

1. Simulations by script (use the function ode45)

From $\det|sI-A|$, we know characteristic eqn. $q(s)$ is

$$q(s) = M \cdot L \cdot s^3 - K_D \cdot s^2 - [(M+m)g + K_P]s - K_I.$$

Utilizing Routh-Hurwitz criterion, we have

$$s^3 \mid \quad M \cdot L \quad - [(M+m)g + K_P]$$

$$s^2 \mid \quad -K_D \quad -K_I$$

$$s^1 \mid \quad b \quad 0$$

$$s^0 \mid \quad -K_I \quad 0$$

where

$$b = [K_D \cdot [(M+m)g + K_P] - (M \cdot L \cdot (-K_I))] / -K_D.$$

For a stable system, we require that the coefficient of $q(s)$ be positive and $b > 0$, which means that

$$-[(M+m)g + K_P] - (M \cdot L \cdot (K_I / K_D)) > 0, \quad -K_D > 0, \quad -K_I > 0.$$

Therefore, the range of PID parameters that stabilize this system are

$$K_P < - (M+m)g - (M \cdot L \cdot (K_I / K_D)), \quad K_D < 0, \quad K_I < 0$$

For my PID controller I choose

$$K_D =$$

$$-100$$

$$K_I =$$

$$-30$$

$$K_P =$$

$$-1.1391e+03$$

PID controller system:

$$C =$$

$$K_p + K_i * \frac{1}{s} + K_d * s$$

$$\text{with } K_p = -1.14e+03, K_i = -30, K_d = -100$$

Continuous-time PID controller in parallel form.

Closed-loop transfer fun.

$T_{\text{sys}} =$

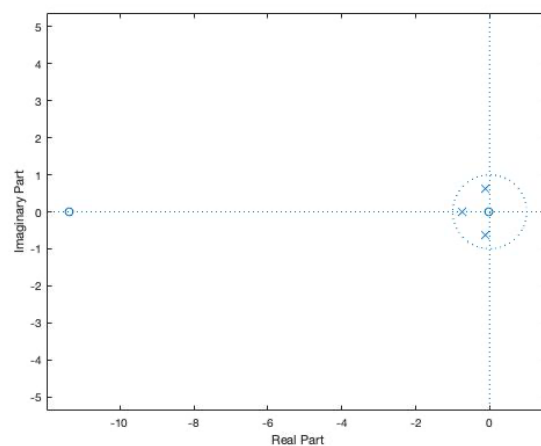
$$\frac{s^2 + 11.39 s + 0.3}{s^3 + s^2 + 0.6 s + 0.3}$$

Continuous-time transfer function.

The roots of my characteristic eqn. $q(s)$ are

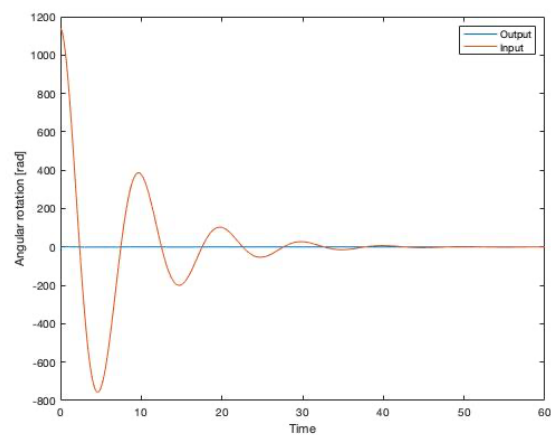
ans =

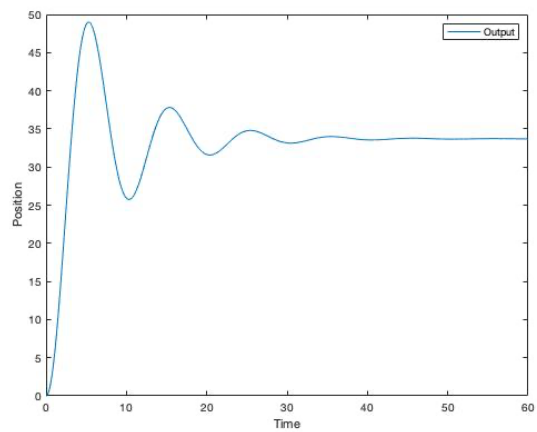
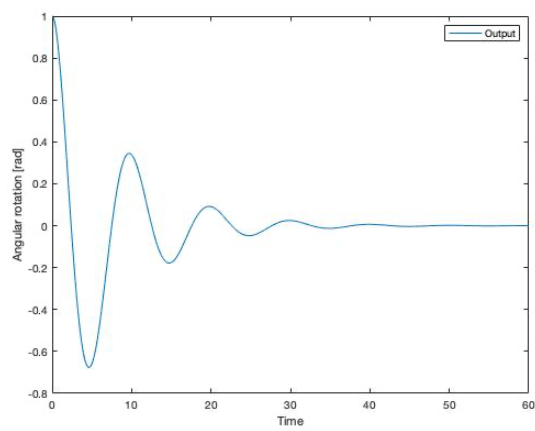
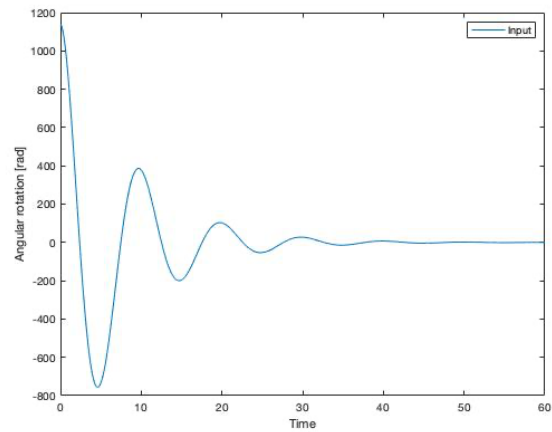
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- 0.131068 - 0.624019i  
- 0.131068 + 0.624019i  
-0.737864
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All roots are located in the left-half s-plane, means system is stable.

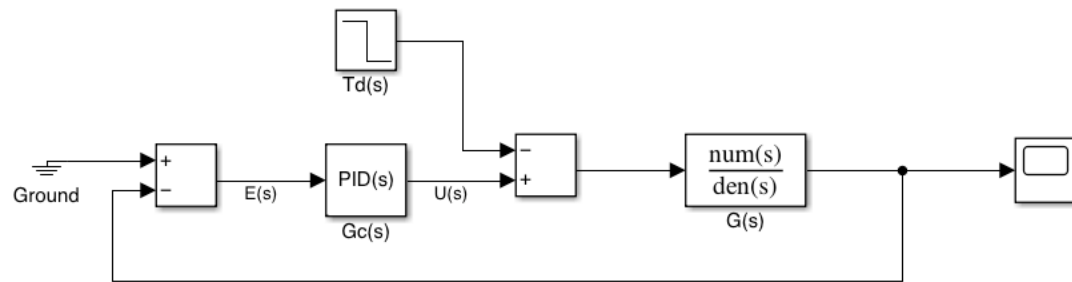
The waveforms using ode45:



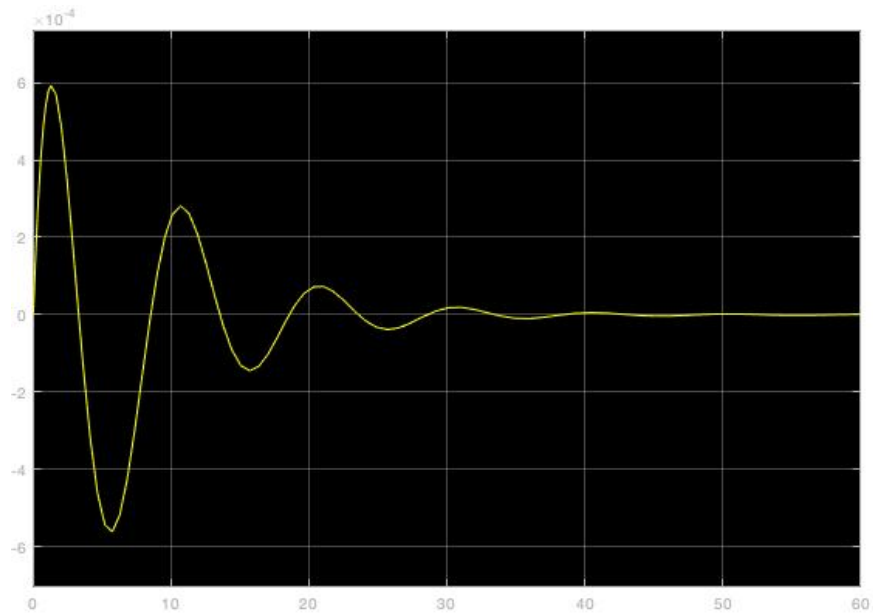


See the waveform of output angular rotation. The output response decays to zero as time approaches infinity, showing that system is stable.

2. Simulations by Simulink



The waveform using Simulink:



The output response also decays to zero as time approaches infinity, showing that system is stable.