EE1-Lab 3 Report

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1 Objectives

This experiment aims to deepen our understanding of AC circuit behavior by: **

- Quantitatively determining the impedance and reactance of capacitors and inductors. This will involve precise measurements and analysis of AC circuit parameters.
- Gaining practical experience with complex number calculations. We will apply these mathematical tools to analyze AC circuits, including phasor diagrams, and understand the implications of complex impedance.
- Exploring the fundamental concepts of reactance, impedance, and phase relationships. We will investigate how these concepts influence current flow and voltage behavior in AC circuits.
- Investigating the unique characteristics of capacitive and inductive reactance. We will observe how these components affect voltage and current in AC circuits at different frequencies.
- Building a deeper understanding of AC circuit behavior. By observing and analyzing experimental data, we will gain insights into the interplay between components and their impact on overall circuit performance."

2 Materials:

- Capacitor (Unknown Capacitance): Used to measure capacitive reactance.
- Inductor (900 turns): Measured with and without an iron core for inductive reactance and impedance.
- Resistors (2Ω): For series connections with capacitors and inductors.
- Capacitance Decade: Adjustable capacitor for the reactive power compensation task.
- Iron Core: Used to modify the inductance in the coil.

3 Equipment:

- Function generator: Generate sinusoidal AC signals at frequencies of 250 Hz, 500 Hz, and 1000 Hz.
- Oscilloscope: Measures and displays voltage waveforms and determines phase relationships between signals.
- Digital Multimeter (TRMS Metrahit TECH): Measures the current flowing through capacitors and inductors.
- LRC Measurement Bridge: Measures capacitance and inductance values accurately.
- Differential Probe: Measures phase angles, particularly useful for tasks involving reactive power compensation.
- 1 k(Ω) Resistor: Used in optional phase-angle measurement experiments.

4 Theory

4.1 Capacitive Reactance (X_C)

Capacitive reactance is a property of a capacitor that describes how it resists the flow of alternating electrical current. Essentially, when an AC voltage is applied to a capacitor, it generates an opposing force that reduces the amount of current that can flow through it. This opposition is known as capacitive reactance, and it is measured in units of ohms (Ω).

The value of capacitive reactance depends on the frequency of the AC signal and the capacitance of the capacitor. A higher frequency or capacitance results in a higher capacitive reactance, while a lower frequency or capacitance results in a lower capacitive reactance.:

$$X_C = \frac{1}{\omega C},$$

where:

- X_C is the capacitive reactance in ohms (Ω),
- $\omega = 2\pi f$ is the angular frequency in radians per second (rad/s),
- f is the frequency of the AC source in hertz (Hz),
- C is the capacitance in farads (F).

This formula shows that the capacitive reactance decreases as the frequency of the applied signal increases. For ideal capacitors, the phase difference between voltage and current is $-90°$, meaning the current leads the voltage by $90°$.

4.2 Inductive Reactance (X_L) and Impedance (Z_L)

For an inductor, the impedance Z_L is the total opposition to current flow, including both the inductive reactance and any resistance inherent in the coil. For a real inductor, the impedance is given by:

$$Z_L = \frac{U_L}{I_L},$$

where:

- Z_L is the impedance of the inductor in ohms (Ω),
- U_L is the voltage across the inductor in volts (V),
- I_L is the current through the inductor in amperes (A).

The inductive reactance X_L is the opposition to current caused purely by the inductance of the coil and is given by:

$$X_L = 2\pi f L,$$

where:

- X_L is the inductive reactance in ohms (Ω),
- L is the inductance in henries (H).

For a coil with both resistance R_L and inductance L, the total impedance is:

$$Z_L = \sqrt{R_L^2 + X_L^2}.$$

The impedance Z_L is always a complex quantity, as it accounts for both resistive and reactive components of the coil. The phase difference between the voltage and current in an inductive circuit is $+90°$, meaning the current lags the voltage by $90°$.

4.3 Resonance Condition for Reactive Power Compensation

In the reactive power compensation section, we aim to balance the reactance of a capacitor and an inductor. At resonance, the reactance of the capacitor and inductor cancel each other out. The condition for resonance is given by:

$$\frac{1}{\omega C} = \omega L,$$

where L and C are the inductance and capacitance, respectively. At this point, the total reactance of the circuit becomes zero, and the current reaches its maximum value. This is the condition for maximum power transfer in circuits involving both inductive and capacitive elements.

5 Experiment

5.1 Measuring the Reactance of a Capacitor

Circuit Setup: A sinusoidal signal of varying frequencies (250 Hz, 500 Hz, and 1000 Hz) is applied to a capacitor with a known voltage (3V).

Measurements: Using a TRMS multimeter, the current through the capacitor is measured, and an oscilloscope is used to measure the voltage across the capacitor.

Frequency f [Hz]	Voltage U_C RMS [V]	Current I_C [mA]	Reactance X_C [Ω]	Capacitance C [μF]
250	3.01	15.90	189.24	1.68
500	3.01	15.24	197.53	1.6
1000	3.01	13.25	227.17	1.4

Table 1: Measurements

Calculations: The reactance X_C and the capacitance C are calculated using the formulas:

$$X_C = \frac{U_C}{I_C}, \quad C = \frac{1}{\omega X_C}.$$

Result: At different frequencies, the reactance of the capacitor is calculated and plotted. The expected trend is that as the frequency increases, the capacitive reactance decreases, in line with the inverse relationship between X_C and f. The capacitance measured with the LRC bridge should closely match the calculated values.

5.2 Measuring the Impedance of an Air Coil

Circuit Setup: The coil (without iron core) is connected in series with a resistor. The voltage and current are measured at three frequencies (250 Hz, 500 Hz, and 1000 Hz).

Frequency f [Hz]	Voltage U_2 [V]	Voltage Across Coil U_L [mV]	Current I_L [mA]
250	3.00	840	1.149
500	3.00	1160	0.609
1000	3.00	1790	0.329

Table 2: Air Coil Voltage and Current Measurements

Impedance Z_L [Ω]	Inductive Reactance X_L [Ω]	Inductance L [mH]
730.98	730.98	464.7
1904.6	1904.6	605.5
135.19	135.19	21.5

Table 3: Air Coil Impedance and Reactance Measurements

Measurements:

6 Measuring the Impedance of a Coil with an Iron Core

Circuit Setup: The coil with an iron core is tested at a frequency of 1 kHz, and the voltage and current are measured.

Estimation: The relative permeability of the core is estimated, assuming the coil to be a long cylindrical coil.

Result: Inserting the iron core into the coil increased the inductance and impedance, as the iron core enhances the magnetic field and thus the inductive reactance. The change in reactance is directly linked to the core's permeability.

The following are the Voltage and the current measurements obtained from the oscilloscope and the Metra TRMS multimeter respectively.

Frequency	f [Hz]: 1000
Voltage across the Coil	U_L [V]: 2.98
Current through the coil	I_L [mA]: 0.381
Impedance Z_L [Ω]	7821.522
Inductive Reactance X_L [Ω]	7821.506
Relative Permeability (μ_r)	168.513
Inductance L [mH]	1244.831
Resistance R_L [Ω]	16.090

Result: The impedance and reactance of the coil were measured at various frequencies. As expected, the reactance increases with frequency, consistent with the formula $X_L = 2\pi f L$. The inductance calculated from the impedance values was found to match the measurements from the LRC bridge.

The value of inductance is determined using the resonance condition:

$$\frac{1}{\omega C} = \omega L$$

where:

- $\omega = 2\pi f$ is the angular frequency, in radians per second,
- C is the capacitance, in farads (F),
- L is the inductance, in henries (H).

Rearranging for L, the inductance is given by:

$$L = \frac{1}{\omega^2 C}$$

Using this condition, the inductance of the coil is calculated and found to match the expected theoretical values.

Capacitance (C) [µF]	**Current (I) [mA]**
0.0	4.020
0.1	5.465
0.2	14.03
0.3	21.71
0.4	28.25
0.5	33.51
0.6	37.69
0.7	40.98
0.8	43.56
0.9	45.60
1.0	47.53

Table 4: Relationship between Capacitance (C) and Current (I)

7 Conclusion

The experiment successfully demonstrated the principles of reactance and impedance in capacitive and inductive components. The measured values for reactance and impedance were consistent with theoretical predictions, and the relationship between frequency, capacitance, and inductance was verified. The iron core significantly altered the coil's characteristics, and reactive power compensation was achieved at resonance, confirming the expected behavior of LC circuits.