Author： 黃仕鈞

Autoregressive Modeling

```
clear all; clc;
```

Initial parameter

Sampling parameters

Samples quantity , N = 100

```
N = 100; % quantity of samples
```

Gaussian noise

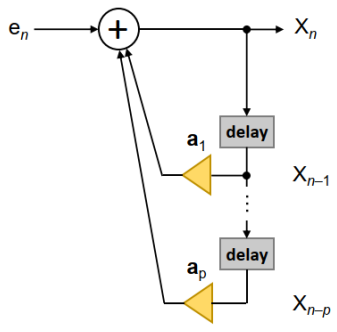
$e_n = \text{randn}(N)$

```
e = randn(N);
e = e / max(e);
```

AR model AR(0)

$$X_n = c + \sum_{i=1}^{p} a_i X_{n-i} + e_n$$

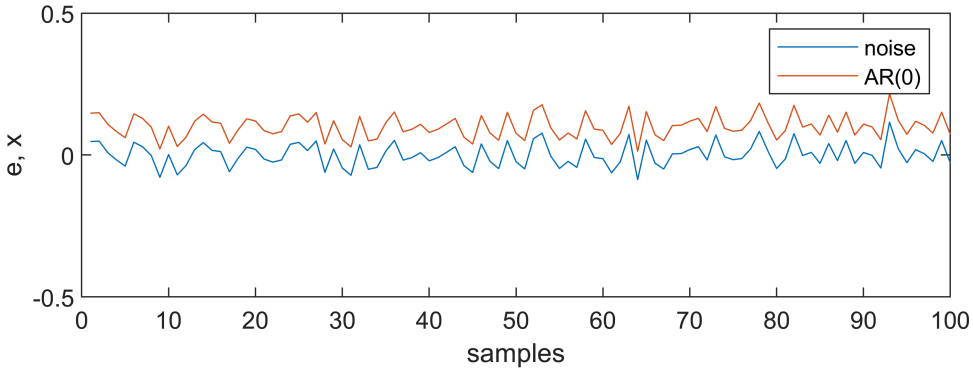
```
a = [];
p = length(a); % matlab counter 不能從零開始 所以不用 `i = p:N`
x = zeros(N,1); % 初始化矩陣大小
for i = (p+1):N
    x(i) = 0.1+e(i);
end
```



Plot

```
figure();
subplot(2, 1, 1);
plot(e);
```

```
hold on;
plot(x);
ylim([0, N]); %xlim(0, N);
ylim([-0.5, 0.5]); %ylim(-2, 2);
xlabel('samples');
ylabel('e, x') ;
% legend(['noise', 'AR(0)'], loc='best');
% tight_layout(pad=0.5, w_pad=0.5, h_pad=1.0);
legend('noise', 'AR(0)')
```



AR(1)

$$X_n = c + \sum_{i=1}^{p} a_i X_{n-i} + e_n$$

透過改變a_i增益來做，AR(1)，表示找前一個採樣 X_{n-i}與一個增益a_i乘得到結果, delay 1 sampe。

Autoregressive for forecast error: Youtube

delay 不能超過sample數量

Only for short-term forecasts

To forcast k-steps ahead(F_{t+k})

Use AR(p) model only if k>p, p lag p sequence, k is steps

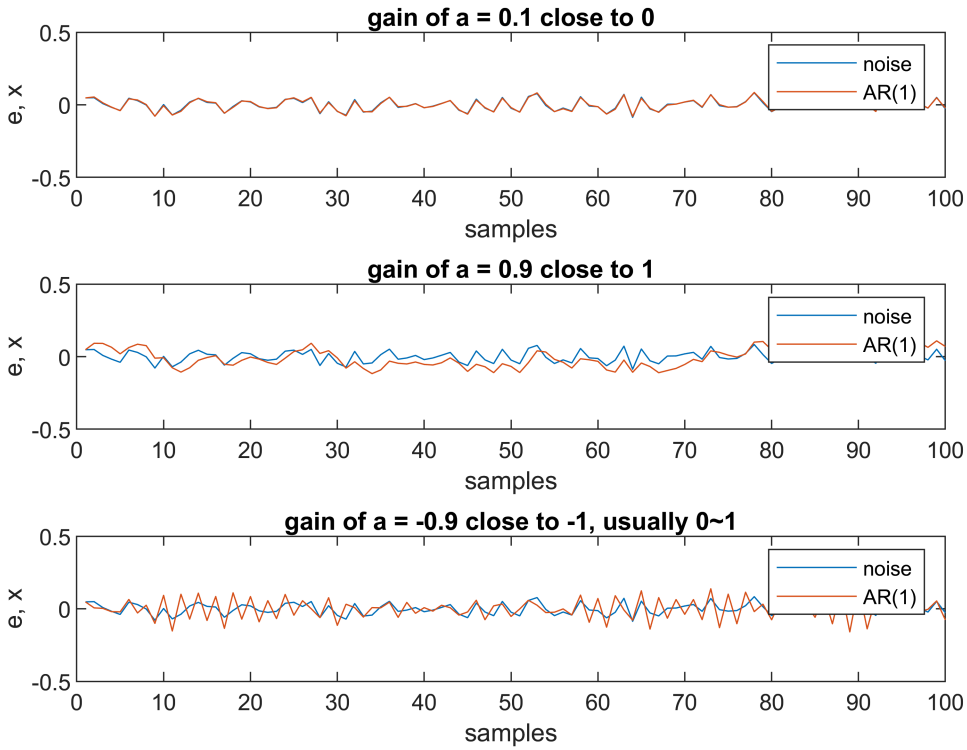
```
a = [];
```

```
p = length(a)+1; % matlab counter 不能從零開始 所以不用 `i = p:N`

a = [0.1, 0.9, -0.9, 0.3, 0.5]; % close to 0, 1
x = zeros(N,5); % 初始化矩陣大小

x(1,1) = a(1).*x(1,1)+e(1);
x(1,2) = a(2).*x(1,2)+e(1);
x(1,3) = a(3).*x(1,3)+e(1);
x(1,4) = a(4).*x(1,4)+e(1);
x(1,5) = a(5).*x(1,5)+e(1);
lag = 1;
for i = (p+lag):N
    x(i,1) = a(1).*x(i-1,1)+e(i);
    x(i,2) = a(2).*x(i-1,2)+e(i);
    x(i,3) = a(3).*x(i-1,3)+e(i);
    x(i,4) = a(4).*x(i-1,4)+e(i);
    x(i,5) = a(5).*x(i-1,5)+e(i);
end
```

Plot

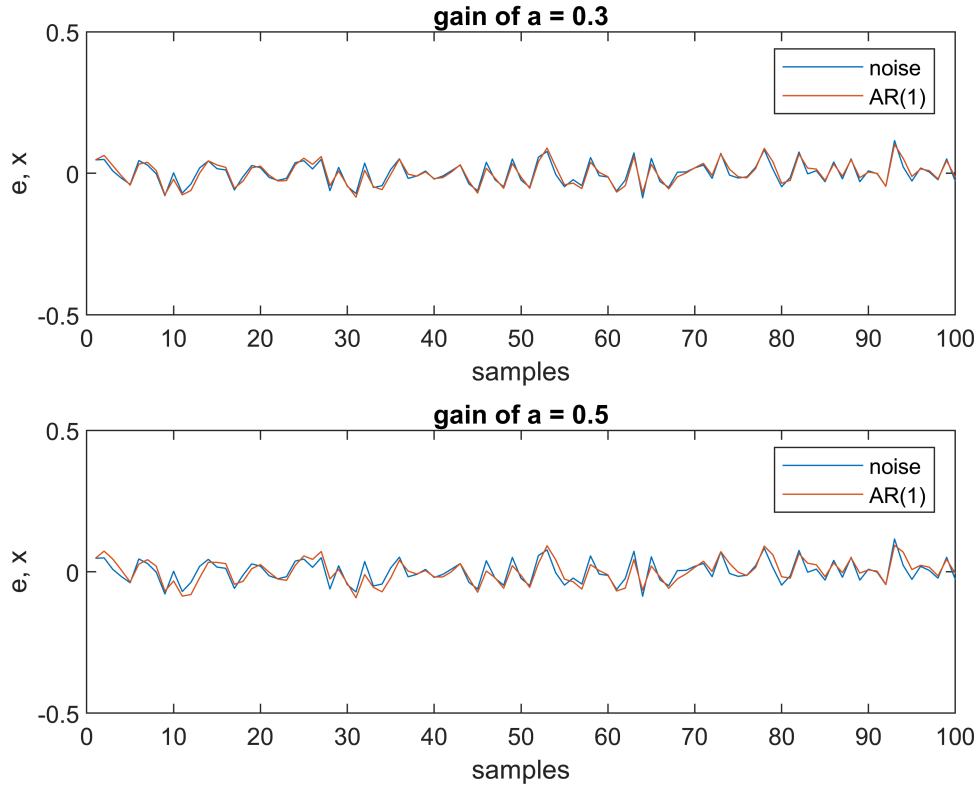
```
figure();
subplot(3, 1, 1);
plot(e);
hold on;
plot(x(:,1));
title('gain of a = 0.1 close to 0');
ylim([0, N]); %xlim(0, N);
ylim([-0.5, 0.5]); %ylim(-2, 2);
xlabel('samples');
ylabel('e, x') ;
% legend(['noise', 'AR(0)'], loc='best');
% tight_layout(pad=0.5, w_pad=0.5, h_pad=1.0);
legend('noise', 'AR(1)')
subplot(3, 1, 2);
plot(e);
hold on;
plot(x(:,2));
title('gain of a = 0.9 close to 1')
xlim([0, N]); %xlim(0, N);
ylim([-0.5, 0.5]); %ylim(-2, 2);
xlabel('samples');
ylabel('e, x') ;
% legend(['noise', 'AR(0)'], loc='best');
% tight_layout(pad=0.5, w_pad=0.5, h_pad=1.0);
legend('noise', 'AR(1)')
subplot(3, 1, 3);
plot(e);
hold on;
plot(x(:,3));
title('gain of a = -0.9 close to -1, usually 0~1')
xlim([0, N]); %xlim(0, N);
ylim([-0.5, 0.5]); %ylim(-2, 2);
```

```
xlabel('samples');
ylabel('e, x') ;
% legend(['noise', 'AR(0)'], loc='best');
% tight_layout(pad=0.5, w_pad=0.5, h_pad=1.0);
legend('noise', 'AR(1)')
```




```
figure();
subplot(2, 1, 1);
plot(e);
hold on;
plot(x(:,4));
title('gain of a = 0.3');
xlim([0, N]); %xlim(0, N);
ylim([-0.5, 0.5]); %ylim(-2, 2);
xlabel('samples');
ylabel('e, x') ;
% legend(['noise', 'AR(0)'], loc='best');
% tight_layout(pad=0.5, w_pad=0.5, h_pad=1.0);
legend('noise', 'AR(1)')
subplot(2, 1, 2);
plot(e);
hold on;
plot(x(:,5));
title('gain of a = 0.5')
xlim([0, N]); %xlim(0, N);
ylim([-0.5, 0.5]); %ylim(-2, 2);
xlabel('samples');
ylabel('e, x') ;
```

```
% legend(['noise', 'AR(0)'], loc='best');
% tight_layout(pad=0.5, w_pad=0.5, h_pad=1.0);
legend('noise', 'AR(1)')
```

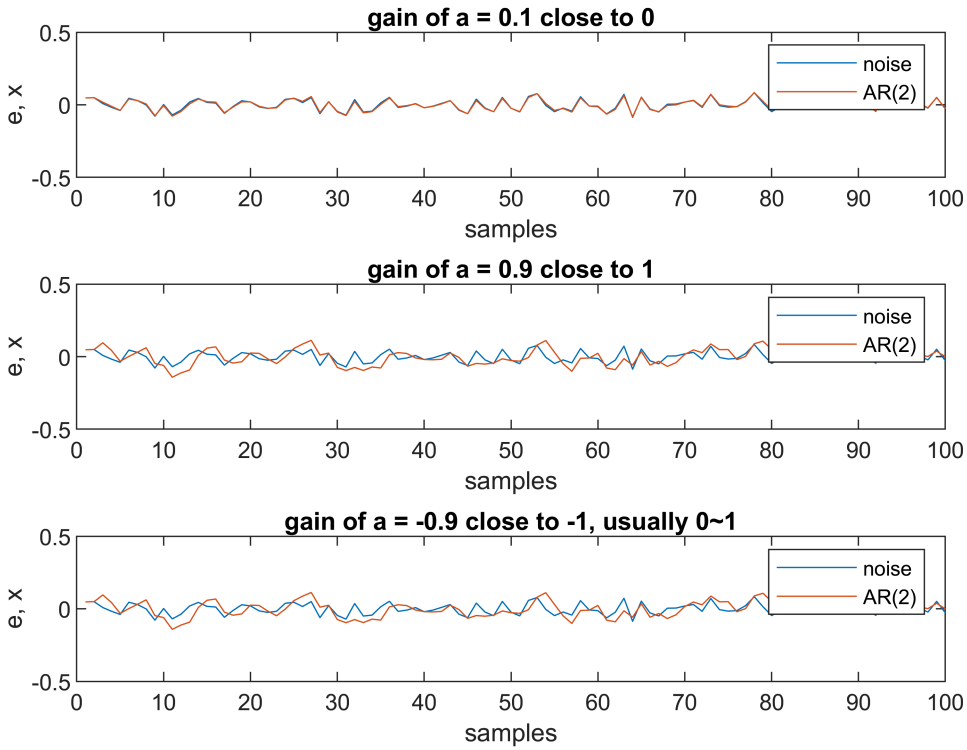


AR(2) lag 2 seq

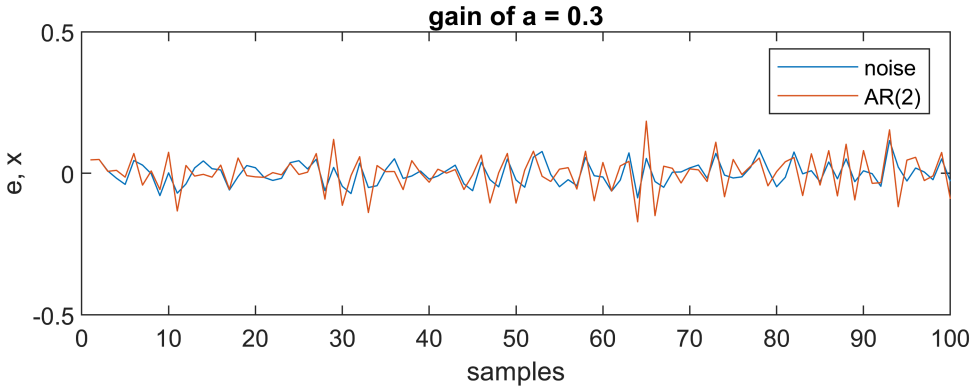
```
a = [];
p = length(a)+1; % matlab counter 不能從零開始 所以不用 `i = p:N`

a = [0.1, 0.9, -0.9 ,0.3, 0.5]; % close to 0, 1
x = zeros(1,N); % 初始化矩陣大小
xc = x; xc1 = x; xc2=x; xc3 = x;
lag = 2;
for i =1:lag % lag not calc
    x(i) = e(i); % a(i).*x(j,i)+e(j);
    xc(i) = e(i); % ac(i).*x(j,i)+e(j);
    xc1(i) = e(i); % ac(i).*x(j,i)+e(j);
    xc2(i) = e(i); % ac(i).*x(j,i)+e(j);
    xc3(i) = e(i); % ac(i).*x(j,i)+e(j);
end

for i = (1+lag):N % calc start from AR(p->lag) --> lag+2
    x(i) = a(1).*x(i-lag)+a(1).*x(i-lag+1)+e(i); % a = 0.1
    xc(i) = a(2).*x(i-lag)+a(2).*x(i-lag+1)+e(i); % 對照組 constant a  = 0.9
    xc1(i) = a(2).*x(i-lag)+a(2).*x(i-lag+1)+e(i); % 對照組 constant a  = -0.9
    xc2(i) = a(2).*x(i-lag)+a(3).*x(i-lag+1)+e(i); % 對照組 constant a  = 0.3
    xc3(i) = a(2).*x(i-lag)+a(4).*x(i-lag+1)+e(i); % 對照組 constant a  = 0.5
end
```

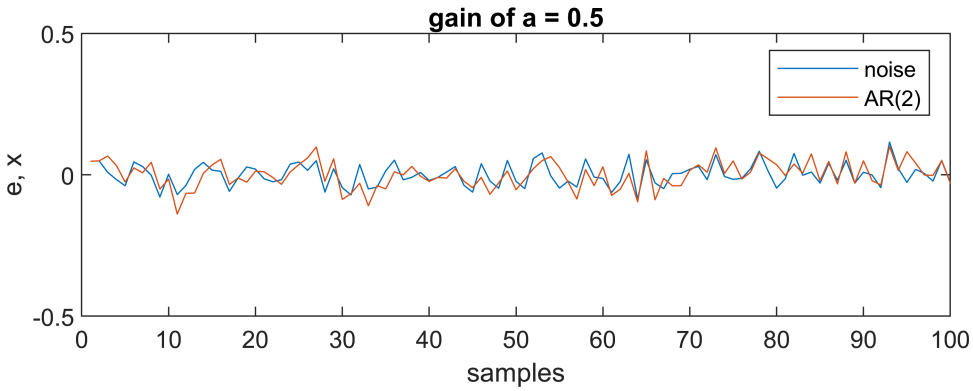
Plot

```
figure();
subplot(3, 1, 1);
plot(e);
hold on;
plot(x);
title('gain of a = 0.1 close to 0');
xlim([0, N]); %xlim(0, N);
ylim([-0.5, 0.5]); %ylim(-2, 2);
xlabel('samples');
ylabel('e, x') ;
% legend(['noise', 'AR(2)'], loc='best');
% tight_layout(pad=0.5, w_pad=0.5, h_pad=1.0);
legend('noise', 'AR(2)')
subplot(3, 1, 2);
plot(e);
hold on;
plot(xc);
title('gain of a = 0.9 close to 1')
xlim([0, N]); %xlim(0, N);
ylim([-0.5, 0.5]); %ylim(-2, 2);
xlabel('samples');
ylabel('e, x') ;
% legend(['noise', 'AR(2)'], loc='best');
% tight_layout(pad=0.5, w_pad=0.5, h_pad=1.0);
legend('noise', 'AR(2)')
subplot(3, 1, 3);
plot(e);
hold on;
plot(xc1);
title('gain of a = -0.9 close to -1, usually 0~1')
xlim([0, N]); %xlim(0, N);
ylim([-0.5, 0.5]); %ylim(-2, 2);
xlabel('samples');
ylabel('e, x') ;
% legend(['noise', 'AR(2)'], loc='best');
% tight_layout(pad=0.5, w_pad=0.5, h_pad=1.0);
legend('noise', 'AR(2)')
```




```
figure();
subplot(2, 1, 1);
plot(e);
hold on;
plot(xc2);
title('gain of a = 0.3');
xlim([0, N]); %xlim(0, N);
ylim([-0.5, 0.5]); %ylim(-2, 2);
xlabel('samples');
ylabel('e, x') ;
% legend(['noise', 'AR(0)'], loc='best');
% tight_layout(pad=0.5, w_pad=0.5, h_pad=1.0);
legend('noise', 'AR(2)')
subplot(2, 1, 2);
plot(e);
hold on;
plot(xc3);
title('gain of a = 0.5')
xlim([0, N]); %xlim(0, N);
ylim([-0.5, 0.5]); %ylim(-2, 2);
xlabel('samples');
ylabel('e, x') ;
% legend(['noise', 'AR(0)'], loc='best');
% tight_layout(pad=0.5, w_pad=0.5, h_pad=1.0);
legend('noise', 'AR(2)')
```


gain of a = 0.3
noise
AR(2)
e, x
samples
0.5
0
-0.5
0
10
20
30
40
50
60
70
80
90
100


gain of a = 0.5
noise
AR(2)
e, x
samples
0.5
0
-0.5
0
10
20
30
40
50
60
70
80
90
100