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The newsvendor problem with barter exchange☆

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ABSTRACT

Barter exchange, as an alternative to move distressed inventory, has become increasingly popular in business. Many companies barter their unsold product for the product they need via barter exchange platforms at full prices. In this paper we consider the newsvendor problem with the barter exchange option. A retailer (the newsvendor) facing stochastic demand not only sells its product, but also buys other product that it needs from the market. It either trades its unsold product for the product it needs on a barter platform or disposes of its unsold product at discounted prices at the end of the selling season like in the classical newsvendor model. We derive the retailer's optimal order quantity, then analytically and numerically examine the impacts of barter on the retailer's inventory decisions and profit. We find that barter exchange can help the retailer to manage demand uncertainty and improve profit. The optimal order quantity decreases with barter commission and barter uncertainty, while increases with demand uncertainty and the value of the product that the retailer needs. Barter is more advantageous with lower barter commission, larger demand uncertainty, lower barter uncertainty, and higher value of the product it needs.

1. Introduction

With the continuing development of e-business, one of the most popular business modes that has emerged is the B2B e-marketplace. The modern barter exchange platform is such a B2B e-marketplace, which is an emerging Internet-based transaction means and has rapidly developed over the past two decades. It is estimated that the trade volume of barter accounts for about 30% of the global trade volume. The total value of the product bartered on barter platforms is US $ 12 billion in 2011 [14]. Now, there are in excess of 500,000 companies engaged in barter trade exchange and about 65% of the Fortune 500 companies engage in barter business. Also, it reports that 65% of the companies listed on the New York Stock Exchange move their distressed inventory using barter exchange.[1]

There are two basic types of barter, i.e., personal barter and retail barter or commercial barter between companies. For example, U-Exchange is a personal barter platform for exchanging various products and services. BookMooch and Home Exchange are personal barter platforms for bartering books and apartments, respectively. We can also find some commercial barter platforms such as iBarter, IMS Barter, b2b-barter, and barterxyz. In this paper we focus only on commercial barter. The trade sequence of barter exchange goes is as follows: Firm A registers on a barter exchange platform and provides some essential information, including what product and how many it will put up for barter? What's the price? And what product it needs? The broker of the platform will help Firm A to find Firm B that exactly needs the product of Firm A and that provides the product that Firm A exactly needs. Then Firm A and B can trade their product without money, and pay the broker a commission that typically is about 5%-15% of the trade value. The collection of barter trade exchange is called the barter pool. In modern barter exchanges, if the broker cannot find a single Firm B for Firm A, e.g., Firm B needs the product of Firm A, but Firm A does not need the product of Firm B, it can find more firms to conduct a multilateral barter, which involves more than two firms. Suppose that Firm C exactly needs the product of Firm B, and Firm A needs the product of Firm C, then the trade is consummated. Multilateral barter exchanges which are conducted through cycles or chains is one of the typical characteristics of the modern barter [1]. Another typical characteristic of the modern barter is that the

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[1] The data is provided by The International Reciprocal Trade Association (IRTA) reports, NATE (National Association of Trade Exchanges) and the U.S. Department of Commerce. Refer to www.barternewsweekly.com.

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trade uses barter currency or barter credit, which can be used as money, but only used on the barter platform, i.e., barter trade does not use money. Therefore, the barter pool is a relatively closed economy, which can maintain a general profit for each firm on the barter platform. For example, the barter price is the same as the market price.

From the above description of commercial barter, we see that barter exchange has many advantages over the traditional methods to move distressed inventory, such as disposal of unsold product at very low cost. Using barter, a firm can buy the product it needs without money, which helps preserve valuable liquidity. In addition, a firm can use barter to move distressed inventory, increase sale volume, find new customers, develop new markets, and cope with the stochastic demand. This is why increasing firms are engaged in barter exchanges.

Barter exchange is different from the secondary market in many ways. First, the barter price is the same as the selling price, whereas the price in the secondary market is a discounted price. Second, a firm can obtain the product it needs by trading its product on the barter platform, not money, while a firm needs to make monetary payment for the product it needs from the secondary market.

Although barter exchange has many advantages and is pertinent to operations management, the related literature in operation management is sparse. In this paper we study single-period (newsvendor) inventory with barter exchange and examine the impacts of barter on inventory decisions and profit. To the best of our knowledge, this is the first research on inventory management with barter exchange.

We organize the rest of the paper as follows: In Section 2 we give a brief review of the newsvendor problem, barter exchange, and secondary market. In Sections 3 we formulate the newsvendor problem with certain barter exchange mathematically, and analyze the impacts of barter exchange on the newsvendor's decisions and profit in Section 4. Section 5 extends our model by considering the uncertain barter and we investigate the newsvendor model with uncertain barter. We give the experimental study to investigate both the demand uncertainty and barter uncertainty on the retailer's optimal decisions and profit in Section 6. Finally we conclude the paper and suggest topics for future research.

2. Literature review

Our study is related to the newsvendor problem, barter exchange, and secondary market. Since the newsvendor problem is the basic model in inventory management, there are many studies on it and its variants. There are some studies on barter exchange, but there is no quantitative research on it from the perspective of operations management at present. Barter exchange is similar to secondary market in form, but entirely different from it, and we also briefly review the literature on secondary market.

2.1. Newsvendor model and its extensions

The newsvendor model has been widely used to make optimal inventory decisions that maximize the expected profit with stochastic demand. Numerous publications have provided a wide variety of extensions to the classical newsvendor model. Several authors propose extensions to the newsvendor model in which constrains on budget [5,20], capacity [6], service and loss [12], fairness [41] etc are considered. There are papers that extend the newsvendor model with consideration of behavioural characteristics of risk preference [4], loss aversion [21,42], regret [29], strategic consumers [43], consumer search cost [30], and bounded rationality [39]. There is a stream of research on extensions of the newsvendor model that consider various inventory strategies such as transshipment [36], centralized inventory [11,26], substitution [15], and order postponement [9]. Some studies consider extensions in which marketing strategy, especially the pricing policy of the newsvendor or supplier, is incorporated in the model and examine how operational problems interact with marketing issues. Monahan et al. [23], Prasad et al. [33] and Zhang et al. [44] gave examples of this extension by considering the advance selling strategy, dynamic pricing and gift cards respectively.

Extensions of newsvendor pricing policies in detail were provided by Petruzzi and Dada [31], and Khouja [13]. Several studies analyze the extension in which the newvendor sells products in multiple markets [17,37]. Robotis et al. [37] modelled the case where a reseller that procures used products using older generations of technology from an advanced market and then sells them in a secondary market with or without re-manufacturing. Except for the above variants, Khouja [13] also reviewed and analyzed the newsvendor problem with extensions that consider different objectives, random yields, and multi-echelon systems. Qin et al. [35] considered extensions concerning different modelling of customer demand, supplier costs, and the buyer's risk profile. Despite the many extensions of the newsvendor model in the literature, to the best of our knowledge, there is no study that considers the case where the newsvendor can barter products on a barter exchange platform.

2.2. Barter exchange

The aim of barter is to maintain trade volumes and balance them while maximizing the utility of the participants, which motivates a wide range of studies on deriving the optimal allocation and efficient exchange algorithms for barter in various industries. Examples are barter exchange markets in the medical industry to exchange kidneys, in the education field where students can exchange courses, in the power industry to exchange energy, as well as in the tourism and material processing industries, spanning secondary car market, house exchange, knowledge exchange, airlines exchange etc. Barter is indeed pervasive in every aspect of business covering the international arena [10], domestic B2B barter [28], and community C2C barter [19]. Besides, some specific barter requirements such as product indivisibility, fairness, reputation etc. are also taken into consideration in barter exchange. For example, Anderson et al. [1] studied the dynamic matching policies aiming to reduce the average waiting time. Considering matching technology and prioritization, Ashlagi et al. [2] studied different dynamic matching policies in heterogeneous markets to improve barer efficiency.

Another related stream of literature focuses on examining the effects of barter on the economic, social, marketing, and logistic issues of commercial barter. Most of the literature on the economic effect of barter focuses on the liquidity problem [40], capitalist economic crisis reduction [18], and lightening the tax burden [38]. Little literature explores the social effect of barter concerning social equity [18] and community building [18,40]. Limited research examines barter as a tool of market segmentation and price discrimination [8,38]. There are a few works in literature analyzing the effect of barter on logistics issues such as purchasing, inventory, and supply chain management [7,28] through case studies. For example, Plank et al. [28] found that barter transactions are beneficial to purchasing and dealing with excess or distressed inventory. They analyzed the potential of barter in reducing the overall inventory cost through case studies. Following [7,28] examined the opportunity of purchasing contributing to strategic goals through counter trade based upon an empirical study of a small number of UK companies.

Whilst there is an abundance of empirical works on barter with the primary objective of moving excess inventory [28], there are few modelling studies focusing specifically on the implications of barter for inventory decisions and other logistics issues.

2.3. Secondary market

Secondary market has increasingly become the means to dispose of excess inventory and improve the mismatch between supply and demand. Research about secondary market is also related to our topic. There are B2B and B2C transactions in the secondary market. The majority of the literature focuses on B2C transactions, in which buying from the secondary market is not considered and the firm merely wants to dispose of its excess inventory by selling to customers in the secondary market [3]. The secondary market may be geographically dispersed with a different selling season, have different customer values for fashion product [32], and have a different level of technology acceptance from the primary market [37]. Some research on the secondary market is motivated by the re-manufacturing behaviour [27], which can reduce the number of units procured from the reseller [22,37].

The B2B secondary market is often motivated by online exchanges between businesses that can dispose and acquire materials. Studies on the B2B secondary market are few. Motivated by the inventory exchange platform of TradingHubs.com, Lee and Whang [16] considered a two-level supply chain scenario under which resellers order and receive products from the manufacturer in the first period and then trade inventories among themselves in the secondary market in the second period. They showed that the secondary market affects the sales volume for the manufacturer and increases allocation efficiency. Angelus [3] studied the joint replenishment and disposal policy in the secondary market for a multi-period and multi-echelon supply chain system. Pilehvar et al. [34] empirically explore the effect of market pricing information and bidder heterogeneity on the B2B auctions in the secondary market. Their work provided implications for the auctioneers to design and manage the information on the platform. The study on newsvendor problems with second selling options are also related to our work [24,25]. For example, Khouja and Zhou [24] considered a supply chain of a manufacturer who sells a product to a retailer, and the retailer can sell excess products to the off-price retailer.

3. The model

In this section we consider the single-period inventory (newsvendor) problem with barter exchange from the retailer's perspective, in which the customer demand is stochastic and characterized by a random variable, and the retailer determines the optimal stocking policy to satisfy the customer demand at the beginning of the selling season. As in the classical model, the retailer will dispose of the unsold product at a low price at the end of the selling season. However, in our model, the retailer also has to buy some product from the market for its firm or employees, such as office supplies or other product that it needs. So the retailer has one more option, i.e., it can barter its unsold product at almost the full selling price (not a discounted price) for the product it needs on a barter exchange platform. Therefore, when some products are unsold, it is better for the retailer to barter them for the product it needs than to dispose of them at a very low price.

We suppose that the retailer purchases only a single product A from the supplier, and the retail price is set by the supplier or the market. The retailer's decision problem is to decide the optimal order quantity to maximize its own expected profit.

The sequence of the event is as follows: The retailer orders Q units of the product A from the supplier at a fixed price at the beginning of the selling season, then it sells the product A to its customers at the retail price. If customer demand is greater than Q, then it will pay the shortage penalty cost for the unsatisfied demand. If customer demand is less than Q, the retailer will trade the unsold product A for the product it needs on a barter platform. If there are still some unsold product A after barter exchange, the retailer will dispose of them at a very low price as in the classical newsvendor model.

The notation used in the paper is as follows:

- x = stochastic demand, a random variable;
- $f(x)$ = the probability density function of x;
- $F(x)$ = the cumulative distribution function of x;
- c = the supplier's wholesale price;
- p = the retailing price per unit ($p > c$);
- v = the salvage value per unit ($v < c$);
- s = the shortage penalty cost per unit;
- Q = the order quantity, a decision variable;
- Q_0= the value of the product the retailer needs on the barter platform is equal to the value of Q_0 units of the product that the retailer sells;
- Q_c^* = the optimal order quantity in the classical newsvendor model;
- Q^* = the optimal order quantity in the newsvendor model with barter;
- r= the retailer pays r percent of retail price to the barter platform as the commission of per unit product;
- $\pi(Q, x)$ = the total profit if the order size is Q units and the customer demand is x.

Since the retailer will trade its product for other product at the retail price p on the barter platform, and it pays a commission rp per unit product, the value of per unit product for the retailer is $(1-r)p$. And $r \in [0.05, 0.15]$ in practice. Besides, we assume $(1-r)p > c$ to make sure the retailer can have positive profit on the platform.

3.1. The classical newsvendor model with consumption

In order to compare with the classical newsvendor model, we briefly review the classical newsvendor model and its optimal solution. For comparability, we incorporate the retailer's consumption into the model, i.e., it will buy some product whose value is equal to that of Q_0 units of the product it sells.

The retailer's profit per period is

$$\pi_c(Q,x) = \begin{cases} (p-c)Q - s(x-Q) - pQ_0, & \text{if } Q \le x, \\ px + v(Q-x) - cQ - pQ_0, & \text{if } Q > x. \end{cases} \quad (1)$$

And the retailer's expected profit is shown in Eq. (2) as follows:

$$E[\pi_c(Q)] = (p-c+s)Q\int_Q^{+\infty} f(x)dx - s\int_Q^{+\infty} xf(x)dx + (p-v)\int_0^Q xf(x)dx - (c-v)Q\int_0^Q f(x)dx - pQ_0. \quad (2)$$

The retailer's optimal order quantity satisfies the following well-known fractile formula in Eq. (3):

$$F(Q_c^*) = \frac{p+s-c}{p+s-v}. \quad (3)$$

3.2. The newsvendor model with barter exchange

In this section we present the newsvendor model with barter exchange and its solution. First, we present the retailer's profits for the following three cases.

Case 1. When $Q \le x$, i.e., the retailer's order quantity Q is less than the customer demand x, the retailer pays the shortage penalty cost s for the unsatisfied demand, so the retailer's profit is

$$\pi(Q,x) = (p-c)Q - s(x-Q) - pQ_0.$$

Case 2. When $x < Q \le x + Q_0$, since the order quantity Q is greater than the customer demand x, $Q - x$ units of the product are unsold. The retailer trades $Q - x$ units of the product A for those the retailer needs on the barter exchange platform, pays the commission $rp(Q - x)$, and buys product from the market, whose value is equal to that of $Q_0 - (Q - x)$ units of the product A. So the retailer's profit is

$$\pi(Q, x) = px - rp(Q - x) - p(Q_0 - Q + x) - cQ.$$

Case 3. When $Q > x + Q_0$, the retailer barters its product A for all the product it needs at the cost of the commission rpQ_0 and disposes of the rest the product A at v. So the retailer's profit is

$$\pi(Q, x) = px - rpQ_0 + v(Q - Q_0 - x) - cQ.$$

Therefore, after simplifying, we have

$$\pi(Q, x) = \begin{cases} (p - c + s)Q - sx - pQ_0, & \text{if} \quad Q \le x, \\ [(1 - r)p - c]Q + rpx - pQ_0, & \text{if} \quad x < Q \le x + Q_0, \\ (v - c)Q + (p - v)x - (rp + v)Q_0, & \text{if} \quad Q \ge x + Q_0. \end{cases} \tag{4}$$

Taking the expected value of π gives the retailer's expected profit as follows:

$$\begin{aligned} E[\pi(Q)] = {} & [(p - c + s)Q - pQ_0] \int_Q^{+\infty} f(x)dx - s\int_Q^{+\infty} xf(x)dx \\ & + [[(1 - r)p - c]Q - pQ_0] \int_{Q-Q_0}^{Q} f(x)dx \\ & + rp\int_{Q-Q_0}^{Q} xf(x)dx \\ & + [(v - c)Q - (rp + v)Q_0] \int_0^{Q-Q_0} f(x)dx \\ & + (p - v)\int_0^{Q-Q_0} xf(x)dx \\ = {} & (p - c + s)Q - sQF(Q) - (p - v)\int_0^{Q-Q_0} F(x)dx \\ & - rp\int_{Q-Q_0}^{Q} F(x)dx - s\int_Q^{+\infty} xf(x)dx - pQ_0. \end{aligned} \tag{5}$$

Taking the first and second derivatives of $E[\pi(Q)]$ with respect to Q gives

$$E'[\pi(Q)] = p - c + s - (rp + s)F(Q) - [(1 - r)p - v]F(Q - Q_0),$$
$$E''[\pi(Q)] = -(rp + s)f(Q) - [(1 - r)p - v]f(Q - Q_0).$$

Since $(1 - r)p > c > v$, it is obvious that $E''[\pi(Q)] < 0$, which indicates that $E[\pi(Q)]$ is concave in Q. Therefore, the first order condition is necessary and sufficient to find the optimal solution. Letting $E'[\pi(Q)] = 0$, we have the following result.

Theorem 1. *The retailer's optimal order quantity Q^* satisfies*

$$(rp + s)F(Q^*) + [(1 - r)p - v]F(Q^* - Q_0) = p + s - c. \tag{6}$$

From Theorem 1, we derive the following corollaries.

Corollary 1. *When $(1 - r)p = v$, then*

$$Q^* = Q_c^*. \tag{7}$$

Proof. When $(1 - r)p = v$, i.e., $rp = p - v$, from Eq. (6), we get $(p - v + s)F(Q^*) = p - c + s$ and $F(Q^*) = F(Q_c^*)$, and the result in Eq. (7) holds. Corollary 1 implies that the classical newsvendor model is a special case of our model, and when the net value of the product on the barter platform is equal to the salvage value, then our model reduces to the classical newsvendor model. □

Corollary 2. *1) When the customer demand follows the uniform distribution, i.e., $x \sim U[a, b]$, then*

$$Q^* = \frac{(p + s - c)b + (c - v)a}{p + s - v} + \frac{(1 - r)p - v}{p + s - v}Q_0. \tag{8}$$

(2) When the customer demand follows the exponential distribution with the rate parameter λ, then

$$Q^* = \frac{ln[p + s - v + [(1 - r)p - v](e^{\lambda Q_0})] - ln(c - v)}{\lambda}. \tag{9}$$

From Eq. (8) and Eq. (9) in Corollary 2, we can make some observations. Notice that in the classical newsvendor model $Q_c^* = \dfrac{(p + s - c)b + (c - v)a}{p + s - v}$ and $Q_c^* = \dfrac{ln(p + s - v) - ln(c - v)}{\lambda}$ when the customer demand follows the uniform distribution and exponential distribution, respectively. We can easily find that $Q^* > Q_c^*$ and the increment increases in Q_0. Given that $(1 - r)p > v$, the increment is less than Q_0.

4. The impact of barter on the newsvendor's decisions

In this section we examine the impacts of barter on the newsvendor's decisions and its profit. Specifically, we theoretically analyze the impact of the commission and Q_0 on the newsvendor's decisions and its profit, which are summarized in the Theorems 2 and 3.

Theorem 2. *Barter exchange can help to manage demand uncertainty and increase retailer's profit with $Q_c^* < Q^* < Q_c^* + Q_0$ and $E[\pi(Q^*)] > E[\pi_c(Q_c^*)]$.*

Proof. 1) Eq. (6) can be re-written as follows:

$$\frac{rp + s}{p + s - v}F(Q^*) + \frac{(1 - r)p - v}{p + s - v}F(Q^* - Q_0) = \frac{p + s - c}{p + s - v} = F(Q_c^*) \tag{10}$$

Notice that $\dfrac{rp + s}{p + s - v} + \dfrac{(1 - r)p - v}{p + s - v} = 1$, so $F(Q_c^*)$ is the convex combination of $F(Q^*)$ and $F(Q^* - Q_0)$. And one of $F(Q^*)$ and $F(Q^* - Q_0)$ must be greater than $F(Q_c^*)$, another must be less than $F(Q_c^*)$. Since $F(Q)$ increases with Q, we have $F(Q^* - Q_0) < F(Q_c^*) < F(Q^*)$, i.e., $Q_c^* < Q^* < Q_c^* + Q_0$.

2) Eq. (1) can be re-written as

$$\pi_c(Q, x) = \begin{cases} (p - c + s)Q - sx - pQ_0, & \text{if } Q \le x, \\ (v - c)Q + (p - v)x - pQ_0, & \text{if } x < Q \le x + Q_0, \\ (v - c)Q + (p - v)x - pQ_0, & \text{if } Q > x + Q_0. \end{cases} \tag{11}$$

Notice that $(1 - r)p > v$, we have $p - v > rp$ and $rp + v < p$. Comparing (4) and (12), we have $\pi(Q, x) > \pi_c(Q, x)$, so $E[\pi(Q)] > E[\pi_c(Q)]$ and $E[\pi(Q^*)] > E[\pi_c(Q_c^*)]$. The result holds. □

Theorem 2 indicates that the retailer's optimal order quantity in the newsvendor model with barter exchange is greater than that in the classical newsvendor model, but the increment is less than Q_0, which is totally determined by the value of the product that the retailer buys. This theorem also states that the retailer's profit in the newsvendor model with barter exchange is greater than that in the classical newsvendor model. In other words, barter exchange can induce the retailer to order more product. On the one hand, with higher order quantity, the stock-out rate decreases and the shortage cost becomes lower. On the other hand, the unsold products can be bartered on the platform which increases the salvage value of the product. Then the expected profit increases. Therefore, we conclude that barter exchange can help to manage demand uncertainty and increase retailer's profit.

Theorem 3. *The retailer's order quantity and profit decreases with barter commission, while the order quantity increases and profit decreases with the value of the product that the retailer will buy. Furthermore, the profitability of barter increases with barter commission and decreases with the value of the product that the retailer will buy.*

Proof. 1) Taking the first derivative of Eq. (6) with respect to r and simplifying, we have

$$\{(rp+s)f(Q^*) + [(1-r)p-v]f(Q^*-Q_0)\}\frac{dQ^*}{dr} = p[F(Q^*-Q_0)-F(Q^*)].$$

Since $(1-r)p-v>0, F(Q^*-Q_0)-F(Q^*)<0$, we have $\frac{dQ^*}{dr}<0$. From Eq. (5), $\frac{dE[\pi(Q)]}{dr} = -p\int_{Q-Q_0}^{Q} F(x)dx < 0$. Furthermore, the expected profit $E[\pi_c(Q_c)]$ is independent with the barter commission r, therefore $\frac{d(E[\pi(Q)]-E[\pi_c(Q_c)]}{dr} < 0$. The result holds.

2) Taking the first derivative of Eq. (6) with respect to Q_0 and simplifying, we have

$$\{(rp+s)f(Q^*) + [(1-r)p-v]f(Q^*-Q_0)\}\frac{dQ^*}{dQ_0} = [(1-r)p-v]f(Q^*-Q_0).$$

Since $(1-r)p-v>0$, we have $\frac{dQ^*}{dQ_0}>0$. From Eq. (5), $\frac{dE[\pi(Q)]}{dQ_0} = [(1-r)p-v]F(Q-Q_0)-p<0$. Furthermore, $\frac{dE[\pi(Q_c)]}{dQ_0} = -p < 0$. Then, the decline rate with increasing Q_0 is faster than $E[\pi(Q)]$. Besides $E[\pi(Q)] > E[\pi_c(Q)]$, therefore $\frac{d(E[\pi(Q)]-E[\pi_c(Q_c)]}{dQ_0} > 0$. Then the result holds. □

The optimal order quantity and the profit of the newsvendor with barter compared with the traditional newsvendor are related with the barter commission and the value of the product that the retailer will buy. The theorem is not difficult to understand. When offered with the chance to barter on the platform, the retailer will balance the salvage value and the barter cost to make the optimal order decision. With increasing commission of the barter platform, the cost of barter increases and then the retailer should decrease the amount of product bartered on the platform, which results in decreases in the retailer's order quantity and profit.

Similarly, given certain barter commission, when the value of the product that the retailer buys increases, the retailer can barter more unsold product on the platform, and can move more distressed inventory, which induce it to order more product. When ordering more products, obviously the profit will decrease. However, barter can still benefit the retailer with higher profit compared with the traditional newsvendor. In practice, the retailer should consider the purchasing decisions of multiple products simultaneously to take advantage of barter exchange.

5. The impact of barter uncertainty

In this section, we extend the model by considering that the supply on the barter platform is uncertain. We present the newsvendor model with uncertain barter exchange and explore the effect of barter uncertainty on the optimal inventory decision and resulting profit. The barter supply is characterized by wQ_0, w is random with mean u_w and variance of σ_w. $g(w)$ is the probability density function of w. The optimal order quantity in the model with uncertain barter is denoted by Q_u^*, and the expected profit is denoted by $E[\pi_u(Q_u)]$. We present the retailer's profits for the following two cases, namely the barter supply is lower than the barter demand ($0 \le w \le 1$) and supply is higher than the barter demand ($w > 1$).

Case 1: $0 \le w \le 1$

1) When $Q \le x$, i.e., the retailer's order quantity Q is less than the customer demand x, the retailer pays the shortage penalty cost s for the unsatisfied demand, so the retailer's profit is

$$\pi_u(Q,x) = (p-c)Q - s(x-Q) - pQ_0.$$

2) When $x < Q \le x + wQ_0$, since the order quantity Q is greater than the customer demand x, $Q-x$ units of the product are unsold. The retailer trades $Q-x$ units of the product A, pays the commission $rp(Q-x)$, and buys Q_0-Q+x units of the product A from the market. So the retailer's profit is

$$\pi_u(Q,x) = px - rp(Q-x) - p(Q_0-Q+x) - cQ.$$

3) When $Q > x + wQ_0$, the retailer barters its product A for all the product it needs at the cost of the commission $rpwQ_0$, buys $(1-w)Q_0$ units of the product A from the market, and disposes of the rest the product A at v. So the retailer's profit is

$$\pi_u(Q,x) = px - rpwQ_0 + v(Q-wQ_0-x) - cQ - (1-w)pQ_0.$$

Case 2: $w > 1$

4) When $Q \le x$, i.e., the retailer's order quantity Q is less than the customer demand x, the retailer pays the shortage penalty cost s for the unsatisfied demand, so the retailer's profit is

$$\pi_u(Q,x) = (p-c)Q - s(x-Q) - pQ_0.$$

5) When $x < Q \le x + Q_0$, since the order quantity Q is greater than the customer demand x, $Q-x$ units of products are unsold. Supply is wQ_0 and all the unsold products can be traded on the platform. Then the retailer buys Q_0-Q+x units of the product A, trades $Q-x$ units on the platform. So the retailer's profit is

$$\pi_u(Q,x) = px - rp(Q-x) - p(Q_0-Q+x) - cQ.$$

6) When $Q > x + Q_0$, since the order quantity Q is greater than the customer demand x, the supply is wQ_0, then $Q-x$ units of the product are unsold and can be traded on the platform. So the retailer's profit is. So the retailer's profit is

$$\pi_u(Q,x) = px - rpQ_0 + v(Q-Q_0-x) - cQ.$$

Taking the expected value of $\pi_u(Q_u, x)$ gives the retailer's expected profit as follows:

$$\begin{aligned} E[\pi_u(Q_u)] = {} & \int_0^{+\infty}\int_{Q_u}^{+\infty}[(p-c+s)Q_u - sx - pQ_0]f(x)g(w)dxdw \\ & + \int_0^1\int_{Q_u-wQ_0}^{Q_u}(((1-r)p-c)Q_u \\ & + rpx - pQ_0)f(x)g(w)dxdw \\ & + \int_0^1\int_0^{Q_u-wQ_0}[(v-c)Q_u + (p-v)x - (rp+v)wQ_0 \\ & - (1-w)pQ_0]f(x)g(w)dxdw \\ & + \int_1^{+\infty}\int_{Q_u-Q_0}^{Q_u}[px - rp(Q_u-x) - p(Q_0-Q_u+x) \\ & - cQ_u]f(x)g(w)dxdw \\ & + \int_1^{+\infty}\int_0^{Q_u-Q_0}[px - rpQ_0 + v(Q_u-Q_0-x) \\ & - cQ_u]f(x)g(w)dxdw. \end{aligned} \tag{12}$$

Similarly, taking the first and second derivatives of $E[\pi_u(Q_u)]$ with respect to Q_u gives

$$\begin{aligned} E'[\pi_u(Q_u)] = {} & p-c+s-(rp+s)F(Q_u) \\ & - [(1-r)p-v]\left(\int_0^1 F(Q_u-wQ_0)g(w)dw \right. \\ & \left. + \int_1^{+\infty} F(Q_u-Q_0)g(w)dw\right), \end{aligned}$$

$$E''[\pi_u(Q_u)] = -(rp+s)f(Q_u)$$

$$- [(1-r)p - v]\left(\int_0^1 f(Q_u - wQ_0)g(w)dw + \int_1^{+\infty} f(Q_u - Q_0)g(w)dw\right) < 0.$$

Obviously that $E[\pi_u(Q_u)]$ is concave in Q_u. Therefore, the first order condition is necessary and sufficient to find the optimal solution.

Theorem 4. *The retailer's optimal order quantity with uncertain barter Q_u^* satisfies*

$$(rp+s)F(Q_u^*) + ((1-r)p - v)\left(\int_0^1 F(Q_u^* - wQ_0)g(w)dw + \int_1^{+\infty} F(Q_u^* - Q_0)g(w)dw\right) = p + s - c. \quad (13)$$

The optimal order quantity with uncertain barter is similar with the one in Eq. (6). Different from the order quantity condition in Eq. (6), the optimal order quantity with uncertain barter is dependent on the expected fulfillment rate, i.e., the fulfillment rate when barter supply cannot meet the barter demand, i.e., $F(Q_u^* - wQ_0)$, and fulfillment rate when supply exceeds the barter demand, i.e., $F(Q_u^* - Q_0)$.

Theorem 5. *The newsvendor with barter uncertainty can still benefit the retailer with higher profit compared with the traditional newsvendor. However, the barter uncertainty will decrease the order quantity and the profitability, i.e., $Q_c^* < Q_u^* < Q^*$ and $E[\pi(Q^*)] > E[\pi_u(Q_u^*)] > E[\pi_c(Q_c^*)]$.*

Proof. 1) Eq. (13) can be re-written as $\frac{rp+s}{p+s-v}F(Q_u^*) + \frac{(1-r)p-v}{p+s-v}(\int_0^1 F(Q_u^* - wQ_0)g(w)dw + \int_1^{+\infty} F(Q_u^* - Q_0)g(w)dw) = \frac{p+s-c}{p+s-v} = F(Q_c^*)$. Notice that $\frac{rp+s}{p+s-v} + \frac{(1-r)p-v}{p+s-v} = 1$, so $F(Q_c^*)$ is the convex combination of $F(Q_u^*)$ and $(\int_0^1 F(Q_u^* - wQ_0)g(w)dw + \int_1^{+\infty} F(Q_u^* - Q_0)g(w)dw)$. And one of $F(Q_u^*)$ and $\int_0^1\int_0^{Q_u^*-wQ_0} f(x)g(w)dxdw$ must be greater than $F(Q_c^*)$, another must be less than $F(Q_c^*)$. Since $F(Q)$ increases with Q, we have $\int_0^1 F(Q_u^* - wQ_0)g(w)dw + \int_1^{+\infty} F(Q_u^* - Q_0)g(w)dw) < F(Q_c^*) < F(Q_u^*)$, i.e., $Q_c^* < Q_u^*$.

Besides, $\int_0^1 F(Q_u^* - wQ_0)g(w)dw + \int_1^{+\infty} F(Q_u^* - Q_0)g(w)dw > F(Q_u^* - Q_0)$. Then compared with Eq. (10), $\frac{(1-r)p-v}{p+s-v}(\int_0^1 F(Q_u^* - wQ_0)g(w)dw + \int_1^{+\infty} F(Q_u^* - Q_0)g(w)dw)$ is larger than $\frac{(1-r)p-v}{p+s-v}F(Q_u^* - Q_0)$. Therefore, $\frac{rp+s}{p+s-v}F(Q_u^*)$ is smaller than $\frac{rp+s}{p+s-v}F(Q^*)$ and $Q_u^* < Q^*$.

2) Comparing Eq. (11) and Eq. (4), we have $E[\pi_u(Q_u)] - E[\pi_c(Q)] > 0$ as shown in Eq. (14). Comparing Eqs. (11) and (12), we have $E[\pi_u(Q_u)] - E[\pi(Q)] < 0$ as shown in Eq. (15).

$$\begin{aligned} E[\pi_u(Q_u)] - E[\pi_c(Q)] &= \int_0^1\int_{Q-wQ_0}^{Q} ((1-r)p - v)(Q-x)f(x)g(w)dxdw \\ &+ \int_0^1\int_0^{Q-wQ_0} ((1-r)p - v)wQ_0 f(x)g(w)dxdw \\ &+ \int_1^{+\infty}\int_{Q-Q_0}^{Q} ((1-r)p - v)(Q-x)f(x)g(w)dxdw \\ &+ \int_1^{+\infty}\int_0^{Q-Q_0} ((1-r)p - v)Q_0 f(x)g(w)dxdw. \quad (14) \end{aligned}$$

$$E[\pi_u(Q_u)] - E[\pi(Q)] = \int_0^1\int_0^{Q-wQ_0} ((1-r)p - v)(wQ_0 + x - Q)f(x)g(w)dxdw. \quad (15)$$

Then $E[\pi(Q)] > E[\pi_u(Q_u)] > E[\pi_c(Q)]$, and $E[\pi(Q^*)] > E[\pi_u(Q_u^*)] > E[\pi_c(Q^*)]$. The result holds. □

Theorem 5 demonstrates that uncertain barter can still benefit the retailer with higher expected profit and lower stock-out rate. And the retailer should order more inventory when he switches from traditional purchasing method to barter exchange. However, when barter supply is uncertain, there is possibility that only a portion of the unsold products can be bartered on the platform. Then the retailer will reduce the optimal order quantity and buy the products from the market, which decreases the salvage value of the unsold products and then the expected profit decreases consequently.

Theorem 6. *In the newsvendor model with uncertain barter exchange,*

1. *the sensitivities of retailer's optimal order quantity and profit with respect to barter commission and value of the product that the retailer needs are consistent with the results when barter supply is certain;*
2. *the retailer's optimal order quantity decreases as the probability that barter supply is smaller than the retailer's barter demand on the barter platform, i.e., $\int_0^1 g(w)dw$, increases.*

Proof. 1) Taking the first derivative of Eq. (13) with respect to r and simplifying, we have $\{(rp+s)f(Q_u^*) + [(1-r)p - v](\int_0^1 f(Q_u^* - wQ_0)g(w)dw + \int_1^{+\infty} f(Q_u^* - Q_0)g(w)dw)\}\frac{dQ_u^*}{dr} = p[\int_0^1 F(Q_u^* - wQ_0)g(w)dw + \int_1^{+\infty} F(Q_u^* - Q_0)g(w)dw - F(Q_u^*)]$. Since $(1-r)p - v > 0$, $\int_0^1 F(Q_u^* - wQ_0)g(w)dw + \int_1^{+\infty} F(Q_u^* - Q_0)g(w)dw - F(Q_u^*) < 0$, we have $\frac{dQ_u^*}{dr} < 0$. From Eq. (12), $\frac{dE[\pi(Q)]}{dr} = \int_0^1\int_{Q-wQ_0}^{Q}(-pQ + px)f(x)g(w)dxdw - \int_0^1\int_0^{Q-wQ_0}(pQ_0)f(x)g(w)dxdw - \int_1^{+\infty}\int_{Q-Q_0}^{Q} p(Q-x)f(x)g(w)dxdw - \int_1^{+\infty}\int_0^{Q-Q_0} pQ_0 f(x)g(w)dxdw < 0$. The result holds.

2) Taking the first derivative of Eq. (13) with respect to Q_0 and simplifying, we have $\{(rp+s)f(Q_u^*) + [(1-r)p - v](\int_0^1 f(Q_u^* - wQ_0)g(w)dw + \int_1^{+\infty} f(Q_u^* - Q_0)g(w)dw)\}\frac{dQ_u^*}{dQ_0} = [(1-r)p - v](\int_0^1 wf(Q_u^* - wQ_0)g(w)dw + \int_1^{+\infty} f(Q_u^* - Q_0)g(w)dw)$. Since $(1-r)p - v > 0$, we have $\frac{dQ_u^*}{dQ_0} > 0$. From Eq. (12), $\frac{dE[\pi(Q)]}{dQ_0} = -p + \int_0^1\int_0^{Q-wQ_0}((1-r)p - v)wf(x)g(w)dxdw + \int_1^{+\infty}\int_0^{Q-Q_0}((1-r)p - v)f(x)g(w)dxdw < 0$. The result holds.

3) Note that w is distributed along $(0, \infty)$. $F(Q)$ is increasing in Q, then $F(Q_u^* - wQ_0)$ is larger than $F(Q_u^* - Q_0)$ when $0 < w < 1$. Denote $\int_0^1 F(Q_u^* - wQ_0)g(w)dw + \int_1^{+\infty} F(Q_u^* - Q_0)g(w)dw$ with R. If the probability that supply on the barter platform exceeds the retailer's maximized barer demand, i.e., $\int_1^{+\infty} g(w)dw$, increases, then $\int_0^1 g(w)dw$ decreases, resulting R decreases. Therefore, we deduce that Q_u^* increases when $\int_1^{+\infty} g(w)dw$ increases. □

Theorem 6 demonstrates that the profitability of barter decreases with the barter commission, and increases with the value of the product that the retailer will buy on the platform. Furthermore, the retailer should order less product when the probability that barter supply is smaller than the barter demand increases. Though the retailer has some unsold products to be bartered, the supply on the platform is limited. Then the retailer will reduce order quantity in case of increasing inventory cost.

6. Experimental study

Next, assuming both the demand and barter follow the normal distribution, we present a barter example to illustrate and verify the above theoretical analysis. Furthermore, we also conduct sensitivity analysis to examine the demand uncertainty and

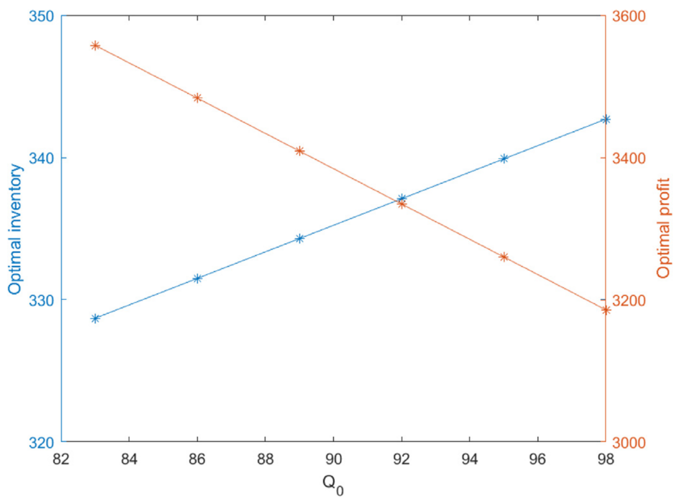

Fig. 1. Impact of Q_0 on the optimal inventory and profit.

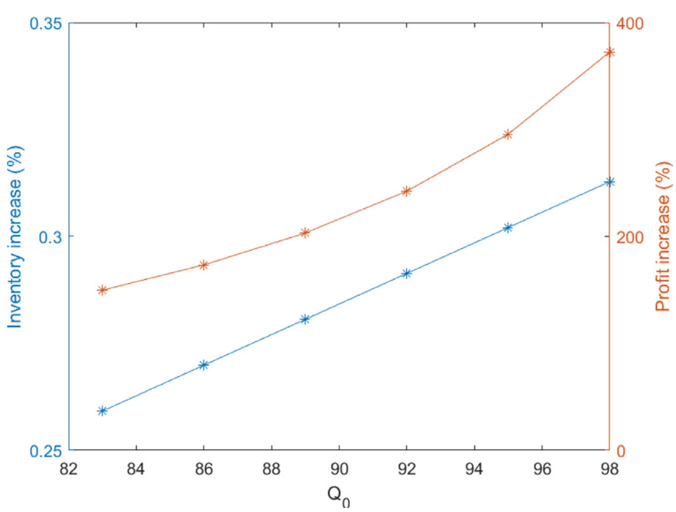

Fig. 2. Impact of Q_0 on inventory and profit increase.

barter uncertainty on the newsvendor's decisions and profit. Suppose that the customer demand follows a normal distribution, i.e., $x \sim N(u_x, \sigma_x)$, successful barter probability w also follows a normal distribution with mean u_w and standard deviation σ_w. And we set $u_w = 1$ and $\sigma_w \in [0.01, 0.5]$. Besides, $r \in [0.05, 0.15]$, $\sigma_x \in [30, 60]$, $u_x = 200$, $p = 50, c = 20, s = 10, v = 15, Q_0 = 50$. Inventory increase is measured by $\frac{Q_u^* - Q_c^*}{Q_c^*}$ and profit increase is measured by $\frac{E[\pi_u(Q_u^*)] - E[\pi_c(Q^*)]}{E[\pi_c(Q^*)]}$.

Figs. 1–4 verify Theorem 3. Fig. 1 shows that with increasing Q_0, the retailer's order quantity increases while the profit decreases. As shown in Fig. 3, compared with the classical newsvendor, barter can improve the profit and the optimal order quantity, and increment rates also increase with Q_0. Fig. 3 and 4 show that with decreasing commission, the retailer's order quantity and profit increase, and their increment rates also increase.

Fig. 5 shows that with increasing demand variance, the retailer's order quantity increases, while the profit decreases. Compared with the classical newsvendor in Fig. 6, barter can improve the retailer's profit with higher order quantity. Besides, increment rate of inventory is decreasing while the increment rate of profit increasing with demand uncertainty, which indicates that barter can effectively cope with variance in demand.

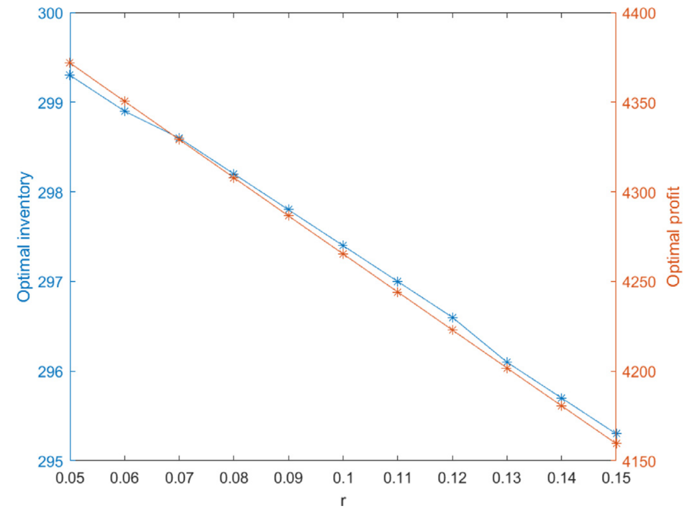

Fig. 3. Impact of the commission on optimal inventory and profit.

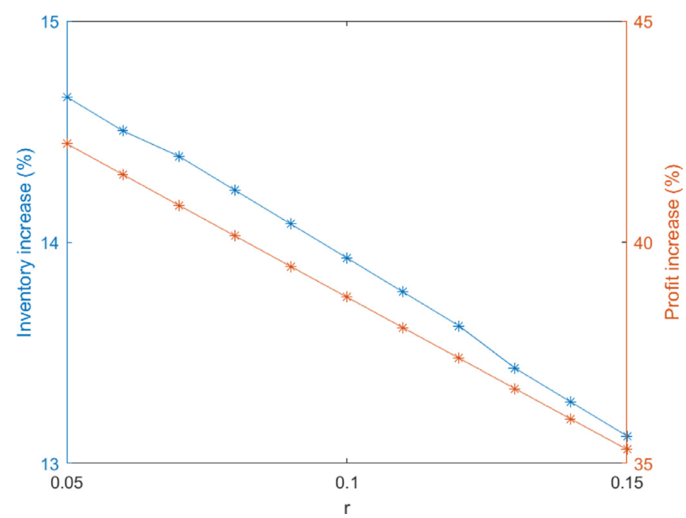

Fig. 4. Impact of the commission on the inventory and profit increase.

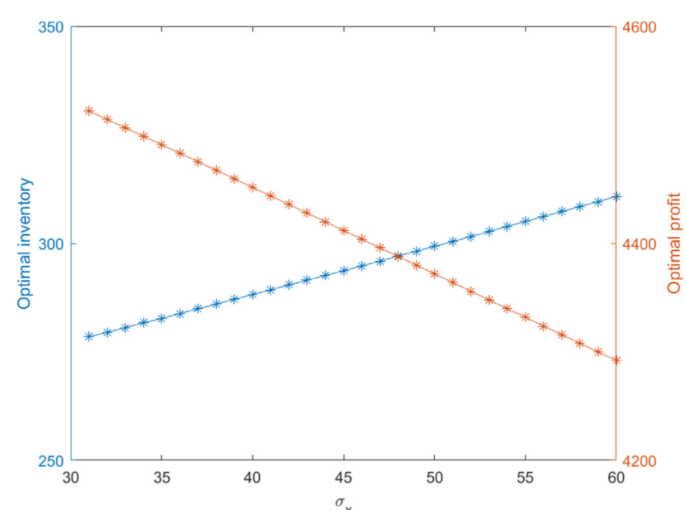

Fig. 5. Impact of demand uncertainty on the optimal inventory and profit.

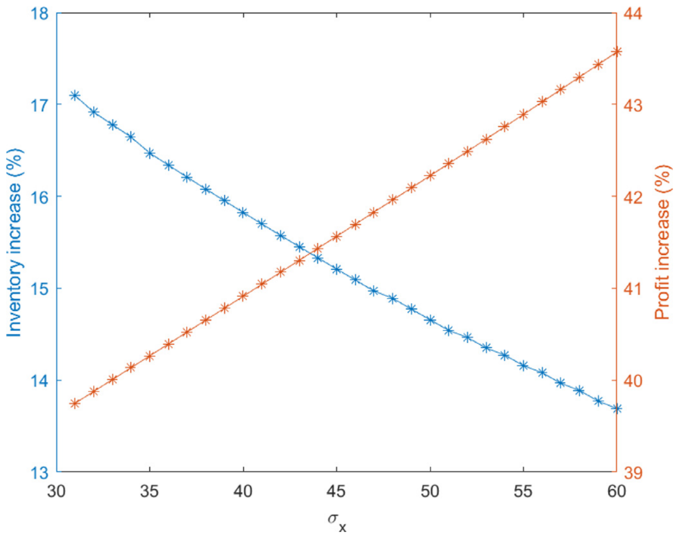

Fig. 6. Impact of demand uncertainty on inventory and profit increase.

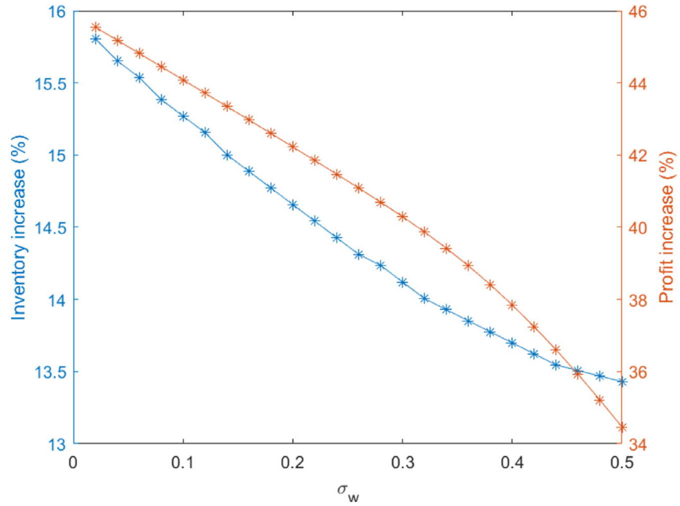

Fig. 8. Impact of barter uncertainty on inventory and profit increase.

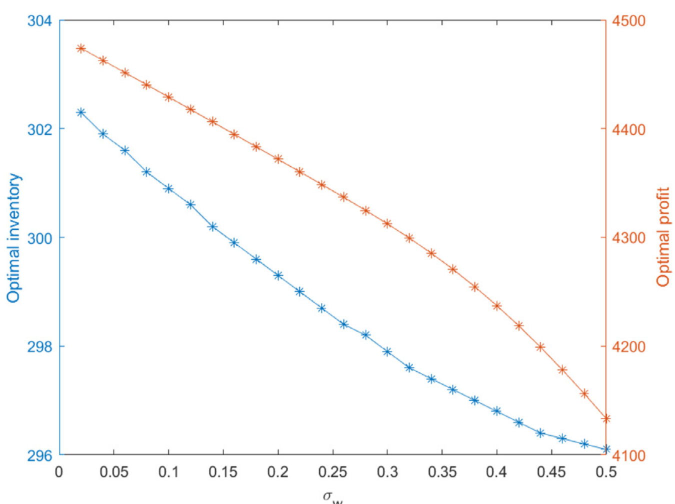

Fig. 7. Impact of barter uncertainty on the optimal inventory and profit.

Different from the effect of demand uncertainty, Fig. 7 shows that with increasing barter supply uncertainty, both the retailer's order quantity and profit decrease. Compared with the classical newsvendor, barter can still improve the retailer's profit with higher order quantity. However, as shown in Fig. 8, increment rates of inventory and profit are decreasing, which indicates that barter uncertainty will dampen the profitability of barter. Since with barter uncertainty, the successful barter quantity can be lower or higher than the barter demand of the retailer. Besides, the barter demand is also random due to the random unsold products. The barter demand can be fully satisfied when the barter supply on the platform is relatively higher than the retailer's barter demand. If the supply is relatively lower, the barter demand can only be partially satisfied. Then when the barter supply quantity is uncertain, the retailer may order excess inventory which cannot be bartered for what he wants on the platform. Consequently, the unreliable barter will increase inventory cost and make barter less attractive. And the retailer should decrease inventory when barter uncertainty becomes larger.

7. Conclusions

With the development of barter exchange platforms, more and more firms move their distressed inventory using barter. But some managerial issues are yet to be addressed. Such as what is the optimal inventory policy for the retailer? And what is the impact of barter on the retailer's inventory decisions? We apply the newsvendor model incorporated with barter exchange to address these issues.

We present a newsvendor model with barter exchange and compare its behaviour with that of the classical newsvendor model to generate some managerial insights. We find that barter exchange can effectively induce the retailer to order more product and is beneficial to it. The optimal order quantity decreases with barter commission and barter uncertainty, while increases with demand uncertainty and the value of the product it needs. Barter is more advantageous with lower commission, larger demand uncertainty, lower barter uncertainty, and higher value of the product it needs. The managerial implications of our work are 1) Our work provides guidance for the retailer on how to make inventory decision when using the barter platform. With higher demand uncertainty, the retailer should increase order quantity, while with higher barter uncertainty the retailer should order less products. 2) Barter can be used by the retailer to cope with demand uncertainty. Especially when the value of the product he needs is larger, and barter uncertainty is lower. Then in practice, the retailer should consider the joint purchasing decisions of multiple products to take full advantage of barter exchange. For example, purchasing one product A with higher demand uncertainty and another product B that can be efficiently bartered with unsold product A will be more profitable. 3) For the barter platform, decreasing the barter uncertainty is significant to attract more barter platform users.

We consider that the products on the platform can be partially bartered, we believe that barter in batch will decrease barter efficiency and undermine the profitability of barter. Since the retailer may trade more unwanted products which increases inventory cost. There are several topics for further research. In this paper we consider a single-period inventory problem. A natural extension is to study the multi-period inventory problem with barter. The quantity of the product that the retailer needs Q_0 is certain in our model. Further research can consider the effect of demand uncertainty of multiple products in newsvendor model with barter exchange. Endogenizing the barter pricing scheme in the newsvendor model with barter choice is also interesting.

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Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.omega.2019.102149.

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