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1.

$A_s = -20\log 0.0001 = 80$

$W_c = \frac{W_p + W_s}{2} = \frac{0.8\pi}{2} = 0.4\pi$

$\Delta W = 0.5\pi - 0.3\pi = 0.2\pi$

2. (a)

$W[n] = 0.5W_R[n] - 0.5\cos\left(\frac{2\pi n}{M}\right)W_R[n]$

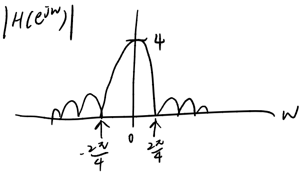
$W(e^{jw}) = 0.5W_R(e^{jw}) - 0.5\left(\frac{1}{2}\left(W_R(e^{j(w-\frac{2\pi}{M})}) + W_R(e^{j(w+\frac{2\pi}{M})})\right)\right)$

(b) 因為在time domain Hann window的波形比較smooth，較少高頻的部分，rectangular has a jump discontinuous 所以在frequeny domain Hann的mainlobe就會比rectangular來的大，且has the faster decay of the sidelobe.

3.

$H(e^{jw}) = \sum_{n=0}^{3} e^{-jwn} = \frac{1-e^{-j4w}}{1-e^{-jw}} = e^{-jw\frac{3}{2}}\frac{\sin(2w)}{\sin(\frac{w}{2})}$

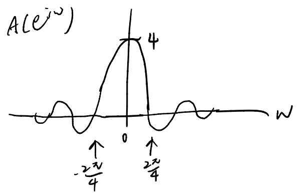
$|H(e^{jw})| = \sqrt{\left(\frac{\sin(2w)}{\sin(\frac{w}{2})}\right)^2}$



(b)

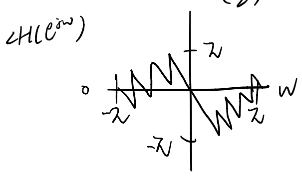
$A(e^{jw}) = \frac{\sin(2w)}{\sin(\frac{w}{2})}$

$A(e^{jw})$與$|H(e^{jw})|$只差在$|H(e^{jw})|$把負數值翻為正數

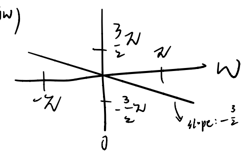


(c)

$\angle H(e^{jw}) = -\frac{3}{2}w + \angle\frac{\sin(2w)}{\sin(\frac{w}{2})} = -\frac{3}{2}w + \tan^{-1}\left(\frac{0}{\sin(2w)}\right) - \tan^{-1}\left(\frac{0}{\sin(\frac{w}{2})}\right)$



(d)



$\psi(e^{jw}) = -\frac{3}{2}w$

phase response is not continuous, but angle response is continuous.

4. (a)

$H(e^{jw}) = \sum_{k=0}^{M} h[k]e^{-jwk} = \left(h[0] + h[1]e^{-jw} + h[2]e^{-2jw} + \cdots + -h[2]e^{-j(M-2)w} + -h[1]e^{-j(M-1)w} + -h[0]e^{-Mjw}\right) = 2j\left(\frac{h[0]e^{j\frac{M}{2}w} + h[1]e^{j(\frac{M}{2}-1)w} + \cdots + -h[0]e^{-j\frac{M}{2}w}}{2j}\right)e^{-j\frac{M}{2}w}$

$= 2j\left(h[0]\sin\left(\frac{M}{2}w\right) + h[1]\sin\left(\frac{M}{2}-1\right)w) + \cdots + h\left[\frac{M+1}{2}\right]\sin\left(-\frac{1}{2}w\right)\right)e^{-jw\frac{M}{2}} = \sum_{k=1}^{\frac{M+1}{2}} d[k]\sin\left[w\left(k-\frac{1}{2}\right)\right]je^{-jw\frac{M}{2}}$, $d[k] = 2h\left[\frac{(M+1)}{2} - k\right]$ $k = 1, 2, \cdots, \frac{M+1}{2}$

$H(e^{jw}) = jA(e^{jw})e^{-jw\frac{M}{2}}$, $A(e^{jw}) = \sum_{k=1}^{\frac{M+1}{2}} d[k]\sin\left(w\left(k-\frac{1}{2}\right)\right)$

(b)

$A(e^{jw}) = \sum_{k=1}^{\frac{M+1}{2}}\left(d[k]\sin\left[w\left(k-\frac{1}{2}\right)\right]\right) = d[1]\sin\left(w-\frac{w}{2}\right) + d[2]\sin\left(2w-\frac{w}{2}\right) + \cdots + d\left[\frac{M+1}{2}\right]\sin\left(\frac{M+1}{2}w - \frac{w}{2}\right)$

$\sin\left(w-\frac{w}{2}\right) = 2\sin\frac{w}{2}\cos w - \sin\left(w+\frac{w}{2}\right)$, $\sin\left(2w-\frac{w}{2}\right) = 2\sin\frac{w}{2}\cos 2w - \sin\left(2w+\frac{w}{2}\right)$, $\sin\left(\frac{M+1}{2}w - \frac{w}{2}\right) = 2\sin\frac{w}{2}\cos\frac{M+1}{2}w - \sin\left(\frac{M+1}{2}w + \frac{w}{2}\right)$

$A(e^{jw}) = d[1]\sin\left(\frac{w}{2}\right) + d[2]\sin\left(\frac{3w}{2}\right) + d[3]\sin\left(\frac{5w}{2}\right) + \cdots$

$= \sin\left(\frac{w}{2}\right)\left[(d[1] - d[2] + d[3] + \cdots) + 2(d[2] - d[3] + d[4] + \cdots)\cos w + 2(d[3] - d[4] + d[5] + \cdots)\cos 2w + \cdots\right]$

$= \sin\left(\frac{w}{2}\right)\sum_{k=0}^{\frac{M-1}{2}}\hat{d}[k]\cos wk$, $d[k] = \begin{cases} \frac{1}{2}(2\hat{d}[k] - \hat{d}[1]), & k=1 \\ \frac{1}{2}(2\hat{d}[k-1] - \hat{d}[k]), & 2 \le k \le \frac{M-1}{2} \\ \frac{1}{2}(2\hat{d}[\frac{M-1}{2}]/2), & k = \frac{M+1}{2} \end{cases}$