

# Markov Games with Time-variant Types as a Framework for Human-robot Coordination

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**Abstract**—Coordination between humans and robots commonly happens when they co-exist in a shared workspace. Such scenario includes human-robot teaming on collaborative tasks, and human-robot conflict-resolving on limited shared resources. Mis-coordination reduces efficiency of both parties, and reduces the human’s trust and patience with the robot. In this work, we propose a game-theoretic framework to analyze convergence in human-robot coordination. We also propose human behavior hypotheses on their decision-making mechanisms under this framework, to capture how the human 1) perception of the robot’s capabilities, 2) personal preferences, 3) level of self-interest, and 4) social trust affect their policies and adaptability in dynamic environments. We provide human-robot path crossing as an instantiation of our framework, and use the hypothesized human behaviors to simulate real-world observed interaction patterns. Lastly, we simulate humans acting adaptively to their observed robot policies, as an initiative to incorporate effects on humans when designing robot algorithms using human-human interaction data.

## I. INTRODUCTION

Human-robot interaction has received increased attention in recent years due to the emerging interest in deploying robots in human environments. Such environments may involve human-robot collaboration on given tasks, humans and robots working in a shared workspace, or service robots deployed in human environments. In such environments, robots may need to coordinate with humans with partially shared information and partially shared objectives; agents may need to reach agreement on one solution among multiple feasible choices, which makes the coordination non-trivial to settle. For a motivating example, see Fig. 1, which shows three agents coordinating at an intersection, where each agent has an individual goal to reach.

For a robot to engage coordination without confusing the human, it first requires the basic capabilities to understand human intent, and to respond in a legible manner [7]. Beyond those, to negotiate and agree on the coordinating solutions, the robot needs knowledge of human agents’ behaviors to predict the outcome of its own actions. It also needs to know the reaction time humans need to update their policies [27], and to consider potential impacts of its own actions on humans’ future decisions [10], [9]. Then the robot can plan and coordinate with humans in an intent-consistent fashion.

Past research has sought to improve human-robot coordination in a variety of ways, including: intent-expressive robot motion generation [7], [19], human preference-aware behavior modeling and its use for coordination [11], [5], human-robot mutual adaptation [23], [22], human expectations on robot capability [2], [17], as well as trust and comfort for long-term deployment[32].

While these topics share a focus on factors which affect

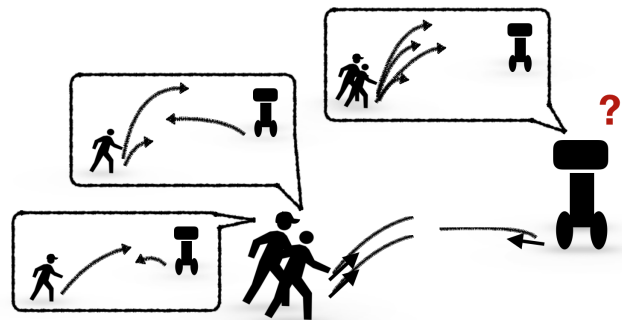


Fig. 1. Human-robot coordination in a crossing interaction. Since people have different prior assumptions on robot behaviors, their choice of actions differ.

human behaviors and their decision-making mechanisms, they lack a unifying framework to keep track of how those factors correlate with each other in different situations, how they affect human decisions and motions, and how they evolve over time as a function of robot interaction policies. Those questions are important for designing intent-consistent robots to interact with humans in daily situations. And they are important for designing robots that are aware of human adaptation to their long-term deployments. To fulfill such purposes, the main contribution of this work is a proposed game-theoretic model for human-robot coordination where agents have hidden preferences and time-variant adaptive behaviors based on online observations of others’ behaviors.

To discuss human behaviors while interacting with robots, we point out some important factors that have been proposed in the human-robot interaction community, such as perceived robot capability, personal preferences, and trust on robot ability. We illustrate how they affect human behaviors through our proposed human decision-making model under the game framework, using a receding-horizon approach. We also propose two factors that we observed to be important in the social navigation domain: level of self-interest and social trust. We incorporate them along with others to analyze how people interact with the robot, with different personal preferences and assumptions about robot behaviors. We collect real-world human-robot interaction data to illustrate our model effectiveness.

We also use our decision-making model to discuss human adaptability to robots, and simulate human adaptation to robots after perceiving robot socially trust-worthy behaviors.

## II. RELATED WORK

Despite the growing interest in deploying robots in human populated workspaces, motion planning algorithms for smooth human-robot coordination have remained a challenge. While traditional planning algorithms have shown

success in static environments, deploying such approaches alongside humans has shown insufficient adaptability to highly dynamic environments, and produce awkward motions making interpretability difficult [19], [7], [15]. Approaches considering time-variant factors, such as temporal constraints for multi-agent collision avoidance [31], have drawn attention for such applications; approaches considering human factors, such as human collision-avoidance behavior anticipation [12], have also gained attention for robot planning in human workspaces [28].

On the other hand, another community solves robot planning in human environments as a joint multi-agent planning problem, to incorporate the collective crowd behaviors with explicit modeling of the effect of agent actions on its surround agents [29], [16], [21]. Such joint modeling methodology has shown to be effective at outputting smooth human-mimicking trajectories at the same time acting responsively to its surrounding agents.

One major drawback applying these approaches interacting with humans is that the multi-agent joint dynamics models are typically learnt from data collected by human demonstrations, whereas humans do not act the same way around a robot compared to in pure-human environments. This problem has shown inefficiency of such approaches when humans use unexpected behaviors around robots – behaviors that humans will not present in front of another human [26].

To model joint behaviors among agents – how one’s action affects the other – another way is to incorporate individual’s action values with correlations with other agents’ actions. Such joint behavior formulation is widely studied in Game theory: with different player assumptions, games evolve with different outcomes. In the artificial intelligence community, such game formulation has been incorporated with Markov Decision Process for multi-agent reinforcement learning [20]. Such incorporation enables strategy design for multi-agent robotic systems with mutual learning.

Yet, to simulate human-robot interaction, it requires agents modeled after humans, who have distinctive learning mechanisms and decision-making processes from reinforcement learning agents; moreover, agents have different behavioral types, as humans have personal preferences; agents online adapt to other agents, as humans observe the others and plan accordingly. Those features are widely studied in the human-robot interaction community, but not yet well formulated by one unifying framework. We therefore propose an extensive framework on Markov games with time-variant types to incorporate human-robot interaction with multi-agent learning problems to eliminate the gap.

### III. PRELIMINARIES

In almost every real-life interaction between robots and humans, each agent’s utility is being influenced by the other agents’ actions in addition to his own action. Thus, we choose to model human-robot interaction as a game. We propose a model, extended from Markov Games, to incorporate types as in Bayesian games, which assign game

outcomes based on both agent actions and types<sup>1</sup>. A formal game definition as well as a quick overview about Markov Games are given in the following subsections.

#### A. Game Definition

Consider a game  $G$  with  $k$  players, where each player  $i \in \{1 \dots k\}$  has a finite action set  $A^i$ . The set of action profiles is denoted as  $\Sigma = A^1 \times A^2 \times \dots \times A^k$ . The utility of an agent  $i$  is a function, denoted as  $f^i : \sigma \rightarrow \mathbb{R}$ , evaluated at  $\sigma \in \Sigma$ .

In *repeated games* (i.e., games which repeat for more than one action per player),  $G^T$ , players receive cumulative utilities over a time horizon  $T$ , defined as:

$$V^i = \sum_{t=0}^T f^i(\sigma_t). \quad (1)$$

Where  $\sigma_t \in \Sigma_t$  is the action profile at time  $t$ . We will define a *strategy profile*,  $\sigma_{0:T}$ , to be the series of action profiles  $\sigma_t$  over time, i.e.,  $\sigma_0 \times \dots \times \sigma_T$ . One important feature in repeated games is that agents need to consider not only the current outcome of an action, but also its impact on the other agents’ future actions (which eventually will affect their own expected cumulative rewards).

Let  $A^H$  and  $A^R$  be the action space of humans and robots. A *Nash Equilibrium* in such a game is defined to be such that no agent (human or robot) benefits from unilateral deviation from his current strategy (i.e., set of actions). Formally:

$$\forall i \in \{H, R\}, t, a_t^i \in A^i, f^i(a_t^{*i}, a_t^{*-i}) \geq f^i(a_t^i, a_t^{*-i}). \quad (2)$$

where  $a^R \in A^R$ ,  $a^H \in A^H$ ,  $a_t^{*i}$  is the optimal action of agent  $i$  at time  $t$ , and  $a_t^{*-i}$  refers to actions taken at time  $t$  by all agents but  $i$ . Consider a two-player game with agent  $R$  and  $H$ , here  $a_t^{-H} = a_t^R$  and  $a_t^{-R} = a_t^H$ .

#### B. Markov Games

Markov games [20] are defined on top of Markov Decision Process, with finite state space  $s \in S$ , finite action space  $a \in A$ , and other agents’ action space  $a' \in A'$ . Agent reward function is a function of the state, the action, and the other agents’ actions:  $r(s, a, a')$ . The framework is commonly used for multi-agent reinforcement learning, where agent action value  $Q(x, a)$  is defined to take other agents’ actions in consideration:

$$Q(s, a, a') = r(s, a, a') + \gamma \sum_{s'} \mathcal{T}(s'|s, a, a') V(s'), \quad (3)$$

where  $\gamma$  is the discount rate,  $s'$  is the state transition from  $(s, a, a')$ ,  $\mathcal{T}$  is the transition probability function, and  $V$  is the value function.

### IV. MARKOV GAMES WITH TIME-VARIANT TYPES FRAMEWORK

#### A. The Model

Our model includes  $k$  agents working in a joint state space  $X$ , where each individual has its own state representation  $x_t^i \in X^i$  at time  $t$ , and control input from a bounded input space  $u_t^i \in U^i$ .

<sup>1</sup>The model can also take in continuous-space input space  $U$  to apply to real-world robotics domains

While actions  $a_t^i \in A^i$  define the *high-level, finite* actions an agent can take to affect the game outcome,  $u_t^i \in U^i$  defines the low-level continuous-space realization of such actions, by:

$$u_t^i = g^i(x_t, a_t^i, \theta_t^i), \quad (4)$$

where  $x_t$  stands for all the agents' state representation at time step  $t$ , i.e.,  $x_t = (x_t^1, x_t^2, \dots, x_t^k) \in X$  and  $\theta_t^i \in \Theta^i$  is some parametrization of the agent's (potentially time-variant) behavior.  $g^i : X \times A^i \times \Theta^i \rightarrow U$  can take in any model formulation, possibly stochastic, to sample inputs from  $p(u_t^i | x_t, a_t^i, \theta_t^i)$ . One example for high-level actions is the table-turning directions, clockwise or counterclockwise; the low-level inputs to realize such actions are motor torque commands to control the position and orientation (the state) of the end effector of the manipulator. Note that,  $u_t^i$  is a function of  $x_t$  and not solely of  $x_t^i$ , since agents adjust their motions (the realization of their high-level actions) based on other agents' status in the joint state space. An example could be collision avoidance among two dynamic agents.

Given a control input  $u_t^i$ , agent state transition function  $\mathcal{T}_{low}^i$  can be individually represented by:  $p(x_{t+1}^i | x_t^i, u_t^i)$ . However, since dealing with high-level actions is much more intuitive than dealing with the low-level ones, we will use a different representation for agent  $i$ 's state transition function by marginalizing over  $u_t^i$  as follows:

$$p(x_{t+1}^i | x_t, a_t^i, \theta_t^i) = \int_{u^i \in U^i} p(x_{t+1}^i | x_t^i, u_t^i) p(u_t^i | x_t^i, a_t^i, \theta_t^i) du^i. \quad (5)$$

We note that we use  $X$  as a part of agent  $i$ 's state transition function's new representation ( $\mathcal{T}_{high}^i : X \times A^i \times \Theta^i \rightarrow X^i$ ), due to the fact that the action realization,  $g^i$ , takes in the joint state  $x_t$ . The *joint state transition function*,  $\mathcal{T}$ , however, represents a collective behavior among all agents and therefore takes in the following form:  $\mathcal{T} : X \times \Sigma \times \Theta \rightarrow X$ .

After taking control input  $u_t^i$  at state  $x_t$ , agent  $i$  receives an immediate reward of  $r_t^i$  calculated according to:

$$r_t^i = r_{low}^i(x_t, u_t^i, u_t^{-i}), \quad (6)$$

where  $u_t^{-i}$  is the control input of other agents at time  $t$ , and the agent reward function  $r_{low}^i : X \times U \rightarrow \mathbb{R}$  considers all agents' states  $x_t$  and inputs  $u_t$  ( $\in U = U^1 \times \dots \times U^k$ ) for evaluation.

Using Equation 4, a new representation of  $r_{low}^i$ , using high-level actions ( $r_{high}^i : X \times \Sigma \times \Theta \rightarrow \mathbb{R}$ ), can be used in order to calculate  $r_t^i$  as follows:

$$r_t^i = r_{high}^i(x_t, \sigma_t, \theta_t). \quad (7)$$

## B. Analysis

Given the reward function  $r_{high}^i$  as defined in Equation 7, the optimal policy is to find the strategy  $a_{0:T}^{*i}$  that maximizes the cumulative rewards,

$$a_{0:T}^{*i} = \underset{a_{0:T}^{*i}}{\operatorname{argmax}} \sum_{t=0}^T \mathbb{E}_{x_t, \sigma_t, \theta_t | \mathcal{T}, \sigma_{0:t-1}} [r^i(x_t, \sigma_t, \theta_t)] + V_{T+1}^i(x_T, \sigma_T, \theta_T), \quad (8)$$

where  $V_{T+1}^i$  is some cost-to-go function for terminating, given the action profile  $\sigma_T$  at time  $T$ .

Despite the general formalism of this framework which allows planning directly with the low-level control inputs  $u_t^i$ , for the rest of the paper we will consider the high-level action space for planning. This is due to the fact that high level action space is oftentimes easily obtained for domain-specific applications. In addition, if using a high level actions space (which is discrete) it is much cheaper to perform a search. Discrete action planning also well approximate human planning with hierarchical reasoning.

We present Markov games with time-variant types as a framework to incorporate human adaptiveness and typed behaviors in interactions with robots, and to design algorithms that leverages multi-agent action effects for human-aware robot planning. We first introduce general solutions for the framework in both pre-computation and online setting in Section V; we then introduce our approximation of human decision-making mechanism to incorporate research topics on human behaviors when interacting with robots in Section VI; lastly, we use the proposed model to simulate human-robot navigation with path crossing.

## V. REAL-TIME GAME

### A. Game Setting

To formally discuss the coordinating process between humans and robots, we define the real-time game setting through game *start* and *termination* criteria. Those criteria are very important as they define the time frame in which each agent considers the other agents' behaviors.

1) **Game start criteria** - In order for the game to begin, the following two criteria need to hold:

- $\mathcal{C}_{PI}$  (*potential interaction*) - The criteria is met when agents' actions affect the outcome of each other.
- $\mathcal{C}_{MA}$  (*mutual awareness*) - The criteria is met when agents are aware of each other.

2) **Game termination criteria** - After the interaction starts at time  $t = 0$ , the game repeats *finitely* until a termination criteria  $\beta : X \times \Sigma \rightarrow \{True, False\}$  is satisfied. Termination criteria may be a predefined criteria,  $t > T$ , where  $T$  is a pre-specified time frame of mandatory co-working. Termination criteria may also take a dynamic format, such as finish the game whenever  $\mathcal{C}_{PI}$  no longer holds. An example can be navigation in crowds. When pedestrians are past the route intersections, the game terminates, as no further intervention is expected.

### B. Game Solutions

If both  $\mathcal{C}_{PI}$  and  $\mathcal{C}_{MA}$  are met, the game unfolds until the termination criteria is met. Within this game time frame, agent  $i$  is interested in finding the solution (i.e., strategy) which optimizes his future cumulative rewards (calculated according to Equation 8). This process is formally presented by a simultaneous-move game tree representation (see example in Figure 2), which contains: \*\*\*Shih-Yun, please change the tree description according to the new tree\*\*\*

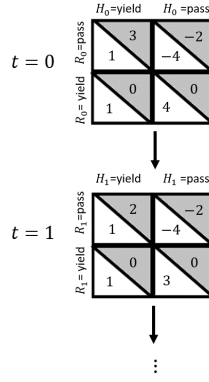


Fig. 2. Two-player human-robot interaction, modeled as a simultaneous-move game tree.

- 1) Decision nodes: nodes where players make action choices,  $a_t^i \in A^i$ , based on current state  $x_t$  (solid circles). Each decision node is associated with only one player out of all the  $k$  players.
- 2) Terminal nodes: nodes where game outcomes  $V(x_T, \sigma_T, \theta_T)$  are assigned (hollow circles at the bottom)
- 3) History set: plays history of all other agents before current time associated with each decision node, i.e.,  $\sigma_t \in \Sigma_t$ .

\*\*\*Shih-Yun, please change it the way we discussed, add number of lines which present the subject you are interested to talk about before each subject and remove the bold headline\*\*\***Action profile prediction, strategy evaluation, and search for solution action sequence:**

We note that the policy of how an agent  $i$  makes his high-level decision regarding to the action he is about to take can be abbreviated, without loss of generality, as a function of his (possibly time-variant) type  $\theta_t^i$ ,

$$a_t^i \sim \pi^i(x_t, \theta_t^i), \quad (9)$$

where  $\theta_t^i$  can be time-variant in order to capture human adaptive behaviors in uncertain environments.

More specifically, at each decision node, agent  $i$  chooses action(s) based on current state  $x_t$  for evaluation. In order to do so, agent  $i$  needs to compute the following:

- 1)  $p(\sigma_t^{-i} | \sigma_{0:t-1}^i, x_t)$ : action profile probability of other agents given history set.
- 2)  $r_t^i$  or  $V_T^i$ : reward estimate of the current action profile or the value estimate if the agent is at termination nodes\*\*\*Shih-Yun, please update this according to the new representation of the tree\*\*\*.

We note that the form of the game outcomes, either defined by the cumulative rewards (as commonly seen in Markov Decision Process) or terminal values (as commonly seen in extensive-form games), has no effect on the solution algorithm.

Finally we note that, for the purpose of action evaluation, there exist other suitable computational models, such as the one appearing in Equation 9, or a direct mapping from  $x_t^i$  to  $u_t^i$ . We chose to present this high-level model for its generality, and further implement the model to simulate

specific human-robot interaction behaviors as described in literature, using behavioral game theory rules.

Due to the forward state transitions in Markov Games, backward induction, a common solution for extensive-form games, is no longer applicable. Instead, forward-search approaches such as Monte Carlo Tree Search can be applied: at each iteration, one would randomly sample an action  $a_t^i$  for evaluation and expands the search tree by sampling the action profile  $p(\sigma_t^i | \sigma_{0:t-1}^i, x_t)$ . To compute the reward  $r_t^i$ , Equation 6 can be applied by sampling  $u_t$  from Equation 4, given prior on other agents' types  $p(\theta_t^{-i})$ . This can also be done for the value at \*\*\*Shih-Yun, we now don't have termination node, we need to think about what to write here\*\*\*termination nodes  $V_T$ .

\*\*\*Shih-Yun, please remove it as before\*\*\***Online solutions with prediction model updates:**

When planning for long-horizon purposes, agents ideally want to optimize their outcome considering full-horizon reward accumulation, as introduced in Equation 8. However, due to modeling errors and typed behaviors among humans, pre-trained models may not well accommodate newly observed behaviors. Pre-computed solutions therefore may not suit well to the newly encountered agents. Thus, online re-plan is needed to well adapt to new situations, using real-time received observation data. \*\*\*Shih-Yun, please move it to the middle of the paragraph as we discussed\*\*\* Inaccurate prediction includes those of other agents' future action profiles, action realizations, and state transitions.

In order to deal with real world interactions (which involve many uncertainties) we are proposing the use in an *online game strategies*. Instead of using the search algorithms for the entire horizon, the agent will use *belief updates* whenever new observations arrive, and will *re-plan for a specific period in the horizon* from the current time step  $t$ , until the longest computationally possible lookahead horizon  $H$ . We assume agents have knowledge of termination time,  $T$ , even in the dynamic setting.

The following provides a detailed explanation regarding the belief update and Receding-horizon planning of each agent.

1) *belief updates*: During real-world interactions, observations can be received from direct measures, such as relative positions and velocities of all agents. Observations can also be received implicitly through ways like eye contacts or body languages, to express an intention [14].

With either form of observations,  $o_{t:t+\tau}^i \in O^i$ , where  $1/\tau$  is the observation update rate and  $O^i$  is his observation data space, agents can better infer about future motions of the other agents. In order to plan in the high-level action space  $A^i$ , given computational models for motion predictions such as  $u_t^{-i} \sim g^{-i}(x_t, a^{-i}, \theta_t^{-i})$  proposed in our framework (but not restricted to such formulation), agent  $i$  needs to update his beliefs regarding the following:

(a) Future strategy profiles of other agents  $b_t^i(\sigma_{t:T}^{-i} | x_t, \sigma_{t:l}^i)$ : the strategy profile prediction based on history plays,  $p(\sigma_l | x_l, \sigma_{t:l}^i)$ , where  $t \leq l \leq T$  is the future time step of interest.

(b) Type estimate of other agents  $b_t^i(\theta_t^{-i})$ : the type estimate  $p(\theta_t^{-i} | o_{t:t-\tau}^i, \theta_{t-\tau}^{-i})$  (in the Bayesian setting), in order to

sample the joint state transition function  $\mathcal{T}$  and to compute action reward  $r_t^i$ .

We note that belief updates are usually computationally cheap, so agents may run it at a high rate (potentially higher than \*\*\*Shih-Yun, you need to explain where we discussed the re-plan in the paper so people will understand what is the re-plan\*\*\*re-plan rate) to deal with noisy observations.

2) *Receding-horizon planning*: Due to the potential demand of fast re-planning in dynamic environments, agents may only be computationally available to plan for finite-step lookahead  $L < T$ . Therefore, at each time step  $t = l, l < T$ , agents will try to optimize for  $t = l : L'$ , where  $L' = \min(l + L, T)$ , based on the updated model of future strategy anticipation of other agents'  $p(\sigma_{l:L'} | x_l, I_l^i)$ . This online re-planning for finite horizon strategy, known as receding-horizon planning, can be formulated as follows:

$$a_{l:L'}^{i*} = \underset{a_{l:L'}}{\operatorname{argmax}} \sum_{t=l}^{L'} \mathbb{E}_{x_t, \sigma_t, \theta_t | \mathcal{T}, \sigma_{0:t}^i} [r^i(x_t, \sigma_t, \theta_t)] + \hat{V}_{L'+1}^i(x_{L'}, \sigma_{L'}, \theta_{L'}^i), \quad (10)$$

where  $\hat{V}_{L'+1}(x_{L'}, \sigma_{L'}, \theta_{L'}^i)$  is some cost-to-go estimate for using action profile  $\sigma_{L'}^i$  from  $t = L' + 1$  to  $T$ .

In coordination games, it is usually the case that as earlier the termination is, the better is the final outcome. An example can be the bargaining game with benefit discounts. In the table-turning task example, the faster the two agents reach agreement on which direction to go, the earlier they start investing their efforts on the actual problem and make progress. This is despite the potentially suboptimal (longer) route selection. Therefore, biased search toward strategies with early termination has empiric advantage; as a result, many times agents will prefer trading off computation depth for breadth to achieve better exploration its action profile.

\*\*\*Shih-Yun, please remove it according to what we discussed\*\*\***Strategy response time and turn-taking formulation**: Since high-level actions or intents are not directly observable by other agents, as suggested by Section \*\*\*Shih-Yun, wrong section, please update\*\*\* V-B.1, strategies take time for other agents to infer; further, agents need time to replan given a newly observed strategy, and to execute the new plan with potential changes in motions. As a result, with a proper waiting time after the initial action, the decision timing of both agents can be clearly separated to prevent oscillations. Thus, the game formulation take the form of turn-taking, as illustrated in Figure 3. We note that turn-taking games simplify the simulation's setting and save computation due to the negligence of current-time action-profile sampling. This is due to the fact that there is no need to predict other agents' concurrent actions.

## VI. HUMAN BEHAVIORS

Since one of the main purposes of our proposed framework is to model *human* interaction with AI agents, we are now introducing our hypotheses regarding human behaviors and their decision-making mechanism; by implementing these hypotheses in the suggested framework, we will achieve a better analysis of the different paradigms in human-robot coordination and how they affect the overall game result.

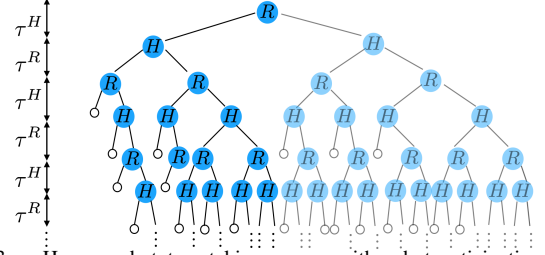


Fig. 3. Human-robot turn-taking games with robot anticipating action value based on finite-horizon predictions.  $\tau^R$  and  $\tau^H$  are approximate time intervals for robot and human response times. The game may terminate at either player's decision nodes.

### A. Adaptation to other agents

In interaction, oftentimes humans observe other agents and adapt their strategies accordingly [22], [32]. This is because humans update their beliefs of the other agents based on past interaction; during which, they acquire samples of others' motion realizations and strategies, which directly affect their own action rewards  $r_t^H$ . Therefore, adaptation can be seen as a dynamic strategy to maximize self interests. We incorporate such behaviors in our proposed framework through the dynamic type parameter  $\theta_t^H$ , and refer human adaptive policy to Equation 9.

Further, we propose that a human's type  $\theta_t^H$  is parameterized by a static parameter  $z^H$ , and a dynamic inference parameter  $b_t^H(\theta_t^{-H})$ . The static parameter,  $z^H$ , is associated with domain-specific personal preferences, such as urgency to travel to destination, which do not change in short period of time; and the dynamic inference parameter,  $b_t^H(\theta_t^{-H})$ , captures the agent's time-evolving *beliefs* of other agents' (potentially dynamic) types. More specifically,

$$\theta_t^H = \begin{bmatrix} z^H \\ b_t^H(\theta_t^{-H}) \end{bmatrix}. \quad (11)$$

As for the detailed formulation in human belief, we hypothesize that humans possess certain *prior* of the other agents' behaviors, such as assuming elderly to move slower and robots to be compliant; when new observations come in, humans make use of certain *information budget*\*\*\*Shih-Yun, this was not defined yet, please talk in more general terms here, the formal definition is in the next paragraph\*\*\* to reason about other agents' behaviors to update their beliefs.

### B. Depth of Inference and Biased Adaptation

We now define the concept of *information budget* to be an agent's computational capability to infer about other agents' strategy, which includes assumptions over their decision-making model. Here, the decision-making model is referred to the policy that outputs the agent's high-level action(s), i.e.,  $a_t^i \sim \pi^i(x_t, \theta_t^i)$ . With different information budgets, agents will update their beliefs based on different assumptions over the other agents' decision-making model. The different categories of *information budget* are outlined below:

- **Zero Information** -  $b_t^i(\theta_t^{-i}) = \emptyset$ . Agents allocate zero budget, therefore keep no information of the other agents' behaviors during the interaction; they parametrize their policies solely based on (static) personal preferences, and make no use of other agents' actions/motions to characterize game outcomes. Such



agents pay no attention to the game, and we assume *aware* humans do *not* belong to this category.

- **One-Layer Inference** -  $b_t^i(\theta_t^{-i}) = \hat{z}^{-i}$ . Agents assume other agents act only according their static preferences,  $z^{-i}$ . This means that they assume others to maintain no information of themselves (i.e.,  $b_t^i(b_t^{-i}(\theta_t^i)) = \emptyset$ ). Therefore, agents adapt the others as if their own actions have no impact on other agents' strategies or decision-making model, parametrized as  $p(a_t^{-i}|x_t, z^{-i})$ . We assume humans belong to this category while interacting with robots. We will apply one level inference as part of planning inference in our framework.
- **Two-Layer Inference** -  $b_t^i(\theta_t^{-i}) = [\hat{z}^{-i}, \hat{z}^i]$ . Agents assume other agents are also adaptive to their actions with one-layer inference (i.e.,  $\theta_t^{-i} = [z^{-i}, \hat{z}^i]$ ). Therefore, when planning for more than one period, agents act adaptively, at the same time evaluating their actions' potential impacts on the other agents' future strategies. For example, when planning for only one game period, agents with two layer inference budget plan according to what they predict the others' predictions about themselves are, i.e.,  $a_t^i \sim \pi^i(x_t, b_t^i(\pi^{-i}(x_t, b_t^{-i}(\pi^i(x_t, z^i)))))$ . When applied for long-horizon planning, agents need to distinguish updates for either  $\hat{z}^{-i}$  or  $\hat{z}^i$ , and replan while considering the others' one-layer inference of their own choices of actions.

Due to the intrinsic complexity, this is the maximal budget we assume humans can afford for real-time inference, and is assumed only affordable for one-shot games but not for long-horizon planning.

Therefore, given the information budget assumption over human inference for long-horizon planning,  $b_t^H(\theta_t^H) = \hat{z}^{-H}$ , we consider human policies being adaptive to their perceived, presumably static, behavior of other agents. The higher the adaptation rate is, the more compliant they appear to the other agents' strategies and therefore the joint policy is more flexible.

\*\*\*Shih-Yun, please add connection to the former part\*\*\**Biased Adaptation*:

Although  $\theta_t^i$  may be partially inferred by other agents  $-i$  through observations, it is assumed to be not directly observable. Type  $\theta_t^H$  can be treated as the *mental state* of the human agents, which are hidden from other agents, and sometimes even from themselves.

The more complex the domain is, the more complicated is the way type is related to an agent's policy. Given prior estimate of such hidden parameter and its belief update rule, sometimes its variables may have coupled effects to the resultant observations. In that case, the agent has extra degrees of freedom to allocate different levels of attention on each variable to apply belief updates, therefore only that the agent most concerns gets updated. We refer this as *biased adaptation* in human behavior, which saves computation for both belief updates and replanning.

\*\*\*Shih-Yun, please add connection to the former part\*\*\**Human Lookahead Planning*:

As introduced in Section \*\*\*Shih-Yun, wrong section,

please update\*\*\* V-B.2, we also assume humans re-plan online with finite-step lookahead,  $L^H < T$ , with belief updates on prediction models such as the strategy profile of other agents  $b_t^H(\pi^{-H}(x_t, \theta_t^{-H}))$ . Finite-step lookahead can be either *0-step lookahead* (i.e.,  $L^H = 0$ ), where agents act as if the current game is the termination game, or *multi-step lookahead* (i.e.,  $L^H > 0$ ), where agents plan as the game has more than one period. We note that if  $L^H > 0$ , adaptive behaviors due to belief updates are expected, assuming one layer inference information budget.

### C. Influential factors in human-robot interaction

As pointed out in Section VI, human policies for high-level decisions are parametrized by  $\theta_t^i$ , which includes both the person's static type,  $z^H$ , and his dynamically perceived types of other agents,  $b_t^H(\theta_t^{-H})$ . We note that with different perceptions on others, humans act differently. For example, pedestrians take over others' roads when they are in a hurry; however, they yield when encountering the elderly. Similar situation applies to human-robot interaction: people have distinctive behaviors based on their prior assumptions regarding robots, which affect their policies and their adaptabilities when new observations come in. Here we discuss how to use the above proposed model in order to describe existing phenomena in human-robot interactions.

1) *Perceived robot capability*: The gap between the true capabilities of a robot and human perceived of those capabilities has shown to deteriorate both joint and individual work efficiency in different task domains [6]. Here, we characterize human perceived robot capability for human-robot interaction using two categories: *functional capability* and *social inference capability*.

- *Functional capability* - This includes the belief of whether a robot is able to *identify interaction*. Before engaging interactions, agents need to ensure that  $C_{PI}$  and  $C_{MA}$  are met by all parties, or else confusions may arise. Due to lack of social signaling capability, such as gazes, humans may be uncertain whether the robot is aware of the potential interactions. This may result in distantly-avoiding behaviors through out the interaction, since people are not sure whether they should engage [6]. This also includes the knowledge of robot action set,  $A^R$ , and the confidence of whether the robot is able to *succeed in its target actions*,  $a_t^R$ , especially when complex domains are considered [3].
- *Social inference capability* - This involves the question whether human perceive the robot to be capable of engaging with. Implicit communication has a big part in most interactions [14] and therefore it is very important to be able to notice it. Robots should have the ability to *identify humans' intended high-level actions* based on the inference from past observations, and to *express its own intended action* in a clear, context-aware, or, legible manner [7]. Failing to present such capabilities deteriorates human patience and overall efficiency on given tasks [2].

The above two perceived capabilities are prerequisites for humans to engage in the coordination process with

robots. In complex domains, those criteria have shown to be challenging [14], therefore prior experiences on cross-training, teaching, and learning [34] may be required to prepare for natural engagement.

2) *Static variables,  $z^H$* : The static variables characterize human behaviors that are assumed constant throughout the interaction, which include the following two categories when we consider specific task domains and shared workspaces among humans and robots.

- *Personal preferences*: Since people have different perception and long-time experience in interactions with the environment, they preserve distinctive characteristics in their behaviors that do not change in short period of time. Therefore, when planning considering joint actions, robots should be aware of such types to plan accordingly for agent comfort and overall efficiency [11]. In our proposed framework, personal preferences contribute to agents' policy realizations, transition functions, and affect the joint performances. They directly affect the agent's reward function,  $r^i$  through individual weighting  $y^i$  on the cost function  $C$  [5]:

$$r^i = -y^{iT} C. \quad (12)$$

Take social navigation for example, time delay is a domain-specific cost function, and personal urgency level is the associated weighting for that cost term.

- *Level of self-interest*: When agents are deployed in a shared workspace, the notion of public welfare plays in to assess policy fairness [8]. While the public welfare is the self-interest in collaborative tasks, in non-cooperative games, agents have incentives to deviate from cooperative behaviors for personal benefits [10]. When individuals plan in a shared workspace with personal objectives to achieve, resource conflicts may occur. While cooperative policies are the most efficient for social welfare, agents may gain more resource allocation when playing selfishly. This notion of fairness can be characterized as weighting on all parties' interest  $\alpha^i$ :

$$r'^i = \alpha^i r^i + (1 - \alpha^i) r^{-i}. \quad (13)$$

We note that the level of self-interest,  $\alpha^i$ , and the personal preferences,  $y^i$ , jointly contribute to the static variables,  $z^i$ . Further, the way they contribute to the agent's behavior may have coupled effects that are not distinguishable through direct observations.

3) *Dynamic variables,  $b_t^H(z^R)$* : The dynamic variables are to describe the agent's perceptions of the other agents' (here, the robot's) static variables. Since perceptions (or belief) are online adjusted throughout the interaction, they are dynamical properties and contribute to the transitions of the agent's policy in real-time.

- *Perceived robot preferences*: As humans have domain-specific preferences, they also infer such preferences of the robot to predict their strategy. Take social navigation for example, it could be the perceived level of urgency of the robot given observed tasks at hand.

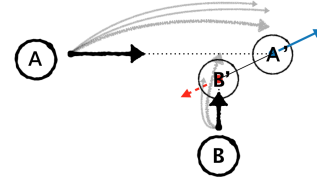


Fig. 4. Collision avoidance actions based on closest position estimate. Red dashed arrow (social force direction): yielding agent with later arrival timing at the intersection; blue solid arrow: passing-first agent.

- *Perceived robot level of self-interest, or, social trust on collaborativity*: Perceived capabilities are preliminary for human trust to interact with robots [32], since the knowledge of the action set of the robot enables human prediction of robot future actions. When there are resource conflicts in shared workspaces, self-interested agents may not be socially compliant to the other agents. In this case, trust on social collaborativity in conflicted situations, captured by humans' belief of robot static type  $b_t^H(z^R)$ , affects human policies through their anticipation of robot policy. While perceiving robots as socially trust-worthy agents, humans predict robots to have non-hostile behaviors and may cooperate at ease.

## VII. PROBLEM INSTANTIATION

In this section we instantiate the framework on 2-player human-robot navigation with path crossing, shown in Figure 1.

Great progress has been made in the past two decades in pedestrian simulations [13], [33], robot navigation with human predictive models [29], [16], and socially-friendly robot planning [21], [4], to smoothly deploy robots in human workspaces. In human-robot interaction community, robot motion legibility has gained attention for human understanding [7], and the concept of trajectory interpretability has been applied on robot navigation in crowded environments to identify goals of pedestrians [1], [30].

In this section we make use of the social force model with explicit collision prediction [33], which was inspired by the social-force model but equipped with explicit avoidance behaviors. Other choices of human behavioral model for action realizations  $g(x_t, a_t^H, \theta_t^H)$  can also fit in the framework for simulation; we choose this model for its underlying motion *legibility* concept in distinguishing choices of avoidance behaviors, as can be seen in Figure 4.

When two agents have intersecting paths and have similar timing to arrive at the intersection, explicit avoidance behaviors are involved. Particularly, an agent has to decide whether to avoid in front or behind the other. This leaves the coordination to agree on two action combinations: (passing\_first, yielding), or (yielding, passing\_first).

We define the state space as following:  $x_t^R = \begin{bmatrix} p_t^R \\ v_t^R \end{bmatrix}$ , and  $x_t^H = \begin{bmatrix} p_t^H \\ v_t^H \end{bmatrix}$  where  $p_t^R, p_t^H$  are the robot's and human's positions, and  $v_t^R, v_t^H$  are the robot's and human's velocities, respectively. All variables are described in 2D, therefore,  $p_t^R, p_t^H, v_t^R, v_t^H \in \mathbb{R}^2$ .

### A. Game start and termination criteria

As pointed out in Section V, to initiate interactive behaviors between agents, the potential-interaction criteria  $C_{PI}$  and the mutual-awareness criteria  $C_{MA}$  are prerequisites to start the game. In path crossing,  $C_{PI}$  is met if:

- 1) two agents have path intersections in near future:  
 $\exists t^H < t^{rH}, t^R < t^{rR}$ , such that  $v_t^R \times t^R + p_t^R = v_t^H \times t^H + p_t^H$
- 2) the arrival timing difference is within certain threshold:  
 $|t^H - t^R| < \delta$

Where  $t^{rH}$  is the reaction time (i.e., the time its take to agents to decide to engage in the scenario). For pedestrian avoidance, experimental results suggest that  $t^{rH}$  is on average 4 secs before reaching the intersection [25]. The arrival timing difference depends on agent velocities, the safety margin to keep from other pedestrians, and the noise in estimation. Here, we use an estimate value of 1.5 sec, which is an upper bound to time difference for which agents respond to the intersecting scenario.

Finally, the game terminates whenever two agents have passed each other, or the minimum relative distance has passed.

### B. Intentions in path crossing and time delay

We will define the agents' action set as following:  $A^R = \{a^{pf}, a^y\}$ ,  $A^H = \{a^{pf}, a^y\}$ , where  $a^{pf}$  is corresponding to the class of trajectory realizations of passing first (i.e., in front of the other agent), and  $a^y$  is corresponding to the class of yielding to the other.

Empirical study on human-robot crossing has suggested distinctive velocity and trajectory profiles among two classes of avoidance actions [24]. Agents who intend to pass first often accelerate and bend their trajectories away from the other. Agents who intend to yield often slow down. Such changes in motion profiles are clear signals for agent passing intents, and one can observe those responses within a short period of time, around 0.7 sec.

The minimum time for human agents to react to their action changes,  $\tau^H$ , is assumed to be between 0.7s to 0.9s for such a model. For more complex domains, higher values should be considered.

### C. Response-time-adapted turn-taking structure

While the interaction process is modeled as a simultaneous-move game in Section V, with the time delays in action realization, it is often played as a *turn-taking* game.

### D. Experiments

We hypothesize that our proposed framework can explain different types of avoidance behaviors when their paths intersect with robots. We use our framework to simulate a particular type, cautious behavior, in comparison with collected trajectory data, to show that the framework can capture distinctive interaction patterns among people. We deploy a mobile robot into a building atrium, and record human crossing trajectories using tracking packages on laser scans [18]. Eight participants were requested to head towards

a set of goals, while the robot followed a given route that intersected with human eight times. The robot was implemented with a local planner with emergent slow-down when sensing an object within 1 meter.

We analyze the results received for 0-step, 1-step, and 2-step lookahead with the same prior on robot strategy profile. With 0-step lookahead, virtual pedestrians act compliantly after observing the other agent's action, due to the assumption of immediate termination. With 1-step lookahead, agents assume the game may end with the robot playing minimax strategy, therefore they play based on conservative value estimate. With 2-step lookahead, virtual pedestrians is capable of planning for the optimistic (2nd step) in the worst case robot strategy (1st step). As a result, their behavior in the third case is less conservative compared to the previous two.

We simulate virtual pedestrians with type  $\theta_t^H = [z^H, b_t^H(z^R)]$ , 2-step lookahead, memory bound on interaction history = 2, in the turn-taking formulation. This is associated with the decision-making capacity to reason: "what would the robot do after seeing my action, and what I could do to in reaction to that action?". Other combinations can be considered. We have the robot choose avoidance actions purely based on arrival timing estimates. It is a simple model for demonstration clarify on human responses; more complex models can be used.

**\*\*\*Comments regarding to Figure 6: (1) You need to write in the figure what each color stands for. (2) You need to explain (you can do this as a footnote) the different movements in the curves that presents the simulated pedestrian velocity.\*\*\***

1) *Perceived capabilities*: The complete knowledge of the robot's capabilities is specified in Section VI-C.1 as prerequisites for humans to engage in the coordination process with robots. When people notice potential collision with a robot, they may be unsure whether the robot sees them or not (i.e.,  $C_{MA}$ ); moreover, they may be unsure whether the robot is capable of sensing the potential collision (i.e.,  $C_{PI}$ ); they may not be certain that the robot can identify their intention when choosing an avoidance action (i.e., social inference capability on perception); finally, they may not be able to identify the intention of the robot if it chooses an avoidance action (i.e., social inference capability on motion generation).

2) *Types in path crossing*: Agents perceived other agents' behaviors and update their own type  $\theta_t^H$ . In social navigation, people's urgency levels to travel to their own goals are common factors to describe navigation preferences. In simulation, we simulate agents to choose avoidance actions based on the following cost formulation:

$$r_t^H = -\eta\alpha C_t^H - (1 - \alpha)C_t^{-H}, \quad (14)$$

where  $C_t^H$  is the estimated time delay,  $\eta \in [0, 1]$  is the urgency level, and  $\alpha \in [0, 1]$  is the level of self-interest.

3) *Social trust*: When navigation around a robot, the assumption of whether it is socially compliant affects humans' avoidance decision. We commonly observe, at the beginning of the experiment, participants slow down to wait for the robot to pass first, even when the robot has a later arrival timing; people speed up and stay far from the robot when



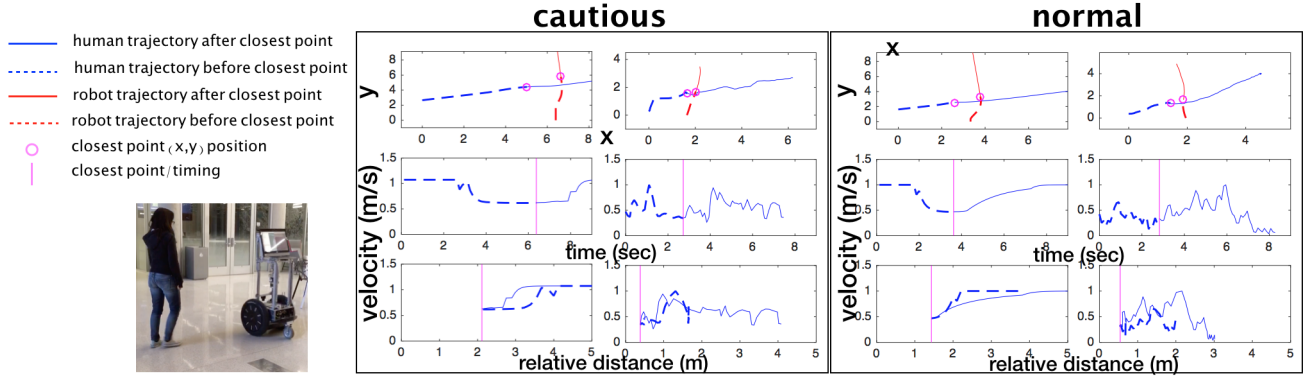


Fig. 5. SIMULATED pedestrian yielding, compared with REAL recording. We observed many types of behaviors during the experiments, and illustrate one common type in human-robot crossing: cautious, shown in the left box. Compared with people with gradual slow-down and speed-up (noted as “normal”, shown in the right box), cautious agents wait for the robot until it passes the intersection (shown in the bottom-left photo). We simulate agent yielding and compare the x-y trajectory (top), the velocity profile over time (middle), and relative distance (bottom) with the true recordings, right-side of each box.

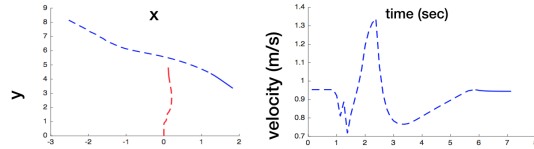


Fig. 6. SIMULATED cautious agent updating belief on robot strategies. While the simulated pedestrian yields at the beginning (with -0.7 sec arrival timing difference), after observing robot yielding behavior, she updates her belief and changes the action in the next time frame.

trying to pass in front, even when the robot is still far from the intersection. We refer to this type of behavior as cautious, simulated in Figure 5 through varying the perceived robot level of self-interest:  $b_t^H(z^R)$ .

\*\*\*Comments regarding to Figure 5: (1) It is not clear which squares present the results of the recording and which squares present the results of the simulation. You should write it next to each relevant column. (2) The axis' titles are not well presented, it looks very messy. I suggest spacing the figure a little bit, so you will have space for the titles. (3) I can not see what is the difference between the cautious and normal agents in the top graphs, it look the same to me. Do you have other recording/simulation in which the normal agent crosses first? Then there will be a difference between the two kinds of agents. (4) Can you add to the figure a map of the routes used in the experiment? I think it will help better understanding of the result and even the experiment itself.\*\*\*

4) *Human exploration and adaptation:* In the above described experiments we observed many conservative decisions made by human regarding to their crossing (e.g., pedestrians constantly yield to the robot). One agent actually tried passing in front of the robot for a couple of times and then decided to pass first. We consider this process as gradually gaining social trust on the robot; the agent updated her belief of the robot behavior and then adapted her own strategy. Similar behavior was simulated on virtual human with high belief update rate, shown in Figure 6.

## VIII. CONCLUSION AND FUTURE WORK

We propose a Markov Game model with time-variant types to analyze human-robot coordination outcome, and propose a human decision-making model to describe phenomena in human-robot interactions. With the framework, we simulate virtual pedestrians with cautious type behaviors

on human-robot crossing and compare the results with real-world recording. We also simulate adaptive human behaviors through belief updates on robot policy. In future work, we will further investigate human adaptability criteria based on different prior assumptions on robot behaviors and game convergence given false human assumptions.

## REFERENCES

- [1] Haoyu Bai, Shaojun Cai, Nan Ye, David Hsu, and Wee Sun Lee. Intention-aware online pomdp planning for autonomous driving in a crowd. In *Robotics and Automation (ICRA), 2015 IEEE International Conference on*, pages 454–460. IEEE, 2015.
- [2] Elizabeth Cha, Anca D Dragan, and Siddhartha S Srinivasa. Perceived robot capability. In *Robot and Human Interactive Communication (RO-MAN), 2015 24th IEEE International Symposium on*, pages 541–548. IEEE, 2015.
- [3] Min Chen, Stefanos Nikolaidis, Harold Soh, David Hsu, and Siddhartha Srinivasa. Planning with trust for human-robot collaboration. *arXiv preprint arXiv:1801.04099*, 2018.
- [4] Yu Fan Chen, Michael Everett, Miao Liu, and Jonathan P How. Socially aware motion planning with deep reinforcement learning. *arXiv preprint arXiv:1703.08862*, 2017.
- [5] Anca D Dragan Dorsa Sadigh, Shankar Sastry, and Sanjit A Seshia. Active preference-based learning of reward functions. In *Robotics: Science and Systems (RSS)*, 2017.
- [6] Anca D Dragan, Shira Bauman, Jodi Forlizzi, and Siddhartha S Srinivasa. Effects of robot motion on human-robot collaboration. In *Proceedings of the Tenth Annual ACM/IEEE International Conference on Human-Robot Interaction*, pages 51–58. ACM, 2015.
- [7] Anca D Dragan, Kenton CT Lee, and Siddhartha S Srinivasa. Legibility and predictability of robot motion. In *Human-Robot Interaction (HRI), 2013 8th ACM/IEEE International Conference on*, pages 301–308. IEEE, 2013.
- [8] Ernst Fehr and Urs Fischbacher. Social norms and human cooperation. *Trends in cognitive sciences*, 8(4):185–190, 2004.
- [9] Jakob N Foerster, Richard Y Chen, Maruan Al-Shedivat, Shimon Whiteson, Pieter Abbeel, and Igor Mordatch. Learning with opponent-learning awareness. *arXiv preprint arXiv:1709.04326*, 2017.
- [10] Takako Fujiwara-Greve. *Non-cooperative game theory*, volume 1. Springer, 2015.
- [11] Matthew C Gombolay, Cindy Huang, and Julie A Shah. Coordination of human-robot teaming with human task preferences. In *AAAI Fall Symposium Series on AI-HRI*, volume 11, page 2015, 2015.
- [12] Dirk Helbing and Peter Molnar. Social force model for pedestrian dynamics. *Physical review E*, 51(5):4282, 1995.
- [13] Ioannis Karamouzas, Peter Heil, Pascal van Beek, and Mark H Overmars. A predictive collision avoidance model for pedestrian simulation. In *International Workshop on Motion in Games*, pages 41–52. Springer, 2009.
- [14] Ross A Knepper, Christoforos I Mavrogiannis, Julia Proft, and Claire Liang. Implicit communication in a joint action. In *Proceedings of the 2017 acm/ieee international conference on human-robot interaction*, pages 283–292. ACM, 2017.

- [15] Thibault Kruse, Patrizia Basili, Stefan Glasauer, and Alexandra Kirsch. Legible robot navigation in the proximity of moving humans. In *Advanced Robotics and its Social Impacts (ARSO), 2012 IEEE Workshop on*, pages 83–88. IEEE, 2012.
- [16] Markus Kuderer, Henrik Kretzschmar, Christoph Sprunk, and Wolfram Burgard. Feature-based prediction of trajectories for socially compliant navigation. In *Robotics: science and systems*. Citeseer, 2012.
- [17] Minae Kwon, Malte F Jung, and Ross A Knepper. Human expectations of social robots. In *The Eleventh ACM/IEEE International Conference on Human Robot Interaction*, pages 463–464. IEEE Press, 2016.
- [18] Angus Leigh, Joelle Pineau, Nicolas Olmedo, and Hong Zhang. Person tracking and following with 2d laser scanners. In *Robotics and Automation (ICRA), 2015 IEEE International Conference on*, pages 726–733. IEEE, 2015.
- [19] Christina Lichtenthäler, Tamara Lorenzy, and Alexandra Kirsch. Influence of legibility on perceived safety in a virtual human-robot path crossing task. In *RO-MAN, 2012 IEEE*, pages 676–681. IEEE, 2012.
- [20] Michael L Littman. Markov games as a framework for multi-agent reinforcement learning. In *Machine Learning Proceedings 1994*, pages 157–163. Elsevier, 1994.
- [21] Christoforos I Mavrogiannis and Ross A Knepper. Decentralized multi-agent navigation planning with braids. In *Proceedings of the Workshop on the Algorithmic Foundations of Robotics. San Francisco, USA, 2016*.
- [22] Stefanos Nikolaidis, Anton Kuznetsov, David Hsu, and Siddharta Srinivasa. Formalizing human-robot mutual adaptation: A bounded memory model. In *The Eleventh ACM/IEEE International Conference on Human Robot Interaction*, pages 75–82. IEEE Press, 2016.
- [23] Stefanos Nikolaidis and Julie Shah. Human-robot cross-training: computational formulation, modeling and evaluation of a human team training strategy. In *Proceedings of the 8th ACM/IEEE international conference on Human-robot interaction*, pages 33–40. IEEE Press, 2013.
- [24] Sébastien Paris, Julien Pettré, and Stéphane Donikian. Pedestrian reactive navigation for crowd simulation: a predictive approach. In *Computer Graphics Forum*, volume 26, pages 665–674. Wiley Online Library, 2007.
- [25] Julien Pettré, Jan Ondřej, Anne-Hélène Olivier, Armel Cretual, and Stéphane Donikian. Experiment-based modeling, simulation and validation of interactions between virtual walkers. In *Proceedings of the 2009 ACM SIGGRAPH/Eurographics Symposium on Computer Animation*, pages 189–198. ACM, 2009.
- [26] Mark Pfeiffer, Ulrich Schwesinger, Hannes Sommer, Enric Galceran, and Roland Siegwart. Predicting actions to act predictably: Cooperative partial motion planning with maximum entropy models. In *Intelligent Robots and Systems (IROS), 2016 IEEE/RSJ International Conference on*, pages 2096–2101. IEEE, 2016.
- [27] Julie Shah, James Wiken, Brian Williams, and Cynthia Breazeal. Improved human-robot team performance using chaski, a human-inspired plan execution system. In *Proceedings of the 6th international conference on Human-robot interaction*, pages 29–36. ACM, 2011.
- [28] Masahiro Shiomi, Francesco Zanlungo, Kotaro Hayashi, and Takayuki Kanda. Towards a socially acceptable collision avoidance for a mobile robot navigating among pedestrians using a pedestrian model. *International Journal of Social Robotics*, 6(3):443–455, 2014.
- [29] Peter Trautman and Andreas Krause. Unfreezing the robot: Navigation in dense, interacting crowds. In *Intelligent Robots and Systems (IROS), 2010 IEEE/RSJ International Conference on*, pages 797–803. IEEE, 2010.
- [30] Vaibhav V Unhelkar, Claudia Pérez-D’Arpino, Leia Stirling, and Julie A Shah. Human-robot co-navigation using anticipatory indicators of human walking motion. In *Robotics and Automation (ICRA), 2015 IEEE International Conference on*, pages 6183–6190. IEEE, 2015.
- [31] Jur Van Den Berg, Stephen J Guy, Ming Lin, and Dinesh Manocha. Reciprocal n-body collision avoidance. In *Robotics research*, pages 3–19. Springer, 2011.
- [32] X Jessie Yang, Vaibhav V Unhelkar, Kevin Li, and Julie A Shah. Evaluating effects of user experience and system transparency on trust in automation. In *HRI*, pages 408–416, 2017.
- [33] Francesco Zanlungo, Tetsushi Ikeda, and Takayuki Kanda. Social force model with explicit collision prediction. *EPL (Europhysics Letters)*, 93(6):68005, 2011.
- [34] Yu Zhang, Sarath Sreedharan, Anagha Kulkarni, Tathagata Chakraborti, Hankz Hankui Zhuo, and Subbarao Kambhampati. Plan explicability and predictability for robot task planning. In *Robotics and Automation (ICRA), 2017 IEEE International Conference on*, pages 1313–1320. IEEE, 2017.