DATASCI W261, Machine Learning at Scale

Assignement: week #6

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Due: 2016-10-18, 8AM PST

HW6.0.

Mathematical optimization is the selection of a best solution with respect to some criteria from some set of available candidates. In general, an optimization problem consists of maximizing or minimizing a real function by systematically choosing input values from an allowed set and finding maximum or minimum value of the function.

In W207 project, we tried to adjust different input parameters to obtain maximum correction rate in NaiveBaysian estimation problem.

HW6.1 Optimization theory:

For unconstrained optimization of a function f, the first order necessary condition (FOC) for a point $x = v$ to be maximum or minimum, is that the first derivative of f at the point v is equal to zero. This condition can be expressed as:

$$\left.\frac{\mathrm{d}f(x)}{\mathrm{d}x}\right|_{x=v} = 0$$

The second order optimality conditions (SOC) require that the second derivative evaluated at v be positive/negative for the candidate point $x = v$ to be a minimum/maximum. A maximum will satisfy:

$$\left.\frac{\mathrm{d}^2f(x)}{\mathrm{d}x^2}\right|_{x=v} < 0$$

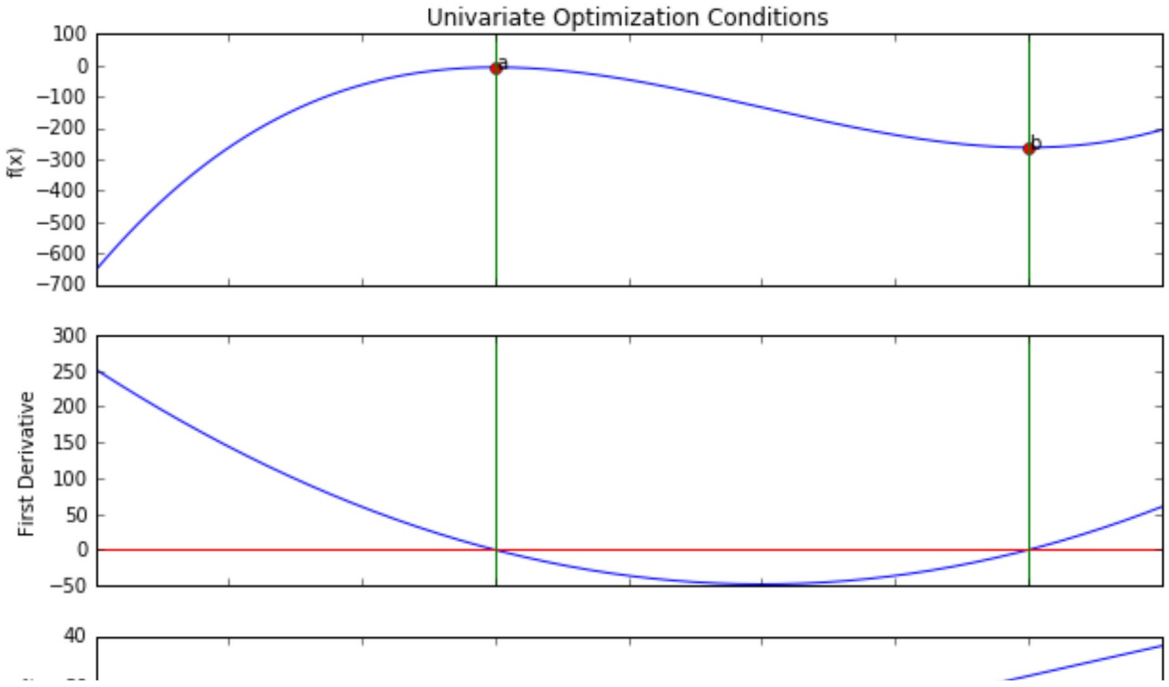
and a minimum will satisfy

$$\left.\frac{\mathrm{d}^2f(x)}{\mathrm{d}x^2}\right|_{x=v} > 0$$

Plot and its code are provided as following:

In [52]:
```
%load_ext autoreload
%autoreload 2
```

In [8]:
```
%matplotlib inline
import matplotlib.pyplot as plt
import numpy as np

n=200
x = np.linspace(-6, 10, n)
y = [pow(i,3)-12*pow(i,2)-6 for i in x]
yd = [3*pow(i,2)-24*i for i in x]
y2d = [6*i-24 for i in x]
# plot
f, axarr = plt.subplots(3, sharex=True)
f.set_size_inches([10,8])
axarr[0].plot(x, y)
axarr[0].plot(0, -6, 'ro')
axarr[0].plot(8, pow(8,3)-12*pow(8,2)-6, 'ro')
axarr[0].set_xlim([-6, 10])
axarr[0].set_ylim([-700, 100])
axarr[0].plot(8*np.ones(n), np.linspace(-700,100,n))
axarr[0].plot(0*np.ones(n), np.linspace(-700,100,n), 'g')
axarr[0].set_title('Univariate Optimization Conditions')
axarr[0].set_ylabel('f(x)')
axarr[0].text(0, -6, 'a')
axarr[0].text(8, pow(8,3)-12*pow(8,2)-6, 'b')

axarr[1].plot(x, yd)
axarr[1].plot(x, np.zeros(n), 'r')
axarr[1].plot(0*np.ones(n), np.linspace(-50,300,n), 'g')
axarr[1].plot(8*np.ones(n), np.linspace(-50,300,n), 'g')
axarr[1].set_ylabel('First Derivative')

axarr[2].plot(x, y2d)
axarr[2].plot(x, np.zeros(n), 'r')
axarr[2].plot(4*np.ones(n), np.linspace(-60,20,n))
axarr[2].set_ylabel('Second Derivative')

plt.show()
```



Discussions

By plotting original f (top plot) and $f'(x)$ (middle plot) derivative over the range $[-6, 10]$, or solving for the FOC:

$$f'(x) = 3x^2 - 24x = 0,$$

we obtain the candidate local maximums and minimums exist at $x = 0, 8$. By checking SOC of the second derivative (bottom plot):

$$f''(x) = 6x - 24$$

at $x = 0, 8$, we are able tell that $x = 0$ is the local maximum, and $x = 8$ is the local minimum.

For unconstrained multi-variate optimization, what are the first order Necessary Conditions for Optimality (FOC)?

For the unconstrained optimization of a multi-varite function f(x), the first order necessary condition (FOC) states that the gradient at a specific point $x = v$ be the zero vector:

$$\nabla f(x)|_{x=v} = \left[\frac{\partial f(x)}{\partial x_1}\bigg|_{x=v}, \cdots, \frac{\partial f(x)}{\partial x_N}\bigg|_{x=v} \right] = (0, \cdots, 0)$$

What are the second order optimality conditions (SOC)? Give a mathematical defintion. What is the Hessian matrix in this context?

For the second order optimality conditions, we must introduce the Hessian matrix of second partial derivatives:

$$H_f = \begin{bmatrix} \frac{\partial^2 f(x)}{\partial x_1^2} & \cdots & \frac{\partial^2 f(x)}{\partial x_1 \partial x_N} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 f(x)}{\partial x_N \partial x_1} & \cdots & \frac{\partial^2 f(x)}{\partial x_N^2} \end{bmatrix}$$

The SOC states that if evaluation at the point $x = v$ renders H_f negative/positive definite, i.e., all eigenvalues of H_f are negative/positive, then the candidate point $x = v$ is a local maximum/minimum.

HW6.2

The update rule for finding x_{n+1} by the Newton-Raphonson method is given by the evaluation of $f(x)$ at the zero of the line tangent to f at x_n:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

So, with expressions for f and f', we have

$$x_{n+1} = x_n - \frac{x_n^3 + 2x_n - 4}{3x_n^2 + 2}$$

into which we can substitute $x_n = 1$ to find:

$$x_2 = x_1 - \frac{x_1^3 + 2x_1 - 4}{3x_1^2 + 2} = 1 - \frac{1^3 + 2 \cdot 1 - 4}{3 \cdot 1^2 + 2} = 1.2$$

HW6.3 Convex optimization

What makes an optimization problem convex?

An optimization problem is convex if it is posed for a convex objection function over a convex set.

A set X in Euclidean space is convex if for every pair of points, $x_1, x_2 \in X$, every other point, x_3, along the straight line segment connecting x_1 and x_2 lies in X. A function $f(x)$ is convex over X if for all $x_1, x_2 \in X$ and for all $t \in [0, 1]$:

$$f(tx_1 - (1-t)x_2) \leq tf(x_1) - (1-t)f(x_2)$$

Intuitively, a convex function lies below its chords.

What are the first order Necessary Conditions for Optimality in convex optimization?

The first order necessary condition for optimality in convex optimization states that for $x = v$ to be an extremum, the gradient evaluated at c must be equal to the zero vector:

$$\nabla f(v) = (0, \cdots, 0)$$

What are the second order optimality conditions for convex optimization?

If the problem is convex, then the Hessian is gauranteed to be positive semi-definite, and a local minimum is a global minimum.

Are both necessary to determine the maximum or minimum of candidate optimal solutions?

The first order optimality condition is both necessary and sufficient for convex optimization. If the objective function is concave, i.e.,

$$f(tx_1 - (1-t)x_2) \geq tf(x_1) - (1-t)f(x_2)$$

then an extremum is a global maximum, and if the objective function is convex, then an extremum will be a global minimum.

HW 6.4

1. cost function $J(W) = \frac{1}{2}\sum_i weight_i(WX_i - y_i)^2$
2. let's assume i is the index for training sample, and j is the index for regression coefficients w_j
3. the gradient function $\nabla J(W)$ is a vector of $\frac{\partial J(w_j)}{\partial w_j}$, where:

$$\frac{\partial J(w_j)}{\partial w_j} = \frac{\partial}{\partial w_j}\frac{1}{2}\sum_i weight_i(WX_i - y_i)^2 = \frac{1}{2}\sum_i weight_i \times \frac{\partial}{\partial w_j}(WX_i - y_i)^2$$

$$= \frac{1}{2}\sum_i weight_i \times 2(WX_i - y_i)\frac{\partial}{\partial w_j}(WX_i) = \sum_i weight_i(WX_i - y_i)X_{ij}$$

Finally, we have

$$\nabla J(W) = \sum_i weight_i(WX_i - y_i)X_i$$

HW 6.5

In [9]:
```
# generate 10^6 data points and save to disk
import numpy as np
size = 1000000
x = np.random.uniform(-4, 4, size)
y = x * 1.0 - 4 + np.random.normal(0,0.5,size)
data = zip(y,x)
```

In [10]:
```
%%writefile MrJobBatchGDUpdate_LinearRegression.py
from mrjob.job import MRJob, MRStep

class MrJobBatchGDUpdate_LinearRegression(MRJob):
    # run before the mapper processes any input
    def read_weightsfile(self):
        # Read weights file
        with open('weights.txt', 'r') as f:
            self.weights = [float(v) for v in f.readline().split(',')]
        # Initialze gradient for this iteration
        self.partial_Gradient = [0]*len(self.weights)
        self.partial_count = 0

    # Calculate partial gradient for each example
    def partial_gradient(self, _, line):
        D = (map(float,line.split(',')))
        # y_hat is the predicted value given current weights
        y_hat = self.weights[0]+self.weights[1]*D[1]
        # Update partial gradient vector with gradient form current exampl
        self.partial_Gradient =  [self.partial_Gradient[0] + D[0]-y_hat,
                                  self.partial_Gradient[1]+ (D[0]-y_hat) *
        self.partial_count = self.partial_count + 1
        #yield None, (D[0]-y_hat,(D[0]-y_hat)*D[1],1)

    # Finally emit in-memory partial gradient and partial count
    def partial_gradient_emit(self):
        yield None, (self.partial_Gradient,self.partial_count)

    # Accumulate partial gradient from mapper and emit total gradient
    # Output: key = None, Value = gradient vector
    def gradient_accumulater(self, _, partial_Gradient_Record):
        total_gradient = [0]*2
        total_count = 0
        for partial_Gradient,partial_count in partial_Gradient_Record:
            total_count = total_count + partial_count
            total_gradient[0] = total_gradient[0] + partial_Gradient[0]
            total_gradient[1] = total_gradient[1] + partial_Gradient[1]
        yield None, [v/total_count for v in total_gradient]

    def steps(self):
        return [MRStep(mapper_init=self.read_weightsfile,
                       mapper=self.partial_gradient,
                       mapper_final=self.partial_gradient_emit,
                       reducer=self.gradient_accumulater)]

if __name__ == '__main__':
    MrJobBatchGDUpdate_LinearRegression.run()
```

```
Writing MrJobBatchGDUpdate_LinearRegression.py
```

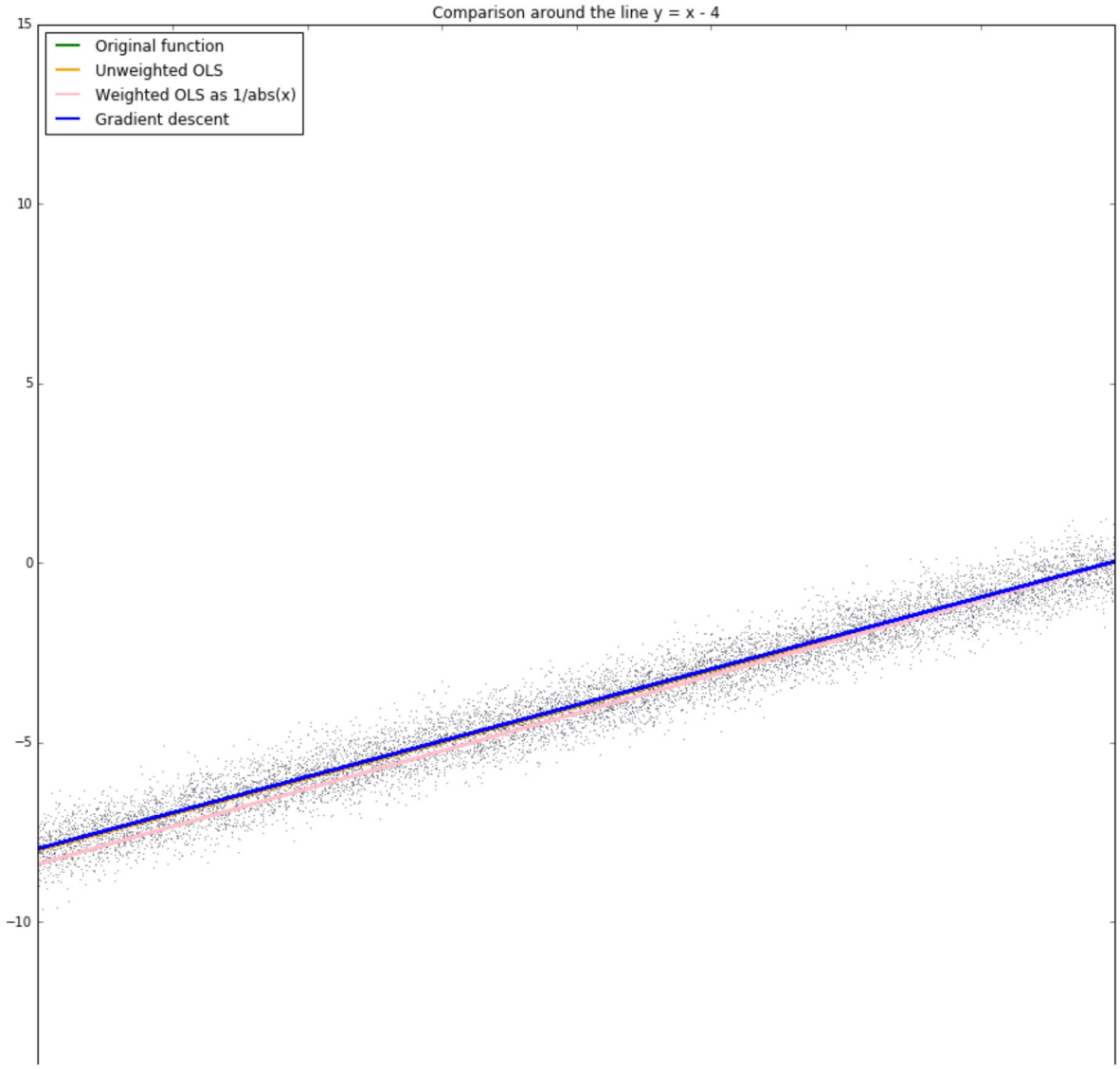
In [11]:
```
from numpy import random,array
from MrJobBatchGDUpdate_LinearRegression import MrJobBatchGDUpdate_LinearR

learning_rate = 0.05
stop_criteria = 0.000005

# Generate random values as inital weights
weights = array([random.uniform(-3,3),random.uniform(-3,3)])
# Write the weights to the files
with open('weights.txt', 'w+') as f:
    f.writelines(','.join(str(j) for j in weights))

# create a mrjob instance for batch gradient descent update over all data
mr_job = MrJobBatchGDUpdate_LinearRegression(args=['LinearRegression.csv',
                                                   '--file', 'weights.txt'
                                                   '--strict-protocols'])
# Update centroids iteratively
i = 0
while(1):
    print 'iteration: {0:4d} weights: {1:3.9f} {2:3.9f}'.format(i, weights
    # Save weights from previous iteration
    weights_old = weights
    with mr_job.make_runner() as runner:
        runner.run()
        # stream_output: get access of the output
        for line in runner.stream_output():
            # value is the gradient value
            key,value =  mr_job.parse_output_line(line)
            # Update weights
            weights = weights + learning_rate*array(value)
    i = i + 1
    # Write the updated weights to file
    with open('weights.txt', 'w+') as f:
        f.writelines(','.join(str(j) for j in weights))
    # Stop if weights get converged
    if(sum((weights_old-weights)**2)<stop_criteria):
        break

print 'Final weights: {0:3.9f} {1:3.9f}'.format(weights[0], weights[1])
```

```
iteration:    0 weights: -1.3786386929 -2.8695571806
iteration:    1 weights: -1.5098207277 -1.8370925893
iteration:    2 weights: -1.6344063174 -1.0800795571
iteration:    3 weights: -1.7527352472 -0.5250313016
iteration:    4 weights: -1.8651276550 -0.1180662770
iteration:    5 weights: -1.9718857229 0.1803222234
iteration:    6 weights: -2.0732950949 0.3991010700
iteration:    7 weights: -2.1696260853 0.5595091757
iteration:    8 weights: -2.2611347244 0.6771192043
iteration:    9 weights: -2.3480636776 0.7633492465
iteration:   10 weights: -2.4306430644 0.8265712002
iteration:   11 weights: -2.5090911951 0.8729234524
iteration:   12 weights: -2.5836152428 0.9069067480
iteration:   13 weights: -2.6544118590 0.9318210899
iteration:   14 weights: -2.7216677432 0.9500860805
iteration:   15 weights: -2.7855601726 0.9634757997
iteration:   16 weights: -2.8462574962 0.9732910220
```

In [27]:
```
from numpy.polynomial.polynomial import polyfit
from numpy.random import choice

size = 1000000
x = np.random.uniform(-4, 4, size)
y = x * 1.0 - 4 + np.random.normal(0,0.5,size)


j = choice(range(len(x)), 0.01*len(x))
## OLS
weights = float(1)/(x*0 + 1)
result1 = polyfit(x[j], y[j], 1, w=weights[j])
print "The intercept, slope from applying OLS without weights:"
print result1[0],result1[1]
print "="*100
## WOLS with w = 1/abs(x)
weights = float(1)/(abs(x))
result2 = polyfit(x[j], y[j], 1, w=weights[j])
print "The intercept, slope from applying weighted OLS as 1/abs(x):"
print result2[0],result2[1]
```

```
The intercept, slope from applying OLS without weights:
-3.99468225057 1.00104174181
====================================================================================
============================
The intercept, slope from applying weighted OLS as 1/abs(x):
-4.18999737701 1.050134644

/home/cloudera/anaconda2/lib/python2.7/site-packages/ipykernel/__main__
.py:9: VisibleDeprecationWarning: using a non-integer number instead of
 an integer will result in an error in the future
```

In [31]:
```
fig = py.figure(figsize = (15,15))
py.ylim(-15, 15)

py.plot(x[idx], y[idx],'o',ms=0.25)
py.plot(
    x[idx],x[idx]-4,
    color='green',linestyle='solid', label="Original function",lw = 2)

py.plot(
    x[idx],result1[1]*x[idx]+result1[0],lw = 2,
    color='orange',linestyle='solid',label="Unweighted OLS")

py.plot(
    x[idx],result2[1]*x[idx]+result2[0],lw = 2,
    color='pink',linestyle='solid',label="Weighted OLS as 1/abs(x)")

py.plot(
    x[idx],1.0001087412*x[idx]-3.9583702910 ,lw = 2,
    color='blue',linestyle='solid', label="Gradient descent")

py.title('Comparison around the line y = x - 4')
py.legend(loc=2)
```



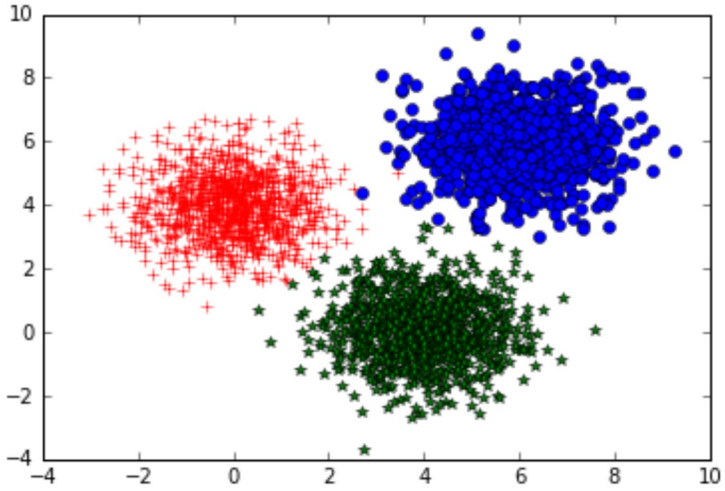
Discussion

From figure, OLS provides a result (orange) that is far from the truth (green), and further, even WLOS with $w(x) = \frac{1}{|x|}$ (pink) does not do good enough to fit desired results.

HW6.6 Clean up notebook for GMM via EM

In [85]:
```
%matplotlib inline
from numpy import random, append
import matplotlib.pyplot as plt
import matplotlib.cm as cm
import json

# generate 3 clusters the size of 1000 points each
size1 = size2 = size3 = 1000

# cluster 1 is centered about (4,0)
samples1 = random.multivariate_normal([4, 0], [[1, 0],[0, 1]], size1)
data = samples1

# cluster 2 is centered about (6,6)
samples2 = random.multivariate_normal([6, 6], [[1, 0],[0, 1]], size2)
data = append(data,samples2, axis=0)

# cluster 3 is centered about (0,4)
samples3 = random.multivariate_normal([0, 4], [[1, 0],[0, 1]], size3)
data = append(data,samples3, axis=0)

# we've appended all the generated data for the three clusters into a sing
# now randomize the list - shaken, not stirred. Save the result to a JSON
data = data[random.permutation(size1+size2+size3),]
with open("data.txt", "w") as f:
    for row in data.tolist():
        json.dump(row, f)
        f.write("\n")
```

In [86]:
```
plt.plot(samples1[:, 0], samples1[:, 1],'*', color = 'green')
plt.plot(samples2[:, 0], samples2[:, 1],'o',color = 'blue')
plt.plot(samples3[:, 0], samples3[:, 1],'+',color = 'red')
plt.show()
```



In [60]:
```
%%writefile mr_GMixEmInitialize.py
from mrjob.job import MRJob, MRStep

from numpy import mat, zeros, shape, random, array, zeros_like, dot, linal
from numpy import mean as npmean
from random import sample
import json
from math import pi, sqrt, exp, pow


class MrGMixEmInit(MRJob):
    DEFAULT_PROTOCOL = 'json'

    def __init__(self, *args, **kwargs):
        super(MrGMixEmInit, self).__init__(*args, **kwargs)
         # number of mappers
        self.numMappers = 1
         # count number of points to be classified
        self.count = 0


        # Configuations for input arguments
    def configure_options(self):
        super(MrGMixEmInit, self).configure_options()
        self.add_passthrough_option(
            '--k', dest='k', default=3, type='int',
            help='k: number of densities in mixture')
        self.add_passthrough_option(
            '--pathName', dest='pathName', default="", type='str',
            help='pathName: pathname where intermediateResults6_6.txt is s

    # each line is a data point of the cluster x_j
    # take the first 2*k lines and emit them then ignore the rest
    def mapper(self, key, xjIn):
        #something simple to grab random starting point
        #collect the first 2*k
        if self.count <= 2*self.options.k:
            self.count += 1
            yield (1,xjIn)

    # every key is the same = 1, so we should have 2*k items show up at th
    # all we want is to accumulate the points into a list from which we wi
    # Using update formula to get next iteration inputs for mean, covarain
    def reducer(self, key, xjIn):
        #accumulate data points mapped to 0 from 1st mapper and pull out k
        initial_point_set = []
        for xj in xjIn:
            x = json.loads(xj)
            initial_point_set.append(x)
            yield 1, xj

        # take k random samples
        centroids = [ initial_point_set[i] for i in sample(xrange(len(init

        # use the covariance of the selected centers as the starting guess
        # first, calculate mean of centers, the mu vector.
```

In [61]:
```
%%writefile mr_GMixEmIterate.py
from mrjob.job import MRJob

from math import sqrt, exp, pow,pi
from numpy import zeros, shape, random, array, zeros_like, dot, linalg
import json

# compute the guassian probability for a point x
def gauss(x, mu, P_1):
    xtemp = x - mu
    n = len(x)
    p = exp(- 0.5*dot(xtemp,dot(P_1,xtemp)))
    detP = 1/linalg.det(P_1)
    p = p/(pow(2.0*pi,n/2.0)*sqrt(detP))
    return p

class MrGMixEm(MRJob):
    DEFAULT_PROTOCOL = 'json'

    def __init__(self, *args, **kwargs):
        super(MrGMixEm, self).__init__(*args, **kwargs)

        with open(self.options.pathName + 'intermediateResults6_6.txt', 'r
            inputJson = fileIn.read()

        inputList = json.loads(inputJson)
        self.phi =   inputList[0]  # prior class probabilities
        self.means = inputList[1]  # means
        self.cov_1 = inputList[2]  # inverse covariance matrices

        #sum of weights - by cluster
        self.new_phi = zeros_like(self.phi)        #partial weighted sum o
        self.new_means = zeros_like(self.means)
        self.new_cov = zeros_like(self.cov_1)
        #number of mappers
        self.numMappers = 1
        #counts for points
        self.count = 0


    def configure_options(self):
        super(MrGMixEm, self).configure_options()

        self.add_passthrough_option(
            '--k', dest='k', default=3, type='int',
            help='k: number of densities in mixture')
        self.add_passthrough_option(
            '--pathName', dest='pathName', default="", type='str',
            help='pathName: pathname where intermediateResults6_6.txt is s

    # each item is a point in the set of data points
    def mapper(self, _, val):
        #accumulate partial sums for each mapper
        xList = json.loads(val)
        x = array(xList)
        wtVect = zeros_like(self.phi)
```

In [62]:
```
from mr_GMixEmInitialize import MrGMixEmInit
from mr_GMixEmIterate import MrGMixEm
import json
from math import sqrt
import matplotlib.pyplot as plt


# Plot each iteration output
def plot_iteration(means):
    plt.plot(samples1[:, 0], samples1[:, 1], '.', color = 'blue')
    plt.plot(samples2[:, 0], samples2[:, 1], '.', color = 'blue')
    plt.plot(samples3[:, 0], samples3[:, 1],'.', color = 'blue')
    plt.plot(means[0][0], means[0][1],'*',markersize =10,color = 'red')
    plt.plot(means[1][0], means[1][1],'*',markersize =10,color = 'red')
    plt.plot(means[2][0], means[2][1],'*',markersize =10,color = 'red')
    plt.show()

def dist(x,y):
    #euclidean distance between two lists
    sum = 0.0
    for i in range(len(x)):
        temp = x[i] - y[i]
        sum += temp * temp
    return sqrt(sum)

# first run the initializer to get starting centroids
filePath = 'data.txt'
mrJob = MrGMixEmInit(args=[filePath,
                           '--k','3',
                           '--pathName','/home/cloudera/',
                           '--strict-protocols'])

# after running this MRJob we will have an initialized  mean, prior probab

with mrJob.make_runner() as runner:
    runner.run()

# pull out the centroid values to begin iterate after one iteration
emPath = "/home/cloudera/intermediateResults6_6.txt"
with open(emPath, "r") as fileIn:
    paramJson = fileIn.read()

delta = 10
iter_num = 0
#Begin iteration on change in centroids
while delta > 0.02:
    print "Iteration" + str(iter_num)
    iter_num = iter_num + 1
    # parse old centroid values
    oldParam = json.loads(paramJson)
    # run one iteration
    oldMeans = oldParam[1]
    mrJob2 = MrGMixEm(args=[filePath,
                           '--k','3',
                           '--pathName','/home/cloudera/',
                           '--strict-protocols'])
```

HW6.7 Implement Bernoulli Mixture Model via EM

In [5]:
```
import json

#Unit test data into the correct format
data = {1: 'hot chocolate cocoa beans',
        2: 'cocoa ghana africa',
        3: 'beans harvest ghana',
        4: 'cocoa butter',
        5: 'butter truffles',
        6: 'sweet chocolate',
        7: 'sweet sugar',
        8: 'sugar cane brazil',
        9: 'sweet sugar beet',
        10: 'sweet cake icing',
        11: 'cake black forest'}

# Collect the vocabulary of these documents
vocab = set()
for doc in data:
    words = data[doc].split()
    for word in words:
        vocab.add(word)
vocabList = list(vocab)

# Binarize the data according to our vocabulary
binaryData = []

for doc in data:
    binarized = [0.0]*len(vocabList)
    words = data[doc].split()
    for word in words:
        binarized[vocabList.index(word)] = 1.0
    binaryData.append(binarized)

with open('BMM_unit_data.txt', 'w') as f:
    for row in binaryData:
        json.dump(row, f)
        f.write('\n')
```

In [14]:
```
%%writefile MR_BMM_EM_Initialize.py
from mrjob.job import MRJob
import numpy as np
from random import sample
import json

class BMM_EM_Initialize(MRJob):


    # Set default protocol
    DEFAULT_PROTOCOL='json'

    # Define initializer
    def __init__(self, *args, **kwargs):
        super(BMM_EM_Initialize, self).__init__(*args, **kwargs)

        # Set number of mappers
        self.numMappers = 1

        # Initialize count
        self.count = 0

    # Configure job options
    def configure_options(self):
        super(BMM_EM_Initialize, self).configure_options()
        self.add_passthrough_option('--k', dest='k', default=4, type='int'
                                    help='k: number of densities in mixtur
        self.add_passthrough_option('--pathName', dest='pathName', default
                                    help='pathName: path name where interm

    def mapper(self, _, line):
        # We are going to just output 2 * k points
        if self.count < 2 * self.options.k:
            self.count += 1
            yield (1, line)


    def reducer(self, key, values):


        # Initialize centroids


        k = self.options.k
        Centroids = []

        # Append each point from the mapper to centroids

        for val in values:
            Centroids.append(json.loads(val))
            yield 1, val

        # Sample k points

        sampleIndex = sample(range(len(Centroids)), k)
```

In [15]:
```
%%writefile MR_BMM_EM_Iterate.py
from mrjob.job import MRJob
import numpy as np
import json
from math import pi, sqrt, exp, pow, log
from decimal import *

# Calculating Bernoulli distribution, expressed by log

def BernouProb(x, mu):
    # Use logs here!
    n = len(x)
    logProb = 0
    for i in range(n):
        if mu[i] ** x[i] * (1 - mu[i]) ** (1 - x[i]) == 0:
            logProb = float('-inf')
            break
        elif mu[i] ** x[i] * (1 - mu[i]) ** (1 - x[i]) == 1:
            logProb += 0.0
        else:
            logProb += x[i] * log(mu[i]) + (1 - x[i]) * log(1 - mu[i])
    return logProb

class BMM_EM_Iterate(MRJob):


    DEFAULT_PROTOCOL='json'


    def __init__(self, *args, **kwargs):
        super(BMM_EM_Iterate, self).__init__(*args, **kwargs)

        # Read input from JSON
        fullPath = self.options.pathName + 'intermediateResults6_7.txt'
        with open(fullPath,'r') as infile:
            inputJSON = infile.read()
        inputList = json.loads(inputJSON)

        # Initialize prior probabilities (phi), centroids
        self.phi = np.array(inputList[0])
        self.centroids = np.array(inputList[1])

        # Initialize partial sums
        self.new_phi = np.zeros_like(self.phi)
        self.new_centroids = np.zeros_like(self.centroids)

        # Set number of mappers
        self.numMappers = 1

        # Initialize count
        self.count = 0

    # Configure job options
    def configure_options(self):
        super(BMM_EM_Iterate, self).configure_options()
        self.add_passthrough_option('--k', dest='k', default=3, type='int'
```

Driver

In [16]:
```
from MR_BMM_EM_Initialize import BMM_EM_Initialize
from MR_BMM_EM_Iterate import BMM_EM_Iterate

import json
from math import sqrt

# Helper to calculate Euclidean distance
def dist(x, y):
    sum = 0.0
    for i in range(len(x)):
        sum += (x[i] - y[i]) ** 2
    return sqrt(sum)

def run_bmm(filePath, k):
    # Run the initializer to get the starting centroids
    myPathName = '/home/cloudera/'
    mrJobInit = BMM_EM_Initialize(args=[filePath, '--pathName', myPathName

    with mrJobInit.make_runner() as runner:
        runner.run()

    # Read the initialized values for first comparison
    intermediatePath = 'intermediateResults6_7.txt'
    with open(intermediatePath, 'r') as f:
        paramJSON = f.read()

    # Initialize delta and iteration
    delta = 10
    iteration = 0
    TotalIterations = []

    # For unit test
    testWords = ['africa', 'brazil', 'cocoa', 'sugar', 'sweet']

    oldParam = json.loads(paramJSON)
    oldCentroids = oldParam[1]

    TotalIterations.append(oldParam)

    while delta > 0.02:
        iteration += 1

        # Parse old centroid values
        oldParam = json.loads(paramJSON)
        oldCentroids = oldParam[1]

        # Run one iteration of algorithm
        mrJobIterate = BMM_EM_Iterate(args=[filePath, '--pathName', myPathI
        with mrJobIterate.make_runner() as runner:
            runner.run()

        # Compare new centroids to old centroids
        with open(intermediatePath,'r') as infile:
            paramJSON = infile.read()

        newParam = json.loads(paramJSON)
```

Results of unit test

In [20]:
```
TotalIterations = run_bmm('BMM_unit_data.txt', 2)
lastResult = TotalIterations[-1]

# Final output
print 'After convergence:'
print '='*20
print '\nIterations:', len(TotalIterations)
print '\nalpha1:', lastResult[0][0]

testWords = ['africa', 'brazil', 'cocoa', 'sugar', 'sweet', 'ghana', 'bean

# For unit test
for word in testWords:
    print '='*40
    print 'q', word, '1: ', lastResult[1][0][vocabList.index(word)]
```

```
After convergence:
====================

Iterations: 11

alpha1: 0.545411881185
========================================
q africa 1:  0.0
q africa 2:  0.199981229483
========================================
q brazil 1:  7.07388301407e-11
q brazil 2:  0.199981229399
========================================
q cocoa 1:  0.166679707175
q cocoa 2:  0.399962455146
========================================
q sugar 1:  0.333281230567
q sugar 2:  0.200075026239
========================================
q sweet 1:  0.666640588763
q sweet 2:  9.38564882318e-05
========================================
q ghana 1:  0.166679703991
q ghana 2:  0.199981229483
========================================
q beans 1:  0.333359407982
q beans 2:  2.04611389861e-177
```

In [52]:
```
import csv

# Get our data into binarize and frequencies format


Y = []
X = []
X_freq = []
outfile = open('BMM_data.txt', 'w')
outfile2 = open('BMM_freq_data.txt', 'w')

with open('topUsers_Apr-Jul_2014_1000-words.txt', 'r') as infile:
    for line in csv.reader(infile):
        Y.append(int(line[1]))

        X_i = [float(int(x) > 0) for x in line[3:]]
        X.append(X_i)

        json.dump(X_i, outfile)
        outfile.write('\n')

        X_freq_i = [int(x) for x in line[3:]]
        X_freq.append(X_freq_i)

        json.dump(X_freq_i, outfile2)
        outfile2.write('\n')

outfile.close()
outfile2.close()
```

In [23]:
```
TotalIterations = run_bmm('BMM_data.txt', 4)
lastResult = TotalIterations[-1]
finalCentroids = lastResult[1]
```

Get the cluster assignments according to the final centroids

In [40]:
```
from decimal import *
from math import log
import numpy as np

def BernouProb(x, mu):
    # Use logs here!
    n = len(x)
    logProb = 0
    for i in range(n):
        if mu[i] ** x[i] * (1 - mu[i]) ** (1 - x[i]) == 0:
            logProb = float('-inf')
            break
        elif mu[i] ** x[i] * (1 - mu[i]) ** (1 - x[i]) == 1:
            logProb += 0.0
        else:
            logProb += x[i] * log(mu[i]) + (1 - x[i]) * log(1 - mu[i])
    return logProb

TotalWeights = []
for x in X:
    weights = np.zeros((len(finalCentroids)))
    for i in range(len(finalCentroids)):
        weights[i] = Decimal(BernouProb(x, finalCentroids[i])).exp()

    if sum(weights) == 0:
        weights += .00001
    weights /= sum(weights)
    TotalWeights.append(weights)
```

In [41]:
```
# Go through all the weights and pick out the index of the max probability
# If there is a tie, go with the class that occurs the most frequently
Result_BMM = []

for i in range(len(TotalWeights)):
    if np.all(TotalWeights[i] == TotalWeights[i][0]):
        Result_BMM.append(np.argmax(sum(TotalWeights)) + 1)
    else:
        Result_BMM.append(np.argmax(TotalWeights[i]) + 1)
```

Repeat this experiment using your KMeans MRJob implementation fron HW4.

In [53]:
```
# Use the centroids generated in HW4 KMeans as one benchmark
# Remember that this KMeans was done on the frequency counts, not the bina

# Calculate find the nearest centroid for data point
def MinDist(datapoint, centroid_points):
    datapoint = np.array(datapoint)
    centroid_points = np.array(centroid_points)
    diff = datapoint - centroid_points
    diffsq = diff*diff
    # Get the nearest centroid for each instance
    minidx = np.argmin(list(diffsq.sum(axis = 1)))
    return minidx

with open('Centroids_1.txt', 'r') as myfile:
    centroids_KMeans = [map(float,s.split('\n')[0].split(',')) for s in my

Result_KMeans = []

for x in X_freq:
    Result_KMeans.append(int(MinDist(x,centroids_KMeans)) + 1)
```

In [55]:
```
from sklearn.metrics.cluster import adjusted_rand_score

print "Comparing to BMM:   ", adjusted_rand_score(Result_BMM, Y)
print "Comparing to KMeans:", adjusted_rand_score(Result_KMeans, Y)
```

```
Comparing Y (truth) to BMM:    0.624275291483
Comparing Y (truth) to KMeans: 0.372943282038
```

In this case, the KMeans algorithm has a smaller Rand index than the Bernoulli Mixture Model via EM. Both algorithms are heavily dependent on the initialization methods used.

HW6.8 (OPTIONAL) 1 Million songs