers is equivalent to £0.264 per hour on average. These results suggest that while over-education is the focus of headlines in the education debate, under-education is a substantially more costly problem when it occurs, which is why workers tend to err on the side of over-investment.

I conduct a number of counterfactual simulation exercises to shed light on optimal policy in this framework. First, I consider the implementation of a compensated graduate tax or subsidy aimed at affecting the share of students who choose to go to university. I show that workers and firms are not aligned in their policy preferences. A compensated graduate tax to reduce the level of education would increase average worker utility by increasing wages when the supply of skilled workers falls. Firms on the other hand would prefer a maximal subsidy as they do not not pay for workers' education and would like to free-ride on workers' human capital investments both in terms of increased productivity of matches and a lower wage because of labour supply. In practice, it seems to be welfare-improving to reduce the level of education by 1.6pp by implementing a small revenue-neutral graduate tax.

Second, I consider the effects of targeting the quality of the information students have about their return to education. I find that, as expected, increasing the informativeness of students' signals about their return to education increases their welfare, but surprisingly, it decreases firms' profits. Increasing informativeness leads to increased inequality, because it allows high return workers to more perfectly sort into education and low return workers to sort out of education. The increased inequality also leads to a more polarised distribution of skilled workers, leading to higher wage costs for firms and a reduced worker quality for less productive firms.

This paper proceeds as follows. I begin by discussing the related literature in subsection 1.1. I then describe some stylised facts about UK higher education in section 2 that my model seeks to explain. I then describe my model in section 3. In section 4, I describe the empirical parameterisation of my model, identification and the data sources which I use. Section 5 describes the results of my estimation, and the model fit. Section 6 analyses individual mismatch in this model given the estimated parameters. I consider whether policy makers can increase welfare using a compensated graduate tax or subsidy in section 7, relating the exercise to current policy proposals being considered by the UK government. Section 8 concludes.

1.1 Related Literature

This paper relates most directly to the literature on the determinants of underemployment. Proposed explanations include labour market frictions leading to educated workers choosing to accept 'non-graduate' jobs (Albrecht & Vroman (2002), Dolado et al. (2009), Jackson (2021)), skilled workers choosing to enter low-skilled jobs in order to escape competition from other skilled workers (Barnichon & Zylberberg (2019)), and workers having preferences for particular low-skilled occupations (Gottschalk & Hansen (2003)). Besides Jackson (2021), these models do not analyse the normative implications of their models for the level of education in the economy. Under-employment in these models is also an optimal choice by workers given the labour market or due to labour market frictions. I contribute to this literature by (1) distinguishing between two senses (individual and aggregate) in which over-education may exist, (2) providing a mechanism for individual overeducation based on incomplete information, and (3) showing that workers' and firms' incentives are not necessarily aligned in the conduct of optimal policy. My paper suggests that this phenomenon could be due not only to ex-post mismatch on the labour market, but also due to ex-post mismatch in education choices instead.

A central argument in my paper is that the information set students have access to at the time of choice can be an important determinant of the extent of mismatch in the education decision. This coheres with recent research arguing that testing results and grades may have an informative purpose in informing students about their underlying ability and helping them make more ambitious course choices (Tan (2022)), as well as research suggesting that removing the SAT in college admissions may have negative efficiency effects (Borghesan (2022)).

Finally, my paper also relates to the literature on analysing matching in the labour market with endogenous education choice (Shephard & Sidibe (2019); Macera & Tsujiyama (2020); Chade & Lindenlaub (2021)). This literature typically focuses on studying the implications of the matching plus education choice model set-up for wage inequality in the labour market. My paper differs from these papers not only in emphasising overeducation, but also in featuring selection into education based on uncertain returns. A major resulting complication is that the distribution of skill, and therefore wages in this economy, become dependent on the education decision. This emphasis also provides a bridge between the micro-econometric literature that focuses on uncertainty and heterogeneity in the returns to education (Altonji (1993), Cunha et al. (2005), Cunha & Heckman (2016), Arcidiacono et al. (2016),Heckman et al. (2018)), and this labour matching literature which has typically abstracted from these concerns.

2 Stylised Facts

I motivated the paper by noting that in many developed country contexts, there is increasing concern about there being too many students selecting into higher education[4]. In this section, I focus on the UK context, where the incidence of higher education has increased substantially over the past decades[5]. I document three stylised facts which motivate my theoretical approach.

1. Many graduates are employed in low-skilled occupations.
2. The incidence and persistence of overeducation is strongly related to the ability of students.
3. Access to more highly paid occupations accounts for just under half of the overall wage premium.

[4]A recent paper, Jackson (2021), has documented similar patterns in the American context, while Girsberger & Meango (2022) documents similar facts in West Africa.

[5]In appendix A, I describe the UK higher education system in more detail and document the change in the number of graduates and wage premium over time.

2.1 Fact 1: There is a significant share of graduates in low-skill occupations

A stylised fact that is often found in the UK context is that there is a significant and increasing share of workers with graduate degrees working in so-called 'non-graduate' jobs[6]. Given the policy interest in the subject, especially since the government implicitly subsidises graduates who cannot fully repay their student loans, the UK Office of National Statistics has published a number of technical reports attempting to quantify overeducation using a variety of methods[7].

In this section, I first show that there are more graduates in occupations classified by the government statistical agencies as non-skilled occupations. The narrative that emerges from this exercise seems consistent; there are more graduates today working in occupations that would traditionally be considered 'non-graduate'. Figure 1a plots the share of graduates working in occupations considered to require a NVQ 1 or 2 level education (equivalent to having a good high school qualification), as classified by the ONS occupational classification. Over the studied time period, the share of workers working in low skill occupations increased from 11.4% in 2002 to a high of 19.4% in 2018. Figure 1b plots a similar graph, but instead uses a classification used by the UK Home Office to classify occupations into different skill levels[8]. Occupations were classified into low skill (requiring less than an A-level qualification), requiring an A-level qualification (RQF 3), requiring one year of a bachelor's degree (RQF 4), requiring an undergraduate degree (RQF 6), and requiring a postgraduate degree. At the start of the period, 9.8% of graduate workers worked in an occupation classified as low skill, but at the end, 14.2% of graduate workers do. 22.1% of workers were in an occupation requiring a high-school qualification or below at the start of the period, and at the end, 27.4% did. Thus, on the face of it, it seems that more graduate workers are now employed in so-called low-skill qualifications requiring a high-school degree or lower. Appendix B.1 shows that while the share of graduates in so-called low-skill occupations do vary between different subjects, non-negligible shares of graduates in most subjects, both in STEM and non-STEM, work in low-skilled occupations[9].

The interpretation of this fact is however not straightforward. As the number of graduates increase, the market readjusts[10], and jobs which were previously seen as non-graduate may be increasing taken up by graduates. This is in fact a feature of the model proposed in the following section, that as the supply of graduates changes, the composition of graduates within particular occupations similarly change. Appendix B.2 documents that the increase in the number of graduates

[6] As discussed in the literature review, the demarcation between graduate and non-graduate jobs is controversial, and a main concern of the large overeducation literature has been the clarification of the demarcation between the two categories. There are two main approaches to this issue. First, some researchers, including Green & Zhu (2010), and Green and Henseke (2016), argue that a classification should be based on some notion of an occupation requiring certain skills. Approaches which make use of existing skills classifications, such as the Office for National Statistics SOC classifications or the Home Office Appendix J classifications for occupation skill levels, would fall under this approach, as do approaches which turn to the task content of occupations to stipulate whether a job is 'graduate'. Second, other researchers have tried to classify occupations into 'graduate' and 'non-graduate' categories by examining outcomes, either the college-noncollege wage premium within an occupation (Gottschalk & Hansen (2003), O'Leary & Sloane (2016)) or the share of workers with degrees within that occupation (Elias and Purcell (2003)).

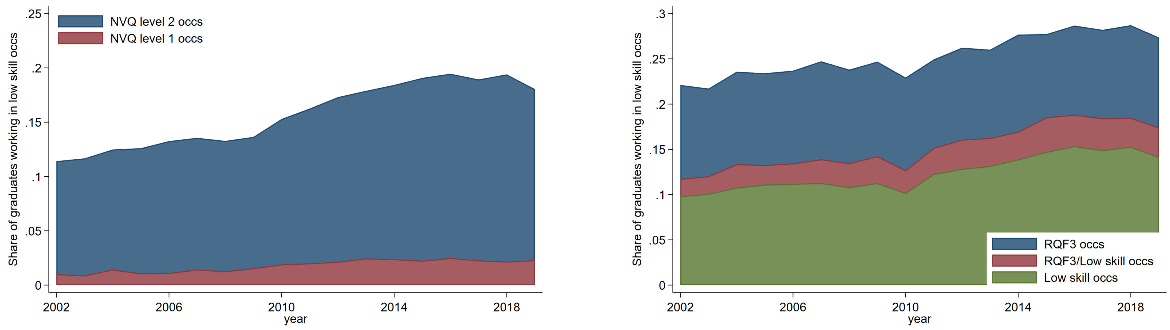
[7] See Clegg (2017) and Saric (2019). Academic work in a UK context attempting to quantify overeducation include Elias & Purcell (2004), Green & Zhu (2010), and O'Leary & Sloane (2016).

[8] This classification has also been used in Aghion et al. (2019)

[9] This does not apply to degrees for vocational subjects, such as medicine, subjects allied to medicine, architecture, and to a lesser extent, education.

[10] Models in which the composition of jobs changes as a result of technological change or an increase in the number of graduates include Acemoglu (1999), Albrecht & Vroman (2002), and Gottschalk & Hansen (2003).

Figure 1: Share of graduates in low-skill jobs by two different classifications



(a) Share of graduates working in occupations requiring NVQ1/2 qualifications

(b) Share of graduates in low skill occupations by Home Office classification

Data from the UK Labour Force Survey 2002-19. The occupational classifications produced by the Office for National Statistics are assigned skill levels relating the qualification level (based on the old NVQ classification) required for the job. The appendix J classifications are produced by the Home Office for the then-purpose of classifying prospective migrants by their skill level according to the Regulated Qualifications Framework.

between 2011-19 led to these graduates finding employment in occupations which previously had low shares of workers with university degrees, further showing that the occupations graduates work in change when the supply of graduates changes.

2.2 Fact 2: The incidence of overeducation is higher for low-ability individuals

What determines the probability of working in low-skilled occupations as a graduate? To examine this question, I use data from the UK Understanding Society household panel study, which contains longitudinal observations of individuals' earnings, their occupation, and a rich set of descriptive variables[11]. The survey also measures respondents' intelligence using a battery of cognitive tests, documented in McFall (2013), which I develop into a scalar measure of ability following the method of Ichino et al. (2022)[12].

I then estimate three models (linear probability, logit, and probit) of the probability of being observed to work in a high-skill occupation on intelligence interacted with whether the respondent has a degree[13]. I present these results in table 1 in terms of the implied change in the probability of working in a high-skill occupation given a 1 standard deviation increase in intelligence. My estimates suggest that a standard deviation increase in intelligence, measured by the first principal component of the age-detrended cognitive tests, is associated with a 4.1-4.6 percentage point increase in the probability of working in a high-skill occupation for non-graduates and a 10.3-11.2 percentage poiint increase for graduates.

[11] I construct a panel dataset of individuals in work, restricting the sample to full-time employees between the ages of 25 and 60, dropping individuals with missing wages, job hours, occupation and degree information. The main earnings variable I use is log net hourly income (after taxes and transfers).

[12] See appendix B.3 for details.

[13] In each regression, I control for a cubic age profile, sex, ethnicity, and parents' occupation and education. Standard errors are clustered at the individual level.

Table 1: Prob of Working in Skilled Occupation by Intelligence

	(1)	(2)	(3)
Non-graduate	0.0413***	0.0460***	.0446***
	(0.00494)	(0.00549)	(0.00542)
Graduate	0.112***	0.103***	0.105***
	(0.00681)	(0.00591)	(0.00600)
Model	Linear	Probit	Logit

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

This table presents the coefficients on the interaction term between intelligence and a degree indicator for models with the probability of working in a high-skilled job as the dependent variable, and the afore-mentioned terms plus a cubic wage profile, sex, ethnicity, parents' education and parents' occupation as independent variables. The sample was restricted to full-time workers between the ages of 25-60 with non-missing data, from the Understanding Society survey. The standard errors were clustered at the individual level.

The relation between over-education of graduates and ability is not accounted for by current approaches to undereducation. Many of the approaches following Albrecht & Vroman (2002) (including Dolado et al. (2009) and Barnichon & Zylberberg (2019)) assume constant types within educational categories. Thus, it is not clear how one can explain this relationship with their preferred mechanism - that graduates may select into under-employment to escape tight educational labour markets or due to labour market friction (unless this friction was higher for low-ability individuals). This is a gap that my model looks to fill.

2.3 Fact 3: A major component of the higher education wage premium is higher access to high paying occupations.

Having documented the role of heterogeneous intelligence in affecting the probability of matching to a high-skilled occupation, I now document that a substantial part of the wage premium operates through the occupational upgrading channel. To fix ideas, consider a simple framework in which the effect of education works by (i) allowing workers to match to more high-paying occupations (matching effect) and (ii) by increasing workers' wages within the occupation (productivity effect). Lemieux (2014) suggests that it may be able to tease apart these two mechanisms by contrasting estimates of the causal effect of education with and without controlling for the student's occupation. Estimates of models without occupation fixed effects should recover the sum of the matching effect and the productivity effect while estimates of models with occupation fixed effects should control for the matching effect and leave only the productivity effect.

I use the UK Understanding Society survey to attempt this exercise[14], using two approaches to control for selection concerns. First, I estimate the propensity of undertaking education with a probit model, controlling for intelligence (using a measure based on cognitive tests described in the previous sub-section) and student characteristics including sex, ethnicity, family background, and

[14] I restrict the sample to individuals between the ages of 25 and 60, dropping individuals with missing wages, job hours, occupation and degree information. I also restrict the analysis to full time employees.

birth year. I then estimate a linear regression with log hourly income as the dependent variable and the degree indicator as the independent variable, with cubic age profile, year fixed effects and a cubic polynomial of the propensity score as controls. Second, I estimate Mincer equations simply including the covariates in the propensity score equation in the linear regression. i also control for year, cohort as well as age profile. Standard errors are clustered at the individual level.

Table 2 below reports the results for four specifications, with and without occupation fixed effects, and by controlling for the propensity score or including a rich set of observable characteristics as covariates. Regardless of the method, I find that the causal effect with occupation controls is 56.5-61.6% of the causal effect without occupation controls, suggesting that the occupational upgrading channel accounts for roughly 40% of the effect of education. This motivates a central mechanism in the proposed model, that a major effect of education is to enable students to access better matches on the labour market.

Table 2: Wage premia with and without occupation FE

Log hourly income	(1)	(2)	(3)	(4)
Degree	0.258***	0.159***	0.299***	0.169***
	(0.0114)	(0.0103)	(0.0103)	(0.0100)
Observations	39393	39393	39394	39394
Occupation FE	No	Yes	No	Yes
Method	Propensity score	Propensity score	Linear regression	Linear regression

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

This table presents the coefficients on the degree indicator with and without occupation controls for two regression models. For the method labelled "propensity score", I estimate the probability of attending university based on intelligence, cohort and personal characteristics, and regress log hourly wages on cubic polynomials in age, the propensity score, and year fixed effects. For the method labelled "linear regression", I regress log hourly earnings on a cubic polynomial in age, intelligence, personal characteristics, and year and cohort fixed effects. Standard errors are clustered at the individual level.

I also replicate the well-documented fact that graduates who are over-educated suffer a wage penalty relative to their 'well-matched' peers[15] using the UK Understanding Society survey. I estimate the Duncan-Hoffman augmented wage equation[16], an augmented Mincer equation interacting the degree dummy with a dummy for whether the graduate is over-educated[17], and replicate the finding that being over-educated substantially reduces the university wage premium. These results are presented in table 3; I find that being underemployed reduces the graduate wage premium by over 50%. This result is in line with the results on the importance of the occupational upgrading channel to the graduate wage premium.

[15]For a review of the empirical literature on the overeducation penalty, see Leuven & Oosterbeek (2011).

[16]I construct a panel of workers using the Understanding Society for full-time workers between the ages of 25 and 60, dropping individuals with missing information on wages, occupations, degree status, sex, ethnicity, cognitive test scores and family background. I construct an intelligence measure using the approach from Ichino et al. (2022). Details on the construction of intelligence are provided in appendix B.3.

[17]Here, I use the Home Office classification of the skill-level of occupations.

Table 3: Overeducation Penalty

	(1)
	Log hourly income
Degree	0.339^{***}
	(0.0115)
Degree × Overeducated	-0.192^{***}
	(0.0133)
Observations	39393

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

This table presents the coefficients on the degree indicator and an interaction between having a degree and working in a low skill occupation. I control for age with a cubic polynomial, sex, ethnicity, parents' occupation and education, and intelligence. Standard errors are clustered at the individual level.

3 A Model of Education Investment and the Labour Market

The stylised facts suggest that (i) underemployment is strongly related to individual ability, and (ii) being able to match to better occupations seems to account for a major share of the education wage premium. In this section, I study a model in which workers first decide in an initial period whether or not to invest in education, and subsequently receive a wage depending on their heterogeneous characteristics and their education choices. The model of education choice in the first period is a Willis-Rosen type model of self-selection into education, but with incomplete information and uncertainty about returns, like in Cunha et al. (2005). The labour market is modelled with a frictionless matching model with transferable utility, like in Sattinger (1993)[18]. Outcomes in the first education stage affect the latter labour market stage by affecting the skill composition of the labour market; when more students are educated, the supply of skilled workers increases. The central aim of the model is to rationalise the patterns in the wages and occupational outcomes of graduate and non-graduate workers with their choices to invest in higher education, as well as to show how both individual and aggregate overeducation can arise in this setting.

This section proceeds as follows. I describe the model and maintained assumptions in section 3.1, including the characterisation of equilibrium in this setting. I consider optimal private investment and optimal social investment in education in this model in section 3.2, and discuss how individual mismatch arises in this setting. Section 3.3 argues that the selection into higher education that occurs based on individual choice is not generally socially optimal, and leads to aggregate mismatch.

[18]Alternative matching models studied in the labour market context include two-sided matching with search frictions (Shimer & Smith (2000)), and models reducing the number of types of either workers or firms (Acemoglu (1999), Shephard & Sidibe (2019)).

3.1 Description of the model

3.1.1 Setting

There are two masses of workers and jobs of equal length. As a primitive, workers are heterogeneous in two respects. First, they differ in their labour market ability, denoted by a and distributed with density function $f_A(a)$. Second, they differ in their preference for higher education, denoted by two random variables η_1 and η_0 representing preference for HE and not going to HE respectively. Their net preference for HE is denoted by $\Delta\eta$ which is a random variable with distribution $f_{\Delta\eta}(\Delta\eta)$.

The random variable a determines a worker's return to education. In the model, workers can choose to select into higher education, denoted by binary variable $e \in \{0, 1\}$. Attending HE increases their skill on the labour market; in the model, it is the skill level s, instead of initial ability a, that determines the worker's return on the labour market. The relation between labour market ability and education, and skill is summarised by the skill function $S(a, e)$. I assume that the skill function $S(a, e)$ is continuous and differentiable in a for both values of e. Furthermore, I assume that $\frac{\partial S(a,1)}{\partial a} > \frac{\partial S(a,0)}{\partial a}$ for all values of a, such that the difference of $S(a, 1) - S(a, 0)$ is increasing. Intuitively, this implies that the skill return to education, denoted by $\Delta S(a) \equiv S(a, 1) - S(a, 0)$, is increasing in ability.

Workers are unable to fully observe their labour market ability a, and instead receive a noisy signal θ, which imperfectly reveal their underlying ability to the worker. I assume that the noisy signal is additive in ability and a white noise term ε which garbles the true ability, as follows.

$$
\begin{aligned}
&\theta = a + \varepsilon \\
&\varepsilon \sim N(0, \sigma_\varepsilon^2)
\end{aligned} \tag{1}
$$

The signal is imperfectly correlated with labour market ability a because (i) the current testing technology available to us cannot perfectly capture worker talent and ability, and (ii) in the time between when the signal was drawn and labour market performance is realised, workers could experience a number of shocks to their labour market performance, e.g. adverse health shocks, or luck in meeting the right opportunities. More generally, the variance of ε governs the correlation between the information workers have at the time they have to make an education decision and their actual labour market ability.

Jobs have varying levels of productivity, denoted by y. Suppose that there are K discrete occupations, indexed by k, where the distribution of productivity of jobs within that occupation is denoted by $f_y^k(y)$, and the share of jobs in that occupation in the economy is denoted by p_k, where $\sum_{k\in\{1,\cdots,K\}} p_k = 1$. The distribution of productivity in the entire economy is given by $f_y(y) = \sum_{k=1}^{K} p_k f_y^k(y)$. In this model, occupations do not serve any direct role in the equilibrium match (i.e. there are no preferences for occupations), but they are important for two reasons. First, they help the estimation by providing information on the productivity of the job the worker is matched to, especially if information on the matched job is not available. Second, they lead to testable implications about the resulting share of workers with degrees and the college wage premium within occupations. I do not make any assumptions about the ordering of occupation-specific productivity distributions. I discuss implications of the model for the occupation structure in the economy further in appendix C.1.

There are two periods in the model. In the first period, workers decide whether to invest in education based on their expected returns, and their preferences for education. The decision to invest in education is described in detail in section 3.1.3. This determines the distribution of skill among workers in the economy. In the second period, workers with skill s and jobs with productivity y can match pairwise with each other to produce joint output $g(s, y)$. I assume that $g(s, y)$ is twice continuously differentiable, increasing and supermodular[19]. A model of this kind is known in the matching literature as a frictionless matching model with transferable utility and has been analysed thoroughly in other contexts. The resulting wage function for workers with skill s is the wage that individuals receive in the second period and is analysed in the following subsection 3.1.2.

In summary, the following assumptions are maintained throughout.

Assumption 1 *In the model, I maintain the following assumptions.*

1. *(Information structure of workers) Workers do not observe their initial ability a, and observe a noisy signal θ, related to a by the following expression. θ is the only information that the worker has about a and their return to education. ε has a mean zero, log-concave distribution.*

$$\theta = a + \varepsilon$$
$$\varepsilon \sim N(0, \sigma_\varepsilon^2)$$

2. *(Assumptions on $S(a, e)$) The skill function $S(a, e)$ is continuous and differentiable in a for both values of e. Furthermore, I assume that $\frac{\partial S(a,1)}{\partial a} > \frac{\partial S(a,0)}{\partial a}$ for all values of a, such that the difference of $\Delta S(a) \equiv S(a, 1) - S(a, 0)$ is increasing in a.*

3. *(Assumptions on joint output $g(s, y)$) The joint output function $g(s, y)$ is increasing in both s and y, and twice continuously differentiable. The function is assumed to be supermodular, which is equivalent to the following condition since it is twice-continuously differentiable: $\frac{\partial^2 g}{\partial s \partial y} \geq 0$.*

3.1.2 The labour market and the wage function

Post-education, workers come to the labour market where they seek employment. I consider a model with a few important abstractions. After completion of higher education, firms are able to observe workers' skill types; this assumption coheres with work by Lange (2007) which shows that employers typically learn quickly about their employees productivity types. I also abstract from a number of mechanisms which will generate overeducation automatically; I assume that there are no labour market frictions, and once matched, workers stay matched forever in the job. This ensures that there are no dynamics in this model that can lead workers to transition between jobs. These abstractions mean that the outcome described here should be seen as a kind of end-state for a potentially dynamic matching process where workers move between jobs early in their careers to find their best match. This coheres with work by Topel & Ward (1992) and Adda & Dustmann (2023) that find more job mobility earlier in workers' careers. I also assume for simplicity that there is no unemployment[20].

[19] In this case, the supermodularity condition is equivalent to $\frac{\partial^2 g}{\partial s \partial y} \geq 0$ (Topkis (1998)).

[20] It is possible to relax this assumption by assuming that there are more workers than jobs. See Chade et al. (2017) for an overview.

To fix ideas, I start by describing this labour market as resolving a discrete assignment problem, where N workers match to N jobs. The results that I need however are expressed in continuous terms with continuums of jobs and workers, which can be thought of as corresponding to the situation when N becomes very large. On the labour market, workers are heterogeneous to job-operators only insofar as that they have different levels of skill. The distribution of worker skill can be described by distribution $f_s(s)$, which is exogenous in the labour market stage but endogenous to workers' earlier education choices[21]. Jobs differ in their productivity (as described in the previous section), which governs the total joint output that a worker with skill s and a job with productivity y produces. The primitives of the matching problem that characterises the labour market are thus the distribution of skills $f_s(s)$, the overall distribution of productivity $f_y(y)$, and the joint output function $g(s, y)$.

Workers and firms simultaneously play what Shapley & Shubik (1971) refer to as an assignment game, that is, workers and jobs have to find a counterparty to match to, and to divide the joint output that results from the match. Because there are no restrictions on how the joint surplus is divided besides feasibility and rationality, workers and firms alike can attempt to induce more productive parties to match with them by offering to accept less of the surplus themselves. The outcome of this game is a matching, which is a specification of which workers match with which jobs, denoted μ, and a set of wages paid to workers w and a set of profits π paid to job-operators which come from the division of the joint output. In the continuous case, the matching function is a continuous function, denoted μ, mapping the domain of s to the domain of y, while the wage ($w(s)$) and profit ($\pi(y)$) functions become continuous functions in s and y respectively; i.e. the outcome of the assignment game in the continuous case is a triple of functions $\{\mu, w, \pi\}$.

This is a well-studied question in the field of matching theory[22], and in this section, I only present the results and intuition relevant to my application, omitting the proofs. In particular, I am interested in what the outcome will be in the assignment game in the labour market. An important notion here is that of stability; a stable outcome is one which no pair of workers and jobs can profitably block. The technical condition related to the no-blocking requirement is that for any values of s, y:

$$w(s) + \pi(y) \geq g(s, y), \quad \forall s, y \tag{2}$$

If this condition did not hold, then worker s and firm y could increase their payoffs by matching with each other and increasing their output by splitting the excess $g(s, y) - w(s) - \pi(y)$. A stable outcome can be defined as a feasible outcome where the no-blocking condition given by equation 2 is satisfied, where feasibility means that all agents are matched and that the sum of the surpluses received by all workers and jobs does not exceed the total output of the economy.

Combining results from Shapley & Shubik (1971) and Becker (1973), it turns out given the supermodularity of the joint output function $g(s, y)$, the unique stable outcome is positive assortative matching, i.e. when the most skilled workers match to the most productive firms. Then the assignment that results in the labour market is given as follows, where $F_s(s)$ and $F_y(y)$ denote the cumulative distribution functions of the skill distribution and the productivity distribution respectively.

$$\mu(s) = F_y^{-1}(F_s(s)) \tag{3}$$

[21]The determination of $f_s(s)$ is discussed in sections 3.1.3 and 3.1.5.

[22]The exposition of the problem, its solution and attendant proofs have been discussed in many other papers and books, including Galichon (2016), and Chade et al. (2017).

What are the payoffs that can sustain this positive assortative matching? One can pin down the pay-offs $\{w, \pi\}$ by demonstrating the properties it has to satisfy if such pay-offs exist, and then in turn show that the pay-offs are feasible[23]. This exercise is often interpreted as showing that the optimal assignment can be sustained in a competitive (Walrasian) market in which agents take the prices they have to pay to match as given, and optimise over who they want to match with. Let $w(s)$ and $\pi(y)$ denote a pay-off that can sustain the unique stable assignment. Then, workers' and job-operators' individual problems are to choose an agent of the opposite kind to match with so as to maximise their individual pay-off, as follows:

$$\text{Worker's problem for worker with skill } s^*: \quad \mu(s^*) = \arg\max_y g(s^*, y) - \pi(y) \tag{4}$$

$$\text{Firm's problem for firm with prod } y^*: \quad \mu^{-1}(y^*) = \arg\max_s g(s, y^*) - w(s) \tag{5}$$

In equilibrium, the first-order condition of the maximisation problem faced by firms needs to be satisfied by the worker with skill s that it matches to. That is, for all values of s, the following condition should hold, pinning down the shape of the resulting wage function. Analogously, the first-order condition of the maximisation problem faced by the worker fixes the shape of the profit function as follows, where $\mu^{-1}(y)$ specifies the skill of the worker that a firm with productivity y matches to. This also suggests that the wage and profit schedules are convex functions[24].

$$\frac{\partial g(s, \mu(s))}{\partial s} = w'(s) \tag{6}$$

$$\frac{\partial g(\mu^{-1}(y), y)}{\partial y} = \pi'(y) \tag{7}$$

We can then solve for the wage and profit functions by integration.

$$w(s) = w_0 + \int_{-\infty}^{s} \frac{\partial g(x, \mu(x))}{\partial x} dx \tag{8}$$

$$\pi(y) = \pi_0 + \int_{-\infty}^{y} \frac{\partial g(\mu^{-1}(x), x)}{\partial x} dx \tag{9}$$

The wage function is thus fixed up to location, given by w_0 which is a constant of integration[25]. The outcome of the labour market is thus the matching $\mu(s)$, which predicts positive assortative matching between workers and jobs (given by equation 3), and the wage and profit functions, given by equations 8 and 9. These wage and profit functions depend on the primitives of the matching problem, in particular the distribution of skill in the economy, which is in turn dependent on workers' education choices. In this way, there is a feedback channel from the education decisions of workers to the wages that they obtain in the model.

[23] As this is not relevant to my application, I refer those interested in the second part of this proof to chapter 4 of Galichon (2016).

[24] To see this, note that for any $s' > s''$, $\mu(s') \geq \mu(s'')$, with non-equality for most continuous distributions of y. Then, by the assumption of supermodularity, $\frac{\partial g(s,\mu(s))}{\partial s}(s') > \frac{\partial g(s,\mu(s))}{\partial s}(s'')$, and so $w'(s') > w'(s'')$.

[25] This indeterminacy is discussed in Sattinger (1993); one intuition is that since the incentives for matching with any type depends on the relative incentives, any wage function that preserves the shape of the function leads to the same assignment in Walrasian equilibrium. The values of w_0 and π_0 must be such that $w_0 + \pi_0 = g(s_0, y_0)$, where s_0 and y_0 denote the minimum (or infimum) value of s and y in the domain of the distribution of both variables. The precise value of w_0 relative to π_0 can be interpreted as the bargaining outcome between the least skilled worker and the least productive firm, possibly reflecting the relative outside options of each agent.

3.1.3 The decision to invest in education

As described before, workers have heterogeneous preferences for higher education, denoted by η_1, and for non education, denoted by η_0, with $\Delta\eta$ denoting their net preference for HE. I assume that the heterogeneous preference shocks for education are independent of the other shocks in the model, i.e. of the information shock ε, as well as the initial distribution of ability a. They also receive utility from their wages in the subsequent period, with time preference factor β. Students also face a fixed cost for investing in education, which I denote by $-\kappa$.

Workers have beliefs about the wage they would receive conditional on the skill that they have. Let $w^B(\cdot)$ denote a wage function which maps a level of skill to a wage on the labour market. $w^B(\cdot)$ defines the wage environment that workers believe will prevail in the labour market at the time of workers' educational choice. For simplicity, I assume that all workers believe that the same wage environment prevails.

Denote the difference between the value functions under HE and no HE by ΔV, where the arguments are θ, $\Delta\eta$ and w^B.

$$\Delta V(\theta, \Delta\eta, w^B(\cdot)) = \kappa + \Delta\eta + \beta \left\{ E\{w^B(S(a,1)) - w^B(S(a,0))|\theta\} \right\} \tag{10}$$

Workers face a discrete choice problem. Their optimal education choice e^* given their beliefs $w^B(\cdot)$ therefore is given as follows.

$$e^*(\theta, \Delta\eta, w^B(\cdot)) = \underset{e\in\{0,1\}}{\arg\max}\, e \times \Delta V(\theta, \Delta\eta, w^B(\cdot)) \tag{11}$$

Intuitively, the addition of endogenous education choice under uncertainty leads to education mismatch in two senses. First, the uncertainty that workers face leads to mismatch in the sense that their ex-ante optimal education choices may not line up with ex-post optimal ones. Second, as I will show subsequently, the endogeneity of education choice in a matching model also results in the aggregate level of education under free choice not being socially optimal in general. These two kinds of mismatch are conceptually separate; mismatch in the first sense occurs with choice under uncertainty even without matching labour markets and mismatch in the second occurs with matching markets even without uncertain education choice. However, I show in subsequent simulations that with higher choice uncertainty, the inefficiency of education choice under equilibrium is more severe.

3.1.4 The distribution of skill in the economy

The distribution of skill in the economy is an important determinant of wages in the economy as it enters the wage equation through the matching function, equation 3. In this model, the distribution of skill in the economy is dependent on who chooses to go into education. When more workers are educated, there is a greater supply of skilled workers in the economy, which affects the wages that workers receive. Let $p(\theta, \Delta\eta)$ denote a function that maps $\mathbb{R} \times \mathbb{R} \to \{0, 1\}$. Intuitively, this function represents an education profile, which maps workers by their observed grade to whether they attend higher education. Using the law of total probability, we can compute the distribution of skill in the economy using the Law of Total Probability.

$$
\begin{aligned}
f_S(s) = \int_{\varepsilon \in \mathbb{R}} \int & p(S^{-1}(s,1) + \varepsilon, \Delta\eta) f_A(S^{-1}(s,1)) \left| \frac{dS^{-1}(s,1)}{ds} \right| + \\
& (1 - p(S^{-1}(s,0) + \varepsilon, \Delta\eta)) f_A(S^{-1}(s,0)) \left| \frac{dS^{-1}(s,0)}{ds} \right| dF_{\Delta\eta}(\Delta\eta) dF_\varepsilon(\varepsilon) \qquad (12)
\end{aligned}
$$

$$
F_S(s) = \int_{-\infty}^{s} f_S(x)dx \qquad (13)
$$

3.1.5 Equilibrium under Rational Expectations

To complete the model and define equilibrium, I first introduce two notions. An education profile p is consistent with wage beliefs w^B if:

$$
\forall \theta \in \mathbb{R}, \forall \Delta\eta \in \mathbb{R} : p(\theta, \Delta\eta) = e^*(\theta, \Delta\eta, w^B(\cdot)) \qquad (14)
$$

Second, a wage function w is generated by an education profile p if the education profile implies a skill distribution (by equation 12),and thus a matching function (by equation 3) which imply w (by equation 8). That is to say, the skill distribution $F_S(\cdot)$ is a functional with p as an argument, the matching function $\mu(\cdot)$ is a functional that takes F_S as an argument, and $w(\cdot)$ is a functional that takes $\mu(\cdot)$ as an argument. Thus, this series of functionals give a mapping between each education profile p to a wage function w. For convenience, I denote this mapping of functions by R in the following text.

To complete the model, I need to specify how students generate beliefs about the wage functions they will face on the labour market. This is an area where much research still need to be done. Many authors now realise that it is the perceived return to higher education that matters for studying students' investment decisions (Jensen (2010), Wiswall & Zafar (2014)), but there is still relatively little work on how students form such beliefs about returns or what statistical approximations allows for reliable structural work involving beliefs. In the absence of more sophisticated theories, I resort to a simple assumption of rational expectations in this paper, but note that in practice, the economy would not be in equilibrium.

In this context, rational expectations imply that workers are able to anticipate the wages that actually obtain when they are on the labour market and set their beliefs to the predicted wages. If workers have rational expectations, then workers' beliefs w^B, the education profile generated by their optimal education choices p, and the resulting wage structure w have to agree.

Definition 1 *(Equilibrium) Rational expectation equilibrium obtains when workers have beliefs w^{B*}, an education profile p^* obtains, and wages w^* are generated such that:*

1. *p^* is consistent with wage beliefs w^{B*},*
2. *w^* is generated by p^*,*
3. *and $w^{B*}(s) = w^*(s), \forall s$.*

Note that the definition of "generated by" specifies a mapping of education profiles to wage functions, denoted by R. Similarly, the notion of consistency with a set of beliefs maps a function representing wage beliefs w^B to an education profile p. Denote this mapping T. The final condition trivially maps wages to wage beliefs; denote this by S. Then the conditions for equilibrium imply the following mappings:

$$R(p) = w$$
$$T(w^B) = p$$
$$S(w) = w^B$$

Putting these mappings together, the equilibrium condition implies a self-mapping of the wage belief function to itself, and an equilibrium wage belief is a Banach fixed point in this self-map:

$$w^B = S(R(T(w^B))) \tag{15}$$

Another way to define equilibrium is thus as follows. It is also possible to analogously define the self-mapping in terms of education profiles p or wages w.

Definition 2 *(An equilibrium wage belief) Let Q denote the set of functions that map $\mathbb{R}_+$ to $\mathbb{R}_+$. A function $w^B \in Q$ is an equilibrium wage belief if it is a fixed point in the self-mapping $w^B = S(R(T(w^B)))$.*

3.2 Optimal education investment

Under the assumptions maintained, it turns out that the solution follows a cut-off structure; workers should invest in higher education if their signal and preference for education is 'high enough'. I first discuss the characterisation of the optimal education decision under both the full-information and incomplete-information settings considering only private utility returns. Finally, I describe ex-post regret in this model, for individuals who made the ex-post welfare-reducing choice despite making the ex-ante rational decision, and show that this is entirely characterised by the relevant investment thresholds.

3.2.1 Full-Information Benchmark

Consider the case when ability is fully observable. Define by $e^P(a, \Delta\eta)$ the optimal education choice under full information for a worker with ability a and net preferences $\Delta\eta$. I assume for the subsequent analysis that workers are in rational expectations equilibrium where their beliefs about wages w^B are equal to actual wages.

$$\begin{aligned} e^P(a, \Delta\eta) &= \underset{e\in\{0,1\}}{\arg\max}\, e \times \{\kappa + \Delta\eta + \beta\{w(s(a,1)) - w(s(a,0))\}\} \\ &= \begin{cases} 1 \text{ if } \kappa + \Delta\eta + \beta\{w(s(a,1)) - w(s(a,0))\} \geq 0 \\ 0 \text{ otherwise} \end{cases} \end{aligned} \tag{16}$$

Denote for convenience $\Delta w(a) \equiv w(s(a,1)) - w(s(a,0))$. The equation defining optimality also implicitly defines a cut-off value of ability $a^P(\Delta\eta)$ for each level of education preference. We can visualise the optimality condition as a line in a-$\Delta\eta$ space. By implicitly differentiating the condition by $\Delta\eta$, we can further show that this line must be downward-sloping, thus defining a boundary, beyond which workers invest in education and before which they do not. This is visualised in figure 2 in the red line.

Proposition 1 *(Optimal private education choice under full-information)*

1. *The optimal education decision under full information is characterised by a cut-off point $a^P(\Delta\eta)$ for each value of $\Delta\eta$, which is the unique point that satisfies the optimality condition $w(s(a,1)) - w(s(a,1)) = -\left\{\frac{\kappa+\Delta\eta}{\beta}\right\}$. This cut-off is decreasing in $\Delta\eta$.*
2. *The optimal education decision under full information $e^P(a,\Delta\eta)$ is increasing in a, and $\Delta\eta$.*

Proof: *See appendix D.1.*

3.2.2 Imperfect information

Now, I consider my main setting where ability is not observable by the worker. Workers face an optimisation decision where they choose their education based on a signal θ, which is related to initial ability a through the structure specified by equation 1. The standard deviation of signal noise, σ_ε, determines how informative the signal is about the worker's underlying ability.

Denote by $e^I(\theta,\Delta\eta)$ the optimal education decision conditional on the signal and education preferences.

$$e^I(\theta,\Delta\eta) = \underset{e\in\{0,1\}}{\arg\max}\, e \times \{\kappa + \Delta\eta + \beta E\{w(s(a,1)) - w(s(a,0))|\theta\}\} \tag{17}$$

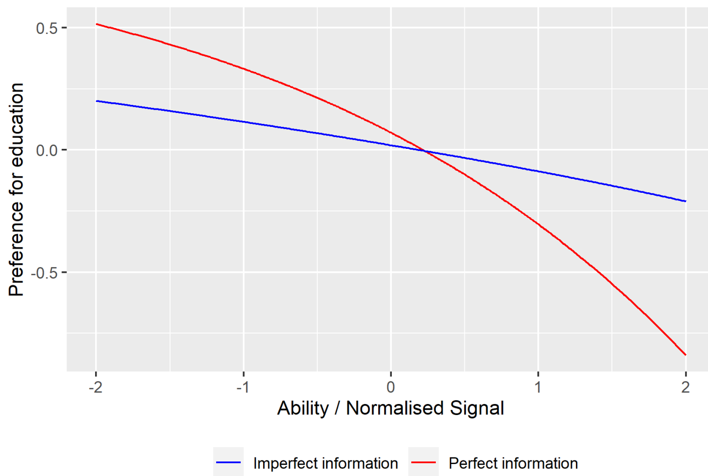
We can show that the optimal decision $e^I(\theta)$ follows an optimal cut-off point structure under the maintained assumptions.

Proposition 2 *(Partial information case under signals with additive, normally distributed noise)*

1. *Denote by $e^I(\theta,\Delta\eta)$ a function that maps θ and $\Delta\eta$ to the optimal education decision of a worker with signal θ and preferences $\Delta\eta$ who maximises their expected utility under imperfect information. Then, conditional on Δeta, $e^I(\theta,\Delta\eta)$ is weakly increasing in θ under the maintained assumptions.*
2. *The optimal education decision under imperfect information is characterised by a cut-off signal $\theta^I(\Delta\eta)$, which is the point that satisfies equation 18 below. This cut-off is decreasing in $\Delta\eta$, such that workers with greater preference for higher education require a lower signal to induce them to choose to invest in education.*

$$E\left\{w(s(a,1)) - w(s(a,0))|\theta^P(\Delta\eta)\right\} = -\left\{\frac{\kappa+\Delta\eta}{\beta}\right\} \tag{18}$$

Figure 2: Boundary Lines Describing Optimal Education Investment in Perfect and Imperfect Info



This graph plots a parameterised example optimality condition under perfect information and imperfect information. The parameterisation comes from the empirical implementation, described in subsequent sections.

Proof: *Direct application of Athey (2002), theorem 2, p. 200. See appendix D.2 for details.*

Propositions 1 and 2 can be interpreted as saying that in a two-dimensional representation of worker heterogeneity, we can draw a continuous downward-sloping line that divides the space of workers into those that select into HE and those who do not. Figure 2 plots a parameterised version the optimality condition with equality under perfect and imperfect information; as discussed, this condition implicitly defines a decreasing function in $\Delta\eta$ and a space which serves as a boundary for whether students will invest in HE. Students will invest in HE in imperfect information if they are to the right of the blue line, and will not invest in education if they are to the left of the line.

3.2.3 Individual mismatch

In the motivation of this paper, I discussed two notions of education mismatch, individual mismatch and aggregate mismatch. It turns out that the cut-off point structure of the solution to individuals' education decision problems offer a natural definition of individual education mismatch in a way that relates to the potential outcomes framework used in causal analysis. It also relates to the notion of ex-post regret as described in Cunha et al. (2005).

A worker experiences individual education mismatch when they choose a level of education ex-ante which leads than a worse outcome ex-post.

Definition 3 *(Education mismatch, over-education, under-education) A worker with ability a, signal θ, and education preferences $\Delta\eta$...*

1. *...experiences individual education mismatch if $e^P(a, \Delta\eta) \neq e^I(\theta, \Delta\eta)$.*

2. *...is over-educated if* $e^P(a, \Delta\eta) = 0$ *and* $e^I(\theta, \Delta\eta) = 1$.

3. *...is under-educated if* $e^P(a, \Delta\eta) = 1$ *and* $e^I(\theta, \Delta\eta) = 0$.

There are two kinds of workers who are ex-ante rational but ex-post worse off in my model. There are workers who do not select into education when they would have received a benefit ex-post; in this sense, they under-invest in education. There is also the corresponding type of worker who do invest in education when they do not receive a benefit ex-post; these type of workers over-invest in education. In both cases, on the labour market, they match 'correctly'; graduates with low ability match to lower productivity jobs and non-graduates with high ability match to high productivity jobs. Rather, they are mismatched in their initial education decision, despite these choices being made ex-ante rationally.

Propositions 1 and 2 show that conditional on a worker's education preferences, their education choice under perfect and imperfect information depends on whether their ability or signal exceeds cutoffs, denoted by $a^P(\Delta\eta)$ and $\theta^I(\Delta\eta)$. Thus, we can redefine education mismatch in terms of these cut-offs. We can also use the cut-offs to computed the shares of workers who are mismatched given the joint distribution of θ and a.

$$\begin{aligned} Pr(\text{overinvest}) &= Pr(a < a^P(\Delta\eta), \theta > \theta^I(\Delta\eta)) \\ &= \int_{-\infty}^{a^P(\Delta\eta)} (1 - F_\varepsilon(\theta^I(\Delta\eta) - a)) f_A(a) da \end{aligned} \tag{19}$$

$$\begin{aligned} Pr(\text{underinvest}) &= Pr(a > a^P(\Delta\eta), \theta < \theta^I(\Delta\eta)) \\ &= \int_{a^P(\Delta\eta)}^{\infty} F_\varepsilon(\theta^I(\Delta\eta) - a) f_A(a) da \end{aligned} \tag{20}$$

Outside of rational expectations equilibrium, mismatch can also occur if workers' beliefs about the wage schedule that they will face does not align with the actual wage schedule. I abstract from this issue in my empirical application, and leave the quantification of mismatch due to mistaken beliefs to future work.

3.3 Socially optimal education investment

Is the education profile that prevails under rational expectations equilibrium efficient? To investigate this question, I formulate a simple version of the planner's problem where the planner chooses an education profile to maximise the sum of worker and firm utility in the economy. It turns out that the decentralised equilibrium is not efficient in general in this sense, and there is aggregate education mismatch in the economy.

Suppose there were a social planner who could choose an education profile, that is, a mapping $p : \mathbb{R} \times \mathbb{R} \to \{0, 1\}$ that specifies a level of education investment for each level of grades and education preference. Denote by $\mathcal{P}$ the set of education profiles p. This planner aims to choose an education profile to maximise the sum of joint output and students' preferences in the economy[26].

[26] In my notation, workers' and firms' utility are equally weighted. It is straightforward to account for different weighting of workers and firms, and does not affect the qualitative conclusions below.

The social planner's objective function is as follows, denoted by W. The social planner's problem is to find the education profile $p \in \mathcal{P}$ that maximises $W[p]$. I denote this optimal profile by p^*.

$$W[p] = \int_{\Delta\eta}\int_{\varepsilon}\int_{a} \{p(a+\varepsilon,\Delta\eta)\,[g(s(a,1),\mu(s(a,1))) + \kappa + \Delta\eta] + (1-p(a+\varepsilon,\Delta\eta))\,[g(s(a,0),\mu(s(a,0)))]\}\, dF(a)dF(\varepsilon)dF(\Delta\eta) \tag{21}$$

Note that besides directly appearing in the equation, $\mu(\cdot)$ is also a functional in p, as p determines the supply of skill in the economy, which affects the match conditional on s. Denote by $\mu(s,p)$ the function with the function p as a secondary argument. By extension, this implies that $g(s,\mu(s))$ is a functional of p.

It is in general difficult to solve the social planner's problem; since the object optimised over is a function, it is not straightforward to use conventional derivative based methods to solve the problem. It is simpler to show however, that the education profile that prevails in equilibrium may not be socially optimal.

Proposition 3 *Suppose the economy is in rational expectations equilibrium, where the education profile in equilibrium is $\bar{p}$. Then, $\bar{p}$ is not the socially optimal education profile, i.e. $\bar{p} \neq p^*$, except in a specific knife-edge case.*

Proof: *See appendix D.4.*

The basic strategy behind the proof is to redefine the problem so that one could take functional derivatives of W, and show that in general, this derivative at $\bar{p}$ is not equal to 0. Inspecting the first-order condition of the derivative of $\tilde{W}$ at $\bar{\psi}$[27] also explains why equilibrium is not efficient. There are two externalities (corresponding to the two terms in the equation) which lead to inefficiency at equilibrium. First, there is a hold-up externality at equilibrium because the equilibrium in the assignment game is not perfectly competitive. This is because workers and firms on either side of the labour market are not fully substitutable, and thus have a degree of market power. Workers are not able to fully appropriate the surplus that they generate when they invest in education, and as such, underinvest in it relative to the social optimum[28].

Second, an indirect effect of choosing to invest in education is that it makes other workers' relatively lower on the overall skill distribution, which leads them to match to less productive firms. This is apparent in a discrete model; if a worker's education choice leads them to leapfrog another worker on the skill ranking, they match with the firm that those workers matched with and reduce those workers' wages by pushing them to match with less productive firms. I call this a congestion externality[29] or a positional externality, using these terms interchangeably. In making

[27] See equation 32 in appendix D.4. This text describes the intuition derived from the proof without going into the mathematical detail.

[28] Note that equation 6 does not imply that workers appropriate their whole marginal surplus, which corresponds to the total derivative with respect to s, not the partial derivative. Note that:

$$\frac{dg(s,\mu(s))}{ds} = \frac{\partial g(s,\mu(s))}{\partial s} + \frac{\partial g(s,\mu(s))}{\partial y}\mu'(s)$$

The latter term represents the job-operator's increase in their profits from the workers' increased productivity, which the worker does not fully appropriate.

[29] The interpretation of the congestion externality is different from that in search and matching models. In those

their education choice, workers do not factor in this indirect effect on other workers' wages. Because of this, they are likely to over-invest in education.

In general, the rational expectations equilibrium is thus not socially efficient. I verify this result by showing that it is possible to improve aggregate utility by implementing flat subsidies or taxes in policy simulations. Because there are two counteracting externalities, it is generally ambiguous whether education is too high in my model in aggregate.

4 Empirical Implementation

I structurally estimate a parametric version of my model to quantify the extent of individual mismatch and to conduct counterfactual analysis. In this section, I will describe my empirical approach. Section 4.1 describes a parametric specification of the model described above. Then, I discuss the identification of my parameters and estimation in subsection 4.2. Finally, section 4.3 describes the data sources I use for my empirical application.

4.1 Parameterization

To bring the model to data, I impose a number of parametric assumptions on the exogenous functions and distributions in my model, summarised below in table 4. I assume that ability a is normally distributed, and normalise the mean and variance to 0 and 1. I assume that the noise term ε is also normally distributed with mean 0 and a standard deviation σ_ε^2 that is to be estimated. σ_ε^2 determines the relative degree of uncertainty that workers face about their true underlying ability. This parameterisation of the prior and the noise is commonly used in the economics of information, and a well-known result is that the resulting posterior distribution, that is the distribution of $a|\theta$, is also normal with a mean and variance that depend on the signal and the signal noise, i.e. $a|\theta \sim N(\frac{\theta}{1+\sigma_\varepsilon^2}, \frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2})$. The normality of the posterior distributions is convenient as it allows the use of quadrature methods to compute objects of interest, particularly the expected return to education conditional on the received signal.

To parameterise the skill function, which relates underlying ability and education to skill, I use a simple exponential form, where a worker with skill a has skill $\exp(a)$ if uneducated and $(1+\delta)\exp(a)$ if educated. This satisfies the assumptions made on the skill function in section 3, particularly that the marginal increase in skill to ability is greater for educated than non-educated workers. To parameterise the joint output function, I follow the Cobb-Douglas parameterisation chosen by Chade & Lindenlaub (2021), where the joint output is given by $qs^{\gamma_1}y^{\gamma_2}$. This implies that the slope of wages as a function of skill is parameterised by $q(\gamma_1)s^{\gamma_1-1}\mu(s)^{\gamma_2}$. This parameterisation satisfies supermodularity if the parameters q, γ_1 and γ_2 are greater than 0, and if the domain of s and y is restricted to $\mathbb{R}_+$. Because only the shape of the wage function is determined, and not the location, I assume a minimum wage that will be estimated from the data, denoted by w_0. I note that the normalisation of the distribution parameters of the ability distribution also normalises

models, the congestion externality is the effect of workers' search or firms' vacancy posting on the value of search or vacancy posting. In my model, the congestion effect comes from education choices of some workers squeezing other workers down the skill ranking.

the skill function. These normalisations are performed because it is not possible to separate these terms from q, the scale parameter of the joint output function.

I also parameterise the distribution of job productivities as a mixture distribution with K components, where K is the number of occupations. The full specification of this distribution involves specifying the weights, a $K \times 1$ vector denoted by $p = \big(p_1, \cdots, p_K\big)$, and the distribution for each component distribution. I assume that the job productivity within each distribution is log-normal with mean μ_y^k and variance σ_y^k. These moments are collected in the two $K \times 1$ vectors μ_y and σ_y respectively. While this parametric assumption may still be restrictive, I argue that it is an improvement on current approaches in the literature, which simply assumes that each occupation is associated with a particular productivity level[30].

Finally, I assume that preferences for higher education are consist of two components, an aggregate net preference for higher education, κ, and non-systematic preferences for higher education are parameterised by $\eta(1)$ and $\eta(0)$, which I assume to be mean 0 type I extreme value distributions with scale parameter $\frac{1}{\xi}$. The difference $\eta(1) - \eta(0)$ is logistically distributed with location parameter 0 and scale parameter $\frac{1}{\xi}$. For simplicity, I assume β in equation 10 is equal to 1. It is not separately identified from the scale of the net preference for education ξ.

Table 4: Parametric Specification

No	Object	Notation	Parametric Form	Parameters
	Exogenous functions			
1	Skill function	$s(a,e)$	$\exp(a)(1+\delta e)$	δ
2	Joint output function	$g(s,y)$	$qs^{\gamma 1}y^{\gamma 2}$	$q, \gamma 1, \gamma 2$
	Exogenous distributions			
3	Ability distribution	$f_A(\cdot)$	$N(0,1)$	-
4	Dist. for signal noise	$f_\varepsilon(\cdot)$	$N(0,\sigma_\varepsilon^2)$	σ_ε
5	Demeaned dist. for net heterogeneous educ pref	$f_{\Delta\eta}(\cdot)$	Logistic with loc 0 and scale $\frac{1}{\xi}$	ξ
6	Aggregate preference for HE	κ	-	κ
	Other parameters			
7	Minimum wage	-	-	w_0
8	Job productivity distribution	$f_Y(\cdot), f_Y^k(\cdot)$	$logN(\mu_y^k, (\sigma_y^k)^2)$	μ_y^k, σ_y^k

There are two aspects of the parameterisation which may be seen as extreme. First, the model starkly predicts that workers with the same ability should receive the same wage, and thus any variation in wages is attributed to the worker's observed signal deviating from their actual ability. A reasonable objection is that there may be sources of variation in observed wages that is not due to this mechanism, particularly data-driven sources of variance like observation error. Unfortunately, adding observation error to the model leads to problems with optimisation, which suggests that the variance of the observation error and the variance of the signal noise is not well identified separately. Thus, my estimates are likely to overstate the true uncertainty in the system if there is significant observation error in the data.

Second, my mechanism for assigning observed occupations to workers assumes that workers are indifferent to the occupation choice beyond the productivity of the job offered. This excludes

[30]This is the case in Chade & Lindenlaub (2021), as well as the occupational mismatch literature (e.g. Guvenen et al. (2020); Lise & Postel-Vinay (2020)).

concerns like preference for particular occupations, e.g. for prestige reasons. Implementation of these elements in my model is unfortunately not straightforward; incorporating these elements requires a more complex matching problem than has been described thus far. As I shall show in the discussion of model fit, this simple assumption seems to fit outcomes for non-graduate workers well but workers seem to have a preference for skilled preferences even when the productivity of the job they match to is the same. Thus, my model captures the dependence of the share of graduate workers in skilled occupations on the observed signal, but misses the level.

4.2 Identification and estimation

In this section, I provide a heuristic discussion of the features of the data which help to identify my model. I start by noting that it is possible to construct the distribution of skill in the economy for parameter values of δ and σ_ε given estimates of the share of workers who get degrees conditional on θ, according to equation 12. If I can observe the components of the productivity distribution by observing the occupation shares, and occupation-specific means and variances, p_k, μ_y^k, σ_y^k for all $k \in \{1, \cdots, K\}$ of jon productivity, I can also construct the distribution of productivity in the economy. This implies that conditional on parameters δ and σ_ε, I know the distributions F_S and F_Y in the economy.

In that case, the identification of the joint output parameters q, γ_1, γ_2 and the minimum wage w_0 is the same as that in Chade & Lindenlaub (2021), and I can use their identification argument in my case. They note that in this case, Ekeland et al. (2004) argues that the shape of the wage function identifies the parameters. Thus, it is possible to identify parameters q, γ_1, γ_2 and the minimum wage w_0 conditional on parameters ε and δ.

Given the joint output parameters, the education technology parameter δ is governed by the wage return to education conditional on a; since a is not observed, the natural counterpart is the wage return conditional on θ. In a straightforward way, if δ is large, there should be a larger gap between the observed wages conditional on θ for $E[w|e = 1, \theta]$ and $E[w|e = 0, \theta]$. The signal noise standard deviation σ_ε is identified by the variance of wages conditional on θ. In the model, wage dispersion conditional on θ is due solely to the variation of a conditional on θ. Thus, if workers with the same signal have significantly varying wages, this must be because the observed signal is fairly weakly correlated with true underlying ability. Conversely, if σ_ε was small, then we should observe that the signal θ is highly predictive of wages, conditional on education. This intuition is illustrated in figure 3, which plots scatterplots of log wages against the signal for three simulations using different values of σ_ε, overlaid with each other. Decreasing the value of σ_ε reduces the dispersion of the points around a common relationship. As the value of σ_ε goes towards 0, the R^2 of a non-parametric regression of log wages on the signal and degree status increases. Thus, estimates of the joint output parameters and w_0 help pin down the education technology parameter δ and the signal noise parameter σ_ε. These parameters in turn determine the empirical skill distribution, which pins down the joint output parameters and w_0.

Finally, the parameters κ and ξ cannot be identified without taking a stand on workers' beliefs about their returns to education. To estimate these parameters, I assume that workers have rational expectations, that is, they have knowledge of the parameters of the model and can accurately assess their likely returns in expectation prior to education investment, and that the model is in equilibrium, i.e. that the observed distribution of workers choosing education is consistent with

Figure 3: Intuition for the relevance of R^2 in identifying σ_ε

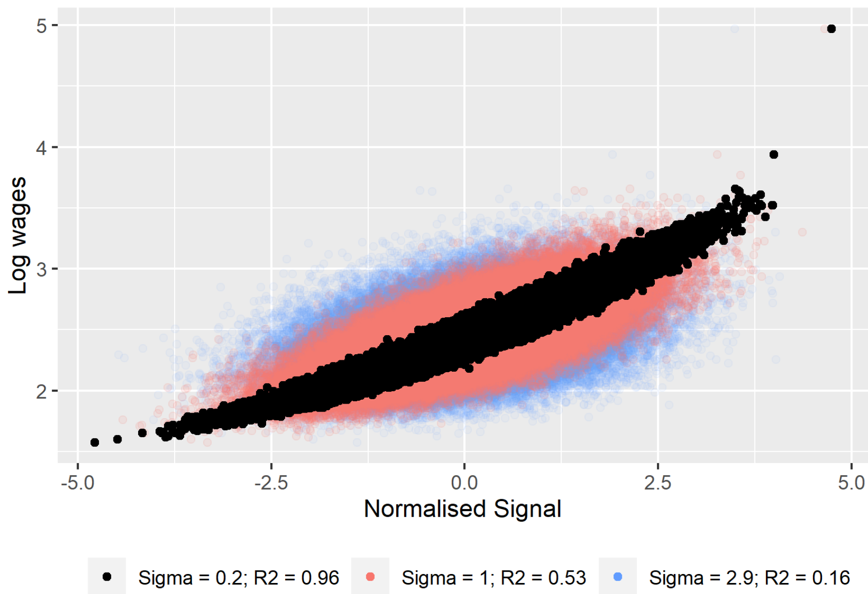


Figure 3 plots the scatterplot of log wages against the normalised signal from three simulations of the model, each with a different value of σ_ε. The R^2 is from regressing log wages on a polynomial of degree 10 on the signal, interacted with degree status.

their incentives to pursue education[31]. In this case, since the parameters $\delta, q, \gamma_1, \gamma_2, \sigma_\varepsilon$ and w_0 are sufficient to construct workers' incentives conditional on their signal θ, I can identify ξ and κ using a binomial logit model, especially since I assume that the heterogeneous preference variables $\eta(1), \eta(0)$ are distributed according to extreme value type I.

This discussion suggests the following procedure for identifying the model parameters:

1. Non-parametrically[32] construct the share of workers who choose higher education in the data conditional on the observed signal θ at a set of points, and denote this $\hat{P}(\theta)$.

2. Estimate the parameters governing wages, the occupational match and the returns to education conditional on $\hat{P}(\theta)$ using simulated method of moments. This recovers the estimated partial parameter vector, $(\hat{\delta}, \hat{q}, \hat{\gamma}_1, \hat{\gamma}_2, \hat{w}_0, \hat{\sigma}_\varepsilon)$.

3. Using $(\hat{\delta}, \hat{q}, \hat{\gamma}_1, \hat{\gamma}_2, \hat{w}_0, \hat{\sigma}_\varepsilon)$ and $\hat{P}(\theta)$, I can calculate the return to education $\Delta(\theta) \equiv E[w(s(a,1)) - w(s(a,0))|\theta]$. Under the assumptions that workers have rational expectations and that the model is in equilibrium, I can identify κ, ξ using a binary logit model, with $\Delta(\theta)$ as the RHS variable.

[31]It is possible that the model is not in equilibrium and thus that a different set of workers would choose education than is actually observed under rational expectations. This would make identification of the preference parameters κ and ξ impossible. However, this cannot be ruled out conclusively ex ante.

[32]I do this by essentially constructing a histogram, and taking the mean share of workers with degrees at 19 points across the signal distribution.

4.2.1 Estimation of the joint output function, education technology function, and signal correlation

To jointly estimate $\delta, q, \gamma_1, \gamma_2, \sigma_\varepsilon$ and w_0, I use the simulated method of moments (Gourieroux et al. (1996); Adda & Cooper (2003)). The basis of this estimation method is to simulate datasets based on the parameters, and to find a set of parameters that produce moments that best approximates moments computed from the data. Let φ denote the vector of parameters to be estimated, and let $\hat{m}$ denote a vector of n_m targeted moments computed from the data. For a given set of parameter vectors, I simulate the model R times with N observations in each simulation, and compute a vector of equivalent moments from the simulated data. I then average the R vectors over the simulations to derive the average simulated counterparts of the moments in $\hat{m}$, which I denoted by $\tilde{m}(\varphi)$[33].

Define by $q(\varphi) \equiv \hat{m} - \tilde{m}(\varphi)$, which is a $n_m \times 1$ vector. Then the estimator is given as follows:

$$\hat{\varphi} = \arg\min_{\varphi} q(\varphi)' W q(\varphi) \tag{22}$$

, where W is a $n_m \times n_m$ positive semi-definite weight matrix. While any choice of W yields consistent estimates of the parameters φ, the optimal weighting matrix is the inverse of the covariance matrix of the moments used, which I compute using the bootstrap. Additional details about computation are available in appendix E.

Using this estimation strategy, it is crucial to choose the moments that can identify the parameters specified. Table 5 summarises the moments that I use for my estimation. There are 126 moments in all, in seven categories. The mean log wage within log-wage deciles (categories 1 and 2) help to identify the joint output function by summarising the shape of the wage distribution. The means, especially conditional on education and signal quintile, help to identify δ, which governs the return to education. The variances as well as the quartiles of log wages conditional on education and signal quintile help to identify σ_ε, which governs how good the signal is of underlying ability. Another important moment is the R^2 of regressing log earnings on a high degree polynomial conditional on education status. In a world without noise, the signal is as good as observing ability and thus the R^2 should be 1. As the degree of noise increases (i.e. σ_ε increases), the R^2 declines as the signal no longer perfectly tracks wages conditional on education status. Thus, this moment is important for identifying the degree of noise in the grade signal of ability.

Table 5: Moments

No.	Moments category	Number of moments
1	Mean log wage within income deciles	10
2	Mean log wage within income deciles cond. on education status	20
3	Log wage quartiles cond. on signal quintile	20
4	Log wage quartiles cond. on signal quintile and education	40
5	Mean and variance of log wages	2
6	Mean and variance of log wages cond. on education	4
7	Mean and variance of wages cond. on signal quintile	10
8	Mean of wages cond. on signal quintile and education	10
9	R^2 of regressing log earnings on a polynomial of grades conditional on degree	2

[33] I use 50 simulations, each with 2000 simulated observations.

4.2.2 Estimation of education preference parameters

Conditional on estimates of δ, σ_ε, the parameters of the joint output function, q, γ_1, γ_2 and w_0 from the estimation procedure described in the previous section, as well as the probability of higher education conditional on the signal, I can construct the expected return to education conditional on the signal, $\Delta(\theta) \equiv E[w(s(a,1)) - w(s(a,0))|\theta]$.

Suppose that workers have rational expectations, and the system is in equilibrium (i.e. the incentives to invest in education align with the observed choices to undergo education). Under this assumption, I then estimate the preference parameters $\psi = \{\kappa, \xi\}$, the location and scale of the extreme value type I distribution of $\eta_1 - \eta_0$, by the Generalised Method of Moments (GMM). The natural moments are the shares of workers with degrees conditional on the worker's signal. Denote by $\hat{\psi}$ the estimated values of ψ. Suppose that $\hat{P}_\theta$ denotes a vector containing the probability of having a degree conditional on $\tilde{\theta}$, where $\tilde{\theta}$ denotes a set of values of θ corresponding to the 5th quantile to the 95th quantile of the distribution of θ. Let $\tilde{P}_\theta(\psi)$ denote the probability of investing in education implied by $E[w(s(a,1)) - w(s(a,0))|\theta]$ and ψ at the points $\tilde{\theta}$. Then, the minimisation problem underlying the estimation is as follows. The standard errors are computed based on the Jacobian of the moments to the parameters (Greene (2003)).

$$\hat{\psi} = \arg\min_{\psi} \left(\tilde{P}_\theta(\psi) - \hat{P}_\theta\right)' W \left(\tilde{P}_\theta(\psi) - \hat{P}_\theta\right) \tag{23}$$

There are two reservations with my education choice set-up as it stands. First, this estimation approach does not account for the possibility that preferences for higher education may differ substantially across demographic groups. Suppose that there are G distinct demographic sub-groups, with the generic sub-group indexed by g. Then, a straightforward strategy is to use data on the observed higher education choice shares for each sub-group g, $\hat{P}_g(\theta)$, to estimate sub-group specific choice parameters. While I do not pursue this strategy in this paper for sample size reasons, it is conceptually straightforward to account for this concern in future research with a bigger dataset.

Second, this method does not allow for psychic costs of education that are correlated with workers' grades. This is, for example, in contrast to Chade & Lindenlaub (2021) which attribute all the correlation between ability scores and education choice to such costs. Without additional variation of earnings not due to differences in grades, such psychic costs are not identified in my model. To the extent that this is a big concern, my estimation method would overstate the degree to which workers are responsive to pecuniary incentives in education choice, and understate the role of non-pecuniary factors in HE choice.

4.3 Data

To estimate the model, I need two sources of data. First, I need the set of moments that are used to identify the parameters in the model, described in table 5, which requires observations on labour market wages, whether the worker has a degree, a measure of their observed ability, and an observation of the occupation they ended up matching to (for analysis of untargeted moments).

I use a sample of individuals from the Understanding Society longitudinal study[34]. This dataset allows administrative linkage to data on high school grades in a national exam, the GCSE, which are the grades that students rely on when applying to higher education. I pool observations from individuals born across five years from 1988-1993, who would have been 18 and considering higher education around the years 2006 to 2011[35], and use this as my main sample. For this sample of students, I have data on their signal, whether they attended higher education, a set of controls, and an unbalanced panel of earnings between the ages of 25 to 32.

For wages, I use hourly labour income, net of taxes and transfers. I face two issues which commonly affect empirical analyses of the relation between education choice and the labour market. First, the 'correct' wage measure to consider in calculating a worker's return to education is their total life-time earnings, which is unfortunately not observed in most datasets. Second, wages are typically the influence of many observable factors, a large number of which is not considered in my model. To address both issues, I run a Mincer regression of log earnings on age, grades, and a set of controls[36], on a panel of earnings for my sample. I adjust earnings using the coefficients from the regression to control for the effects of sex, ethnicity and wave fixed effects. I then take the predicted earnings at age 30 as the relevant earnings that I consider in the structural estimation[37]. Considering earnings at age 30 is common in the literature on higher education given the data typically available and practical constraints on collecting data on the whole life-cycle; see for example similar measures in papers including Blundell et al. (2000); Beffy et al. (2012); Delavande & Zafar (2019); Belfield et al. (2018a). It is helpful to consider these age 30 wages as a proxy for lifetime earnings.

As a measure of the signal that individuals consider, I use a variable recording students' performance at their Key Stage 4 exams[38] (Total GCSE/GNVQ new style point score). I normalise this score within the student's academic year; due to attrition, the distribution of this normalised score in my sample is not standard normal. There is a growing literature documenting that grades play a significant role in providing individuals with information on their ability (Tan (2022)) and their comparative advantage in subject choice (Avery et al. (2018); Li & Xia (2022)). Similarly, my set-up proposes that students learn about their relative aptitude for university and their heterogeneous return from higher education from their grades at GCSE. An important issue is that the researcher does not observe the individual's true information set and thus may be liable to overestimate the

[34]Understanding Society (University Of Essex (2022)) is a longitudinal study which follows households over a long period of time, collecting data on a wide variety of topics for people within the household of all ages. It is a successor of the British Household Panel Survey (BHPS). The linkage to administrative education data is provided by Education & University Of Essex (2022).

[35]The government's policy towards higher education funding changed often in the late 1990s and 2000s, and thus I deliberately choose to pool cohorts which faced similar conditions for higher education funding. In this period (from the 2005/06 academic year to 2012/13), students typically borrowed about £6000 per year, under student loan plan 1. These debt details are taken from a government research report, accessed at https://researchbriefings.files.parliament.uk/documents/SN01079/SN01079.pdf.

[36]The controls I include are wave fixed effects, sex, and ethnicity.

[37]For each individual, I compute the mean residual of the regression summarised in table 6 over the periods that they are observed for in the panel. I add this mean residual to predicted earnings conditional on their grades predicted at age 30.

[38]In the UK, the education system is based on five key stages. Key Stage 4 refers to two years typically called Years 10 and 11, when students are between 14-16. This stage is capped by National Examinations, most typically the GCSE (General Certificate of Secondary Education), although other vocational qualifications are available. The point score used in this paper is a system used to encode students' performance in the range of exams into a numerical point score.

Table 6: Results of a Mincer regression on sample

	(1) Log net hourly labour income
Degree=1	0.0721***
	(3.43)
Normalised KS4 score	0.0752***
	(6.02)
Degree x Normalised KS4 score	0.0462
	(1.86)
Female	-0.0781***
	(-3.95)
Age	0.0363***
	(5.96)
Observations	3122
Individuals	1128

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

This table presents results of a Mincer-style regression of log earnings on degree status interacted with KS4 scores, age, gender, ethnicity and year fixed effects, estimated on an unbalanced panel dataset of workers born between 1988-93. The standard errors were clustered at the individual level.

degree of uncertainty individuals face[39]. Workers may have further private information about their returns than is revealed through grades. I leave a satisfactory resolution of this important issue to future work and note that if private information is indeed significant, my results represent an overstatement of the degree of individual uncertainty.

I thus construct a dataset comprising 1113 observations containing three variables: whether workers have a degree at age 32, their adjusted log wage at age 30, and their normalised KS4 grade, which I interpret as the worker's signal of their underlying return. The summary statistics for the dataset from which the moments are computed are tabulated in table 7.

Table 7: Summary statistics

Variable	N	Mean	Sd
Log hourly labour earnings net of taxes and transfers	1113	2.43	0.26
New style KS4 point score, normalised within student's academic year	1113	0.22	0.91
Whether worker has a degree by age 32	1113	0.49	0.50
Female	1113	0.53	0.50
Non-white ethnicity	1113	0.27	0.44
In 1988 birth cohort	1113	0.17	0.38
In 1989 birth cohort	1113	0.20	0.40
In 1990 birth cohort	1113	0.18	0.39
In 1991 birth cohort	1113	0.16	0.36
In 1992 birth cohort	1113	0.16	0.36
In 1993 birth cohort	1113	0.13	0.34

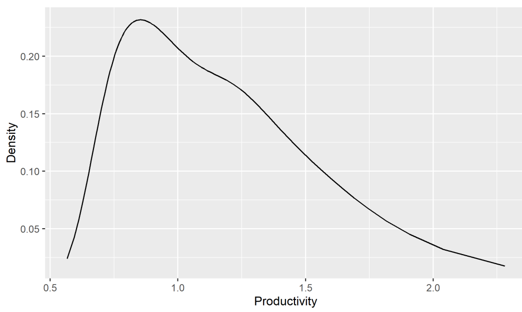
Second, I need data on the distribution of the productivity of jobs that workers match to, that is, the distribution of $f_Y(\cdot)$. As a proxy for productivity, I use the distribution of firm fixed effects within an occupation, calculated from a two-way fixed effect regression of log wages on individual worker and firm-occupation pair fixed effects. I assume that within each occupation (at the SOC00 3 digit level), the distribution of firm productivity is log-normal, with the mean and variance of the log productivity given by the mean and variance of occupation-firm pair fixed effects within the firm. More details about the computation of the fixed effects can be found in Hou & Milsom (2021). The effective distribution of firm productivity is plotted in figure 4. There is precedence for interpreting these fixed effects as informative about productivity (e.g. in Card et al. (2018)); Hou & Milsom (2021) document that these fixed effects are correlated with firm revenue per worker.

5 Empirical Results

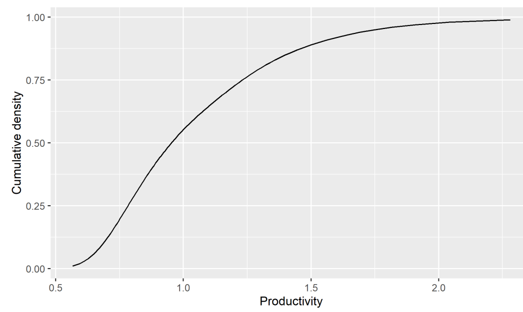
Table 8 summarises the headline parameter estimates that I obtained from my estimation procedure. These results are conditional on a non-parametrically estimated probability of investment in education function conditional on θ, $\hat{P}(\theta)$.

[39]In Cunha et al. (2005); Cunha & Heckman (2016), Cunha and Heckman discuss an empirical strategy for estimating workers' information sets using theoretical restrictions from the permanent income hypothesis and data on consumption. Another approach which is theoretically less onerous is to ask respondents in surveys about their expectations over outcomes, and to compare it against realised outcomes in the future. Unfortunately, these strategies are not available with the data I have.

Figure 4: Productivity distribution in empirical exercise



(a) Productivity density



(b) Productivity cumulative density

Panel 4a plots the density of productivity used in the empirical exercise, while panel 4b plots the cumulative density function of productivity used in the empirical exercise.

Table 8: Parameter Estimates

No.	Parameter	Notation	Value	SE
Stage 1				
1	Signal noise	σ_ε	2.92	0.0000846
2	Skill return to education	δ	0.364	0.00269
3	Joint output function scale	q	7.83	0.172
4	Joint output function - exponent on s	γ_1	0.344	0.00713
5	Joint output function - exponent on y	γ_2	0.0318	0.0226
6	Minimum wage	w_0	4.46	0.0225
Stage 2				
7	Location parameter of het pref for educ relative to no educ	κ	-11.0	0.00447
8	Scale parameter of het pref for educ/no educ	ξ	11.6	0.00417

In subsection 5.1, I discuss the estimation of the variance of signal noise, the parameter governing the education technology function, the parameters of the joint output function, and the constant of the wage function $(\hat{\delta}, \hat{q}, \hat{\gamma}_1, \hat{\gamma}_2, \hat{w}_0, \hat{\sigma}_\varepsilon)$. I loosely refer to this group of parameters as labour market parameters. In subsection 5.2, I discuss the empirical education investment function in θ, $\hat{P}(\theta)$, as well as the estimation of κ and ξ, which I refer to as the choice parameters.

5.1 Estimated labour market parameters and goodness-of-fit

The main feature of these estimates is that the estimate of σ_ε is large, implying that the correlation between actual ability and observed grades is 0.324. As I shall show in the subsequent welfare analysis, this implies that there will be substantial scope for actual returns to education to differ from students' expected returns to education.

The other parameters are not as naturally interpretable. Instead, to evaluate how well the model performs in fitting the data, I analyse how well the simulated moments match the actual moments of the data. Table 9 presents the means and variances of log wages, overall as well as conditional on whether the worker has a degree, for both the actual data and for the simulated outcomes from my model. The simulated means match the actual moments very well. The simulated variances are still within the confidence intervals of the estimated data variances, although my model implies that the log wage variance of graduates is higher than the log wage variance of non-graduates. This is not what I find in this dataset, but is in line with evidence from other papers like Lemieux (2006).

Finally, I plot the R^2 of a regression of log earnings on grades for the subsets of graduates and non-graduates. In my model, if grades are perfect signals of ability, the share of variation captured by grades should be 1, and if grades are non-informative about ability at all, the share of variation captured should be 0. The data suggests that the R^2 of this regression is 0.125 for graduates and 0.0659 for non-graduates, while my model predicts a value of around 0.09 for both groups. While these simulations lie within the 95% confidence intervals, it seems likely that the lower R^2 for non-graduates than graduates is a feature that is not captured by my model (e.g. through multi-dimensional skills).

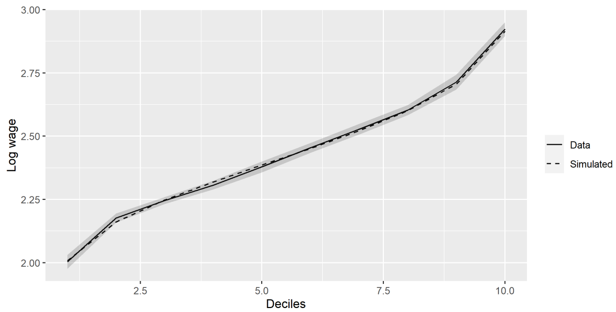
Table 9: Targeted log wage moments, overall and conditional on degree

Statistic	Data	Conf Interval	Simulated
Mean log wage	2.43	[2.42,2.45]	2.43
Mean log wage (e=1)	2.51	[2.49,2.53]	2.51
Mean log wage (e=0)	2.36	[2.34,2.38]	2.36
Variance log wage	0.0688	[0.0634,0.0743]	0.0675
Variance log wage (e=1)	0.062302	[0.0553,0.0693]	0.0652
Variance log wage (e=0)	0.064314	[0.0566,0.0720]	0.0588
R^2 of regressing log wages on grades (e=1)	0.125	[0.0737,0.176]	0.0917
R^2 of regressing log wages on grades (e=0)	0.0659	[0.0251,0.107]	0.0964

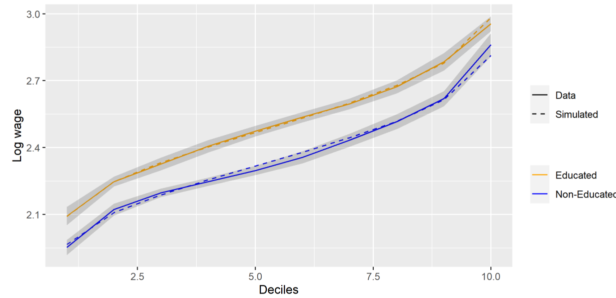
Panel 5 plots the actual and simulated log-wage quantiles, while panel 5b plots the quantiles conditional on education. My model is able to achieve a good match to the wage quantiles, both overall and conditional on education status.

Furthermore, my model is able to capture the interdepence between grades, education status

Figure 5: Actual and simulated wage quantiles



(a) Simulated and actual wage quantiles



(b) Simulated and actual wage quantiles cond. on educ

Panel 5a plots the mean log wage within each wage decile in the data in the solid line and the simulated equivalents in the dashed line. Panel 5b plots the actual and simulated wages for educated workers (in orange) and non-educated workers (in blue). The shaded ribbon plots the confidence intervals of the estimates from the data.

and the individual's wage. Figure 6 plots the mean wage conditional on a worker's signal quintile and whether they are educated. Again, my model fits the data relatively well. In appendix F, I plot the fit of the simulated model to means of log earnings within four quartiles conditional on signal quintile and degree status, and show that the model is able to fit even those more detailed moments well.

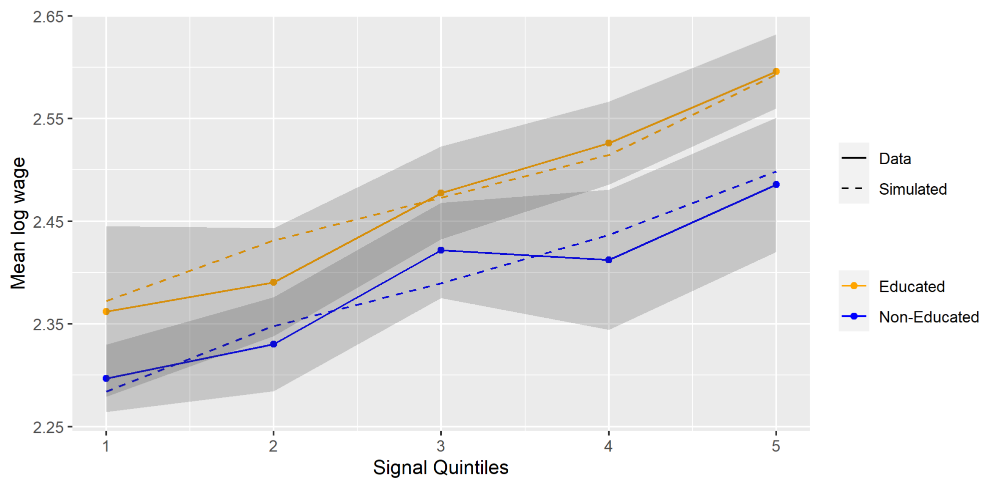
5.1.1 Untargeted moments

Finally, my model makes predictions about the probability of a worker being in particular occupations, conditional on their signal quintile and education. Given a worker's rank in the skill distribution post-education, it specifies a job productivity that the worker matches to. Conditional on this job productivity, a worker randomly matches to an occupation, with the probability of being in occupation k conditional on job productivity s given by equation 28. I then classify these SOC00 3 digit occupations into high skill/low skill categories based on a classification used by the Home Office for immigration guidance purposes.

To analyse how well my model fits the data on the matching of workers to occupations, I plot the share of workers in high-skill occupations, conditional on degree and five signal quintiles, in figure 7. While my model is imperfect in capturing the levels of graduates in high-skill occupations, it captures the qualitative facts that there are substantial shares of workers in both education groups in high-skill jobs and that the share of workers in high-skill jobs increases with the observed signal for both groups. This lends support to the modelling approach that there is an underlying skill index that determines both wages and the occupational match, of which the degree and grades are only indications. However, the significant mismatch in the level of graduates in high-skill jobs suggests that my characterisation does not fully capture sorting into occupations.

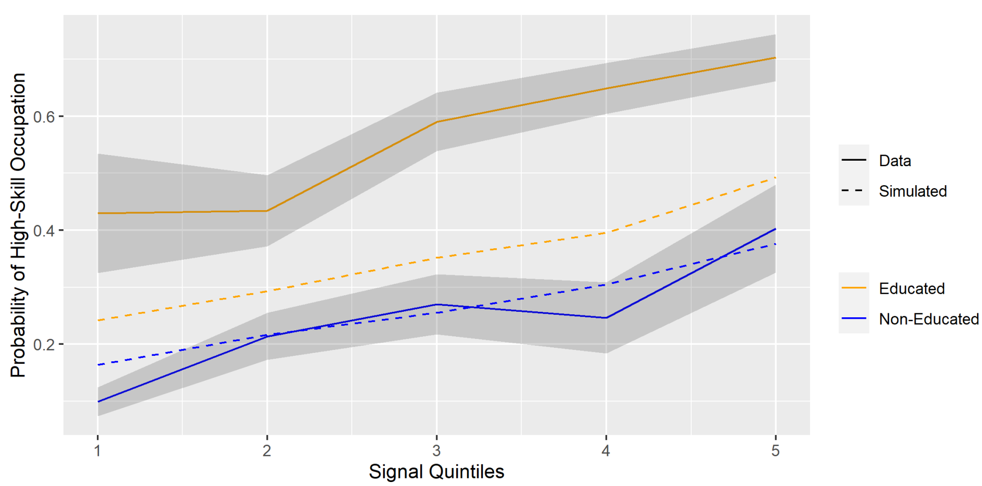
An interesting element to this failure to match the high-skill probability of graduates is that while my model underestimates the probability of working in a high-skill occupation for graduates and overestimates the probability for non-graduates, it captures the wages of both graduates and

Figure 6: Actual and simulated mean wages conditional on education and signal quintile



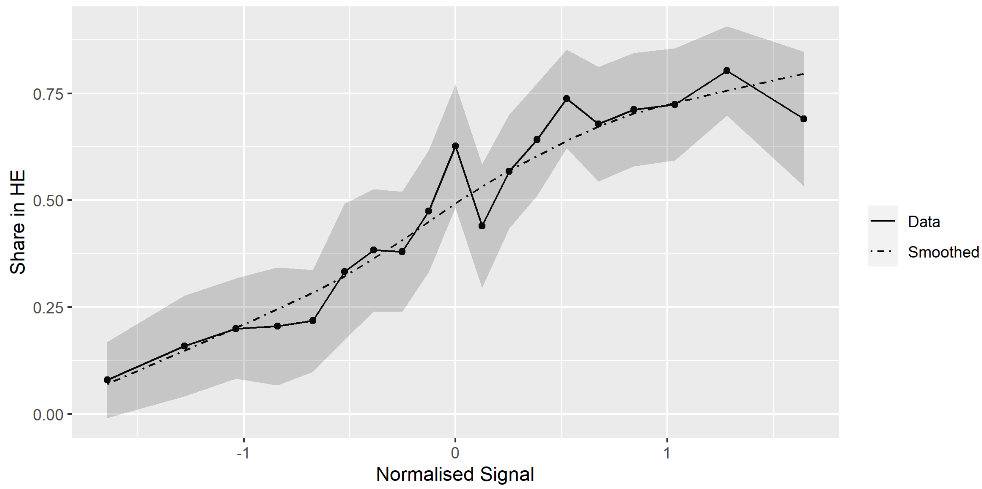
This figure plots the mean log hourly earnings conditional on education (non-graduates in blue and graduates in orange) and five grade quintiles for both the actual data moments (in the solid line) and the simulated data moments (in the dashed line). The shaded ribbon represents the 95% confidence interval for the data moments.

Figure 7: Actual and simulated probabilities of matching to high-skill occupations



This figure plots the probability of being in a high-skill occupation conditional on education (non-graduates in blue and graduates in orange) and signal quintiles for both the actual data moments (in the solid line) and the simulated data moments (in the dashed line). The ribbons give the 95% confidence intervals for the data moments.

Figure 8: Estimated share of workers with degrees conditional on θ



This figure plots the share of workers with degrees conditional on θ at 19 points across the signal distribution. These shares were calculated by averaging the share in a 0.05 interval around each point, corresponding to the 5th to 95th quantiles of θ. The confidence intervals use standard errors calculated by the boostrap. The blue mixed line represents a smoothed approximation of the points, calculated by locally weighted smoothing, which is used to compute the distribution of skill in the estimation.

non-graduates quite well. This suggests that the failure in modelling the match to occupations does not affect the model's specification of wages. This could imply that while the model specifies the right productivity of the job that both graduates and non-graduates match to, it does not correctly specify how graduates choose the occupation conditional on the matched job productivity. This could be indicative that graduates may have non-pecuniary preferences for occupations classified as high-skill (e.g. if they are white collar occupations), even when they are remunerated at the same rate as low-skilled occupations.

5.2 Estimated choice parameters

Figure 8 plots the shares of students who chose to go to higher education within a local region around 19 points across the signal distribution[40]. I compute standard errors by the bootstrap. To summarise this relation between probability of choosing higher education and θ as a function, I smooth the observed points using local polynomial smoothing. As expected, workers are more likely to invest in education if they receive a higher grade, with choice probabilities ranging from 8.0% to 80.3% over the grade distribution.

I follow the procedure set out in section 4.2.2 to estimate the parameters κ and ξ. The parameters are summarised in table 8. The simple base choice model achieves a good fit to most points of the data, besides the point at the median signal and the point at the 95 signal percentile.

[40]For example, the first point summarises the share in higher education from the 2.5th to 7.5th quantile of the signal.

I now turn to considering two extensions to this simple model of education choice. First, in this paper, I have abstracted from risk aversion in my discussion. As a robustness check, I then estimate a version of the choice set-up when workers have CRRA utility as follows, where w denotes their wage:

$$u(w) = \begin{cases} \frac{w^{1-\zeta}-1}{1-\zeta} \text{ if } \zeta \in [0,1) \cup (1,\infty) \\ \log(w) \text{ if } \zeta = 1 \end{cases} \tag{24}$$

The risk aversion parameter is identified by the relative concavity of the log-odds function in θ relative to the expected returns function to θ. If the log-odds function is highly concave when the expected returns function is convex, then, this suggests that workers' choices are driven by a degree of risk aversion. The coefficient of risk aversion parameter is estimated as part of the GMM estimation described before. The results are presented in column 3 of table 10. I estimate a very low degree of risk aversion, consistent with the fairly non-concave shape of the conditional choice probabilities conditional on grades. Although the minimised value is lower than it was for the base model, the improvements are fairly minor.

Table 10: Choice Parameters under Various Specifications

Parameter	Base	Policy	Risk Aversion
ξ	11.5	24.2	17.4
	(0.00447)	(0.00948)	(0.440)
κ	-11.0	-16.2	-12.0
	(0.00417)	(0.00626)	(0.0717)
ζ			0.127
			(0.00765)
Minimised Value	600.	719.	582.

Second, students typically pay for higher education in the UK with an income-contingent loan, where they are not required to repay the loan in full if their income is not sufficiently high. This offers a significant level of insurance to workers and is very generous for workers with lower grades, who have to pay a much lower expected tuition. This changes the incentives that workers face; in appendix G, I describe further how the implementation of the tuition policy changes workers' incentives under the estimated parameters. I estimate a set of parameters where workers face expected returns that would prevail under the tuition policy. These estimates are presented in column 2 of table 10 above. The minimised value suggests that this model is a substantially worse fit to the moments, driven by a bad fit for the middle of the signal distribution. This may suggest that students do not fully internalise the generosity of the income-contingent loan, especially for workers with lower grades.

6 Welfare costs of individual education mismatch

As section 3.2.3 describes, my model implies that imperfect information about ability can lead workers to make ex-ante optimal decisions which nevertheless lead to outcomes that are regretted ex-post. This gives rise to individual mismatch in education choices. In this section, I use the estimates from my structural estimation to analyse (i) whether individual mismatch is a prevalent problem, and (ii) how significant the utility losses from individual mismatch are.

As a measure of welfare, I use the difference between the value a worker places on investing in education and not investing in education measured in terms of foregone pounds per hour earnings (at age 30). Intuitively, this measure can be thought of as the willingness to pay (WTP) for an investment in education; that is the amount in pounds per hour at age 30 equivalent that an uneducated worker is willing to give up to be in higher education (on top of any other tuition or psychic costs already captured by the choice parameters). Conversely, a worker with higher education has to be compensated up to their WTP to be induced out of higher education (on top of being refunded any tuition or psychic costs). A worker's willingness to pay would depend on their net preferences for education $\Delta\eta$ and either their ability a under perfect information or their signal θ under imperfect information. A worker with a positive willingness to pay would select into higher education, and vice versa[41].

With the estimated parameters of my model described in the previous section, I can simulate the joint distribution of WTP under both perfect information and imperfect information. In table 11, I summarise the shares of the simulated population in each of the four quadrants: workers who invest and would benefit, those who invest but would not benefit (over-education), those who do not invest but would have benefited (under-education), and those who do not invest and would not have benefited. The noise around individuals' ability implies significant levels of individual mismatch;

Table 11: Shares of workers by education choice and whether they receive a net benefit

	Would invest	Would not invest	Total
Would benefit	0.286	0.147	0.433
Would not benefit	0.182	0.385	0.567
Total	0.468	0.532	1.00

This table summarises the simulated population by whether they would invest in higher education and whether they would experience a net positive utility from doing so ex-post. The bottom-left quadrant shows the share of workers who are ex-post overeducated, while the top-right quadrant shows the share of workers who are ex-post undereducated.

39.1% of graduates are individually overeducated in the sense that they go to university despite a net negative return and 27.6% of non-graduates are individually undereducated in the sense that they could have benefited from university if they had gone. In total, 32.9% of the population would have been better off if they made a different education choice.

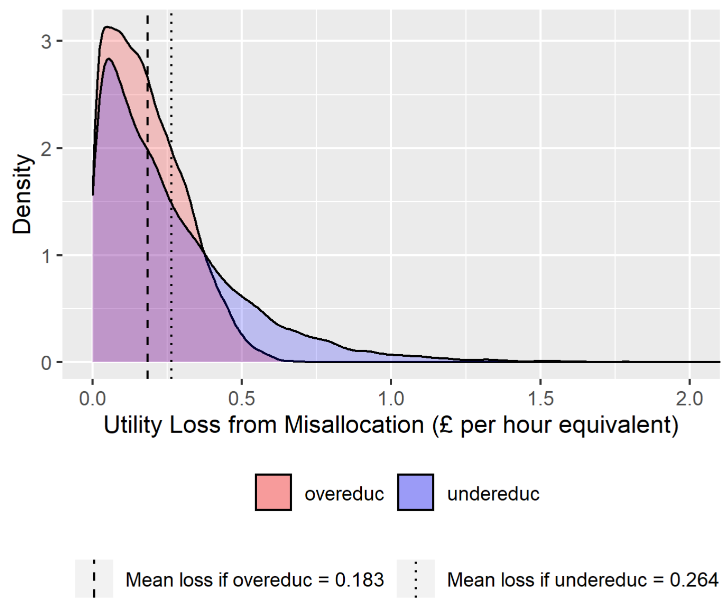
How costly is the misallocation of higher education? In figure 9, I plot the distribution of welfare losses in terms of willingness to pay, conditional on being over-educated and being under-educated. In general, the losses are small and right-skewed, in large part because they are censored to the left. Small deviations between willingness to pay under the perfect information and imperfect information scenarios may not be sufficient to induce the worker to change their education choice, and thus do not lead to misallocation. The mean utility loss in expected return terms comes out

[41] The WTP under perfect information is the difference between the wages under education and no education, and the difference between heterogeneous preferences for education and no education. The WTP under imperfect information is the difference between the value functions.

$$\mathrm{WTP}_{\mathrm{perf}}(a, \Delta\eta) = \beta\left(w(s(a,1)) - w(s(a,0))\right) + \kappa + \Delta(\eta) \tag{25}$$

$$\mathrm{WTP}_{\mathrm{imperf}}(\theta, \Delta\eta) = V(\theta, 1) - V(\theta, 0) \tag{26}$$

Figure 9: Distribution of welfare loss due to misallocation



This figure plots the distribution of the utility loss from misallocation, conditional on being over- and under-educated, from a simulated dataset with the estimated parameters from the structural estimation exercise. The results are presented in terms of willingness to pay, in pounds per hour terms.

to be £0.183 per hour (£323.35 per year) conditional on being over-educated, and £0.264 per hour (£465.22 per year) conditional on being under-educated.

The mean loss from being over-educated is smaller than the mean loss from being under-educated, which suggests that while over-education is a more salient problem because one can observe graduates matched to low-skill jobs in the data, under-education is on average more costly to those who experience it. This is driven by long right tail of utility loss conditional on being under-educated; table 12 below summarises the quantiles of the utility loss in £ per hour money-metric terms. Intuitively, this is driven by the convexity of returns to education with regards to ability. An under-educated worker is a high ability worker who mistakenly thinks that they are low ability, and the counterfactual is that they may miss out on a large return higher education as a result. On the other hand, a over-educated worker is a low ability worker who mistakenly thinks that he is high ability; their counterfactual earnings without education investment is also relatively low, and the degree of loss from mismatch is capped by the low counterfactual earnings. This asymmetry of potential gains and losses lead workers to err in favour of investing in education; thus, the share of over-educated workers exceeds the share of under-educated workers.

There are no directly analogous results to mine in the literature, but this result falls within a plausible range implied by other studies on the UK higher education system. Waltmann et al. (2020) uses an administrative dataset including tax data and education records to estimate lifetime returns to education and find that about 20% of students would experience negative lifetime pecuniary returns to education. They also find that the government is expected to make a loss on the degrees of 40% of men and 50% of women due to the UK's income contingent loan scheme. My finding that

Table 12: Quantiles of WTP loss due to misallocation (£ per hour equivalent)

	25%	50%	75%	90%
Overeducated	0.0797	0.163	0.269	0.365
Undereducated	0.0829	0.194	0.368	0.586

This table summarises four quantiles of utility loss for over- and under-educated workers from a simulated dataset with the estimated parameters from the structural estimation exercise. The results are presented in terms of willingness to pay, in pounds per hour terms.

39.1% of graduates do not benefit from university in aggregate falls within the ballpark of their results. In appendix H, I consider the share of workers who receive a 'sufficiently' high monetary return from higher education; this metric is popular in policy circles interested in ensuring that students' education decisions pass a cost-benefit analysis. I find that particularly in this period (when tuition fees were low at roughly 3000 pounds per year), the cost of education is very low relative to the returns and most workers pass a basic value for money check ex-post (23.2% do not ex-post) and almost all workers do ex-ante (only 0.3% do not ex-ante).

7 Policy counterfactuals

In the previous sections, I have showed how individual and aggregate mismatch can arise in a model with education decisions under uncertainty in a labour market characterised by matching. In this section, I perform a number of counterfactual policy experiments to numerically compute the welfare effects of a number of possible policies to affect the education profile. Are there any policy instruments which can reduce the welfare costs of mismatch and increase aggregate welfare? First, I consider the effects of flat graduate subsidies/taxes which alter workers' incentives to engage in HE, and document how that affects aggregate welfare and the incidence of individual mismatch. Second, I consider an abstract policy to decrease the relative noise in workers' signal of their prospective return to HE, and show that unexpectedly, the policy is not Pareto-improving; while it makes workers unambiguously better off, it can make firms worse off by making skilled workers more expensive.

7.1 Graduate taxes and subsidies

A simple policy instrument to affect the share of students selecting into university is a graduate subsidy or tax, which can be made revenue-neutral by a corresponding lump-sum tax or subsidy on the whole population. Because the lump-sum compensating tax or subsidy would apply regardless of education status, the compensation does not affect the incentives to invest in education. Denote the flat tax by τ, where a negative value implies a subsidy. The policy then modifies the value function for a worker with signal θ and education e as follows:

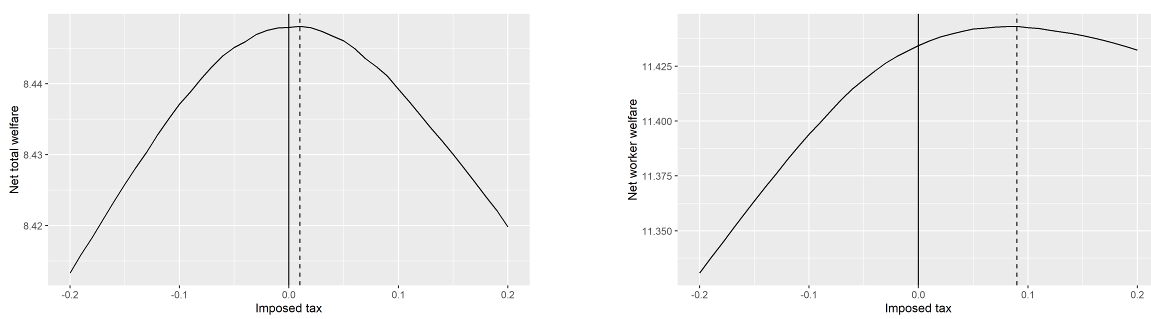
$$V^*(\theta, e) = \left((\kappa - \underbrace{\tau}_{\text{Graduate tax/subsidy}}) \times e \right) + \eta(e) + \underbrace{\left[\int P(\theta) d\theta \right] \tau}_{\text{Compensation}} + \beta E\{w(s(a, e))|\theta\} \tag{27}$$

The tax/subsidy is thus a revenue-neutral method of shifting the aggregate preference for higher education, and can be equivalently thought of as a tool to achieve an optimal level of higher education in the economy. Each possible tax level shifts the economy into a different rational expectations equilibrium, which I compute by iteration. In each resulting equilibrium, I document the incidence of individual mismatch and the aggregate utility for workers and firms.

Figure 10 plots the total net welfare and the net worker welfare for a range of possible tax and subsidy values. I find that quantitatively, the optimal policy for total welfare is to instate a small tax which will slightly reduce the level of education from 46.2% to 44.6%. The optimal policy from a worker welfare perspective is to instate a relatively larger tax[42] that will reduce higher education substantially, from 46.2% to 28.6%.

Qualitatively, the striking result in this case is that what is best in aggregate is not what is best for workers. In the model, over-education is bad for workers but good for firms; they are able to take advantage of the greater stock of skill in the economy to reduce their wages for all skilled workers in equilibrium. Workers on the other hand are better off on average if fewer workers chose to invest in education precisely because skilled work would be more scarce and they would be able to receive higher wages. Thus, the graduate tax or subsidy has to trade off between the welfare of workers and firms.

Figure 10: Net welfare under different compensated tax levels



(a) Net total welfare (b) Net worker welfare

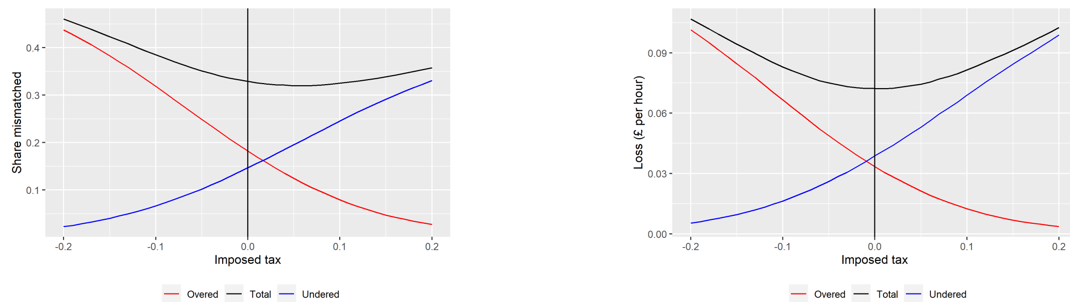
This figure plots the simulated counterfactual net welfare and net worker welfare under various levels of compensated tax. The welfare loss is expressed in willingness to pay. Figure 10a considers wages and utility from higher education, while figure 10b also includes average profits. The dashed vertical lines denote the utility-maximising value of the tax. The optimal tax is 0.00475 in figure 10a and 0.0864 in figure 10b.

Is it possible to reduce the incidence and welfare cost of mismatch through the proposed compensated tax scheme? I simulate the welfare effects of a tax for a range of values between -0.2 and 0.2, and consider minimising the number of workers affected by mismatch, or the total loss in WTP terms of mismatch. Figure 11a plots the share of mismatched workers under various compensated tax levels, while figure 11b plots the total welfare loss in WTP terms under various such levels. The optimal policy if we are aiming to minimise those affected is a tax of 0.06 pounds per hour, reducing the share mismatched from 32.9% to 31.9% under the no-policy baseline. However, this increases the average loss of the mismatch by 4% from 0.0723 pounds per hour to 0.0755. Intu-

[42]This is accomplished by imposing a tax equivalent to approx 9 p per hour at age 30, or adding 3315.36 pounds to the initial cost of education (amortised over 30 years at a 2.5% interest rate).

itively, this follows from the earlier observation that under-education is more costly on average than over-education, and thus there are more over-educated than under-educated workers under ex-ante utility maximisation. Thus, the optimal tax in this situation promotes costly under-education (from 14.7% to 20.5%) to reduce the share of over-educated workers (from 18.2% to 11.4%). The optimal tax to minimise the degree of welfare loss in WTP is 0.01 pounds per hour, which is close to the baseline.

Figure 11: Degree of mismatch under different compensated tax levels



(a) Share affected

(b) WTP Loss

This figure plots the share affected by mismatch and the welfare loss due to the information friction under various levels of compensated tax. The welfare loss is expressed in willingness to pay. The black line plots total loss, and the blue and red lines respectively plot the loss due to undereducation and overeducation.

7.2 Reducing worker uncertainty

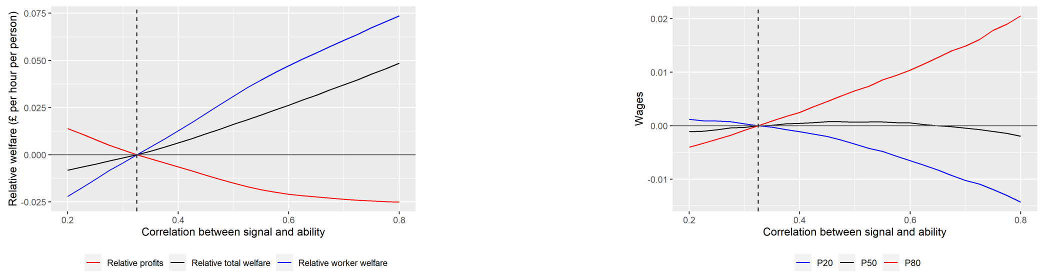
An implication of my theoretical model is that the quality of information that workers have about their return to education is important in determining the extent of individual ex-post mismatch in education choices. If the signal workers receive about their labour market returns was more correlated to their actual labour market return, there would be less individual mismatch in the model. Policy makers may be able to increase the information available to workers about their likely return to education. For example, a policy maker might be able to achieve this by offering more standardised tests to students for them to determine their relative ranking in a cohort, or changing the timing of tests so that workers have access to information before they make important education decisions[43]. More radically, governments may have extensive data on the relation between background and grades, and labour market performance, and can offer data-based recommendations about whether students should go to university (e.g. Athey & Wager (2021)). To my knowledge, there have been few causal studies on policy actions that governments can take to increase the quality of information that workers have about their education decisions.

At present, I consider in an abstract way the value of information about their returns for workers and in aggregate. I consider the resulting rational expectations equilibrium for 25 economies, where all the parameters were as estimated but the degree of noise in the signal σ_ε implies correlations with unobserved ability of 0.2 to 0.8. For each economy, I simulate the equilibrium economies, and

[43]In the UK context, students learn the outcome of their age-18 examinations only after they apply and receive offers for university study.

plot the average worker utility, firm profit and total utility, normalising the value at baseline to 0. The results from this exercise are presented in figure 12a.

Figure 12: Changes in workers' welfare and wages due to changes in noise



(a) Relative changes in welfare as workers' information improves

(b) Relative changes in workers' wages as workers' information improves

Panel 12a plots the relative change in average worker utility, average firm profits, and the sum of the two for 25 values of the implied correlation between labour market ability and the observed signal. Panel 12b plots the relative percent change in average wages for the 20th, 50th, and 80th percentile workers. For each value, I iterate the economy towards the rational expectations equilibrium, and simulate the resulting welfare with 50 repetitions with 2000 observations per draw.

I find that worker welfare increases as the degree of correlation increases. This is to be expected as a result of Blackwell's information theorem, that a better (in a Blackwell sense) information structure results in worker decisions that result in greater wellfare. However, this does not mean that all workers receive higher wages. Rather, wage inequality increases in this economy, in part because the distribution of skill becomes more polarised. With weak signals of ability, the ability distribution of workers who select into higher education is close to the ability distribution of workers who do not. However, if signals were very informative about ability, the former distribution is more right-skewed than the latter one, increasing the dispersion of skill in the economy. Figure 12b plots the wages for the 20th, 50th and 80th percentile workers as the information that workers have becomes more accurate. 20th percentile wages fall, and 80th wages increase as information for workers improve.

Interestingly, overall welfare increases at a slower rate than worker welfare, because firm profits actually decrease as uncertainty decreases. This is for two possible reasons. First, because the wage function is convex in skill, the average share of workers who select into education actually decreases as information improves. The decrease in the share of workers leads to firms, particularly on the lower end of the distribution, facing a less skilled labour market. Second, firms have to pay more to recruit skilled workers with more bargaining power since there are fewer medium skilled workers in the economy as the distribution of skill polarises. These factors lead to profits declining with workers having more information about their returns to education.

8 Conclusion

There is often public concern about the level of investment in higher education, and that too many students go to university when they would not benefit from going. This paper studies the mechanisms behind over-education, focusing on the role of uncertainty about the returns to education at the time of choice. I propose a model in which returns to higher education are heterogeneous and uncertain at the point of university choice, and in which workers match to jobs in a Sattinger-style matching labour market after investing in education. Workers who are observed to work in non-graduate occupations despite getting a degree are interpreted in this framework as experiencing ex-post regret; they correctly invested in higher education with an ex ante expectation of a positive return but ended up with a negative return ex-post. There are two kinds of education mismatch that arises in this model: individual mismatch as outlined above, and aggregate mismatch, when the decentralised equilibrium outcome is not efficient and aggregate utility can be increased by changing the education profile. Aggregate mismatch arises in the model because workers' do not internalise the externalities of their human capital decisions on firms and other workers in the economy.

I estimate a parametric version of my proposed model on a pooled dataset of five cohorts born between 1988 and 1993, using data from the Understanding Society panel survey. I find that the degree of noise inherent in the signal is large; workers only have a signal with a 0.324 correlation to their underlying ability. I find that this leads 32.9% of workers in the economy to end up with ex-post regret. 18.2% of workers end up over-educated, that is, they invest in education but end up with a net negative utility return, and 14.7% are under-educated, missing out on a positive ex-post utility return. On average, the utility loss is in money-metric terms is £323.35 a year for over-educated workers, and £465.22 a year for under-educated workers. Being under-educated is on average more costly than being over-educated, which explains why more workers prefer to over-invest in education rather than vice versa.

I consider the welfare impacts of government policies to (1) impose compensated graduate taxes or subsidies to manipulate the returns to higher education, and (2) reduce worker uncertainty about their individual return to higher education. I find that the optimal policy differs depending on the policy objectives of the government. I find that if the policy is to minimise the incidence or cost of mismatch in the economy, the government should instate a compensated tax to reduce the level of higher education attendance in the economy. For a welfare-maximising social planner, the equilibrium is in general efficient and trades off two kinds of externalities that work in opposite directions. I find that empirically, my estimates imply that the optimal graduate incentive policy would be to reduce the incidence of higher education by 1.6 percentage points by imposing a small graduate tax equivalent to 1 p per hour at age 30 (roughly equivalent to increasing the cost of education by 368 pounds). Reducing worker uncertainty about returns unambiguously benefits workers but surprisingly harms firms by increasing the wage cost. Lower uncertainty about returns also leads to higher inequality by allowing workers to engage in more perfect Roy-type sorting.

There are many unanswered questions arising from my proposed model that I have not been able to address in this paper, which I leave for subsequent research. First, my empirical analysis has been based on observational outcomes data and an assumption of rational expectations. This rules out potentially interesting causes of education relating over-confidence and mistaken beliefs. Another promising avenue of research is to further consider how uncertainty about multiple dimensions of skill could affect the analysis in this model. My framework also leads to the possibility that graduate taxes or subsidies conditional on school grades may be more efficient than flat graduate taxes or subsidies. I hope to eventually tractably extend the welfare analysis to consider a greater policy set than has been considered here.

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A Overview of Trends in UK Higher Education

A.1 Brief Description

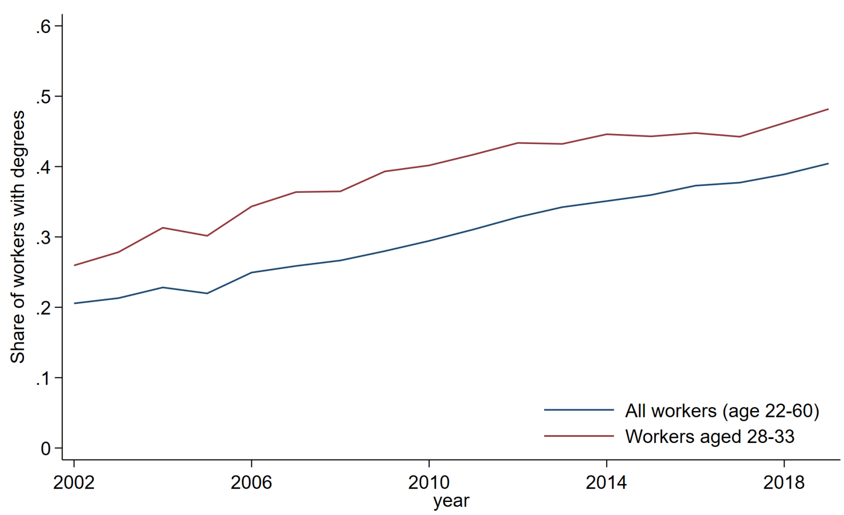
The most typical route for students aiming to go to university is to sit for A-levels, a postsecondary qualification required for entry to most universities. Students typically decide whether to sit for A-levels after compulsory examinations (GCSEs) at the age of 16, and spend typically two years acquiring A-level qualifications. Students typically apply for university in the second year of their A-level study based on their GCSE grades, and are admitted typically with requirements for their A-level grades. Thus, the GCSEs are important to the acquisition of higher education as they are the grades which govern the decision to sit for A-level examinations, and to apply for university, prior to the realisation of the actual A-level grade.

At present and for the time studied in this paper, higher education was financed either by parental contribution, or through income-contingent loans taken by students. Students take out loans made by the Student Loan Company (SLC), and make repayments based on their income when they are working. During the period of analysis, these loans were written off 25-30 years after the first payment. Analysis (e.g. Britton et al. (2019)) has shown that this amounts effectively to an implicit subsidy of higher education. This loan scheme is further described in appendix G. These loans are provided as long as the student is eligible, where the eligibility criteria restricts the courses for which students can take loans and limits these loans largely to first-time students.

A.2 Change over time

There has been a large and sustained increase in higher education in the UK over the last two decades. Figure 13 plots the share of workers with a degree or equivalent in each year from 2002-2019 for the group of 22-60 year old workers and the subset of 28-33 year old workers. The share of workers with degrees increased from 20.6% in 2002 to 40.4% in 2019, an increase of 19.8 percentage points. This fact has been previously documented in Walker & Zhu (2008), Devereux & Fan (2011), and Blundell et al. (2022). Devereux & Fan (2011) argues that this increase in the number of graduates is plausibly exogenous, and Blundell et al. (2022) argues that the increase is due to policy choices by the British government. The data used in this section comes from the UK Labour Force Survey, a nationally representative quarterly survey providing self-reported information on a cross-section of workers, as well as the Understanding Society survey, a longitudinal household panel.

Figure 13: Changes in the share of workers with degrees from 2002-19



Data from the UK Labour Force Survey 2002-19. I consider a worker to have a degree if their highest achieved qualification is recorded as a first degree or a higher degree.

Is there any indication that there are now too many graduates in higher education? In the supply and demand framework used in Katz & Murphy (1992), when the supply of graduates increases faster than demand for graduates, the college wage premium should fall[44]. A falling wage

[44]The calculated wage premium is also not an estimate of the causal wage return to education, since it does not take into account selection into education and other forms of endogeneity. There have been three main papers (Blundell et al. (2000); Belfield et al. (2018a); Waltmann et al. (2020)) estimating the return to higher education in a British context. All these papers estimate Mincer equations, accounting for heterogeneity using a matching approach; they take advantage of detailed observations of schooling and family variables to argue that any residual selection effects

premium may thus be indicative of an excess of graduates. Carneiro & Lee (2011) further argue that when more students select into education, the marginal student is likely to be of lower quality and thus should further lower the wage premium by changing the quality composition of the groups of graduates and non-graduates.

To compute the wage premium, I regress log hourly earnings on a degree indicator, sex and age fixed effects with the LFS sample in each year from 2002-19. Like in Lemieux (2014), I compare this base regression to a second specification with occupation fixed effects. Lemieux (2014) argues that two possible mechanisms for the causal effect of education on earnings are that workers are more productive within the jobs they work at, and that workers match to higher paying occupations due to their training[45]. Controlling for occupation disables the occupation channel, leaving only the human capital channel, as explanations for why graduates earn more than non-graduates. Thus, comparing the coefficient of the base regression to the coefficient of the regression with occupation controls gives a (non-causal) indication of the relative importance of the occupational upgrading story as explanations of why graduates earn more than non-graduates.

Figure 14 plots the coefficient on degree in each year for the base regression in blue and the regression with occupation fixed effects in red. I find that the college wage premium has declined, especially in the latest years from 2013 to 2019, from 0.481 to 0.429[46]. The wage premium conditional on the occupation match has stayed largely stable, falling from 0.184 to 0.171. This seems to suggest that a large part of the fall in the college wage premium comes from a fall in the difference between the occupations that graduates and non-graduates work in. Finally, the absolute size of the fall in the premium, by roughly 10-15 percentage points, is small relative to massive expansion in both the share and numbers of students now in education.

A.2.1 The percentiles of the log wage distributions conditional on education

Many recent studies on the causal impact of higher education has highlighted the heterogeneity of the effect higher education has on students (in the UK, Belfield et al. (2018a); Waltmann et al. (2020); in the US, Heckman et al. (2018); Andrews et al. (2022)).

It is possible that the mean or median wage premia may mask different and divergent trends for graduates at the tails of the wage distribution. To check this, I run a number of quantile regressions at the 10th percentile, median and 90th percentile to study how the earnings gap between graduates and non-graduates have changed at these points. I run quantile regressions of log hourly earnings on degree status, age fixed effects and sex in each year from 2002-19. Figure 15a plots the coefficients on degree status by year for the 10th, 50th and 90th percentiles.

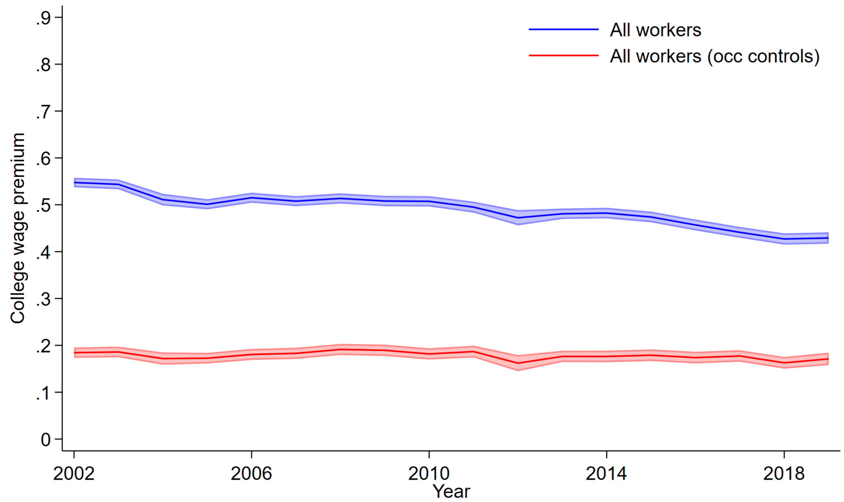
As the blue line (for the 10th percentile) and the red line (for the median) shows, the difference of the conditional 10th percentile and median log wage between graduates and non-graduates has fallen substantially over the years, suggesting that the fall in the wage premium is largely due to the lower quantiles of the graduate wage distribution. Figure 15b plots the indices of the log-wage

do not substantially affect the estimate.

[45] Lemieux also considers a match channel, where workers may earn more if the occupation they work in 'matches' in some sense the education that they achieve. I abstract from this channel in this analysis.

[46] Much of the older literature on the UK higher education premium has tended to conclude that the higher education expansion that started in the 90s did not substantially reduce the observed college wage premium. This seemed to be true from 2002 to 2010, when wage premium stayed at about 0.50.

Figure 14: The college wage premium from 2002-19



Data from the UK Labour Force Survey 2002-19. I consider a worker to have a degree if their highest achieved qualification is recorded as a first degree or a higher degree. The mean college wage premium for each year is calculated as the coefficient on the indicator for whether the individual has a degree of a regression of log hourly earnings on sex, age fixed effects and the degree indicator. The occupational-conditional premium for each year is calculated as the coefficient on the indicator for whether the individual has a degree of a regression of log hourly earnings on sex, age fixed effects, occupation fixed effects and the degree indicator.

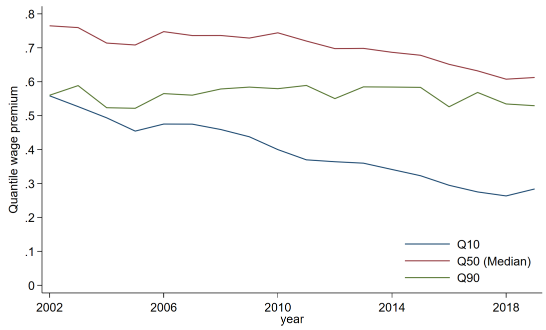
of graduates at the 10th, 50th, and 90th percentiles, with 2002 as the base year. The real log wage has fallen substantially for the median and 10th percentile graduate. Any theoretical model should ideally explain why the wages at the 10th percentile and median have fallen over the course of substantial expansion of higher education.

B Additional Stylised Facts

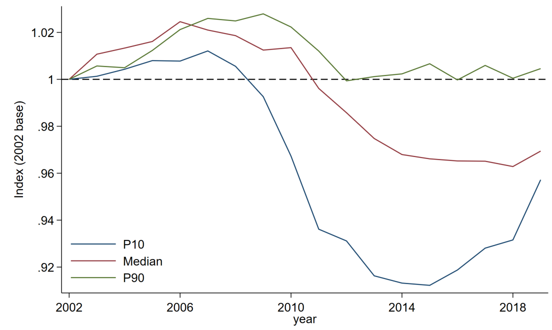
B.1 Employment in low-skilled occupations by subject

Figure 16 plots the share of graduates in non-high-skill occupations by their degree subject. In general, it is true that this rate is higher for workers in non-STEM subjects than in STEM subjects, with biological sciences being an important main exception. However, with the exception of a few vocational subjects like medicine and dentistry, veterinary science and medicine-adjacent subjects like pharmacy, most graduates even in STEM subjects experience at least 10% of workers in non skilled occupations, with the within-science mean being 20%. This suggests that the issues discussed in this paper also apply to STEM fields, even if it is not to a similar extent. Explicitly modelling a multinomial education choice with choice of subject substantially complicates the matching problem considered and is not feasible with the current data used (the Understanding Society which forms the source for the core of the data work does not have information on university subjects). I hope to incorporate it in future work.

Figure 15: Changes in the distribution of earnings conditional on degree status



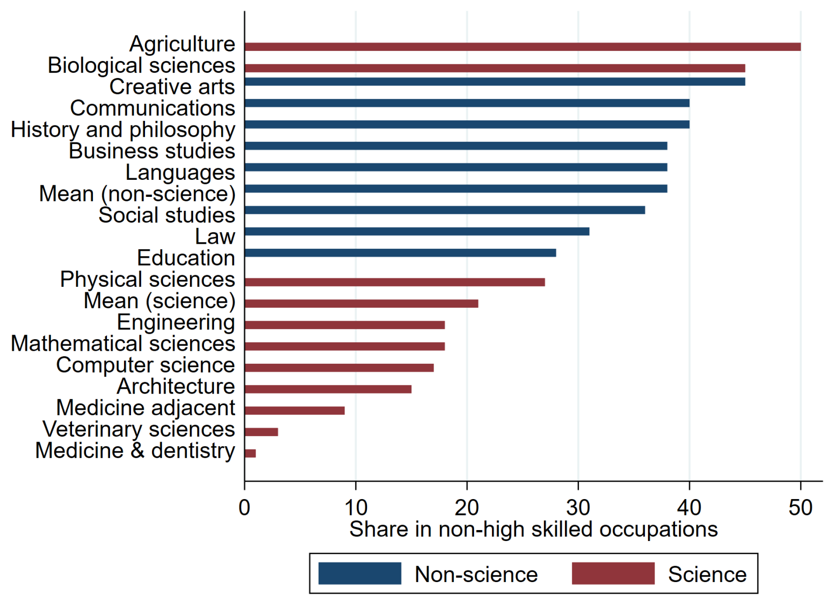
(a) Coefficients on quantile regressions at p10, p50 and p90



(b) Log wages for graduates at p10, p50 and p90

Data from the UK Labour Force Survey 2002-19. Controls for the quantile regressions in figure 15a include sex and age fixed effects. The wage measure used in figure 15b are real log hourly earnings.

Figure 16: Workers by share in non high-skill occupation by degree subject (2018-19)

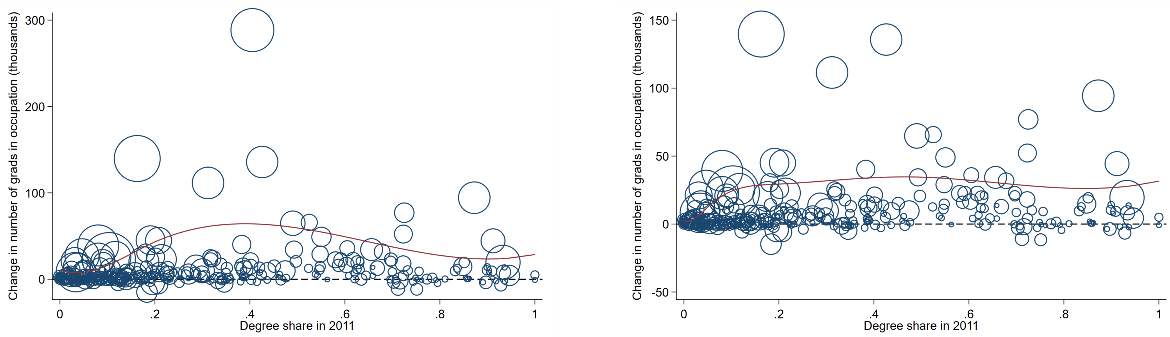


Data from HESA outcomes survey for 2018-19 for graduates one-year from graduation. The red bars represent STEM subjects while the blue pairs represent non-STEM subjects. A high-skilled occupation is defined as an occupation which requires at least a degree, according to the ONS occupation classification, and consists of occupations with occupation codes beginning 1 to 3.

B.2 New graduates are increasingly working in occupations with previously low shares of degree holders

A natural question given the patterns in figure 13 showing a substantial increase in the number of graduates is what occupations these new graduates found employment in. To this end, I use information on the moments of hourly earnings within each occupation from the Annual Survey of Hours and Earnings (ASHE), matched to information about the share of workers with degrees within the occupation. Figure 17a plots a scatter diagram of the occupations, with the share of workers within the occupation with degrees in 2011 (the start of the available period[47]) on the x-axis, and the absolute change in the number of graduates (in thousands) on the y-axis[48]. A non-parametric line of best fit, weighted by the number of workers in that occupation in 2011, is plotted over the points. Figure 17b plots a similar scatterplot, but excludes the occupation with the largest change in figure 17a, nursing[49]. The size of the circles in both diagrams shows the size of the occupation in 2011.

Figure 17: Change in number of graduates within each occupation (2011-19)



(a) Change in number of graduates working in occ

(b) Figure 17a (excluding nurses)

Data on the share of workers with degrees from the UK Labour Force Survey 2011-19. Data on the number of workers in the occupation from the Annual Survey of Hours and Earnings. A non-parametric line of best fit is drawn in red in both diagrams. The size of the bubbles reflects the size of the occupations.

In each figure, I plot a non-parametric line of best fit for the relation between the initial share of workers in the occupation with degrees in 2011 and the change in the number of graduates in the occupation between 2011 and 2019. A general pattern that arises from these figures is that the occupations which received the greatest influx of workers were typically occupations with a fairly medium levels of workers with degrees at the start of the period[50].

[47]To maintain consistent occupational definitions, I used a period, 2011-19, for which SOC10 classifications were available.

[48]The absolute number of graduates was calculated by multiplying the number of workers in the occupation from the ASHE by the share of workers with degrees estimated from the LFS.

[49]From 2008, the minimum award for completing a nursing qualification became a degree, and so, new nursing graduates in subsequent cohorts have substantially increased the share of workers with degrees in the nursing profession. Since this change in classification arguably does not reflect any real economic change yet drives the roughly quadratic shape in figure 17a, it is removed as a robustness check in figure 17b.

[50]The occupation seeing the largest change is nursing, with 288 thousand new graduates entering the profession; this is likely to be a mechanical affair driven by a change in the classification of the nursing qualification. The occupations absorbing the next most graduates are respective "other administrative occupations" (+140 thousand),

Figure 18: Change in the share of workers with degrees within occupations from 2011-19

Data from the UK Labour Force Survey 2011-19. A non-parametric line of best fit is drawn in red. The size of the bubbles reflects the size of the occupations.

The patterns shown in figures 17a and 17b may potentially reflect only the relative sizes of occupations, not that their compositions have changed. Thus to complement those diagrams, I plot a scatterplot of the change in the share of workers with degrees within an occupation from 2011 to 2019 against the intial degree share in 2011 in figure 18. The figure shows that occupations with fairly low shares of workers with degrees experienced a substantial increase in the share of workers in the occupation with degrees, consistent with figure 17. Thus, any theory of the impact of higher education expansion on the graduate labour market has to account for the fact that most of these new graduates end up working in occupations with fairly low shares of workers with degrees.

B.3 Construction of an Ability Measure following Ichino et al. (2022)

This appendix describes the construction of the ability variable used in stylised facts 2 and 3.

In wave c of the Understanding Society study, a cognitive test was administered to respondents above the age of 16, consisting of 6 tests corresponding to specific sub-elements of intelligence: immediate word recall (episodic memory), delayed word recall (episodic memory), subtraction (working memory), number series (fluid reasoning), verbal ability (semantic fluency), and numeric ability (problem solving/numeracy) (for full details, refer to the technical report McFall (2013)). I start by regressing the results of this test (measured in terms of correct answers) on a cubic polynomial of the respondents' age, and take the normalised residual of this regression. Controlling for the age profile is important as intelligence varies over the life-cycle, and the respondents' cognitive abilities are only measured once in 2011/12.

Following Ichino et al. (2022), I use a principal-component analysis to reduce the dimensionality

"sales accounts and business development managers" (+135 thousand), and "production managers and directors in manufacturing" (+111 thousand).

of the multiple administered tests. I extract the first principal component, which has an eigenvalue of 2.64 and explains 44.1% of the variability in the results of the six cognitive tests. The coefficients of all six tests are all positive. I interpret this component as a measure of intelligence. To check that this is sensible, I verify that this component is highly correlate with earnings, the probability of attending university, and the probability of unemployment. This gives me reassurance that this constructed measure is related to economic outcomes.

C Additional Model Description Details

C.1 Implications of the model for the occupational structure

While the solution of the individual's problem and the determination of equilibrium in the model does not involve analysis of occupations, the mixed distribution set-up of the distribution of job productivity allows the model to analyse how graduates match to occupations in this model. Conditional on a worker having skill s, the probability of the worker having a job in occupation k is as follows.

$$Pr(occ = k|S = s) = \frac{p_k f_Y^k(\mu(s))}{\sum_{l\in\{1,\cdots,K\}} p_l f_Y^l(\mu(s))} \tag{28}$$

There are two main advantages to this set-up. First, this allows occupations to be used in the identification of the model in a more natural way. Many past papers have used information on occupations, typically indices constructed from the O*NET dictionary of occupation titles (DOT), to identify structural matching models between workers and jobs. This assumes job heterogeneity within occupations. By allowing for a distribution of job productivities within occupations, I allow both the mean and variance of job productivities to affect the wages and matching to graduate workers in the model.

Second, the model's structure allows the causal effect of education be interpeted as the combination of two effects: the causal effect of education on a worker's productivity in any job, and the causal effect of education on a worker's probability of placing at a better occupation. This coheres with previous research on the channels of the education wage premium, such as Lemieux (2014), which finds that matching to better occupations accounts for at least half of the estimated college wage premium.

It is worth noting that some implicit assumptions made in the analysis. First, an important implicit assumption is that workers have no heterogeneous preferences for occupations. This, of course, is unrealistic; deviations in the predictions of the model for the wages and share of graduate workers in an occupation could plausibly be due to unobserved compensating differentials. Second, I assume that there are no frictions in the labour market part of the model, which implies that there are no occupational mismatch in the labour market. Thus, the entire source of mismatch in this model comes from informational frictions in the educational decision stage. The inclusion of either of these concerns will substantially complicate the matching problem that workers face, and increase the computational costs of the model. I leave the extension of this model to encompass those concerns for future work.

D Proofs

D.1 Proof of proposition 1

The optimality condition is as follows:

$$\kappa + \Delta\eta + \beta\Delta w(a) \geq 0$$

First, note that the LHS of the equation is strictly increasing in both $\Delta\eta$ and a. To show that $\Delta w(a)$ is increasing in a, note that:

$$\begin{aligned}\frac{d\Delta w(a)}{da} &= \frac{dw(s(a,1))}{da} - \frac{dw(s(a,0))}{da} \\ &= w'(s(a,1))\frac{\partial s(a,1)}{\partial a} - w'(s(a,0))\frac{\partial s(a,0)}{\partial a}\end{aligned}$$

By assumption, $\frac{\partial s(a,1)}{\partial a} > \frac{\partial s(a,0)}{\partial a}$. Furthermore, we know that the wage function $w(\cdot)$ is convex, so $w'(s(a,1)) > w'(s(a,0))$. Thus, $\frac{d\Delta w(a)}{da}$ has to be greater than 0; the wage return is increasing in ability. Since the LHS is increasing in a and $\Delta\eta$, it follows that for each value of $\Delta\eta$, there must be a unique value of a which satisfies the optimality condition with equality.

Since the LHS is increasing in a and $\Delta\eta$, it follows that the LHS will only cross the zero threshold once, implying that e^P is increasing in a and $\Delta\eta$.

Let $a^P(\Delta\eta)$ denote the value of a which satisfies the optimality condition with equality, for any given value of $\Delta\eta$. Implicitly differentiate by $\Delta\eta$.

$$\kappa + \Delta\eta + \beta\Delta w(a^P(\Delta\eta)) = 0 \tag{29}$$

$$1 + \beta\frac{d\Delta w(a^P)}{da^P}\frac{da^P}{d\Delta\eta} = 0 \tag{30}$$

$$\frac{da^P(\Delta\eta)}{d\Delta\eta} = -\frac{1}{\beta\frac{d\Delta w(a^P)}{da^P}} \tag{31}$$

The RHS is negative since $\frac{d\Delta w(a^P)}{da^P} > 0$. Thus, we can show that the optimal cut-off $a^P(\Delta\eta)$ is decreasing in $\Delta\eta$.

The solution to each individual's education problem is completely characterised by $a^P(\Delta\eta)$, the cut-off level of ability for each preference state. We know that for any value of a smaller than it, the optimal decision would be to not invest in education, and similarly for any value of a greater than it, the optimal decision would be to invest in education. Also, $\frac{da^P(\Delta\eta)}{d(\Delta\eta)} < 0$, such that when a worker has a greater preference for university, it takes a lower ability threshold to induce them into education. Thus, $e^P(a, \Delta\eta)$ is increasing in a, and $\Delta\eta$.

D.2 Proof of proposition 2

The proof uses results about comparative statics under uncertainty provided in Athey (2002), particularly theorem 2 in her paper. She considers problems where workers aim to maximise a stochastic function $U(x,\theta)$ with respect to $x \in B \subset \mathbb{R}$, where $U(x,\theta)$ can be expressed as follows.

$$U(x,\theta) \equiv \int u(x,s) f(s;\theta) d\mu(s)$$

She provides conditions on the primitive functions $u(x,s)$ and $f(s;\theta)$ such that the argument maximising $U(x,\theta)$ is non-decreasing in θ and B. Theorem 2 states that two conditions are jointly sufficient:

1. The density function f is log-supermodular, which is satisfied when it obeys the monotone likelihood ratio (MLR) order.
2. The payoff function $u(x,s)$ satisfies single-crossing in (x,s); this means that for all $x' > x$, then $u(x',s) - u(x,s)$ crosses 0 at most once and from below.

The first condition is satisfied when $f(s;\theta)$ obeys the monotone-likelihood ratio order. Consider a family of distribution functions $\{\lambda(\cdot,\theta)\}_{\theta\in\mathbb{R}}$; this family of functions obey the MLR order if $\frac{\lambda(a,\theta'')}{\lambda(a,\theta')}$ is increasing in a whenever $\theta'' > \theta'$. It turns out that under the additive error structure assumed in 1, the conditional distribution satisfies MLR order in θ regardless of the initial distribution of a. Thus, given additive noise, the conditional distribution of the underlying random variable conditional on signal θ obeys MLR order with respect to the size of the signal θ as long as the error ε has a log-concave distribution (see a proof of this, see appendix D.3). This is assumed in assumption 3.1.1.

For the second condition to be satisfied, we have to show that $\kappa + \Delta\eta + \beta\{\Delta w(a)\}$ crosses 0 at most once and from below with respect to a. This is satisfied when the term is strictly increasing, which was proved in appendix D.1 as part of the proof of proposition 1. Thus, we can prove that the optimal investment is non-decreasing in θ, conditional on $\Delta\eta$, by directly applying Athey's results.

The corollary of this conclusion is that since the condition for the worker choosing $e = 1$ over $e = 0$ is $V(\theta,1) - V(\theta,0) > 0$, then if $e^I(\theta)$ is non-decreasing in θ, then $V(\theta,1) - V(\theta,0)$ is also weakly increasing in θ. This implies that $E\{w(s(a,1)) - w(s(a,0))|\theta\}$ is also weakly increasing in θ.

Denote by $\theta^P(\Delta\eta)$ the cut-off value of θ such that given heterogeneous preferences $\Delta\eta$, a worker with signal $\theta^P(\Delta\eta)$ would be indifferent between investing and not investing in education. Since $E\{w(s(a,1)) - w(s(a,0))|\theta\}$ is weakly increasing in θ, $\theta^P(\Delta\eta)$ should be weakly decreasing in $\Delta\eta$, and $e^I(\theta)$ would be weakly increasing in $\Delta\eta$. Workers with a greater preference for higher education would be more likely to invest in education across all values of θ. As is the case in the perfect information scenario, the cut-off function defines a boundary in two-dimensional θ-$\Delta\eta$ space which separates students who will invest in HE and those who will not.

D.3 Under additive noise, log-concavity of the density function of the error term is sufficient for MLR order

Under an additive structure as specified in equation 1, the joint distribution of θ and a can be derived by convolution as the product of the density function of a and of ε. The conditional densities can then be computed as follows:

$$
\begin{aligned}
f_{\Theta,A}(\theta,a) &= f_A(a)\cdot f_\varepsilon(\theta-a)\\
f_\Theta(\theta) &= \int_{-\infty}^{\infty} f_A(a)f_\varepsilon(\theta-a)da\\
f_{A|\Theta}(a|\theta=\theta_1) &= \frac{f_{\Theta,A}(\theta_1,a)}{f_\Theta(\theta_1)}\\
f_{\Theta|A}(\theta|A=a) &= \frac{f_{\Theta,A}(\theta,a)}{f_A(a)}\\
&= f_\varepsilon(s-a)
\end{aligned}
$$

Consider the ratio $\frac{f(a|\theta=\theta'')}{f(a|\theta=\theta')}$ where θ'' and θ' are arbitrary values such that $\theta''>\theta'$. We begin by substituting the expression for the conditional density into the expression.

$$
\begin{aligned}
\frac{f(a|\theta=\theta'')}{f(a|\theta=\theta')} &= \frac{f_{\theta,a}(\theta'',a)}{f_{\theta,a}(\theta',a)}\cdot\frac{f_\Theta(\theta')}{f_\Theta(\theta'')}\\
&= \frac{f_A(a)f_\varepsilon(\theta''-a)}{f_A(a)f_\varepsilon(\theta'-a)}\cdot\frac{f_\Theta(\theta')}{f_\Theta(\theta'')}\\
&= \frac{f_\varepsilon(\theta''-a)}{f_\varepsilon(\theta'-a)}\cdot\frac{f_\Theta(\theta')}{f_\Theta(\theta'')}
\end{aligned}
$$

Since $\frac{f_\Theta(\theta')}{f_\Theta(\theta'')}$ is not a function in a, to establish MLR order in θ, we have to show that $\frac{f_\varepsilon(\theta''-a)}{f_\varepsilon(\theta'-a)}$ is increasing in a. Differentiating the expression and signing it, we get the following inequality for $\theta''>\theta'$:

$$\frac{f_\varepsilon'(\theta'-a)f_\varepsilon(\theta''-a)-f_\varepsilon'(\theta''-a)f_\varepsilon(\theta'-a)}{[f_\varepsilon(\theta'-a)]^2}>0$$

Since the denominator must be positive, we can examine the following identity:

$$f_\varepsilon'(\theta''-a)f_\varepsilon(\theta'-a)<f_\varepsilon'(\theta'-a)f_\varepsilon(\theta''-a)$$

Given that distribution functions have a positive range, we can rearrange terms as follows:

$$\frac{f_\varepsilon'(\theta'-a)}{f_\varepsilon(\theta'-a)}>\frac{f_\varepsilon'(\theta''-a)}{f_\varepsilon(\theta''-a)}$$

Thus, the condition for the conditional distributions obeying the monotone likelihood ratio order in the signal θ reduces to the following condition on the distribution of the error f_ε.

$$\theta'<\theta''\rightarrow\frac{f_\varepsilon'(\theta'-a)}{f_\varepsilon(\theta'-a)}>\frac{f_\varepsilon'(\theta''-a)}{f_\varepsilon(\theta''-a)}$$

This condition is equivalent to the condition that $\frac{f'(x)}{f(x)}$ is monotone decreasing in x, and this is sufficient for the distribution f_ε to be log-concave (Bagnoli & Bergstrom (2005)).

D.4 Proof of proposition 3

The broad strategy of the proof is to show that the functional derivative of W with respective to p at the equilibrium education profile $\bar{p}$ is non-zero, and thus the necessary condition for optimality does not hold. However, it is not possible define the derivative in terms of p, because the set of education profiles $\mathcal{P}$ does not constitute a vector space as the education profiles cannot be summed to produce education profiles. Thus, I first define a subset of education profiles which can be represented by a function the set of which is a metric space. If $\bar{\psi}$ is not the maximiser of the counterpart function $\tilde{W}$ within the set Ψ, which corresponds to a subset of $\mathcal{P}$, then $\bar{p}$ also cannot be the maximiser of W within $\mathcal{P}$.

Proposition 2 shows that any equilibrium education profile p can be defined alternatively with a cut-off signal function, which decreases with $\Delta\eta$. A worker selects into education then if their signal exceeds the cut-off signal given their preference level. This means that such education profiles correspond one-to-one to continuous, non-increasing functions which maps the real line to the real line. Then, note that we can re-express the domain of θ and $\Delta\eta$ as $[0,1]$ intervals, by re-expressing any value of θ and $\Delta\eta$ as a quantile. I denote this re-scaled values of θ and $\Delta\eta$ as $\tilde{\theta}$ and $\tilde{\Delta\eta}$, where:

$$\tilde{\theta} \equiv F_\theta(\theta)$$
$$\tilde{\Delta\eta} \equiv F_{\Delta\eta}(\Delta\eta)$$

Denote by $\psi : [0,1] \to \mathbb{R}$ a function which maps a normalised net preference in the interval $[0,1]$ to an un-normalised signal in the real line, and is continuous and non-increasing. Intuitively, this denotes a line in θ-$\Delta\eta$ space, which divides workers who select into education and those who don't. This relates to an education profile p as follows:

$$p(\theta, \Delta\eta) = 1[\theta > \psi(F_{\Delta\eta}(\Delta\eta))]$$

Denote the set of ψ functions by Ψ. Unlike the set $\mathcal{P}$, the set Ψ is a vector space, and we can define a norm for the vector space as follows: $||\psi|| = \int_0^1 |\psi(\tilde{\Delta\eta})| d\theta$. Denote the counterpart of $\bar{p}$ in Ψ by $\bar{\psi}$, where $\bar{p}(\theta, \Delta\eta) = 1[\theta > \bar{\psi}(F_{\Delta\eta}(\Delta\eta))]$ for all values of θ and $\Delta\eta$.

We are now ready to define the derivative. Consider a slightly modified function W, denoted by $\tilde{W} : \Psi \to \mathbb{R}$, which takes ψ as an argument instead of p with the replacement defined above. Denote the Gateaux derivative of $\tilde{W}$ at $\bar{\psi}$ in the direction $\phi \in \Psi$ as follows:

$$d\tilde{W}(\bar{\psi}, \phi) = \lim_{\tau \to 0} \frac{\tilde{W}[\bar{\psi} + \tau\phi] - \tilde{W}[\bar{\psi}]}{\tau}$$

Consider the numerator of the fraction in this limit. For notational clarity, I omit the argument of the function ψ and write $\psi' \equiv \bar{\psi} + \tau\phi$. Noting that the joint output function $g(s, \mu(s))$ is also a functional of the matching function μ, which depends on ψ through F_S, I write $g(s, \mu(s;\psi)) \equiv g(s, \psi)$. I similarly write the wage and profit schedules for a worker with skill s and the firm that matches to a worker with skill s under the education profile corresponding to the function ψ as $w(s,\psi)$ and $\pi(s,\psi)$ respectively. Finally, instead of writing the full expression for the correspondence between p and ψ, I write the function $1[\theta > \psi(F_{\Delta\eta}(\Delta\eta))] \equiv \chi(\theta,\Delta\eta,\psi)$, occasionally suppressing the arguments $\theta, \Delta\eta$ to save space to write $\chi(\psi)$. Denote the difference between joint output functions under ψ by $\Delta g(a,\psi) \equiv g(s(a,1),\mu(s(a,1),\psi)) - g(s(a,0),\mu(s(a,0),\psi))$.

We can rearrange the $\tilde{W}[\bar{\psi}+\tau\phi] - \tilde{W}[\bar{\psi}]$ into the following three terms:

$$\tilde{W}[\psi'] - \tilde{W}[\psi] = \int\int\int (\chi(\psi') - \chi(\bar{\psi}))\{w(s(a,1),p) - w(s(a,0),p) + \kappa + \Delta\eta\}dF(a)dF(\varepsilon)dF(\Delta\eta)$$
$$+ \int\int\int (\chi(\psi') - \chi(\bar{\psi}))\{\pi(s(a,1),p) - \pi(s(a,0),p)\}dF(a)dF(\varepsilon)dF(\Delta\eta)$$
$$+ \int\int\int \chi(\psi')[\Delta g(a,\psi') - \Delta g(a,\bar{\psi})] + [g(s(a,0),\psi') - g(s(a,0),\bar{\psi})]dF(a)dF(\varepsilon)dF(\Delta\eta)$$

Note that at $\bar{p}$, $\bar{p}(\theta,\Delta\eta) = 1$ if and only if $E[w(s(a,1),p) - w(s(a,0),p) + \kappa + \Delta\eta|\theta] \geq 0$. Thus, integrating over θ and $\Delta\eta$, it follows that $\bar{p}$ must maximise $E_{\theta,\Delta\eta}[p\{w(s(a,1),p) - w(s(a,0),p) + \kappa + \Delta\eta\}]$. This means that $\bar{\psi}$ must also be the maximiser of $\int\int\int (\chi(\bar{\psi}))\{w(s(a,1),p) - w(s(a,0),p) + \kappa + \Delta\eta\}dF(a)dF(\varepsilon)dF(\Delta\eta)$, as if a function is the maximiser in a set, it is also the maximiser in any subset of that set.

Furthermore, a necessary condition of $\bar{\psi}$ being the maximiser of the objective in the set Ψ is that the Gateaux derivative of $\int\int\int (\chi(\bar{\psi}))\{w(s(a,1),p) - w(s(a,0),p) + \kappa + \Delta\eta\}dF(a)dF(\varepsilon)dF(\Delta\eta)$ with respect to ψ at $\bar{\psi}$ is equal to 0. Denote:

$$k[\psi] = \int\int\int (\chi(\bar{\psi}))\{w(s(a,1),p) - w(s(a,0),p) + \kappa + \Delta\eta\}dF(a)dF(\varepsilon)dF(\Delta\eta)$$

Then, for all $\phi \in \Psi$,

$$dk(\bar{\psi},\phi) = \lim_{\tau\to 0} \frac{k[\bar{\psi}+\tau\phi] - k[\bar{\psi}]}{\tau} = 0$$

Thus for any direction ϕ, the Gateaux derivative of $\tilde{W}$ at $\bar{\psi}$ consists of the following two remaining terms:

$$d\tilde{W}(\bar{\psi},\phi) = \underbrace{\lim_{\tau\to 0}\left\{\frac{\int\int\int (\chi(\psi') - \chi(\bar{\psi}))\{\pi(s(a,1),\bar{\psi}) - \pi(s(a,0),\bar{\psi})\}dF(a)dF(\varepsilon)dF(\Delta\eta)}{\tau}\right\}}_{\text{Hold-up externality}} +$$
$$\underbrace{\lim_{\tau\to 0}\left\{\frac{\int\int\int \chi(\psi')[\Delta g(a,\psi') - \Delta g(a,\bar{\psi})] + [g(s(a,0),\psi') - g(s(a,0),\bar{\psi})]dF(a)dF(\varepsilon)dF(\Delta\eta)}{\tau}\right\}}_{\text{Positional externality}} \tag{32}$$

In general, the last two terms are not in general non-zero. The surplus profits are positive, and thus for $d\tilde{W}[\bar{\psi}] = 0$, the last term must coincidentally exactly offset the profits term. When this is not the case, the necessary condition for optimality does not hold, implying that the equilibrium education profile is not the optimal education profile from the social planner's point of view.

E Computation and Estimation Details

The estimation procedure requires the simulation of earnings and grades conditional on the distribution of education choices in the economy. To derive the wage function, we require the evaluation of the cumulative density of each value of skill s, which requires numerical integration of a complex object $f_S(s)$, given by equation 12. The probability density function given by equation 12 itself requires integration over the signal noise term ε. I compute the integration in equation 12 using Gauss-Hermite quadrature, since the distribution of ε is assumed to be normal with variance σ_ε^2. I then compute the integration in equation 13 using standard numerical integration.

To compute the wage function, I first compute the derivate of the wage function, which has a simple functional form under my parameterisation: $w'(s) = q\gamma_1 s^{\gamma_1 - 1}\mu(s)^{\gamma_2}$. I approximate this function using a monotone cubic spline using Hyman filtering. To derive the wage condition on s, I integrate this spline approximation according to equation 8. Since each set of trial parameter values requires the computation of only a single wage function, I then approximate this wage function using a monotone cubic spline in s to save on computation when simulating the dataset.

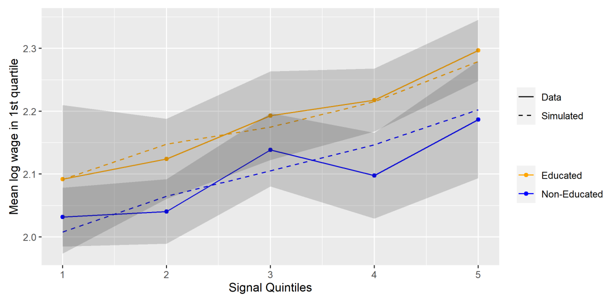
Finally, to compute the difference between the value functions under higher education and not under higher education, it is necessary to take expectations over the wage function conditional on the signal θ. Under the parametric assumption of normality of both the distribution of a and the distribution of ε, the posterior distribution of $a|\theta$ is also normal. I use Gauss-Hermite quadrature to reduce the computational burden of computing these large expectation terms.

F Additional Targeted Moment Fits

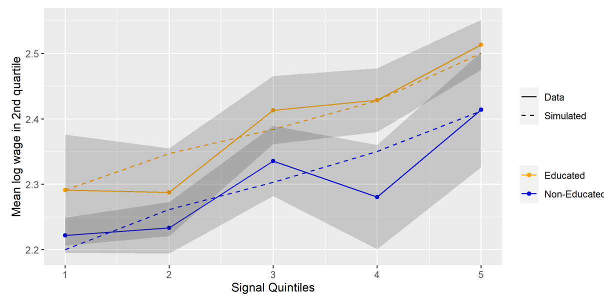
The main figure in this appendix, figure 19 plots the mean log wage within four quartiles of log hourly wages, conditional on degree status and signal quintiles. These quartiles were used as moments in the structural estimation and are used to pin down both the degree of uncertainty about the signal, and the shape of the wage function. The simulated data seems to fit all the moments well, although the some moments are estimated with significant variance.

Figure 19: Simulated versus actual means of log wage within wage quartiles conditional on degree and grade quintile

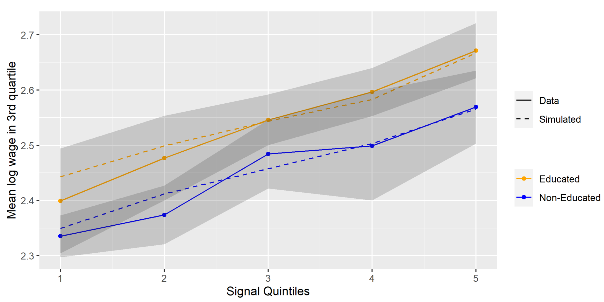
(a) First Quartile



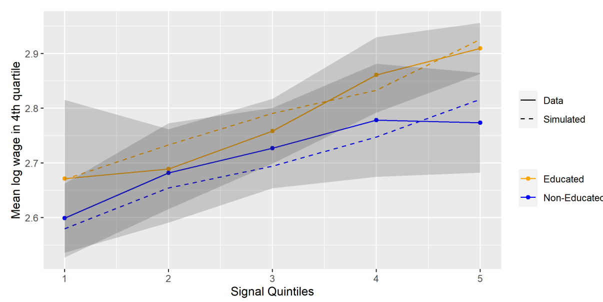
(b) Second Quartile



(c) Third Quartile



(d) Fourth Quartile



Notes: This figure plots the mean log-wage within each of four log wage quartiles, conditional on degree status and five signal quintiles.

G Details about implementation of the income-contingent loan policy

In the UK, students typically pay for higher education in the UK with an income-contingent loan, first introduced under the Labour government in 1998. Under the terms of the loan, students borrow the sum required to pay for tuition fees. They then are obliged to make repayments, where the minimum repayment is a certain percentage of their income above some income threshold. They do not have to make any repayments if their income is below the specified threshold. Repayments stop after the initial loan plus interest is fully repaid, or a certain time period has elapsed[51].

The actual loan policy is implemented over a lifetime, which creates scope for dynamic complexities around the optimal repayment of the loan given a certain wage path. This is not captured in my framework as I focus on effectively a single period; in my implementation, I simplify it for a single period as follows. The parameters of the policy are the repayment threshold ι_1, the repayment rate above the threshold ι_2, and the initial loan sum ι_3. Then, the post-policy wage, given

[51]More details can be found on government websites such as https://www.gov.uk/government/statistics/student-loans-in-england-2021-to-2022/income-contingent-student-loan-repayment-plans-interest-rates-and-calculations-england. The UK income contingent loan policy has also been analysed in Britton et al. (2019).

an initial wage of w, is given by the following expression.

$$\text{post policy wage}(w) = \begin{cases} w \text{ if } w < \iota_1 \\ \iota_1 + (1-\iota_2)(w-\iota_1) \text{ if } w \in [\iota_1, \iota_1 + \frac{\iota_3}{\iota_2}] \\ w - \iota_3 \text{ if } w > \iota_1 + \frac{\iota_3}{\iota_2} \end{cases} \quad (33)$$

The expression says that a worker pays nothing under the scheme if their wage is below the income threshold ι_1. Past the threshold, they pay a share ι_2 of their income in excess of the threshold. Once this payment exceeds the initial loan sum ι_3, then the total payment is simply ι_3. I abstract away from the possibility of strategic repayment of the debt to minimise total repayment for high income workers, in this application. I calibrate the parameters as follows. I set the repayment threshold, ι_1, to be 15711 pounds per year, which is the post-tax average of repayment thresholds from 2016-19[52]. I convert this to pounds per hour terms by assuming that workers work for 44 weeks per year and 40 hours per week. The repayment rate is 9% as has been the policy rate for the loan plan taken by workers in my sample. Finally, I set the initial loan sum to be 22,229 pounds, amortised over 30 years at a 2.5% interest rate and converted to pounds per hour terms. The initial sum assumes on average 3.5 years in university, adding together the sum of the mean maintenance and tuition payments for 2006-2009[53].

Figure 20 plots the return to higher education after factoring in the payment of the income contingent loan under perfect information conditional on ability (in the blue line), and in expectation conditional on grades (in the black line). The blue line shows that the details of the policy creates two kinks, which create non-linearity. The black line is substantially flatter but also kinked at around -0.8. The increase in the expected return is lower up to that point because the relative upset is depressed by the upside of being higher ability than expected is depressed by the flat portion of the blue line between approximately -1.2 and 1.2. Past that point however, the larger upside of the part of the blue line to the right of approximately 1.2 becomes more relevant, and increases the rate at which the expected return increases.

The policy effectively offers a significant level of insurance to workers, and workers who end up with sufficiently low earnings pay almost nothing for higher education. Importantly, the degree of insurance is greater for workers with low grades than those with high grades, as they are more likely than the latter to have low earnings later in life. Furthermore, workers are not liable to pay more than the original sum that they borrowed. Thus, the amount that a worker would pay stops rising after a certain level of income. In this sense, the greatest relative contribution comes from workers in an intermediate range of income.

H Welfare in terms of value for money

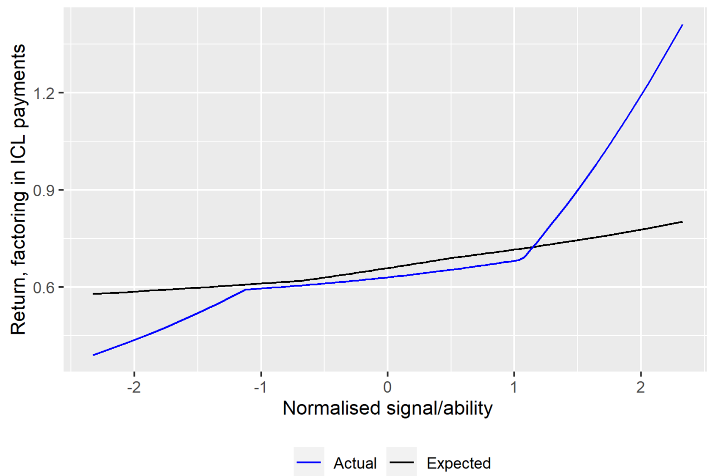
Although willingness to pay is a natural concept of welfare to focus on, the policy debate, especially in the UK, has focused on the concept of 'value for money' or 'earnings potential'[54]. Intuitively,

[52] See https://www.gov.uk/guidance/previous-annual-repayment-thresholds

[53] The loan sizes are taken from student loan statistics produced for the British parliament by Paul Bolton for the Commons Library. See Bolton (2019). The report was accessed online at https://researchbriefings.files.parliament.uk/documents/SN01079/SN01079.pdf.

[54] Earnings potential was discussed by the former Chancellor of the UK, and Conservative party leader hopeful Rishi Sunak, who pledged in the 2022 leadership contest, that he would crack down on university courses, assessing

Figure 20: Post-income contingent loan return to higher education, conditional on grades



This figure plots the relation between ability and the return to higher education net of payments due because of the ICL policy in the blue line, under the parameters obtained in the estimation procedure (described in section 5). The black line plots the relation between ability and the expected return to higher education, post the implementation of the ICL policy.

this seems to refer to the exclusion of non-pecuniary motivations for attending university, roughly implying that students should attend higher education if their decision passes a cost-benefit analysis considering only pecuniary returns to university net of pecuniary costs.

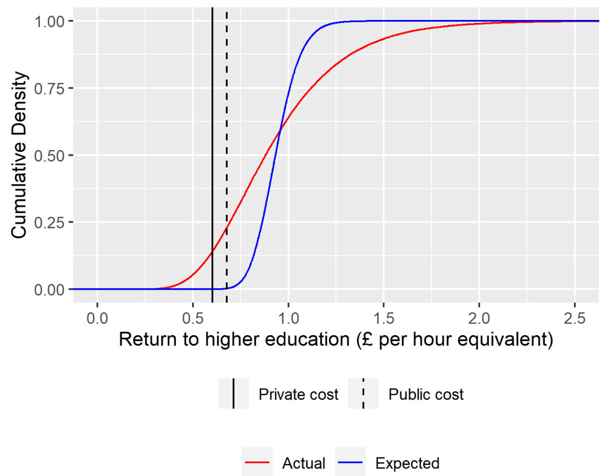
In this paper, I interpret value for money roughly as the pecuniary wage return to attending university exceeding the pecuniary costs of doing so, excluding the consumption value that workers may receive from higher education. In this sense, a worker may attend university in my model despite not receiving value for money for doing so because of (1) they have a positive non-pecuniary preference for attending university, or (2) because the information friction leads them to believe that they will receive a positive pecuniary return despite not actually doing so.

Figure 21 plots the distribution of actual and expected wage returns to higher education in a simulated dataset with 20000 workers as cumulative density functions. A striking feature of these distributions is that expected returns (conditional on the grade signal) are much less variable than average returns (conditional on ability). This is to be expected, especially given the substantial noise with which grades correlates to actual underlying ability.

To conduct a full cost-benefit analysis, I need to specify the pecuniary cost of higher education. I consider two alternatives. First, I take an estimate of the cost of providing higher education from Belfield et al. (2018b), £25,000, and amortise this over 30 years at a 2.5% interest rate. This calculation implies the per-year payment to be £1194.441 per year, or £0.679 per hour (assuming a 40 hour work week for 44 weeks). Second, I take the total student loan that students take out on average, similar amortised over 30 years at a 2.5% interest rate. The average loan is £22,230, and is taken from Bolton (2019). The implied per year per hour equivalent cost is 0.599. Two

them "through their drop-out rates, numbers in graduate jobs and salary thresholds, with exceptions for nursing and other courses with high social value". See https://www.theguardian.com/politics/2022/aug/07/rishi-sunak-vows-to-end-low-earning-degrees-in-post-16-education-shake-up.

Figure 21: Cumulative density of simulated actual and expected returns to higher ed



This figure plots the distribution of actual (conditional on unobserved ability) and expected returns (conditional on the signal) in a simulated dataset generated from the model and the estimated parameters. The red line plots the cumulative density of actual returns and the blue line plots the cumulative density of expected returns. The two vertical lines represent two cost benchmarks.

vertical lines are plotted on the figure, representing two cost benchmarks. The dashed vertical line presents the cost of providing a higher education course during the period, including the private cost borne by the student and the public cost borne by the Treasury in terms of teaching grants given to universities. The solid vertical line represents the cost borne directly by workers, including tuition costs and living costs covered under the maintenance loan.

The figure implies that most workers in this cohort would receive a benefit net of either cost, in part because this cost, especially spread over a lifetime, is not particularly high. Only 14.3% of workers would not have received a return exceeding their private cost of education, and only 23.2% of workers would not have received a return exceeding the total cost of provision. Only 0.3% workers would not have had an expected income above the total cost of provision, implying that if workers considered only the pecuniary accounting of costs and benefits, more workers would have selected into higher education[55]. From this point of view, higher education was an extremely valuable investment for workers relative to its cost for the studied cohorts[56].

In practice, only 46.8% of workers do invest in education. I find that going by the returns net of the cost of provision, most graduates receive a positive return to higher education, and only 18.3% of graduates would not have a positive return. On the other hand, most non-graduates are under-investing in education, and 72.5% of non-graduates could have benefited.

In my model, this misallocation can be generated by two channels. First, the noisiness with which grades measures the actual labour market performance leads to an information imperfection

[55]Note however that if all workers selected into higher education, the prospective return under the resulting skill distribution might not sustain all of them selecting into higher education after wages fall.

[56]Note that the per-year tuition cap was 3000 pounds for the studied cohorts. Since then, this cap has tripled to 9250 pounds per year in 2021.

which impedes choice. Second, heterogeneous net preferences, such as a high consumption value for university or particularly low costs due to family endowments, may lead some workers to find higher education value for money even with a lower labour market return. In table 13, I first summarise the share of workers who would invest in higher education, and then divide the population into workers who invest and receive a return greater than the mean preference benchmark, those who invest but receive a return lower than the benchmark, those who do not invest but would have received a return greater than the benchmark, and those who do not invest and would not have received a return higher than the benchmark. Then, I perform a counterfactual exercise, where I isolate each of these two channels in turn to analyse which factor contributes more to the mismatch between returns and choice (keeping the skill environment constant).

Table 13: Breakdown of investment choices and whether return exceeds mean costs benchmark, under four counterfactual scenarios

	Het + Noise channels	Het channel only	Noise channel only	No channels
Share choosing degree	0.468	0.433	0.997	0.768
Would invest, VfM	0.382	0.429	0.767	0.768
Would invest, not VfM	0.0856	0.004	0.230	0
No invest, VfM	0.386	0.340	0.00108	0
No invest/Not VfM	0.146	0.228	0.00187	0.232

This table summarises the share of workers in higher education, and a breakdown by whether they would experience "sufficiently" high returns to higher education. I use as a benchmark for "sufficiently high" the level of returns that would cover the cost of providing higher education, and is computed to be £0.679 in this setting. This is denoted as VfM, short for "value for money". These shares are computed for four scenario, the base scenario with both uncertainty of returns and heterogeneous preferences, and three counterfactual scenarios alternative with heterogeneous preferences only, uncertain returns, and one with neither.

First, column 1 suggests that there is significant mismatch between wage returns in the system; of the 46.8% of workers who would choose university in the model, 38.2 percentage points, or 81.6% would generate a return greater than the cost of provision benchmark. Furthermore, 38.6 percentage points, or 72.5% of non-graduates, would have received a return higher than the mean preference benchmark but do not end up in higher education; this suggests that too few workers invest in education from a value for money point of view. Heterogeneous non-pecuniary preferences for higher education seems to drive under-investment in higher education, since even knowing the actual returns to higher education, 44.3% who would have received a net positive return would not have attended higher education. On the other hand, the uncertainty about true returns seems to drive over-education, leading workers who might not have benefited to invest in higher education. Thus, both channels are important in different ways in driving the mismatch between the net wage returns to education and workers' choices in the cohort studied.