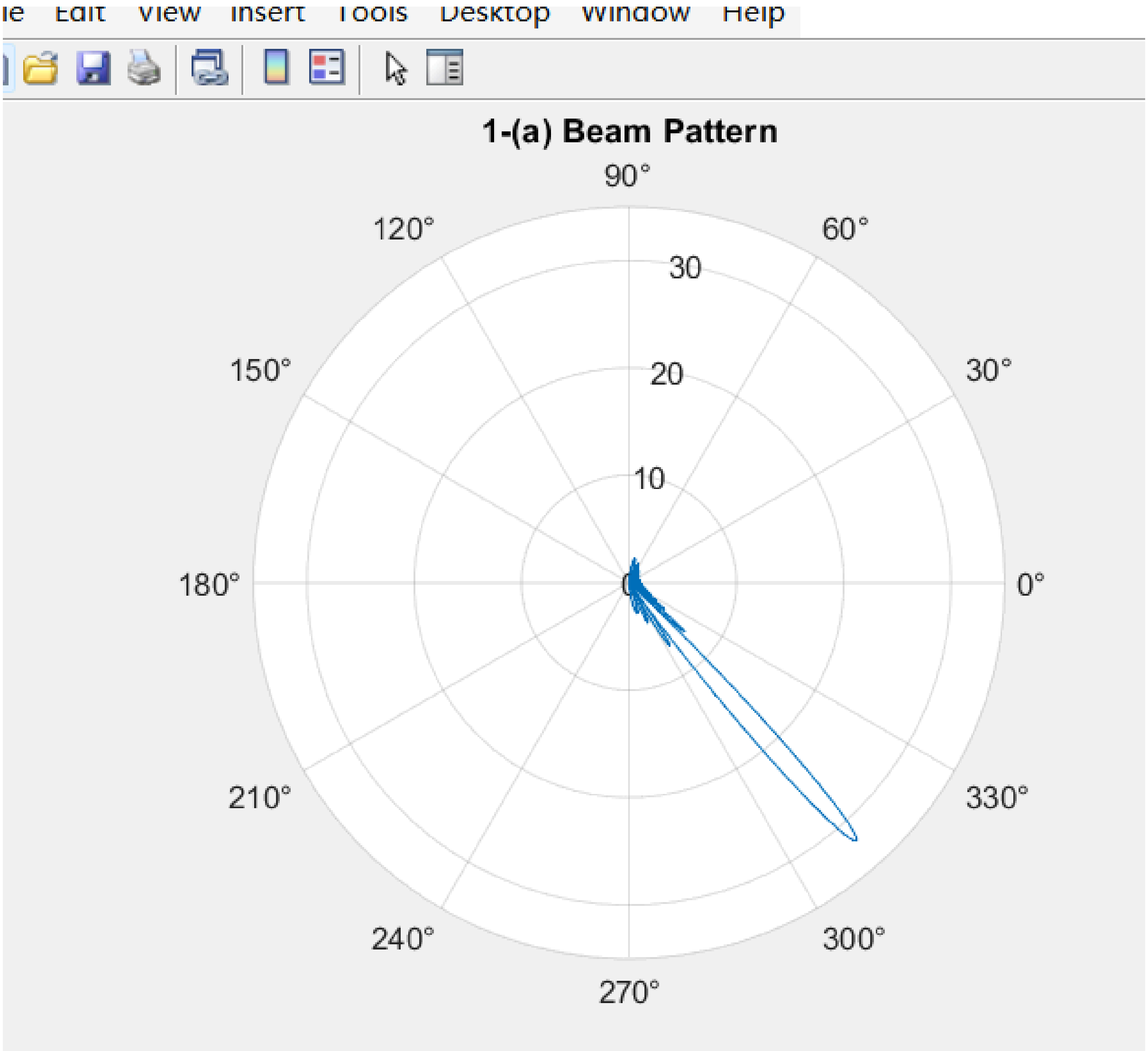
Digital communication IC design　　r13943124 施伯儒
HW4

1.

(a)　for m1 = 4



(b)

we can see the estimated theta based on m is:
(By the way, my theta_r and theta_t are random generated, so you may not get the exact same result when running my code, but they should share the same property)

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>> answer1
theta_r is:
    0.9887

Our theta based on the best result of m
    0.9484
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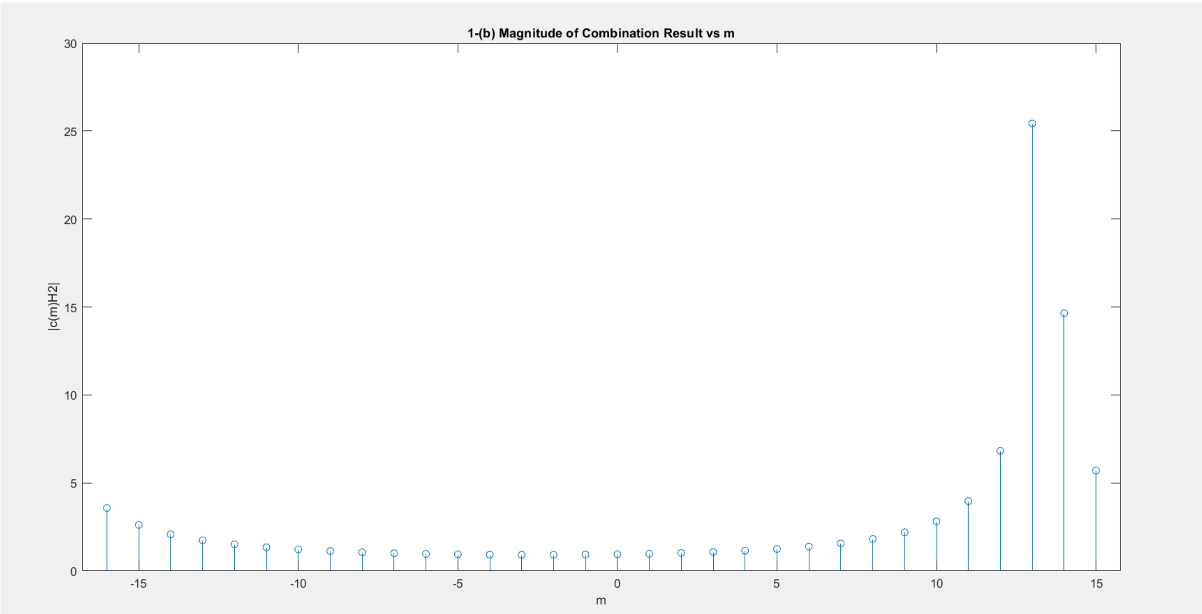
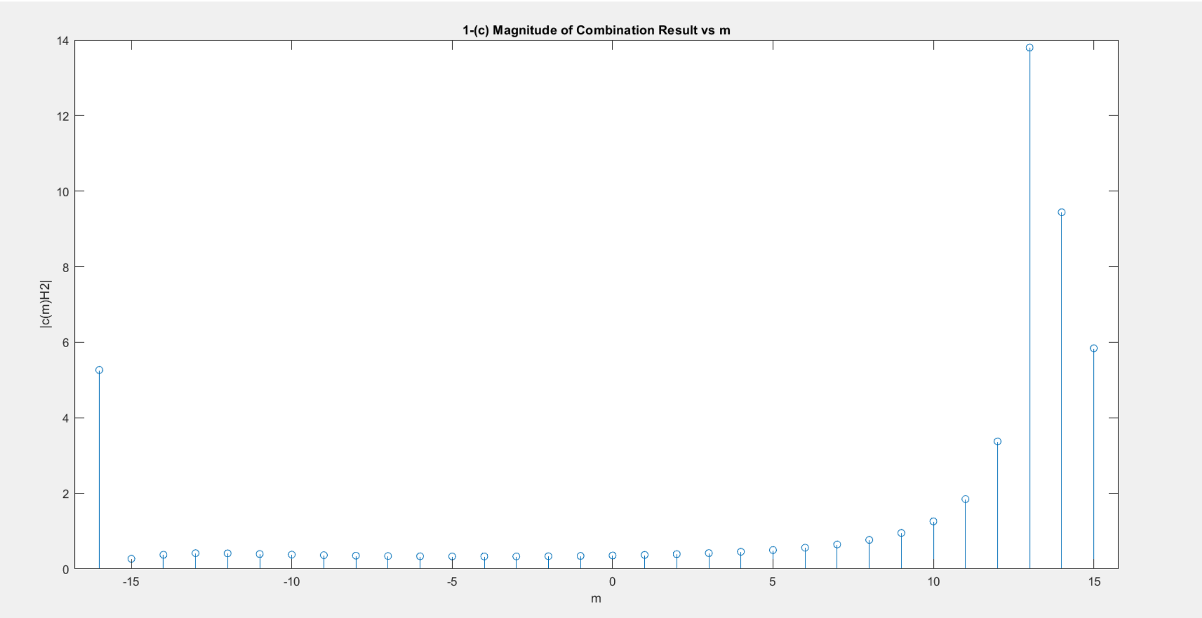


(c)
we can see there is another peak on the left
the best matched theta is still the peak on the left



(d)
If I only have one RF chain, I will pick the m corresponding to the highest peak for the sake of the best gain.

If I have two RF chains, I will pick two m corresponding to the highest and the second highest peak for the sake of the best gain they have.

2.

2. $y_0 = A\cos(2\pi f_c t + \phi_k + \psi)\cos(2\pi f_c t)$

$= A\left[\cos(2\pi f_c t)\cos(\phi_k+\psi) - \sin(2\pi f_c t)\sin(\phi_k+\psi)\right] \cdot \cos(2\pi f_c t)$

$= A\left[\frac{1}{2}\cos(\phi_k+\psi) + \frac{1}{2}\cos(4\pi f_c t)\cos(\phi_k+\psi) - \frac{1}{2}\sin(4\pi f_c t)\sin(\phi_k+\psi)\right]$

$= A\left\{\frac{1}{2}\cos(\phi_k+\psi) + \frac{1}{2}\cos(4\pi f_c t + \phi_k + \psi)\right\}$

$y_0 \rightarrow \boxed{LPF} \rightarrow \frac{1}{2}A\cos(\phi_k+\psi)$

$y_{45} \rightarrow \boxed{LPF} \rightarrow \frac{1}{2}A\cos(\phi_k+\psi-\frac{\pi}{4})$

$y_{90} \rightarrow \boxed{LPF} \rightarrow \frac{1}{2}A\cos(\phi_k+\psi-\frac{\pi}{4})$

$y_{135} \rightarrow \boxed{LPF} \rightarrow \frac{1}{2}A\cos(\phi_k+\psi-\frac{3\pi}{4})$

$$\theta_e = \frac{1}{256} A^4 \left(e^{j(\phi_k+\psi)} + e^{-j(\phi_k+\psi)} \right) \cdot$$

$$\left(e^{j(\phi_k+\psi-\frac{\pi}{4})} + e^{-j(\phi_k+\psi-\frac{\pi}{4})} \right) \cdot$$

$$\left(e^{j(\phi_k+\psi-\frac{2\pi}{4})} + e^{-j(\phi_k+\psi-\frac{2\pi}{4})} \right) \cdot$$

$$\left(e^{j(\phi_k+\psi-\frac{3\pi}{4})} + e^{-j(\phi_k+\psi-\frac{3\pi}{4})} \right) \cdot$$

$$= \frac{1}{256} A^4 \left(e^{j(2\phi_k+2\psi-\frac{1}{2}\pi)} + e^{+j\frac{\pi}{2}} + e^{-j\frac{\pi}{2}} + e^{-j(2\phi_k+2\psi-\frac{1}{2}\pi)} \right) \cdot$$

$$\left(e^{j(2\phi_k+2\psi-\pi)} + e^{j\frac{\pi}{2}} + e^{-j\frac{\pi}{2}} + e^{-j(2\phi_k+2\psi-\pi)} \right)$$

$$= \frac{1}{64} A^4 \left(\cos\left(2\phi_k + 2\psi - \frac{1}{2}\pi\right) + \cancelto{0}{\cos\left(\frac{\pi}{2}\right)} \right) \cdot$$

$$\left(\cos\left(2\phi_k + 2\psi - \pi\right) + \cancelto{0}{\cos\left(\frac{\pi}{2}\right)} \right)$$

$$= \frac{1}{64} A^4 \cdot \cos(2\phi_k + 2\psi) \sin(2\phi_k + 2\psi) \cdot (-1)$$

$$= \frac{1}{128} A^4 \sin(4\phi_k + 4\psi) \cdot (-1)$$

$$= \frac{1}{128} A^4 \sin(4\phi_k + 4\psi) \cdot (-1)$$

Since $4\phi_k = \pi$ or 3π or $-\pi$ or -3π

$$\sin(4\phi_k + 4\psi) = \sin(4\psi + \pi)$$

$$= (-) \sin(4\psi)$$

$$\Rightarrow \underline{\theta_e = \frac{1}{128} A^4 \sin(4\psi)}$$

#.

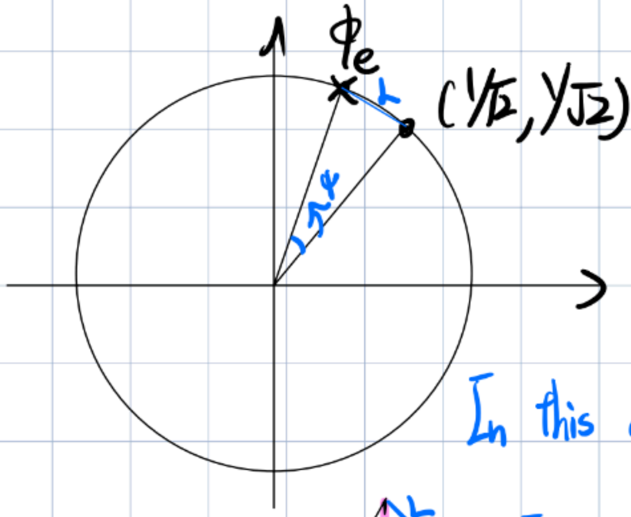
3.

(a)

3. (a) $s_n^* \hat{d}_n = (s_{I,n} - j s_{Q,n})(\hat{d}_{I,n} + j \hat{d}_{Q,n})$

$= (s_{I,n} \hat{d}_{I,n} + s_{Q,n} \hat{d}_{Q,n}) + j (s_{I,n} \hat{d}_{Q,n} - s_{Q,n} \hat{d}_{I,n})$

$\Rightarrow \theta_e = \mathrm{Im}\{ s_n^* \hat{d}_n \}$

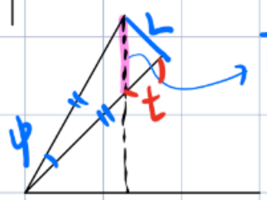


when ψ is small

$\psi \approx L$

$\theta_e = \mathrm{Im}\{ s_n^* \hat{d}_n \}$

In this case, θ_e means:



This part is $|\theta_e|$

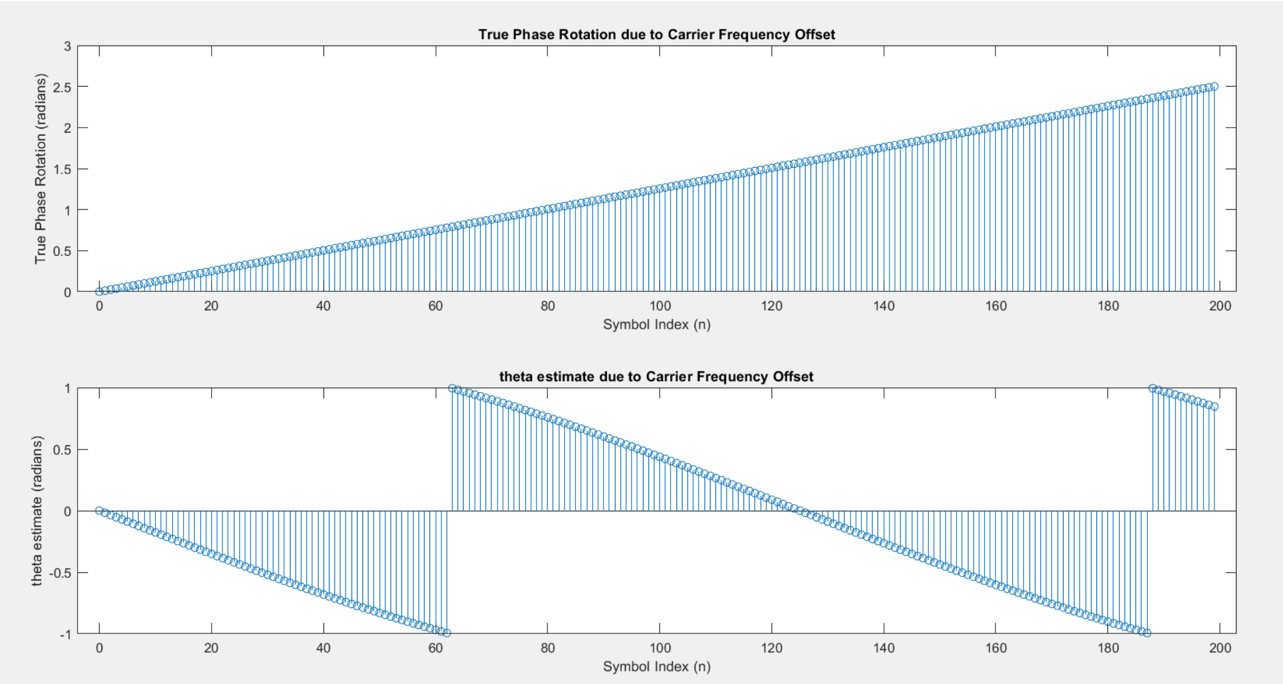
When ψ is small

We can find that $|\theta_e| \approx L \approx \psi$

because t will be very small

$\Rightarrow$ Can use θ_e to Approximate

(b)



(c)

when n is small, we can find out abs(theta) and the true phase rotation are very closed.

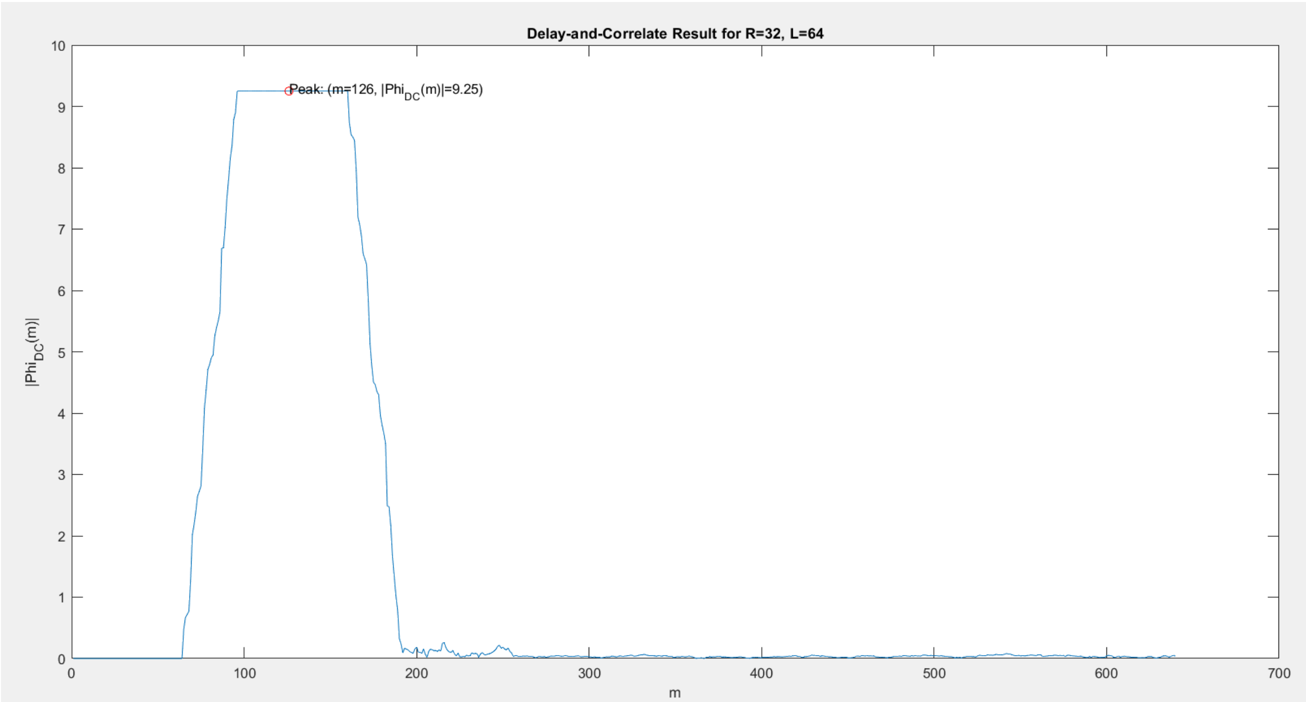
But when the estimated theta(radian) gradually reaches -1, phase ambiguity will happen, it wrongly decides the result to be 1.

4.

(a)(b)

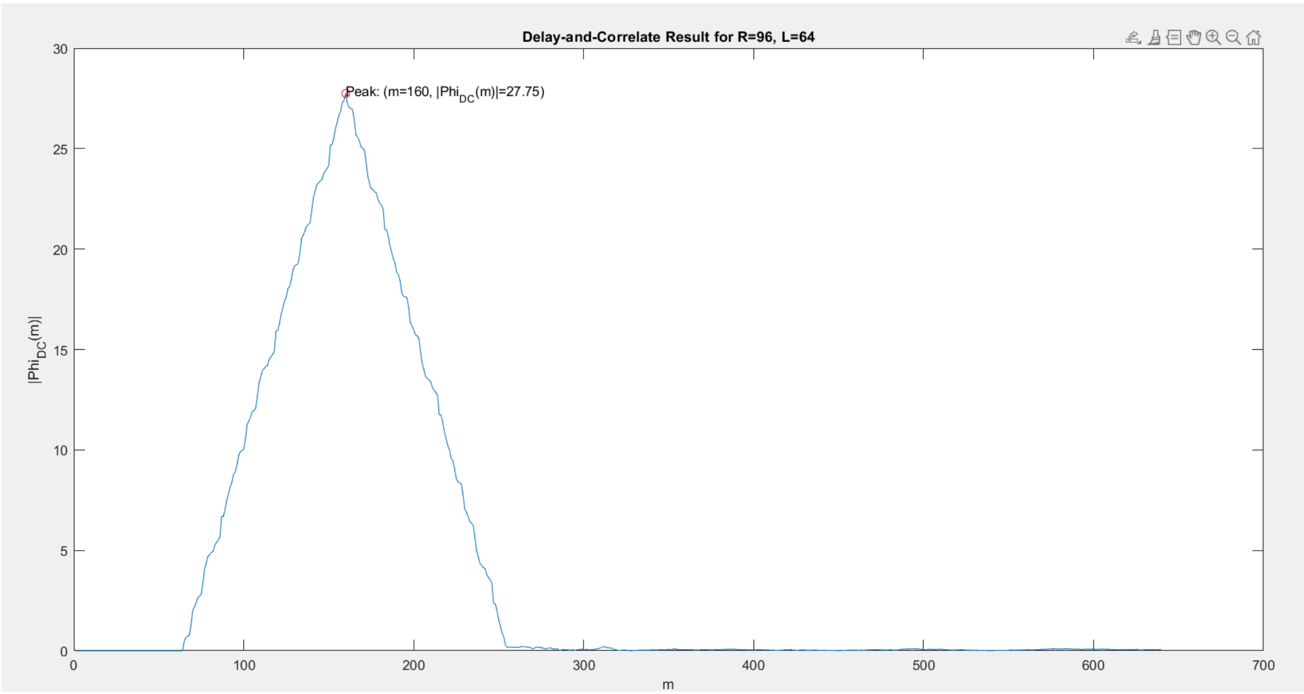
有好幾個值都可以讓peak最大，如下圖:

m 最小從96開始



(c)(d)

很明顯m=160時有唯一的peak



(e)

d is better, because it has a higher peak and shorter band, so the range of m is narrowed down.