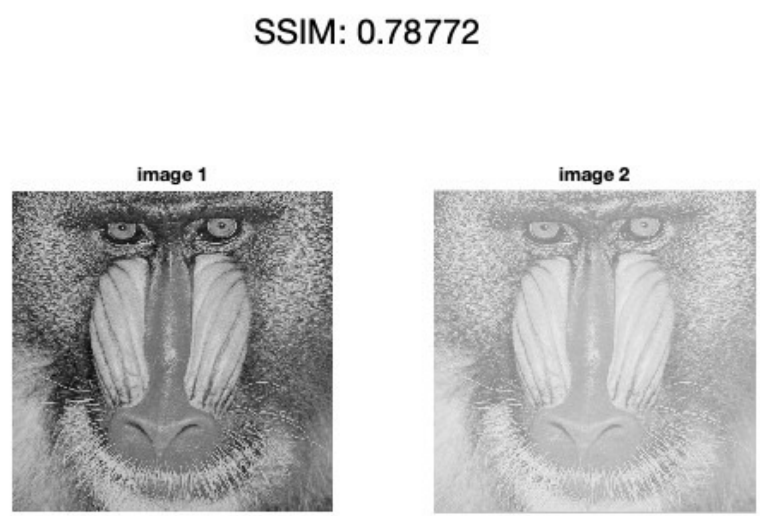
Homework 4 (Due: 5/24)

(1) Write a Matlab or Python program to measure the structural similarity (SSIM) of two images A and B. The sizes of A and B are equivalent.

SSIM(A, B, c1, c2)

where c1 and c2 are some adjust constants.

The Matlab or Python code should be handed out by NTUCool. (20 scores)



(2) (a) How do we use three real multiplications to implement a complex multiplication? (10 scores)

(b) Suppose that

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} b_1 & -b_2 & -b_3 & -b_4 \\ b_2 & b_1 & b_4 & -b_3 \\ b_3 & b_4 & b_1 & -b_2 \\ -b_4 & b_3 & b_2 & b_1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$$

How do we implement above matrix operation with the least number of real multiplications? (10 scores)

(a) 根據講義 p.349，

$$(a+jb)(c+jd) = ac - bd + j(ad+bc)$$

$$令\ ac - bd = e, ad + bc = f$$

$$\begin{bmatrix} e \\ f \end{bmatrix} = \begin{bmatrix} c & -d \\ d & c \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} c & c \\ c & c \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} 0 & -c-d \\ d-c & 0 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

$$① \quad g_1 = c(a+b),\ h_1 = g_1$$

$$② \quad g_2 = (-c-d)b,\ h_2 = (d-c)a$$

$$③ \quad e = g_1 + g_2,\ \ f = h_1 + h_2$$

=> 總共需要 3 MULs, 5 ADDs

(b)

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} b_1 & -b_2 & -b_3 & -b_4 \\ b_2 & b_1 & b_4 & -b_3 \\ b_3 & b_4 & b_1 & -b_2 \\ -b_4 & b_3 & b_2 & b_1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$$

$$\begin{bmatrix} c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} b_2 & -b_3 \\ b_3 & -b_2 \end{bmatrix} \begin{bmatrix} a_1 \\ a_4 \end{bmatrix} + \begin{bmatrix} b_1 & b_4 \\ b_4 & b_1 \end{bmatrix} \begin{bmatrix} a_2 \\ a_3 \end{bmatrix}$$

$$\begin{bmatrix} c_1 \\ c_4 \end{bmatrix} = \begin{bmatrix} b_1 & -b_4 \\ -b_4 & b_1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_4 \end{bmatrix} + \begin{bmatrix} -b_2 & -b_3 \\ b_3 & b_2 \end{bmatrix} \begin{bmatrix} a_2 \\ a_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} b_2 & b_3 \\ b_3 & b_2 \end{bmatrix} \begin{bmatrix} a_1 \\ -a_4 \end{bmatrix} + \begin{bmatrix} b_1 & b_4 \\ b_4 & b_1 \end{bmatrix} \begin{bmatrix} a_2 \\ a_3 \end{bmatrix}$$

$$case3: z_1 = \frac{b_2 + b_3}{2}(a_1 - a_4), z_2 = z_1, z_3 = \frac{b_2 - b_3}{2}(a_1 + a_4), z_4 = -z_3, y_1 = z_1 + z_3, y_2 = z_2 + z_4$$

$$case3: z_5 = \frac{b_1 + b_4}{2}(a_2 + a_3), z_6 = z_5, z_7 = \frac{b_1 - b_4}{2}(a_2 - a_3), z_8 = -z_7, y_3 = z_5 + z_7, y_4 = z_6 + z_8$$

$$c_2 = y_1 + y_3, c_3 = y_2 + y_4,$$

$$\Rightarrow \begin{bmatrix} c_1 \\ c_4 \end{bmatrix} = \begin{bmatrix} b_1 & -b_4 \\ -b_4 & b_1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_4 \end{bmatrix} + \begin{bmatrix} -b_2 & -b_3 \\ b_3 & b_2 \end{bmatrix} \begin{bmatrix} a_2 \\ a_3 \end{bmatrix}$$

$$case3: z_1 = \frac{b_1 - b_4}{2}(a_1 + a_4), z_2 = z_1, z_3 = \frac{b_1 + b_4}{2}(a_1 - a_4), z_4 = -z_3, y_1 = z_1 + z_3, y_2 = z_2 + z_4$$

$$y_3 = -b_2 a_2 - b_3 a_3, \ y_4 = b_3 a_2 + b_2 a_3$$

$$c_1 = y_1 + y_3, c_4 = y_2 + y_4,$$

=> 總共 10 MULs

(3) Determining <u>the numbers of real multiplications</u> for the (a) 125-point DFT, (b) the 147-point DFT, and (c) the 385-point DFT. (15 scores)

(a) 125 = 5 x 25，使用 case2:
N = P1 x P2，5-point DFT 的乘法量為 10，25-point DFT 的乘法量為 148，有 24*4 個值不為 125/12 及 125/8 的倍數，則 125-point DFT 的乘法量為 25*10 + 5*148 + 3*24*4 = 1278。

(b) 147 = 3 x 49
49 = 7 x 7，使用 case2:
7-point DFT 的乘法量為16，則 49-point DFT 的乘法量為 7*16 + 7*16 + 3*6*6 = 332。
再使用 case1:
3-point DFT 的乘法量為 2，49-point DFT 的乘法量為 332，則 147-point DFT 的乘法量為 49*2 + 3*332 = 1094。

(c) 385 = 11 x 35，使用 case1:
11-point DFT 的乘法量為 40， 35-point DFT 的乘法量為 150，則 385-point DFT 的乘法量為 35*40 + 11*150 = 3050 。

(4) What is the complexity of the 3D DFT as follows? Express the solution in terms of the big order. (10 scores)

$$Y[p,q,r]=\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\sum_{k=0}^{K-1}e^{-j2\pi\frac{pm}{M}-j2\pi\frac{qn}{N}-j2\pi\frac{rk}{K}}x[m,n,k]$$

Complexity of the 1-D N-point DFT:

$$O(NlogN)$$

Complexity of the 2-D MN-point DFT:

$$M(NlogN)+N(MlogM)=MN(logN+logM)=MNlog(MN)$$

Complexity of the 3-D MNK-point DFT:

$$\begin{aligned}&KM(NlogN)+KN(MlogM)+MN(KlogK)\\&=MNK(logN+logM+logK)\\&=MNKlog(MNK)\end{aligned}$$

$$\Rightarrow O(MNKlog(MNK))$$

(5) Suppose that there are 1200 cars in a dataset and an algorithm detects 1000 cars. However, among the detected cars, 100 of them are in fact other objects. Determine the precision, the recall, and the F-score of the algorithm.

(10 scores)

True positive (TP): 事實上為真，而且被我們的方法判斷為真的情形

TP = 1000 - 100 = 900

False negative (FN): 事實上為真，卻未我們的方法被判斷為真的情形

FN = 1200 - 900 = 300

False positive (FP): 事實上不為真，卻被我們的方法誤判為真的情形

FP = 100

Precision:

$$\frac{TP}{TP+FP} = \frac{900}{900+100} = \frac{900}{1000} = 0.9$$

Recall:

$$\frac{TP}{TP+FN} = \frac{900}{900+300} = \frac{900}{1200} = 0.75$$

F-score:

$$2\frac{preceision\ *\ recall}{precision\ +\ recall} = 2\frac{0.9 * 0.75}{0.9 + 0.75} = 0.8182$$

(6) Suppose that length($x[n]$) = 1100. What is the best way to implement the convolution of $x[n]$ and $y[n]$ if

(a) length($y[n]$) = 500, (b) length($y[n]$) = 40,

(c) length($y[n]$) = 6, and (d) length($y[n]$) = 2 ? (25 scores)

Please show (i) the convolution method (direct, sectioned convolution, or non-sectioned convolution), (ii) the number of points of the FFT, (iii) and the number of real multiplications for the best implementation method. Also, consider the general case where $x[n]$ and $y[n]$ are complex sequences and the FFT of $y[n]$ can be computed in prior.

(a)

(i) 因為$M \approx N$，所以 convolution method 選擇 case3： non-sectioned convolution

(ii)

$$P \geq M + N - 1 \Rightarrow P \geq 500 + 1100 - 1 = 1599$$

(iii)

$$\text{P choose 1680, } \mathrm{MUL}_{1680} = 10420$$

Number of real multiplications:

$$2MUL_p + 3P = 2 * 10420 + 3 * 1680 = 25880$$

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(b)

(i) 因為$N \gg M$，所以 convolution method 選擇 case2： sectioned convolution

(ii)

$$L_0 = 248, P_0 = L_0 + M - 1 = 287$$
$$\text{P choose 288, MUL}_{288} = 1160$$

(iii)

$$L = P - M + 1 = 249, S = \left\lceil \frac{N}{L} \right\rceil = \left\lceil \frac{1100}{249} \right\rceil = 5$$

Number of real multiplications:

$$2S \times MUL_P + 3S \times P = 2 * 5 * 1160 + 3 * 5 * 288 = 15920$$

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(a) length($y[n]$) = 500, (b) length($y[n]$) = 40,

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Please show (i) the convolution method (direct, sectioned convolution, or non-sectioned convolution), (ii) the number of points of the FFT, (iii) and the number of real multiplications for the best implementation method. Also, consider the general case where $x[n]$ and $y[n]$ are complex sequences and the FFT of $y[n]$ can be computed in prior.

(c)

(i) 因為$N \gg M$，所以 convolution method 選擇 case2： sectioned convolution

(ii)

$$L_0 = 19, P_0 = L_0 + M - 1 = 24$$
$$\text{P choose 24, } \mathrm{MUL}_{24} = 28$$

(iii)

$$L = P - M + 1 = 19, S = \left\lceil \frac{N}{L} \right\rceil = \left\lceil \frac{1100}{19} \right\rceil = 58$$

Number of real multiplications:

$$2S \times MUL_P + 3S \times P = 2 * 58 * 28 + 3 * 58 * 24 = 7424$$

(6) Suppose that length($x[n]$) = 1100. What is the best way to implement the convolution of $x[n]$ and $y[n]$ if

(a) length($y[n]$) = 500, (b) length($y[n]$) = 40,

(c) length($y[n]$) = 6, and (d) length($y[n]$) = 2 ? (25 scores)

Please show (i) the convolution method (direct, sectioned convolution, or non-sectioned convolution), (ii) the number of points of the FFT, (iii) and the number of real multiplications for the best implementation method. Also, consider the general case where $x[n]$ and $y[n]$ are complex sequences and the FFT of $y[n]$ can be computed in prior.

(d)

(i) 因為M是很小的整數，所以 convolution method 先選擇 case1： direct

$$3NM = 3 * 1100 * 2 = 6600$$

但使用 sectioned convolution 計算會發現，它的 real multiplications 是最佳的。

(ii)

$$L_0 = 2, P_0 = L_0 + M - 1 = 3$$

$$\text{P choose 4, } \mathrm{MUL}_4 = 0$$

(iii)

$$L = P - M + 1 = 3 \; S = \left\lceil \frac{N}{L} \right\rceil = \left\lceil \frac{1100}{3} \right\rceil = 367$$

Number of real multiplications:

$$2S \times MUL_P + 3S \times P = 2 * 367 * 0 + 3 * 367 * 4 = 4404$$

(Extra): Answer the questions according to your student ID number.

(ended with (2, 7), (3, 8), (4, 9), (0, 5))