Searching and Sorting

Part One

Outline for Today

- ***Gauss's Sum***
  - A famous, ubiquitous sum.
- ***Sorting Algorithms***
  - How quickly can we get things in order?
- ***Inventing an Algorithm***
  - Building better sorts with big-O.

Recap from Last Time

```
double averageOf(const Vector<int>& vec) {
  double total = 0.0;

  for (int i = 0; i < vec.size(); i++) {
      total += vec[i];
  }

  return total / vec.size();
}
```

Assume any individual statement takes one unit of time to execute. If the input `Vector` has *n* elements, how many time units will this code take to run?

```
double averageOf(const Vector<int>& vec) {
1 double total = 0.0;

          1               n+1       n
    for (int i = 0; i < vec.size(); i++) {
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One possible answer: $3n + 4$.

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  }         n


  return total / vec.size(); 1
}
```

~~One possible answer: $3n + 4$.~~

More useful answer: **O(*n*)**.

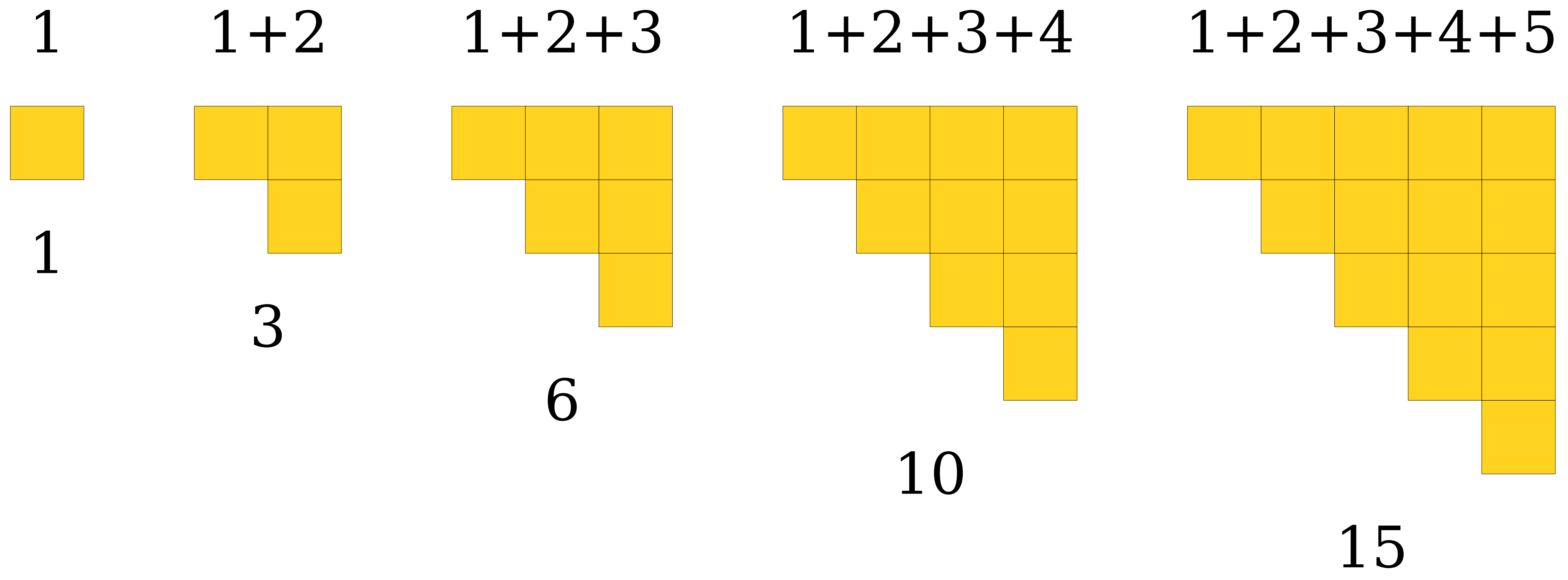
```
void printStars(int n) {
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++) {
            cout << '*' << endl;
        }
    }
}
```

Work Done: **$O(n^2)$**.

New Stuff!

Gauss’s Sum

Gauss's Sum



How does the sum $1 + 2 + 3 + \ldots + n$ scale as n increases?

n	$1+2+\ldots+n$
10	55
20	210
30	465
40	820
50	1275
60	1830
70	2485
80	3240
90	4095
100	5050

Which best describes the rate at which the quantity $1 + 2 + \ldots + n$ grows as a function of n?

A. $O(n)$
B. $O(n^2)$
C. $O(n^3)$

Answer at
https://pollev.com/cs106bwin23

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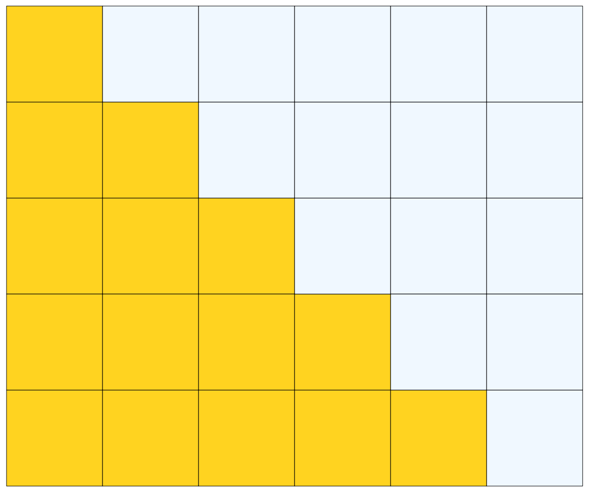
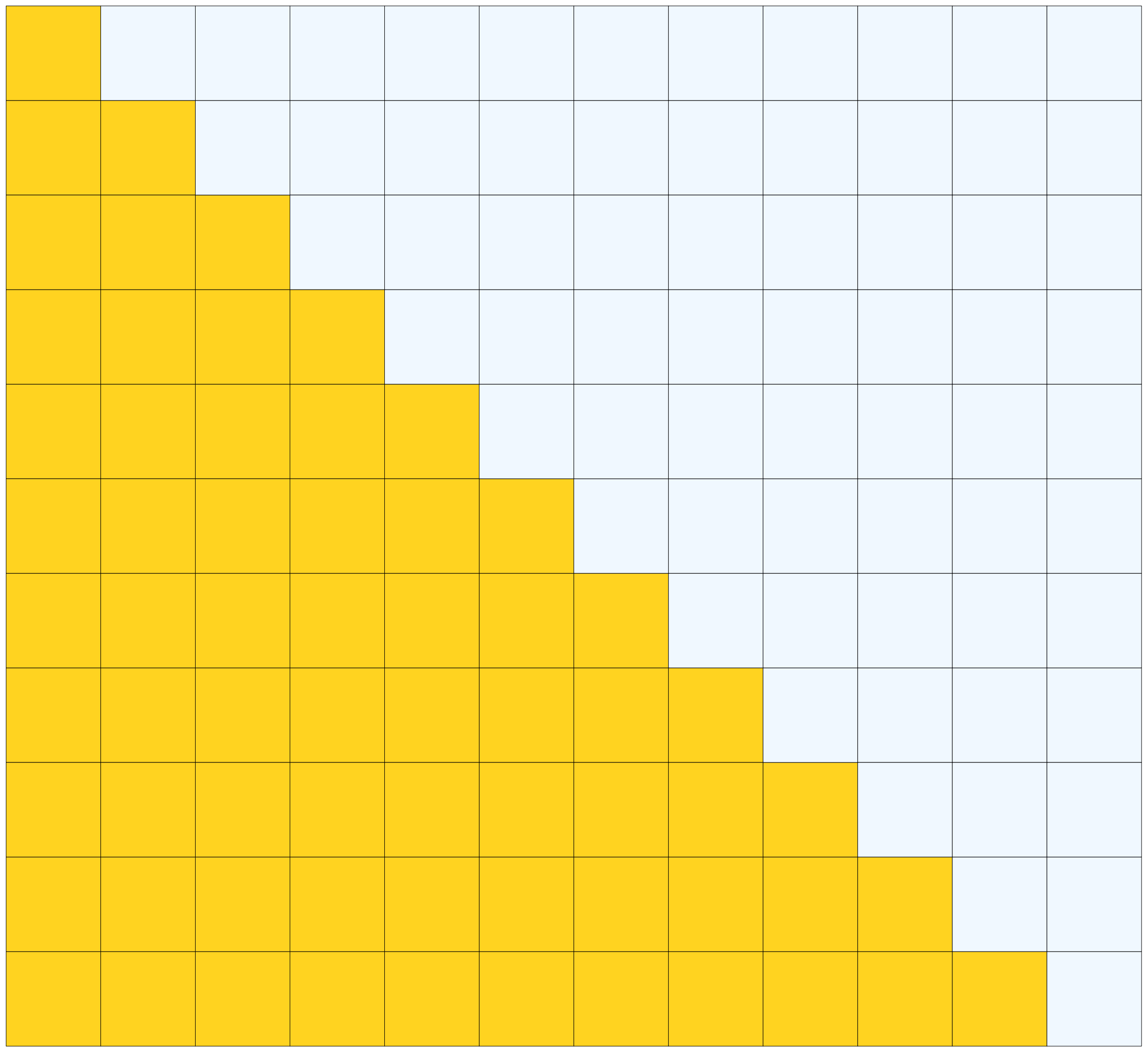
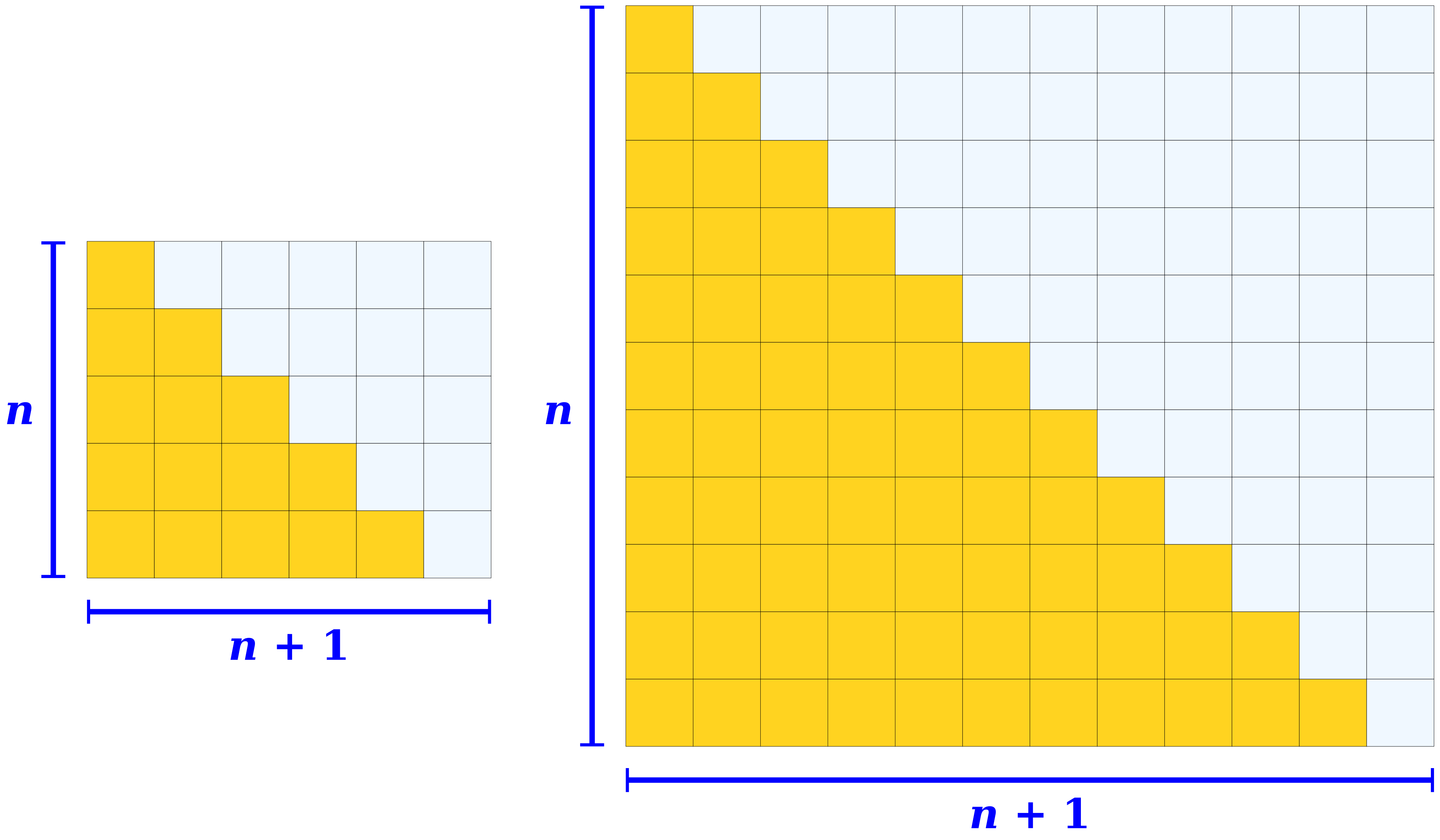
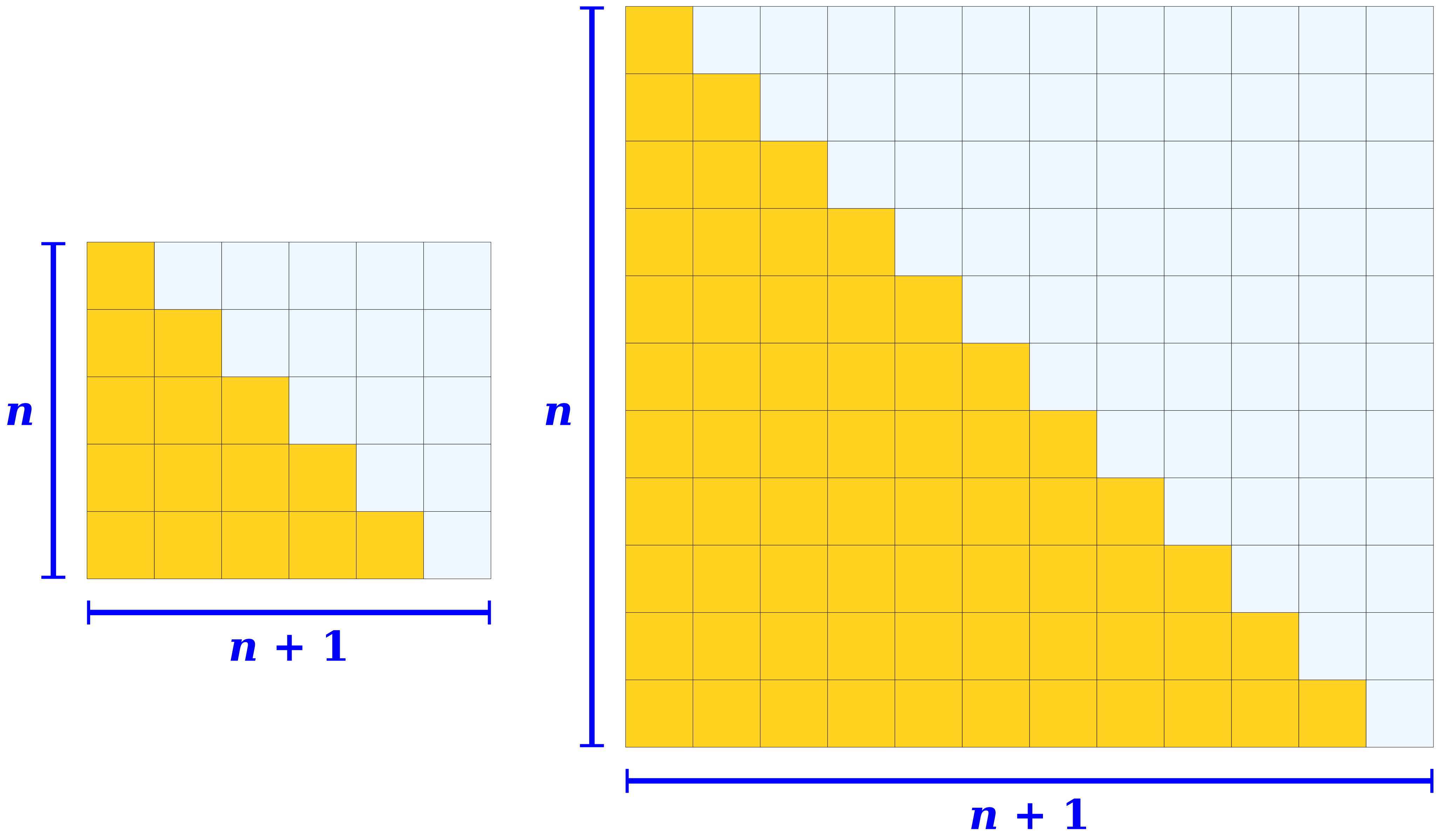




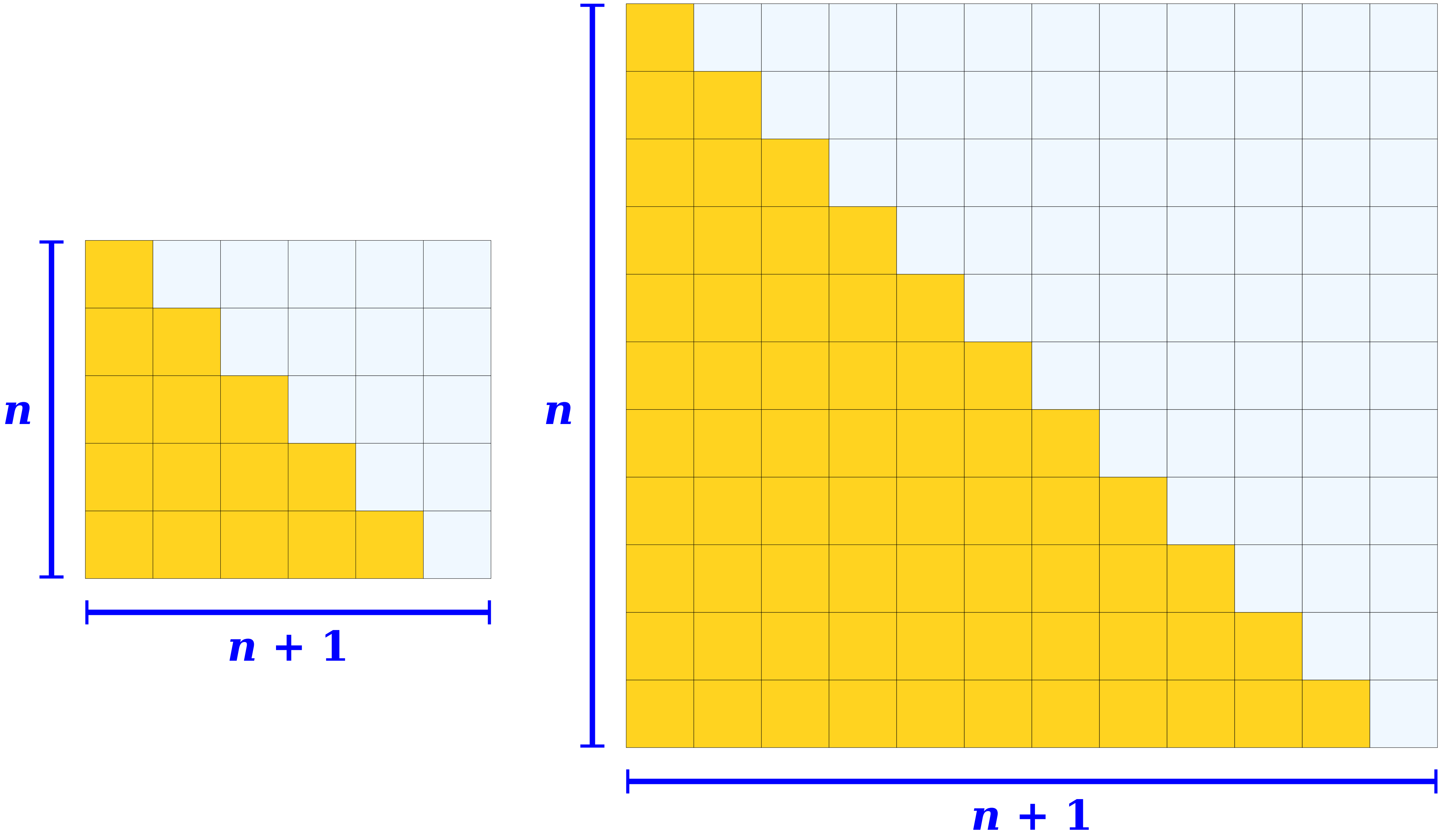
How does the sum 1 + 2 + 3 + … + *n* scale as *n* increases?



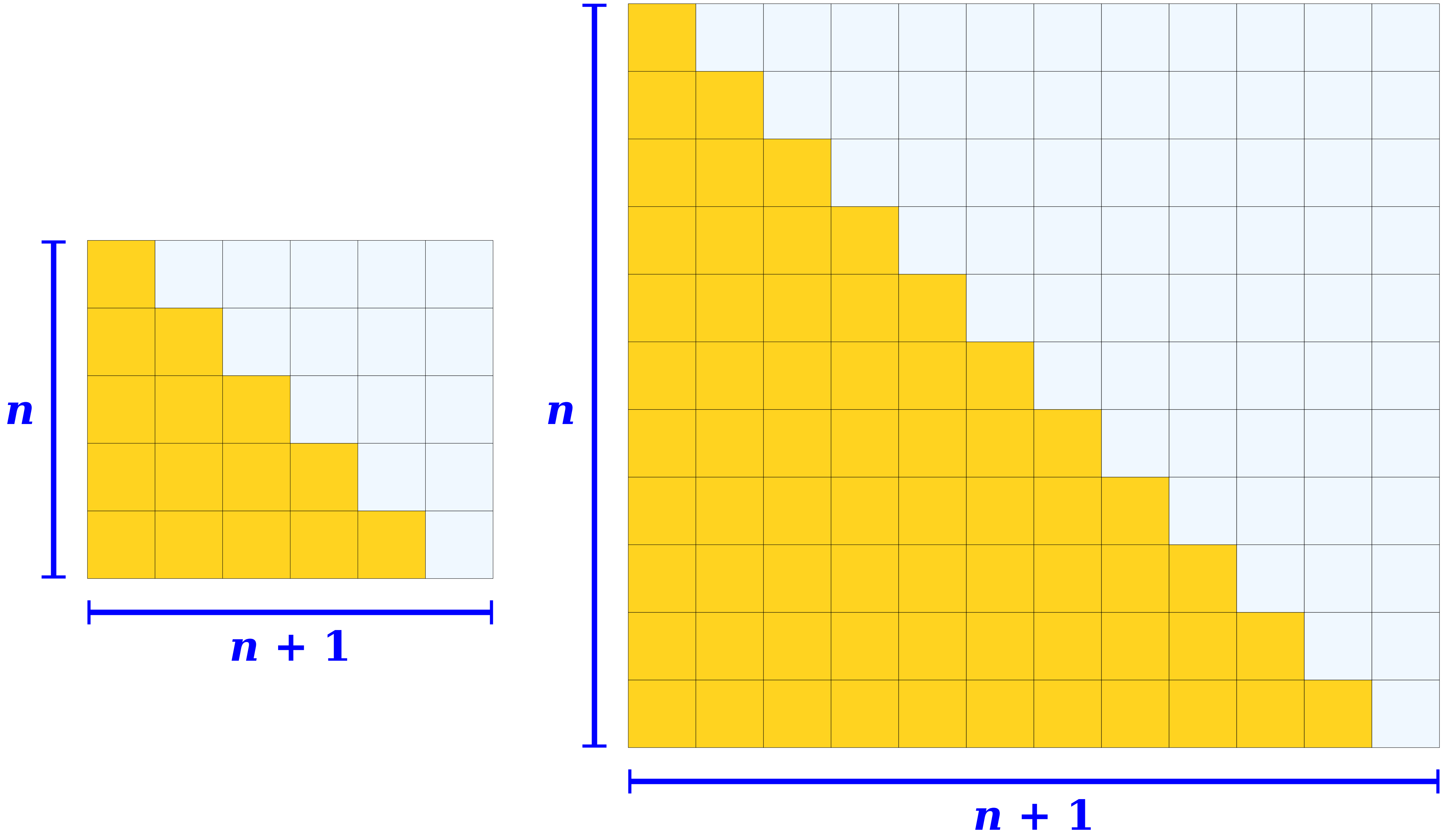
Each figure has area $n(n + 1) = n^2 + n$.



Half that area is the gold figure, which is $1 + 2 + 3 + \ldots + n$.



So $1 + 2 + 3 + \ldots + n = n^2 / 2 + n / 2$.
But big-O ignores leading coefficients
and low order-terms.

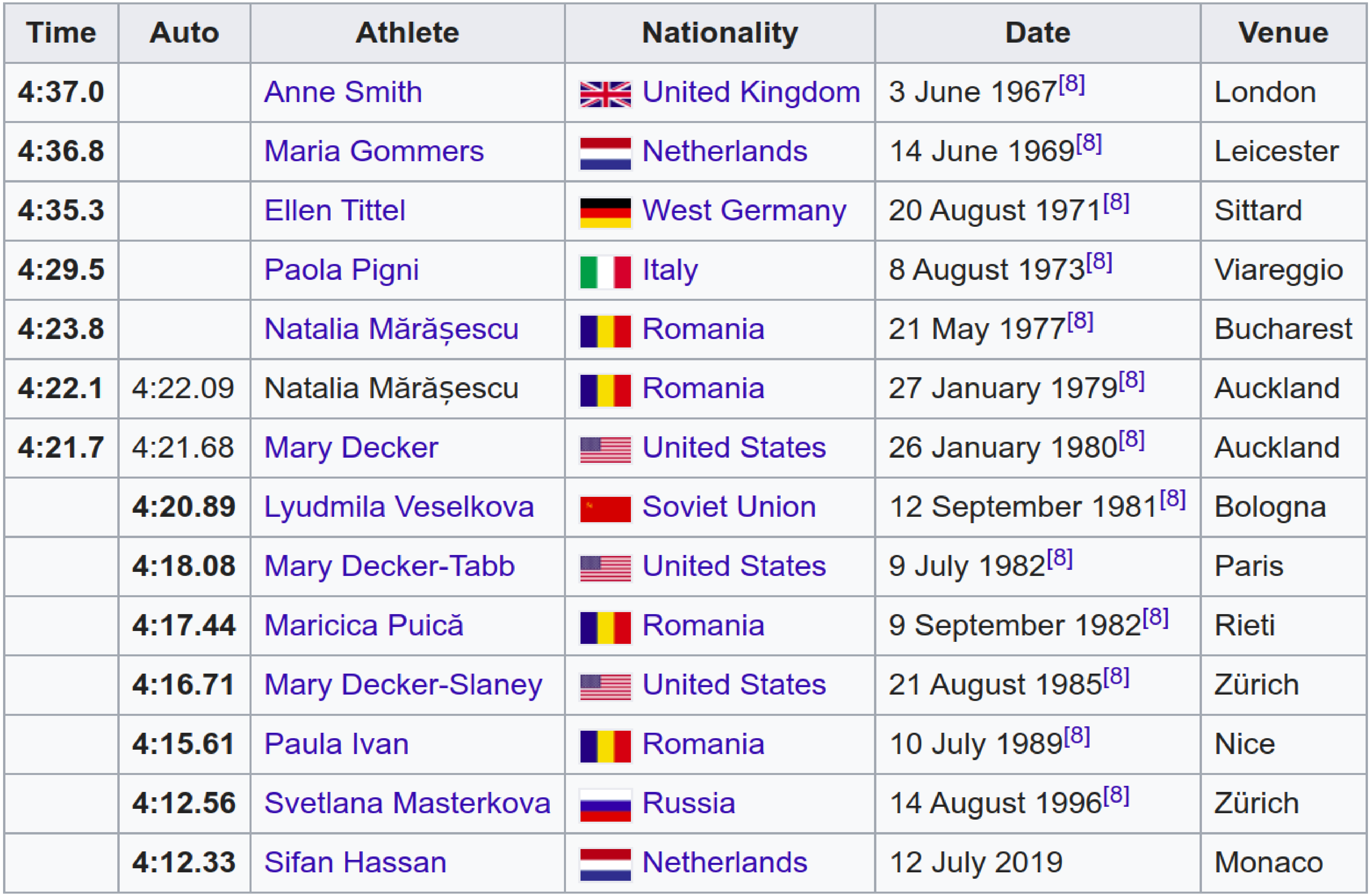


So $1 + 2 + 3 + \dots + n = \mathbf{O}(\boldsymbol{n}^2)$.

Sorting Algorithms

What is sorting?

One style of “sorting,” but not the one we’re thinking about…



Time	Auto	Athlete	Nationality	Date	Venue
4:37.0		Anne Smith	United Kingdom	3 June 1967[8]	London
4:36.8		Maria Gommers	Netherlands	14 June 1969[8]	Leicester
4:35.3		Ellen Tittel	West Germany	20 August 1971[8]	Sittard
4:29.5		Paola Pigni	Italy	8 August 1973[8]	Viareggio
4:23.8		Natalia Mărășescu	Romania	21 May 1977[8]	Bucharest
4:22.1	4:22.09	Natalia Mărășescu	Romania	27 January 1979[8]	Auckland
4:21.7	4:21.68	Mary Decker	United States	26 January 1980[8]	Auckland
	4:20.89	Lyudmila Veselkova	Soviet Union	12 September 1981[8]	Bologna
	4:18.08	Mary Decker-Tabb	United States	9 July 1982[8]	Paris
	4:17.44	Maricica Puică	Romania	9 September 1982[8]	Rieti
	4:16.71	Mary Decker-Slaney	United States	21 August 1985[8]	Zürich
	4:15.61	Paula Ivan	Romania	10 July 1989[8]	Nice
	4:12.56	Svetlana Masterkova	Russia	14 August 1996[8]	Zürich
	4:12.33	Sifan Hassan	Netherlands	12 July 2019	Monaco

Problem: Given a list of data points, sort those data points into ascending / descending order by some quantity.

Suppose we want to rearrange a sequence to put elements into ascending order.

What are some strategies we could use?

How do those strategies compare?

Is there a “best” strategy?

An Initial Idea: ***Selection Sort***

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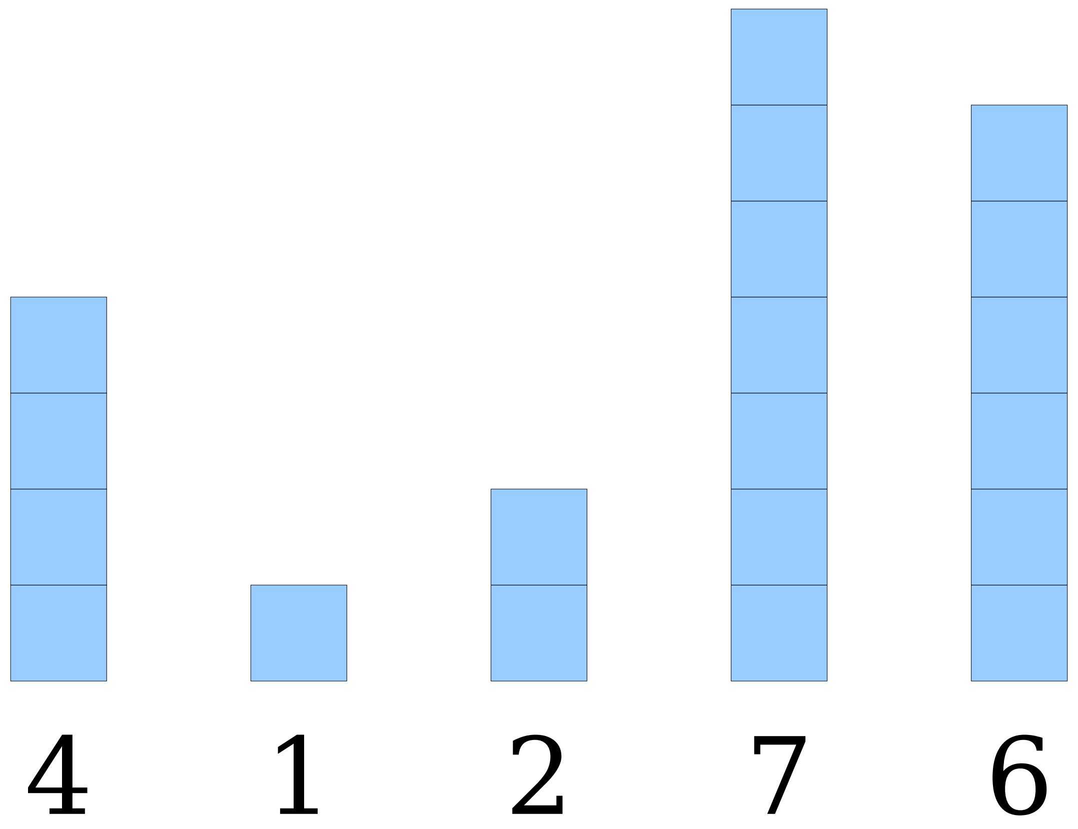

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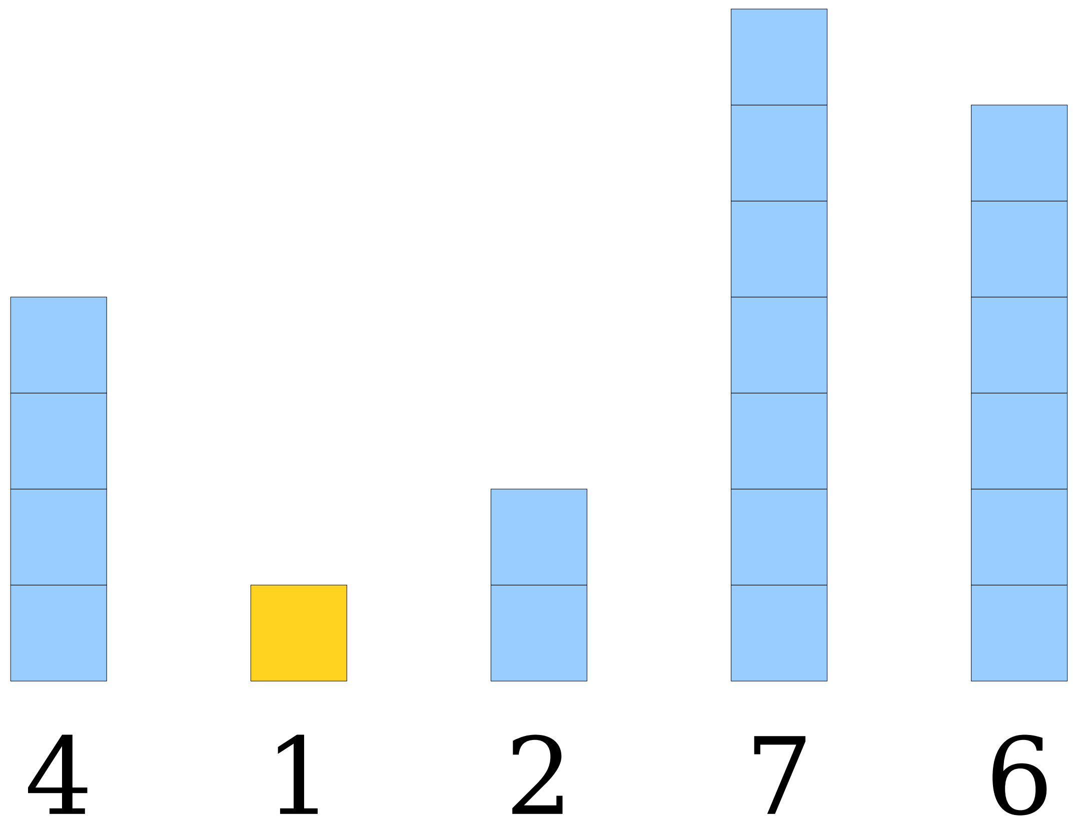
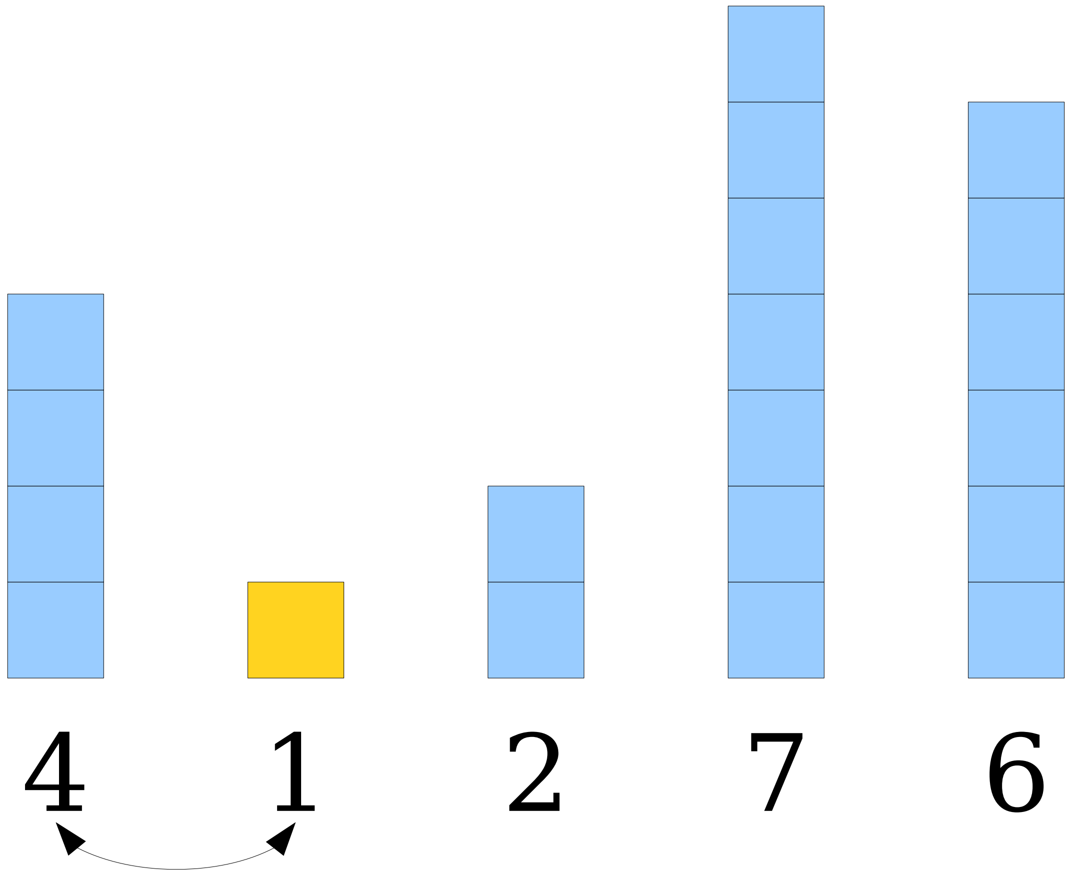


The smallest element should go in front.

An Initial Idea: ***Selection Sort***



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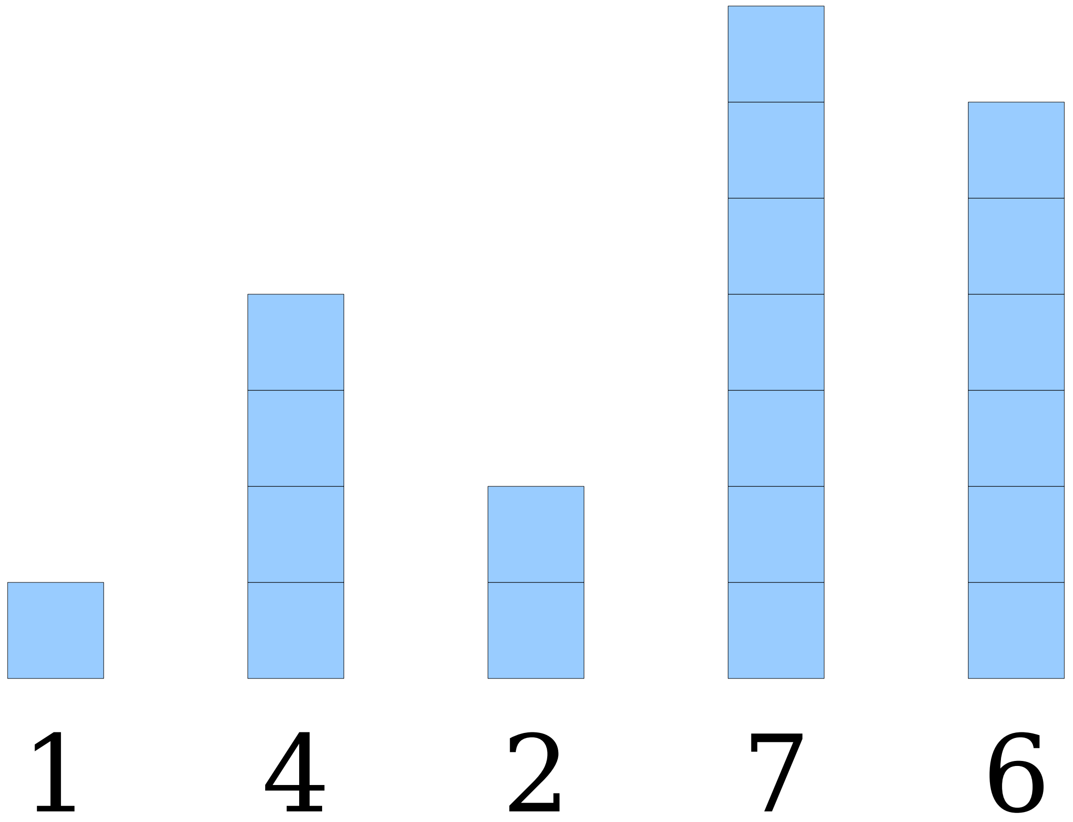

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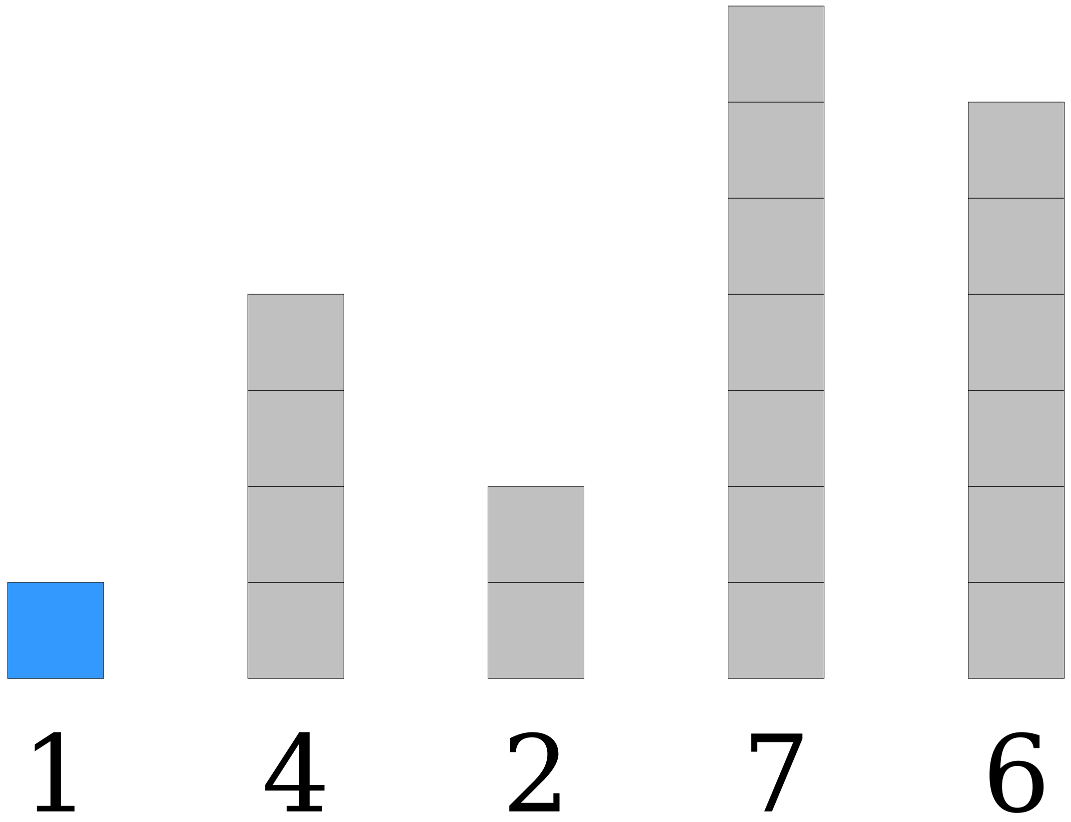
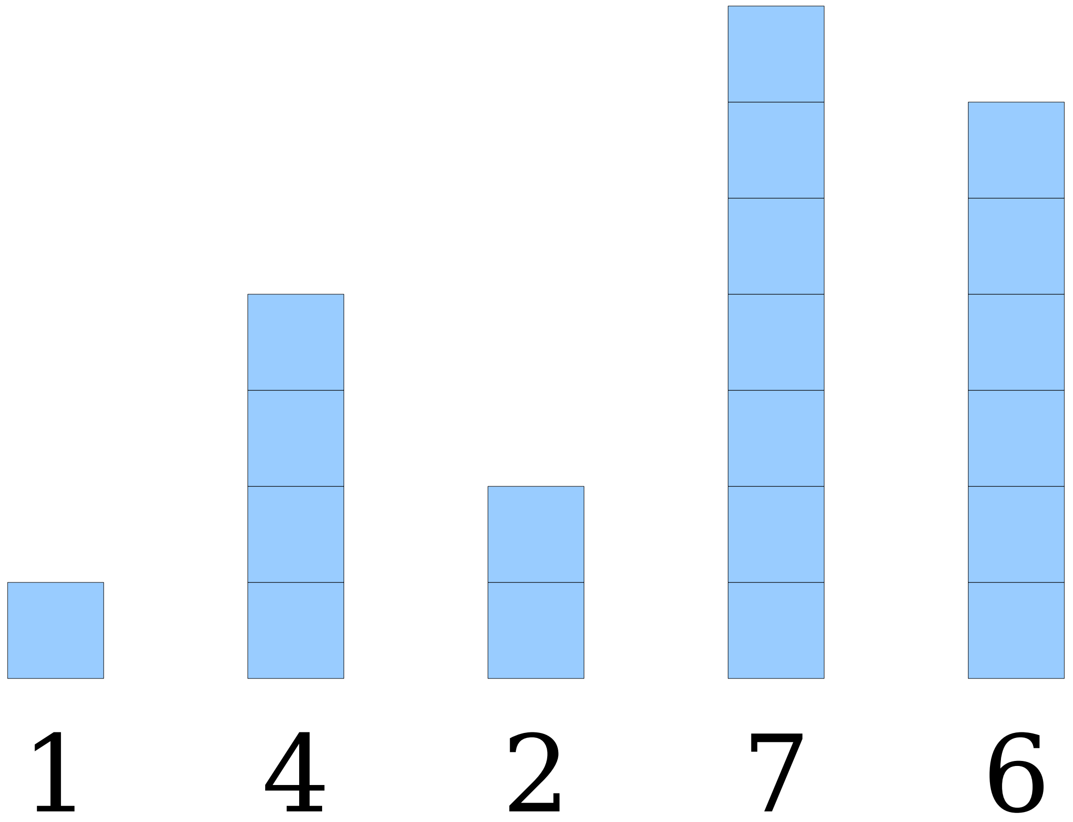


This element is in the right place now.

The remaining elements are in no particular order.

An Initial Idea: ***Selection Sort***



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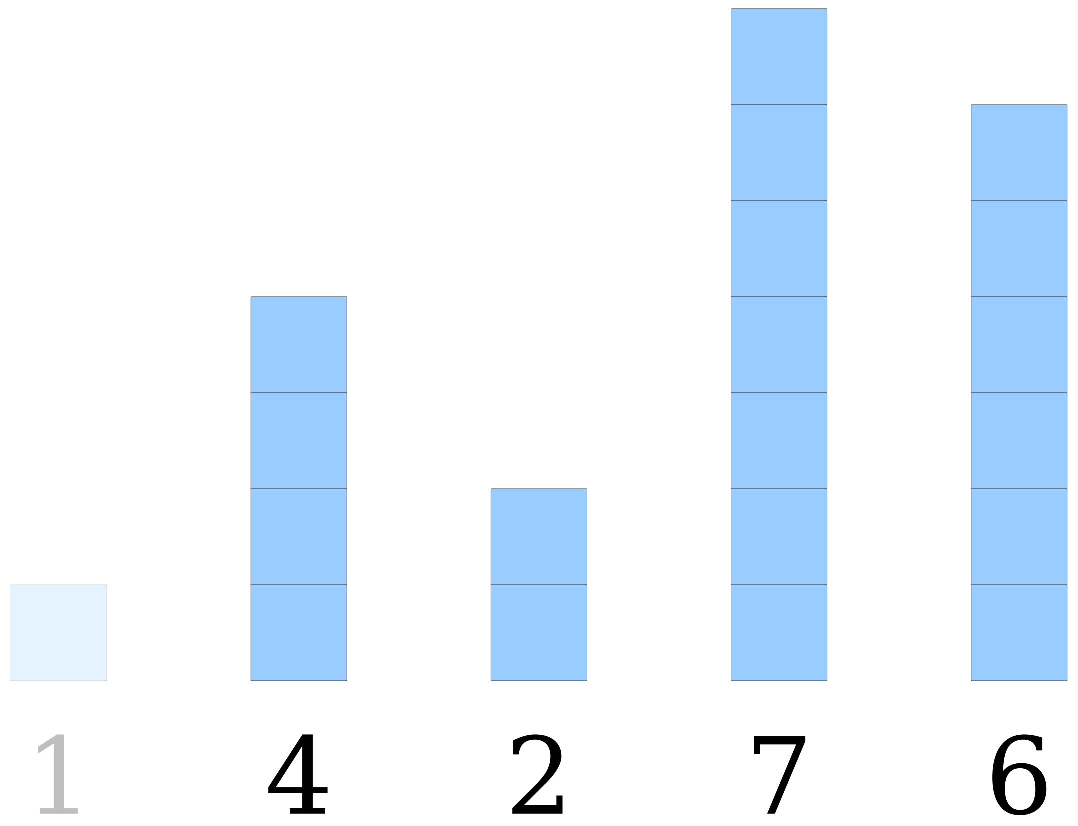

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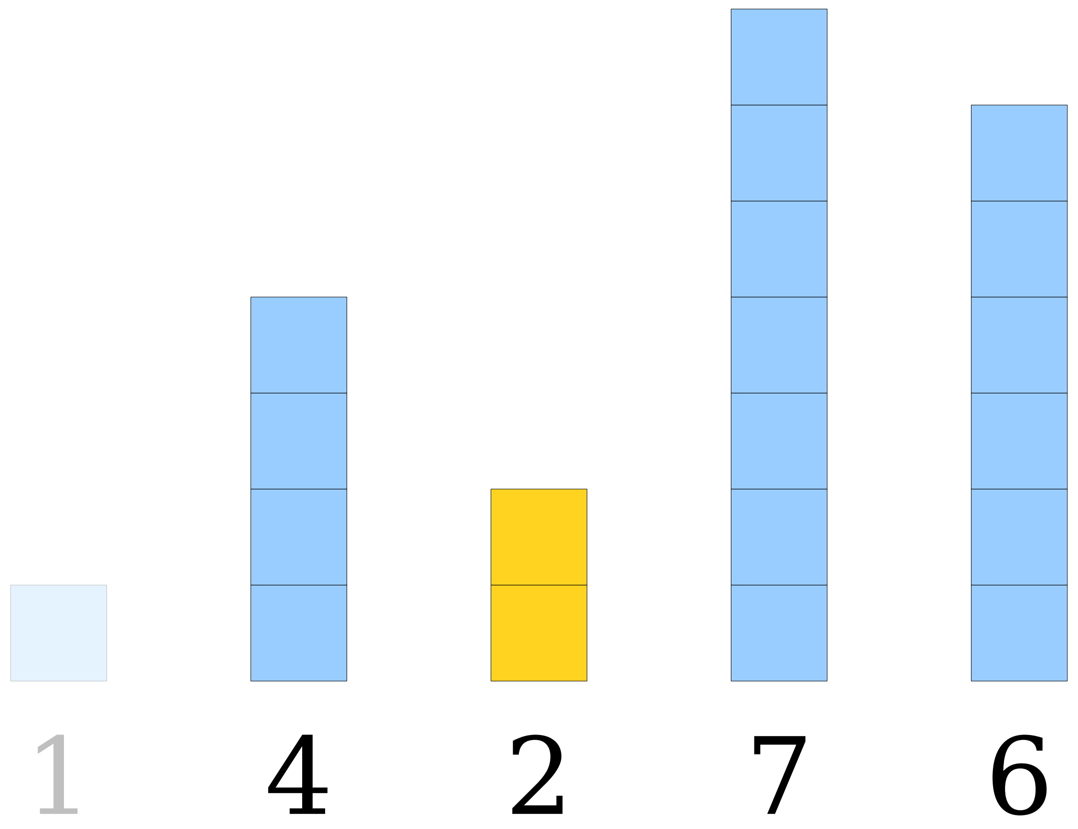
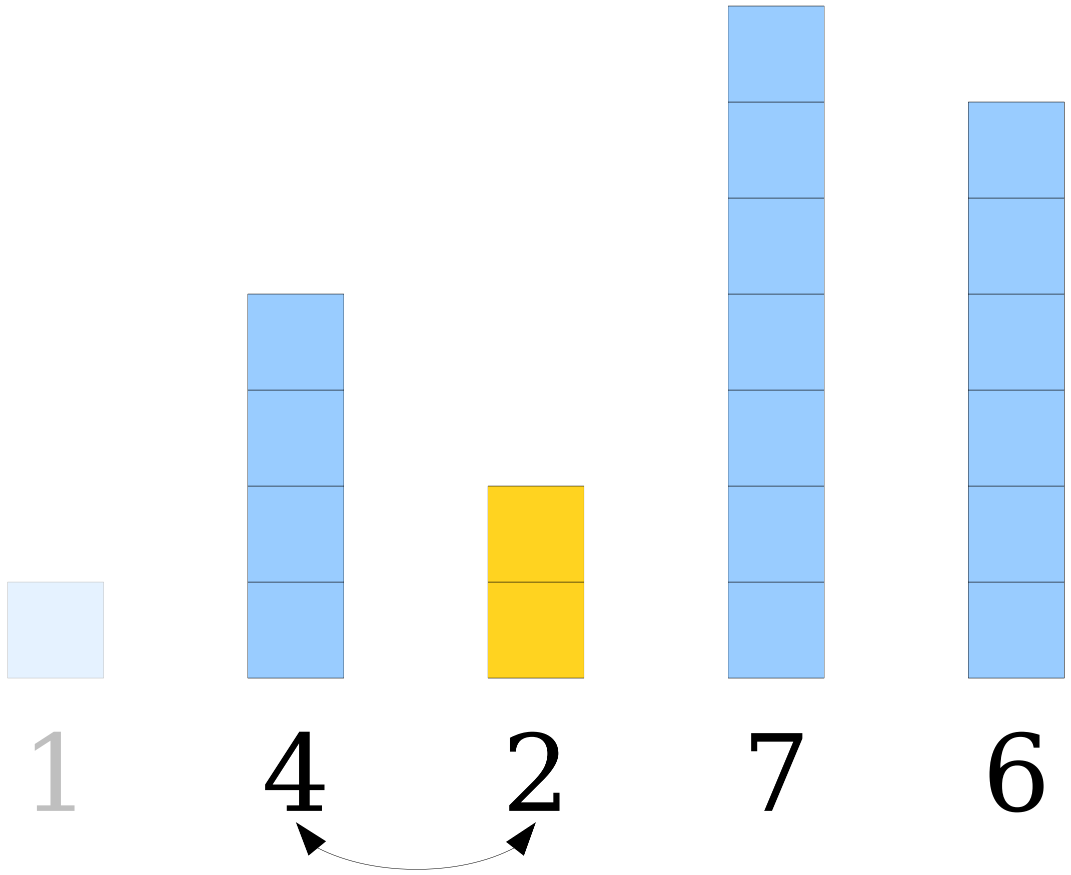


The smallest element of the remaining elements goes at the front of the remaining elements.

An Initial Idea: ***Selection Sort***



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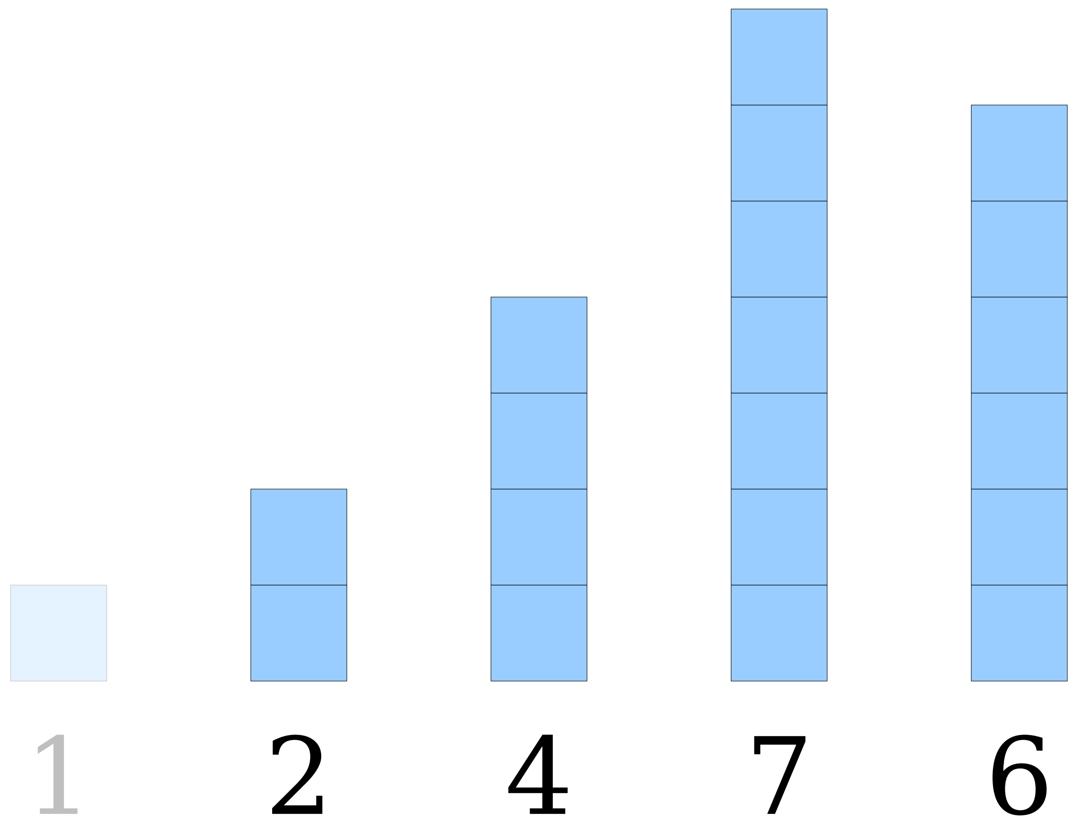

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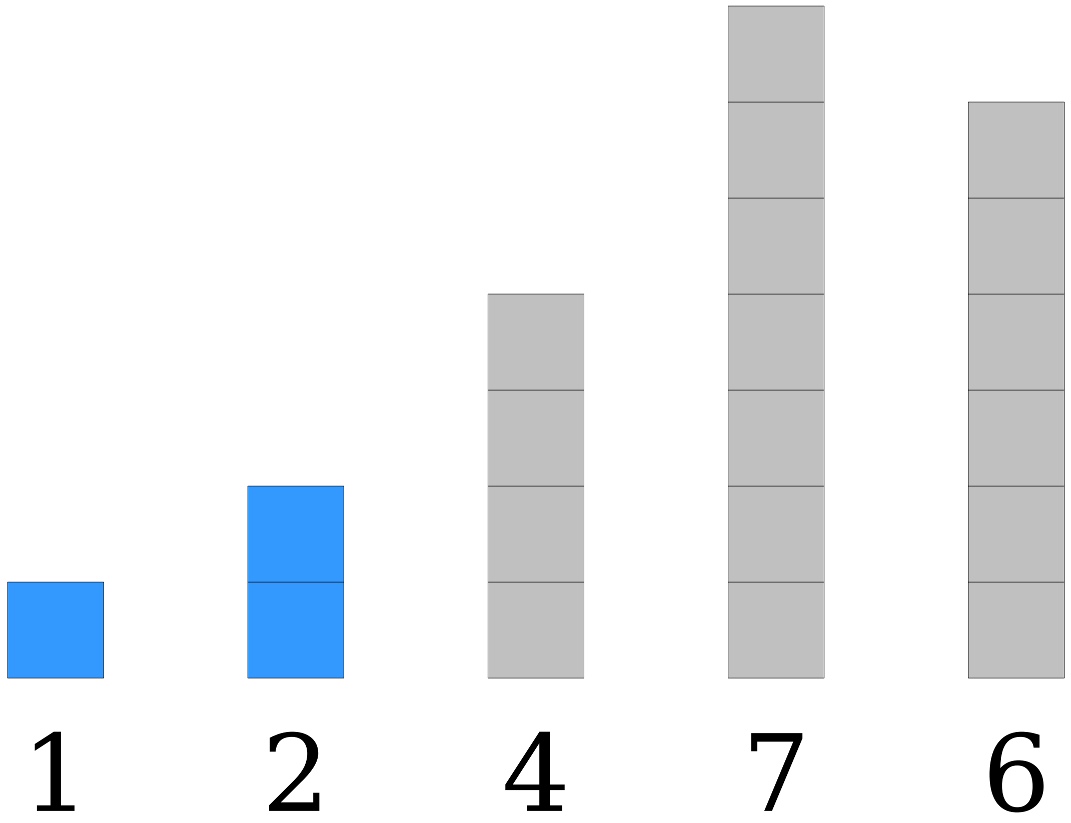

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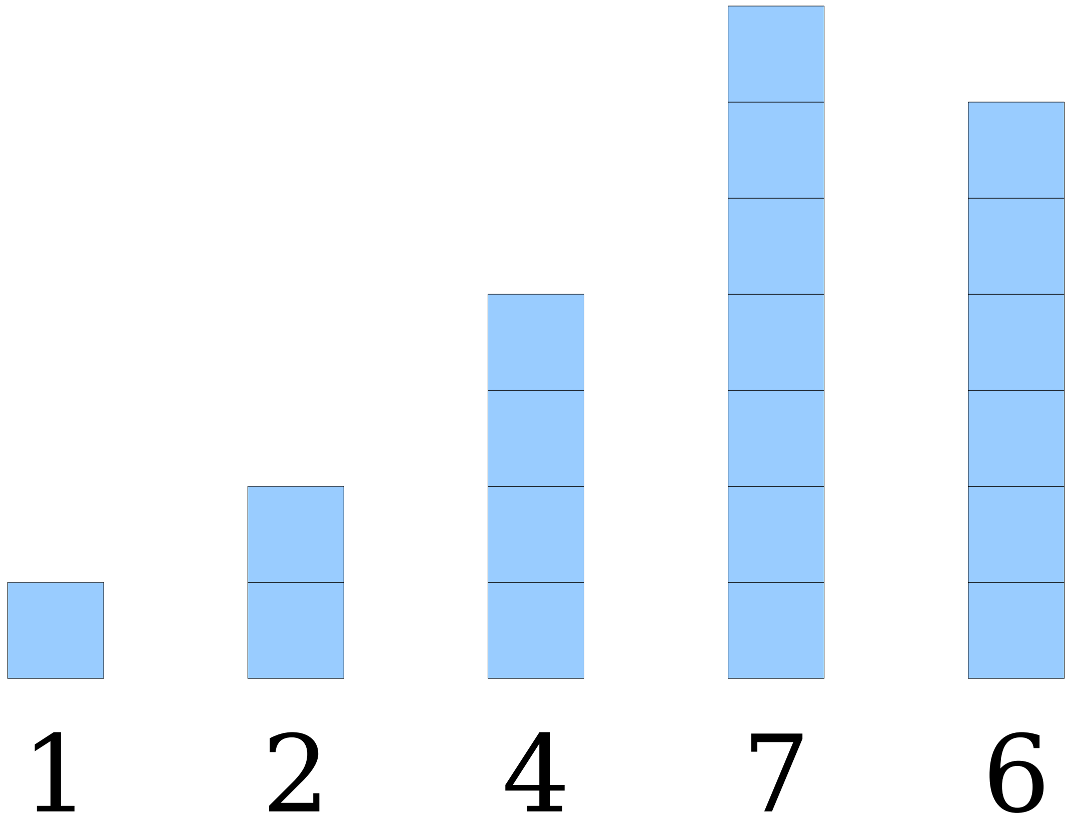

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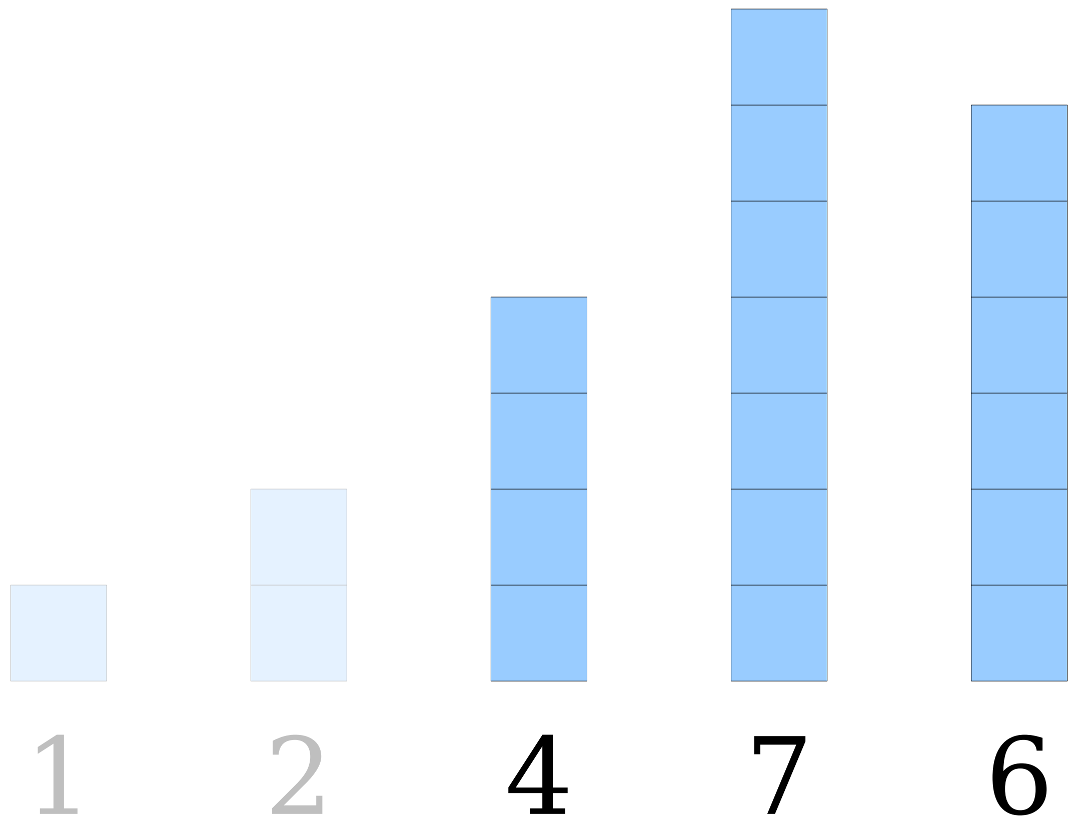

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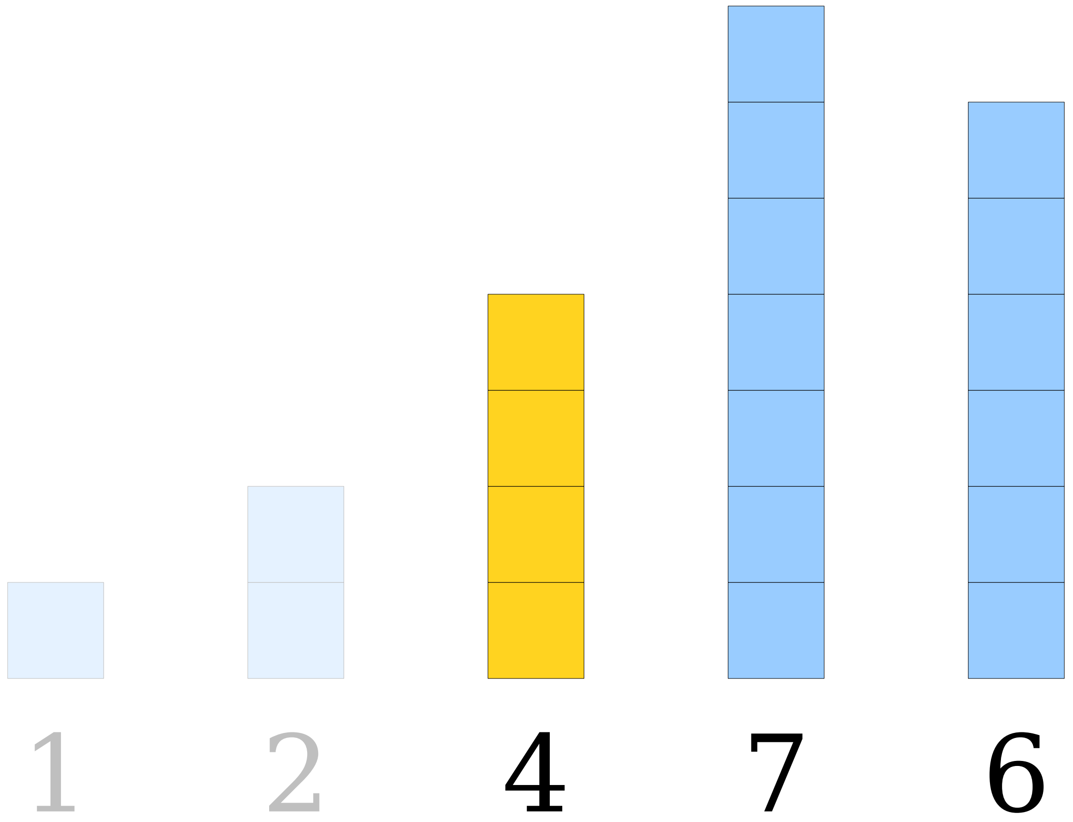

The smallest of these remaining elements goes at the front of the remaining elements.

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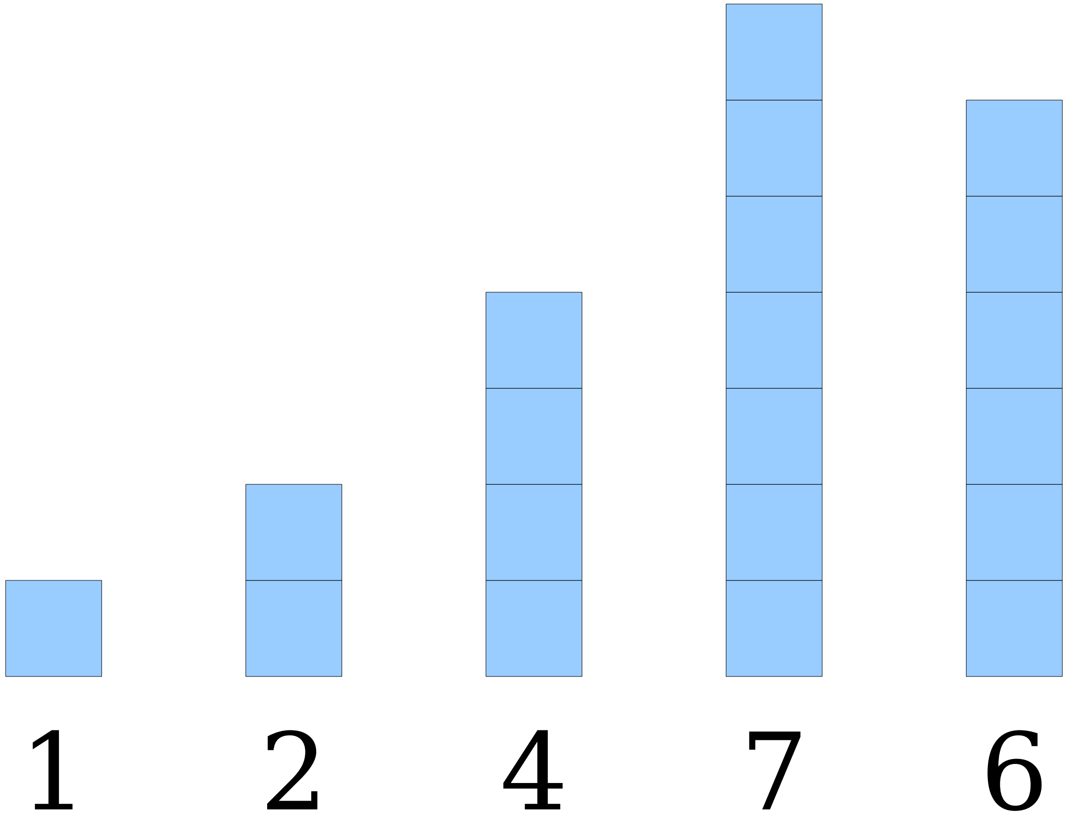

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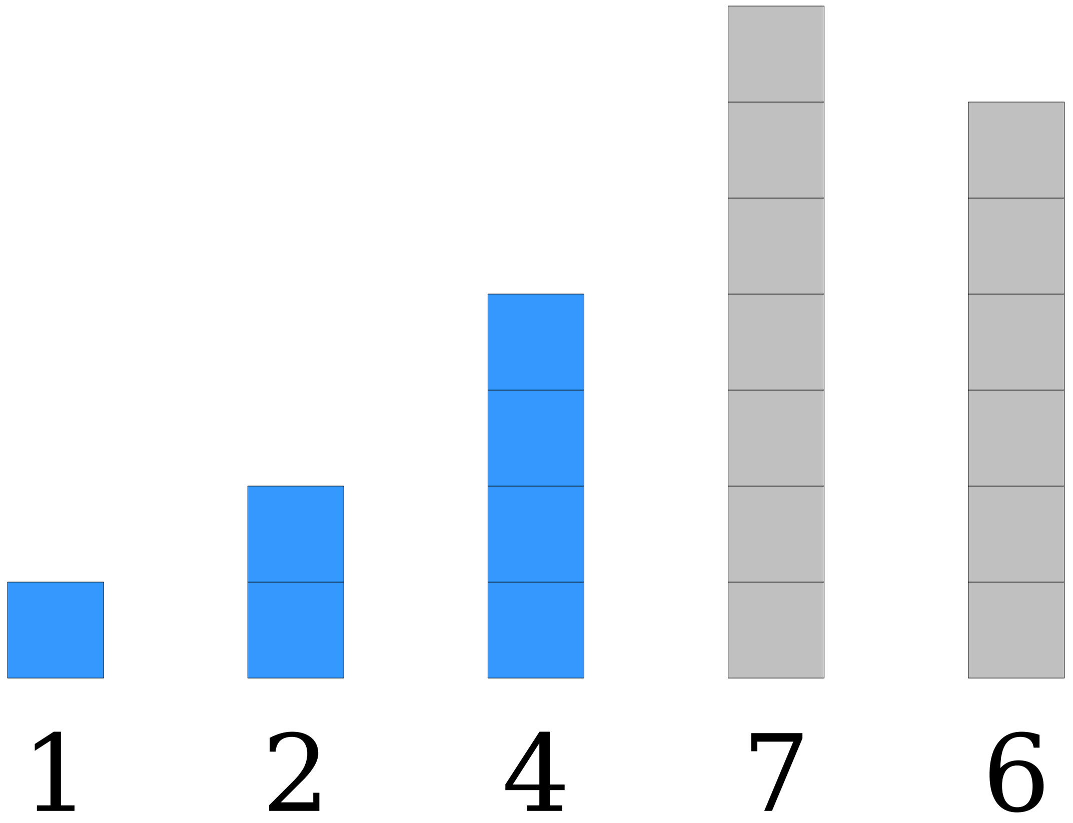

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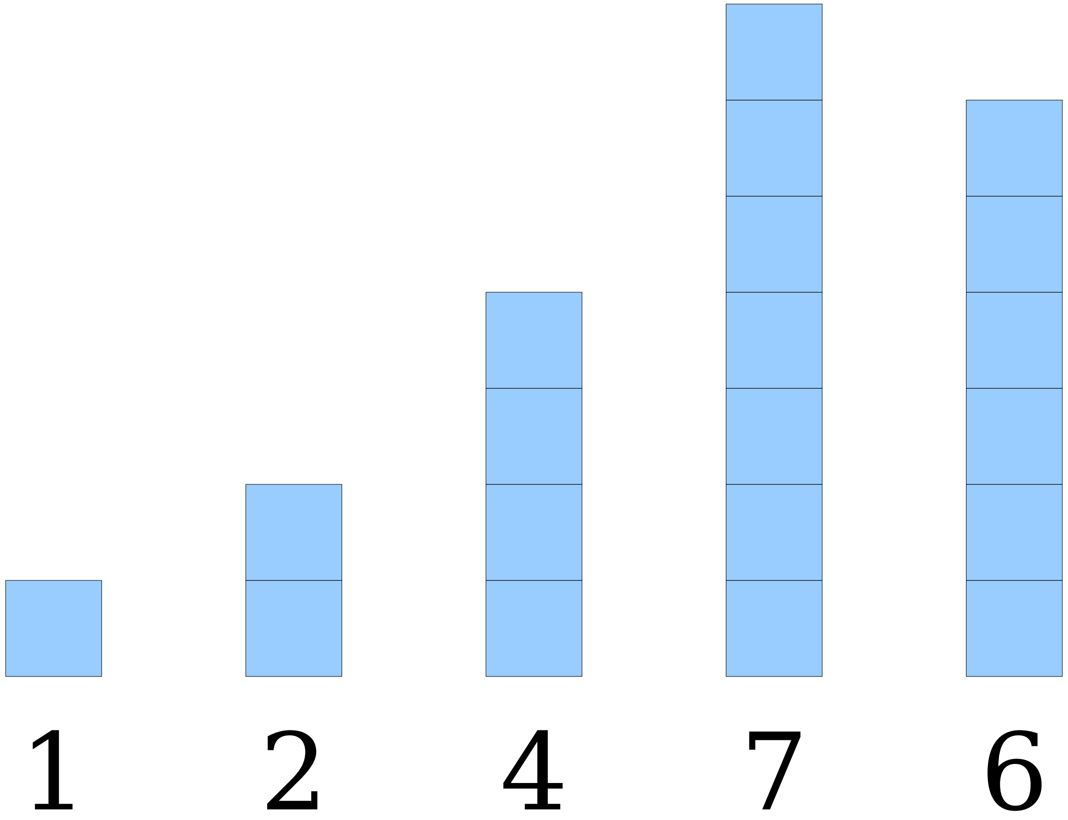

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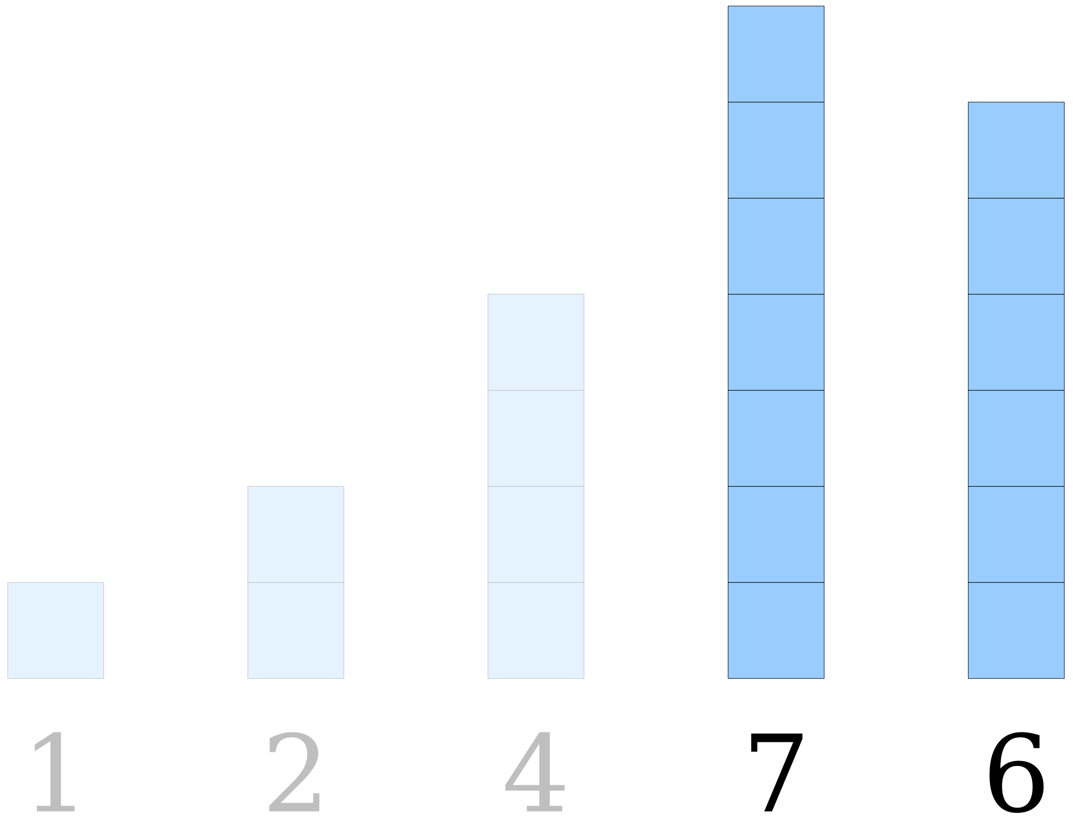

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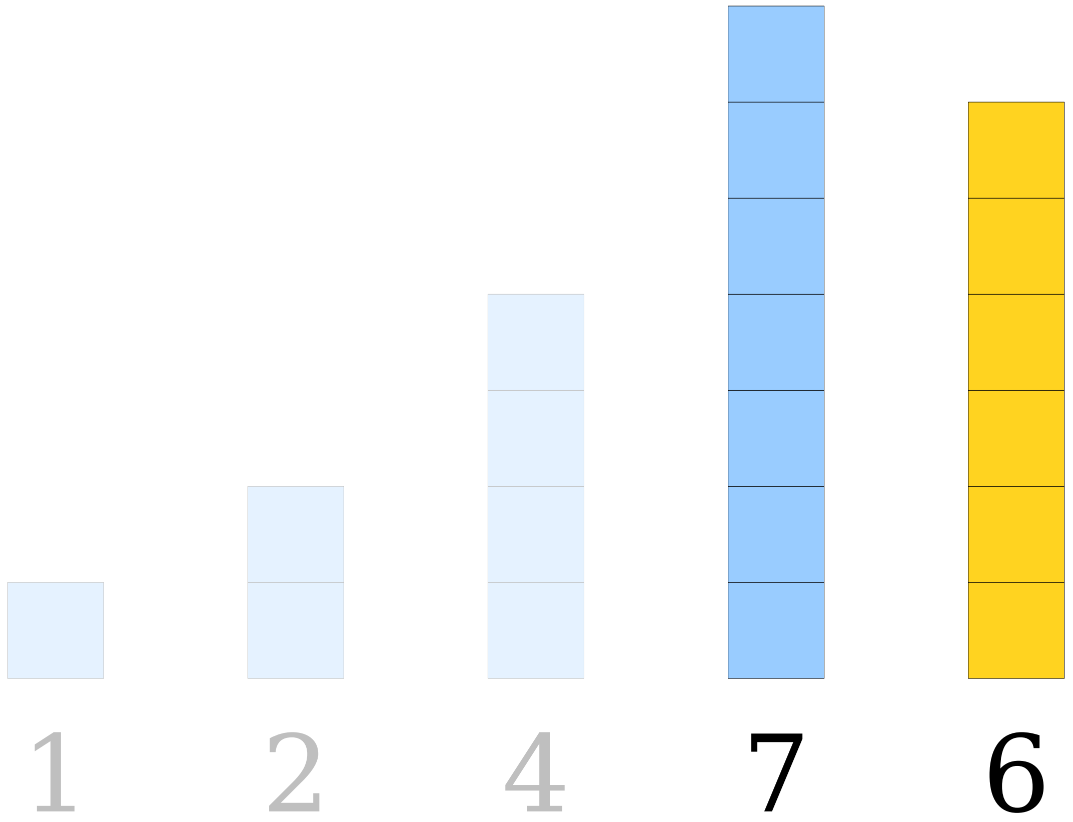
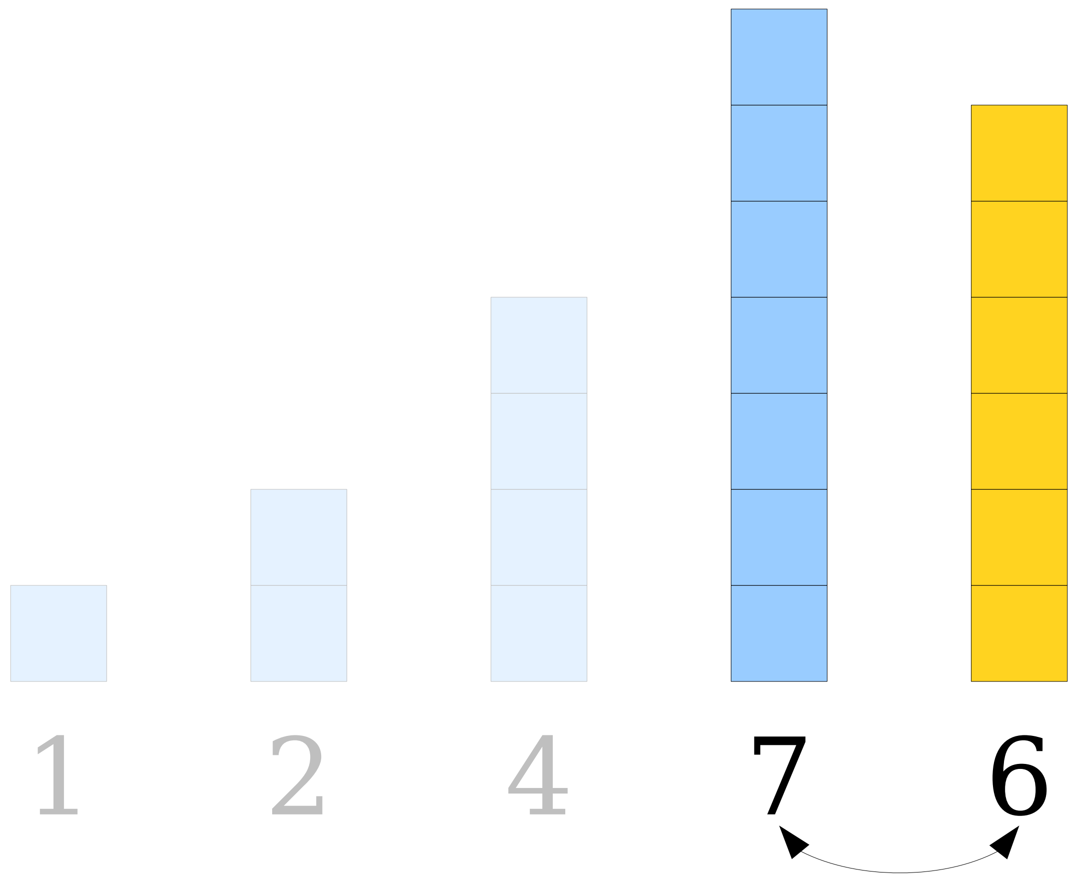


The smallest of these elements needs to go at the front of this group of elements.

An Initial Idea: ***Selection Sort***



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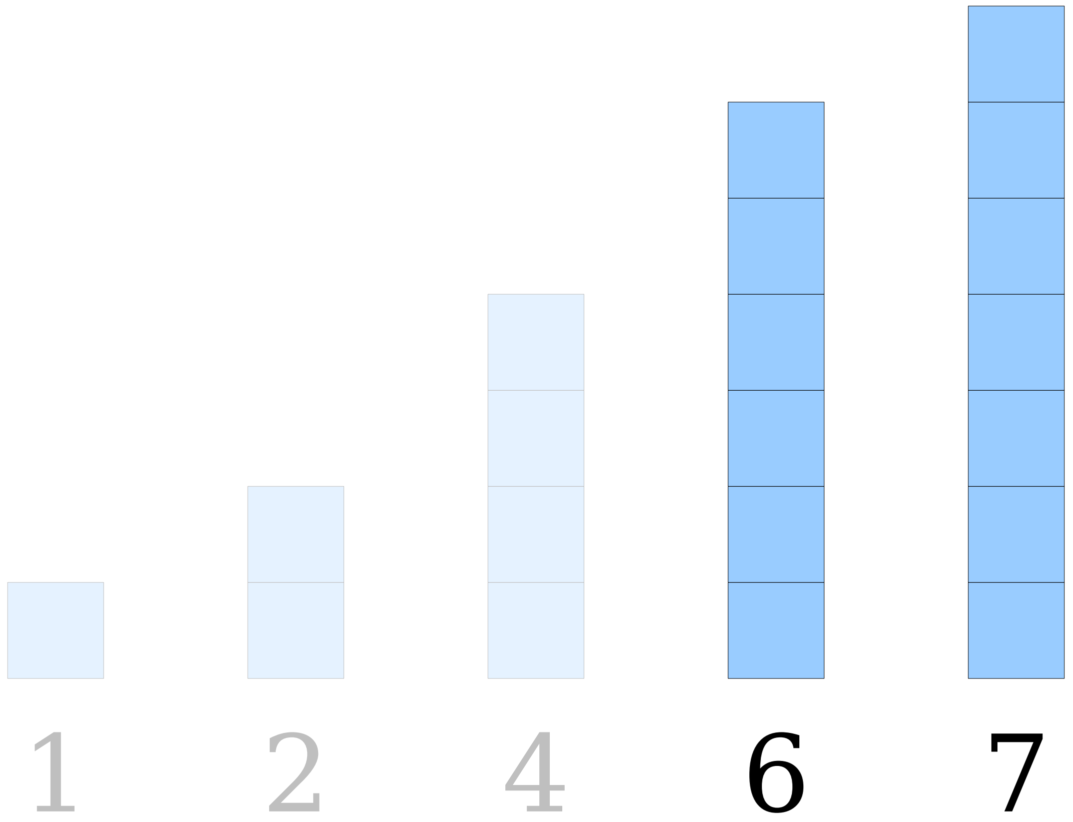

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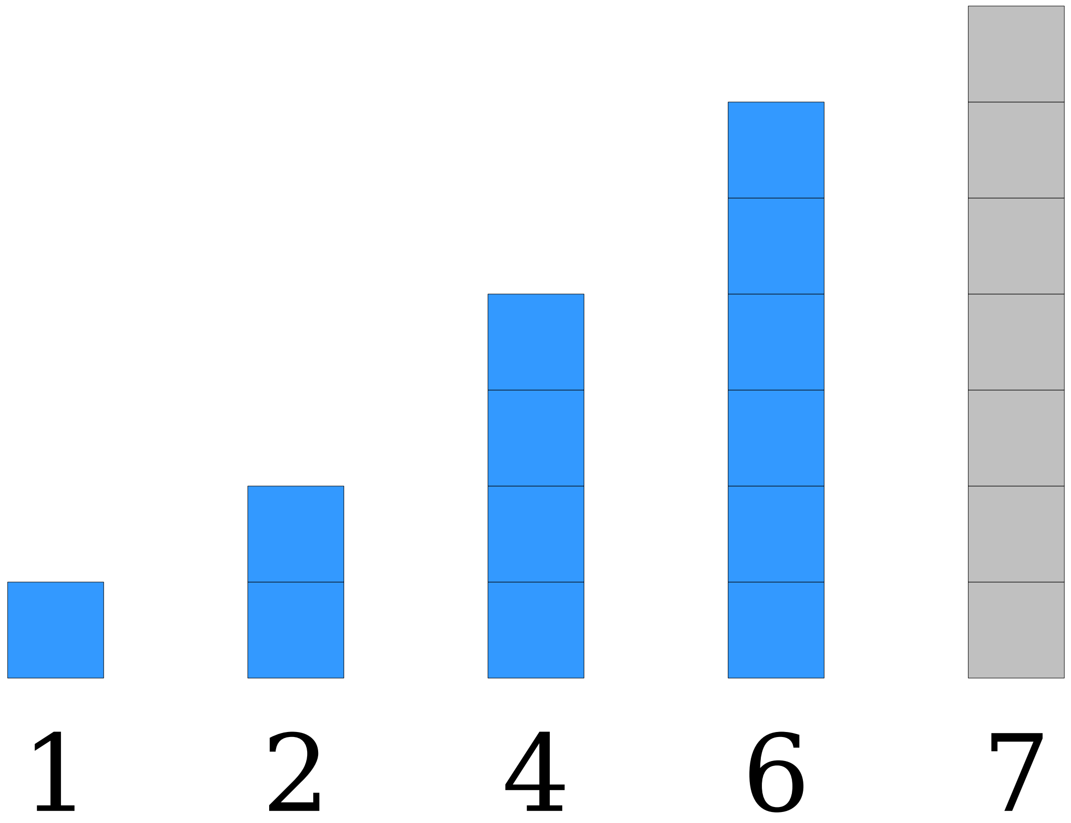

These elements are in the right place now.

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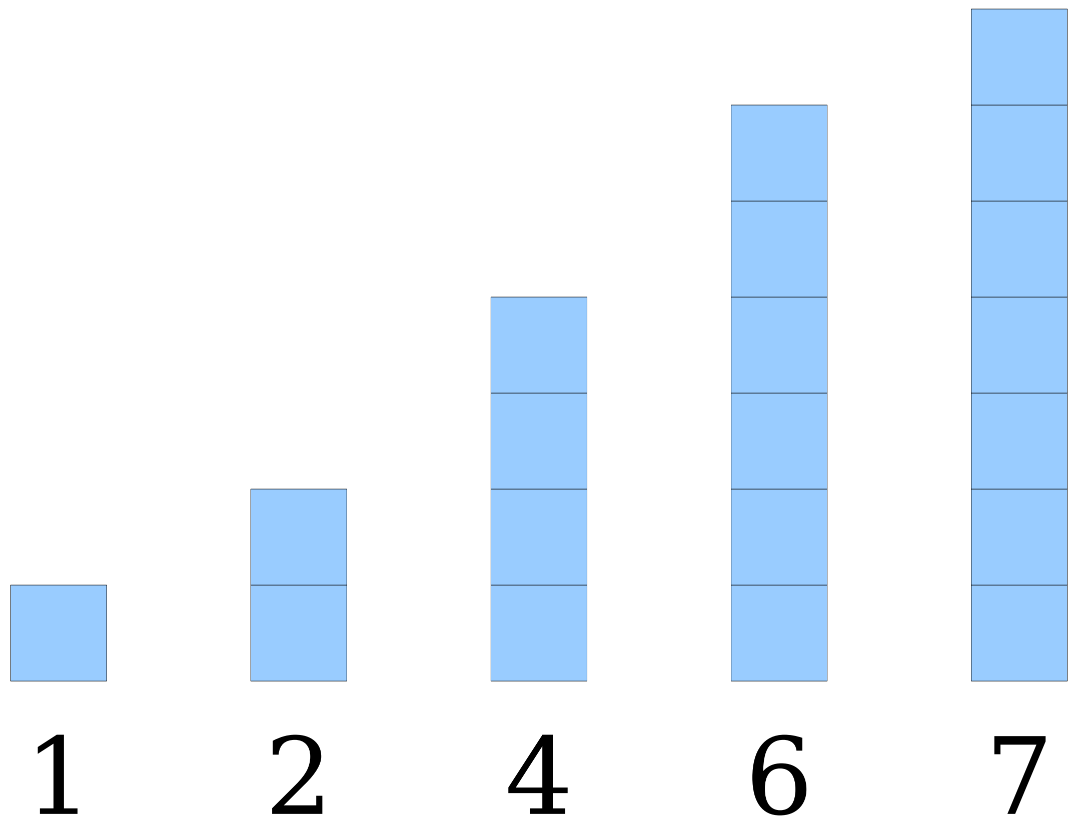

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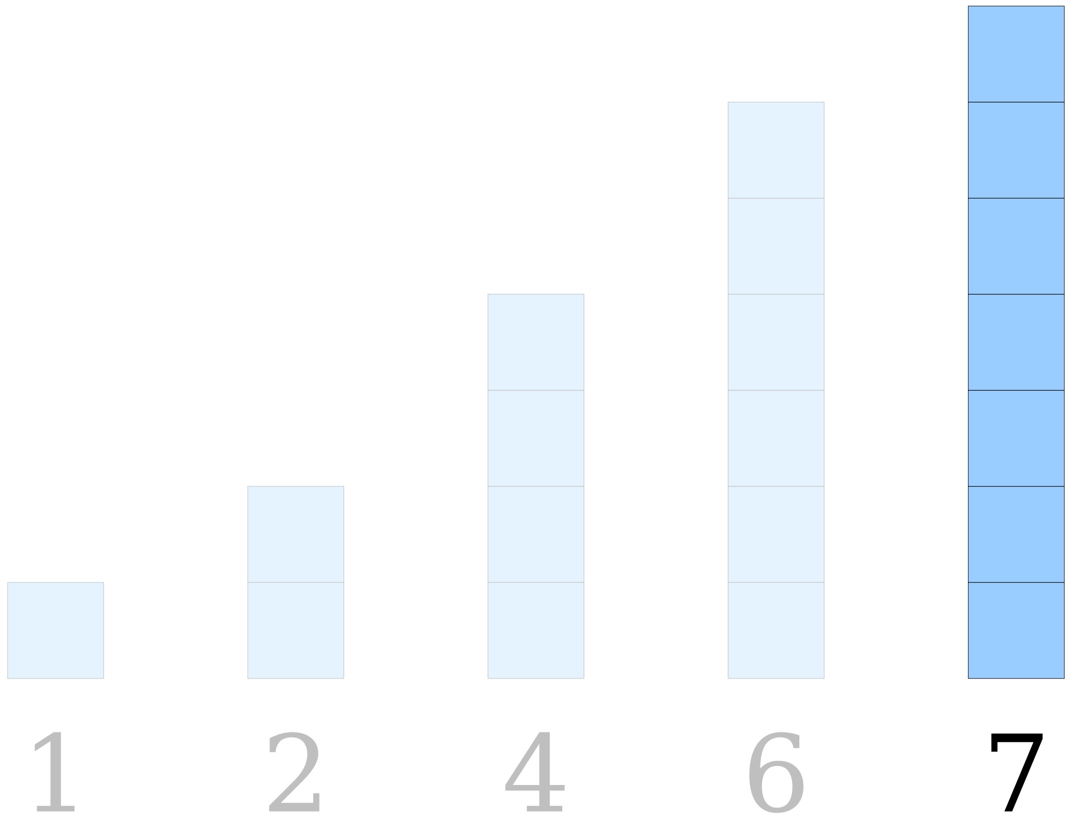

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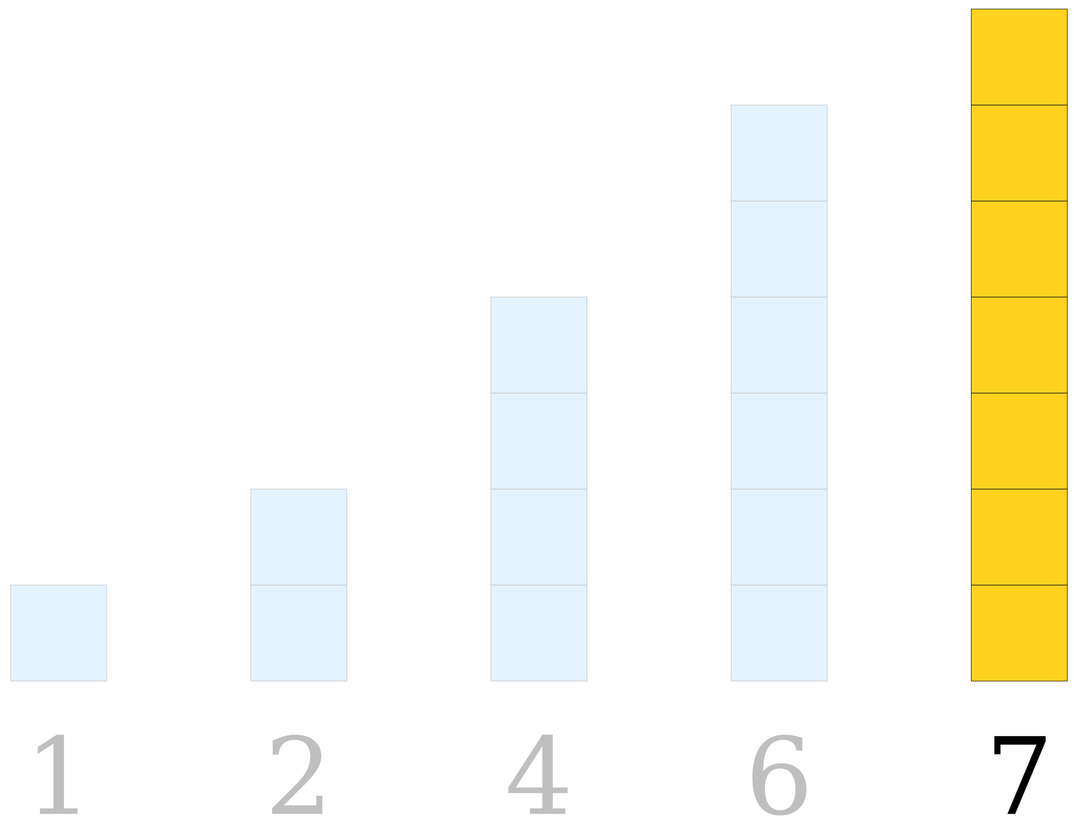
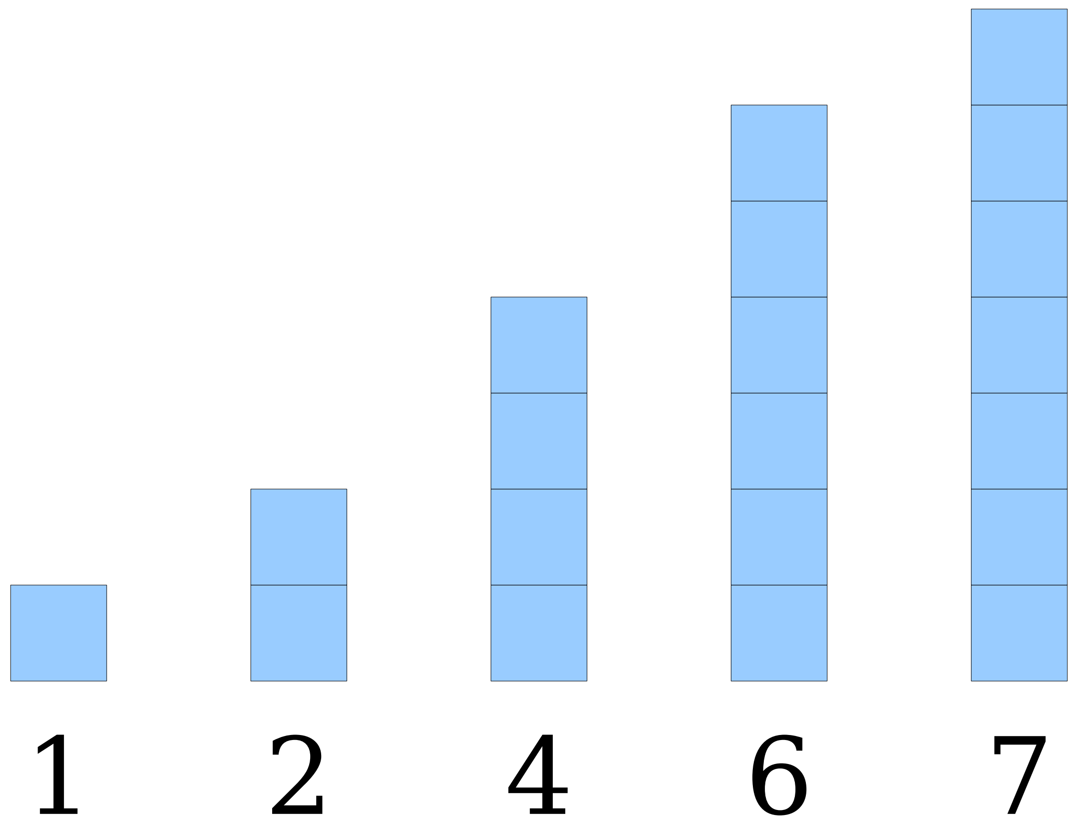


The smallest element from this group needs to go at the front of the group.

An Initial Idea: ***Selection Sort***



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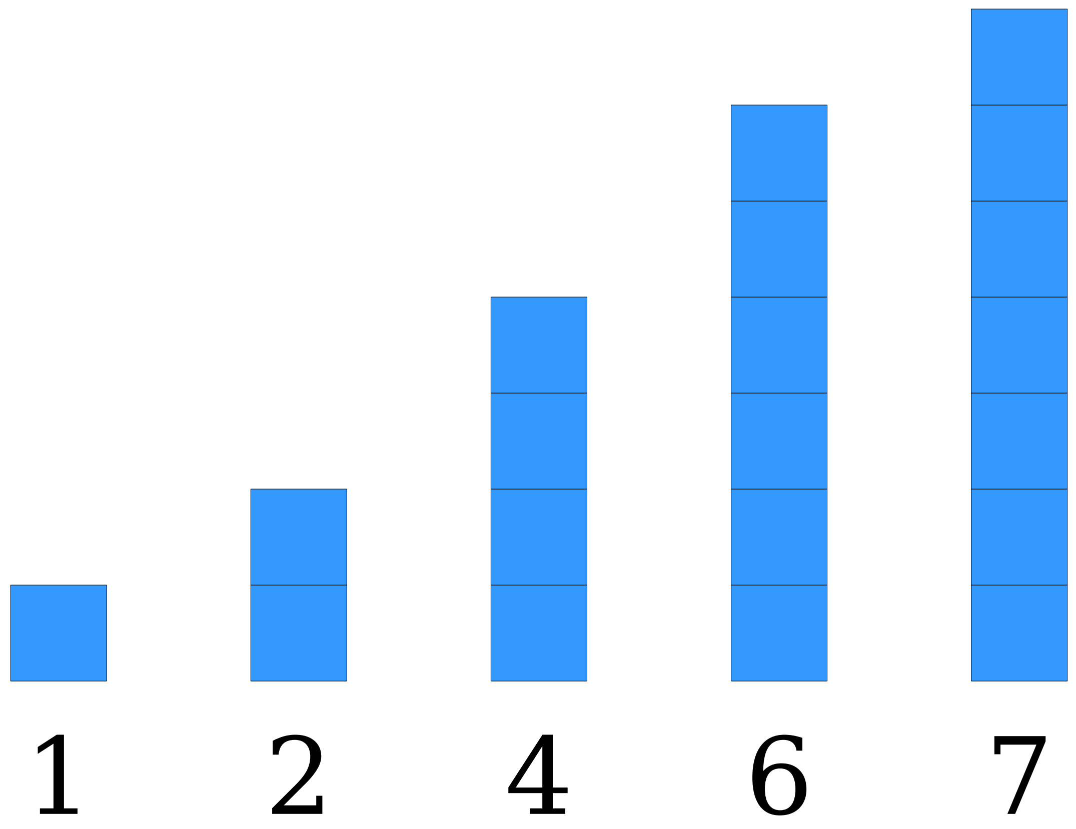

These elements are in the right place now.

Selection Sort

- Find the smallest element and move it to the first position.
- Find the smallest element of what’s left and move it to the second position.
- Find the smallest element of what’s left and move it to the third position.
- Find the smallest element of what’s left and move it to the fourth position.
- (etc.)

```
/**
 * Sorts the specified vector using the selection sort algorithm.
 */
void selectionSort(Vector<int>& elems) {
  for (int index = 0; index < elems.size(); index++) {
    int smallestIndex = indexOfSmallest(elems, index);
    swap(elems[index], elems[smallestIndex]);
  }
}

/**
 * Given a vector and a starting point, returns the index of the
 * smallest element in that vector at or after the starting point.
 */
int indexOfSmallest(const Vector<int>& elems, int startPoint) {
  int smallestIndex = startPoint;
  for (int i = startPoint + 1; i < elems.size(); i++) {
    if (elems[i] < elems[smallestIndex]) {
      smallestIndex = i;
    }
  }
  return smallestIndex;
}
```

```
{ 46, 69, 20, 16, 09, 10, 29, 90, 67, 18, 53, 20, 38, 20, 46 }
  ⬆
```

How fast is selection sort?

```
{ 46, 69, 20, 16, 09, 10, 29, 90, 67, 18, 53, 20, 38, 20, 46 }
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Finding the element that goes in position 0 requires us to scan all *n* elements.

How fast is selection sort?

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      ↑
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{ 09, 10, 20, 16, 46, 69, 29, 90, 67, 18, 53, 20, 38, 20, 46 }
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Finding the element that goes in position 0 requires us to scan all *n* elements.

Finding the element that goes in position 1 requires us to scan *n* – 1 elements.

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Finding the element that goes in position 0 requires us to scan all *n* elements.

Finding the element that goes in position 1 requires us to scan *n* – 1 elements.

Finding the element that goes in position 2 requires us to scan *n* – 2 elements.

How fast is selection sort?

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{ 09, 10, 16, 20, 46, 69, 29, 90, 67, 18, 53, 20, 38, 20, 46 }
```

Finding the element that goes in position 0 requires us to scan all *n* elements.

Finding the element that goes in position 1 requires us to scan *n* – 1 elements.

Finding the element that goes in position 2 requires us to scan *n* – 2 elements.

…

Number of elements scanned:

$$\boldsymbol{n + (n\text{-}1) + (n\text{-}2) + \ldots + 2 + 1}.$$

How fast is selection sort?

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{ 09, 10, 16, 20, 46, 69, 29, 90, 67, 18, 53, 20, 38, 20, 46 }
```

Finding the element that goes in position 0 requires us to scan all *n* elements.

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Finding the element that goes in position 2 requires us to scan *n* – 2 elements.

…

Number of elements scanned:

$$\mathbf{O}(\boldsymbol{n}^2)$$

How fast is selection sort?

Our theory predicts that the runtime of selection sort is O(n^2).

Does that match what we see in practice?

What should we expect to see when we look at a runtime plot?

Time-Out for Announcemnets!

Midterm Exam Logistics

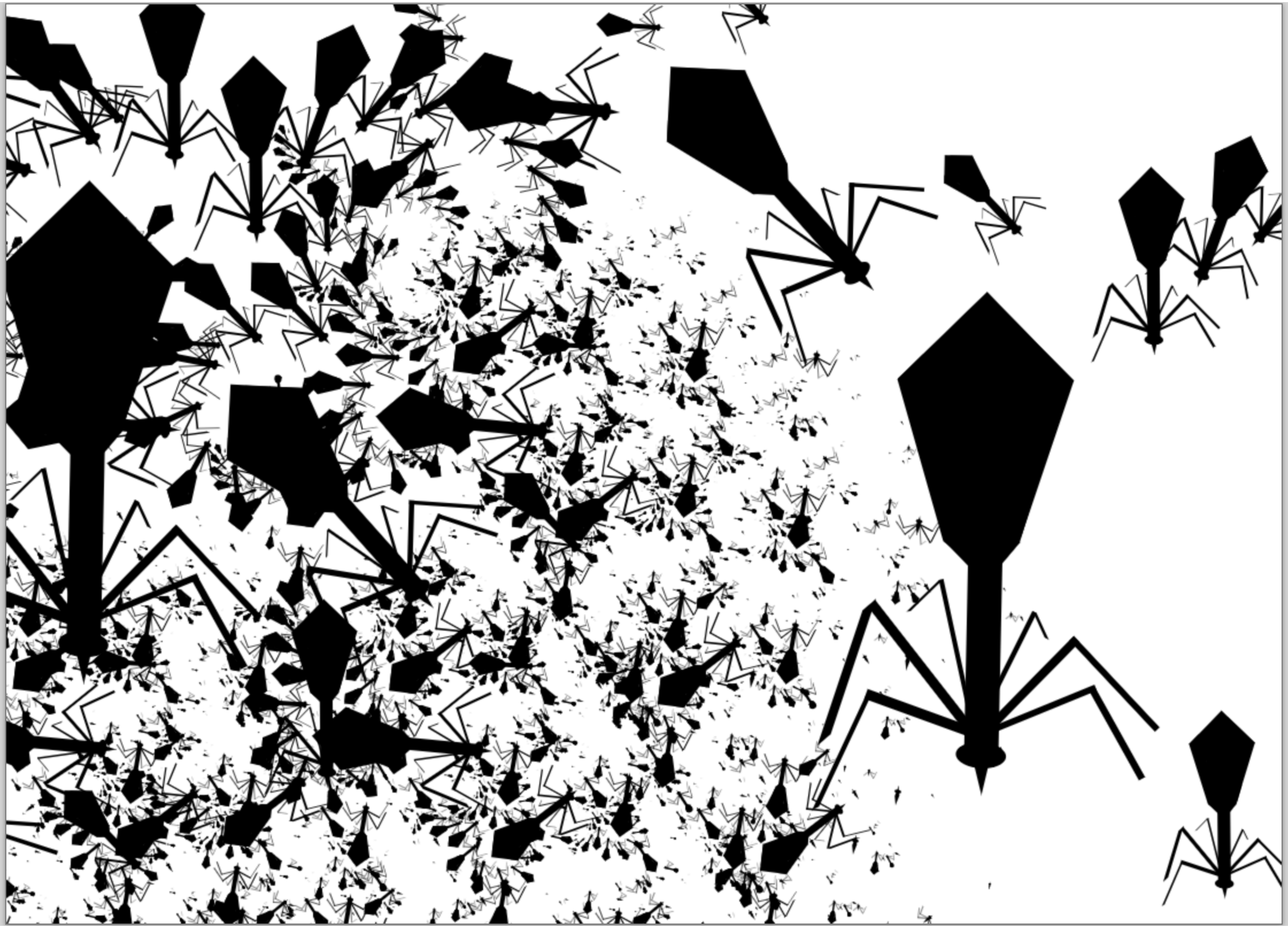
- Our midterm exam will be on Monday, February 13th from 7:00PM – 10:00PM. Locations are assigned by last (family) name:
  - Abdelrahman-Lakkis: Go to Bishop Auditorium
  - Langevine-Zhou: Go to Hewlett 200.
- Exam format:
  - The exam covers L00 – L09 (basic C++ up through but not including recursive backtracking) and A0 – A3 (debugging through recursion).
  - It's a traditional sit-down, pencil-and-paper exam.
  - It's closed-book, closed-computer, and limited-note. You can bring an 8.5" × 11" sheet of notes with you to the exam. We will prove a syntax reference sheet for container types; it'll be on the course website later today.
- We've posted a huge searchable bank of practice problems to the course website, along with three practice exams made from questions selected from that bank.
- Students with OAE accommodations: If you need exam accommodations, please contact us ASAP if you haven't yet done so.

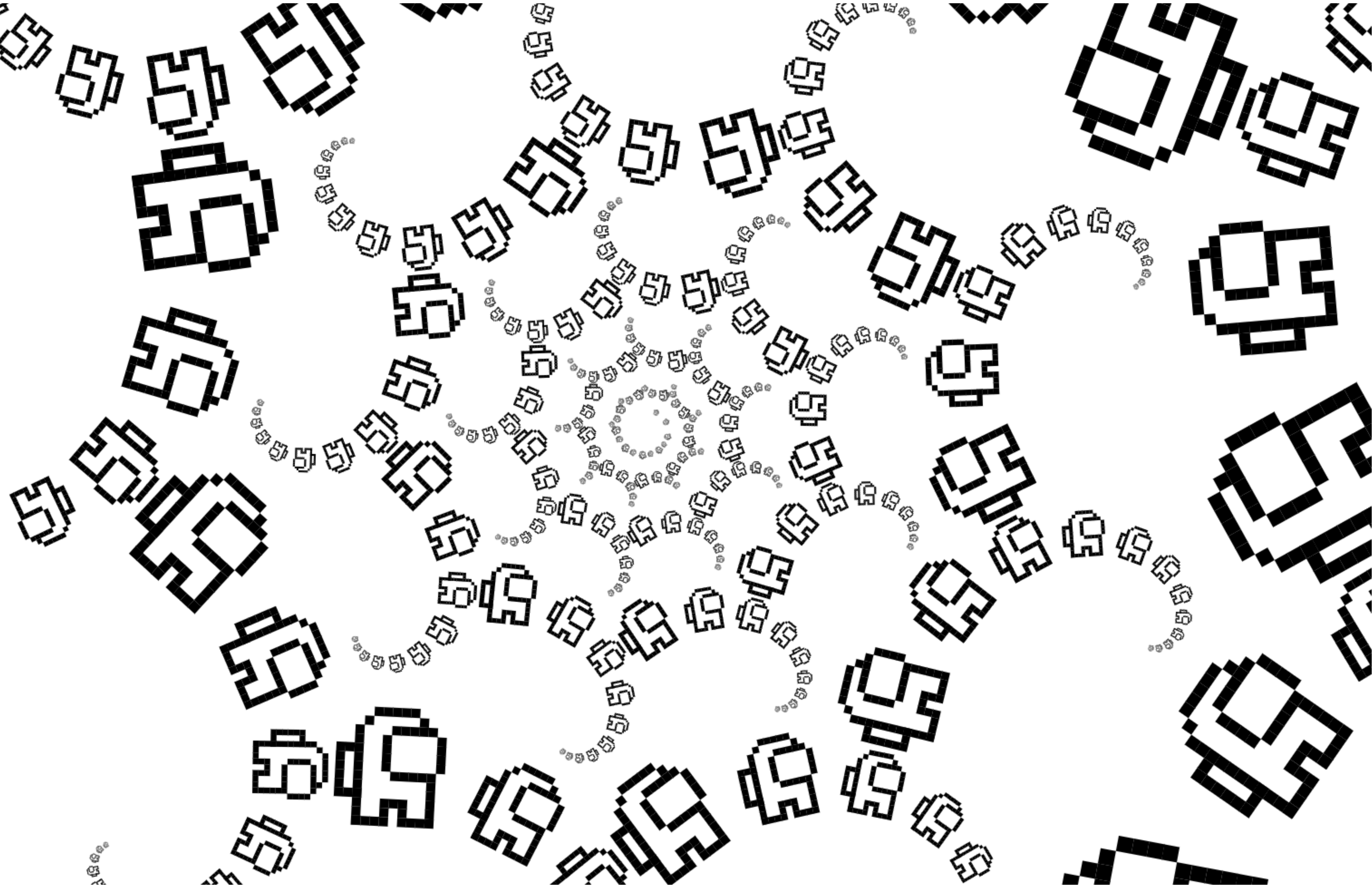
Recursive Drawing Prizes

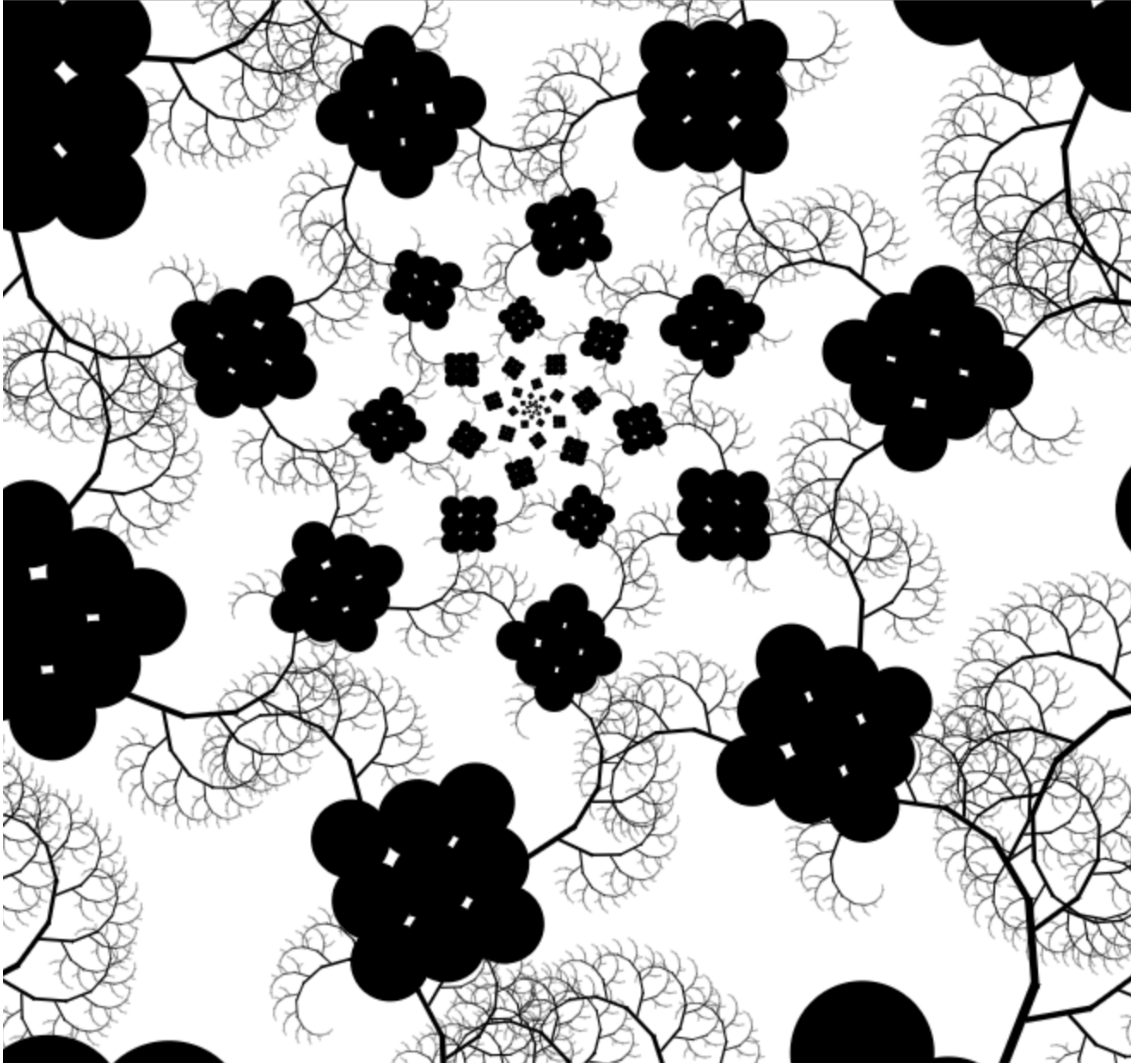
Recursive Cocoa!

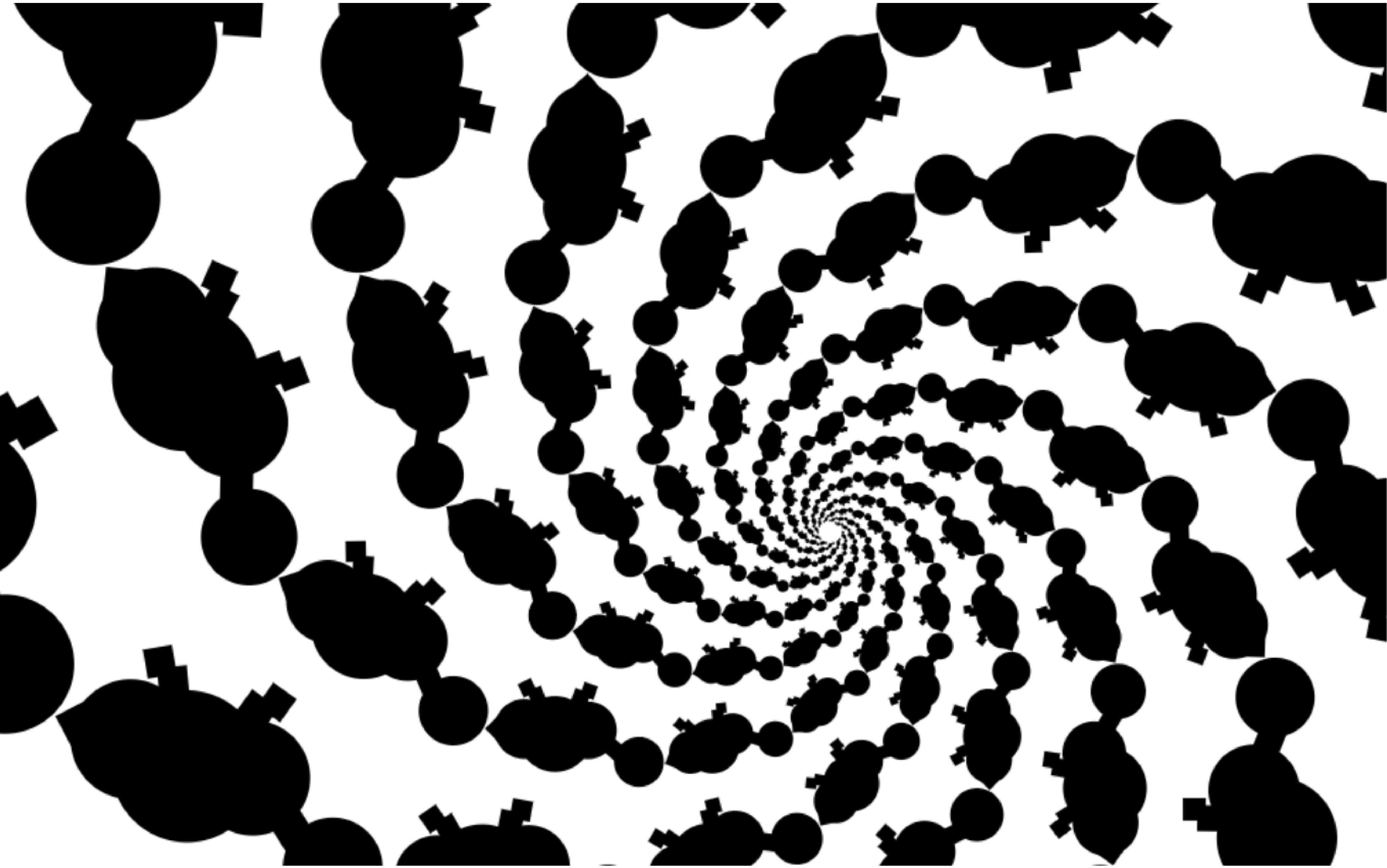
- We have five boxes of Droste Cacao that we'll be awarding as prizes.
- We figured it's a nice recursive art prize for our recursive art contest.

The Awardees









Honorable Mention

Back to CS106B!

Another Sorting Algorithm

Our Next Idea: ***Insertion Sort***

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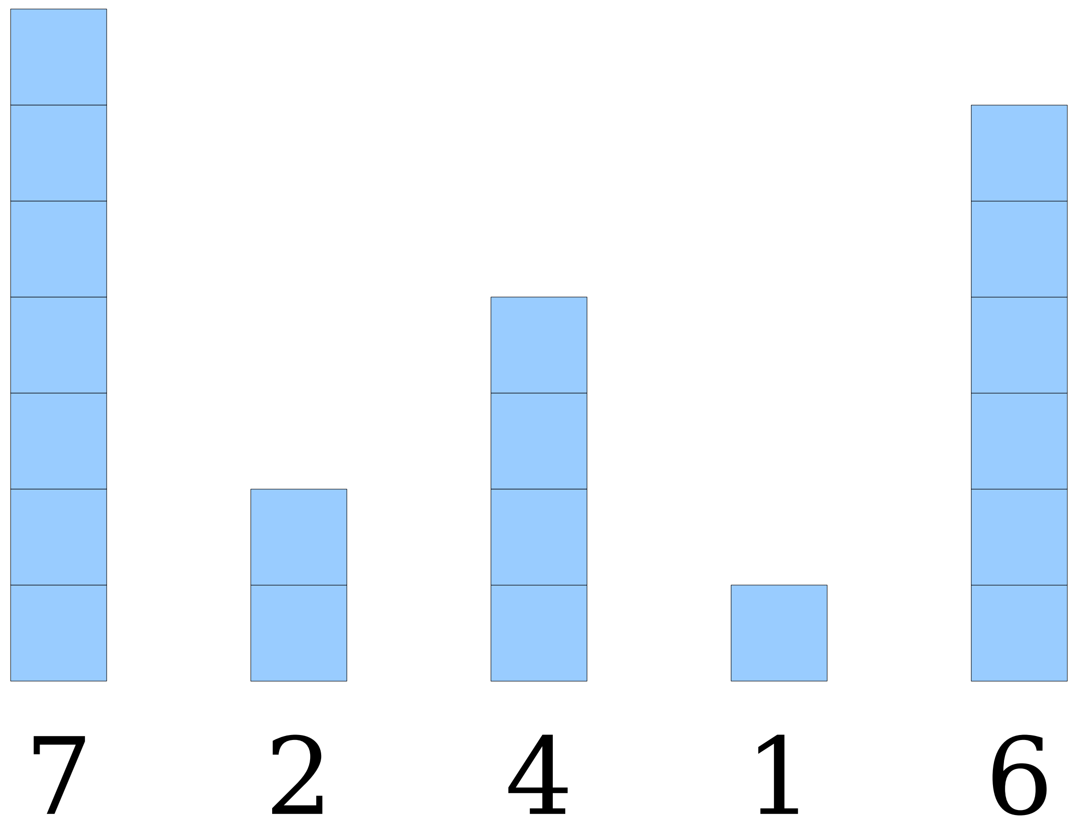

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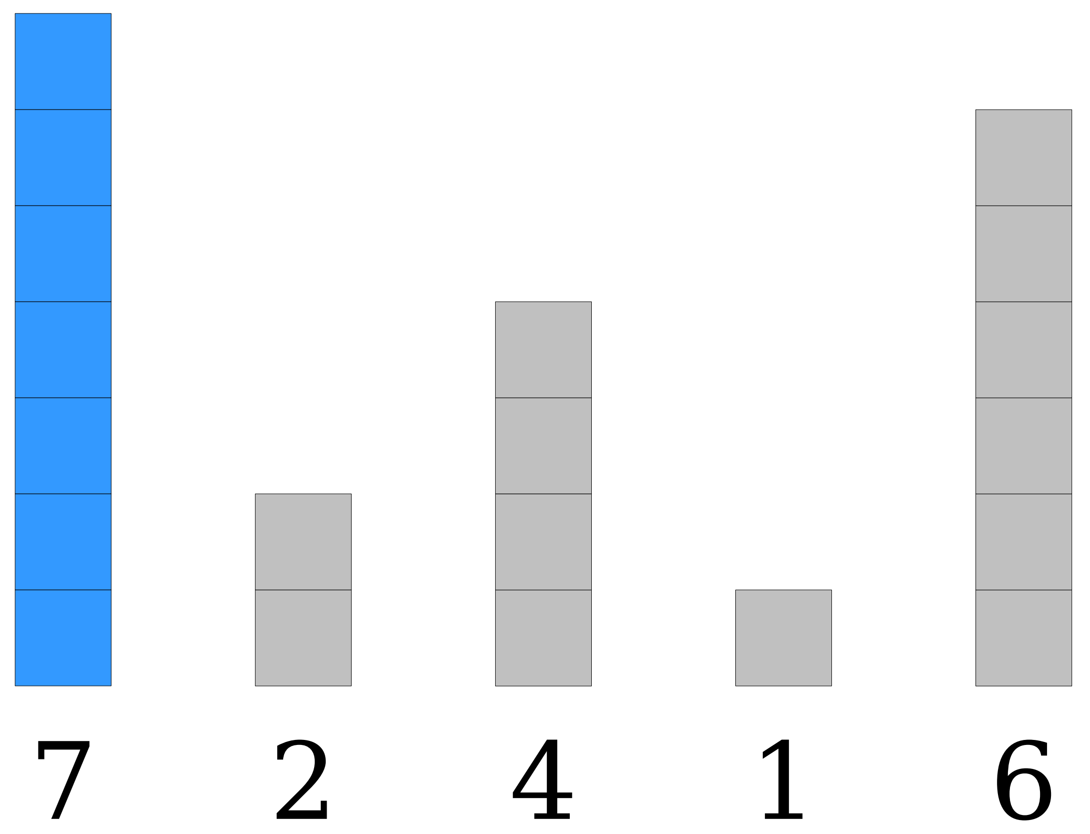

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This sequence in gray is in no particular order.

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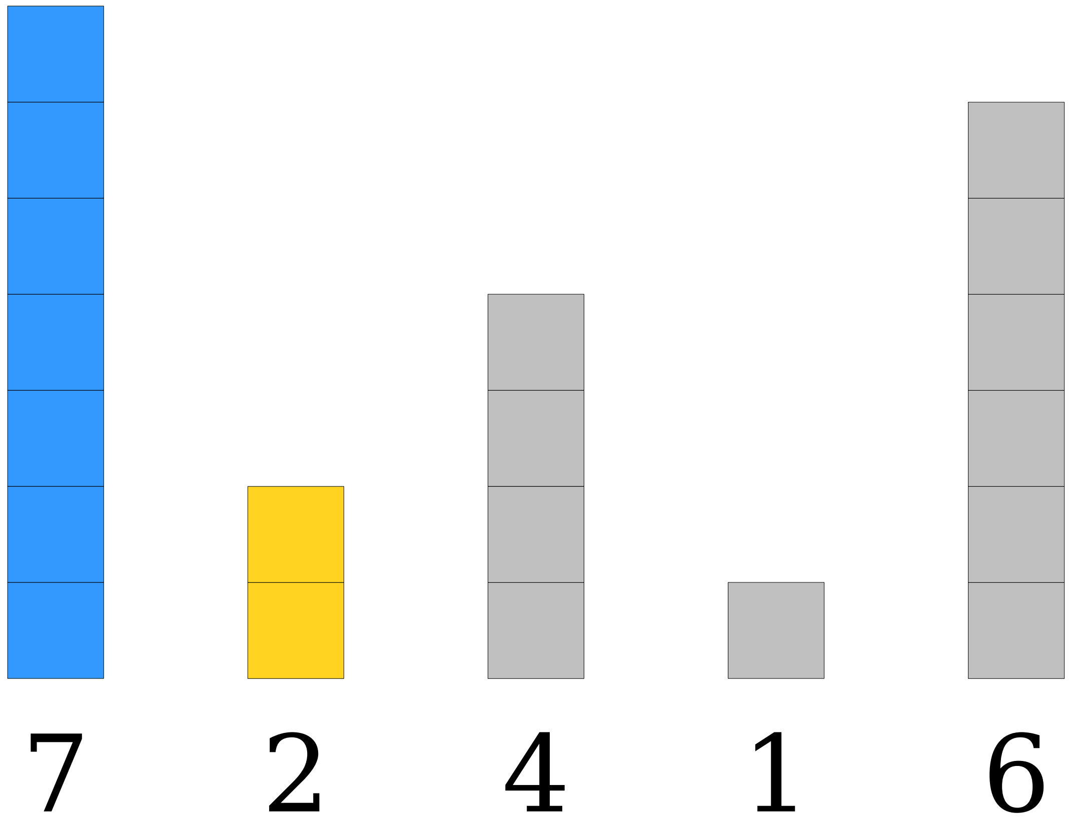

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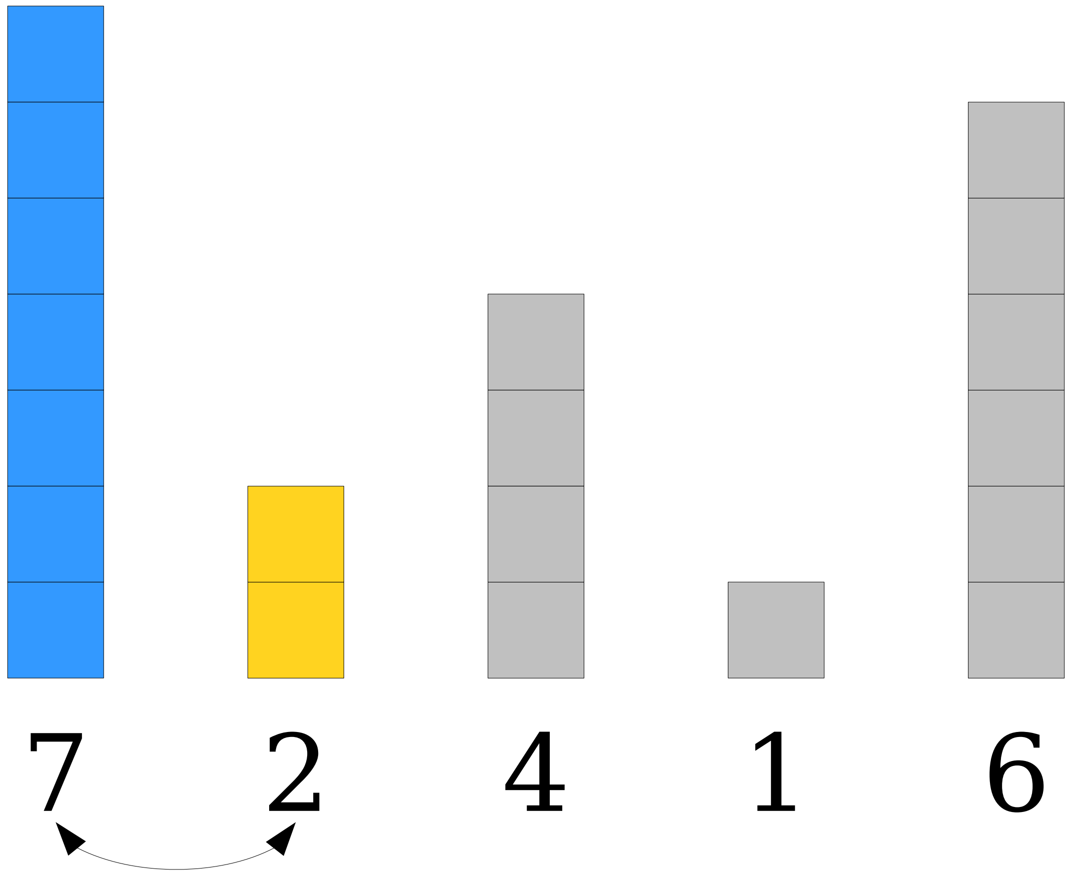

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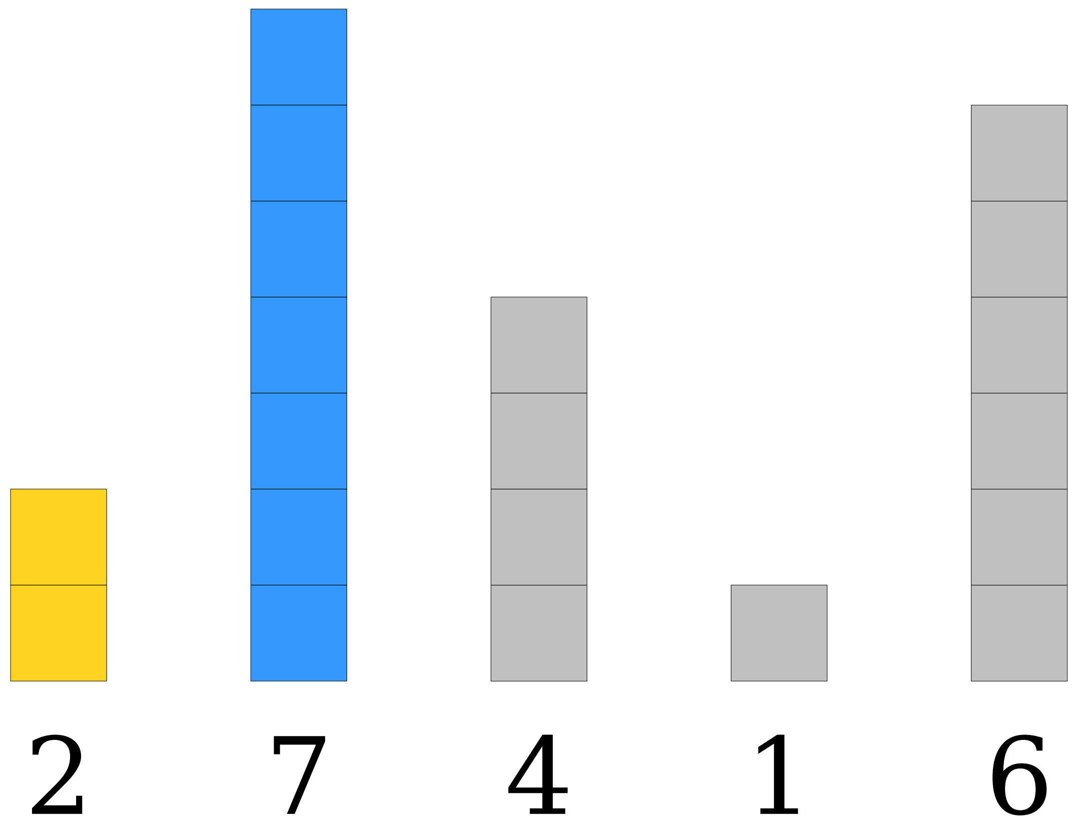

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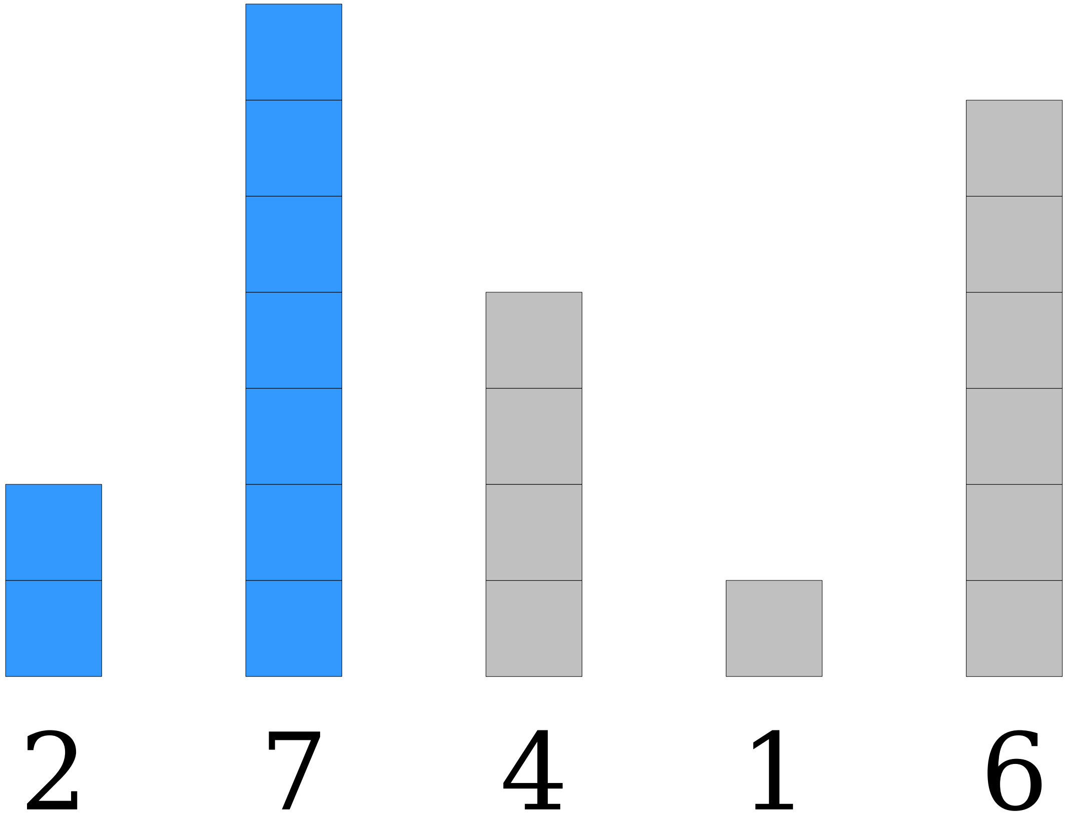

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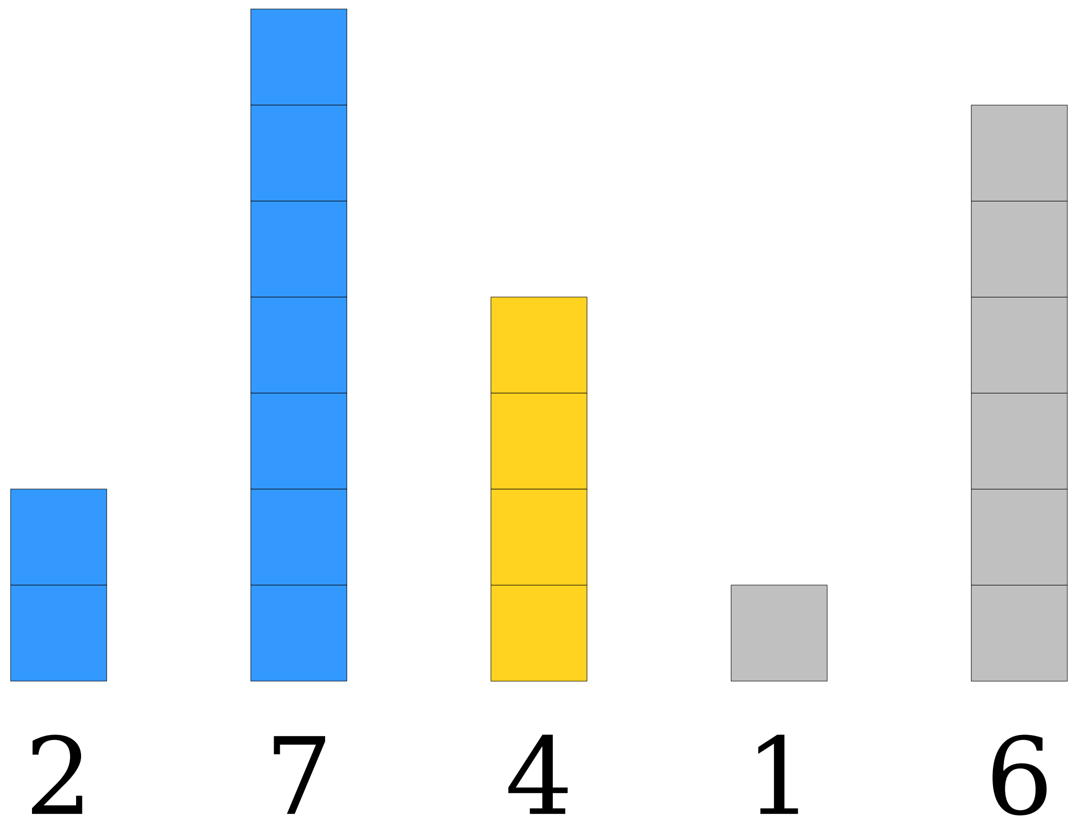

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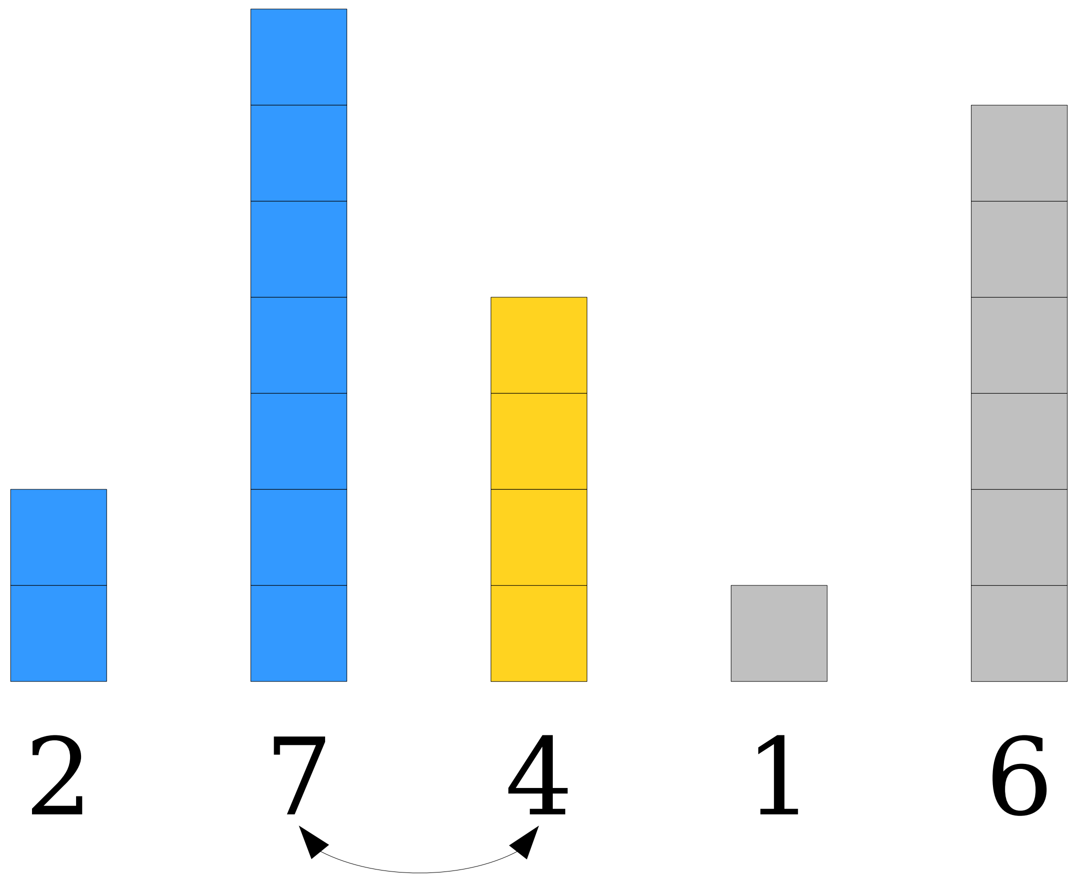

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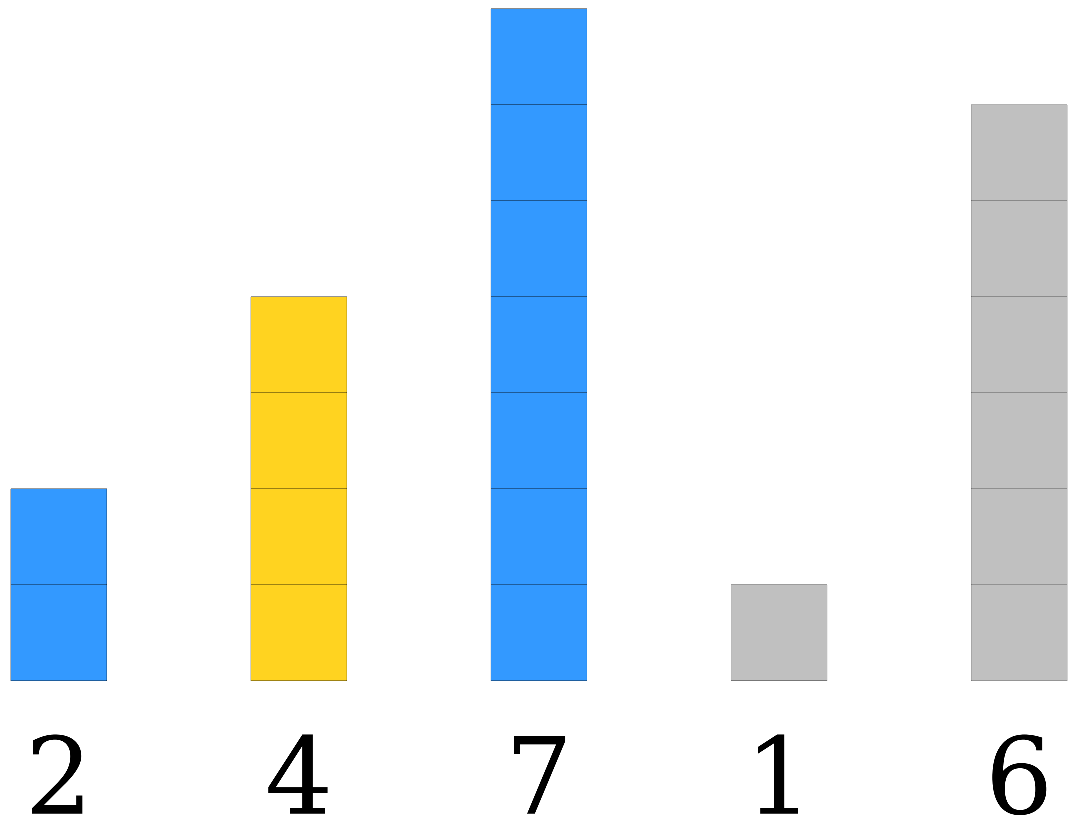

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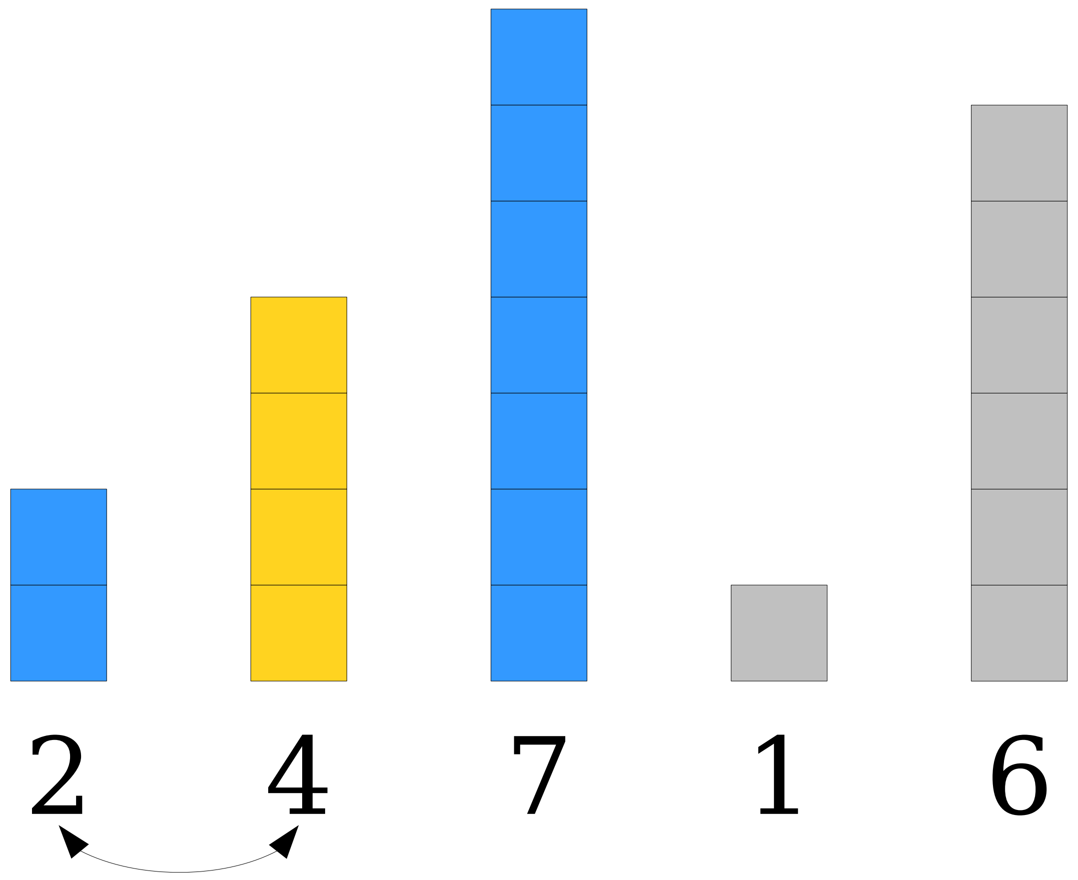

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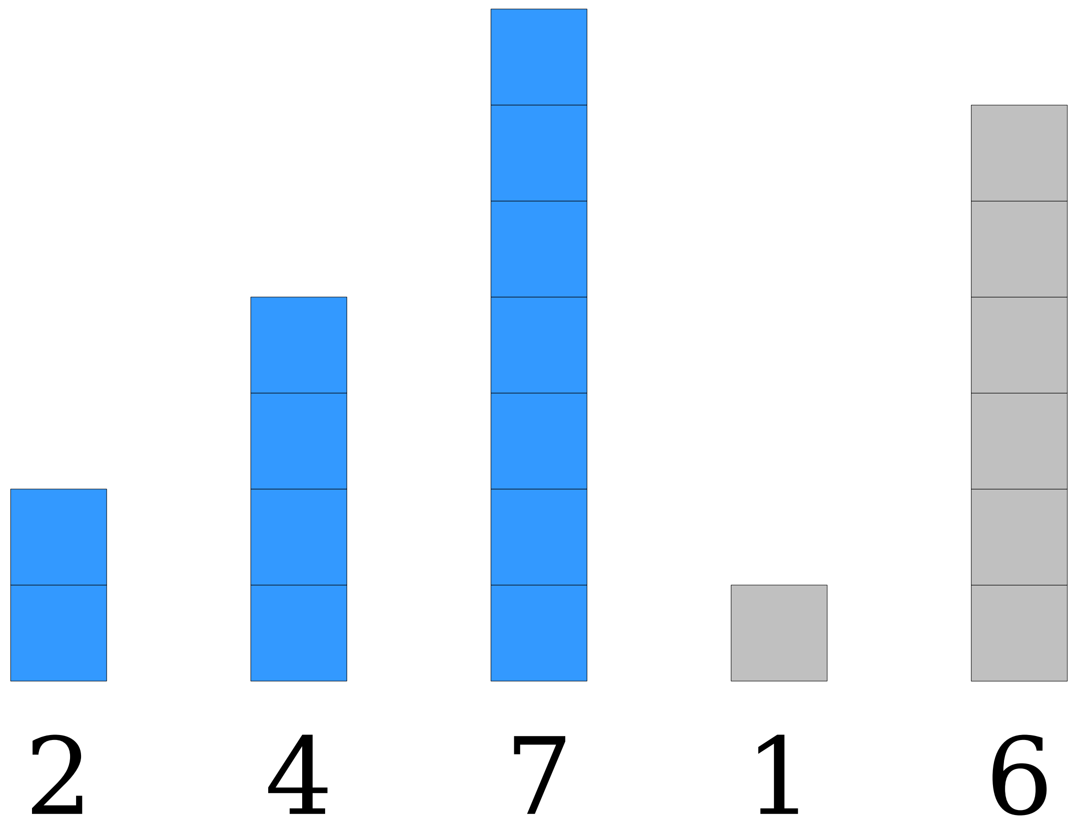

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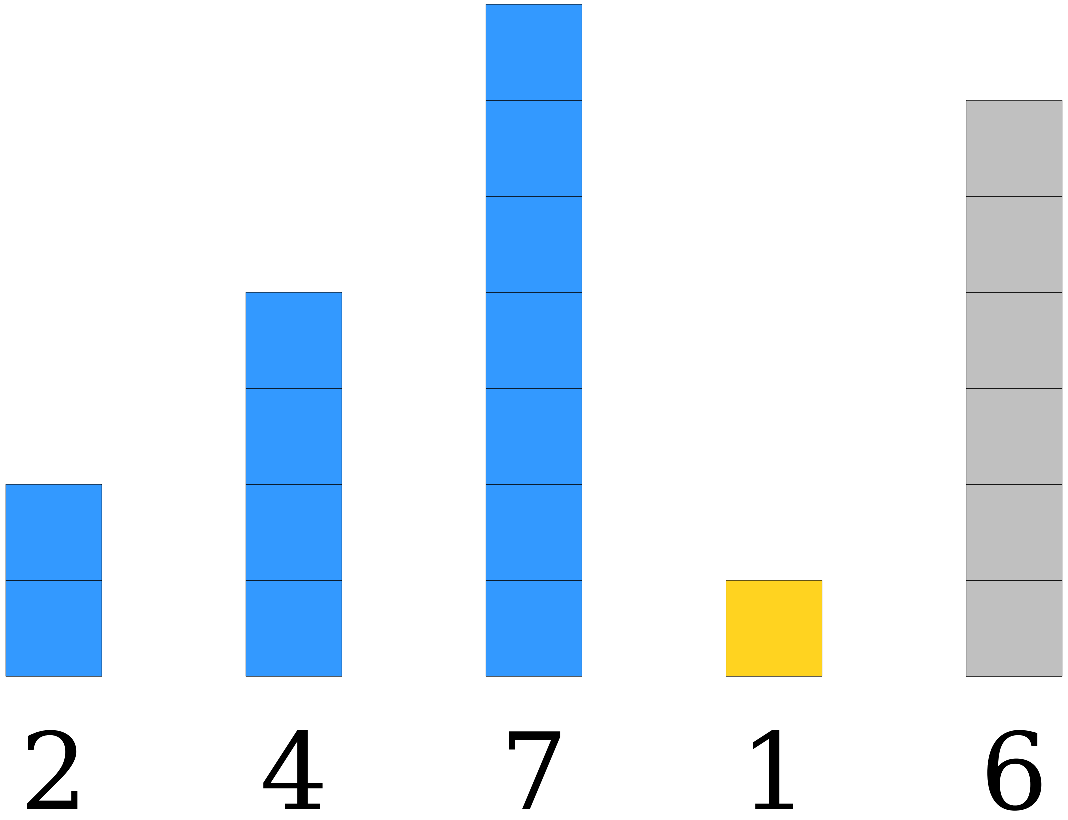

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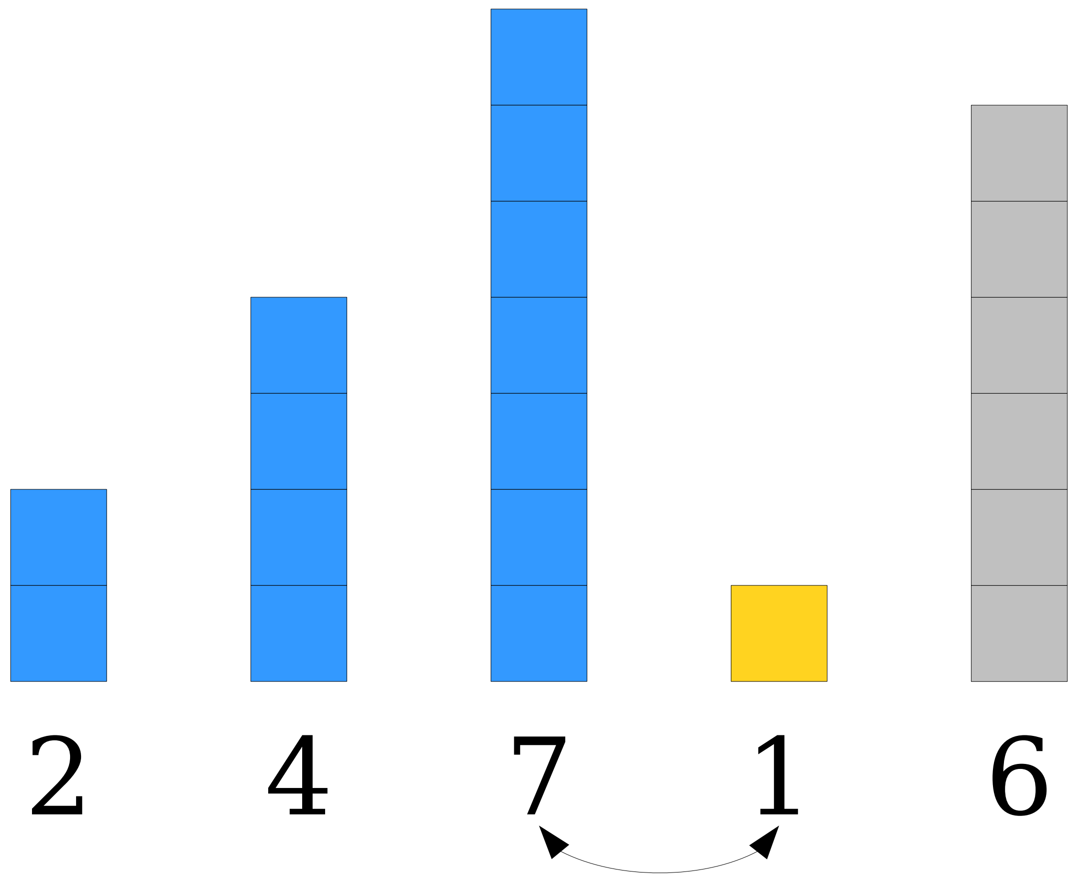

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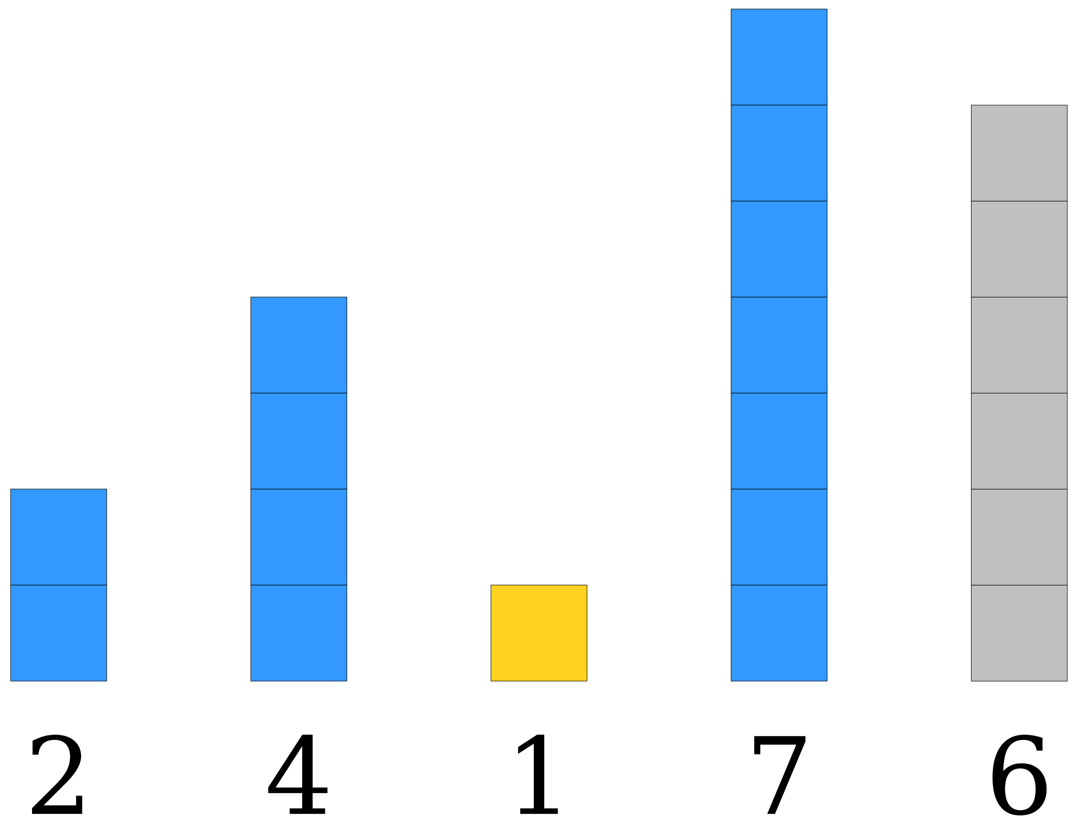

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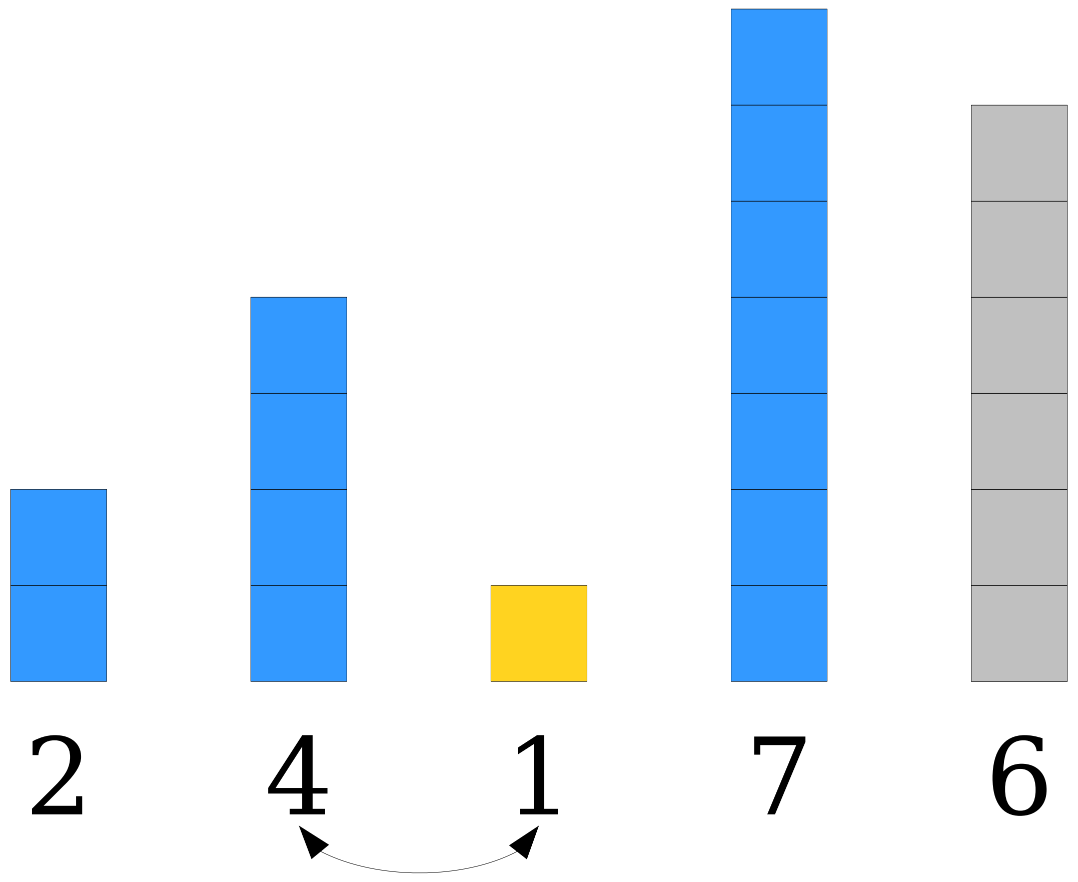

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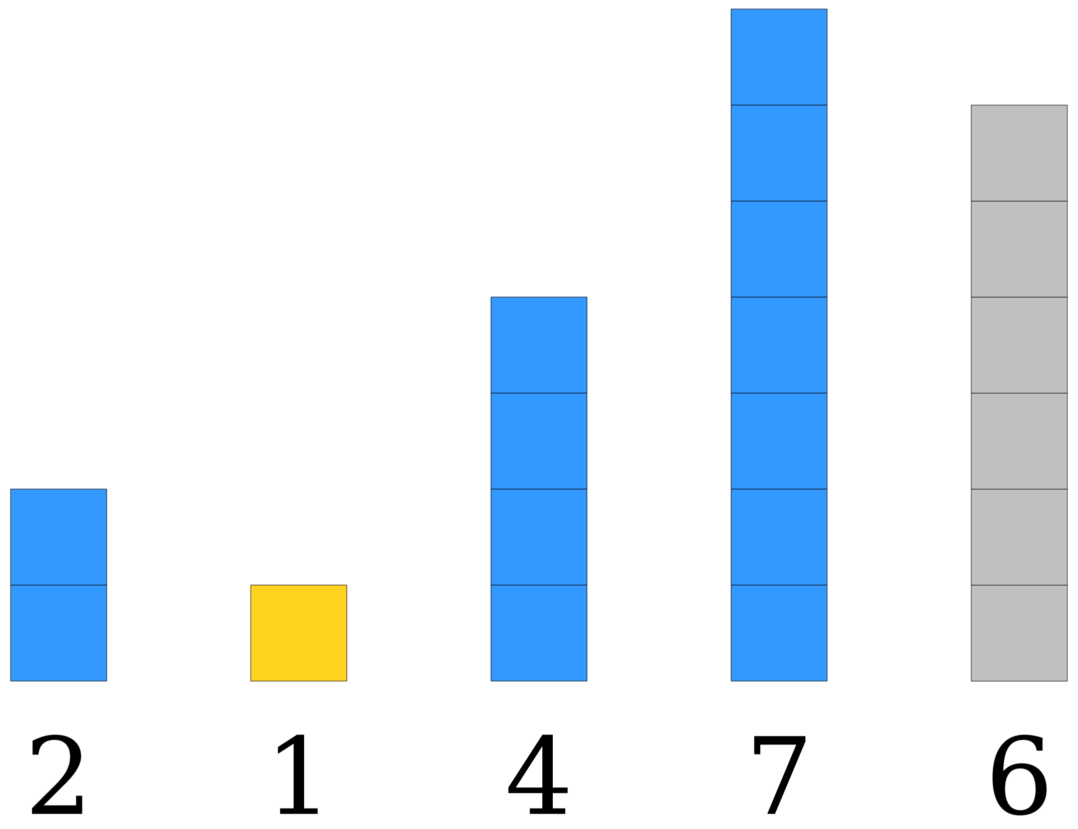

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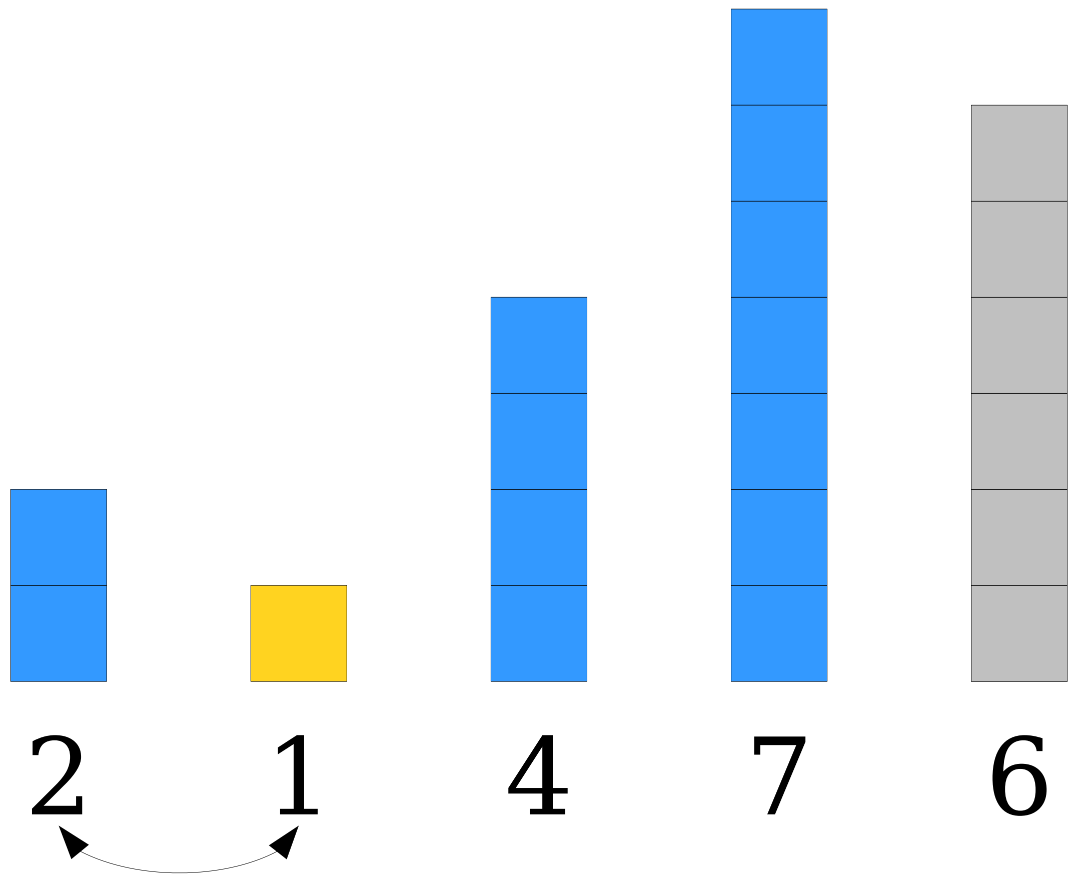

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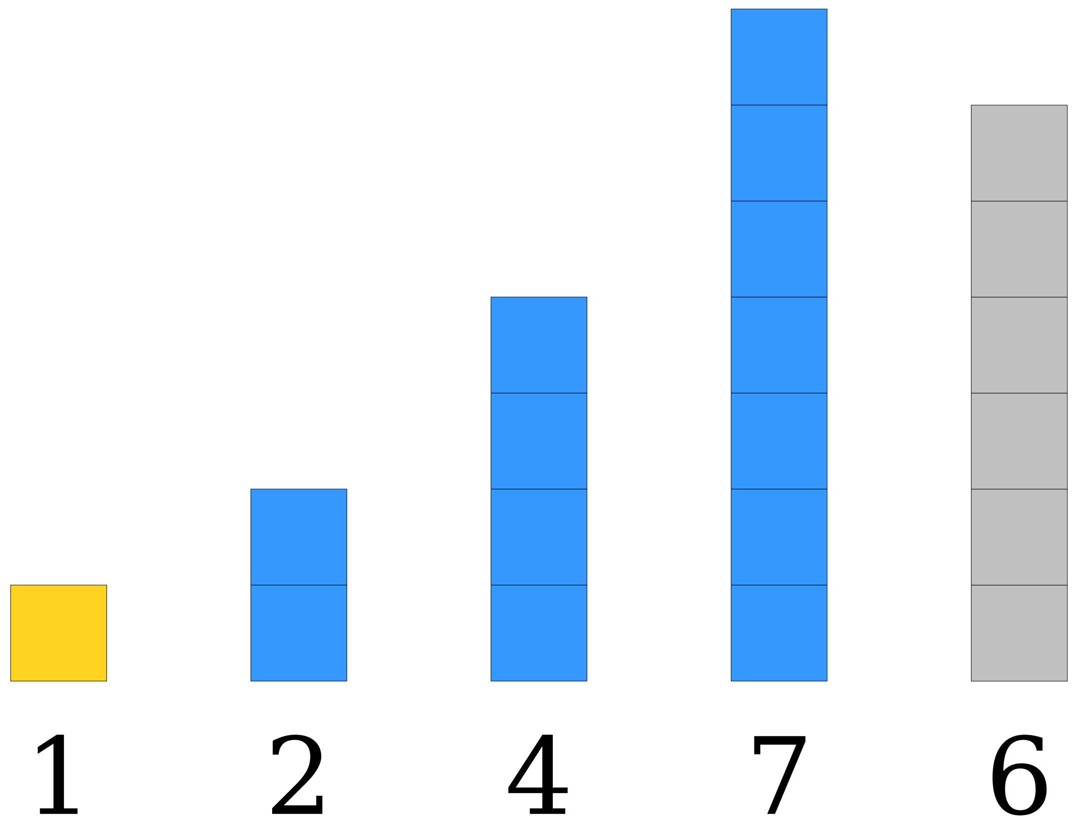

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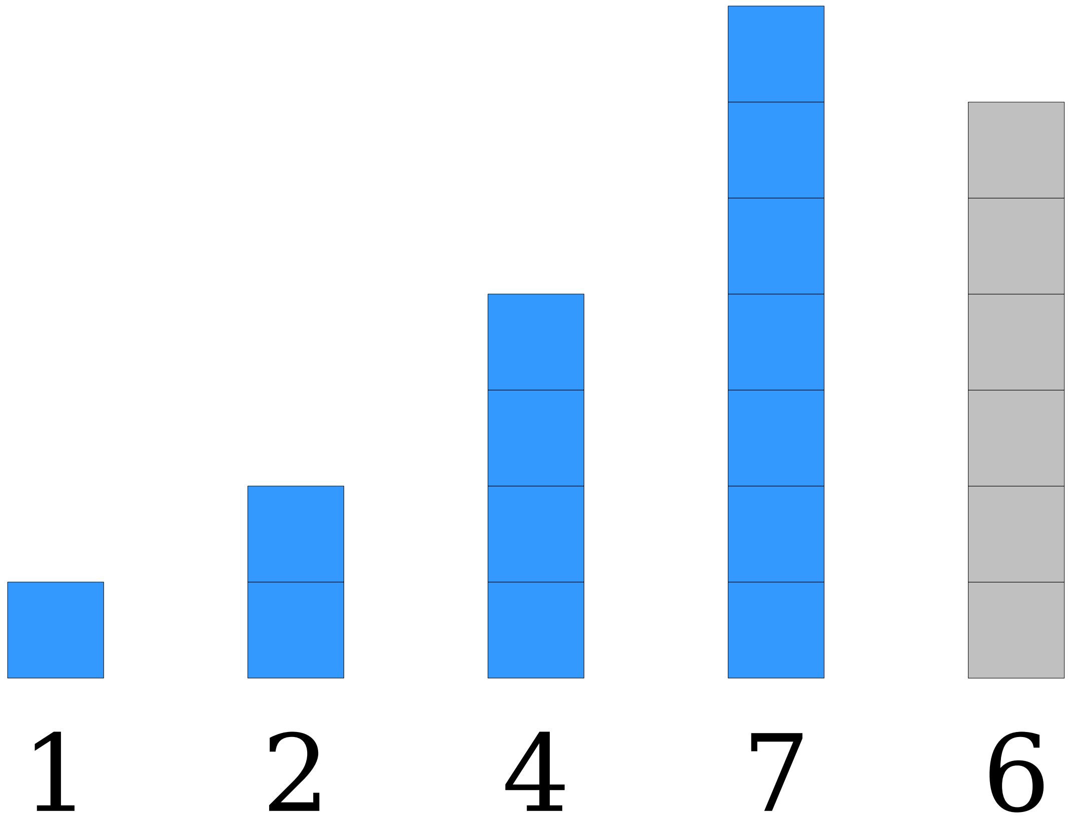

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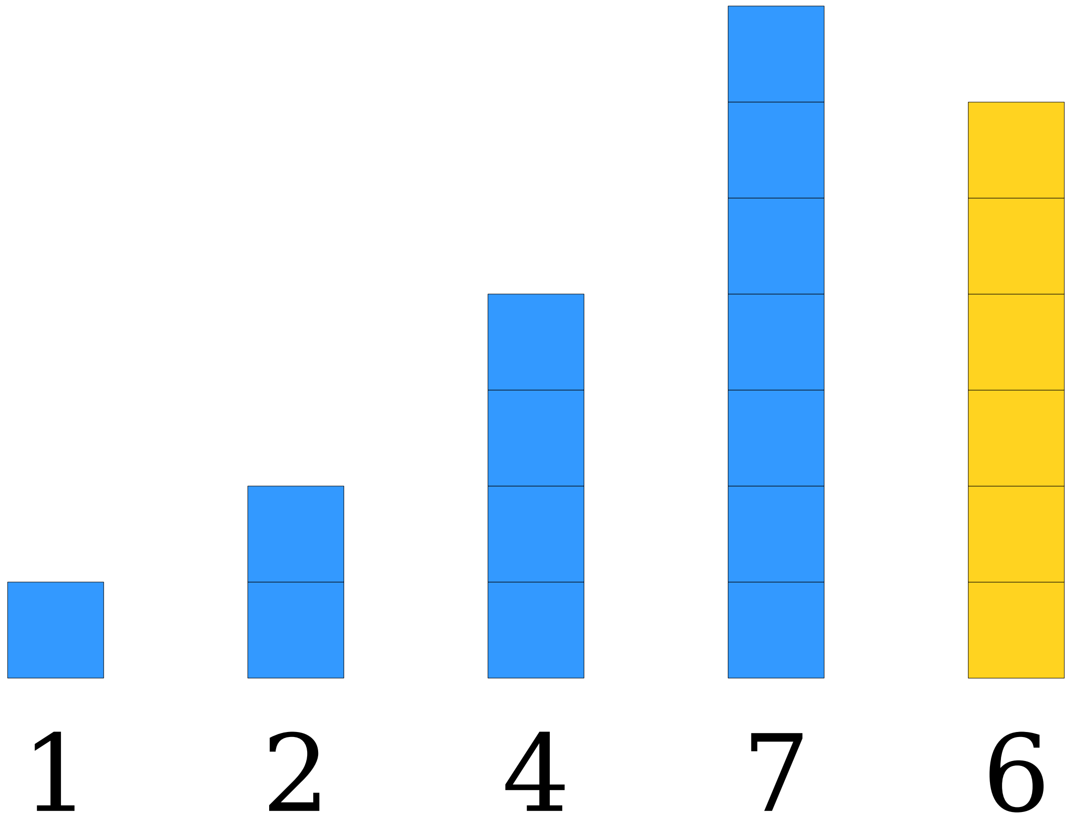

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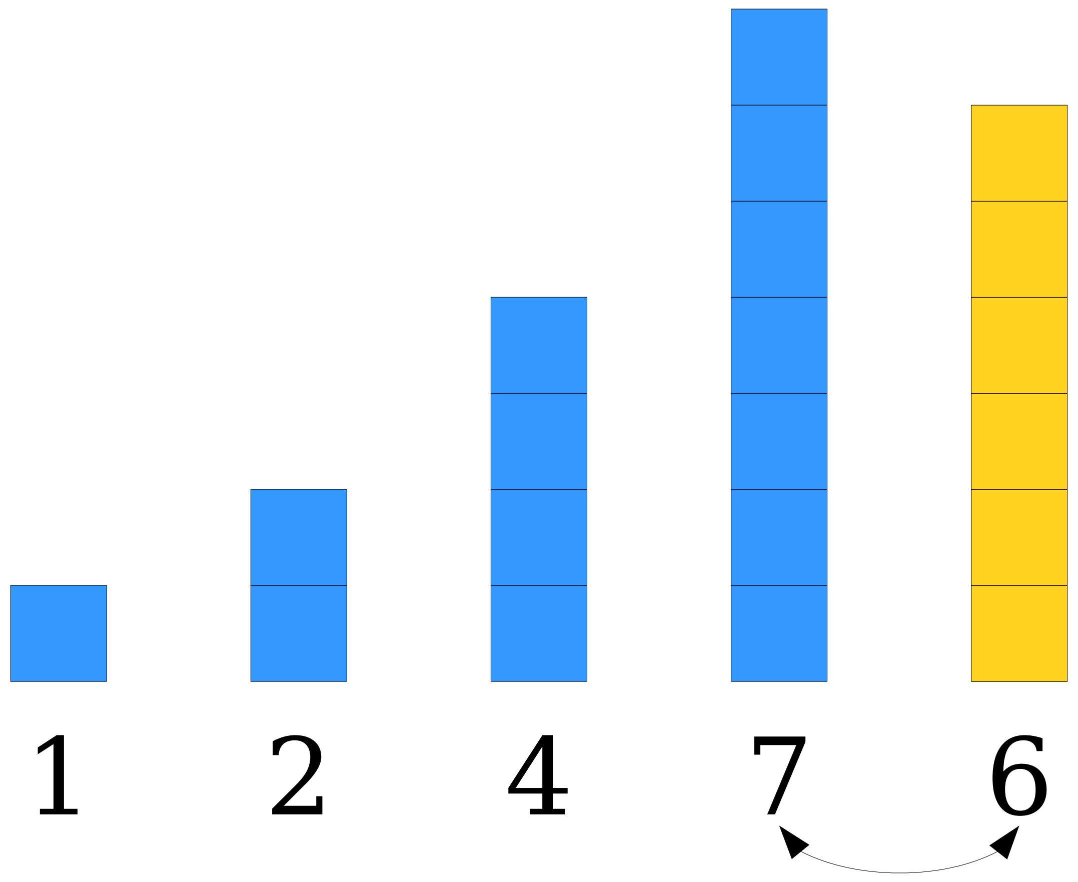

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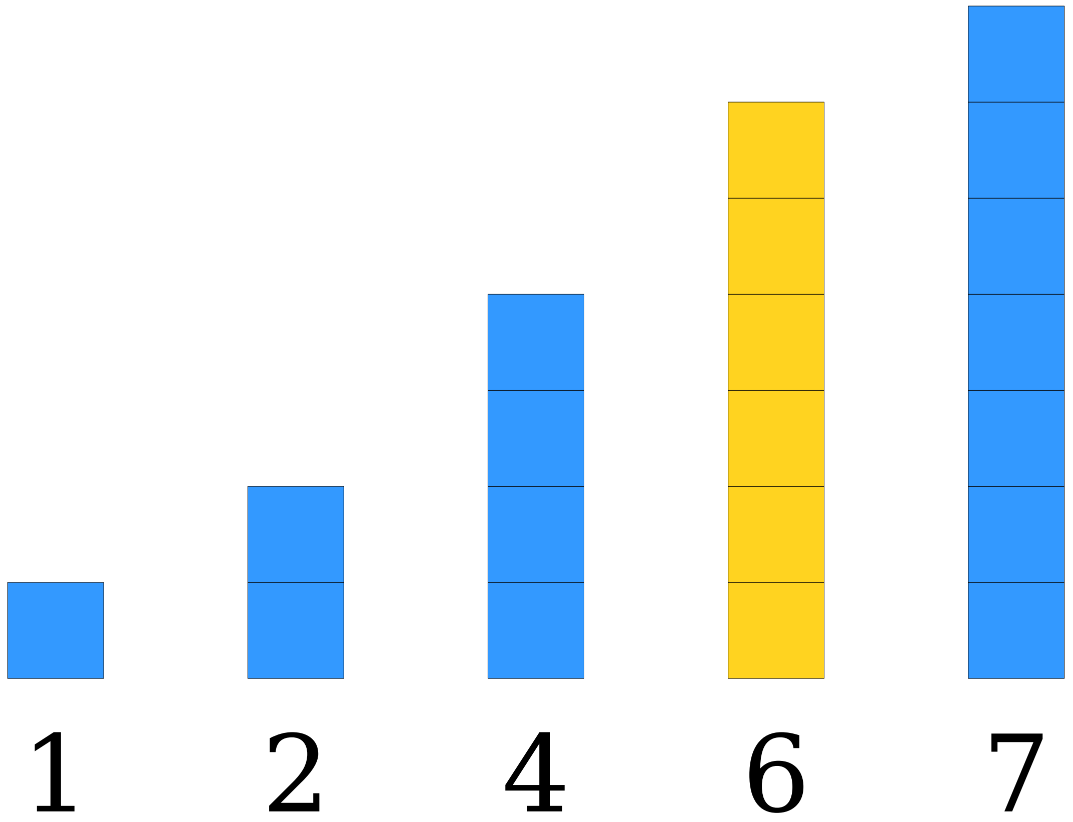

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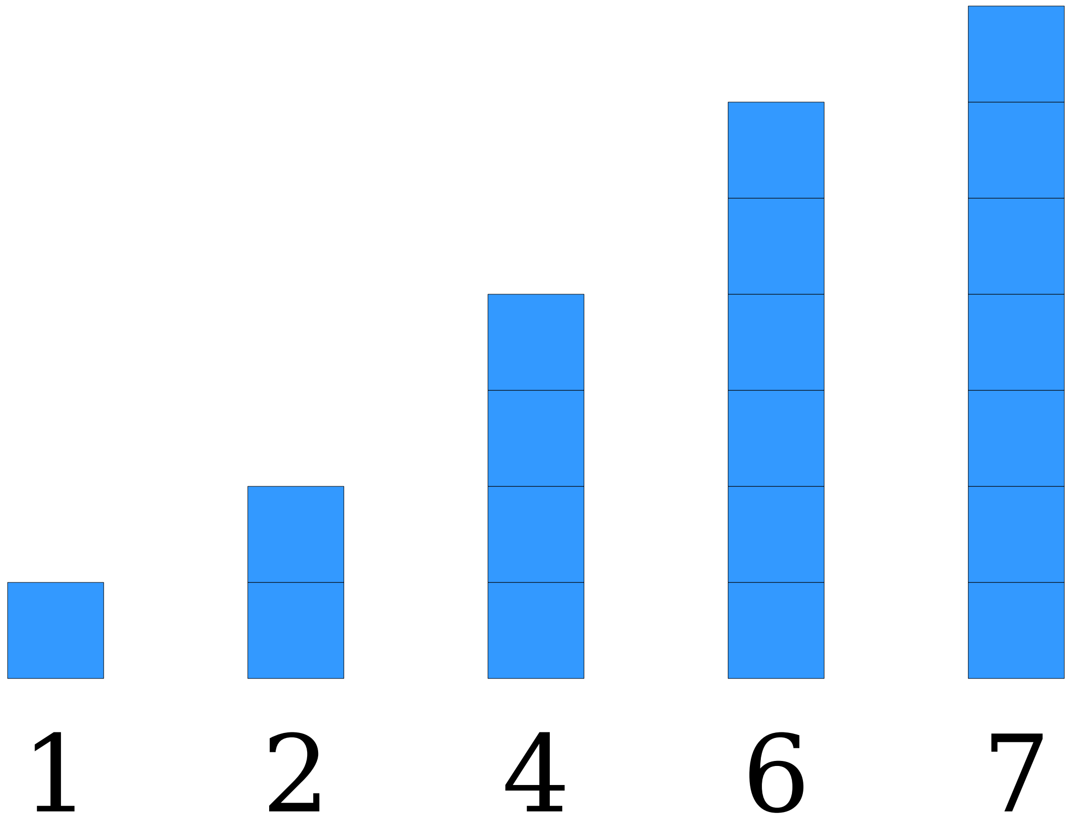


This sequence in blue, taken in isolation, is in sorted order.

There are no more gray elements, so the sequence is sorted!

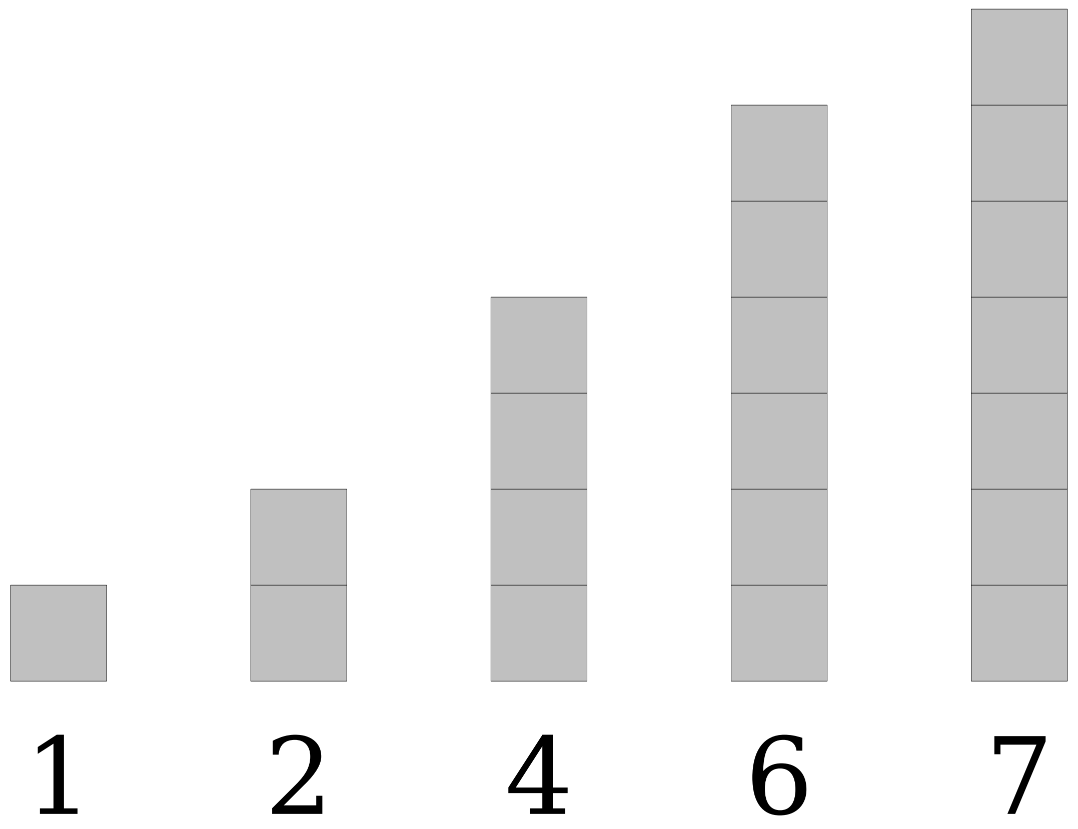
Insertion Sort

- Repeatedly *insert* an element into a sorted sequence at the front of the array.
- To *insert* an element, swap it backwards until either
  - (1) it’s at least as big as the element before it in the sequence, or
  - (2) it’s at the front of the array.

```
/**
 * Sorts the specified vector using insertion sort.
 *
 * @param v The vector to sort.
 */
void insertionSort(Vector<int>& v) {
  for (int i = 0; i < v.size(); i++) {
    /* Scan backwards until either (1) there is no
     * preceding element or the preceding element is
     * no bigger than us.
     */
    for (int j = i - 1; j >= 0; j--) {
      if (v[j] <= v[j + 1]) break;

      /* Swap this element back one step. */
      swap(v[j], v[j + 1]);
    }
  }
}
```

How Fast is Insertion Sort?



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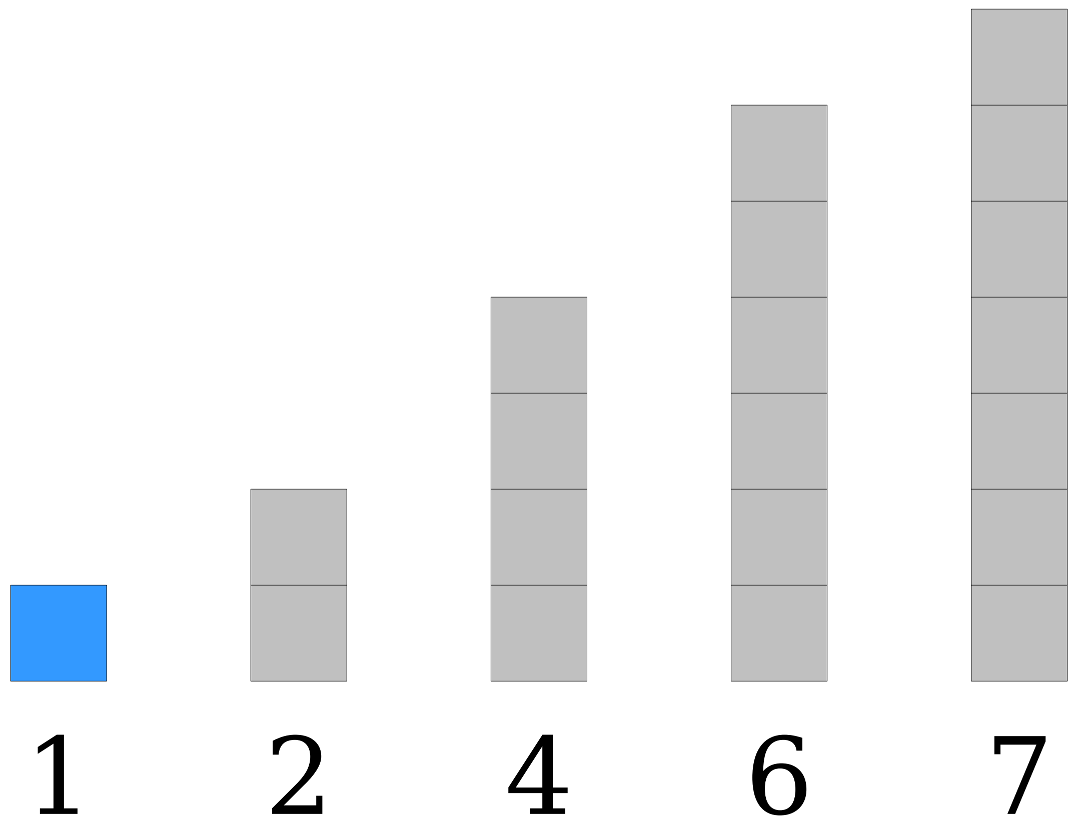

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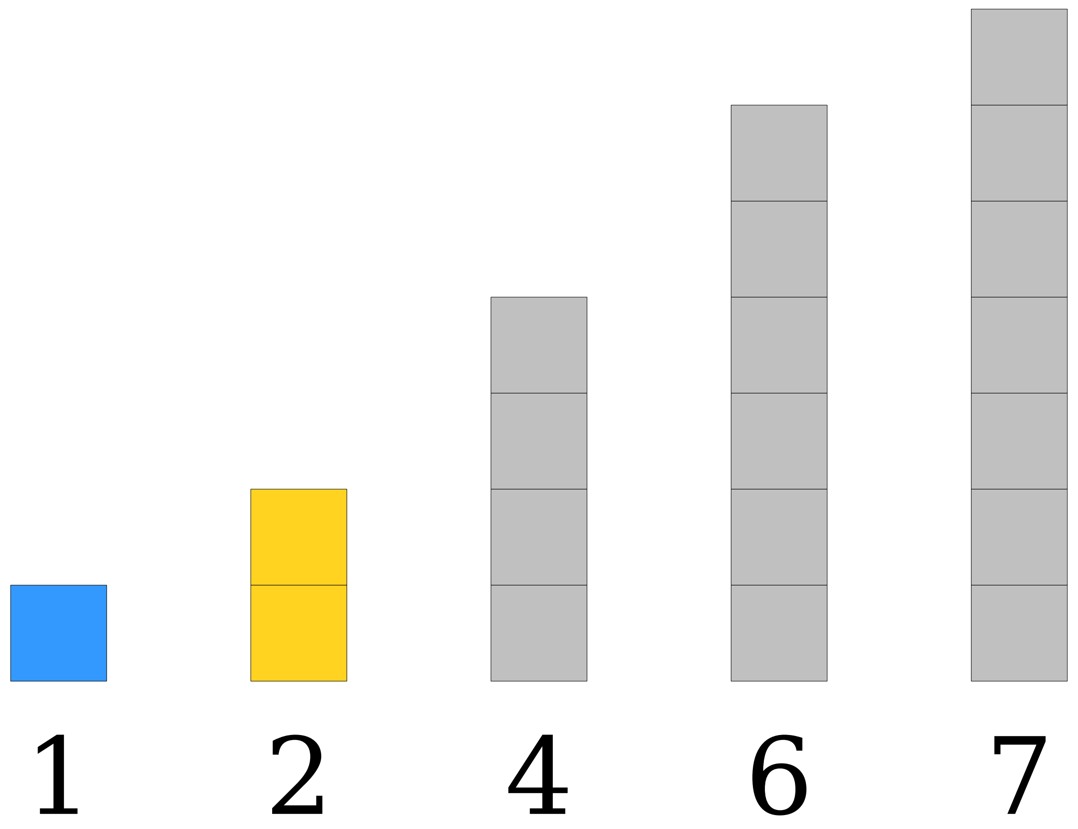

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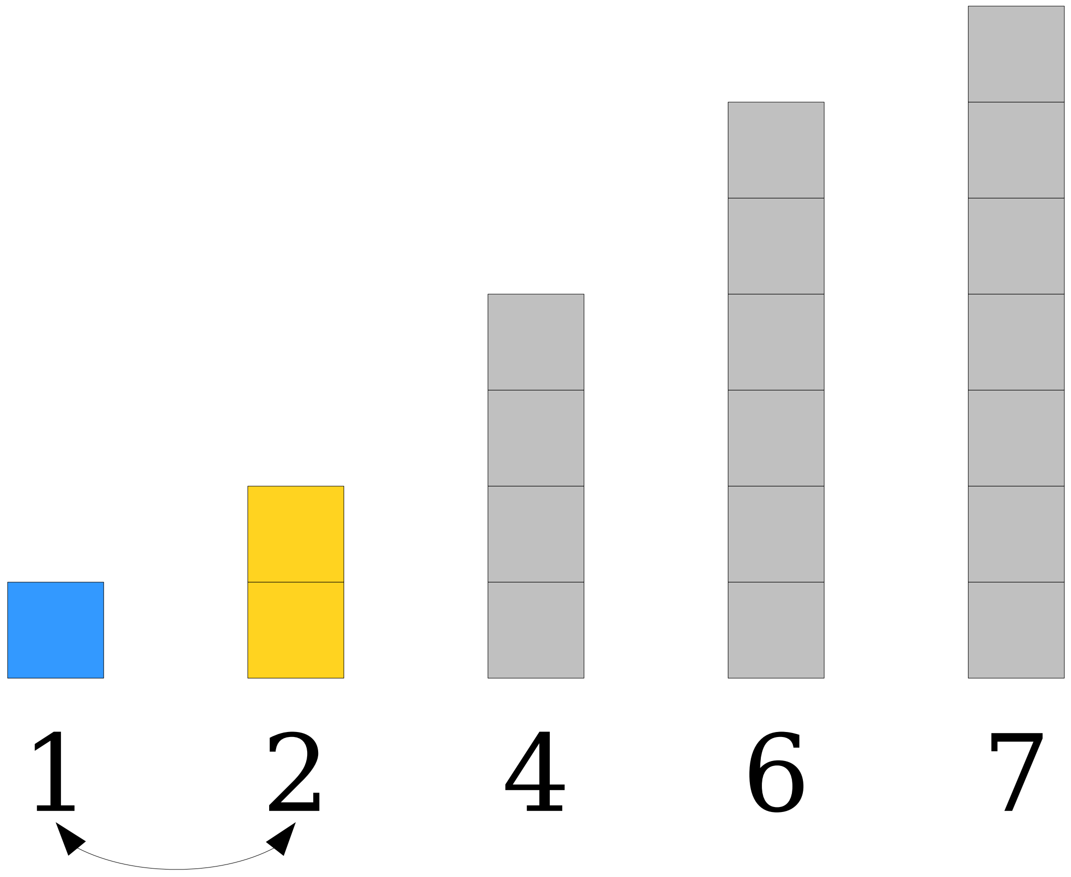

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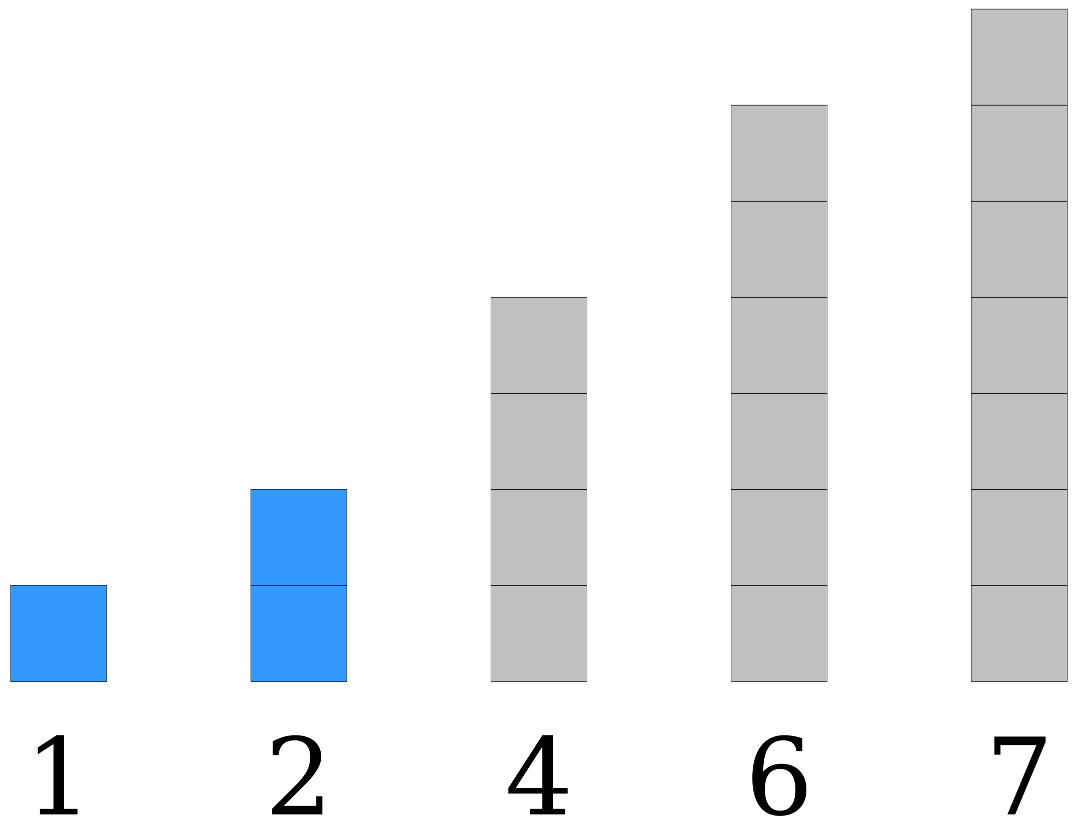

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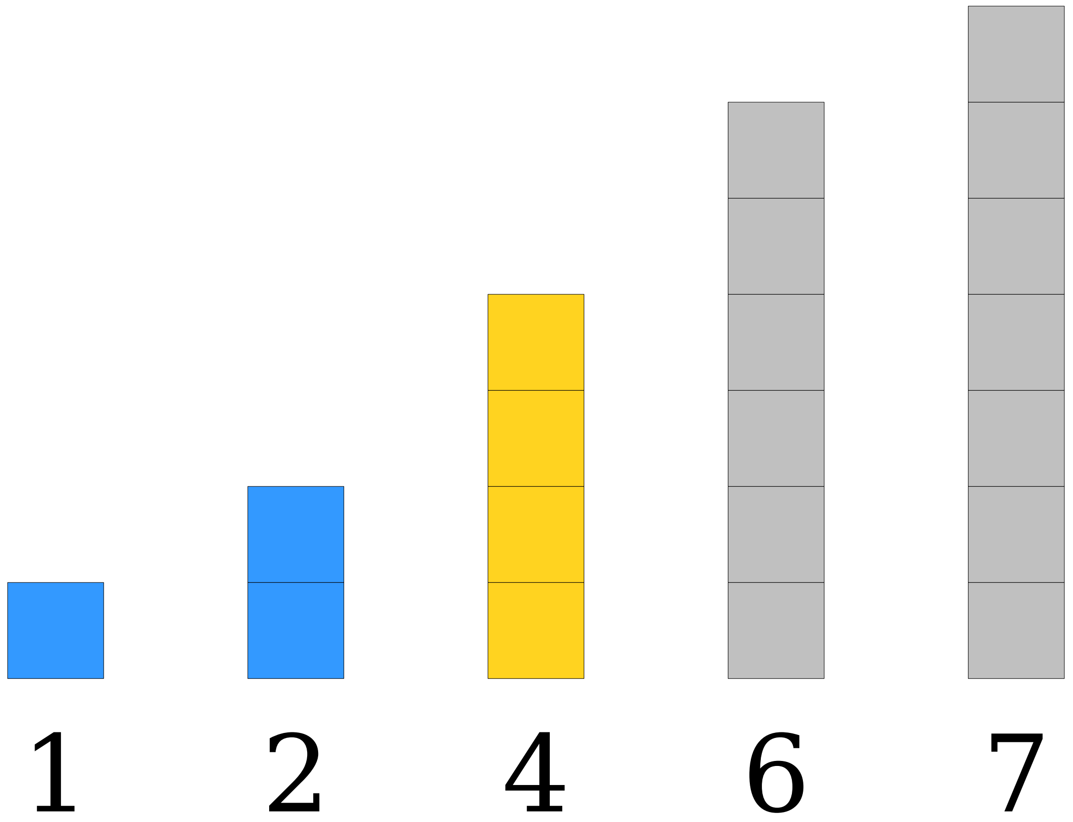

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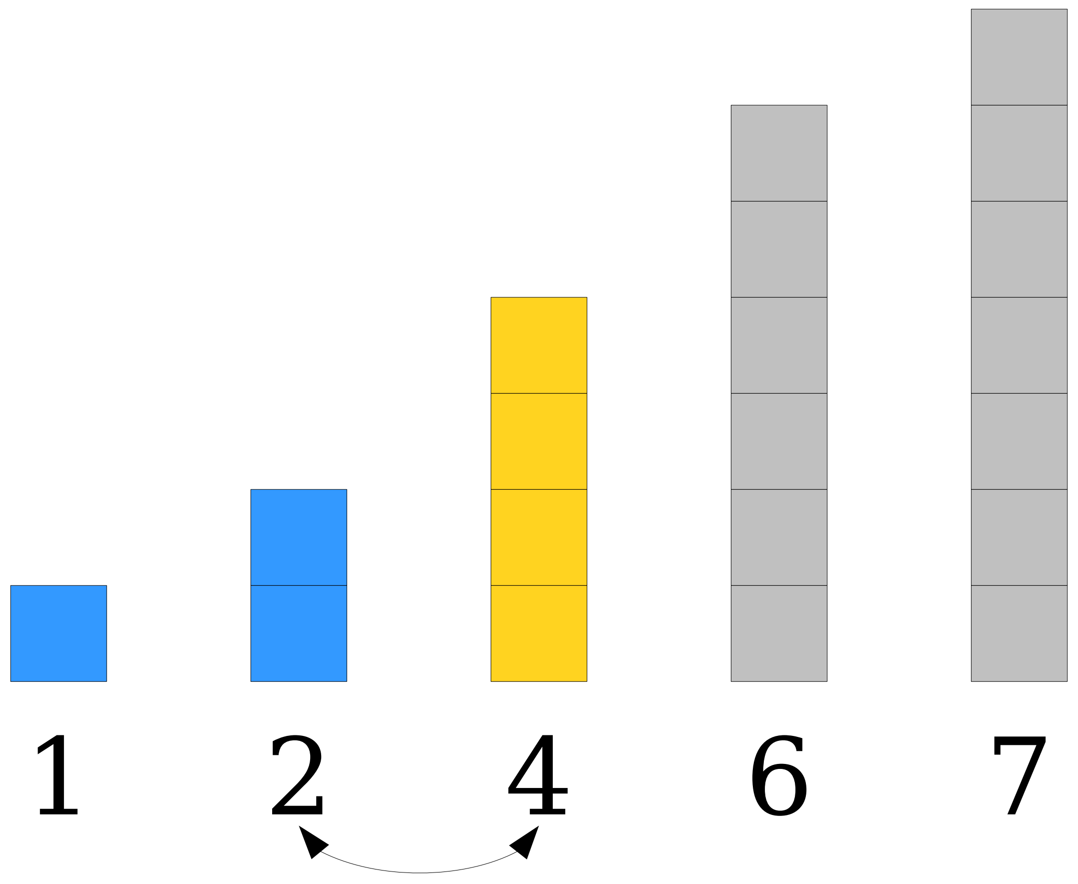

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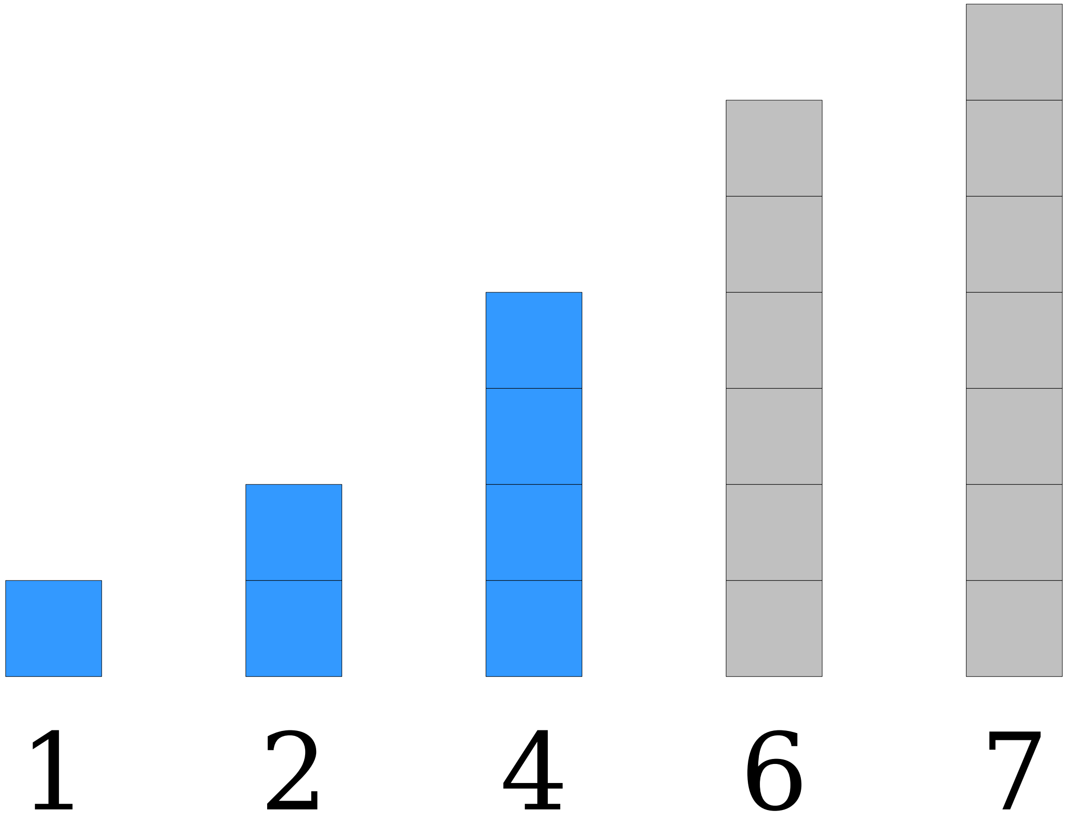

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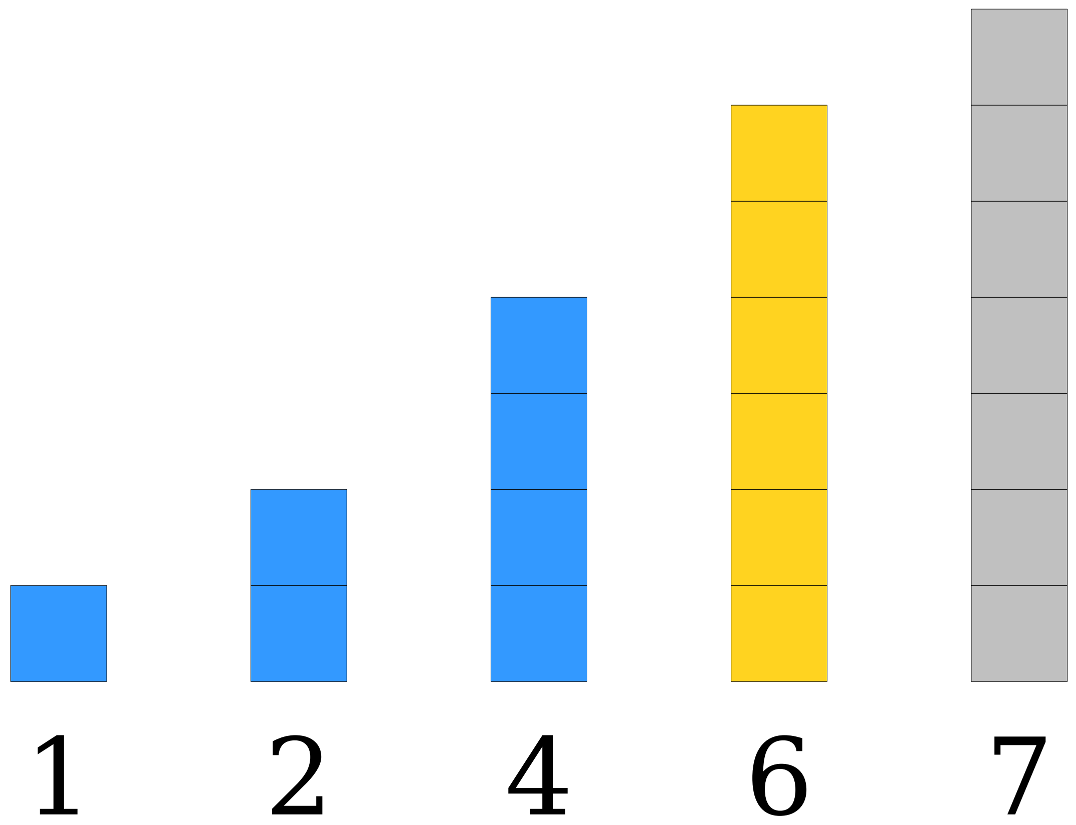

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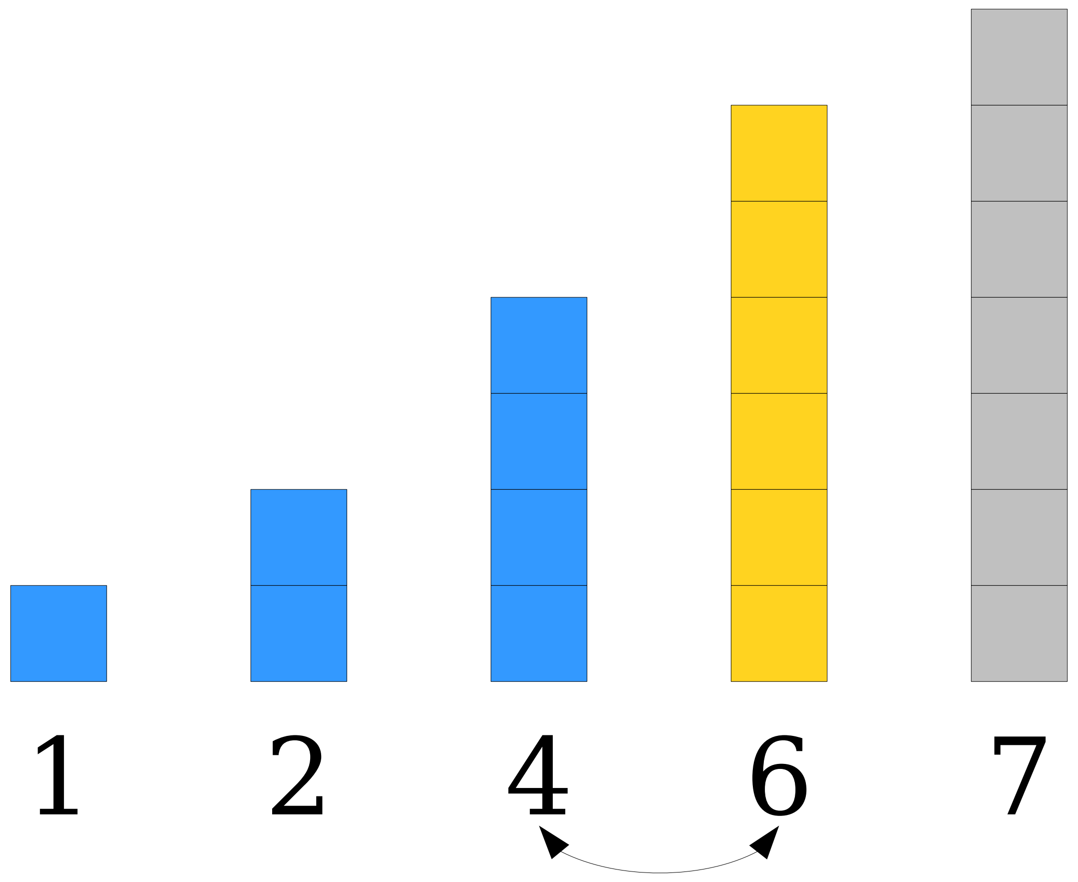

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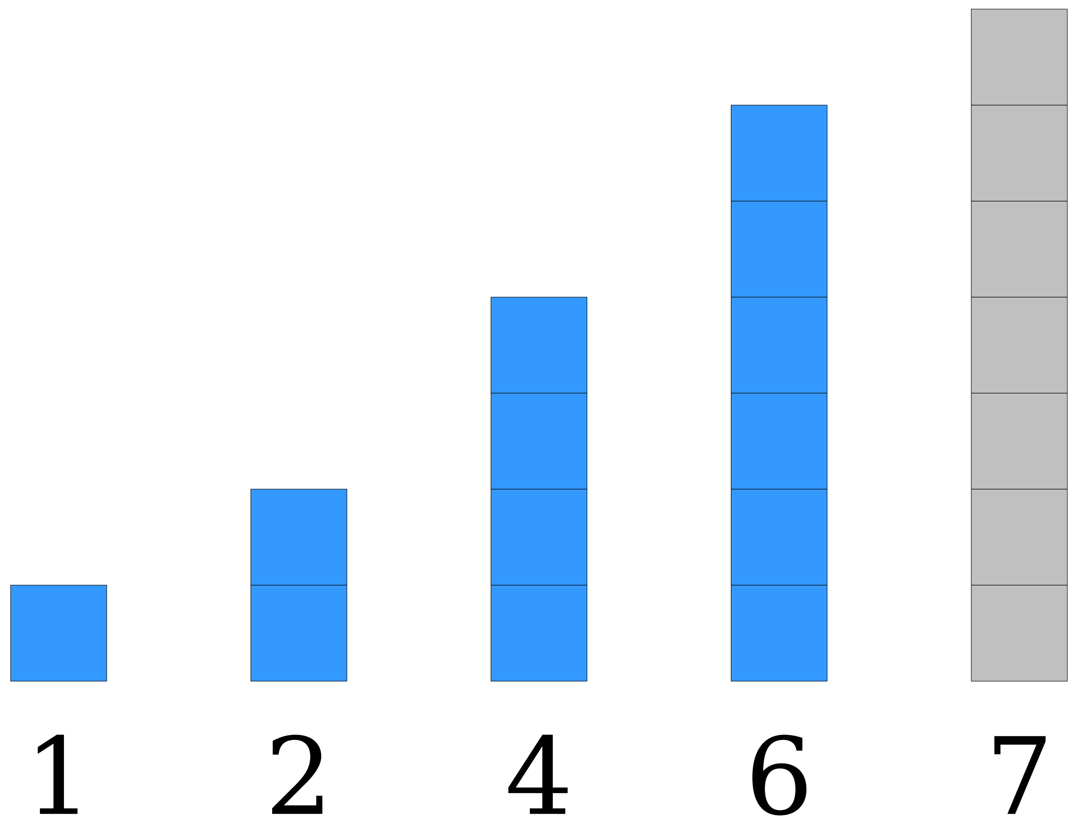

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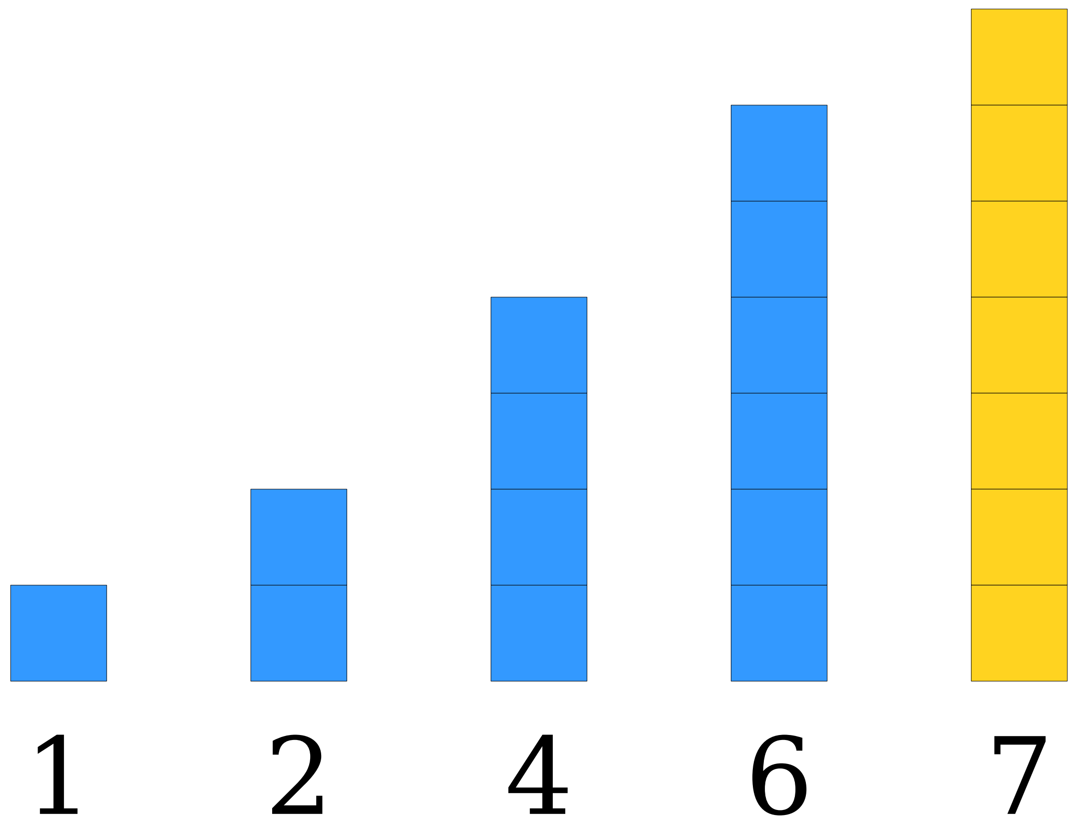

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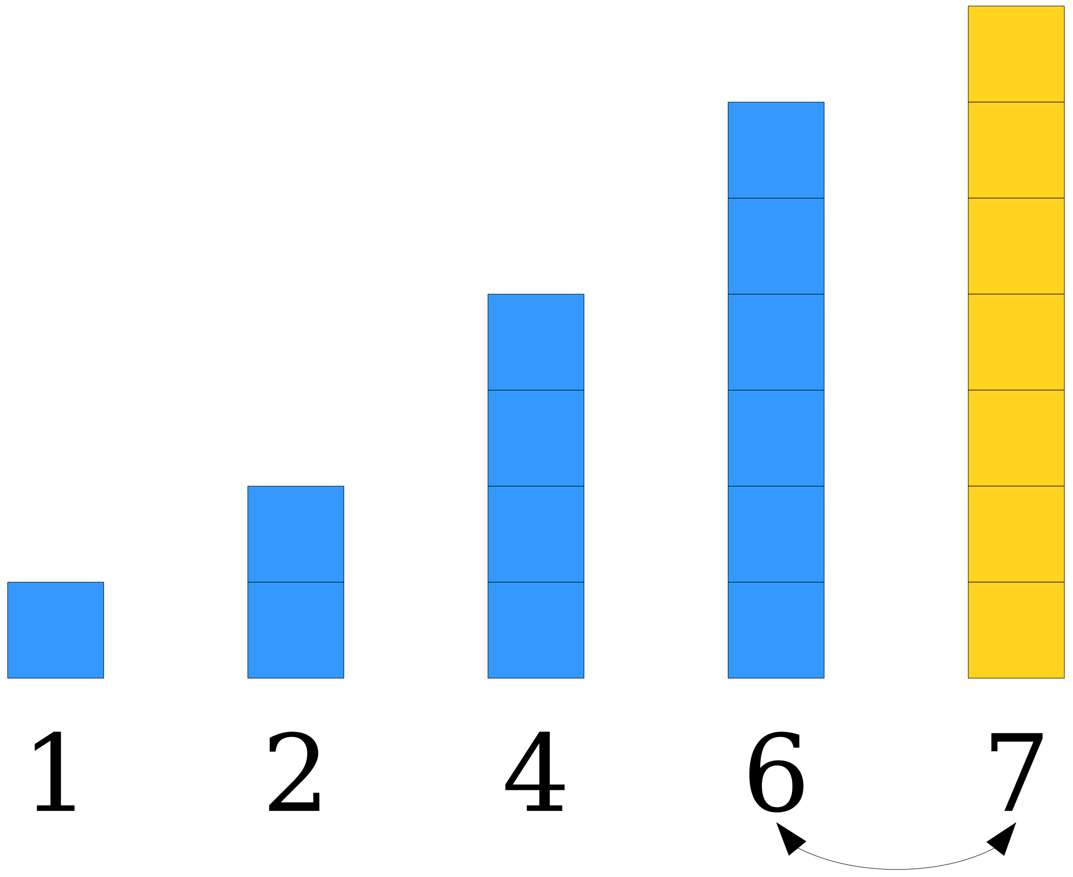

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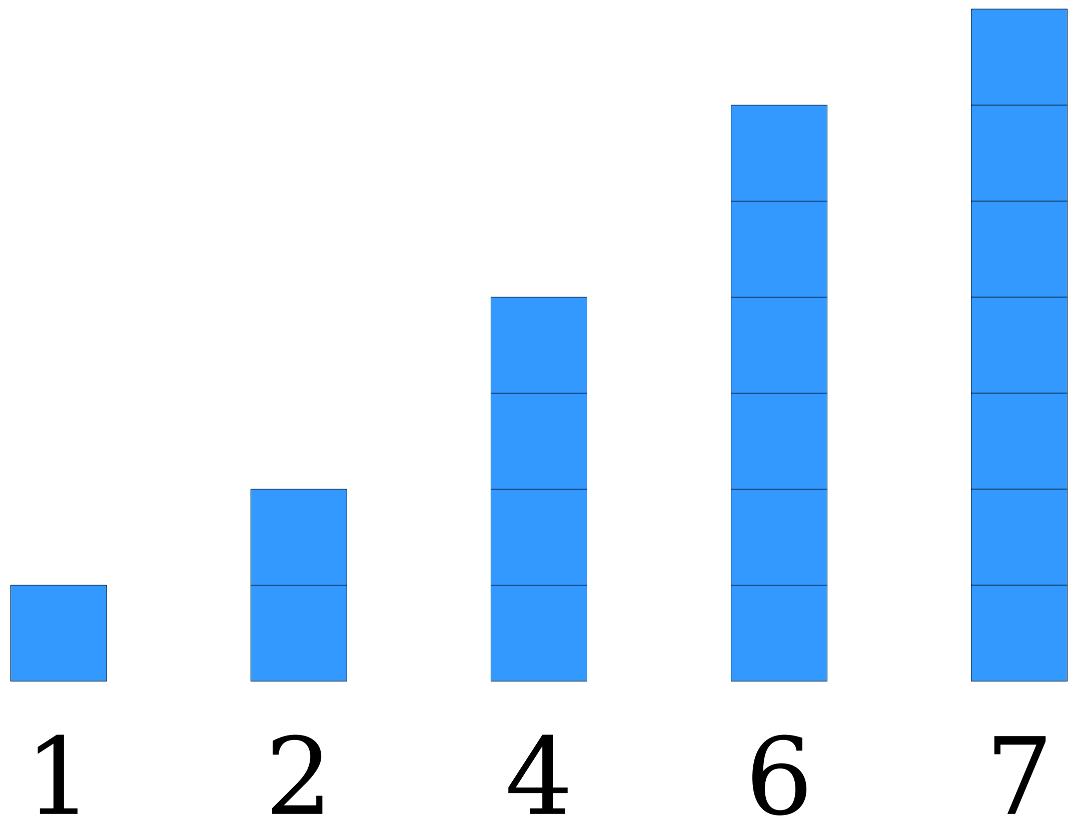

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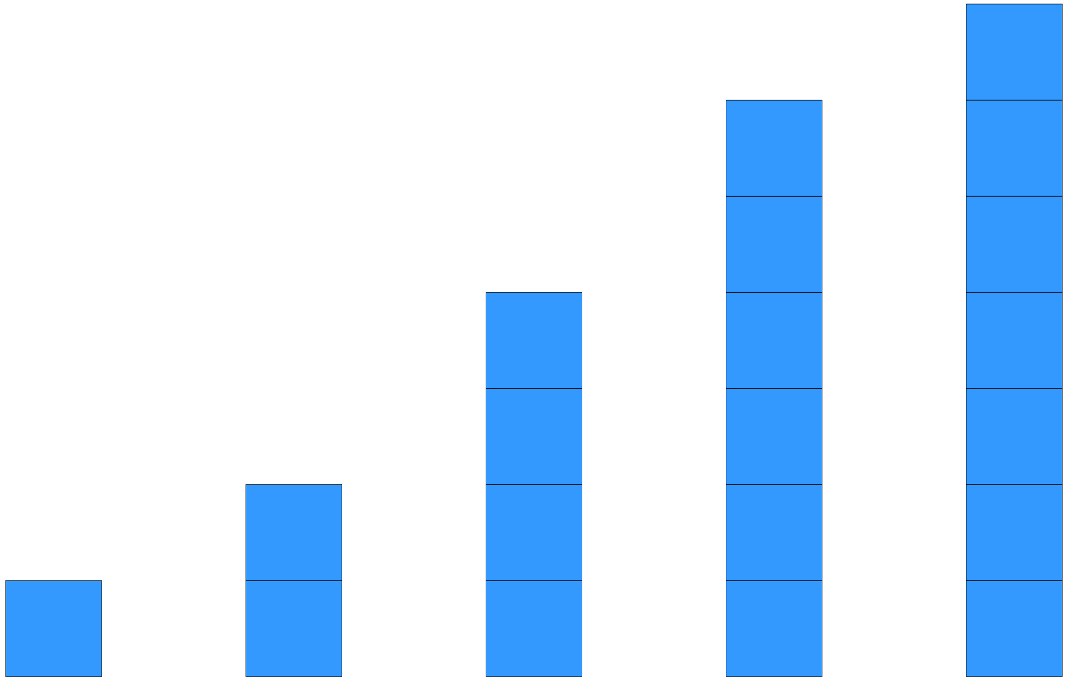
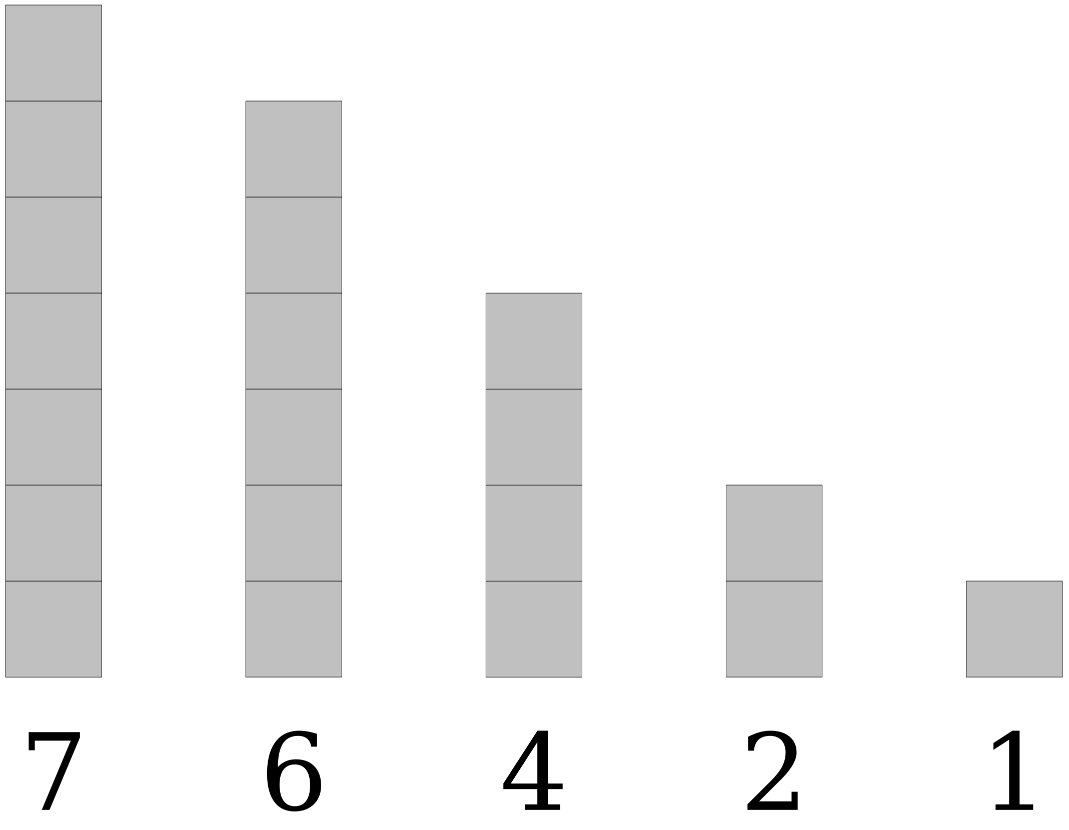


1 2 4 6 7

Work done: **O(*n*)**

How Fast is Insertion Sort?



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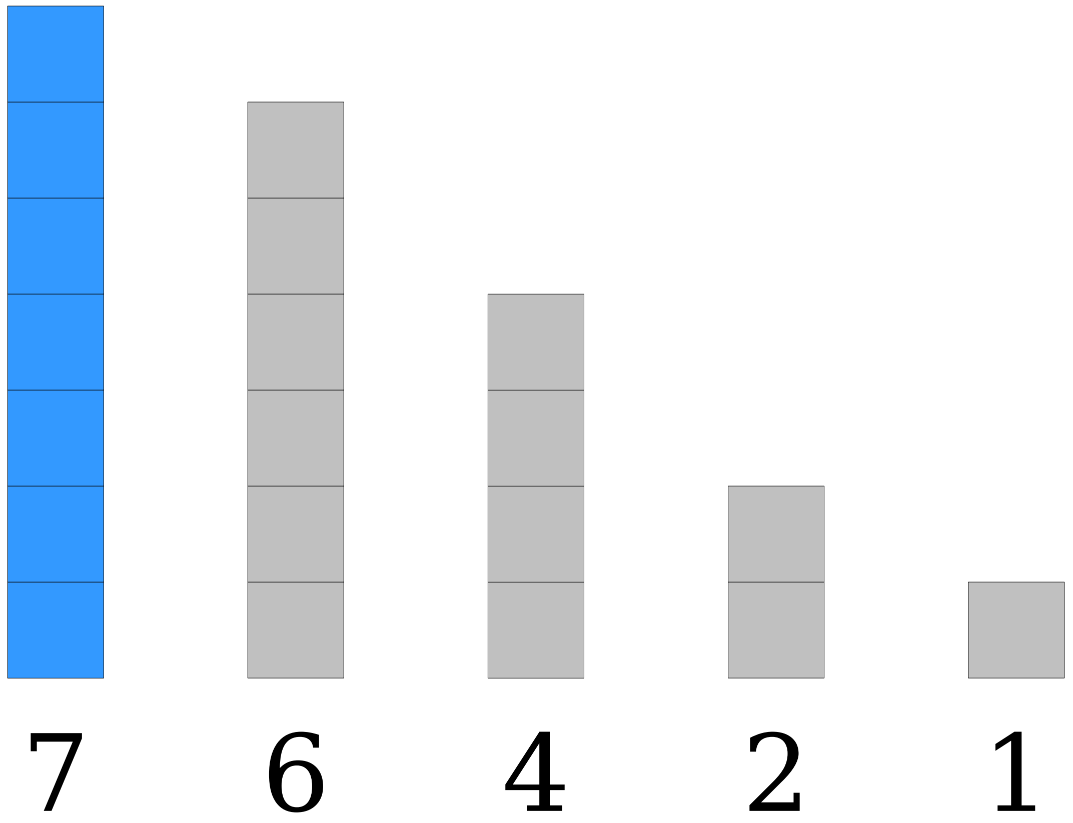

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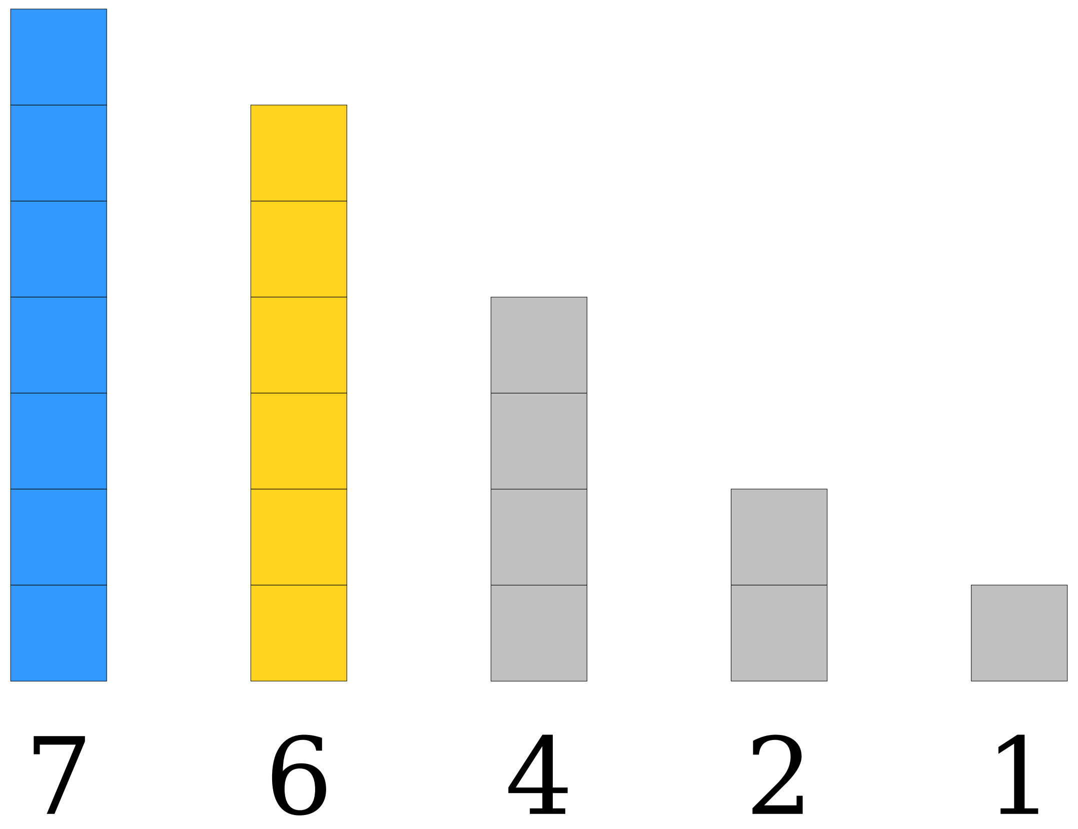

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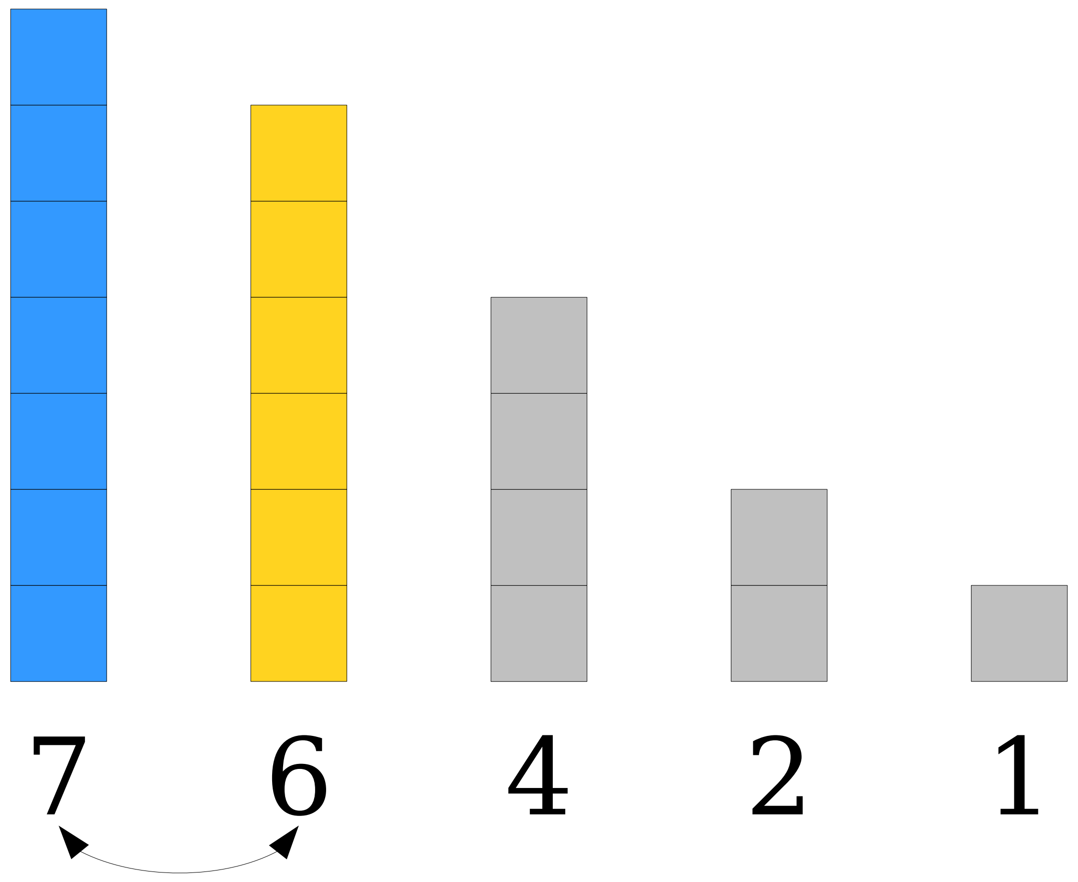

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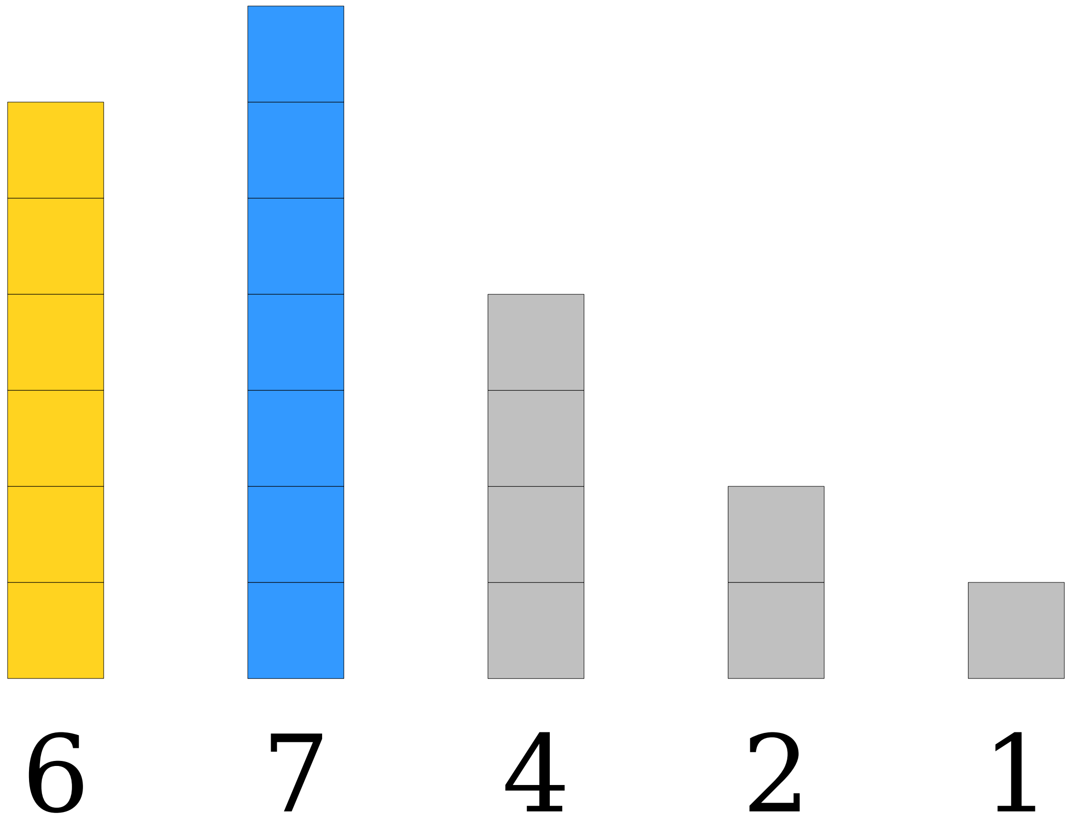

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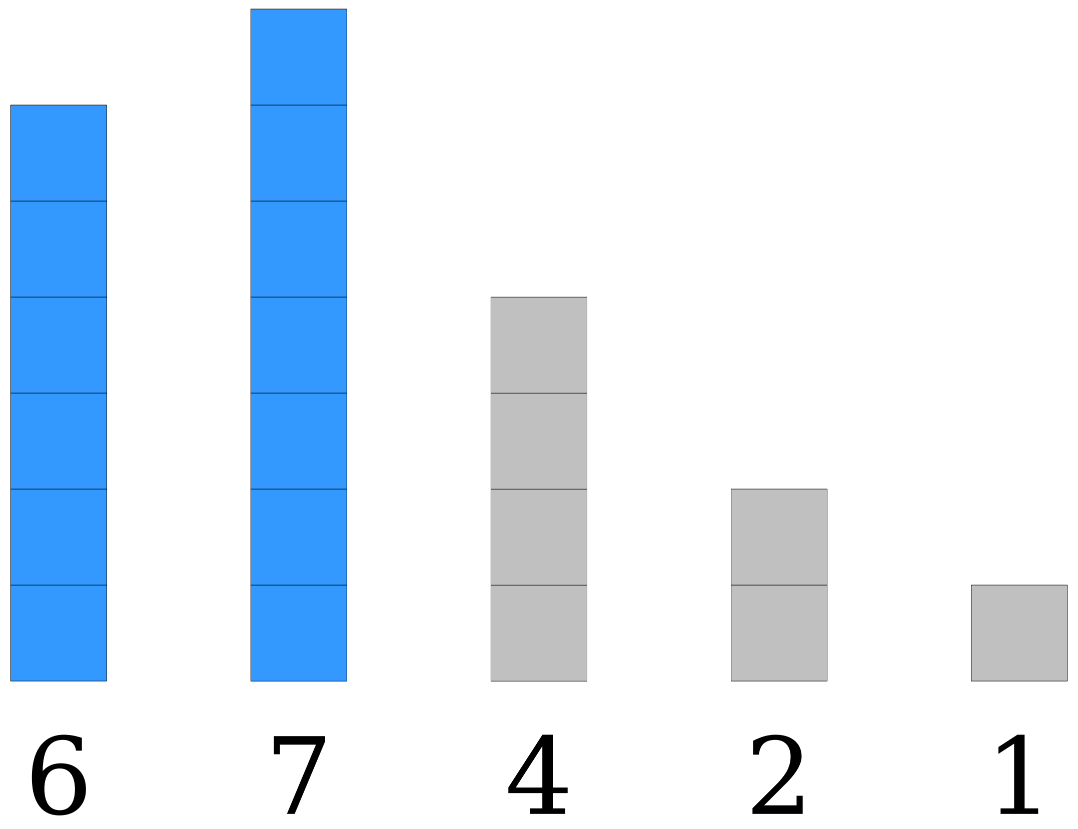

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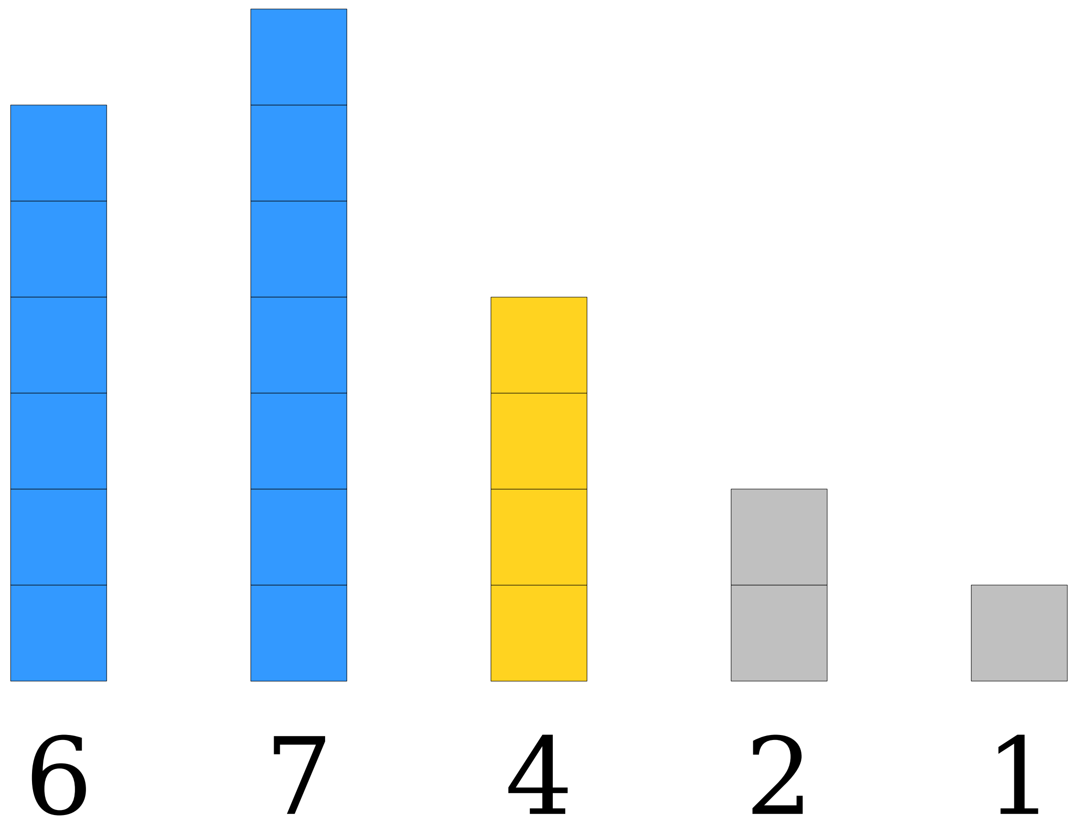

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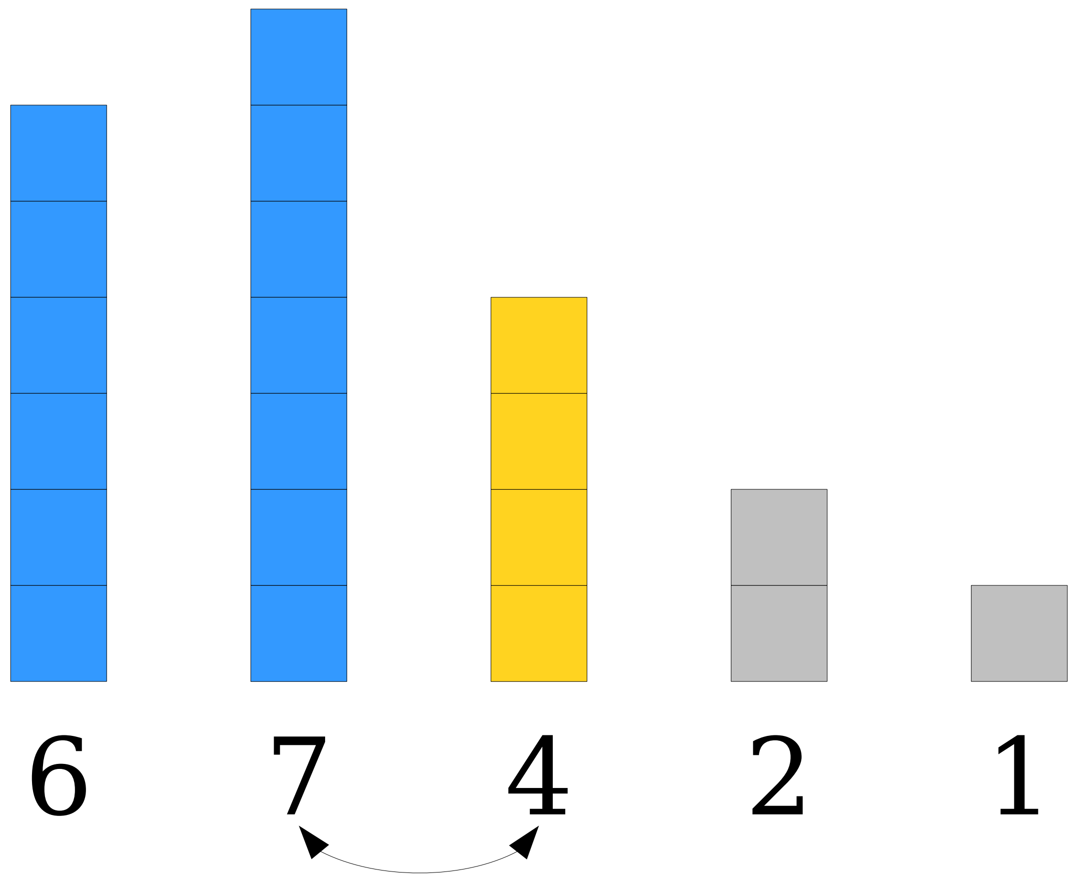

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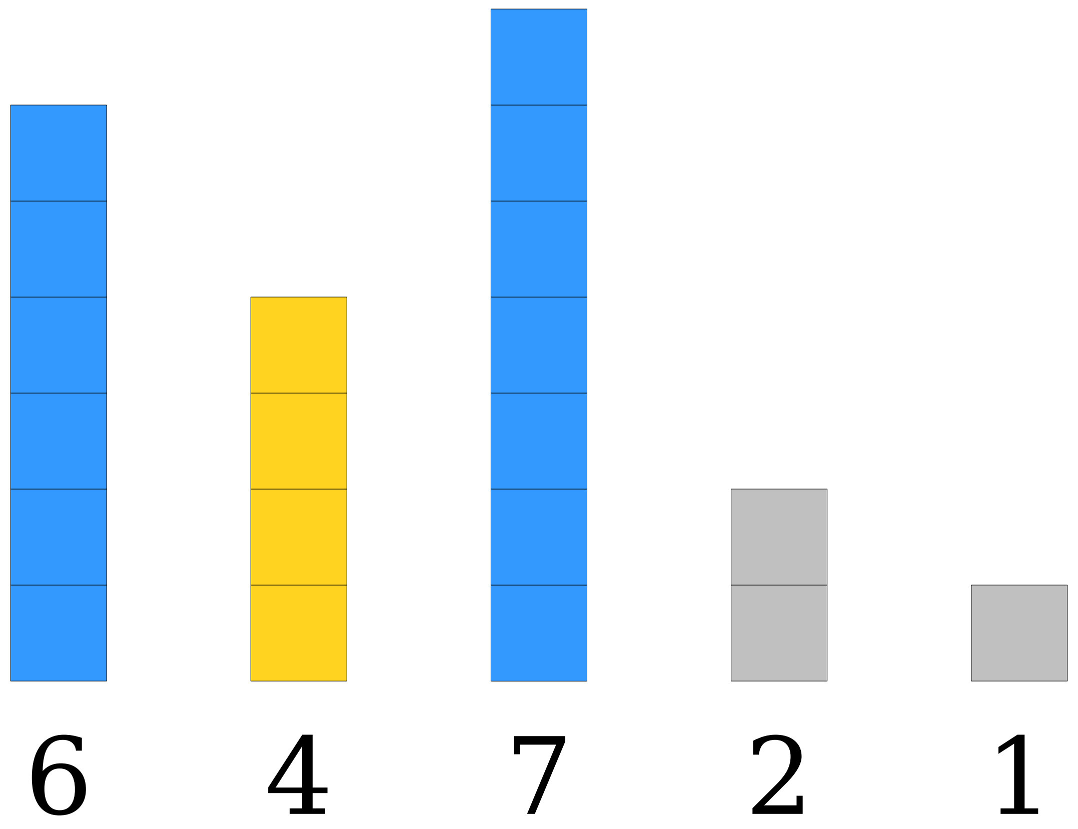

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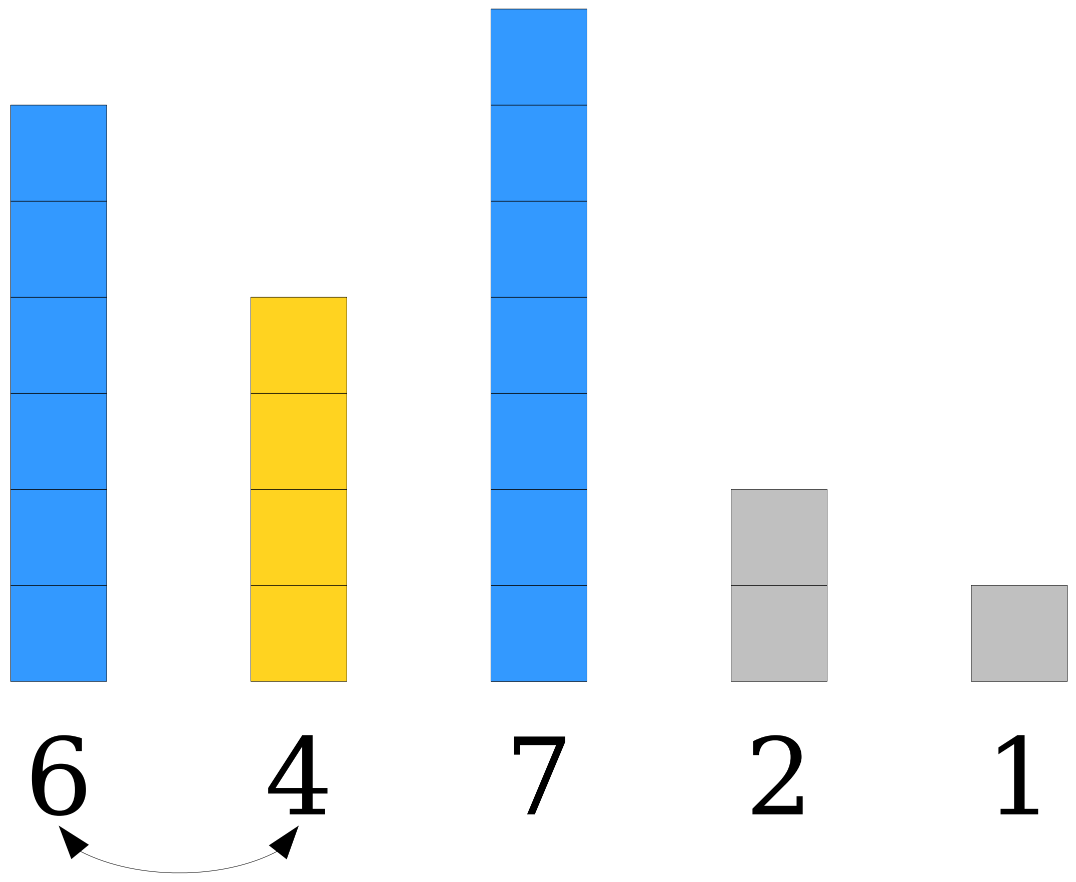

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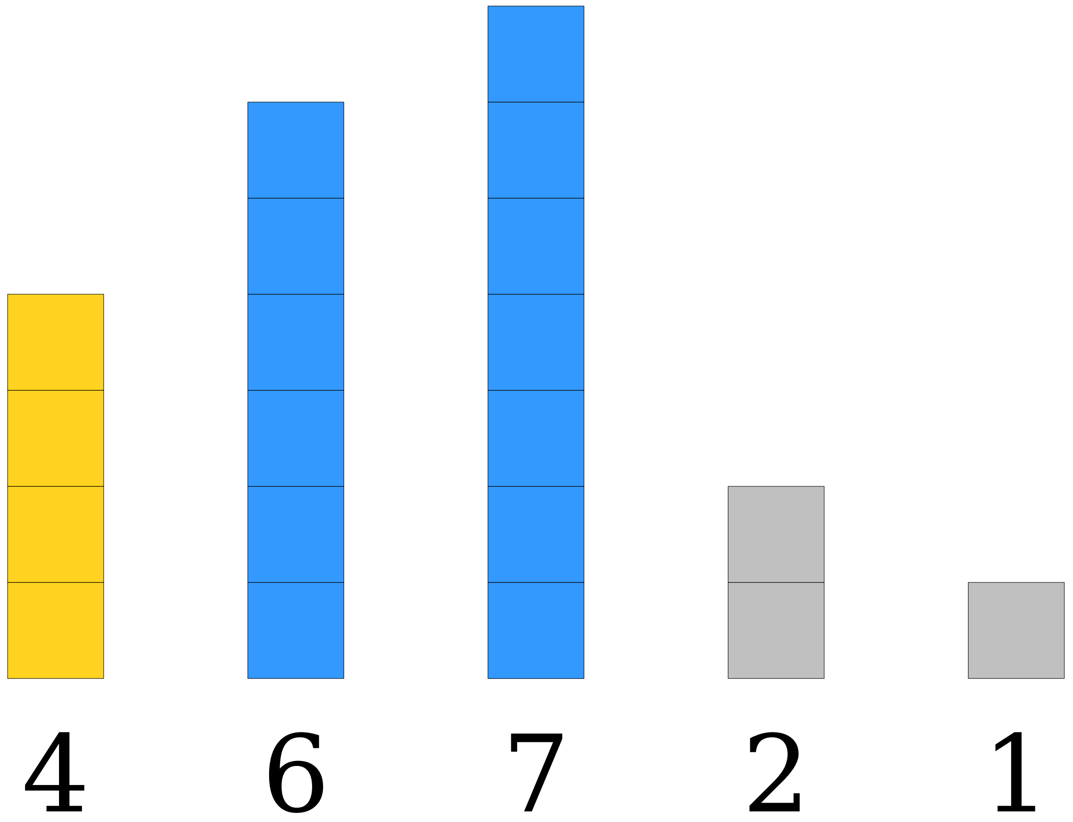

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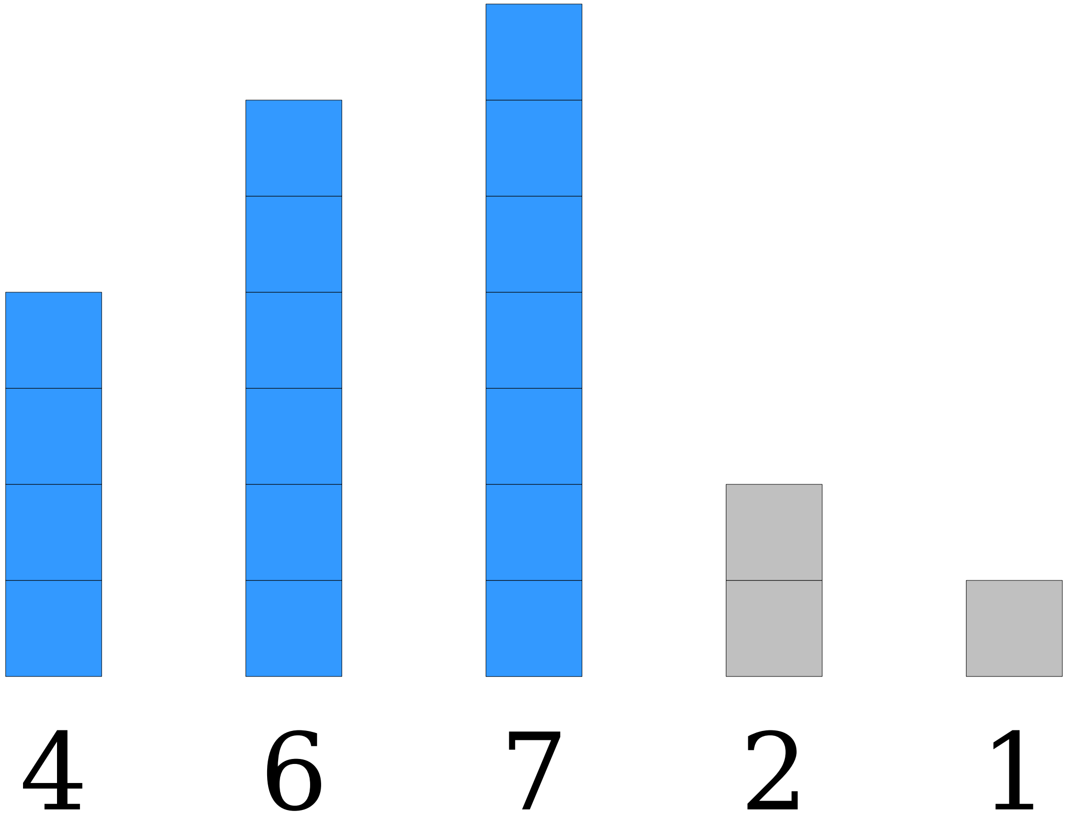

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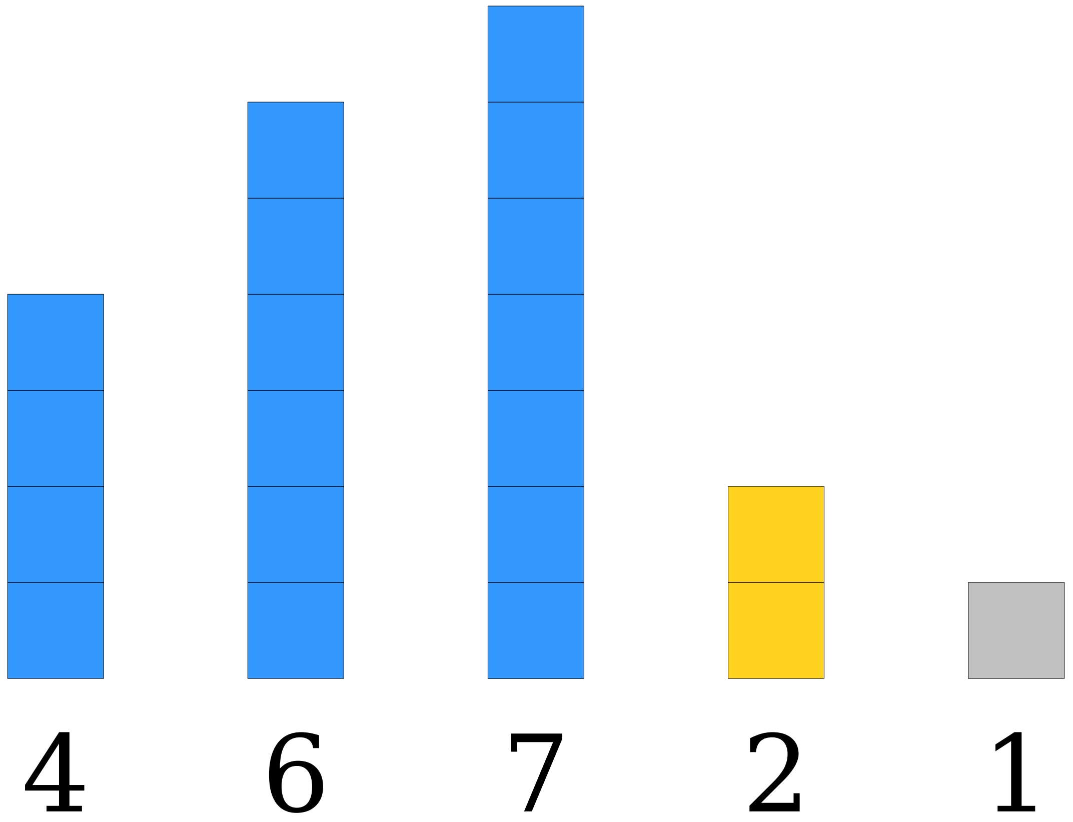

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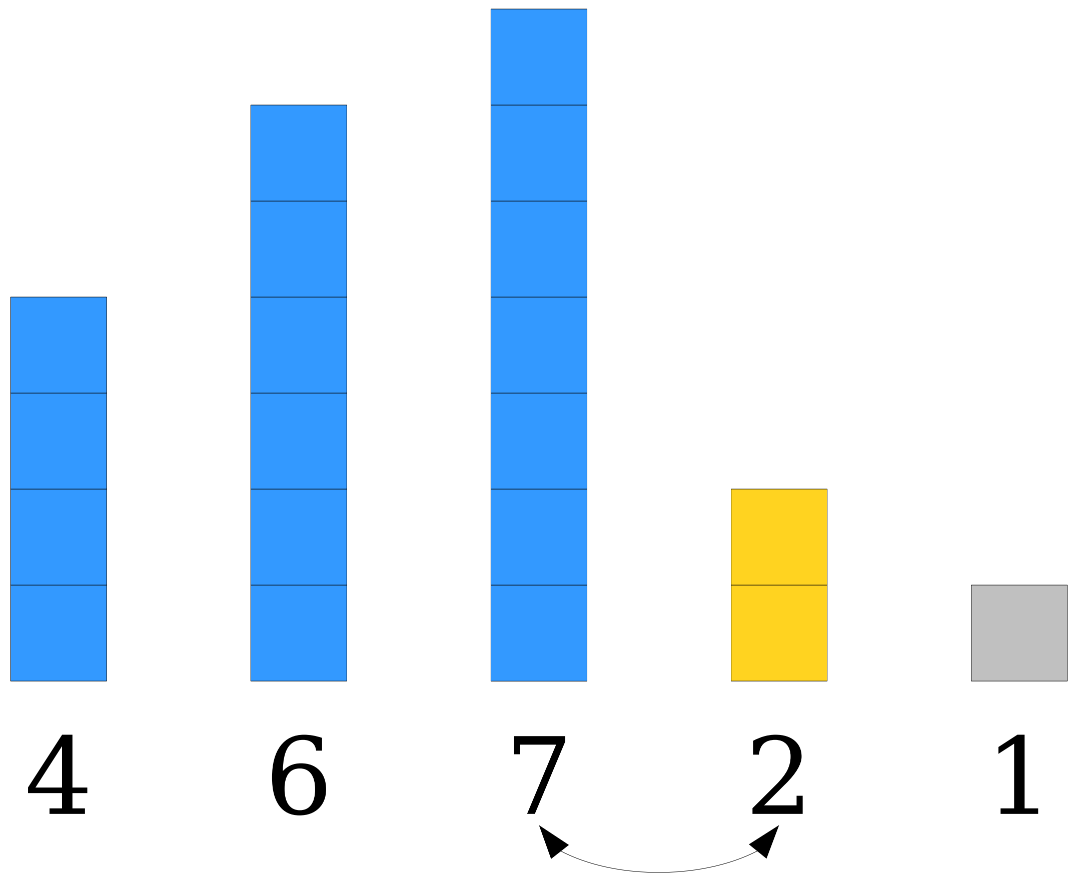

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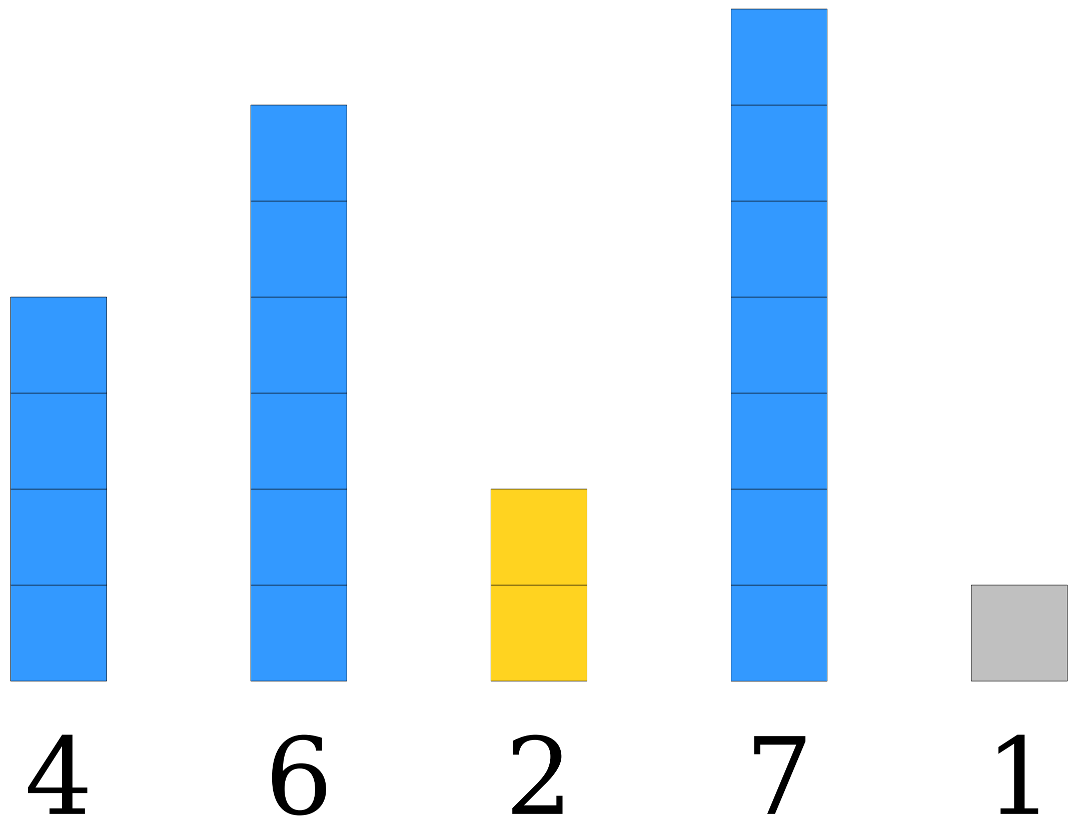

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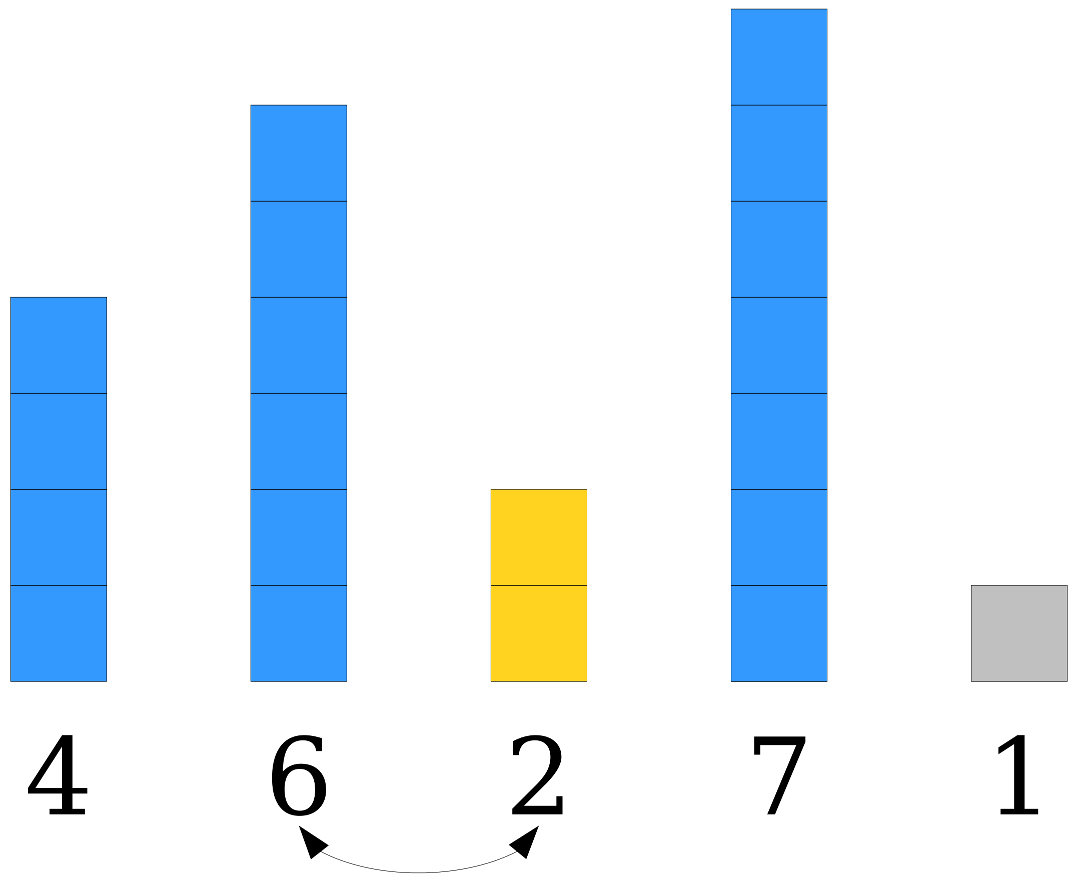

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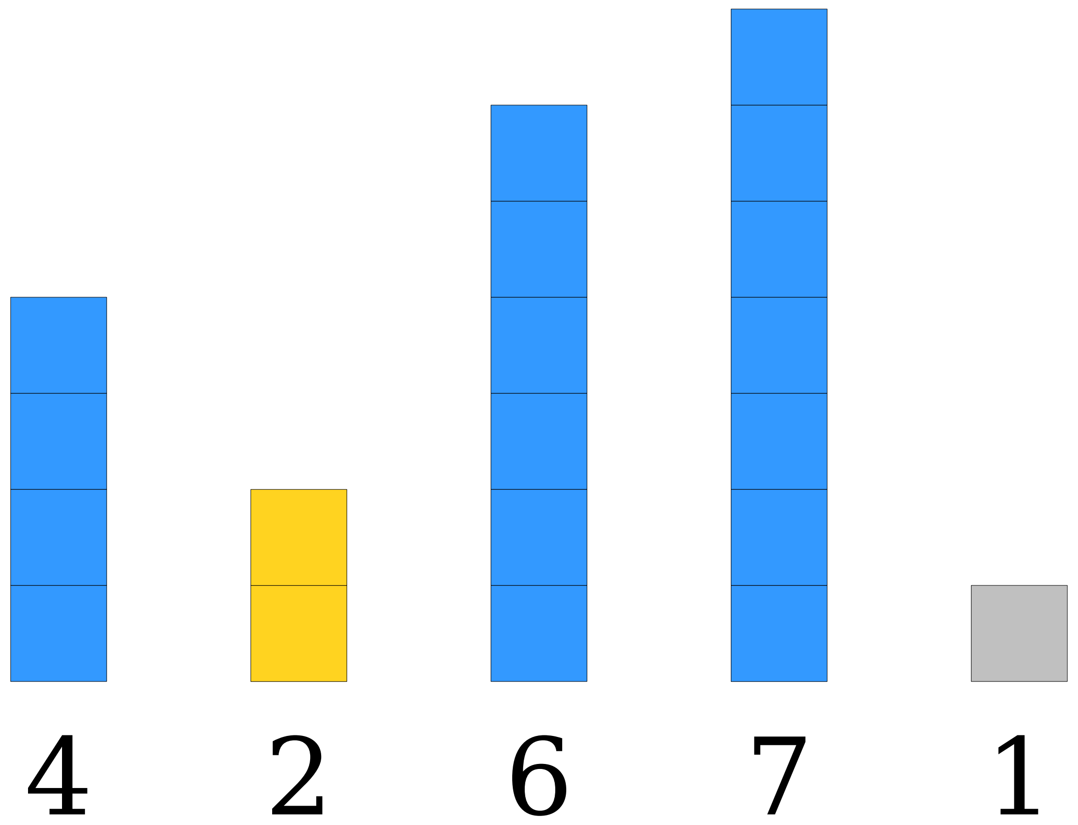

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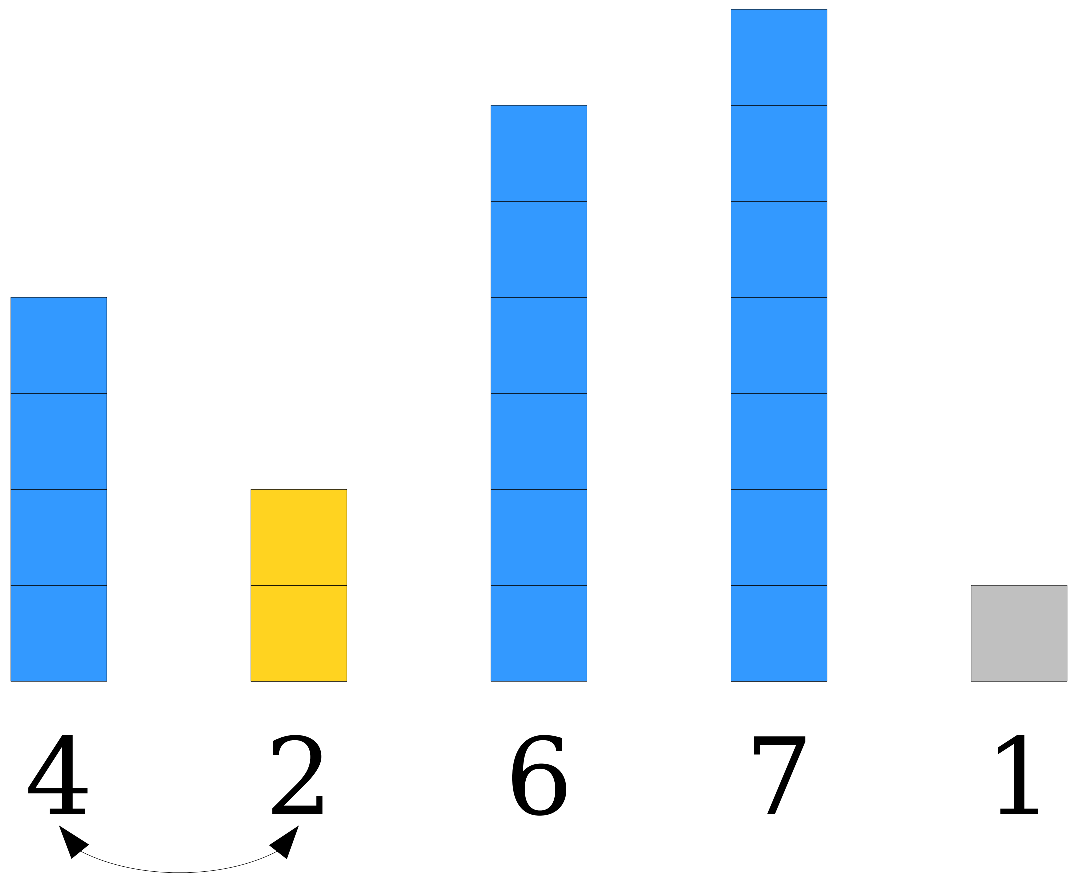

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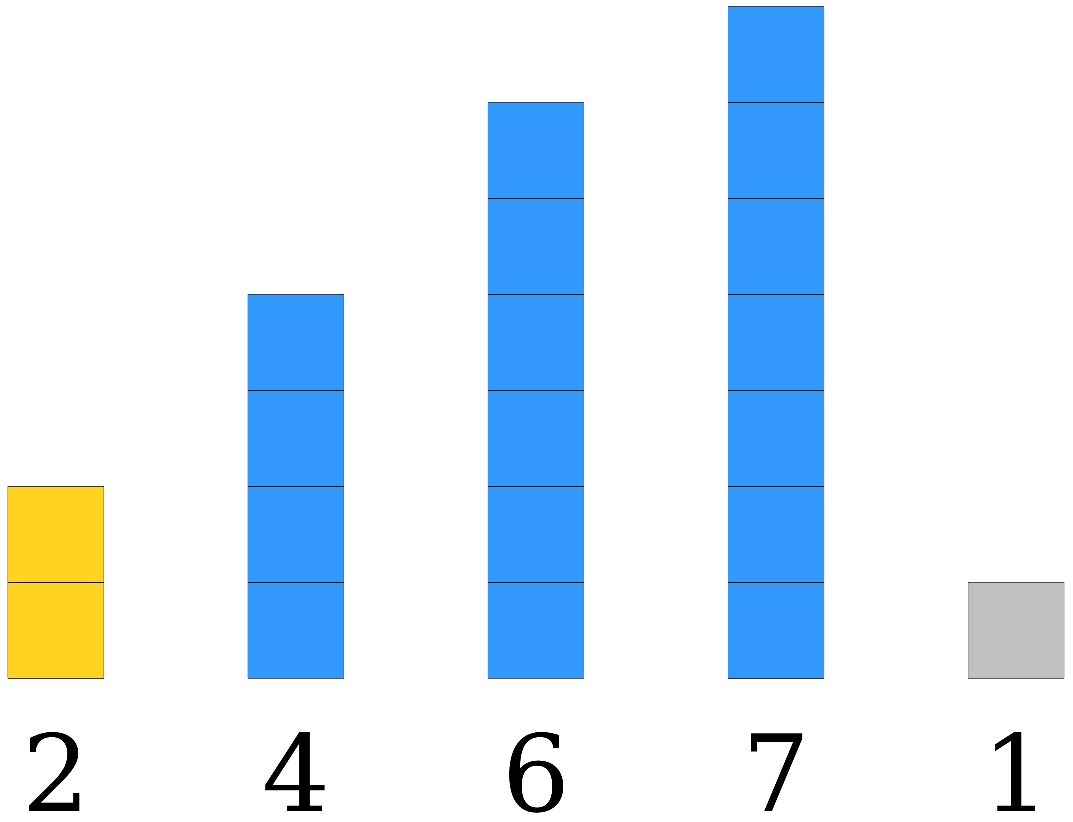

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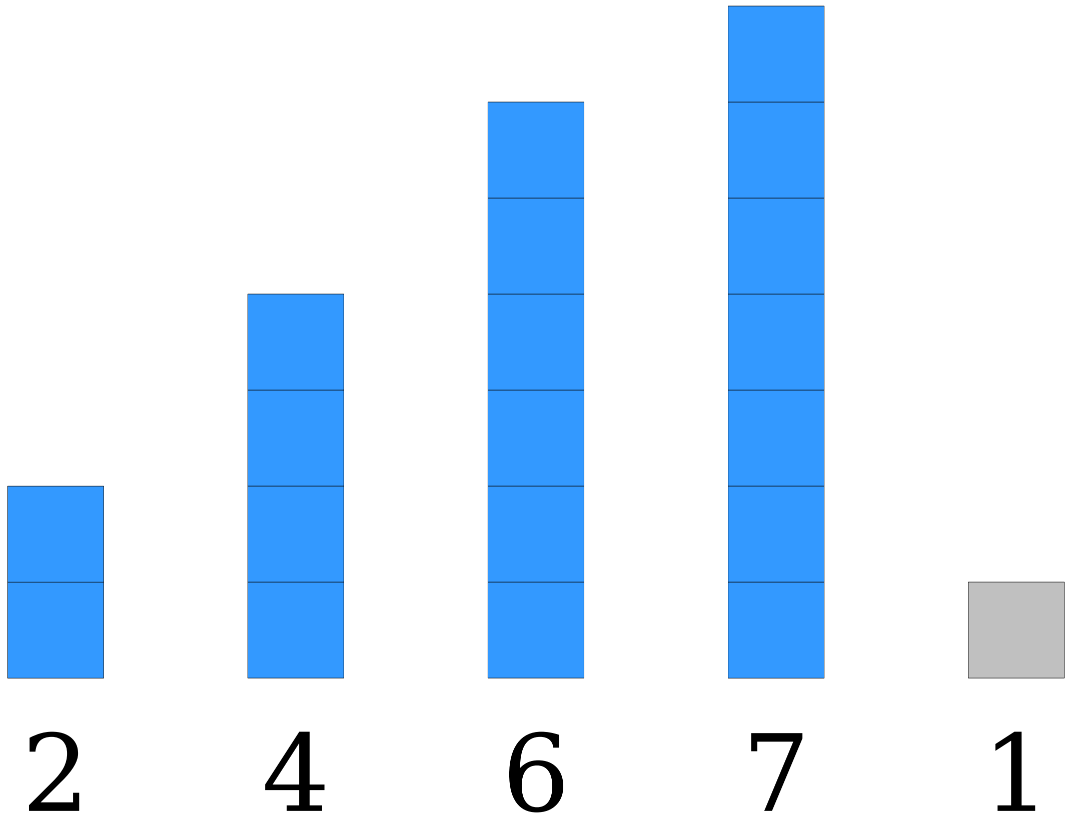

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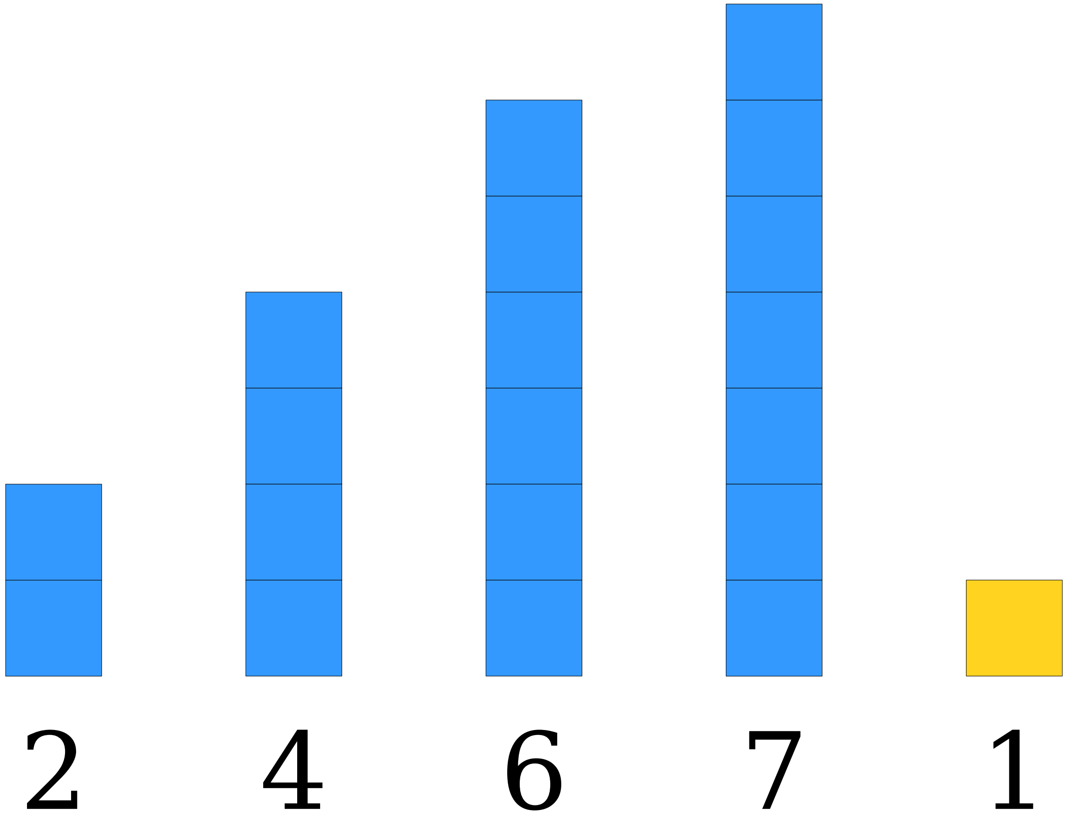

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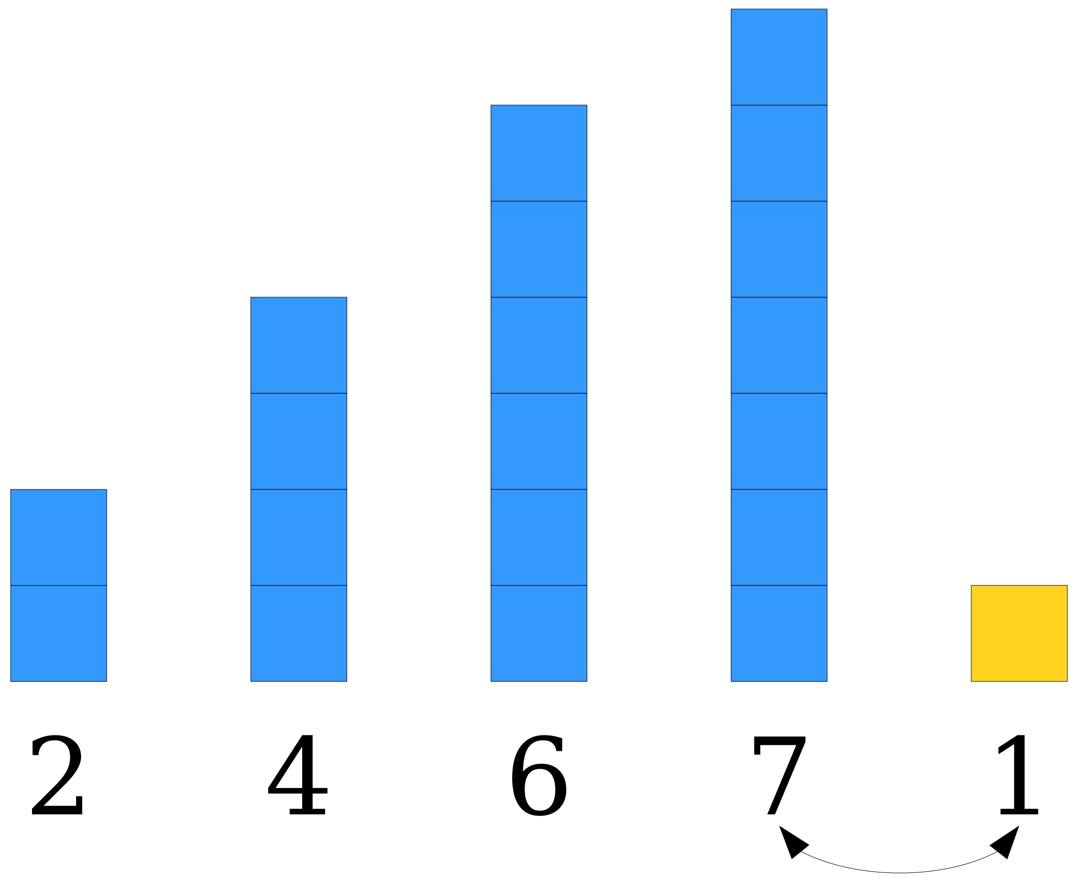

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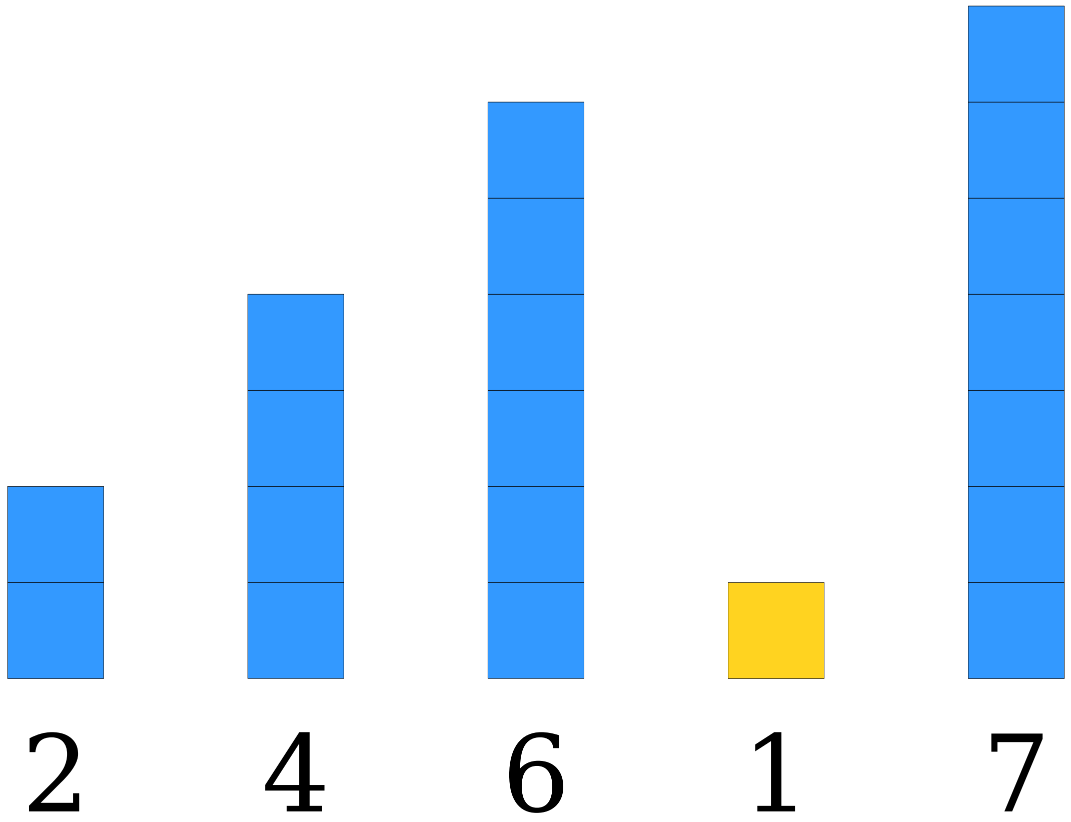

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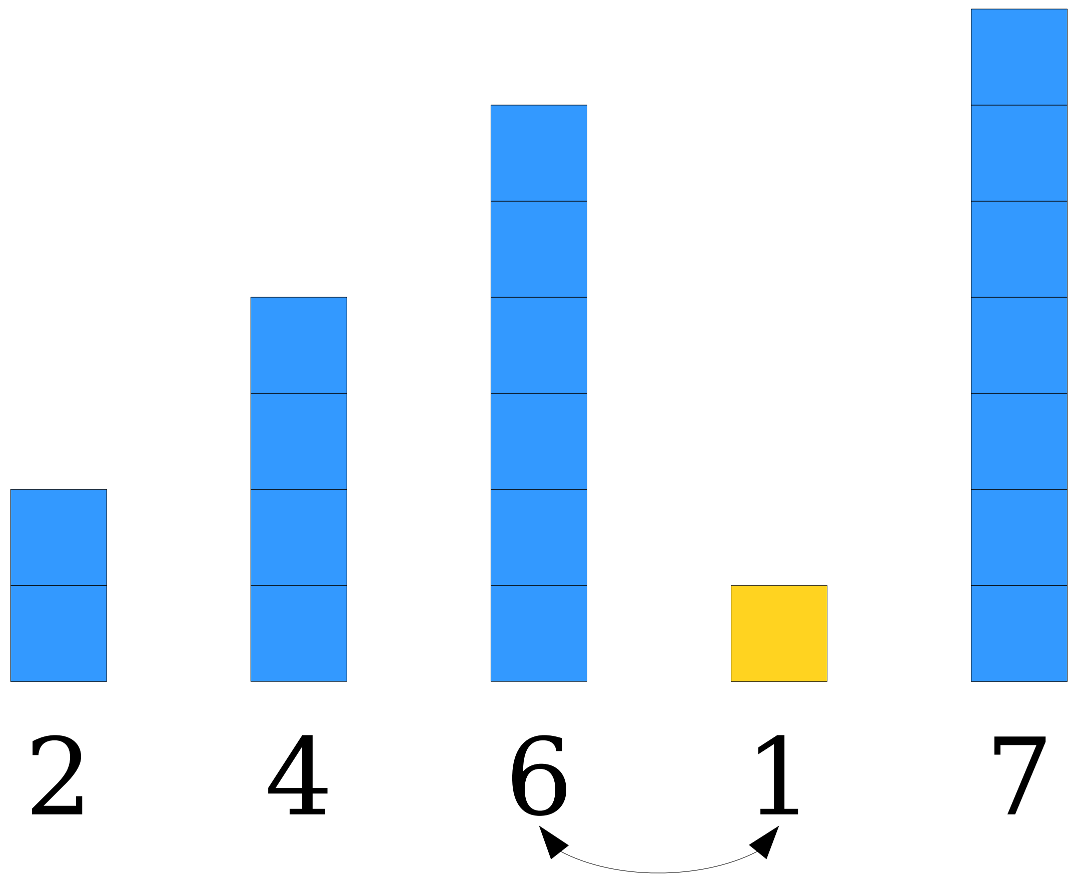

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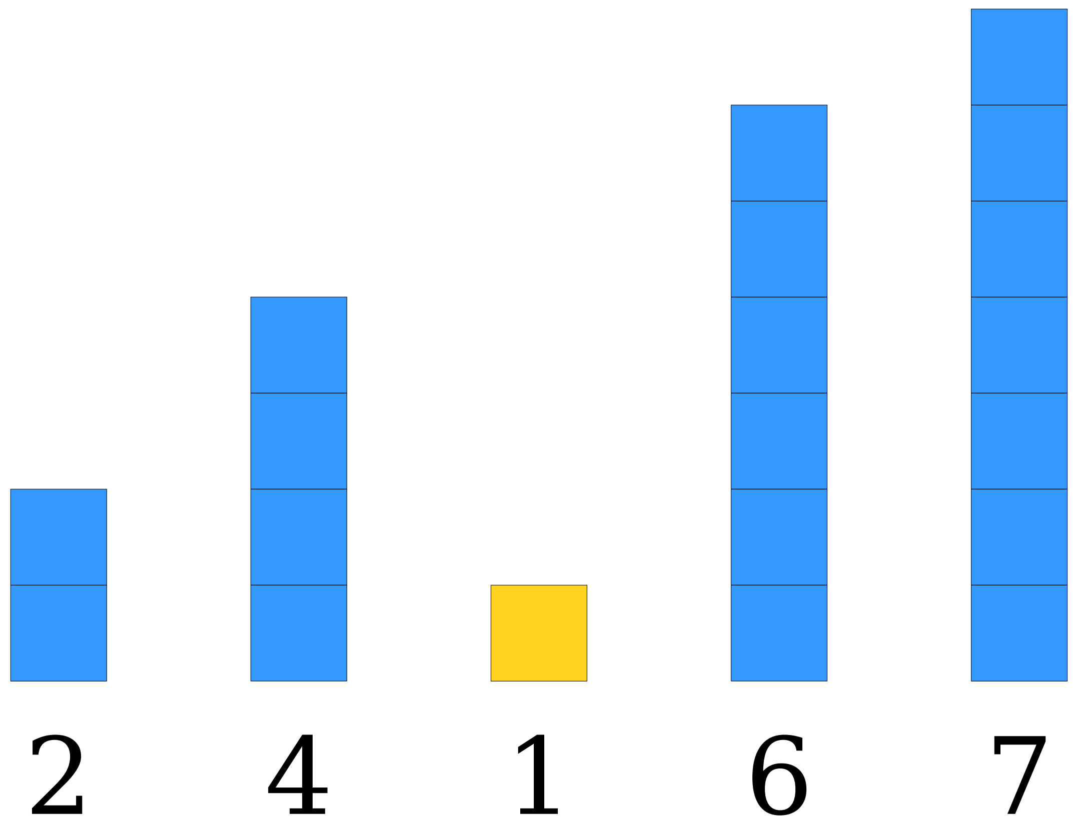

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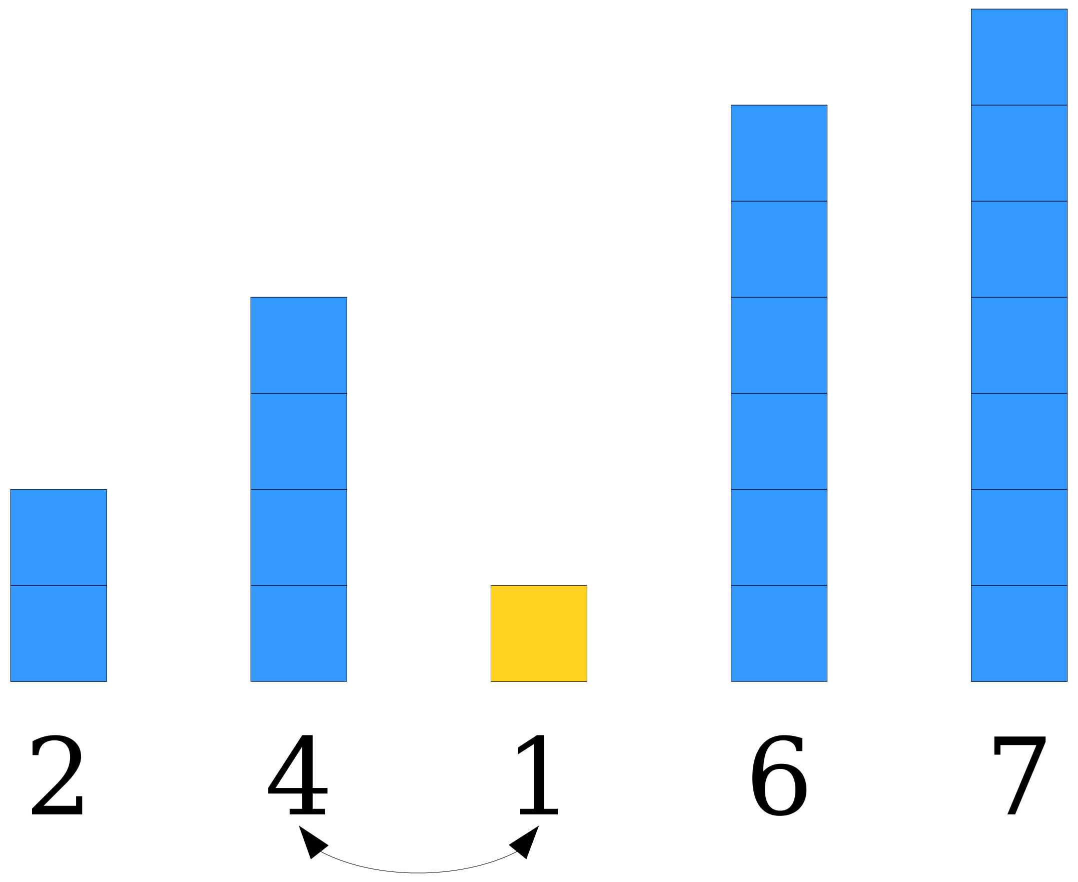

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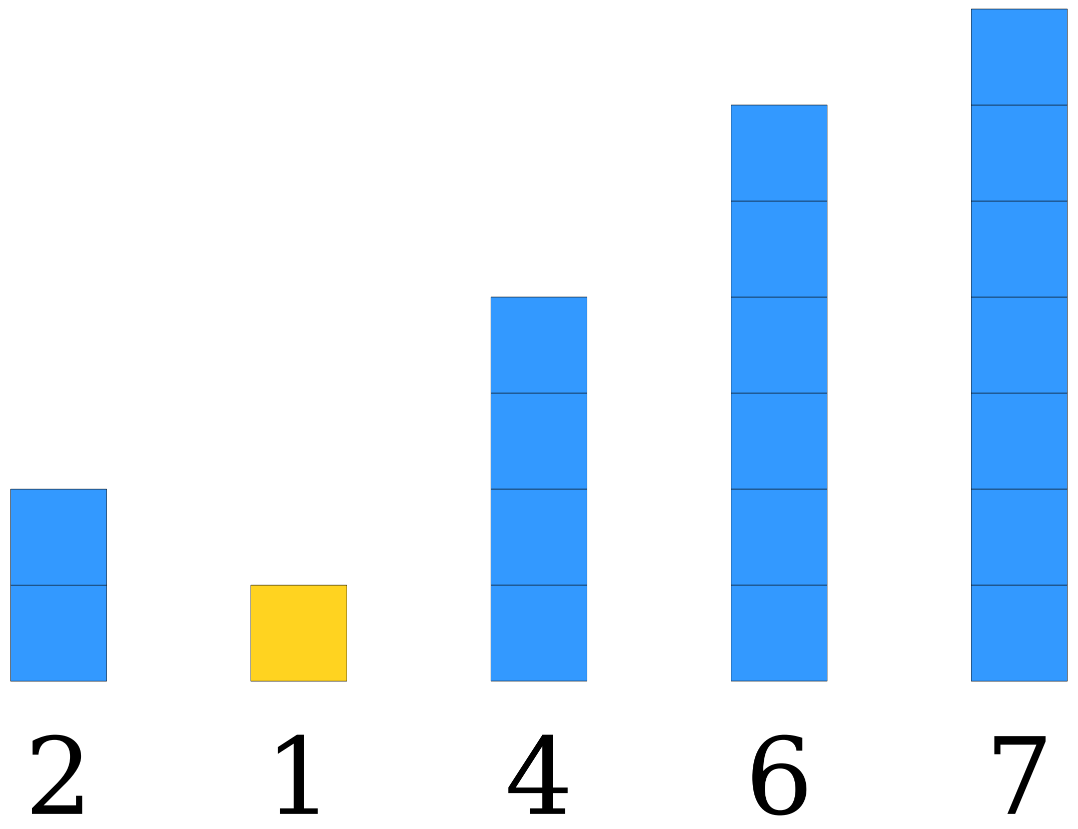

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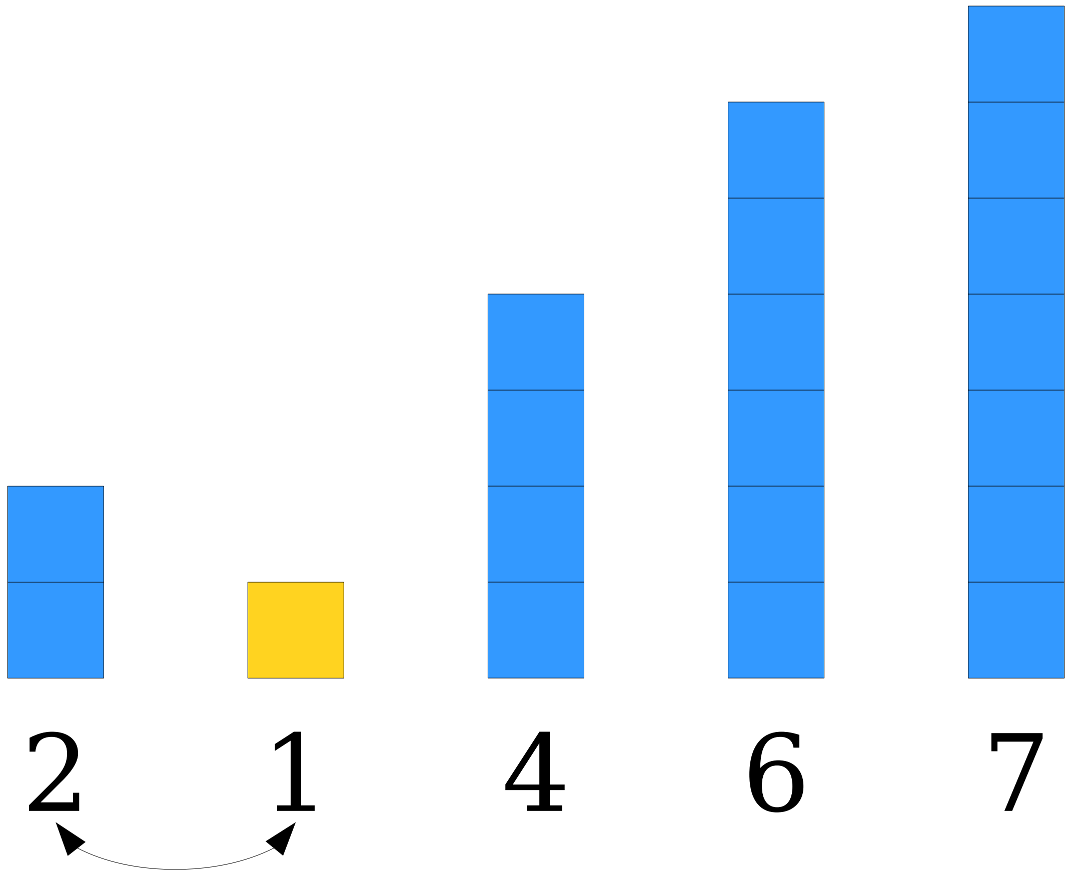

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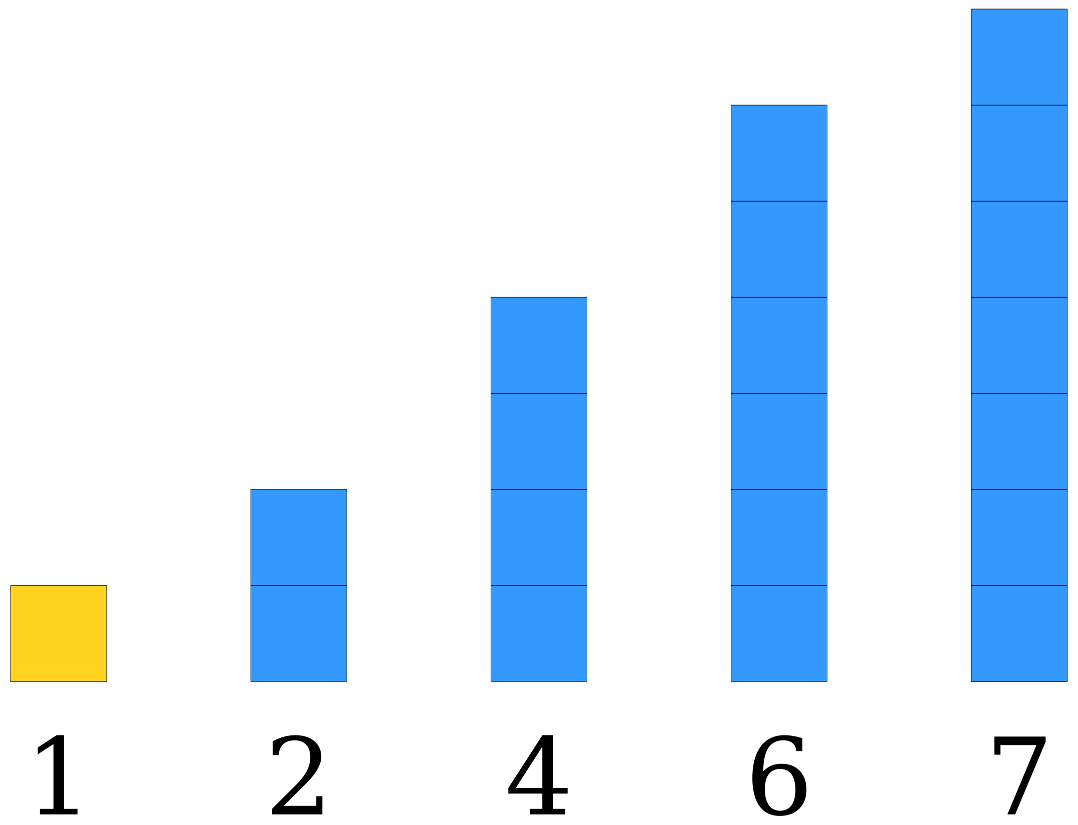

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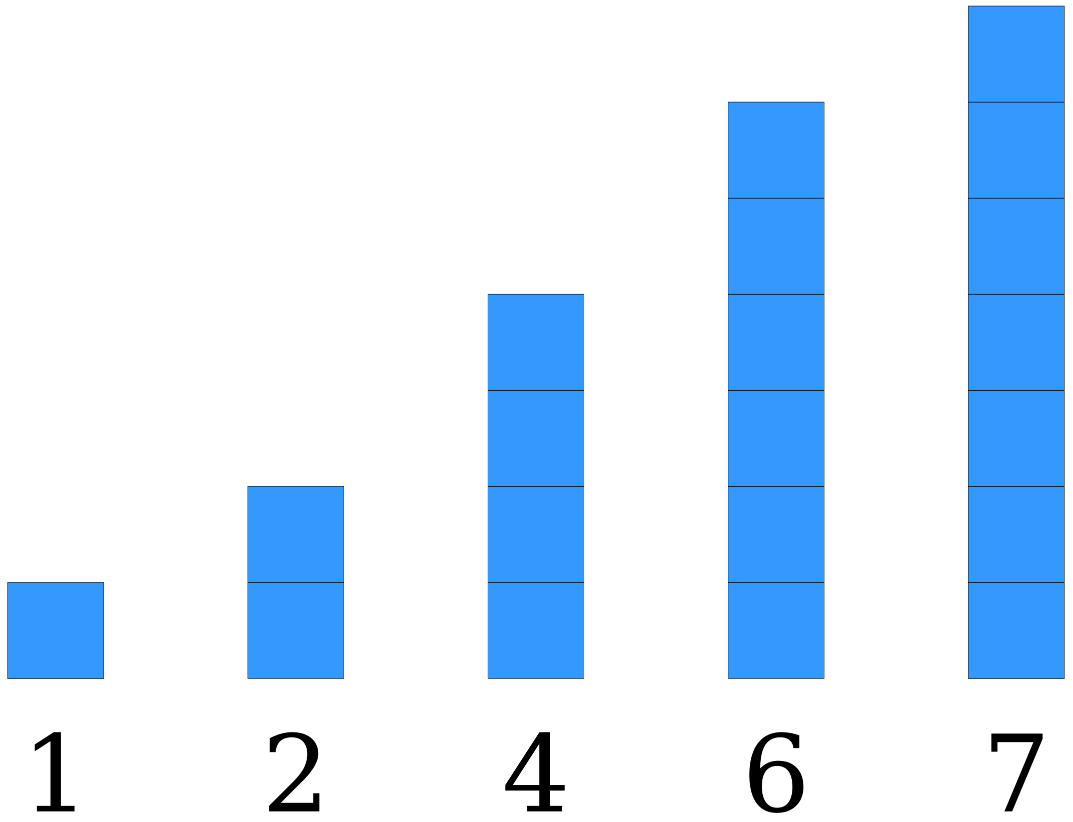

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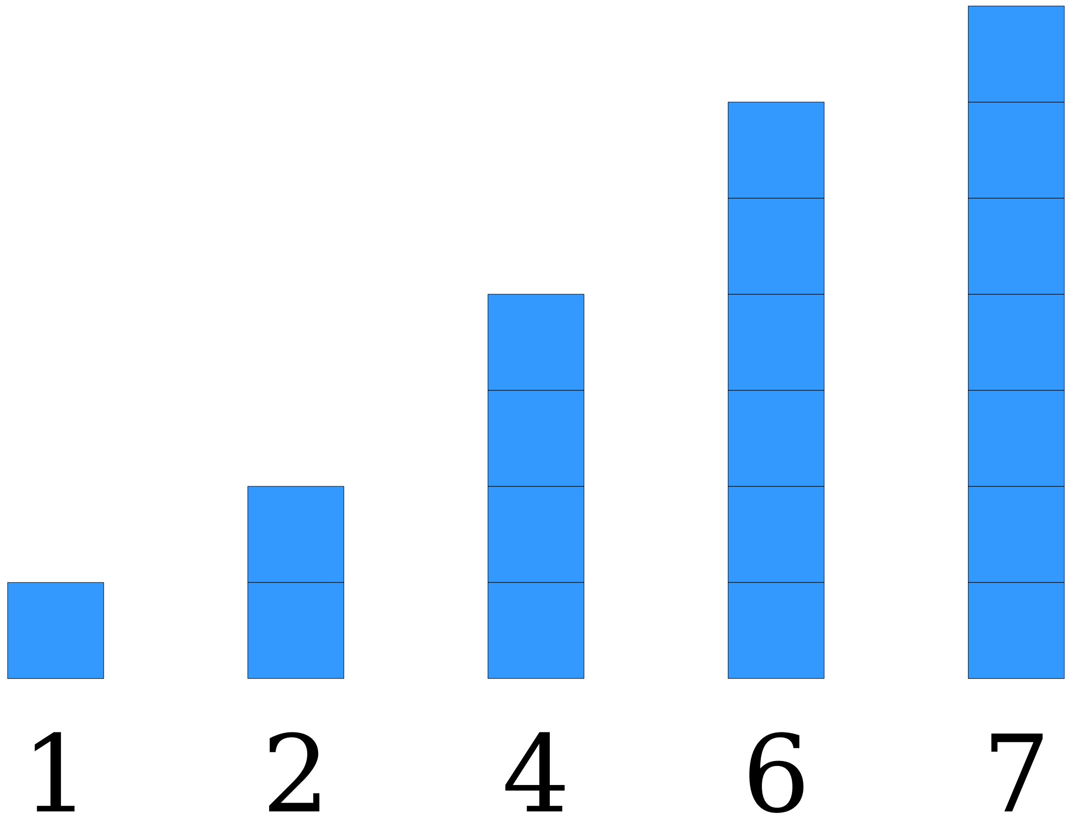


Work Done: **$1 + 2 + 3 + \ldots + n - 1 = O(n^2)$**

Three Analyses

- Worst-Case Analysis
  - What's the *worst* possible runtime for the algorithm?
  - Useful for “sleeping well at night.”
- Best-Case Analysis
  - What's the *best* possible runtime for the algorithm?
  - Useful to see if the algorithm performs well in some cases.
- Average-Case Analysis
  - What's the *average* runtime for the algorithm?
  - Far beyond the scope of this class; take CS109, CS161, or CS265 for more information!

The Complexity of Insertion Sort

- In the best case (the array is sorted), insertion takes time **O(n)**.
- In the worst case (the array is reverse-sorted), insertion sort takes time **O(n^2)**.
- ***Fun fact:*** Insertion sorting an array of (uniformly) random values takes, on average, **O(n^2)** time.
  - Curious why? Come talk to me after class!

How do selection sort and insertion sort compare against one another?

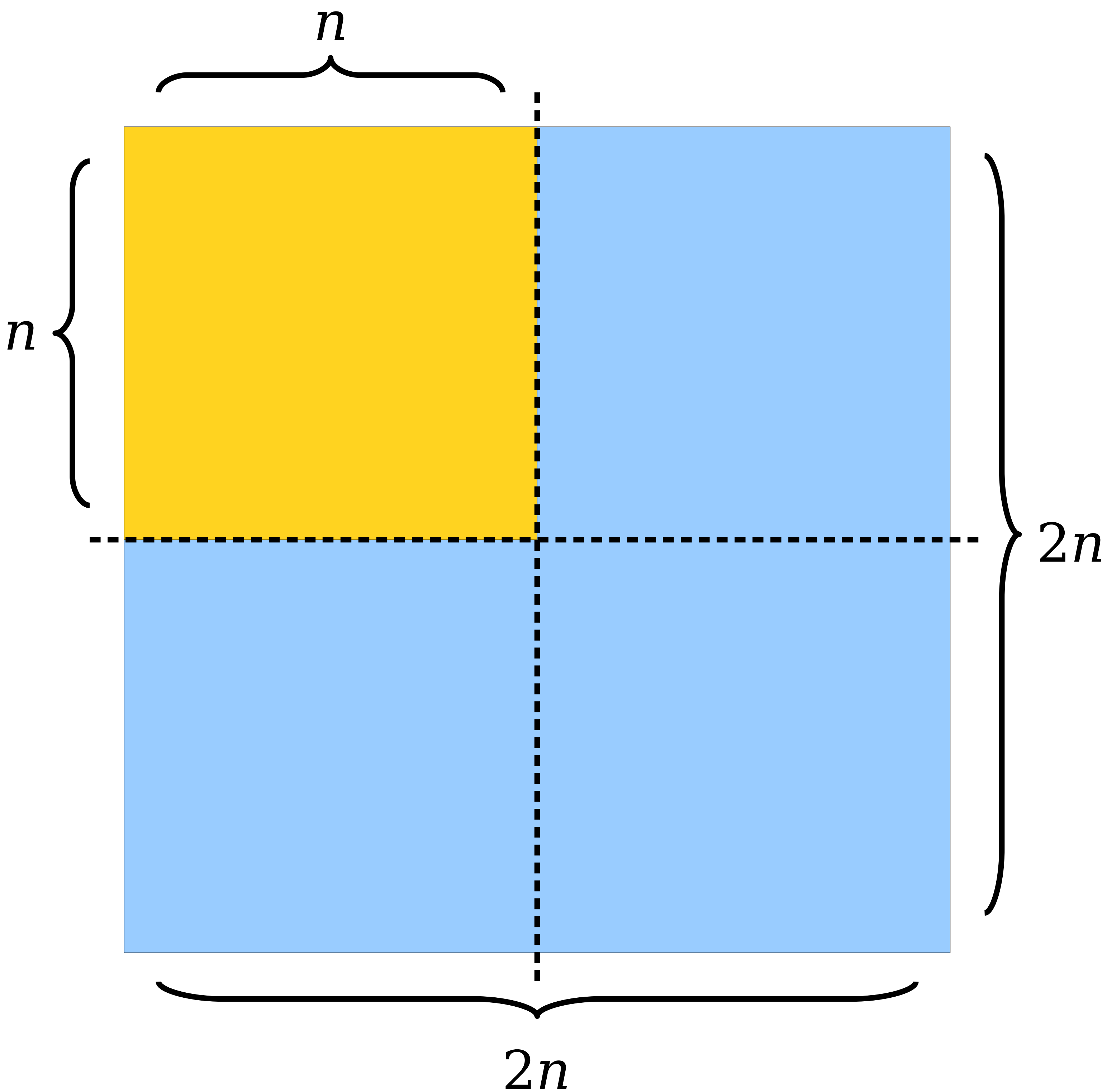
Building a Better Sorting Algorithm

A Thought Experiment

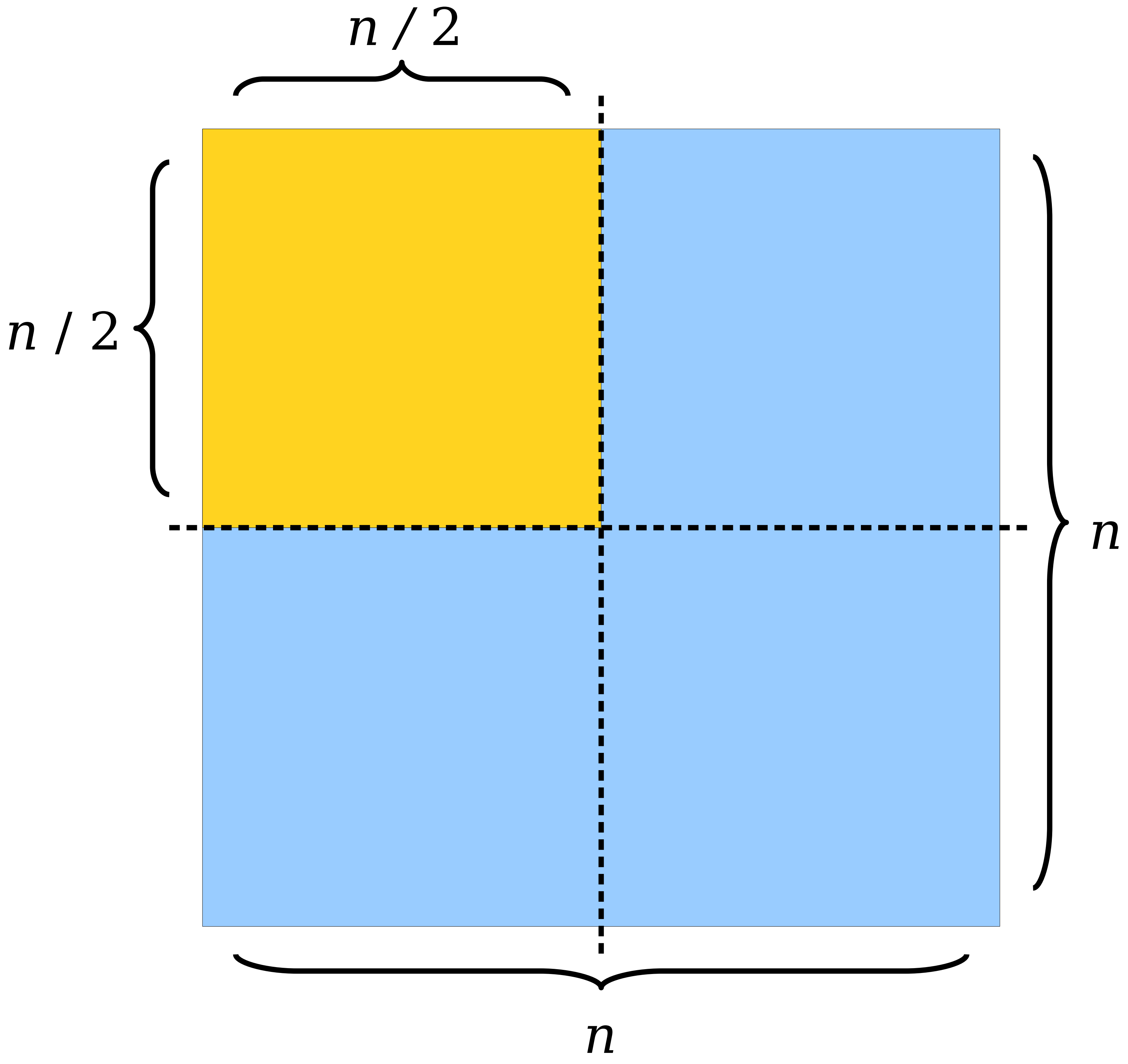
- Suppose it takes 100ms to insertion sort an array of 20,000 random elements.
- Approximately how long will it take to insertion sort two smaller arrays, each of which has 10,000 random elements?

Answer at

https://pollev.com/cs106bwin23

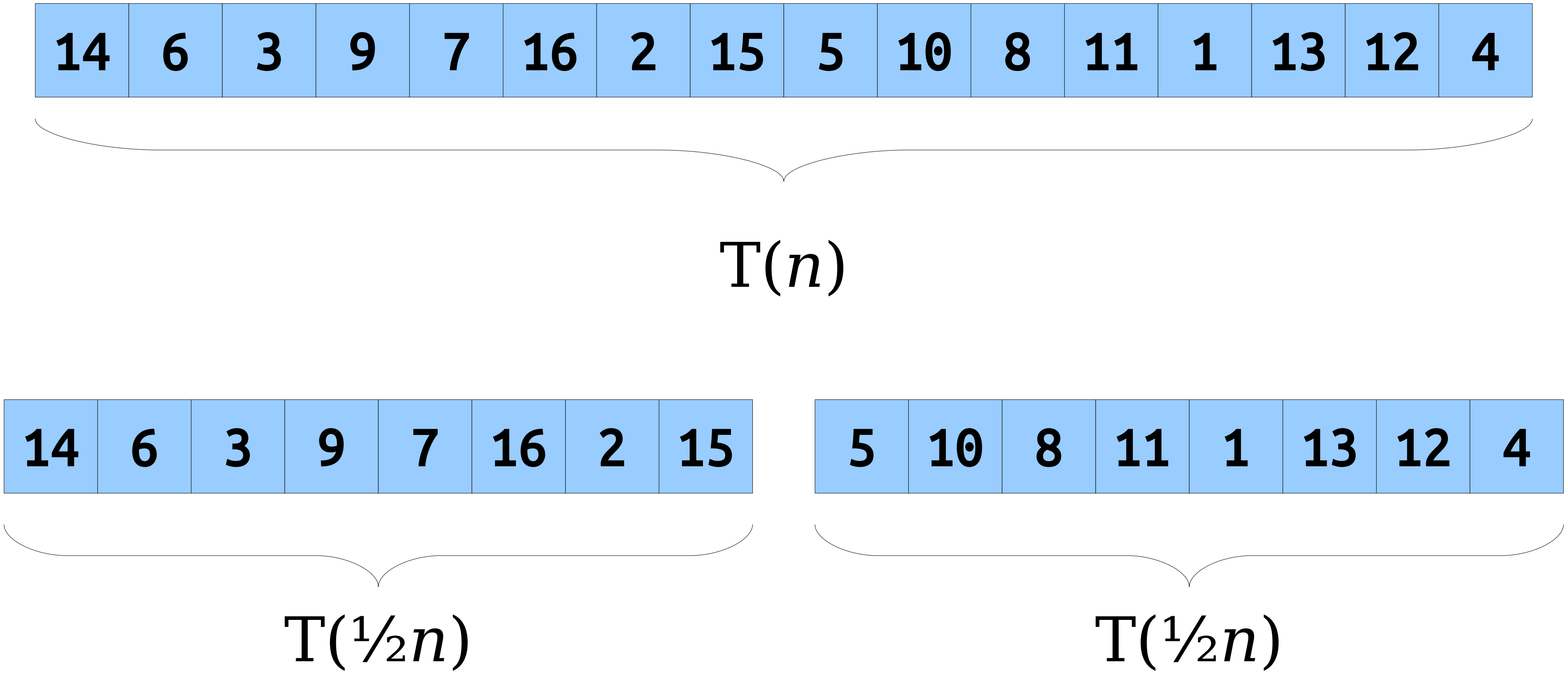


n
n
$2n$
$2n$



n / 2
n / 2
n
n

Thinking About O(n^2)



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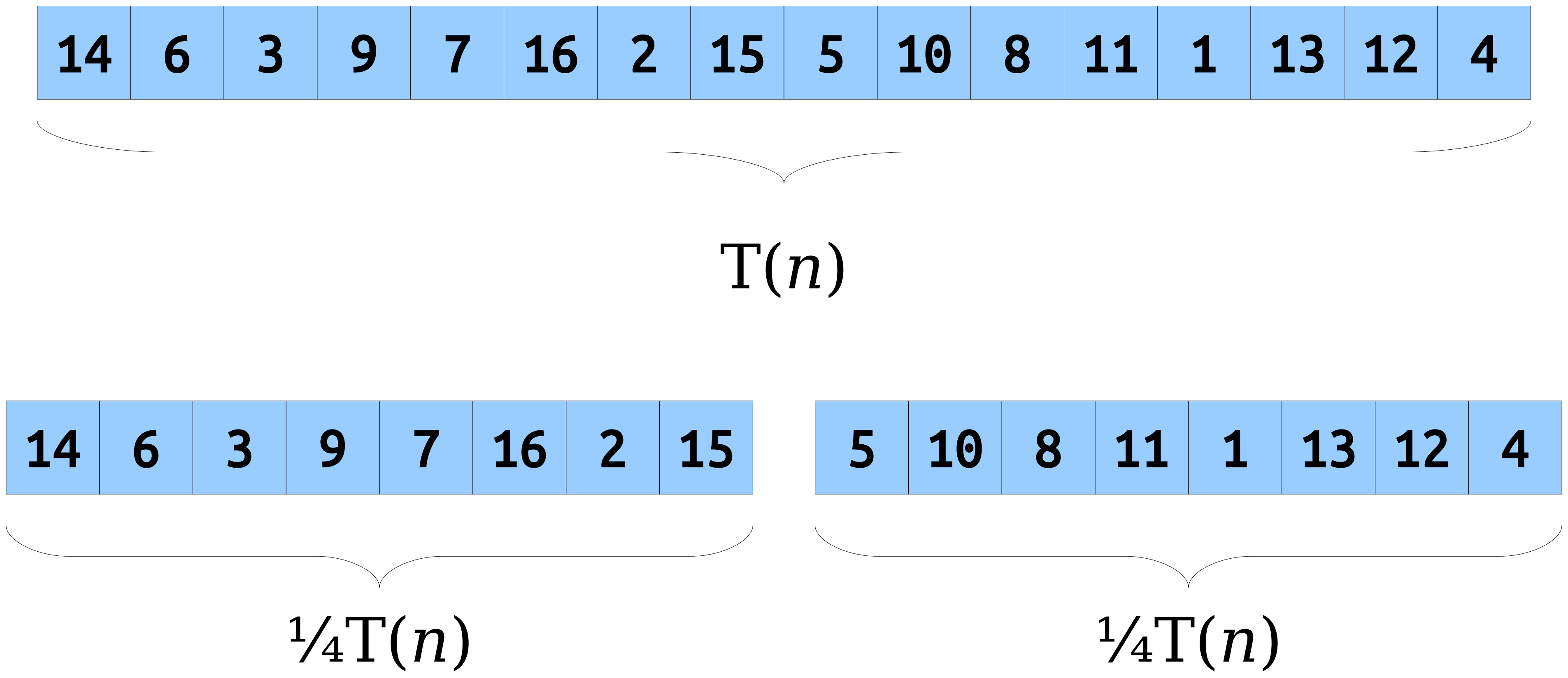

Thinking About O(n^2)

$$\underbrace{\boxed{14}\boxed{6}\boxed{3}\boxed{9}\boxed{7}\boxed{16}\boxed{2}\boxed{15}\boxed{5}\boxed{10}\boxed{8}\boxed{11}\boxed{1}\boxed{13}\boxed{12}\boxed{4}}_{\mathrm{T}(n)}$$

$$\underbrace{\boxed{2}\boxed{3}\boxed{6}\boxed{7}\boxed{9}\boxed{14}\boxed{15}\boxed{16}}_{\frac{1}{4}\mathrm{T}(n)} \quad \underbrace{\boxed{1}\boxed{4}\boxed{5}\boxed{8}\boxed{10}\boxed{11}\boxed{12}\boxed{13}}_{\frac{1}{4}\mathrm{T}(n)}$$

$$2 \cdot \tfrac{1}{4}\mathrm{T}(n) = \tfrac{1}{2}\mathrm{T}(n)$$

With an O(n^2)-time sorting algorithm, it takes twice as long to sort the whole array as it does to split the array in half and sort each half.

Can we exploit this?

The Key Insight: ***Merge***

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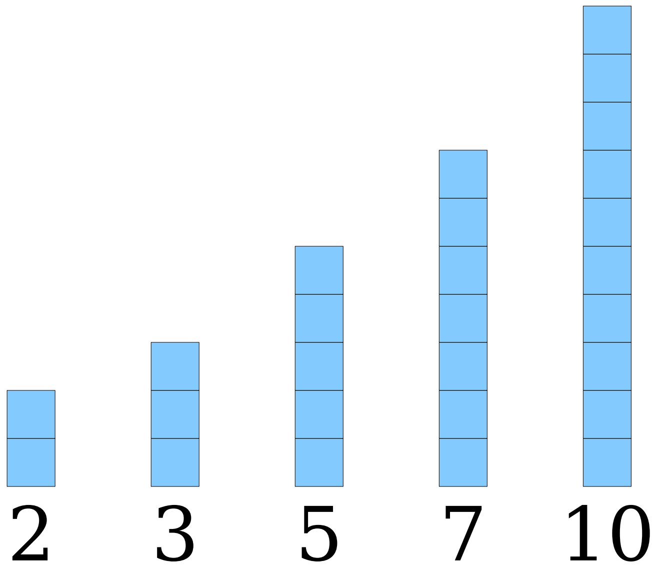

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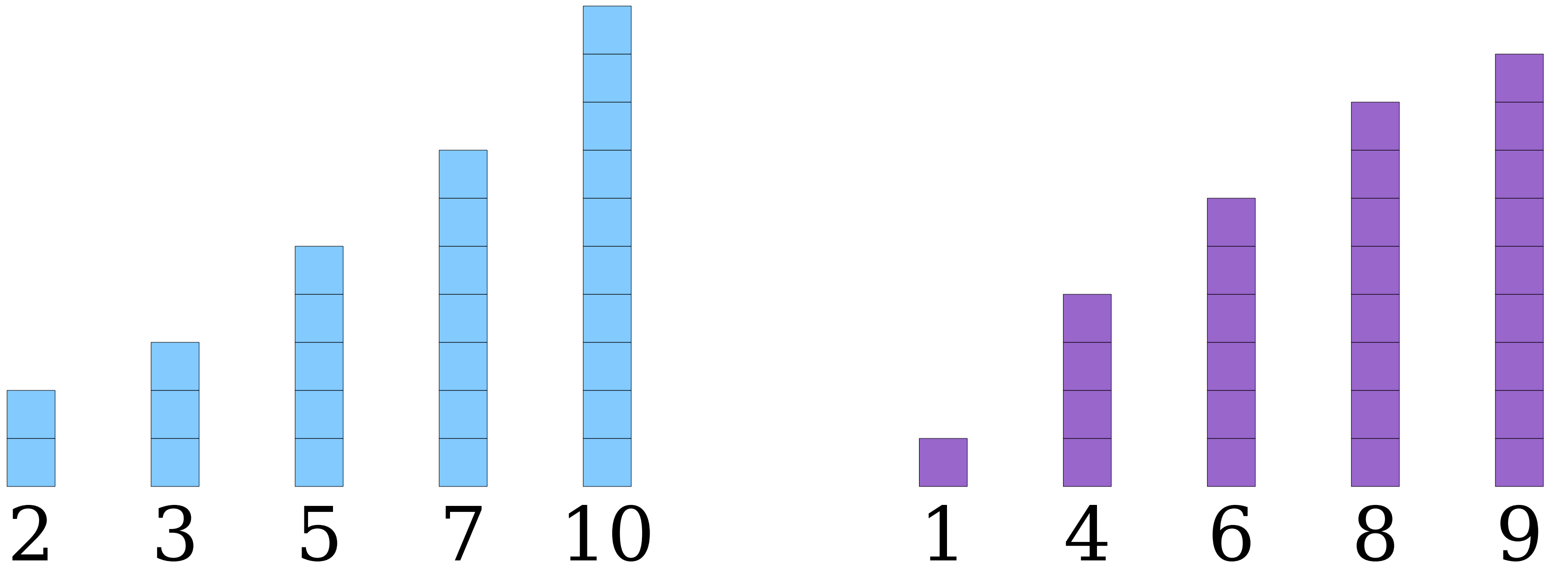

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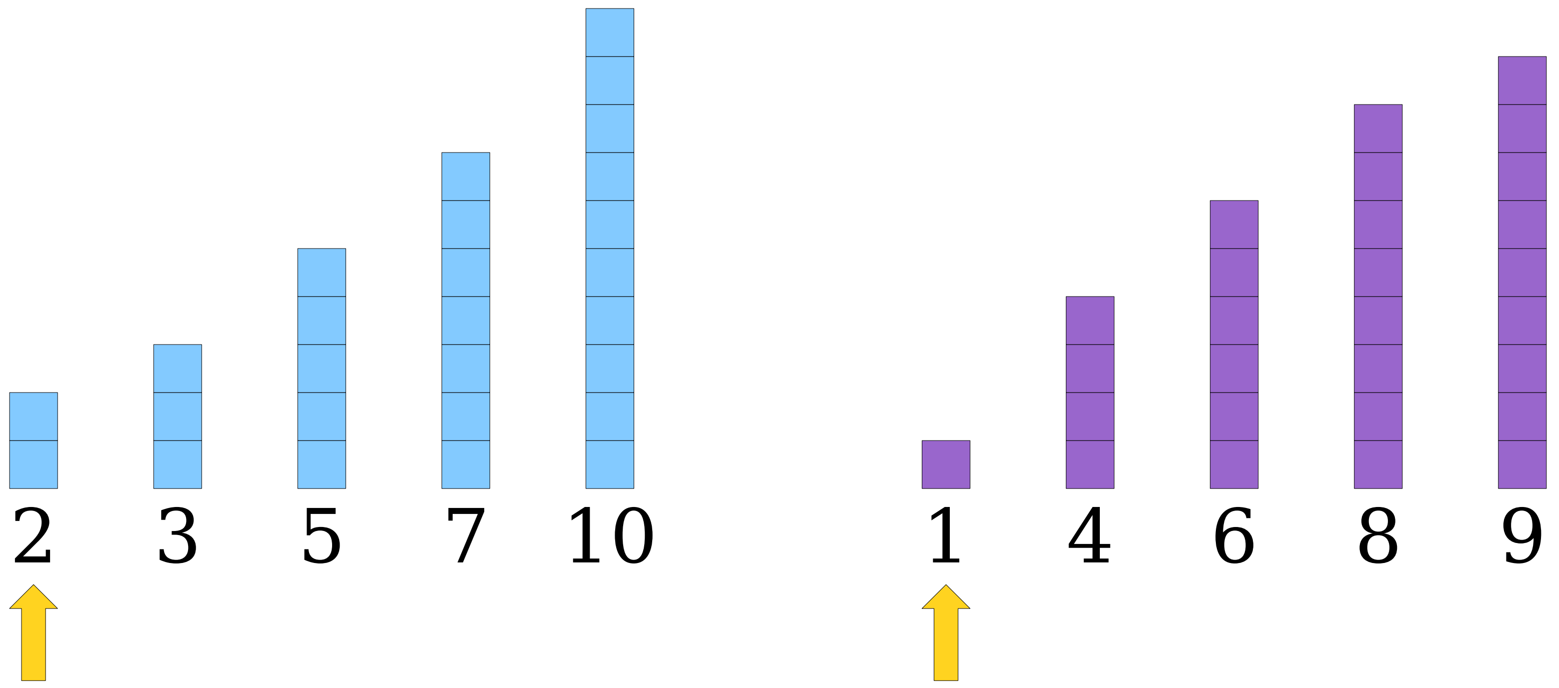

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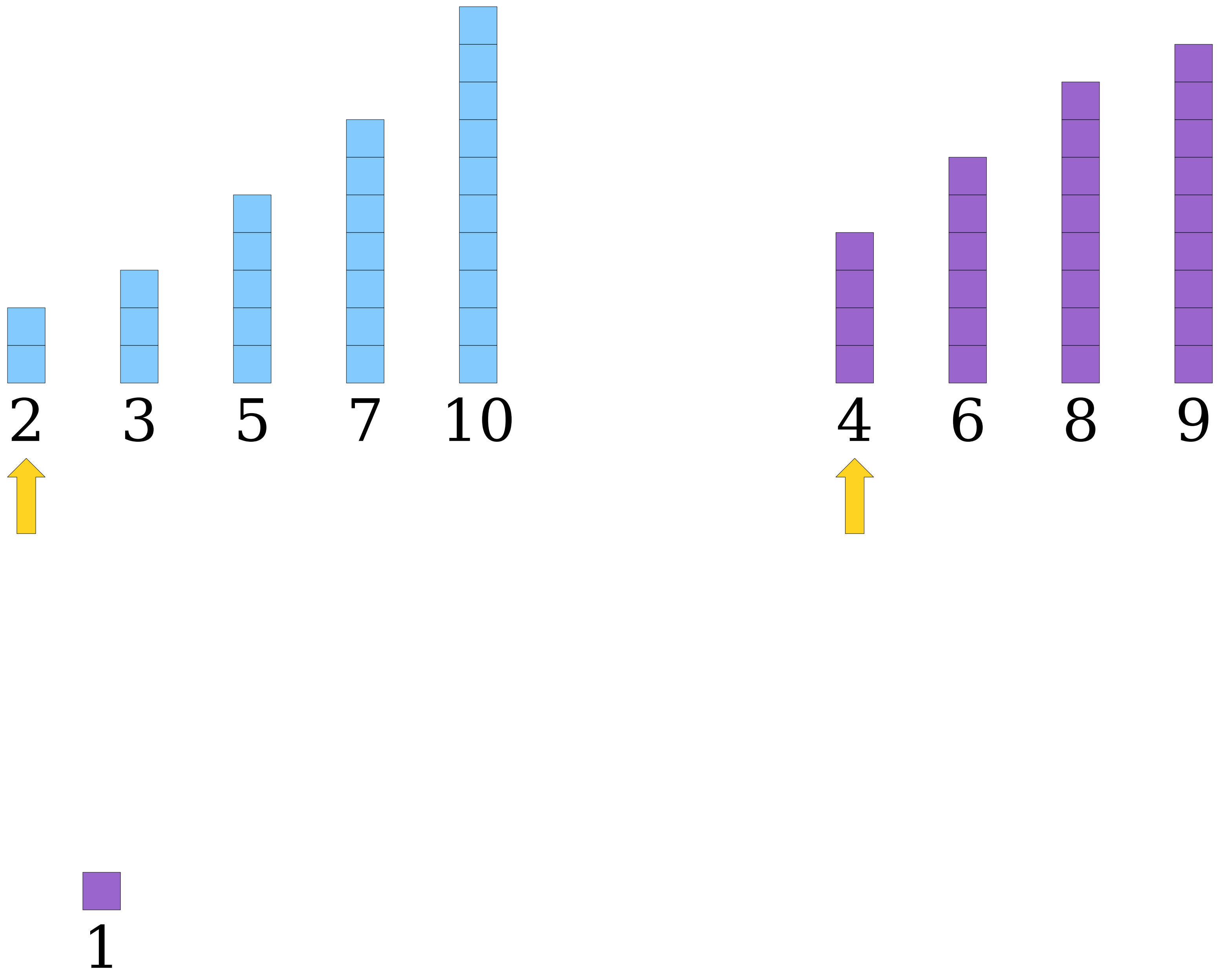

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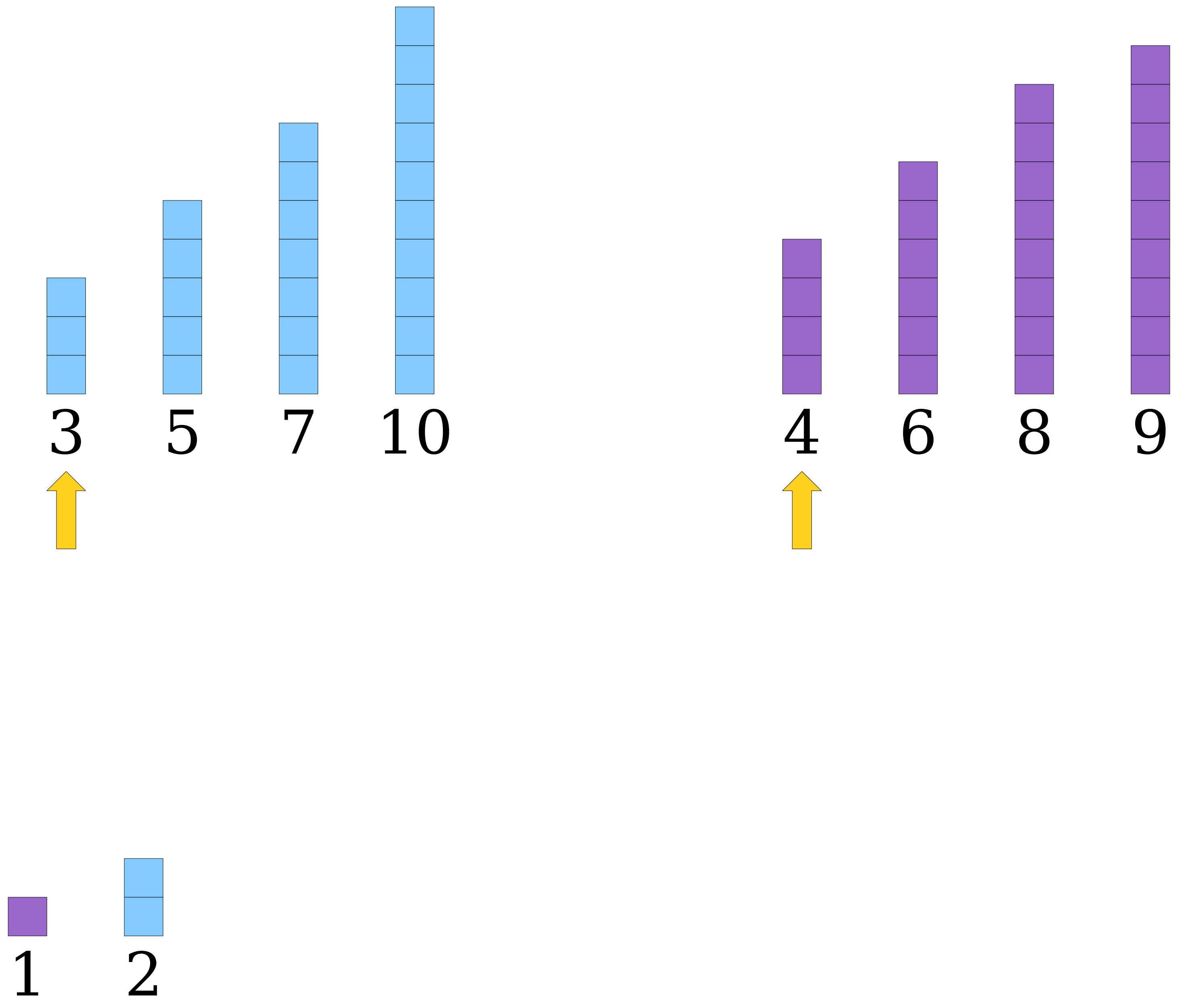

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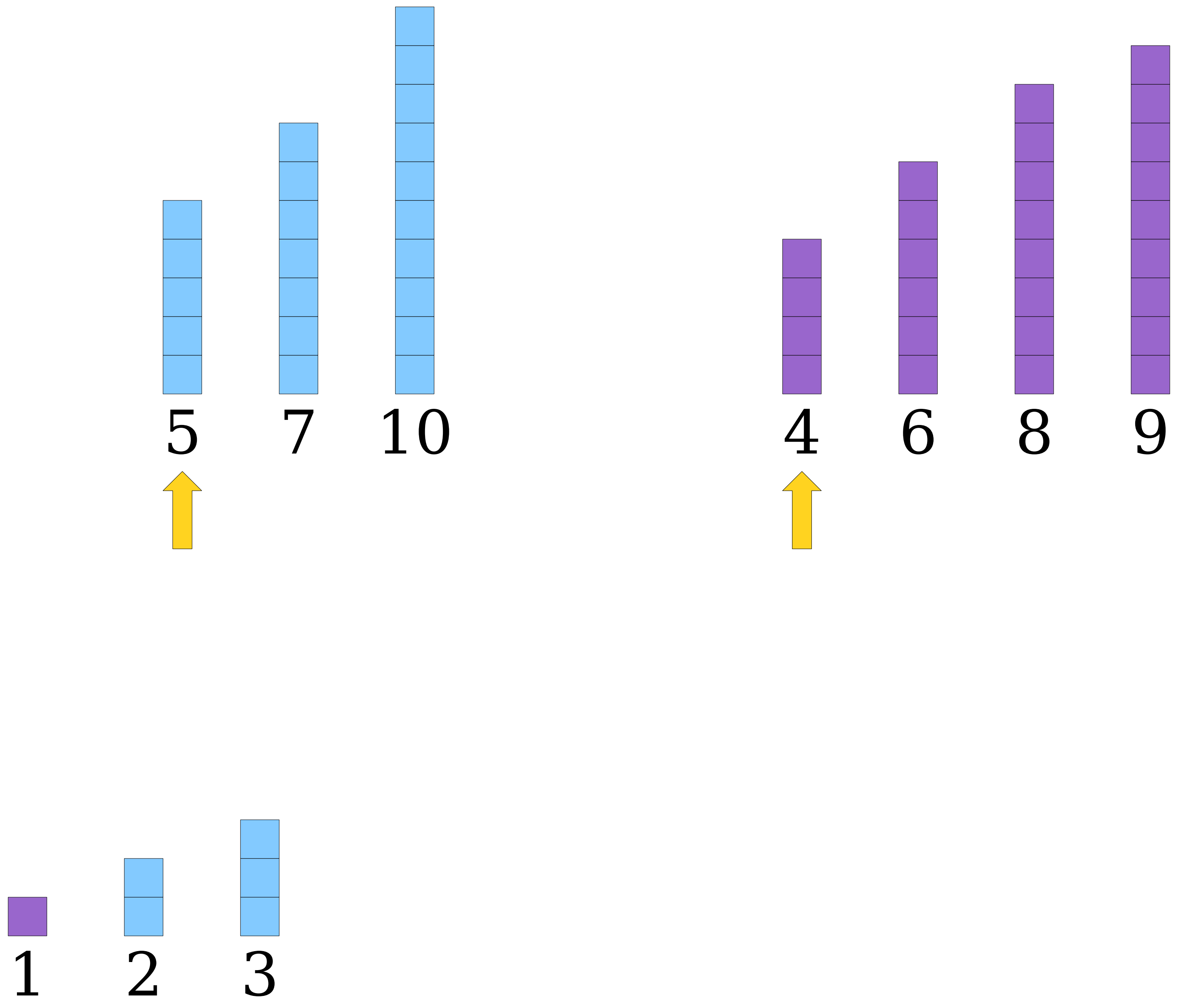

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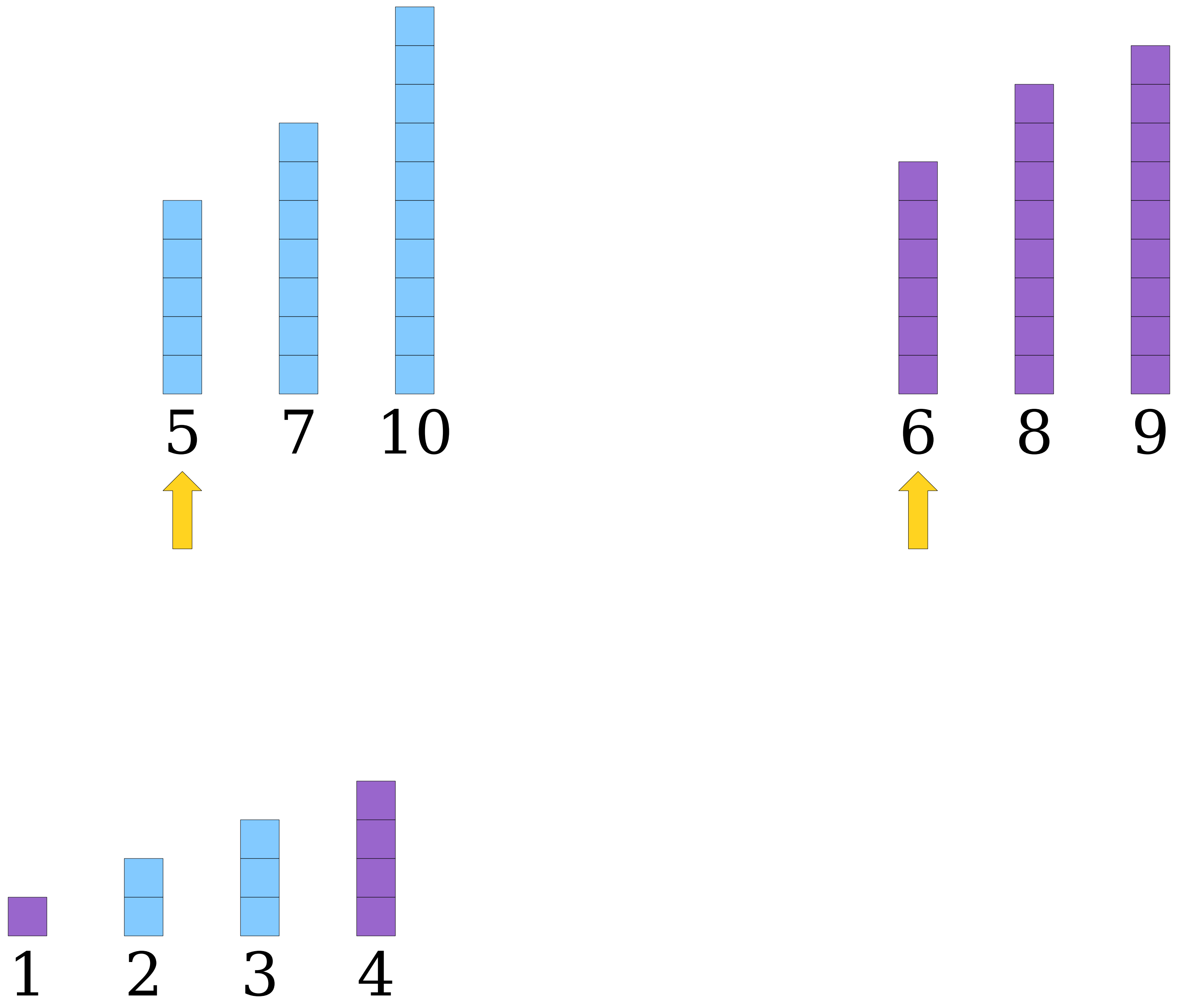

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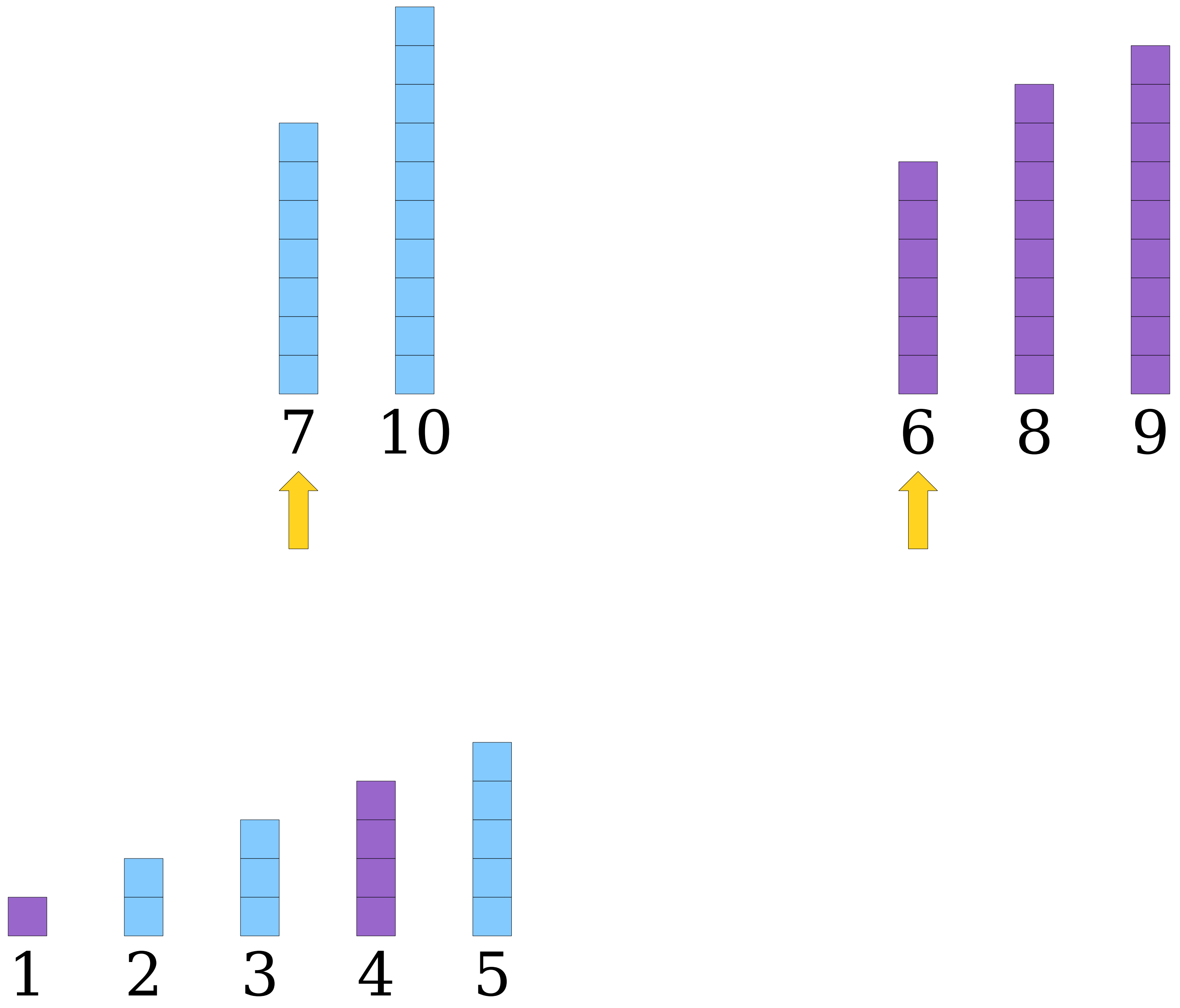

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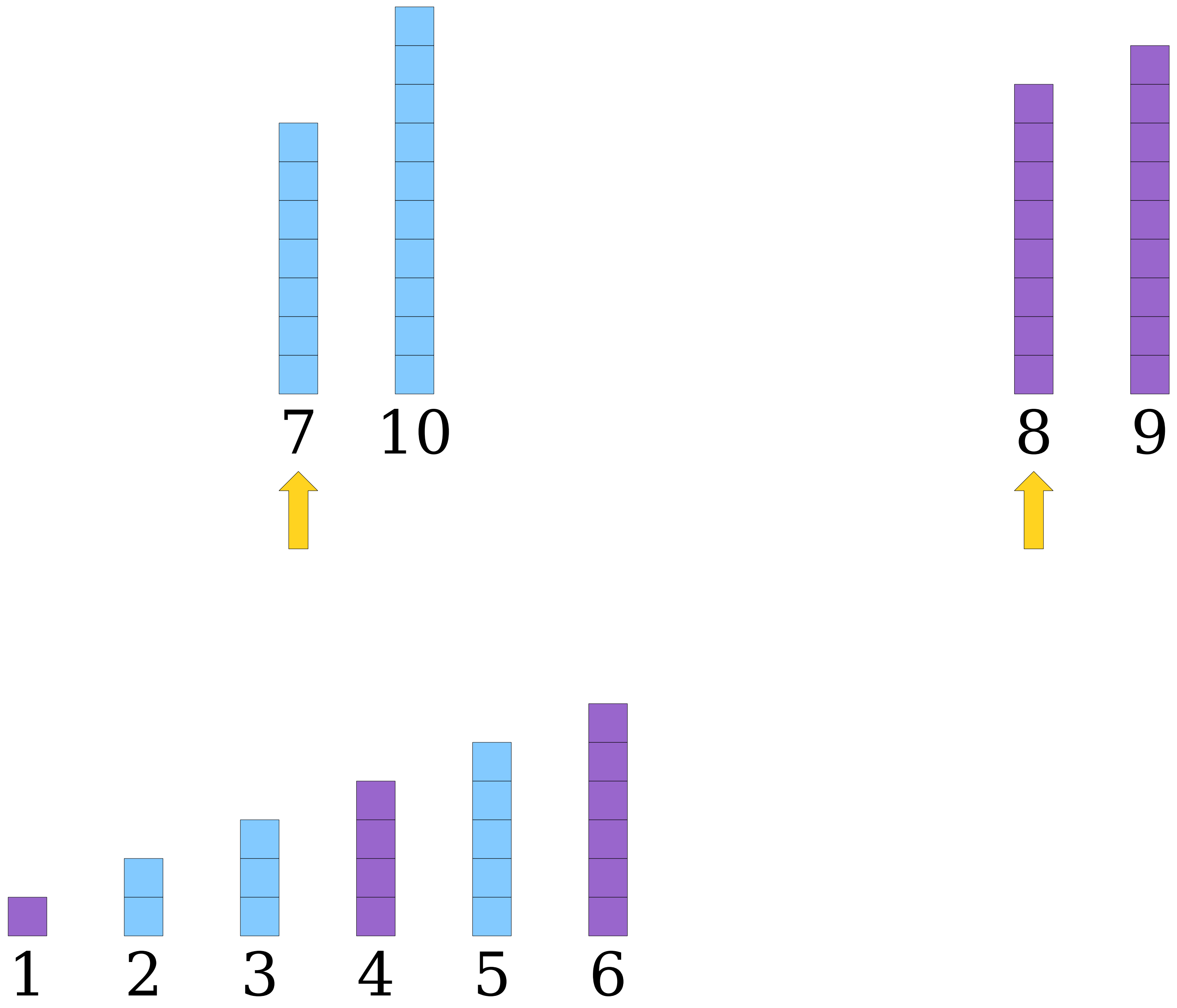

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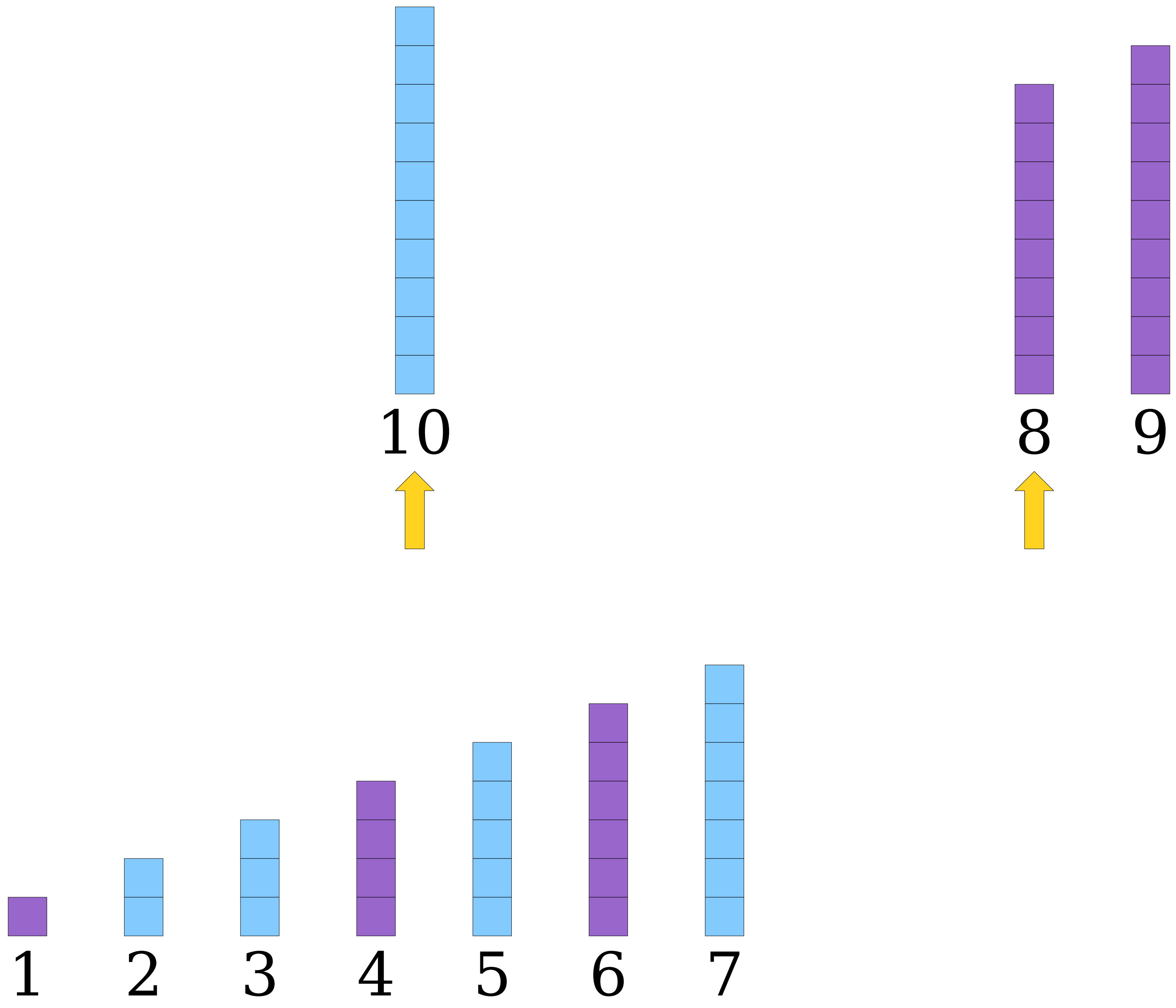

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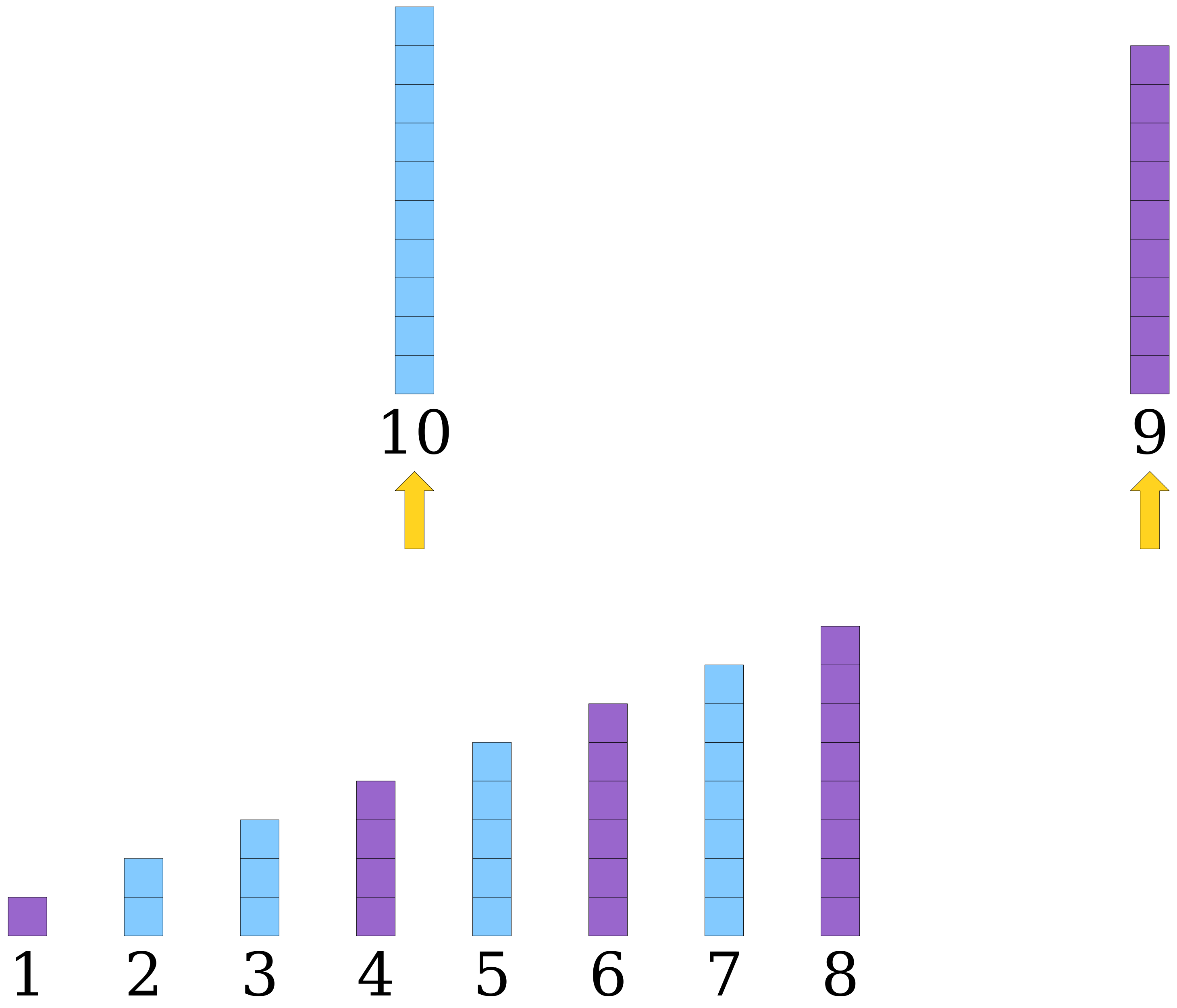

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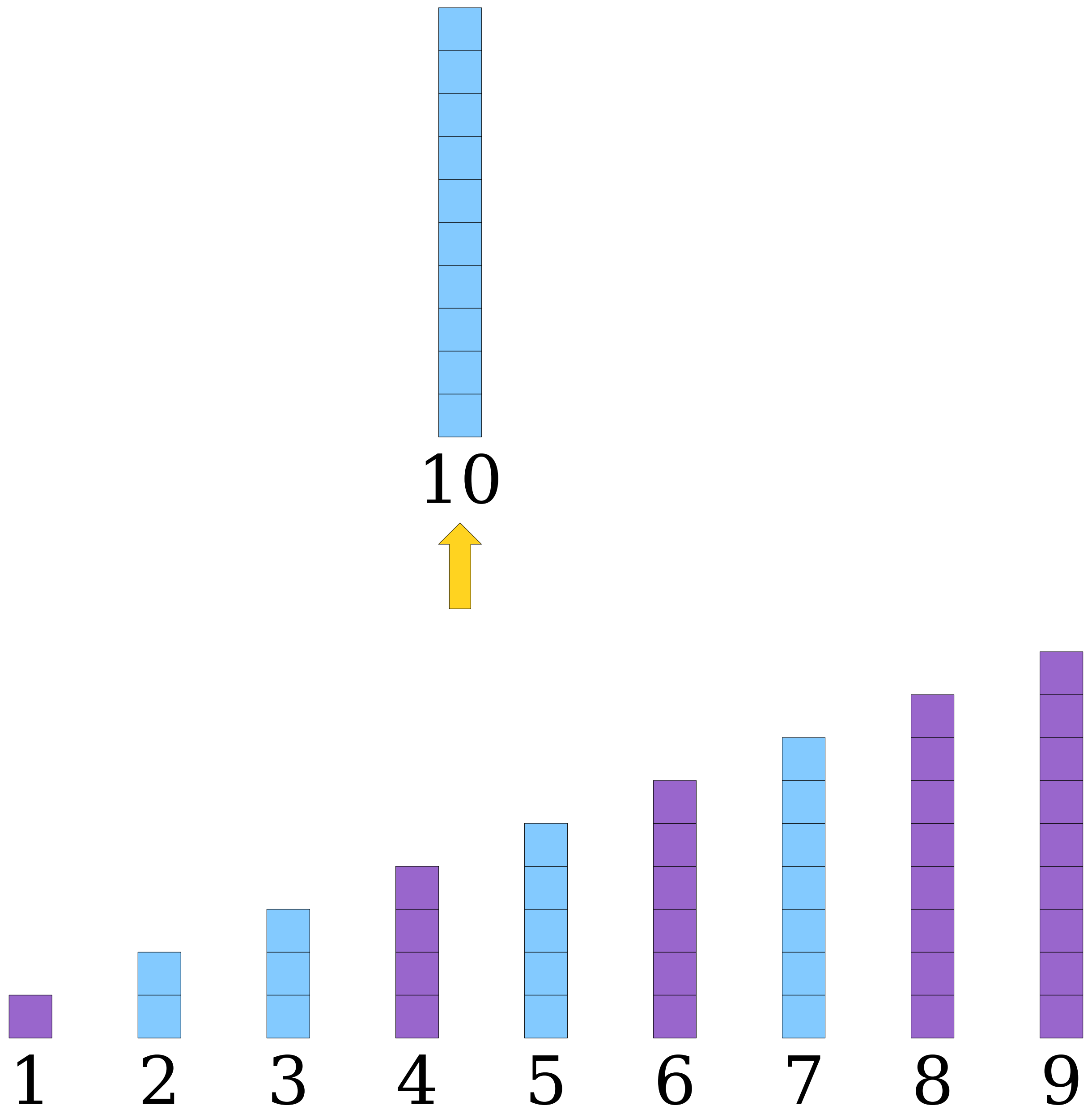

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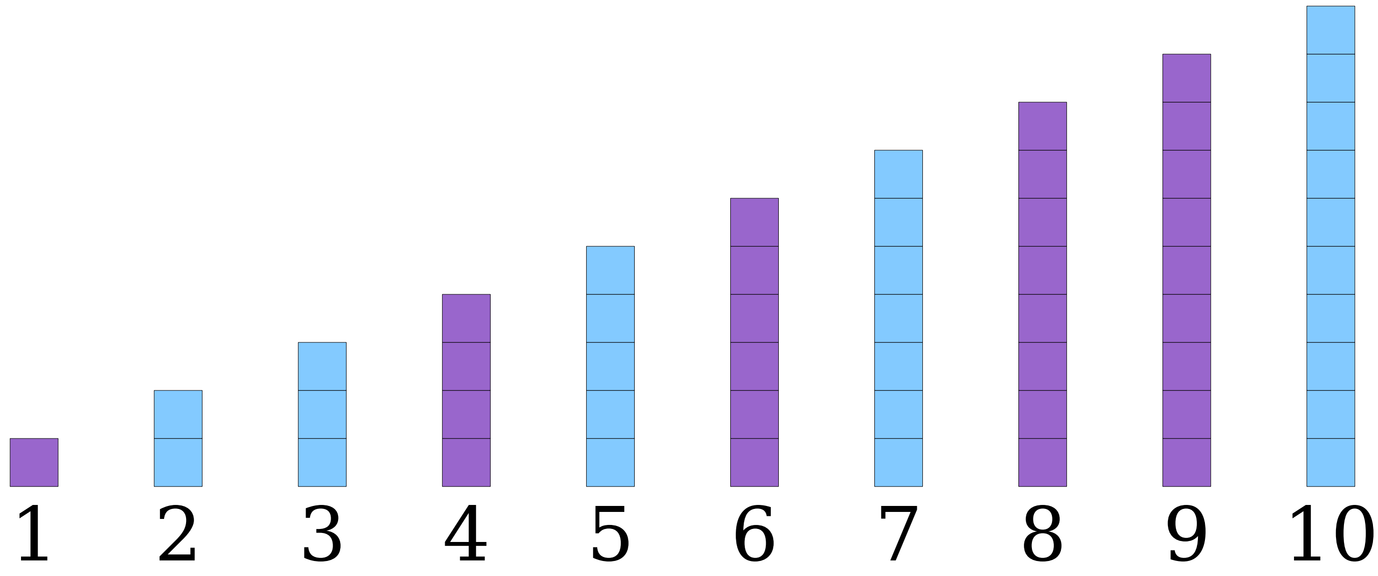
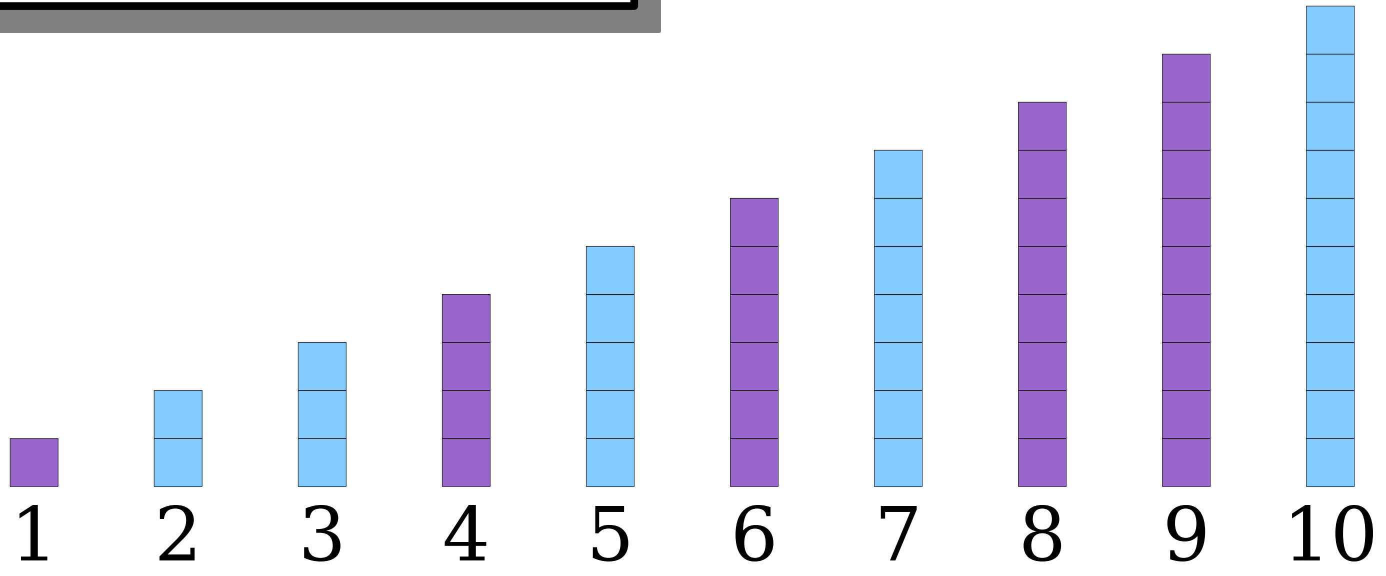


The Key Insight: ***Merge***

Each step makes a single comparison and reduces the number of elements by one.

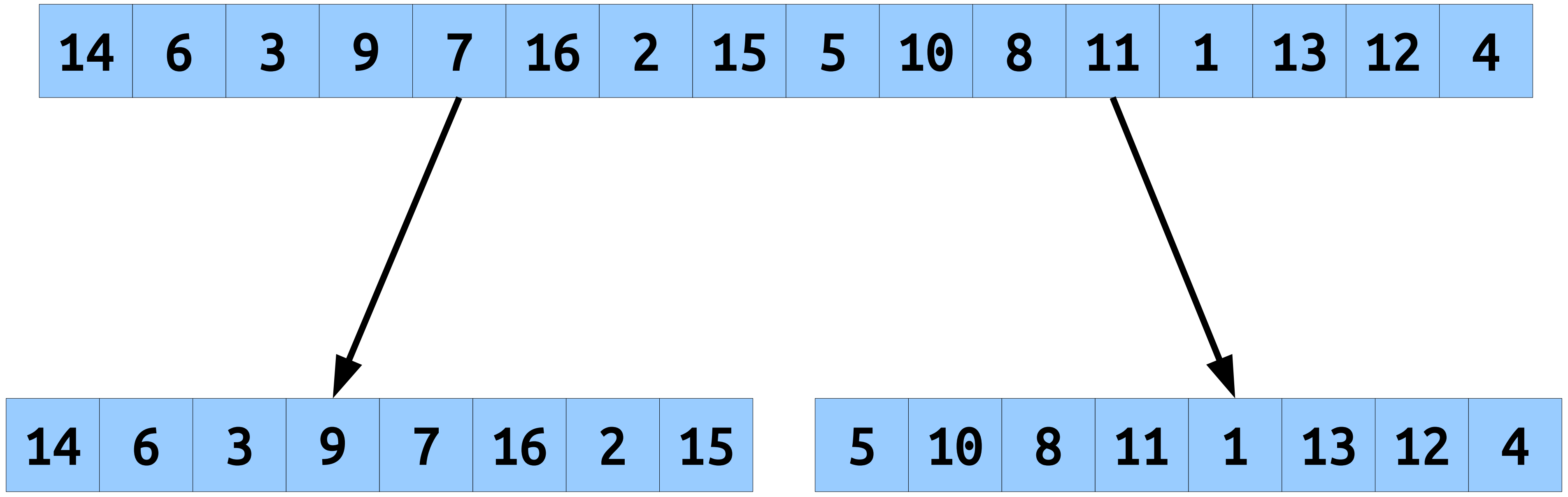
If there are *n* total elements, this algorithm runs in time **O(*n*)**.



The Key Insight: ***Merge***

- The ***merge*** algorithm takes in two sorted lists and combines them into a single sorted list.
  - While both lists are nonempty, compare their first elements. Remove the smaller element and append it to the output.
  - Once one list is empty, add all elements from the other list to the output.
- It runs in time O(n), where n is the total number of elements being merged.

“Split Sort”



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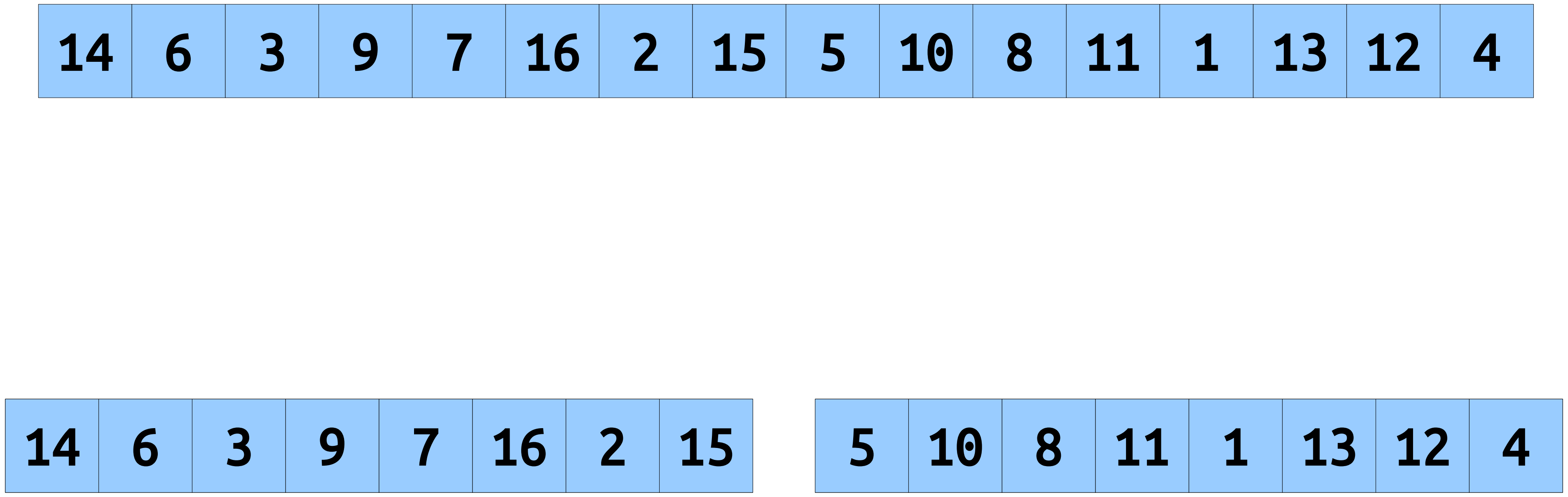

1. Split the input in half.
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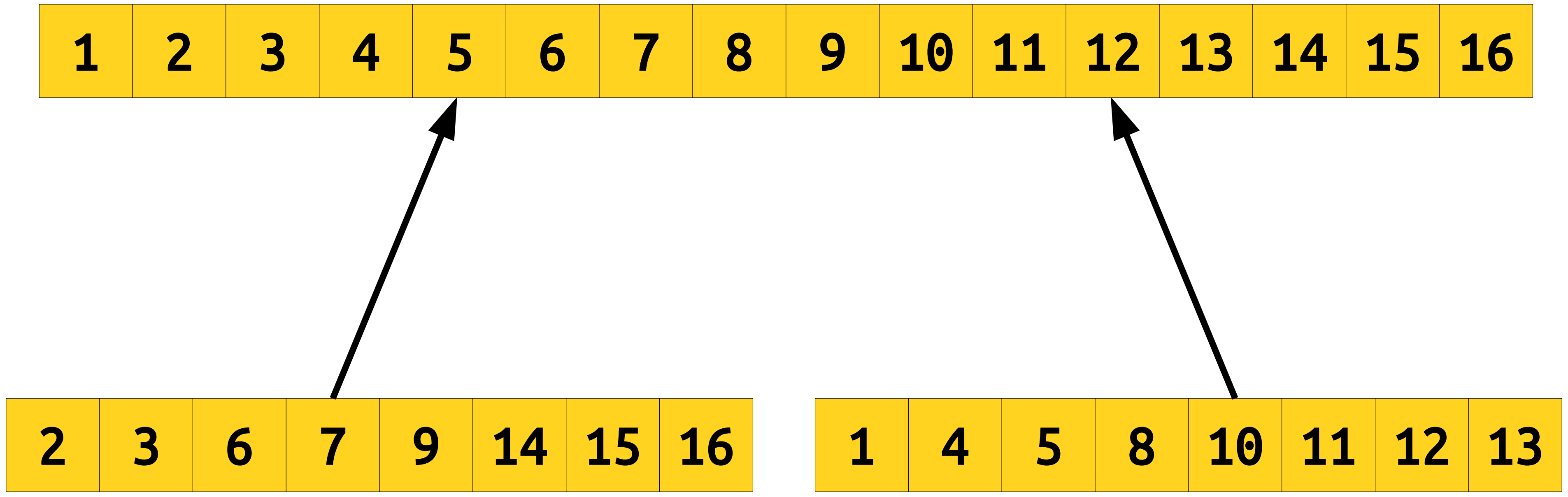

1. Split the input in half.
2. Insertion sort each half.
3. Merge the halves back together.

“Split Sort”

```
void splitSort(Vector<int>& v) {
    /* Split the vector in half */
    int half = v.size() / 2;
    Vector<int> left  = v.subList(0, half);
    Vector<int> right = v.subList(half);

    /* Sort each half. */
    insertionSort(left);
    insertionSort(right);

    /* Merge them back together. */
    v = merge(left, right);
}
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Takes O(*n*) time, since we copy all *n* elements into new `Vectors`.

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Takes O(n^2) time, but about half as much as what we did before.

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Takes O(n) time.

Prediction: This should still take time O(n^2), but be about twice as fast as insertion sort.

Next Time

- ***Mergesort***
  - A beautiful, elegant sorting algorithm.
- ***Analyzing Mergesort***
  - An unusual runtime analysis.
- ***Hybrid Sorting Algorithms***
  - Improving on mergesort.
- ***Binary Search***
  - Finding things fast!