Basic Signal and Image Processing Knowledge (1)

基礎信號與影像處理知識 (一)

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Signal Processing and Image Processing 基礎知識

(1) 自修資料推薦
(2) Edge
(3) Corner
(4) Color and Intensity
(5) MSE and SSIM

(1) 自修資料推薦

(1-1) 影像處理合適的入門書籍

[1] Rafael C. Gonzalez and Richard E. Woods, *Digital Image Processing*, 3rd Edition, 2010

(淺顯易懂，不同科系背景的人都可以看得懂)

[2] Rafael C. Gonzalez, Richard E. Woods, Steven L. Eddins, *Digital Image Processing Using MATLAB*, 2010

(使用 Matlab 來處理影像的方法)

(1) 自修資料推薦

(1-2) 信號處理合適的入門書籍

[1] Alan V. Oppenheim, Alan S. Willsky, with Hamad Nawab, *Signals and Systems*, 2nd Ed., Prentice Hall, 1997

[2] Alan V. Oppenheim, Alan S. Willsky, with Hamad Nawab and Jian-Jiun Ding, *Signals and Systems*, 3rd ed., Prentice Hall, 2016

(信號與系統)

[3] Alan V. Oppenheim, Ronald W. Schafer, *Discrete-Time Signal Processing*, 3rd Edition, Prentice Hall, 2010

(數位信號處理)

(1) 自修資料推薦

(1-3) Matlab

Matlab 的入門書籍

洪維恩，Matlab 7 程式設計，旗標，台北市，2010.

(合適的入門書, 自學一週之內可完成)

張智星，Matlab 程式設計入門篇，第三版，碁峰，2011.

蒙以正，數位信號處理：應用 Matlab，旗標，台北市，2007.

繆紹綱譯，數位影像處理：運用-Matlab，東華，2005.

Matlab 的 commands 查詢

MathWorks http://www.mathworks.com/

或是直接用 Google 查亦可

(1) 自修資料推薦

使用 Matlab Code 來讀與寫聲音檔、影像檔、Video 檔

http://disp.ee.ntu.edu.tw/class/聲音檔和%20Video%20檔的讀與寫.ppt

(1) 自修資料推薦

Images

Read images by Matlab

`A = double(imread(‘filename’));`

- For a gray-level image, A is a matrix.

$$0 \le A[m, n] \le 255$$

- For a color image, A contains 3 matrices.

$$0 \le A[m, n, c] \le 255$$

c = 1, 2, 3, which correspond to R, G, and B

(1) 自修資料推薦

(1-4) 其他鼓勵學習的程式語言

C++

Python

推薦入門書

B. Lubanovic, *Introducing Python*

中文版：

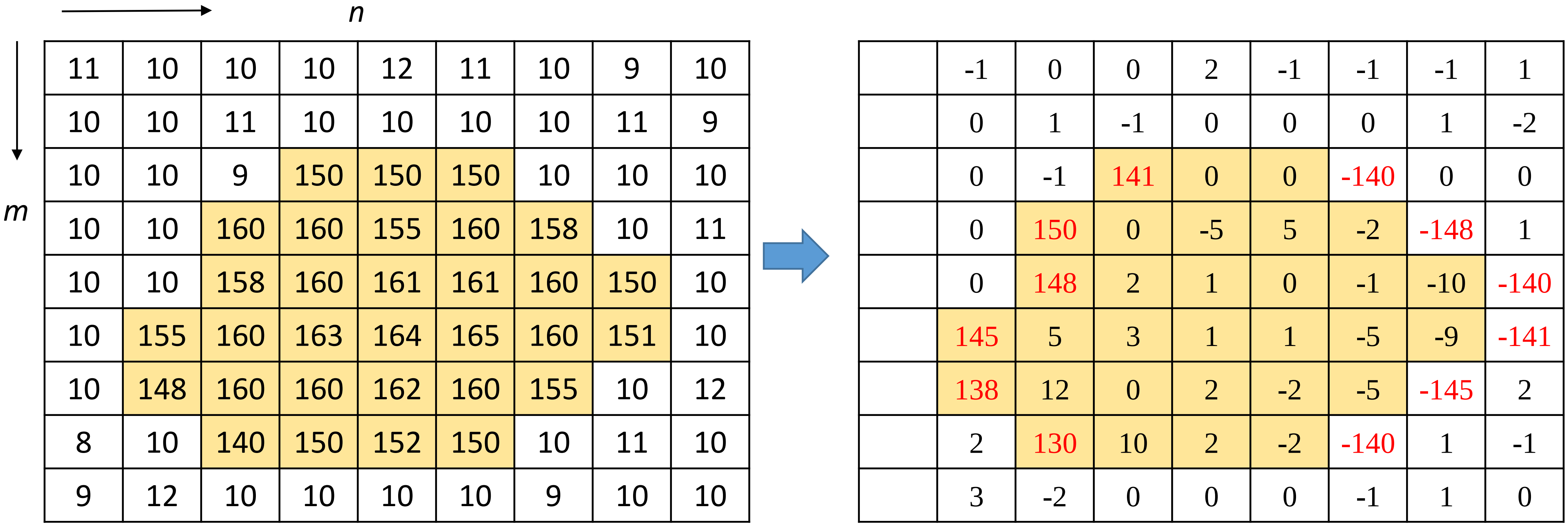
賴屹民譯, *精通Python : 運用簡單的套件進行現代運算*, 基峰資訊股份有限公司，台北市

林大貴，TensorFlow+Keras深度學習人工智慧實務應用，博碩，2017

(2) Edge

Simplest Method: Difference

$A[m, n] - A[m, n\text{-}1]$ (horizontal axis)



11	10	10	10	12	11	10	9	10
10	10	11	10	10	10	10	11	9
10	10	9	150	150	150	10	10	10
10	10	160	160	155	160	158	10	11
10	10	158	160	161	161	160	150	10
10	155	160	163	164	165	160	151	10
10	148	160	160	162	160	155	10	12
8	10	140	150	152	150	10	11	10
9	12	10	10	10	10	9	10	10

	-1	0	0	2	-1	-1	-1	1
	0	1	-1	0	0	0	1	-2
	0	-1	141	0	0	-140	0	0
	0	150	0	-5	5	-2	-148	1
	0	148	2	1	0	-1	-10	-140
	145	5	3	1	1	-5	-9	-141
	138	12	0	2	-2	-5	-145	2
	2	130	10	2	-2	-140	1	-1
	3	-2	0	0	0	-1	1	0

(2) Edge

Simplest Method: Difference

$A[m\text{-}1, n] - A[m, n]$ (vertical axis)

11	10	10	10	12	11	10	9	10
10	10	11	10	10	10	10	11	9
10	10	9	150	150	150	10	10	10
10	10	160	160	155	160	158	10	11
10	10	158	160	161	161	160	150	10
10	155	160	163	164	165	160	151	10
10	148	160	160	162	160	155	10	12
8	10	140	150	152	150	10	11	10
9	12	10	10	10	10	9	10	10

→

-1	0	1	0	-2	-1	0	2	-1
0	0	-2	140	140	140	0	-1	1
0	0	151	10	5	10	148	0	1
0	0	-2	0	6	1	2	140	-1
0	145	2	3	3	4	0	1	0
0	-7	0	-3	-2	-5	-5	-141	2
-2	-138	-20	-10	-10	-10	-145	1	-2
1	2	-130	-140	-142	-140	-1	-1	0

(2) Edge Sobel Operator

$$
\begin{aligned}
&\{2A[m, n+1] - 2A[m, n-1] + \\
&A[m+1, n+1] - A[m+1, n-1] + \\
&A[m-1, n+1] - A[m-1, n-1]\}/4
\end{aligned}
$$

$$A * \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix} / 4$$

(horizontal axis)

11	10	10	10	12	11	10	9	10
10	10	11	10	10	10	10	11	9
10	10	9	150	150	150	10	10	10
10	10	160	160	155	160	158	10	11
10	10	158	160	161	161	160	150	10
10	155	160	163	164	165	160	151	10
10	148	160	160	162	160	155	10	12
8	10	140	150	152	150	10	11	10
9	12	10	10	10	10	9	10	10

	0	35	35.25	0.25	-35.5	-35	-0.5	
	37.25	107.5	69	0	-69.25	-107.25	-37	
	111.75	147.5	33.5	0.25	-33.75	-112.75	-111	
	149	114.5	1.25	1	-0.75	-46.5	-149.25	
	149.5	44.5	3.25	1.25	-4	-47.25	-148.25	
	145.5	43	5	0.5	-40	-113.25	-109	
	103.75	72.5	6.5	0	-73	-107	-35.5	

(2) Edge Sobel Operator

$\{2A[m+1, n] - 2A[m-1, n]+$
$A[m+1, n+1] - A[m-1, n+1]+$
$A[m+1, n-1] - A[m-1, n-1]\}/4$

$$A * \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix} / 4$$

(vertical axis)

11	10	10	10	12	11	10	9	10
10	10	11	10	10	10	10	11	9
10	10	9	150	150	150	10	10	10
10	10	160	160	155	160	158	10	11
10	10	158	160	161	161	160	150	10
10	155	160	163	164	165	160	151	10
10	148	160	160	162	160	155	10	12
8	10	140	150	152	150	10	11	10
9	12	10	10	10	10	9	10	10

	-0.5	34.5	104.25	138.75	104	35	0.5	
	37.25	112	148.5	147.5	148.25	111.25	37	
	37.25	77	45	10.75	45.75	112.75	107.5	
	72.5	37	3.75	6.5	5.25	37.5	70.75	
	69.5	35.5	0.75	0.25	-1.5	-37.75	-70.75	
	-78	-49.5	-14.5	-13	-48	-113.75	-107.5	
	-105.75	-146.5	-150.5	-151	-149.5	-110.5	-37	

(2) Edge Sobel Operator

$$\{2A[m\text{-}1, n+1] - 2A[m+1, n\text{-}1]+ A[m\text{-}1, n] - A[m+1, n]+ A[m, n+1] - A[m, n\text{-}1]\}/4 \qquad (45°)$$

$$A * \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix} / 4$$

n

m

11	10	10	10	12	11	10	9	10
10	10	11	10	10	10	10	11	9
10	10	9	150	150	150	10	10	10
10	10	160	160	155	160	158	10	11
10	10	158	160	161	161	160	150	10
10	155	160	163	164	165	160	151	10
10	148	160	160	162	160	155	10	12
8	10	140	150	152	150	10	11	10
9	12	10	10	10	10	9	10	10

	0.25	0.25	-33.75	-104	-104.75	-70.25	-0.5	
	0.25	-2.25	-77.25	-111.25	-145	-146.5	-74.25	
	37	70.25	-7.75	-7.75	-77.5	-150.5	-146.75	
	75.75	40	-2.5	-3.5	-4.5	-80.75	-147.25	
	77	7.5	1.5	0.75	-1.75	-7.25	-75	
	149.75	84.5	15.75	10.5	6	0.5	-0.75	
	142.5	146.5	116.5	113	74.5	1.75	1.5	

(2) Edge Sobel Operator

$$\{2A[m-1, n-1] - 2A[m+1, n+1] + A[m-1, n] - A[m+1, n] + A[m, n-1] - A[m, n+1]\}/4 \quad (135°)$$

$$A * \begin{bmatrix} -2 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix} / 4$$

n →

m ↓

11	10	10	10	12	11	10	9	10
10	10	11	10	10	10	10	11	9
10	10	9	150	150	150	10	10	10
10	10	160	160	155	160	158	10	11
10	10	158	160	161	161	160	150	10
10	155	160	163	164	165	160	151	10
10	148	160	160	162	160	155	10	12
8	10	140	150	152	150	10	11	10
9	12	10	10	10	10	9	10	10

	0.75	-69.75	-104.75	-104.5	-33.75	0.25	0	
	-74.75	-147.25	-144.75	-111.25	-76.5	-2	-0.25	
	-111.5	-149.75	-77.25	-8.25	-8.5	0	1.75	
	-148.25	-114	-3.5	-5	-3.5	6.75	76.25	
	-147	-77.5	-3	-0.75	4.25	80.25	146.5	
	-66.25	4.5	6.75	9.5	82.5	152	145.75	
	1	71.5	109.5	113	149.5	146.25	72.5	

(2) Edge

Laplacian Operator

$$\{8A[m, n] - A[m+1, n] - A[m-1, n] \\ - A[m+1, n-1] - A[m, n-1] - A[m-1, n-1] \\ - A[m+1, n+1] - A[m, n+1] - A[m-1, n+1]\}/8$$

$$A * \begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix} / 8$$

n →

m ↓

11	10	10	10	12	11	10	9	10
10	10	11	10	10	10	10	11	9
10	10	9	150	150	150	10	10	10
10	10	160	160	155	160	158	10	11
10	10	158	160	161	161	160	150	10
10	155	160	163	164	165	160	151	10
10	148	160	160	162	160	155	10	12
8	10	140	150	152	150	10	11	10
9	12	10	10	10	10	9	10	10

⇨

	-0.13	-16.38	-35.25	-52.88	-35.38	-17.63	1.25	
	-18.75	-56.13	66.88	49.38	67.13	-54.88	-18.63	
	-37.13	76.63	22.13	-1.50	21.88	56.63	-54.88	
	-74.13	35.75	-0.13	0.00	0.63	20.63	66.25	
	71.75	20.75	2.38	2.50	4.63	21.00	67.63	
	66.38	24.25	3.63	4.00	20.25	52.88	-54.88	
	-52.13	57.50	49.50	50.50	66.50	-54.38	-17.25	

(2) Edge

General form of the 1-D edge detection filter

(1) $h[n] = -\,h[-n]$

(2) $|h[n_1]| \le |h[n_2]| \quad \text{if } |n_1| > |n_2|$

General form of the 2-D edge detection filter

(1) $h[m, n] = -\,h[-m, -n]$

(2) $|h[cn_1, cn_2]| \le |\,h[n_1, n_2]| \quad \text{if } \; c > 1$

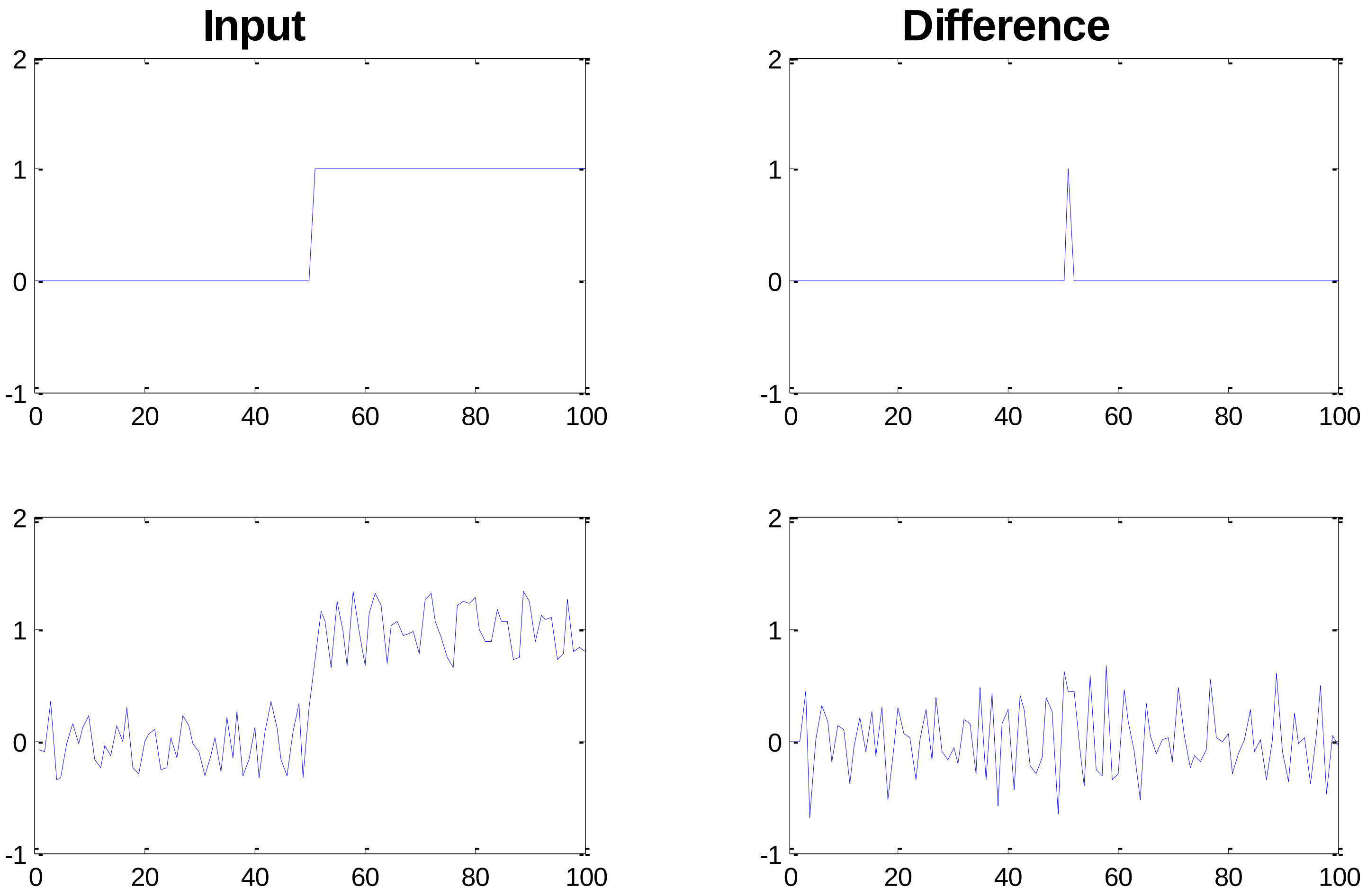
(1) 任何能量隨著 $|n|$ 遞減的 odd function，都可以當成 edge detection filter

(2) The edge detection filter is in fact a matched filter.

(2) Edge

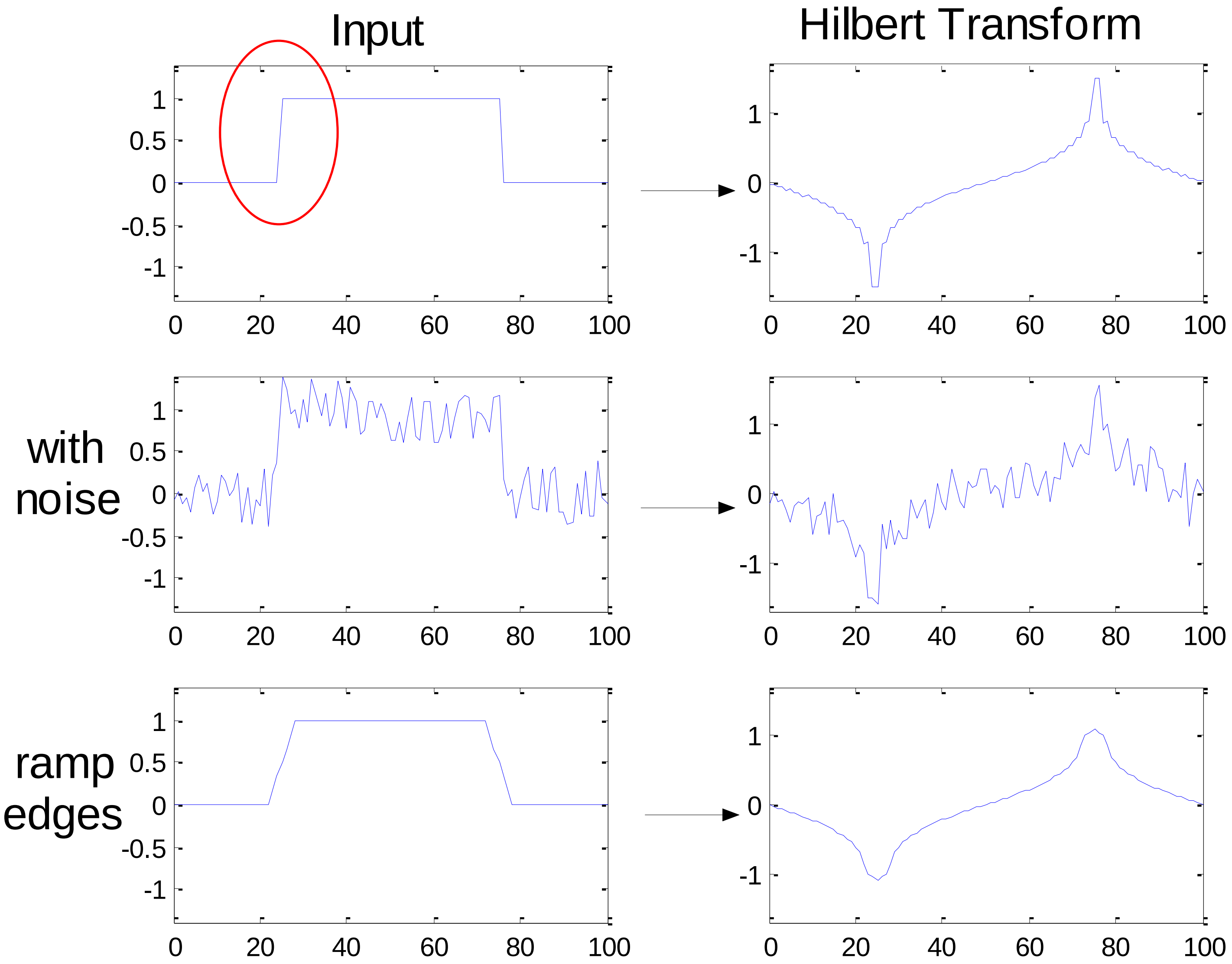
without noise

with noise



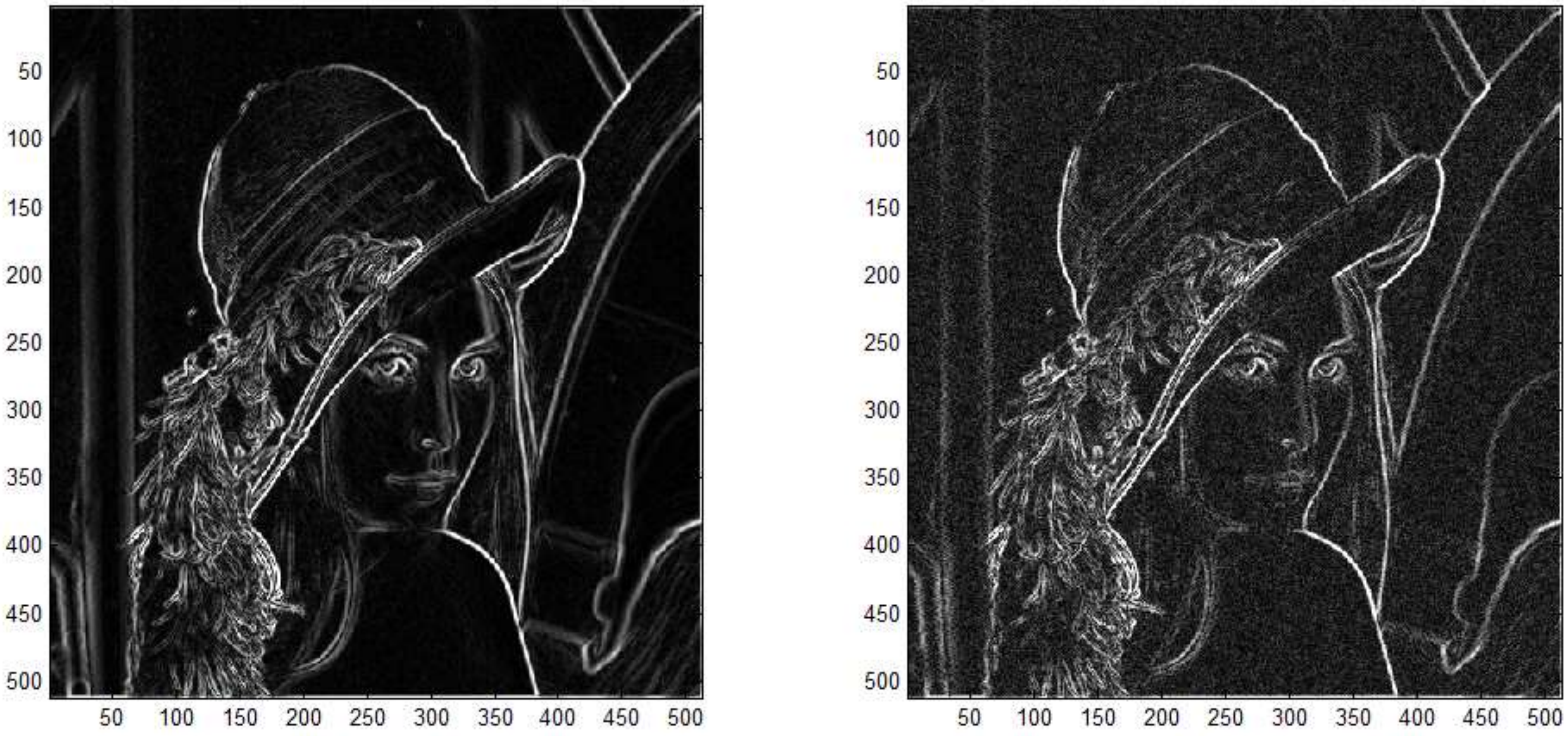
(2) Edge

(long impulse response)



(2) Edge

Short impulse response



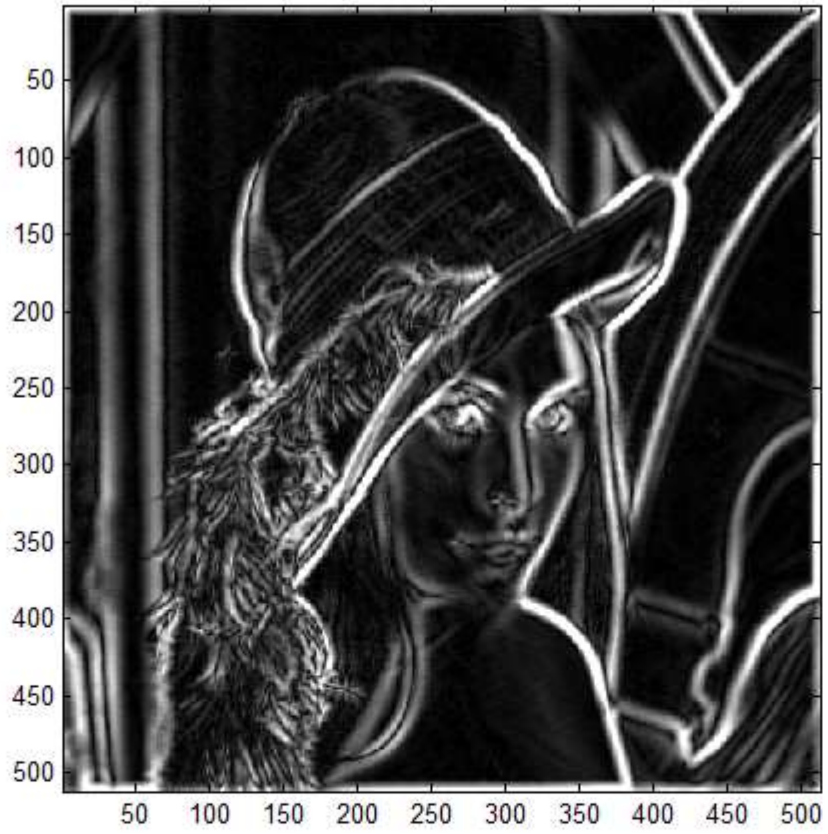
without noise

with noise

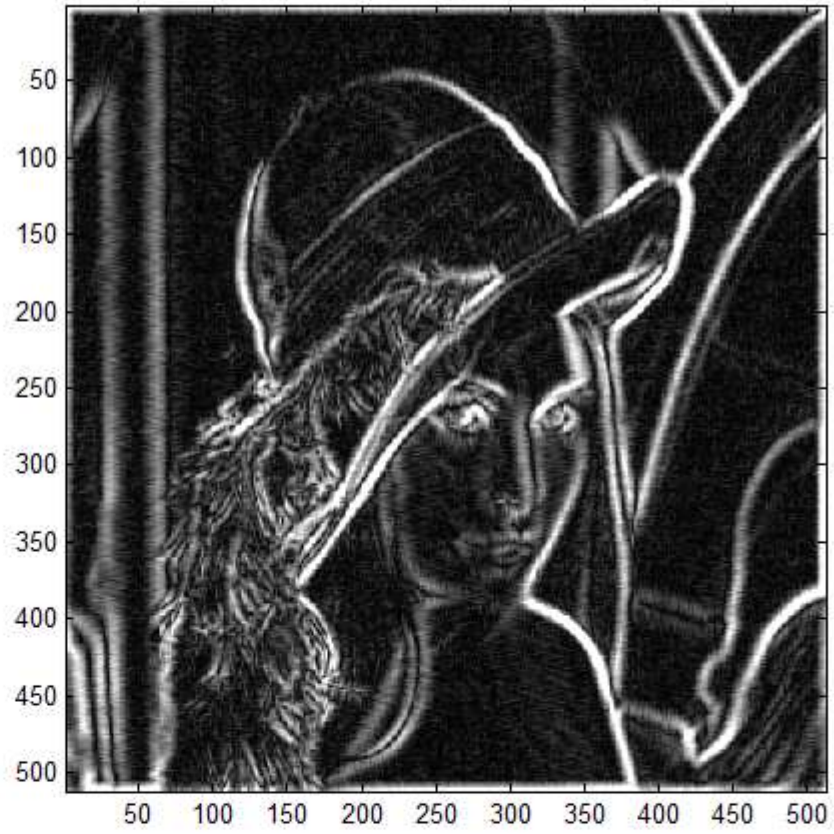
(2) Edge

Long impulse response

without noise



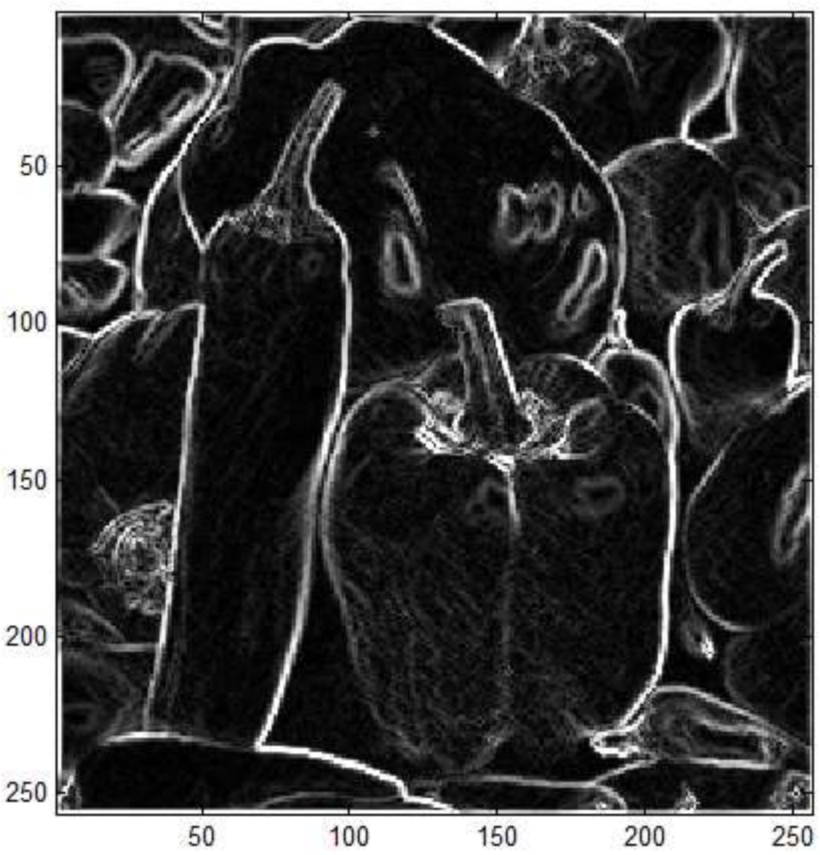
with noise



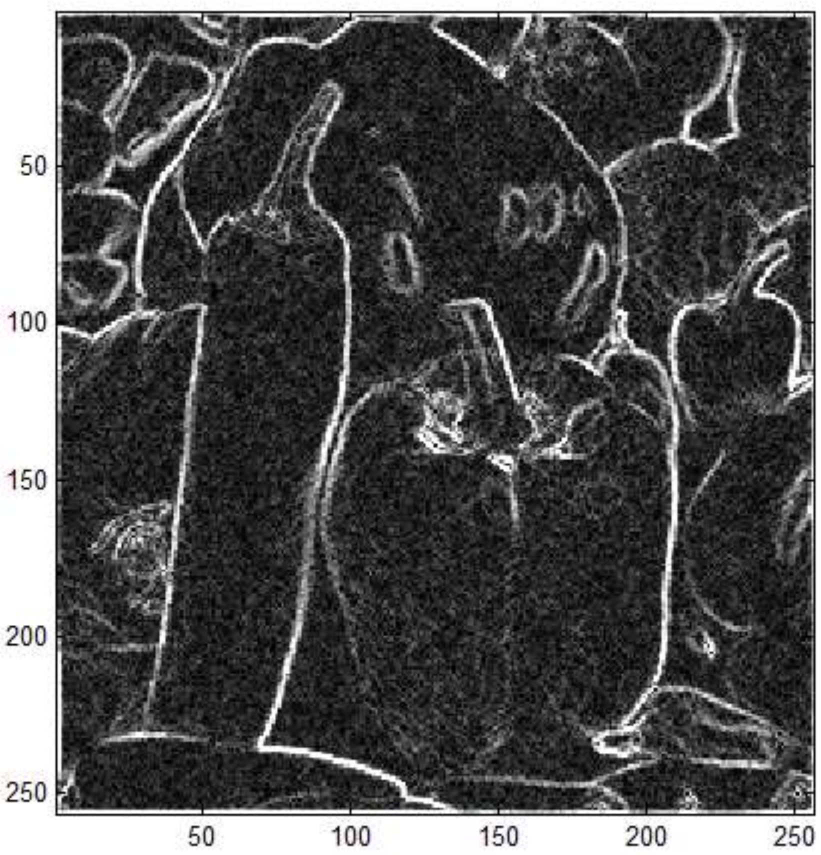
(2) Edge

Short impulse response

without noise



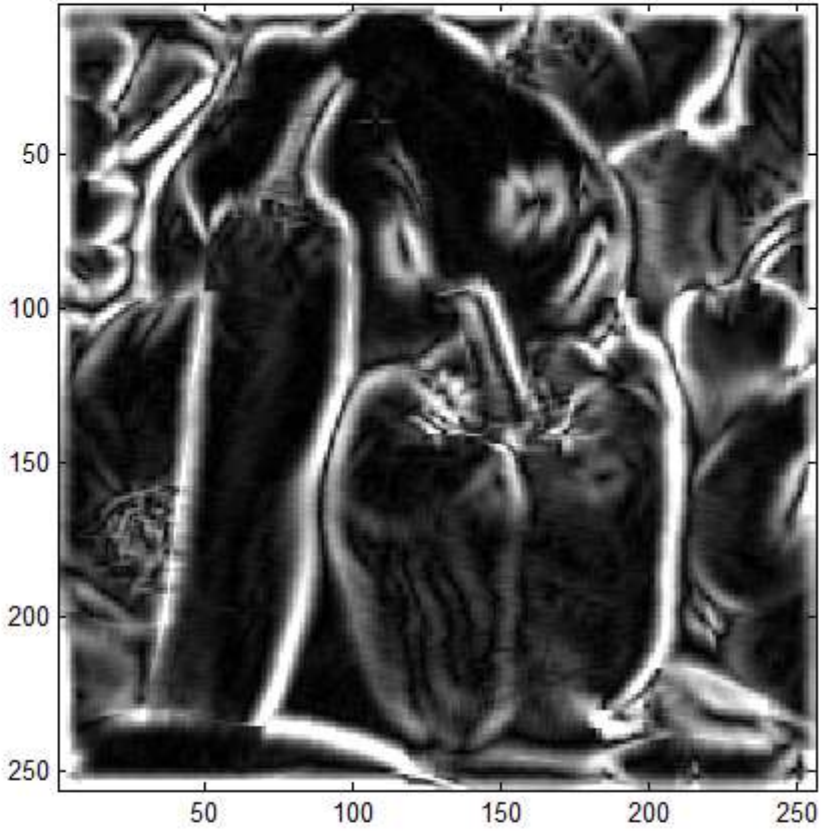
with noise



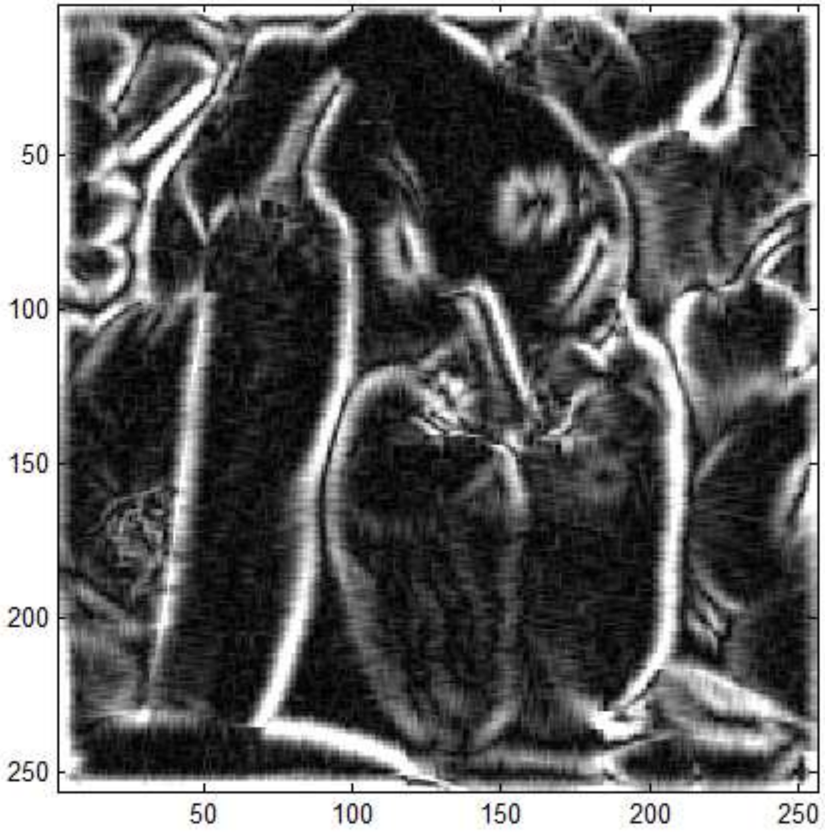
(2) Edge

Long impulse response

without noise



with noise



(2) Edge

Well-known Advanced Edge Detection Operator

(a) Difference of Gaussian (DoG)

$$y[n] = x[n] * h[n]$$

$$h[n] = \exp\left(-\frac{(n+1)^2}{\sigma}\right) - \exp\left(-\frac{(n-1)^2}{\sigma}\right)$$

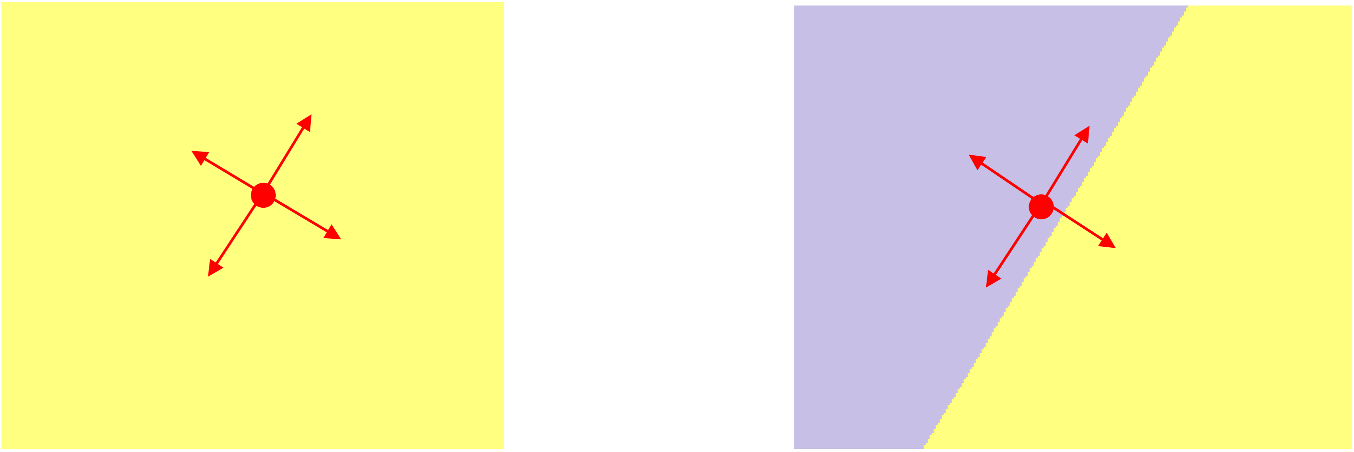
(b) Canny’s Edge Detection Filter

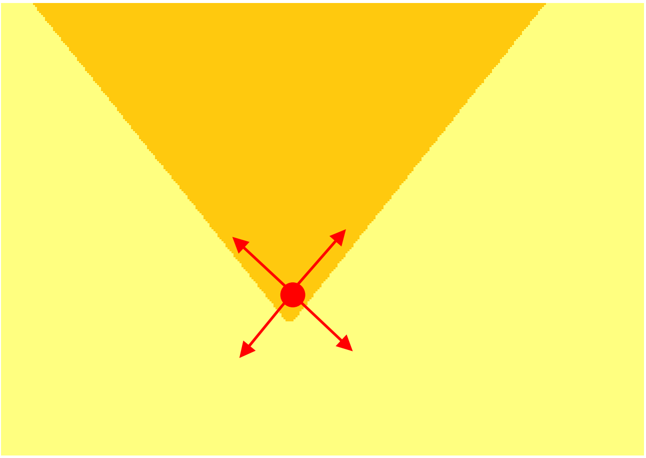
```
edge(I, 'Canny');
```

Canny, J. (1986). A computational approach to edge detection. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, (6), 679-698.

(3) Corner

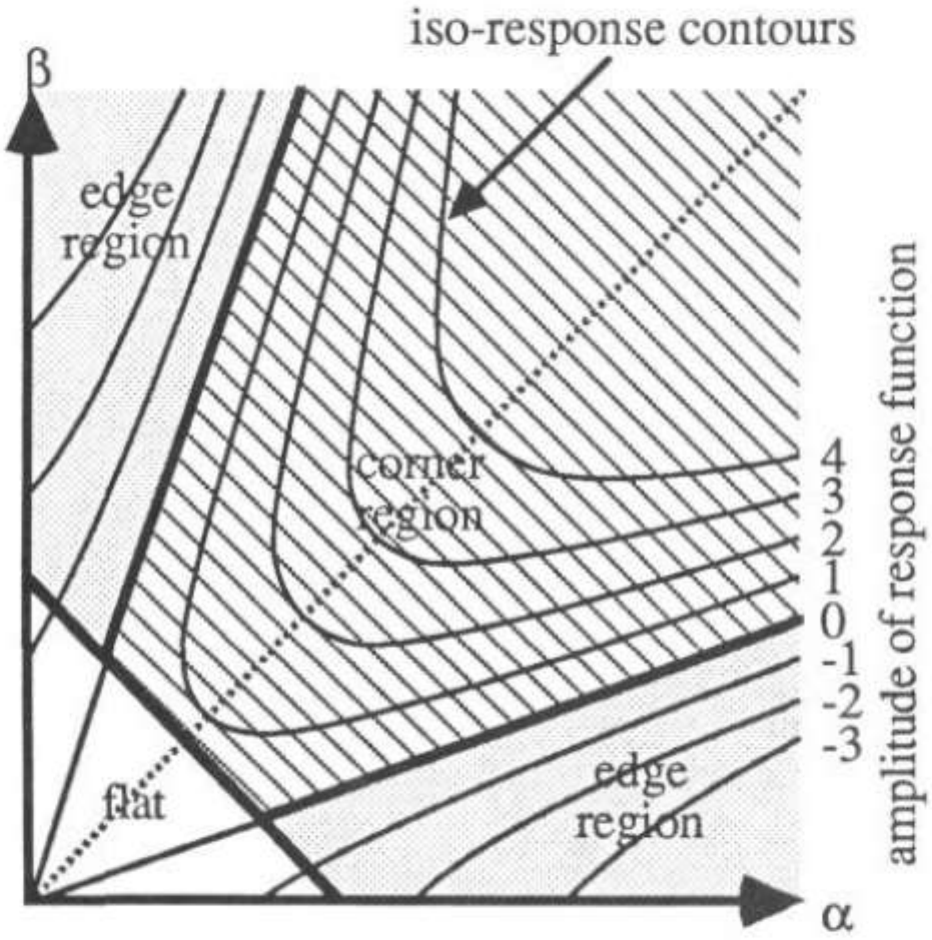
The edge pixel has large variation along one direction





The corner pixel has large variation along two directions

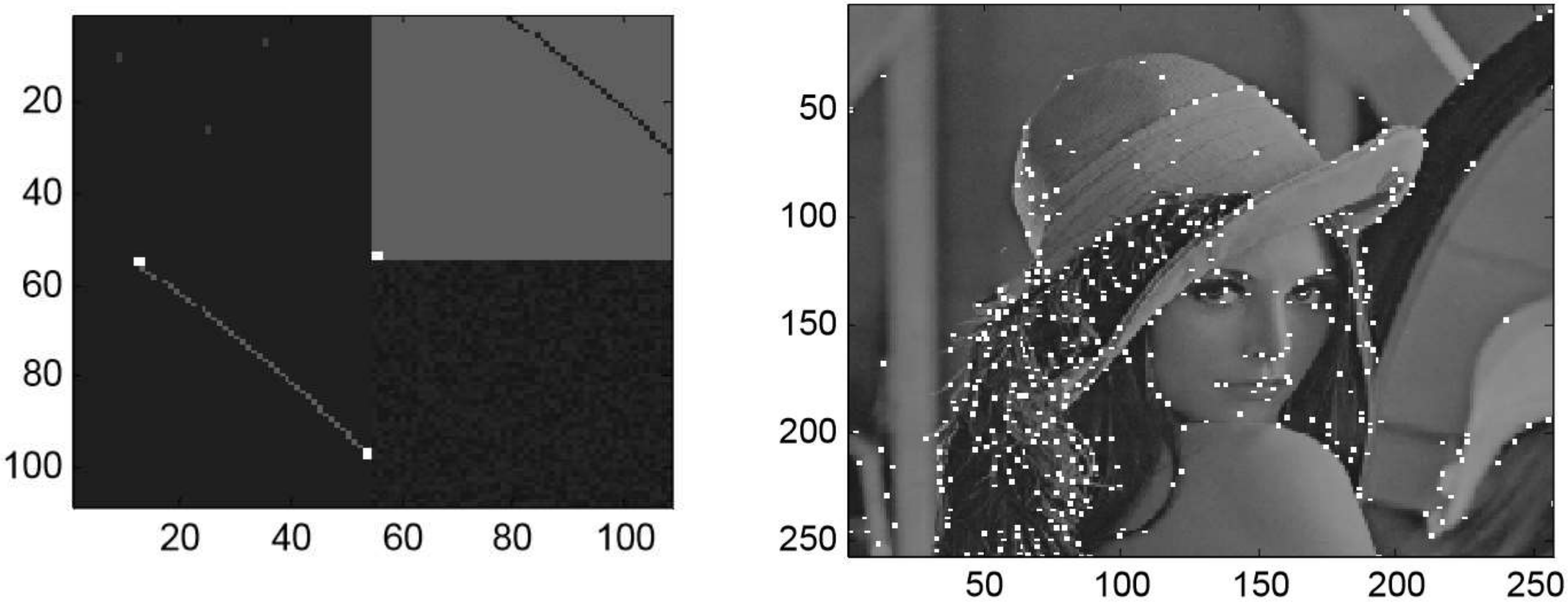
(3) Corner



α, β:
the variation along two axes

C. Harris and M. Stephens, “A combined corner and edge detection”, Proc. 4th Alvey Vision Conf., 1988, pp. 147-151.

(3) Corner



PEI, Soo-Chang; DING, Jian-Jiun. Improved Harris' Algorithm for Corner and Edge Detections. In: *2007 IEEE International Conference on Image Processing*. IEEE, 2007, p. III-57-III-60.

Pei, S. C., & Ding, J. J. (2005, September). New corner detection algorithm by tangent and vertical axes and case table. In *IEEE International Conference on Image Processing 2005* (Vol. 1, pp. I-365). IEEE.

(4) Color and Intensity

三原色：R, G, B

對應人類感光細胞的三種錐狀體

波長 R > G > B

頻率 B > G > R

(4) Color and Intensity

亮度 Y 和 R, G, B 之間的關係

$$Y = 0.299R + 0.587G + 0.114B$$

RGB 不是唯一的座標系

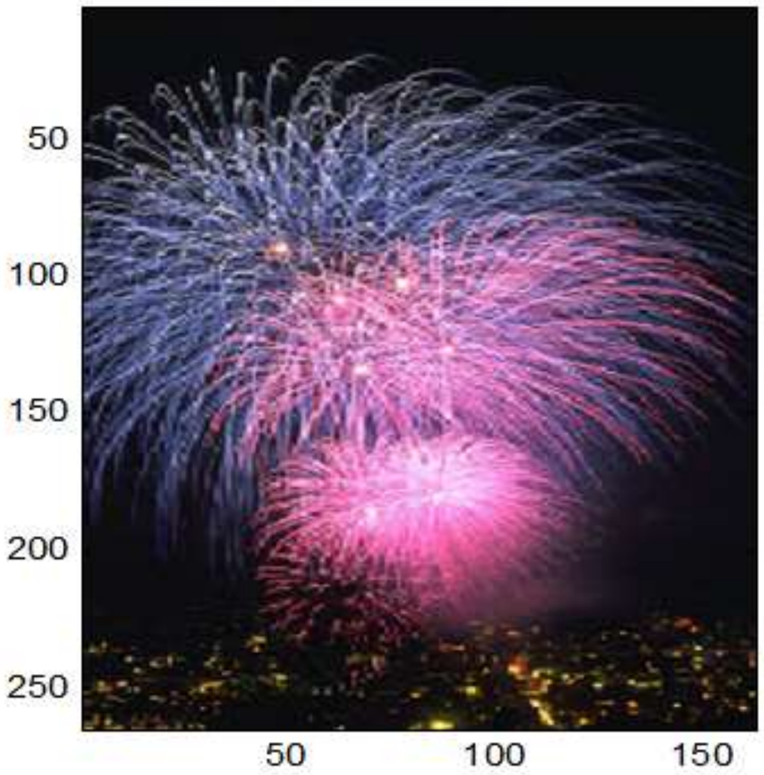
RGB to YCbCr 轉換

$$\begin{bmatrix} Y \\ C_b \\ C_r \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ -0.169 & -0.331 & 0.500 \\ 0.500 & -0.419 & -0.081 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

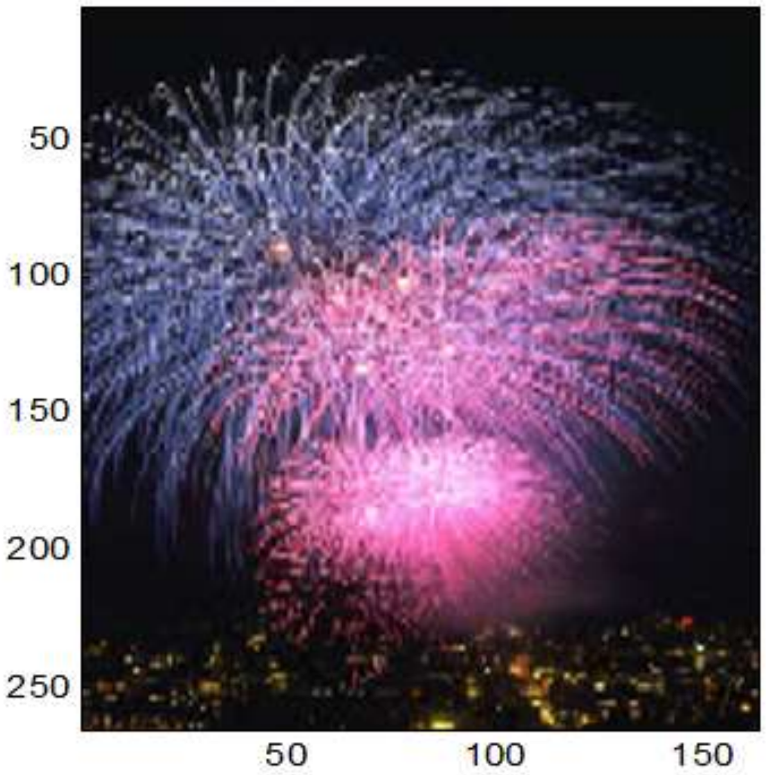
此外，尚有 RGB to HSV (hue, saturation, intensity)
RGB to XYZ
RGB to Lab 等轉換

(4) Color and Intensity

原圖

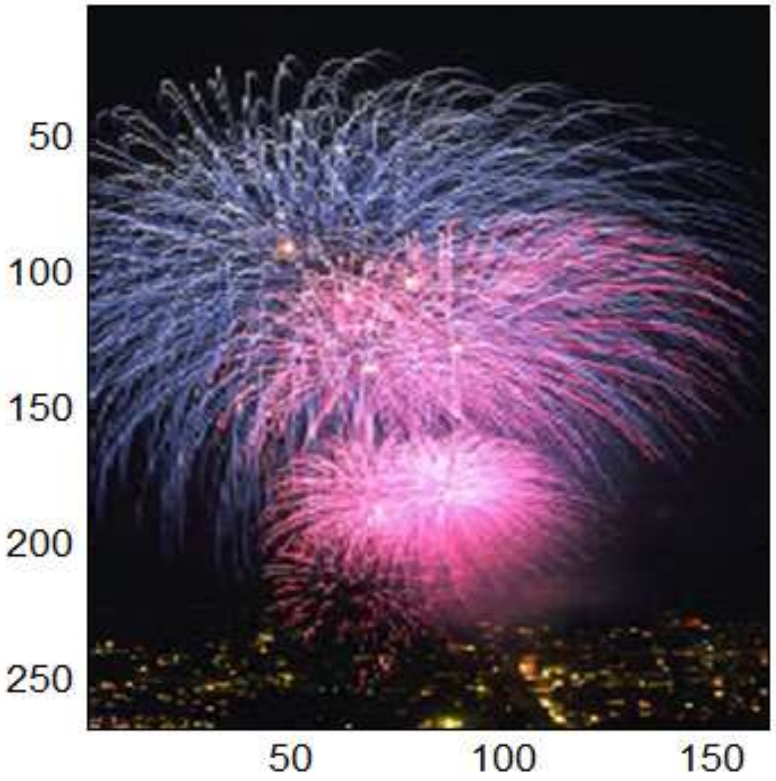


直接在縱軸取一半的pixels 再還原

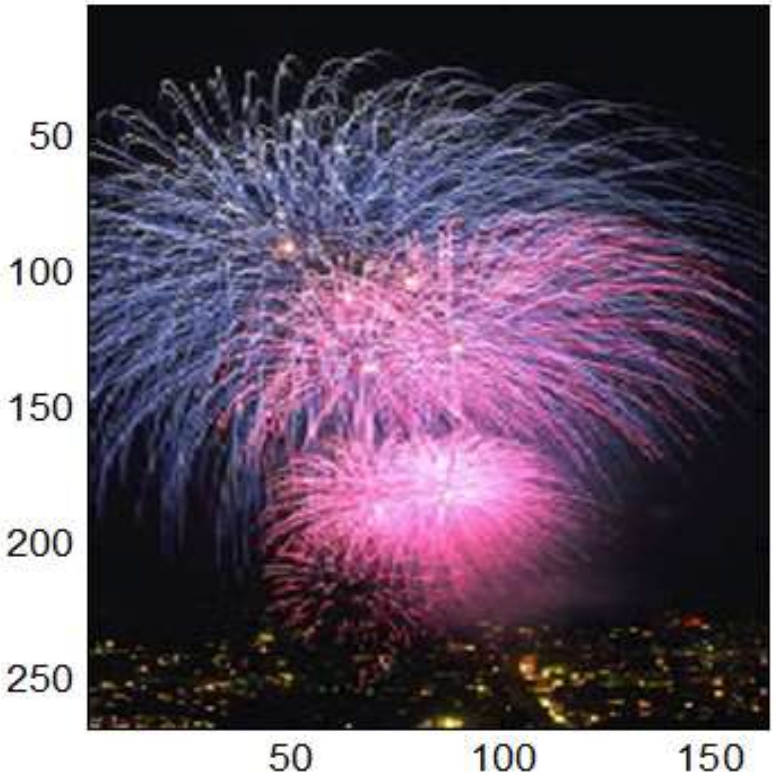


(4) Color and Intensity

Y 不變
CbCr在縱軸取一半的pixels 再還原



Y 不變
CbCr在縱軸及橫軸取一半的pixels 再還原



(4) Color and Intensity

Lighten and Darken

$$Y_o = f(Y)$$

Lighten

$f(0) = 0,\qquad f(255) = 255,$

$f(Y_1) > f(Y_2)\qquad \text{if } Y_1 > Y_2$

$f(Y) > Y\qquad \text{if } 0 < Y < 255$

Darken

$f(0) = 0,\qquad f(255) = 255,$

$f(Y_1) > f(Y_2)\qquad \text{if } Y_1 > Y_2$

$f(Y) < Y\qquad \text{if } 0 < Y < 255$

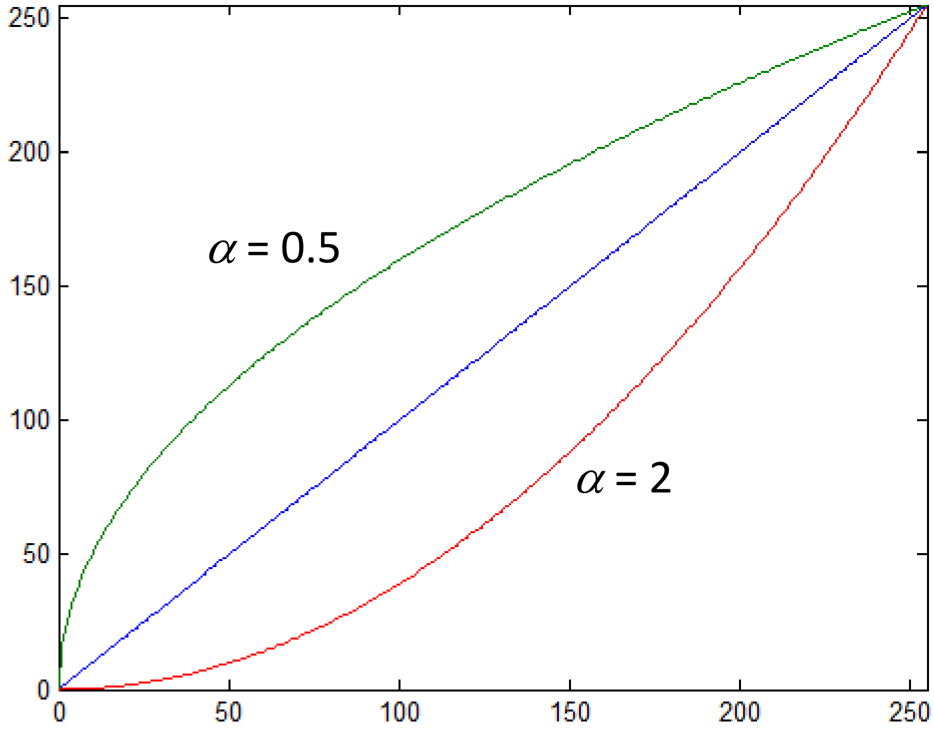
(4) Color and Intensity

Examples of the mapping function for Lighten and Darken

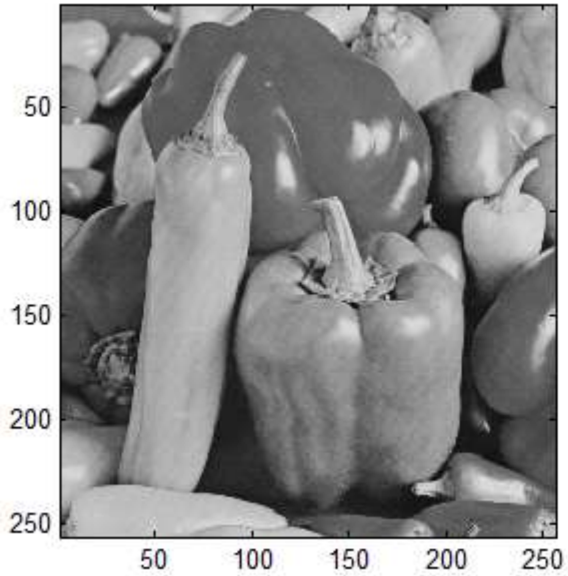
$$f(Y) = 255\left(\frac{Y}{255}\right)^{\alpha}$$

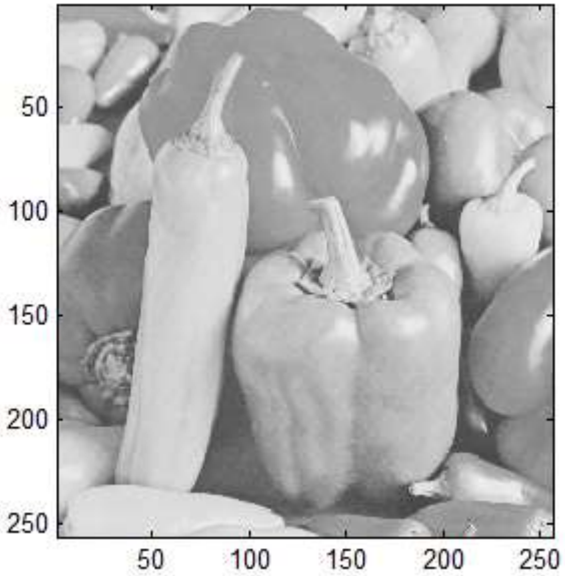
$\alpha < 1$: lighten, $\alpha > 1$: darken

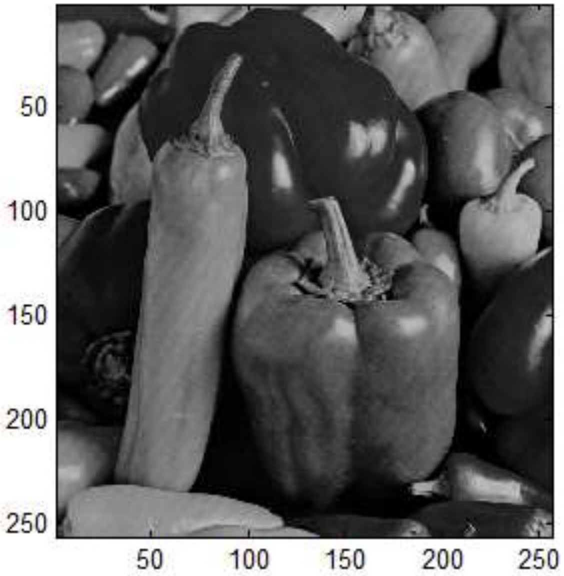
The cosine function, the hyperbolic tangent function, and many other functions have similar effect.



(4) Color and Intensity

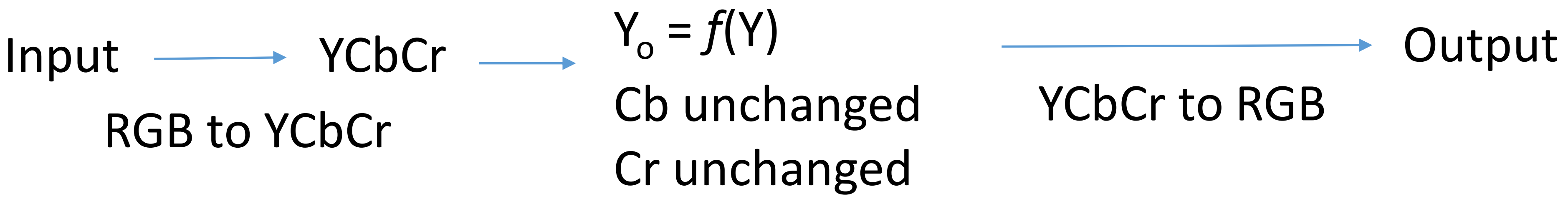

Original image


lighten


darken

(4) Color and Intensity

Lighten and Darken for color image



Input

YCbCr

RGB to YCbCr

$Y_o = f(Y)$

Cb unchanged

Cr unchanged

YCbCr to RGB

Output

Lighten

$f(0) = 0,$ $f(255) = 255,$

$f(Y_1) > f(Y_2)$ if $Y_1 > Y_2$

$f(Y) > Y$ if $0 < Y < 255$

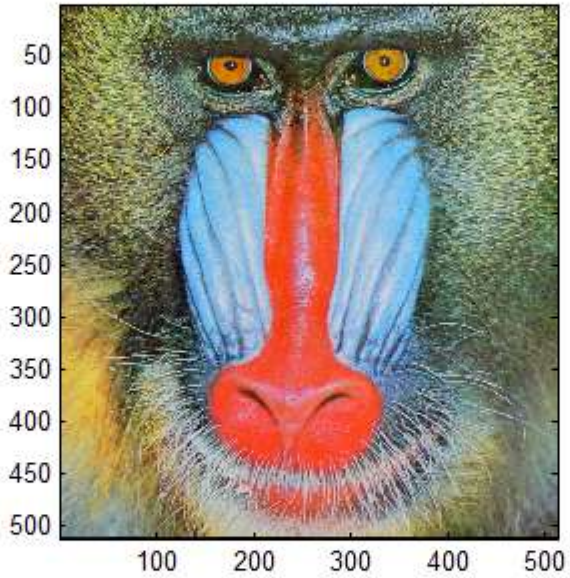
Darken

$f(0) = 0,$ $f(255) = 255,$

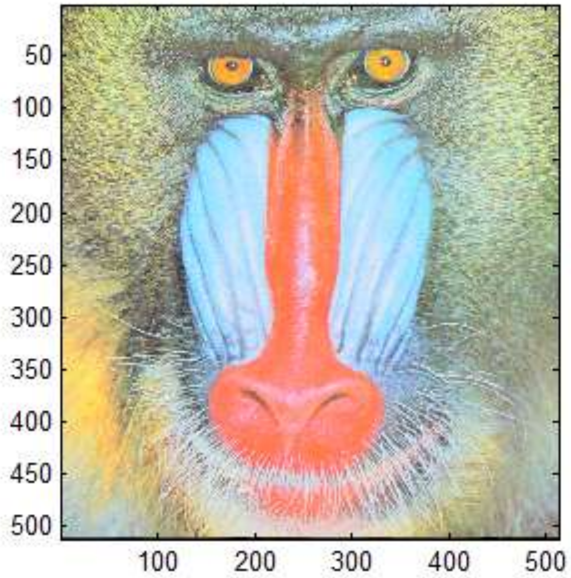
$f(Y_1) > f(Y_2)$ if $Y_1 > Y_2$

$f(Y) < Y$ if $0 < Y < 255$

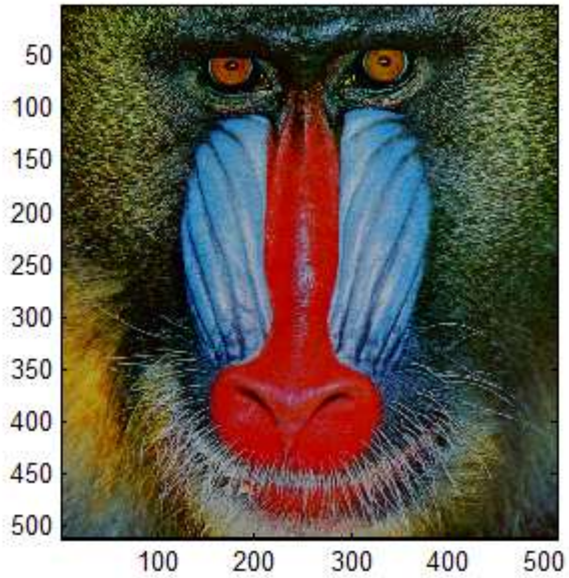
(4) Color and Intensity



Original image



lighten



darken

(4) Color and Intensity

視覺和 intensity

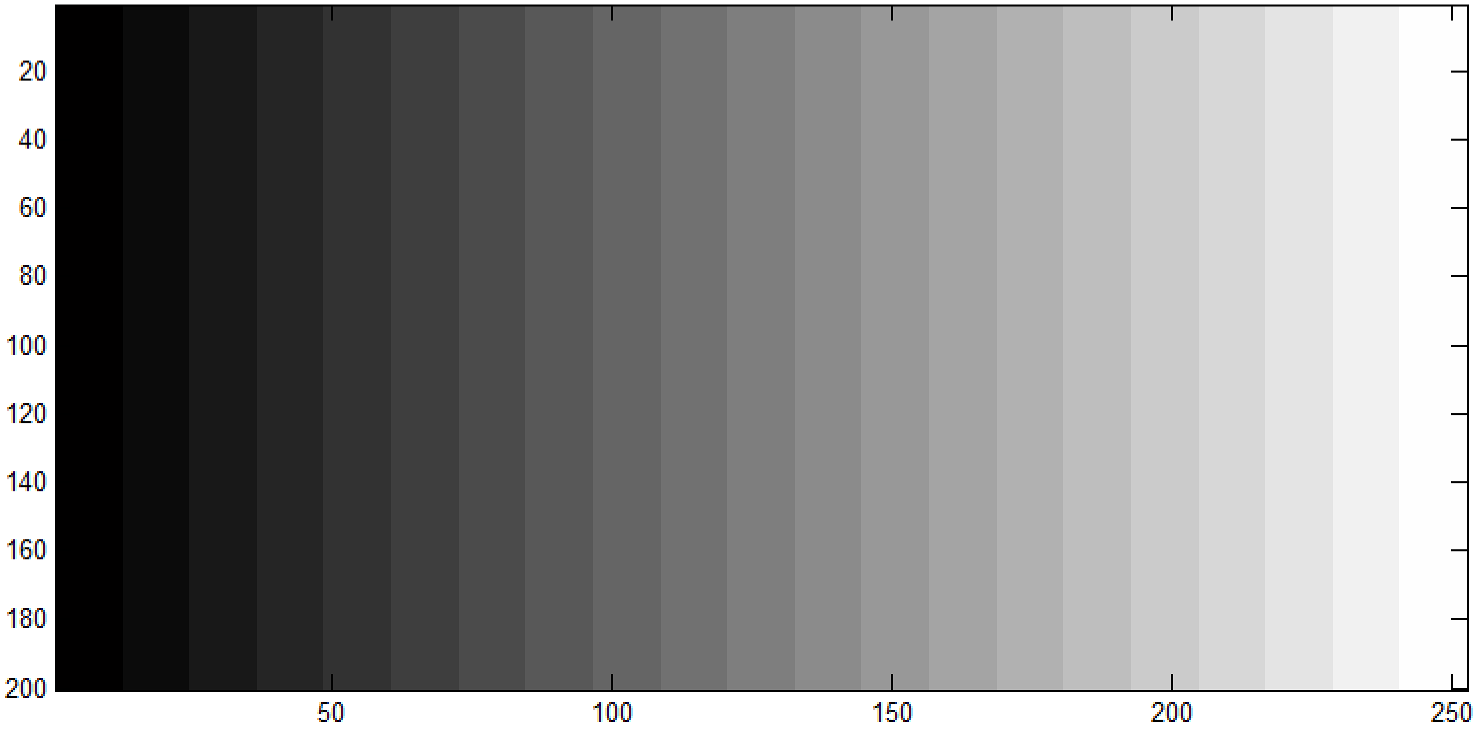
Intensity 範圍是從 0 至 255

但人眼會覺的 intensities 150 和 200 之間的差異，會比 intensities 0 和 50 之間的差異明顯

(Intensities 中間的部分的差異會較為明顯)

(4) Color and Intensity

Intensity 由 0 至 255, 每一個相鄰長條的 intensities 差 12.75



(5) MSE and SSIM

There are several ways to measure the difference / similarity between two images.

傳統量測兩個信號 (including images, videos, and vocal signals) 之間相似度的方式：

(1) maximal error $Max\left(\left|y[m,n]-x[m,n]\right|\right)$

(2) mean square error (MSE) $\frac{1}{MN}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\left|y[m,n]-x[m,n]\right|^2$

(3) normalized mean square error (NMSE) $\frac{\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\left|y[m,n]-x[m,n]\right|^2}{\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\left|x[m,n]\right|^2}$

(4) normalized root mean square error (NRMSE) $\sqrt{\frac{\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\left|y[m,n]-x[m,n]\right|^2}{\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\left|x[m,n]\right|^2}}$

(5) MSE and SSIM

(5) signal to noise ratio (SNR)，信號處理常用

$$10\log_{10}\left(\frac{\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\left|x[m,n]\right|^2}{\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\left|y[m,n]-x[m,n]\right|^2}\right)$$

(6) peak signal to noise ratio (PSNR)，影像處理常用

$$10\log_{10}\left(\frac{X_{Max}^{2}}{\frac{1}{MN}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\left|y[m,n]-x[m,n]\right|^2}\right)$$

X_{Max}: the maximal possible value of $x[m, n]$

In image processing, $X_{Max} = 255$

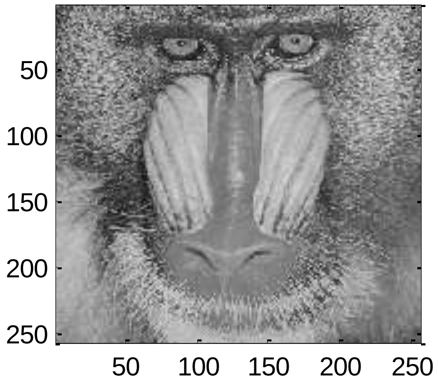
for color image: $$10\log_{10}\left(\frac{X_{Max}^{2}}{\frac{1}{3MN}\sum_{R,G,B}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\left|y_{color}[m,n]-x_{color}[m,n]\right|^2}\right)$$

color = R, G, or B

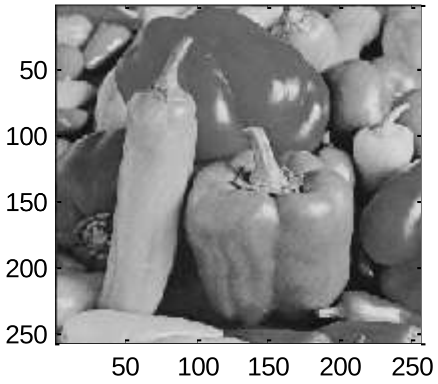
(5) MSE and SSIM

然而，MSE 和 NRMSE 雖然在理論上是合理的，但卻無法反映出實際上兩個影像之間的相似度

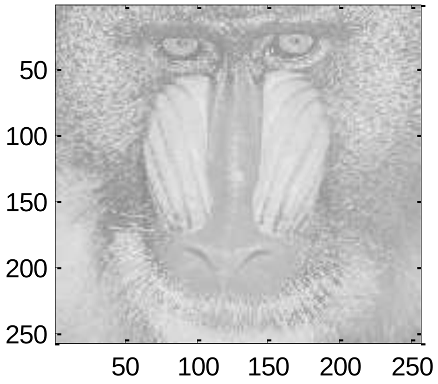
例如：以下這三張圖



圖一



圖二



圖三

圖三 = 圖一 × 0.5 + 255.5 × 0.5

照理來說，圖一和圖三較相近

然而，圖一和圖二之間的 NRMSE 為 0.4411

圖一和圖三之間的 NRMSE 為 0.4460

(5) MSE and SSIM

(7) Structural Similarity (SSIM)

有鑑於 MSE 和 PSNR 無法完全反映人類視覺上所感受的誤差，在 2004 年被提出來的新的誤差測量方法

$$SSIM\left(x,y\right)=\frac{\left(2\mu_x\mu_y+(c_1L)^2\right)}{\left(\mu_x^2+\mu_y^2+(c_1L)^2\right)}\frac{\left(2\sigma_{xy}+(c_2L)^2\right)}{\left(\sigma_x^2+\sigma_y^2+(c_2L)^2\right)}$$

$$DSSIM\left(x,y\right)=1-SSIM(x,y)$$

μ_x, μ_y: means of x and y

σ_x^2, σ_y^2: variances of x and y

σ_{xy}: covariance of x and y

c_1, c_2: adjustable constants

L: the maximal possible value of x − the minimal possible value of x

Z. Wang, A. C. Bovik, H. R. Sheikh, and E. P. Simoncelli, "Image quality assessment: from error visibility to structural similarity," *IEEE Trans. Image Processing*, vol. 13, no. 4, pp. 600−612, Apr. 2004.

(5) MSE and SSIM

若使用 SSIM，且前頁的 c_1, c_2 皆選為 $\sqrt{1/L}$

圖一、圖二之間的 SSIM 為 0.1040

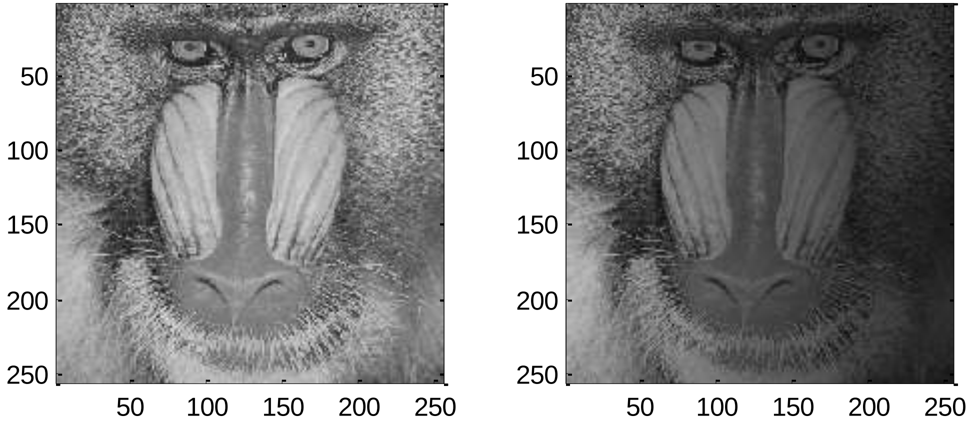
圖一、圖三之間的 SSIM 為 0.7720

反映出了圖一、圖三之間確實有很高的相似度

(5) MSE and SSIM

其他幾個用 MSE 和 NRMSE 無法看出相似度，但是可以用 SSIM 看出相似度的情形

影子 shadow



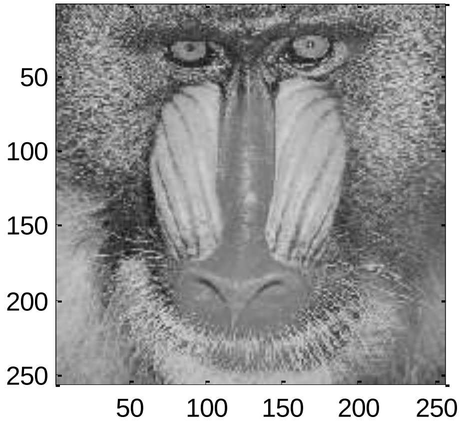
圖四 圖五

NRMSE = 0.4521 (大於圖一、圖二之間的 NRMSE)

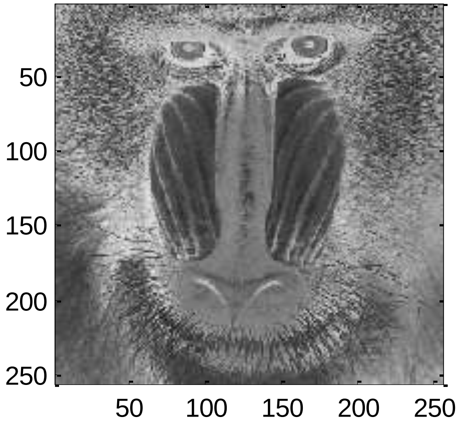
SSIM = 0.6010

(5) MSE and SSIM

底片 the negative of a photo



圖六



圖七

圖七 = 255 − 圖六

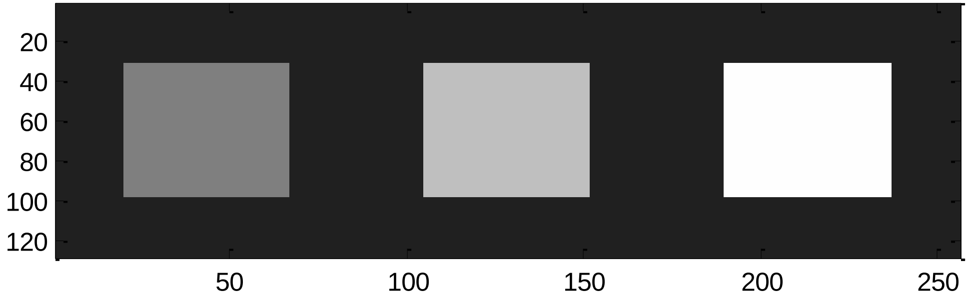
NRMSE = 0.5616 (大於圖一、圖二之間的 NRMSE)

SSIM = -0.8367 (高度負相關)

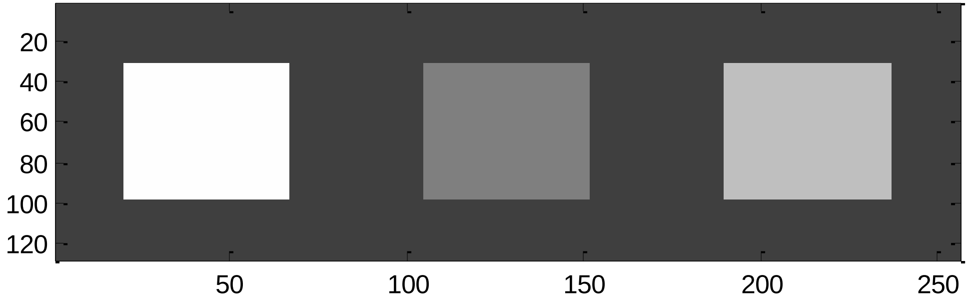
(5) MSE and SSIM

同形，但亮度不同 (Same shape but different intensity)

圖八



圖九



NRMSE = 0.4978 (大於圖一、圖二之間的 NRMSE)

SSIM = 0.7333

練習

(1) 看過這份投影片

(2) 依據 pages 9-15 所述，找出影像不同方向的邊緣

(將結果取絕對值、乘常數之後，用 graylevel 的方式畫出)
image(C*abs(E)); % E: edge detection result
colormap(gray(256))

(3) 依據 page 34 和 page 32 的方式，將影像調亮、調暗

(4) 用 pages 38, 39 的式子，寫出量測兩個信號 NRMSE 和 PSNR 的程式

下次討論要帶隨身碟，讓我看 Matlab 程式碼