Homework 1 (Due: March 22nd)

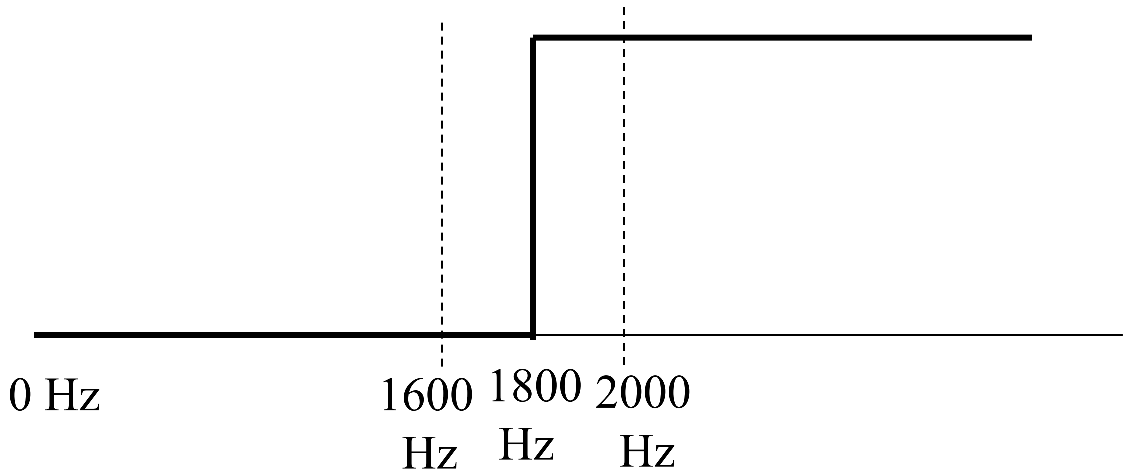
(1) Design a Mini-max **highpass** FIR filter such that (40 scores)

① Filter length = 21, ② Sampling frequency f_s = 8000Hz,

③ Pass Band 1800~4000Hz ④ Transition band: 1600~2000 Hz,

⑤ Weighting function: $W(F)$ = 1 for passband, $W(F)$ = 0.8 for stop band .

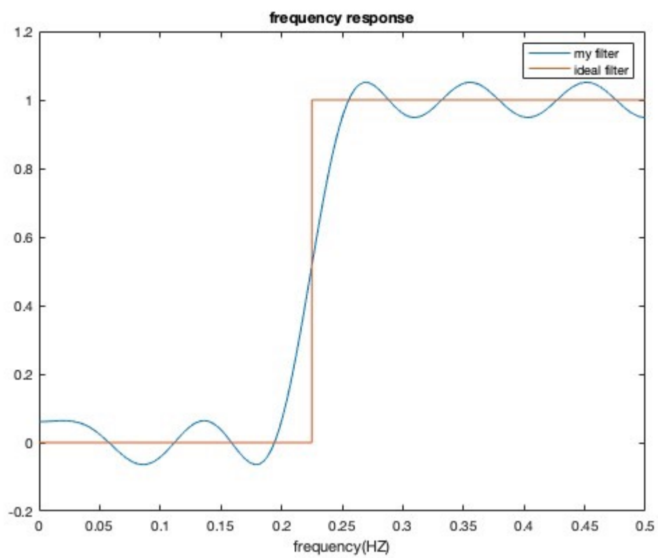
⑥ Set Δ = 0.0001 in Step 5.



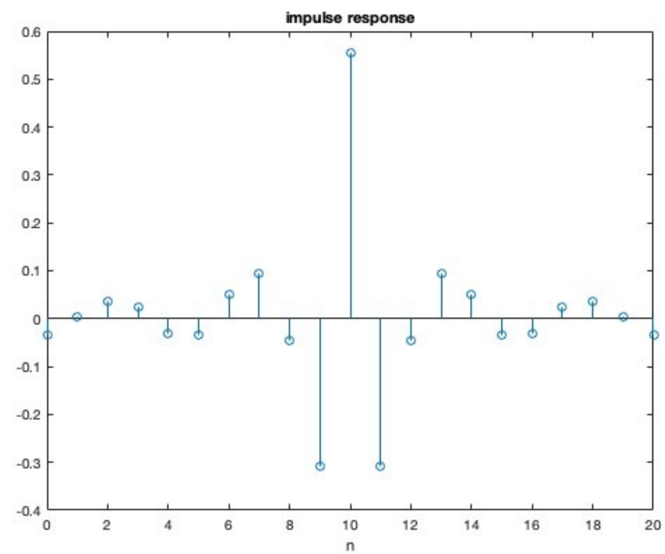
※ The code should be handed out by NTUCool, too.

Show (a) the frequency response, (b) the impulse response $h[n]$, and (c) the maximal error for each iteration.

(a) the frequency response



(b) the impulse response $h[n]$



(c) the maximal error for each iteration.

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E0_all =

    0.1367    0.1013    0.1105    0.0529    0.0511    0.0511
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(2) (a) Which type of systems can be implemented by convolution?

(b) How do we convert convolution into an addition operation? (10 scores)

(a) Linear time-invariant systems.

對於 LTI system，符合以下兩點：

Linear(Scaling + Superposition)：

給予 input x1(t) 得到 output y1(t)，給予 input x2(t) 得到 output y2(t)，則給予 input ax1(t)+bx2(t) 會得到 output 為 ay1(t)+by2(t)。

Time-invariant：

給予 input x(t) 得到 output y(t)，則給予 input x(t-s) 會得到 output為 y(t-s)。

若 input 為 Impulse function δ(t) 而 output 為 Impulse response h(t)，我們可以先將 input x(t) 變成由 Impulse function δ(t) 所組成的式子：$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau$ ，則 output y(t) 會得到 $y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$ 。

這就等於 x(t) 與 h(t) 做 convolution。

(b) 在 time domain 若為 convolution，經過 Fourier transform 可轉為相乘，再兩邊取 log 就可以變成 addition operation。

$$y[n] = x[n] * h[n]$$

$$\Downarrow \text{FT}$$

$$Y(f) = X(f)H(f)$$

$$\Downarrow \log$$

$$\hat{Y}(f) = \hat{X}(f) + \hat{H}(f) \qquad \hat{Y}(f) = \log(Y(f))$$

(3) (a) Describe three advantages of the FIR filter.

(b) How do we implement $y[n] = x[n] * (0.7^n u[n] + 0.2^n u[n])$ using the recursive method where * means the convolution and $u[n]$ is the unit step function?

(10 scores)

(a) 1. Output has finite length.

2. Usually less computation loading.

3. Stable. Because output is a sum of finite number of the input.

(b) Do z transform.

$$Y(Z) = X(Z)\left[\frac{1}{1-0.7Z^{-1}} + \frac{1}{1-0.2Z^{-1}}\right]$$

$$= X(Z)\left(\frac{1-0.2Z^{-1}+1-0.7Z^{-1}}{(1-0.7Z^{-1})(1-0.2Z^{-1})}\right) = X(Z)\left(\frac{2-0.9Z^{-1}}{1-0.9Z^{-1}+0.14Z^{-2}}\right)$$

$$\Rightarrow Y(Z)(1-0.9Z^{-1}+0.14Z^{-2}) = X(Z)(2-0.9Z^{-1})$$

Do inverse z transform.

$$y[n] - 0.9y[n-1] + 0.14y[n-2] = 2x[n] - 0.9x[n-1]$$

$$\Rightarrow y[n] = 0.9y[n-1] - 0.14y[n-2] + 2x[n] - 0.9x[n-1]$$

(4) What are the roles of (a) the transition band and (b) the weight function for minimax FIR filter design? (10 scores)

(a)
在不增加濾波器的點數的情況下，transition band可以讓誤差減少。 transition band越寬，設計出來的濾波器就相對平緩，passband 和 stopband的誤差就可以變小。有transition band的話，才有機會讓誤差<0.5。

(b)
可以透過給予passband 和stopband不同加權，在特定頻帶規定誤差要比較小，weight 就給大一點，在某些頻帶誤差允許大一些， weight就給小一點。

(5) Suppose that $x[n] = y(0.001n)$ and the length of $x[n]$ is 6000. If $X[m]$ is the FFT of $x[n]$, determine m such that $X[m]$ correspond to the frequencies of (a) 200Hz and (b) -100Hz. (10 scores)

$$sampling\ interval\ \Delta_t = 0.001, \text{ sampling frequency } f_s = \frac{1}{\Delta_t} = 1000\ Hz$$

$$N = 6000$$

$$f = \frac{m}{N} f_s\ , for\ m \le \frac{N}{2}$$

$$f = \frac{m}{N} f_s - f_s\ , for\ m > \frac{N}{2}$$

$(a)\ 200 = \frac{m}{6000} 1000 \Rightarrow m = 1200$

$(b)\ -100 = \frac{m}{6000} 1000 - 1000 \Rightarrow 5400$

(6) Use the MSE method to design the 7-point FIR filter that approximates the band filter of $H_d(F) = 1$ for $0.1 < |F| < 0.4$ and $H_d(F) = 0$ for $|F| < 0.1$ or $|F| > 0.4$.

(10 scores)

$$N = 7, k = \frac{N-1}{2} = 3$$

$$s[0] = \int_{-\frac{1}{2}}^{\frac{1}{2}} H_d(F)dF = \int_{-0.4}^{-0.1} 1\, dF + \int_{0.1}^{0.4} 1\, dF = 0.6$$

$$s[n] = 2\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos(2\pi nF)\, H_d(F)dF = 2\int_{-0.4}^{-0.1} \cos(2\pi nF)\, dF + 2\int_{0.1}^{0.4} \cos(2\pi nF)\, dF$$

$$= \frac{\sin(-0.2\pi n) - \sin(-0.8\pi n)}{\pi n} + \frac{\sin(0.8\pi n) - \sin(0.2\pi n)}{\pi n}$$

$$h[3] = s[0] = 0.6$$

$$h[2] = h[4] = \frac{s[1]}{2} = 0$$

$$h[1] = h[5] = \frac{s[2]}{2} = -0.3027$$

$$h[0] = h[6] = \frac{s[3]}{2} = 0$$

$$R(F) = 0.6 + 2\cdot -0.3027 \cdot \cos(4\pi F)$$

(7) Estimate the length of the digital filter if both the passband ripple and the stopband ripple are smaller than 0.01, the sampling interval $\Delta_t = 0.0001$, and the transition band is from 3000Hz to 3300Hz. (10 scores)

$$\delta_1 = \delta_2 = 0.01$$

$$f_s = \frac{1}{\Delta_t} = \frac{1}{0.0001} = 10000$$

$$\Delta F = \frac{\Delta f}{f_s} = \frac{3300 - 3000}{10000} = 0.03$$

$$N = \frac{2}{3} \cdot \frac{1}{\Delta F} \cdot log_{10}\left(\frac{1}{10 \cdot \delta_1 \cdot \delta_2}\right) = \frac{2}{3} \cdot \frac{1}{0.03} \cdot log_{10}\left(\frac{1}{10 \cdot 0.01 \cdot 0.01}\right)$$

$$= \frac{2}{3} \cdot \frac{1}{0.03} \cdot 3 = 66.66 \approx 67$$

(Extra): Answer the questions according to your student ID number.
(ended with 0, 1, 2, 3, 5, 6, 7, 8)

Q:
$f_s = 8000, N = 120000 = 15f_s$
$If\ m = 96000, f = ?$

$$Because\ m > \frac{N}{2} = 60000$$
$$\Rightarrow f = \frac{m}{N} f_s - f_s = \frac{96000}{120000} 8000 - 8000 = -1600\ \text{Hz}$$