# Markov Games with Time-variant Types as a Framework for Human-robot Coordination

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***Abstract*— Coordination between humans and robots commonly happens when they co-exist in a shared workspace. Such scenario includes human-robot teaming on collaborative tasks, and human-robot conflict-resolving on limited shared resources. Mis-coordination reduces efficiency of both parties, and reduces the human's trust and patience with the robot. In this work, we propose a game-theoretic framework to analyze convergence in human-robot coordination. We also propose human behavior hypotheses on their decision-making mechanisms under this framework, to capture how the human 1) perception of the robot's capabilities, 2) personal preferences, 3) level of self-interest, and 4) social trust affect their policies and adaptability in dynamic environments. We provide human-robot path crossing as an instantiation of our framework, and use the hypothesized human behaviors to simulate real-world observed interaction patterns. Lastly, we simulate humans acting adaptively to their observed robot policies, as an initiative to incorporate effects on humans when designing robot algorithms using human-human interaction data.**

## I. Introduction

Human-robot interaction has received increased attention in recent years due to the emerging interest in deploying robots in human environments. Such environments may involve human-robot collaboration on given tasks, humans and robots working in a shared workspace, or service robots deployed in human environments. In such environments, robots may need to coordinate with humans with partially shared information and partially shared objectives; agents may need to reach agreement on one solution among multiple feasible choices, which makes the coordination non-trivial to settle. For a motivating example, see Fig. 1, which shows three agents coordinating at an intersection, where each agent has an individual goal to reach.

For a robot to engage coordination without confusing the human, it first requires the basic capabilities to understand human intent, and to respond in a legible manner [7]. Beyond those, to negotiate and agree on the coordinating solutions, the robot needs knowledge of human agents' behaviors to predict the outcome of its own actions. It also needs to known the reaction time humans need to update their policies [27], and to consider potential impacts of its own actions on humans' future decisions [10], [9]. Then the robot can plan and coordinate with humans in an intent-consistent fashion.

Past research has sought to improve human-robot coordination in a variety of ways, including: intent-expressive robot motion generation [7], [19], human preference-aware behavior modeling and its use for coordination [11], [5], human-robot mutual adaptation [23], [22], human expectations on robot capability [2], [17], as well as trust and comfort for long-term deployment[32].

While these topics share a focus on factors which affect human behaviors and their decision-making mechanisms, they lack a unifying framework to keep track of how those factors correlate with each other in different situations, how they affect human decisions and motions, and how they evolve over time as a function of robot interaction policies. Those questions are important for designing intent-consistent robots to interact with humans in daily situations. And they are important for designing robots that are aware of human adaptation to their long-term deployments. To fulfill such purposes, the main contribution of this work is a proposed game-theoretic model for human-robot coordination where agents have hidden preferences and time-variant adaptive behaviors based on online observations of others' behaviors.

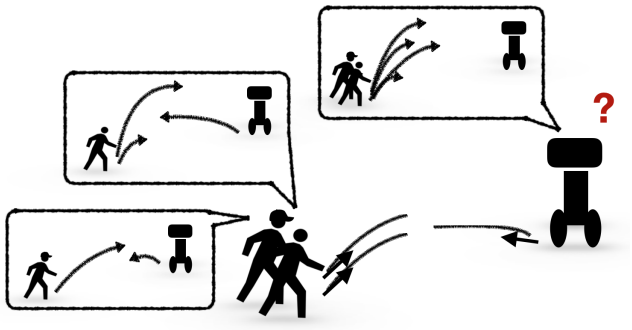


Fig. 1. Human-robot coordination in a crossing interaction. Since people have different prior assumptions on robot behaviors, their choice of actions differ.

To discuss human behaviors while interacting with robots, we point out some important factors that have been proposed in the human-robot interaction community, such as perceived robot capability, personal preferences, and trust on robot ability. We illustrate how they affect human behaviors through our proposed human decision-making model under the game framework, using a receding-horizon approach. We also propose two factors that we observed to be important in the social navigation domain: level of self-interest and social trust. We incorporate them along with others to analyze how people interact with the robot, with different personal preferences and assumptions about robot behaviors. We collect real-world human-robot interaction data to illustrate our model effectiveness.

We also use our decision-making model to discuss human adaptability to robots, and simulate human adaptation to robots after perceiving robot socially trust-worthy behaviors.

## II. Related Work

Despite the growing interest in deploying robots in human populated workspaces, motion planning algorithms for smooth human-robot coordination have remained a challenge. While traditional planning algorithms have shown

success in static environments, deploying such approaches alongside humans has shown insufficient adaptability to highly dynamic environments, and produce awkward motions making interpretability difficult [19], [7], [15]. Approaches considering time-variant factors, such as temporal constraints for multi-agent collision avoidance [31], have drawn attention for such applications; approaches considering human factors, such as human collision-avoidance behavior anticipation [12], have also gained attention for robot planning in human workspaces [28].

On the other hand, another community solves robot planning in human environments as a joint multi-agent planning problem, to incorporate the collective crowd behaviors with explicit modeling of the effect of agent actions on its surround agents [29], [16], [21]. Such joint modeling methodology has shown to be effective at outputting smooth human-mimicking trajectories at the same time acting responsively to its surrounding agents.

One major drawback applying these approaches interacting with humans is that the multi-agent joint dynamics models are typically learnt from data collected by human demonstrations, whereas humans do not act the same way around a robot compared to in pure-human environments. This problem has shown inefficiency of such approaches when humans use unexpected behaviors around robots – behaviors that humans will not present in front of another human [26].

To model joint behaviors among agents – how one's action affects the other – another way is to incorporate individual's action values with correlations with other agents' actions. Such joint behavior formulation is widely studied in Game theory: with different player assumptions, games evolve with different outcomes. In the artificial intelligence community, such game formulation has been incorporated with Markov Decision Process for multi-agent reinforcement learning [20]. Such incorporation enables strategy design for multi-agent robotic systems with mutual learning.

Yet, to simulate human-robot interaction, it requires agents modeled after humans, who have distinctive learning mechanisms and decision-making processes from reinforcement learning agents; moreover, agents have different behavioral types, as humans have personal preferences; agents online adapt to other agents, as humans observe the others and plan accordingly. Those features are widely studied in the human-robot interaction community, but not yet well formulated by one unifying framework. We therefore propose an extensive framework on Markov games with time-variant types to incorporate human-robot interaction with multi-agent learning problems to eliminate the gap.

## III. Preliminaries

In almost every real life daily interaction between robots and humans, each agent's utility is being influenced by the other agents' actions in addition to his own action. Thus, we choose to model human-robot interaction as a game. We propose a model, extended from Markov Games, to incorporate types as in Bayesian games, which assign game outcomes based on both agent actions and types [1]. A formal game definition as well as a quick overview about Markov Games are given in the following subsections.

### A. Game Definition

Consider a game $G$ with $k$ players, where each player $i \in \{1...k\}$ has a finite action set $A^i$. The set of action profiles is denoted as $\Sigma = A^1 \times A^2 \times ... \times A^k$. The utility of an agent $i$ is a function, denoted as $f^i : \sigma \rightarrow \mathbb{R}$, evaluated at $\sigma \in \Sigma$.

In *repeated games* (i.e., games which repeat for more than one action per player), $G^T$, players receive cumulative utilities over a time horizon $T$, defined as:

$$V^i = \Sigma_{t=0}^{T} f_t^i(\sigma_t). \quad (1)$$

Where $\sigma_t$ is the action profile at time $t$. We will define a *strategy profile* to be the series of action profiles $\sigma_t$ over time, i.e., $s = \sigma_0 \times \cdots \sigma_T \in S$. We note that in repeated games, agents need to consider not only the current outcome of an action, but also its impact on the other agents' future actions (which eventually will affect their own expected cumulative rewards).

Let $a^H \in A^H$ and $a^R \in A^R$ be the action space of humans and robots, respectively. A *Nash Equilibrium* in such a game is defined to be such that no agent (human or robot) benefits from unilateral deviation from his current strategy (i.e., set of actions). Formally:

$$\forall i \in \{H, R\}, t, a_t^{i*} \in A^i, f^i(a_t^{i*}, a_t^{-i*}) \geq f^i(a_t^i, a_t^{-i*}). \quad (2)$$

where $a_t^{-i}$ refers to actions taken at time $t$ by all agents but $i$. Consider a two-player game with agent $R$ and $H$ and a strategy profile $s^* = (a_{0:T}^{R*}, a_{0:T}^{H*})$. Here $a_t^{-H} = a_t^R$ and $a_t^{-R} = a_t^H$.

### B. Markov Games

Markov games [20] are defined on top of Markov Decision Process, with finite state space $s \in S$, finite action space $a \in A$, and other agents' action space $a' \in A'$. Agent reward function is a function of the state, the action, and the other agents' actions: $r(s, a, a')$. The framework is commonly used for multi-agent reinforcement learning, where agent action value $Q(x, a)$ is defined to take in the other agents' actions:

$$Q(s, a, a') = r(s, a, a') + \gamma \Sigma_{s'} \mathcal{T}(s'|s, a, a') V(s'), \quad (3)$$

where $\gamma$ is the discount rate, $s'$ is the state transition from $(s, a, a')$, $\mathcal{T}$ is the transition probability function, and $V$ is the value function.

## IV. Markov Games With Time-Variant Types Framework

### A. The Model

Our model includes $k$ agents working in a joint state space $X$, where each individual has its own state representation $x_t^i \in X^i$ at time $t$, and control input from a bounded input space $u_t^i \in U^i$.

[1]The model can also take in continuous-space input space $U$ to apply to real-world robotics domains

While actions $a_t^i \in A^i$ define the *high-level, finite* actions an agent can take to affect the game outcome, $u_t^i$ defines the low-level continuous-space realization of such options, by:

$$u_t^i = g^i(x_t, a_t^i, \theta_t^i), \tag{4}$$

where $\theta_t^i \in \Theta^i$ is some parametrization of the agent's (potentially) time-variant behavior. $g^i : X \times A^i \times \Theta^i \to U$ can take in any model formulation, possibly stochastic, to sample inputs from $p(u_t^i|x_t, a_t^i, \theta_t^i)$. One example for high-level actions is the table-turning directions; the low-level inputs to realize such actions are manipulator torque inputs ***Shih-Yun -please elaborate a little bit more about this example***. We will use $x_t$ to denote all agents' state representation at time $t$, i.e., $x_t = (x_t^1, \dots, x_t^k) \in X$. Note that, $u_t^i$ is a function of $x_t$ and not solely of $x_t^i$, since agents adjust their motions based on other agents' status in the joint state space.

Since dealing with high level actions is much more intuitive than dealing with the low level ones we will use a different representation of the state transition function of agent $i$, $\mathcal{T}^i : X \times U^i \to X$ using $p(x_{t+1}^i|x_t, a_t^i, \theta_t^i)$, by marginalizing over $u_t^i$:

$$p(x_{t+1}^i|x_t, a_t^i, \theta_t^i) = \int_{u^i \in U^i} p(x_{t+1}^i|x_t^i, u_t^i) p(u_t^i|x^t, a_t^i, \theta_t^i) du^i. \tag{5}$$

We note that we use $X$ as a part of agent $i$'s new representation of the state transition function ($\mathcal{T}^i : X \times A^i \times \Theta^i \to X$) due to the fact that agent $i$'s states are part of a joint state space. The *joint state transition function* $\mathcal{T}$ however, represent a collective behavior among all agents and therefor takes in the following form: $\mathcal{T} : X \times \Sigma \times \Theta \to X$.

After taking input $u_t^i$ at state $x_t$, agent $i$ receives an immediate reward of $r_t^i$ calculated according to:

$$r_t^i = r(x_t, u_t^i, u_t^{-i}), \tag{6}$$

where $u_t^{-i}$ is the control input of other agents at time $t$.

The reward function $r : X \times U \to \mathbb{R}$ considers all agents' states $x_t$ and inputs $u_t \in U = U^1 \times \cdots \times U^k$ for evaluation. Using Equation 4, a new representation of $r$ ($r : X \times \Sigma \times \Theta \to \mathbb{R}$) can be used:

$$r_t^i = r(x_t, \sigma_t, \theta_t). \tag{7}$$

### B. Analysis

Given the reward function $r$ defined in Equation 7, the optimal policy is to find the strategy $s^i$ that maximizes the cumulative rewards,

$$a_{0:T}^{i*} = \underset{a_{0:T}^i}{\operatorname{argmax}} \Sigma_{t=0}^{T} \mathbb{E}_{x_t, \sigma_t, \theta_t | \mathcal{T}, \sigma_{0:t-1}} \left[ r_t^i(x_t, \sigma_t, \theta_t) \right] + V_{T+1}^i(x_T, \sigma_T, \theta_T), \tag{8}$$

where $V_{T+1}^i$ is some cost-to-go function for terminating using $\sigma_T$ at time $T$.

***Shih-Yun - please make sure that the following paragraph says what you intended it to say*** Despite the general formalism of this framework which allows planning directly with the low-level control inputs $u_t^i$, and although discrete actions are computationally more expensive for plan evaluation, for the rest of the paper we will use the high level action planning. This is due to the fact that using a high level actions space (which is discrete) is much cheaper to perform a search and oftentimes easily obtained for domain-specific applications. Discrete action planning also well approximate human planning with hierarchical reasoning.

***This is not a full description of what we are doing in the paper. Please update it after we will talk*** We present the framework of Markov games with unknown time-variant types as a tool to analyze the convergence criteria of human-robot coordinating process. We first introduce general solutions for the framework in both pre-computation and online setting in Section V; we then introduce our approximation of human decision-making mechanism to incorporate research topics on human interaction behaviors with robots in Section VI-A; lastly, we use the proposed model to simulate human-robot navigation with path crossing.¡¿¡¿¿

## V. Real-time Game

### A. Game Setting

To formally discuss the coordinating process between humans and robots, we define the real-time game setting through game *start* and *termination* criteria. Those criteria are very important as they define the time frame in which each agent considers the other agents' behaviors.

1) **Game start criteria -** In order for the game to begin two criteria need to hold:
   - $\mathcal{C}_{PI}$ (*potential interaction*) - The criteria is met when agents' actions affect each other outcome.
   - $\mathcal{C}_{MA}$ (*mutual awareness*) - The criteria is met when agents become aware of each other and start acting according to the perceived strategy of the other agents.
2) **Game termination criteria -** When interactions start at time $t = 0$, the game repeats *finitely* until a termination criteria $\beta : X \times \Sigma \to \{True, False\}$ is satisfied. Termination criteria may be a predefined criteria, $t > T$, where $T$ is a pre-specified time frame of mandatory co-working. Termination criteria may also take a dynamic format, such as finish the game whenever $\mathcal{C}_{PI}$ no longer holds. One example can be crowd interactions, when pedestrians are past the route intersections with one another, the game terminates; as no further intervention is expected.

### B. Game Solutions

If both $\mathcal{C}_{PI}$ and $\mathcal{C}_{MA}$ are met and the termination criteria has not met yet, the game time frame has started. Within this time frame agent $i$ is interested in finding the solution (i.e., strategy) which will optimize his cumulative rewards (calculating according Equation 8). This process is being formally presented by the following simultaneous-move game tree representation (see example in Figure 2):

1) Decision nodes: solid circles where players make choices, $a_t^i \in A^i$, based on current state $x_t$
2) Terminal nodes: hollow circles at the bottom where game outcomes $V(x_T, \sigma_T, \theta_T)$ are assigned
3) History set: observed history plays before current time, $I_t^i$. Decision nodes connected with a dashed line share

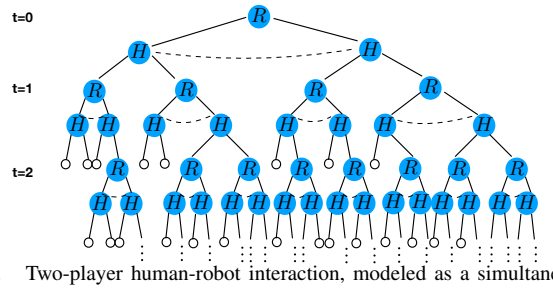

Fig. 2. Two-player human-robot interaction, modeled as a simultaneous-move game.

the same history set. Players cannot distinguish nodes with the identical history sets. Therefore, in games with simultaneous moves, players share the same history set in one period.

We note that the policy of how an agent $i$ makes his high-level decision can be abbreviated as a function of his type $\theta_t^i$,

$$a_t^i \sim \pi^i(x_t, \theta_t^i | \sigma_{0:t-1}), \tag{9}$$

without loss of generality.

More specifically, at each decision node, the player chooses an action based on current state $x_t$, and the policy such that Equation 8 will be optimize. In order to do so, the agent need to compute the following at each decision node:

1) $p(\sigma_t | I_t^i, x_t)$: game actions profile probability given history set
2) $r_t$ or $V_T$: reward estimate, or value estimate at termination nodes

We note that the form of game outcomes has no effect on the solution algorithm. Thus, the game outcomes computation can be done both by using cumulative rewards, as commonly seen in Markov Decision Process, and terminal values, as commonly seen in extensive-form games.

Please explain the following paragraph to me Due to the continuous-space state space formulation, backward induction, a common solution for extensive-form games, is no longer applicable. Instead, forward-search approaches such as Monte Carlo Tree search can be applied: at each iteration, randomly sample an action $a_t^i$ and then expand the search tree by sampling the action profile $p(\sigma^t | I_t^R, x_t)$. To compute the reward of a stage game $r_t$, or the value at termination nodes $V_T$, Equation 6 can be applied by sampling $u_t$ from Equation 4, given prior on other agents' types $p(\theta_t^{-i})$.

When planning for long-horizon purposes, agents ideally want to optimize their outcome considering full-horizon accumulation, as introduced in Equation 8. However, due to the high uncertainty in dynamic environments, pre-computed solutions may not fit well to newly received observation data, which affects agent's inference of the other agents' future action profiles, action realizations, or state transitions. Therefore, when playing in real-time, agents need constant online re-planning to adapt to unmodeled dynamic situations.

Therefore we are proposing the solution of using an *online game strategies*, instead of running search algorithms to solve for the total horizon at $t = 0$, agents will run *belief updates* whenever new observations will arrive, and *re-plan for certain horizon* from current time $t$, for as much lookahead horizon as computational resource allows. We assume agents have knowledge of the termination timing, $\beta_t$, even in the dynamic setting.

The following provide a detailed explanation regarding each agent's belief update and Receding-horizon planning.

*1) belief updates:* During real-world interactions, observations can be from direct measures, such as the relative positions and velocities of all agents. Observations can also be implicit as to infer private messages, through ways like eye contacts or body languages, to express messages such as intent [14].

In either form of observations $o_{0:t}$, for long-horizon planning on Markov games with time-variant types, agents will update their beliefs of the following:

(a) Type estimate of other agents $b_t^i(\theta_t^{-i})$: $p(\theta_t^{-i} | o_{0:t})$, in order to sample the state transitions function $\mathcal{T}$ and to compute reward function $r_t$.

(b) Future strategy profiles of other agents $b_t^i(\sigma_{t:T})$: $p(\sigma_h | I_h^i)$, where $t \leq h \leq T$ is the future time step of interest.

We note that belief updates are usually computationally cheap, so agents may run it at a high rate (potentially higher than re-plan frequency) to deal with noisy observations.

*2) Receding-horizon planning:* Due to the potential demand of fast re-planning in dynamic environments, agents may only be computationally available to plan for finite-step lookahead $H < T$. Therefore, at each time step $t = h$,$h < T$, agents instead try to optimize for $t = h : H'$, where $H' = min(h + H, T)$, based on updated model of future strategy anticipation of other agents' $p(\sigma_{h:H'} | I_h^i)$. This online re-planning for finite horizon strategy, known as receding-horizon planning, can be formulated as the following:

$$a_{h:H'}^{i*} = \underset{a_{h:H'}^i}{\operatorname{argmax}} \Sigma_{t=h}^{H'} \mathbb{E}_{x_t, \sigma_t, \theta_t | \mathcal{T}, I_t^i} \left[ r^i(x_t, \sigma_t, \theta_t) \right] + \hat{V}_{H'+1}^i(x'_H, \sigma'_H, \theta'_H), \tag{10}$$

where $\hat{V}_{H+1}(x_H, \sigma_H, \theta_H)$ is the cost-to-go estimate for the following $\sigma'_H$ from $t = H' + 1$ to $T$.

For coordination games, it is usually true that the earlier the termination, the better the final outcome is: similar to games with benefit discounts. Take the table turning task for example, the faster the two agents reach agreement on the direction to go, the faster progress they make, despite the potentially longer routes. Therefore, biased search to strategies with early termination has its empirically advantage; agents can even trade off computation depth with breadth to better explore its action profile.

## VI. Human-Robot Game

### A. Human Behavior and Decision-Making Model

Since one of the main purposes of our proposed framework is to model *human* interaction with AI agents, we are now introducing our hypotheses regarding human behaviors and decision-making mechanism; by implementing these hypotheses into the suggest framework, we will achieve better analysis of how different paradigms in human-robot coordination affect the overall convergence.

*1) Adaptability to other agents:* In interaction, it is many times the case that humans observe other agents and adapt their strategies accordingly [22], [32]. Our modeling of this behavior, in the proposed framework, suggests that an agent's type $\theta_t^H$ is parameterized by a static parameter $z^H$, which is associated with his personal preferences that do not change in short period of time, and by a *time-variant* parameter $b_t^H(\theta_t^{-H})$, to capture the agent's dynamic *beliefs* of other agents' types. More specifically,

$$\theta_t^H = \begin{bmatrix} z^H \\ b_t^H(\theta_t^{-H}) \end{bmatrix} \tag{11}$$

the static type parameter, $z^H$, features personal preferences such as travel efficiency versus travel energy in navigation domains. The time-variant parameter, $b_t^H(\theta_t^{-H})$, features humans' perception on other agents' types. As for the detailed form in the belief, we hypothesize that humans possess certain *information budget* to reason about other agents' behaviors.

We now define the concept of *information budget* to be an agent's computational capability to infer about other agents' decision-making model. The decision-making model is the policy that outputs the agent's high-level action(s), i.e., $a_{t:t+H}^i \sim \pi^i(x_t, \theta_t^i)$. There are several possible categories of *information budget* as will be outlined below:

- **No Information** - $b_t^i(\theta_t^{-i}) = \emptyset$. Agents keep no information of the other agents' behavior; they parametrize their policies solely based on personal preferences, but make no use of other agents' behaviors to characterize the game outcome. We assume human behaviors to not belong to this category while interacting with robots.
- **One-Layer Inference** - $b_t^i(\theta_t^{-i}) = \hat{z}^{-i}$. Agents assume the other agents maintain no information of themselves, but only act according to their static preferences $\theta_t^{-i} = z^{-i}$. Therefore, agents adapt to the others as if their own actions have no impact on other agents' decision-making model: $p(a_{t+H}^{-i}|x_t, z^{-i})$. We assume human behaviors to belong to this category while interacting with robots and will apply it for planning in our framework.
- **Two-Layers Inference** - $b_t^i(\theta_t^{-i}) = [\hat{z}^{-i}, \hat{\hat{z}}^i]$. Agents assume the other agents are also adaptive to themselves, $\theta_t^{-i} = [z^{-i}, \hat{z}^i]$, with one-layer inference. Therefore, when planning for more than one period, agents act adaptively, at the same time evaluating their actions' potential impacts on the other agents' future strategies. When planning for one period, Agents have the budget to compute two-layers inference: to plan according to what they predict the others' predictions about themselves $a_t^i \sim \pi^i(x_t, b_t^i(\pi^{-i}(x_t, b_t^{-i}(\pi^i(x_t, \hat{\theta}^i)))))$. This is the maximal budget we assume humans can afford for real-time inference, and is not applied for planning due to its intrinsic complexity.

Therefore, with the information budget assumption $b_t^H(\theta_t^H) = \hat{z}^{-H}$, we consider human policies being adaptive to their perceived, presumably static, behavior of other agents. The higher adaptation rate is, the more flexible they appear in the joint policy.

*2) Bounded memory belief updates:* ***I AM NOT SURE WHY WE ARE TALKING ABOUT THIS***As introduced in Section V-B.1, we also assume humans maintain their beliefs of other agents' types as well as their future strategy profile through out online planning. Here, we further assume humans to either run Bayesian updates, or possess *bounded* memory on past observations and interaction history for belief updates[22]: $b_t^H(\sigma_t|I_{t-(t-n)}^i)$, and $b_t^H(\theta_t^{-i}|o_{t-n:t})$.

*3) Finite-step lookahead:* As introduced in Section V-B.2, we also assume humans re-planning online with finite-step lookahead. Finite-step lookahead can be either *0-step lookahead* (i.e., $H = 0$), where agents act as if the current game is the termination game or *multi-step lookahead* (i.e., $H > 0$), where agents plan as the game has more than one period. We note that if $H > 0$, adaptive behaviors due to belief updates are expected [22].

*4) Anticipation of other agents' policy:* As pointed out in Section VI-A.1, humans adapt their policies based on their beliefs of other agents' behavior. When planning at time $t$, agents predict other agents' action profile $\sigma_{t:t+H}$ based on past interactions:

$$a_t^H \sim \pi^H(x_t, b_t^H(a_t^{-H}|\sigma_{0:t-1})). \tag{12}$$

To anticipate other agents' actions $a_t^{-H}|s_{0:t}$, one may assume their policies are non-adaptive, $a_t^{-H} \sim \pi^{-H(x_t, z^{-H})}$, associated with the one-layer inference.

### B. Influential factors in human-robot interaction

As pointed out in Section VI-A, human policies for high-level decisions are parametrized by $\theta_t^i$, which include that person's static type $z^H$ and dynamically perceived types $b_t^H(\theta_t^{-H})$ of other agents. With different perceptions on the other, humans act differently. For example, pedestrians take over others' roads when they are in a hurry; however, they yield when encountering the elderly. Similar situation applies to human-robot interaction: people have distinctive behaviors based on their prior assumptions of robots, which affect their policies and their adaptabilities when new observations come in. In this section we discuss how to use the above proposed model in order to describe existing phenomena in human-robot interactions.

*1) Static preferences:*

- *Personal preferences:* Since people have different perception and long-time experience in interactions with the environment, they preserve distinctive characteristics in their behaviors that do not change in short period of time. Therefore, when planning considering joint actions, robots should be aware of such types to plan accordingly for agent comfort and overall efficiency [11]. In our proposed framework, personal preferences contribute to agents' policy realizations, transition functions, and affect the joint performances. Regarding the reward function, they can be characterized as feature weighting $y^i$ [5]:

$$r^i = -y^{iT} C, \tag{13}$$

where $C$ is some vector of cost function.

- *Level of self-interest:* When agents are deployed in public environments, the notion of public welfare plays in to assess policy fairness [8]. While the public welfare is the self-interest in collaborative tasks, in non-cooperative games, agents have incentives to deviate from cooperative behaviors for personal benefits [10]. When individuals plan in a shared workspace with personal objectives to achieve, resource conflicts may occur. While cooperative policies are the most efficient for social welfare, agents may gain more resource allocation when playing selfishly. This notion of fairness can be characterized as weighting on all parties' interest $\alpha^i$:

$$r'^i = \alpha^i r^i + \frac{1-\alpha^i}{k-1} \sum r^{-i}. \tag{14}$$

We note that the level of self-interest, $\alpha^i$, and the personal preferences, $y^i$, jointly contribute to the static preferences, $z^i$.

*2) Perceived robot capability:* The gap between true robot capability and human perceived robot capability has shown to deteriorate both joint and individual work efficiency in different task domains [6]. Here, we characterize human perceived robot capability for human-robot interaction into two categories which are functional capability and social inference capability.

- *Functional capability* - This includes the belief of whether a robot is able to *identify interaction*. Before engaging interactions, agents need to ensure that $\mathcal{C}_{PI}$ and $\mathcal{C}_{MA}$ are met by all parties, or else confusions may arise. Due to lack of social signaling capability, such as gazes, humans may be uncertain whether the robot is aware of the potential interactions. This may result in distantly-avoiding behaviors through out the interaction, since people are not sure whether they should engage [6]. This also includes the knowledge of robot action set, $A^R$, and the confidence of whether the robot is able to *succeed in its target actions*, $a_t^R$, especially when complex domains are considered [3].
- *Social inference capability* - This involves the question whether human perceive the robot to be capable of engaging with. Implicit communication has a big part in most interactions [14] and therefore it is very important to be able to notice it. Robots should have the ability to *identify humans' intended high-level actions* based on the inference from past observations, and to *express its own intended action* in a clear, context-aware, or, legible manner [7]. Failing to present such capabilities deteriorates human patience and overall efficiency on given tasks [2].

The above two perceived capabilities are prerequisites for humans to engage in the coordination process with robots. In complex domains, those criteria have shown to be challenging [14], therefore prior experiences on cross-training, teaching, and learning [34] may be required to prepare for natural engagement.

*3) Social trust:* Perceived capabilities are preliminary for human trust to interact with robots [32], since the knowledge of the action set of the robot enables human prediction of robot future actions. When there are resource conflicts in shared workspaces, self-interested agents may not be socially compliant to the other agents. In this case, trust on social collaborativity in conflicted situations, captured by humans' belief of robot static type $b_t^H(z^R)$, affects human policies through their anticipation of robot policy. While perceiving robots as socially trust-worthy agents, humans predict robots to have non-hostile behaviors and may cooperate at ease.

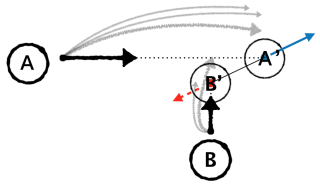


Fig. 3. Collision avoidance actions based on closest position estimate. Red dashed arrow (social force direction): yielding agent with later arrival timing at the intersection; blue solid arrow: passing-first agent.

## VII. Problem Instantiation

In this section we instantiate the framework on 2-player human-robot navigation with path crossing, shown in Figure 1.

Great progress has been made in the past two decades in pedestrian simulations [13], [33], robot navigation with human predictive models [29], [16], and socially-friendly robot planning [21], [4], to smoothly deploy robots in human workspaces. In human-robot interaction community, robot motion legibility has gained attention for human understanding [7], and the concept of trajectory interpretability has been applied on robot navigation in crowded environments to identify goals of pedestrians [1], [30].

In this section we make use of the social force model with explicit collision prediction [33], which was inspired by the social-force model but equipped with explicit avoidance behaviors. Other choices of human behavioral model for action realizations $g(x_t, a_t^H, \theta_t^H)$ can also fit in the framework for simulation; we choose this model for its underlying motion *legibility* concept in distinguishing choices of avoidance behaviors, as can be seen in Figure 3.

When two agents have intersecting paths and have similar timing to arrive at the intersection, explicit avoidance behaviors are involved. Particularly, an agent has to decide whether to avoid in front or behind the other. This leaves the coordination to agree on two action combinations: (passing_first,yielding), or (yielding,passing_first). ***shouldn't it be:(passing_first,yielding), or (yielding_first,passing)?***

We define the state space as following: $x_t^R = \begin{bmatrix} p_t^R \\ v_t^R \end{bmatrix}$, and $x_t^H = \begin{bmatrix} p_t^H \\ v_t^H \end{bmatrix}$ where $p_t^R, p_t^H$ are the robot's and human's positions, and $v_t^R, v_t^H$ are the robot's and human's velocities, respectively. All variables are described in 2D, therefore, $p_t^R, p_t^H, v_t^R, v_t^H \in \mathbb{R}^2$.

### A. Game start and termination criteria

As pointed out in Section V, to initiate interactive behaviors between agents, the potential-interaction criteria $\mathcal{C}_{PI}$ and the mutual-awareness criteria $\mathcal{C}_{MA}$ are prerequisites to start the game. In path crossing, $\mathcal{C}_{PI}$ is met if:

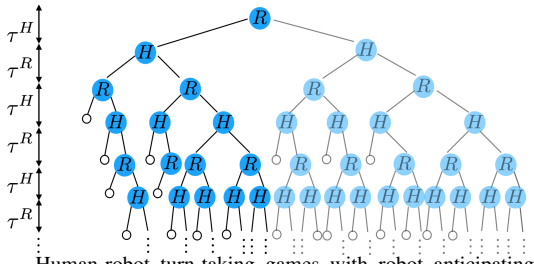


Fig. 4. Human-robot turn-taking games with robot anticipating action value based on finite-horizon predictions. $\tau^R$ and $\tau^H$ are approximate time intervals for robot and human response times. The game may terminate at either player's decision nodes.

1) two agents have path intersections in near future: $\exists t^H < t^{rH}, t^R < t^{rR}$, such that $v_t^R \times t^R + p_t^R = v_t^H \times t^H + p_t^H$
2) the arrival timing difference is within certain threshold: $|t^H - t^R| < \delta$

Where $t^{rH}$ is the reaction time (i.e., the time its take to agents to decide to engage in the scenario). For pedestrian avoidance, experimental results suggest that $t^{rH}$ is on average 4 secs before reaching the intersection [25]. The arrival timing difference depends on agent velocities, the safety margin to keep from other pedestrians, and the noise in estimation. Here, we use an estimate value of 1.5 sec, which is an upper bound to time difference for which agents respond to the intersecting scenario.

Finally, the game terminates whenever two agents have passed each other, or the minimum relative distance has passed.

### B. Intentions in path crossing and time delay

We will define the agents' action set as following: $A^R = \{a^{pf}, a^y\}$, $A^H = \{a^{pf}, a^y\}$, where $a^{pf}$ is corresponding to the class of trajectory realizations of passing first (i.e., in front of the other agent), and $a^y$ is corresponding to the class of yielding to the other.

Empirical study on human-robot crossing has suggested distinctive velocity and trajectory profiles among two classes of avoidance actions [24]. Agents who intend to pass first often accelerate and bend their trajectories away from the other. Agents who intend to yield often slow down. Such changes in motion profiles are clear signals for agent passing intents, and one can observe those responses within a short period of time, around 0.7 sec.

The minimum time for human agents to react to their action changes, $\tau^H$, is assumed to be between 0.7s to 0.9s for such a model. For more complex domains, higher values should be considered.

### C. Response-time-adapted turn-taking structure

While the interaction process is modeled as a simultaneous-move game in Section V, with the time delays in action realization, it is often played as a *turn-taking* game. With proper waiting time after the initial action, the decision timing of both agents can be clearly separated to prevent oscillations. Despite the fact that a simultaneous-move setting is more natural when dealing with interactions, turn-taking games simplify the simulation setting and therefore save computation. The overall turn-taking game used in this instantiation is illustrated in Figure 4.

### D. Experiments

We hypothesize that our proposed framework can explain different types of avoidance behaviors when their paths intersect with robots. We use our framework to simulate a particular type, cautious behavior, in comparison with collected trajectory data, to show that the framework can capture distinctive interaction patterns among people. We deploy a mobile robot into a building atrium, and record human crossing trajectories using tracking packages on laser scans [18]. Eight participants were requested to head towards a set of goals, while the robot followed a given route that intersected with human eight times. The robot was implemented with a local planner with emergent slow-down when sensing an object within 1 meter.

***I need you to explain this paragraph to me*** We analyze the results received for 0-step, 1-step, and 2-step lookahead with the same prior on robot strategy profile. With 0-step lookahead, virtual pedestrians act ***What does it mean "act compliantly, how?*** compliantly after observing the other agent's action, due to the assumption of immediate termination. With 1-step lookahead, agents assume the game may end with the robot playing minimax strategy, therefore they play based on conservative value estimate. With 2-step lookahead, virtual pedestrians is capable of planning for the optimistic (2nd step) in the worst case robot strategy (1st step). As a result, their behavior in the third case is less conservative compared to the previous two.

We simulate virtual pedestrians with type $\theta_t^H = [z^H, b_t^H(z^R)]$, 2-step lookahead, memory bound on interaction history = 2, in the turn-taking formulation. This is associated with the decision-making capacity to reason: "what would the robot do after seeing my action, and what I could do to in reaction to that action?". Other combinations can be considered. We have the robot choose avoidance actions purely based on arrival timing estimates. It is a simple model for demonstration clarify on human responses; more complex models can be used.

***Comments regarding to Figure 6: (1) You need to write in the figure what each color stands for. (2) You need to explain (you can do this as a footnote) the different movements in the curves that presents the simulated pedestrian velocity.***

*1) Perceived capabilities:* ***What are you trying to say in this subsubsection?*** The complete set of perceived capabilities are specified in Section VI-B.2 as prerequisites for humans to engage in the coordination process with robots. When dealing with social navigation, people, if noticing robot coming toward them, may be unsure regarding to whether the robot sees them or not (i.e., $\mathcal{C}_{MA}$); whether the robot is aware of the potential collision (i.e., $\mathcal{C}_{PI}$); whether the robot can identify the underlying intention if they choose an avoidance action (i.e., social inference capability on perception); whether they can identify the underlying intention of the robot if it chooses an avoidance action (i.e., social inference capability on motion generation).

*2) Types in path crossing:* ***I did not understand what are you saying in this paragraph***Agents perceived other agents' behaviors and update their own type $\theta_t^H$. In social

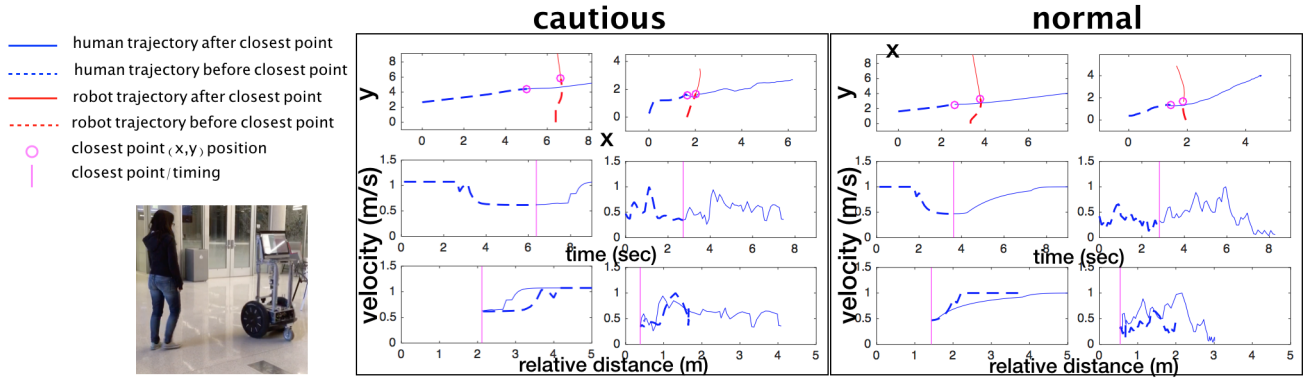


Fig. 5. SIMULATED pedestrian yielding, compared with REAL recording. We observed many types of behaviors during the experiments, and illustrate one common type in human-robot crossing: cautious, shown in the left box. Compared with people with gradual slow-down and speed-up (noted as "normal", shown in the right box), cautious agents wait for the robot until it passes the intersection (shown in the bottom-left photo). We simulate agent yielding and compare the x-y trajectory (top), the velocity profile over time (middle), and relative distance (bottom) with the true recordings, right-side of each box.

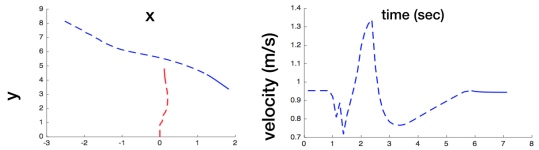


Fig. 6. SIMULATED cautious agent updating belief on robot strategies. While the simulated pedestrian yields at the beginning (with -0.7 sec arrival timing difference), after observing robot yielding behavior, she updates her belief and changes the action in the next time frame.

navigation, personal spacing is a common example, as people act repellently more to unfamiliar agents. People's urgency to travel to their own goals are common factors to describe navigation preferences. In simulation, we simulate agents to choose avoidance actions based on the following cost formulation:

$$r_t^H = -\eta\alpha C_t^H - (1-\alpha)C_t^{-H}, \tag{15}$$

where $C_t^H$ is the estimated time delay, $\eta \in [0,1]$ is the urgency level, and $\alpha \in [0,1]$ is the level of self-interest.

*3) Social trust:* When navigation around a robot, the assumption of whether it is socially compliant affects humans' avoidance decision. We commonly observe, at the beginning of the experiment, participants slow down to wait for the robot to pass first, even when the robot has a later arrival timing; people speed up and stay far from the robot when trying to pass in front, even when the robot is still far from the intersection. We refer to this type of behavior as cautious, simulated in Figure 5 through varying the perceived robot level of self-interest: $b_t^H(z^R)$.

***Comments regarding to Figure 5: (1) It is not clear which squares present the results of the recording and which squares present the results of the simulation. You should write it next to each relevant column. (2) The axis' titles are not well presented, it looks very messy. I suggest spacing the figure a little bit, so you will have space for the titles. (3) I can not see what is the difference between the cautious and normal agents in the top graphs, it look the same to me. Do you have other recording/simulation in which the normal agent crosses first? Then there will be a difference between the two kinds of agents. (4) Can you add to the figure a map of the routes used in the experiment? I think it will help better understanding of the result and even the experiment itself.***

*4) Human exploration and adaptation:* In the above described experiments we observed many conservative decisions made by human regarding to their crossing (e.g., pedestrians constantly yield to the robot). One agent actually tried passing in front of the robot for a couple of times and then decided to pass first. We consider this process as gradually gaining social trust on the robot; the agent updated her belief of the robot behavior and then adapted her own strategy. Similar behavior was simulated on virtual human with high belief update rate, shown in Figure 6.

## VIII. Conclusion and Future Work

We propose a Markov Game model with time-variant types to analyze human-robot coordination outcome, and propose a human decision-making model to describe phenomena in human-robot interactions. With the framework, we simulate virtual pedestrians with cautious type behaviors on human-robot crossing and compare the results with real-world recording. We also simulate adaptive human behaviors through belief updates on robot policy. In future work, we will further investigate human adaptability criteria based on different prior assumptions on robot behaviors and game convergence given false human assumptions.

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