Magnetic Levitation System

Modern Control Systems

Phase 2

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Contents

1. Introduction

Magnetic levitation system is known as electro-mechanical systems in which an object is levitate in specific region without using any support. Now a days this technology is spreading over a wide range because of its contact-less and friction-less properties as it removes energy losses which occur due to friction. Since this technology was developed, researchers have analyzed a large number of such systems. In the analysis of such systems most of the work has been done on designing of controller as they require appropriate control action and are more complex, such as State space controller, feedback linearization is used to control the position of ball. However, there are mostly highly non-linear systems and open loop unstable. This nonlinearity and unstability feature of magnetic levitation systems makes their control and modeling very challenging.

The problem of controlling the magnetic field by control method is taken up to levitate a metal hollow sphere, here. The control problem is to supply controlled position of the ball such that the magnetic force on the levitated body and gravitational force acting on it are exactly equal. Thus the magnetic levitation system is inherently unstable without any control action.

The objective of the work can be directed to design controller considering the linear model as well as nonlinear model. The controller designed is validated by simulating and implementing on real time system.

2. State-Feedback

"Intuitively, the state may be regarded as a kind of information storage or memory or accumulation of past causes. We must, of course, demand that the set of internal states Σ be sufficiently rich to carry all information about the past history of Σ to predict the effect of the past upon the future. We do not insist, however, that the state is the least such information although this is often a convenient assumption."

R. E. Kalman, P. L. Falb and M. A. Arbib, 1969

State feedback, or pole placement, is a method employed in feedback control system theory to place the closed-loop poles of a plant in pre-determined locations in the s-plane. Placing poles is desirable because the location of the poles corresponds directly to the eigenvalues of the system, which control the characteristics of the response of the system. The system must be considered controllable in order to implement this method. We have checked the controllability of the system in the first phase of this project. Therefore this method can be implemented on this system. The block diagram of state-feedback in a system is shown in Fig. 1.

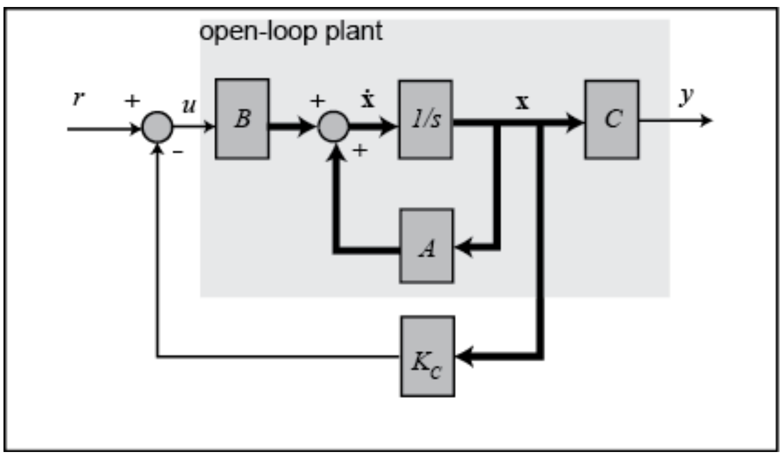

Figure. 1. State Feedback Block Diagram

2.1. Stability Analysis

First, system's poles should be separated by their speed. A pole that is very close to the imaginary axis will take a long time to settle; one that is very far from the imaginary axis will settle very quickly. The magnetic system is described by the following state- variable models:

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 245 & -0.0814 & 24.27 \\ 0 & 0 & -250 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} u$$

The poles of the system are equal to the eigenvalues of matrix "A," which are -250, -15.7, and 15.61. Therefore the system is inherently unstable based on the root locus shown in Fig. 2.

To observe what happens to this unstable system when there is a non-zero initial condition, we use the code in (Appendix B.1); the Fig. 3 shows the ball's behavior.

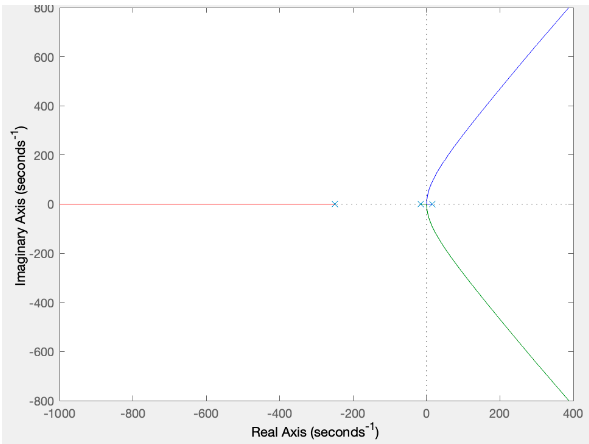

Figure. 2. Root Locus of System

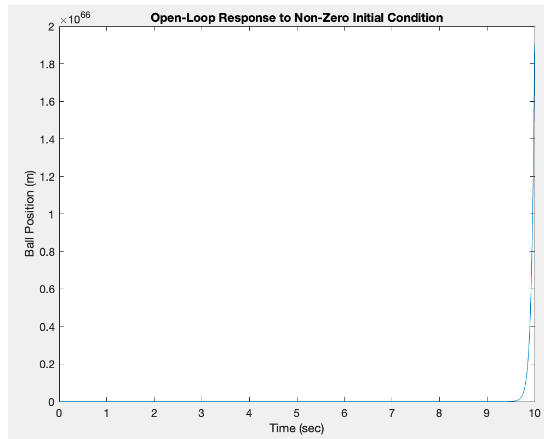

Figure. 3. Open Loop Response to Non-zero Initial Condition

2.2. Control Design using Pole Placement

For simplicity, we assume the reference is zero, $r = 0$. The input is then $u = -Kx + r$. The state-space equations for the closed-loop feedback system are, therefore:

$$\dot{x} = Ax + B(-Kx + r) = (A - BK)x + Br,$$
$$y = Cx$$

$a(s) = (s + 250)(s + 15.7)(s - 15.61) = -61269.2 - 222.577s + 250.09s^2 + s^3$
$\Rightarrow a = [250.09 \quad -222.577 \quad -61269.2]$

2.2.1. Slow Poles

The slow chosen poles are: -3, -4, -5
Therefore the characteristic equation is: $\alpha_s(s) = (s + 3)(s + 4)(s + 5)$
We use the "place" function in MATLAB to obtain the gain (K_s):

```
I=eye(3,3);


c=[B A*B A*A*B];
k_slow=[0 0 1]*c^-1*((A+5*I)*(A+4*I)*(A+4*I))
k_fast=[0 0 1]*c^-1*((A+30*I)*(A+35*I)*(A+45*I))
```

By running this code in the Fig. 4 shows the step response after the designed state feedback is added to the closed loop system.

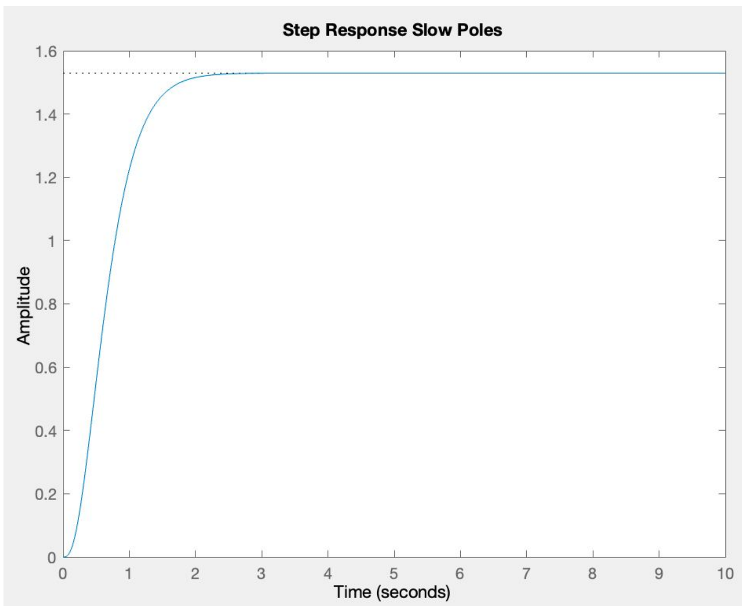

Figure. 4

2.2.1. Fast Poles

The slow chosen poles are: -30, -35, -45

Therefore the characteristic equation is: $\alpha_f(s) = (s+30)(s+35)(s+45)$

We use the "place" function in MATLAB to obtain the gain (K_f):

```
    %Fast Poles
p11 = -30;
p22 = -35;
p33 = -45;

        %Gain
K_f = place(A,B,[p11 p22 p33])

        %step
step(ss(A-B*K_f,B,C,D),t)
title('Step Response Fast Poles')
```

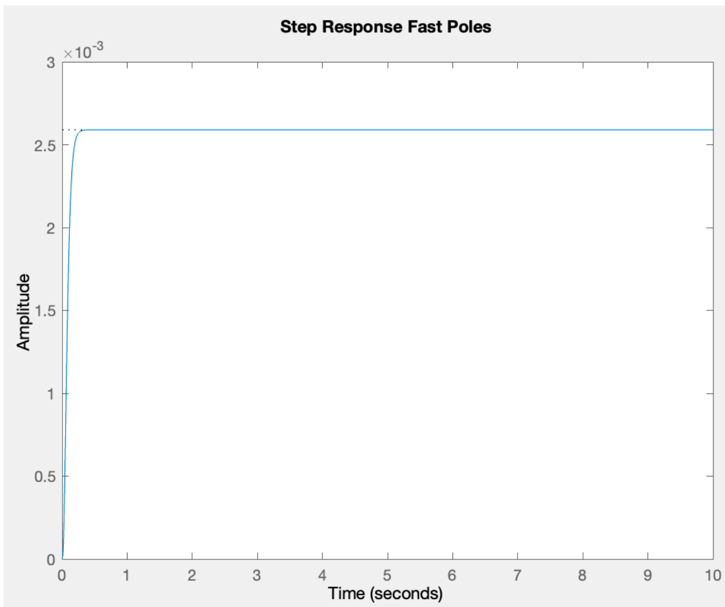

Figure. 5

2.3. Add Gain

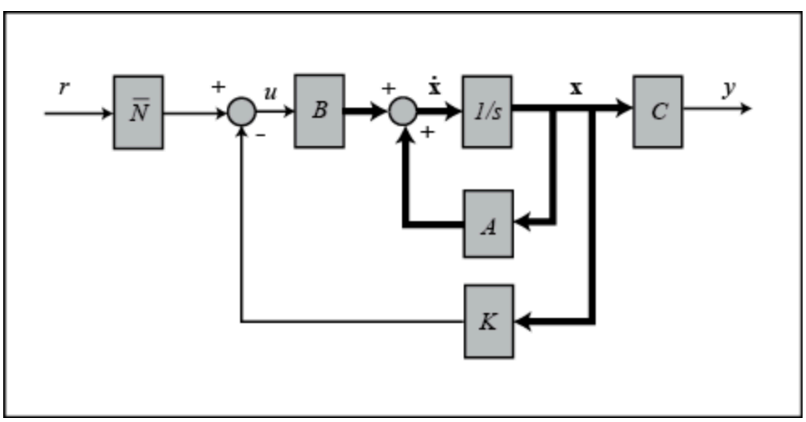

Figure. 6. State-Feedback with Gain for Reference Signal

As can be seen in the diagrams below, the controlled systems do not follow the input (the DC gain of the controlled system is not one) in order for the system to follow the input and the step response converges to one, a gain "*P*" is putted in the input so that the step response of both systems is equal to one and we can compare them better. We'll design a system in Simulink (Fig. 7.)named "State-Feedback" which is shown in Fig. 8. (The blue line is for the fast poles and the yellow one is for the slow poles.)

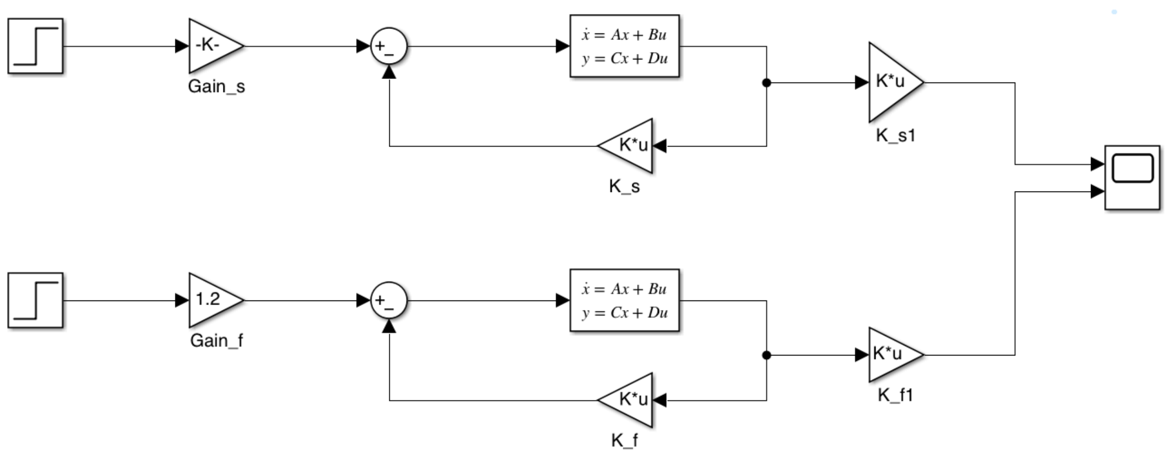

Figure 7

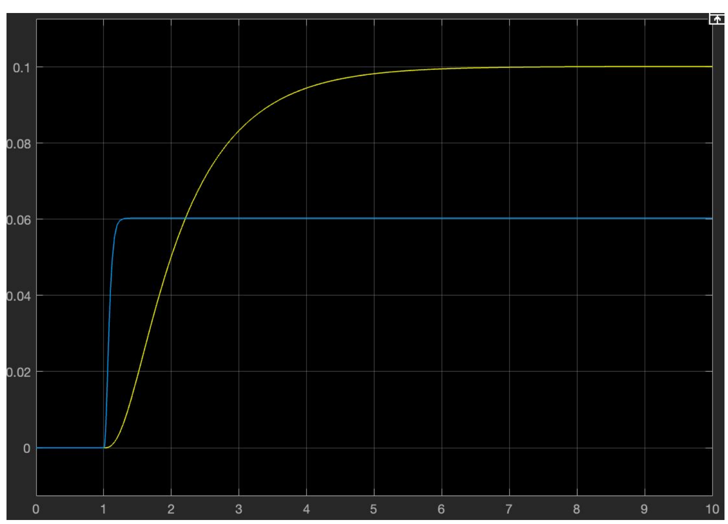

Figure. 8.

As can be seen, the system for which the faster poles are considered has a significant overshoot. If the system does not overshoot with slower poles, it converges slowly to the final value. Therefore, the effort we paid for increasing the speed is actually increasing the system overshoot.

3. State-Feedback with Additive Disturbance

Disturbance signals represent unwanted inputs which affect the control-system's output, and result in an increase of the system error. Assume *w* is the disturbance parameter, the state equations are:

$$\dot{x} = Ax + Bu + D$$

The schematic of the system is shown in Fig. 9.

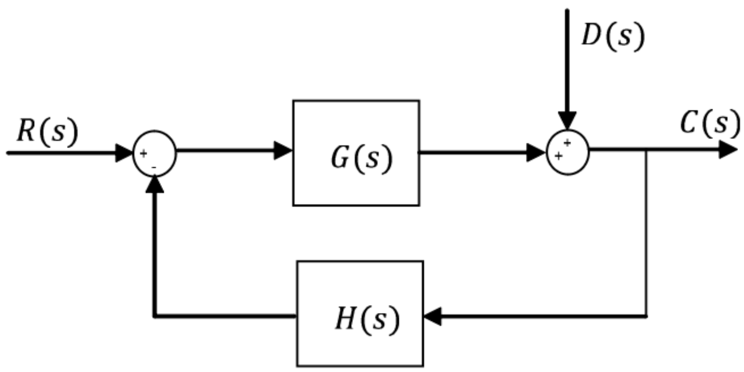

Figure. 9. Generalized Feedback Control System with Additive Disturbance

We'll implement this controller using the "Band-Limited White Nosie" block in Simulink as Fig. 10.

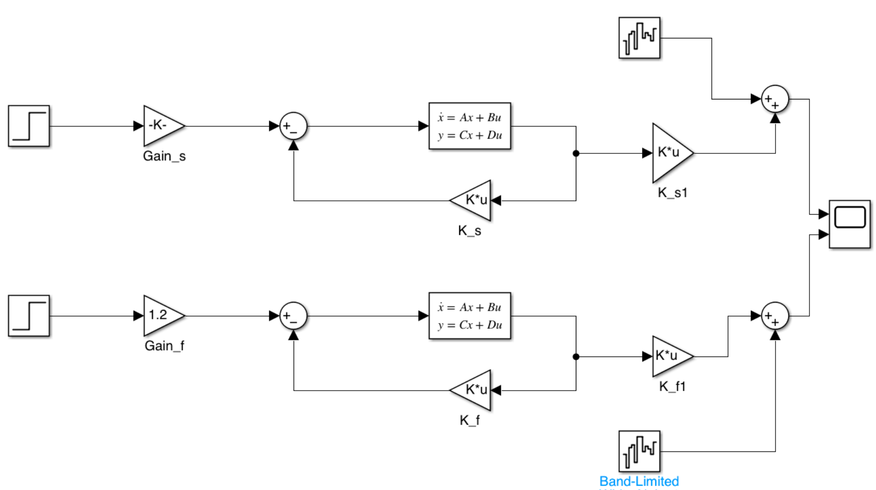

Figure. 10

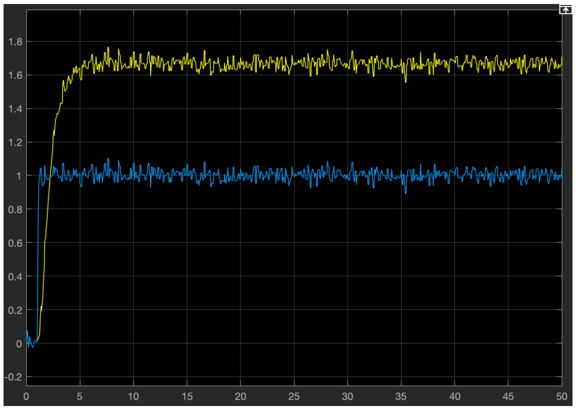

Figure. 11: State-Feedback +Noise

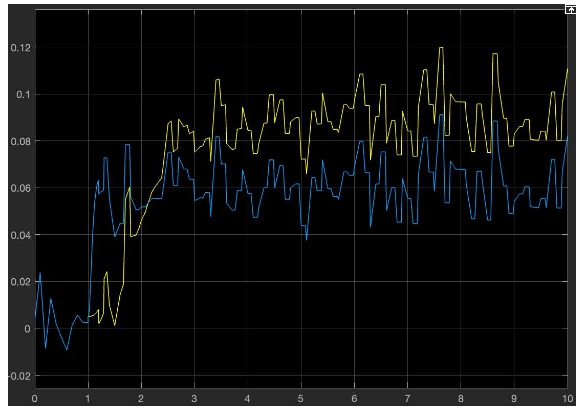

Figure. 12: State-Feedback + Gain + Noise

As we can see, the system is not robust to noise (due to the large coefficients in the conversion function). But relatively, in the case where the poles are slower, the system has less fluctuations compared to the mode and has better relative stability. In both systems, after removing the noise, the system reaches a steady state and becomes stable.

4. Tracker using State-Feedback with Integral Action

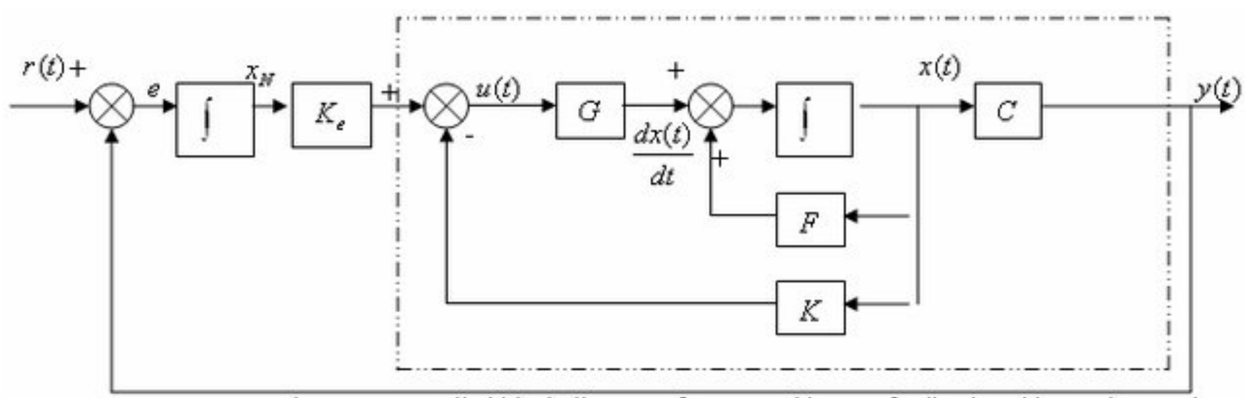

Figure. 13.

We have:

$$u = -Kx - K_e q = -\begin{bmatrix} K & K_e \end{bmatrix} \begin{bmatrix} x \\ q \end{bmatrix}$$

$$\dot{q}(t) = r - y(t) = r - Cx(t)$$

The purpose is to have $\dot{q} \to 0$ when $t \to \infty$. Therefore we have:

$$\dot{x} = Ax + Bu$$
$$\dot{q} = -Cx + r$$

The state-space is:

$$\begin{bmatrix} \dot{x} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x \\ q \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ I \end{bmatrix} r$$
$$y = [C \quad 0] \begin{bmatrix} x \\ q \end{bmatrix}$$

4.1. Controllability

In order to use this method, the matrices $\bar{A}$ and $\bar{B}$ should be controllable.

$$\bar{A} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 245 & -0.0814 & 24.47 & 0 \\ 0 & 0 & -250 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}, \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 5 \\ 0 \end{bmatrix}$$
$$\Rightarrow \bar{\Phi}_c = \begin{bmatrix} \bar{B} & \bar{A}\bar{B} & \bar{A}^2\bar{B} & \bar{A}^3\bar{B} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 122.35 & -30597.459 \\ 0 & 122.35 & -30597.459 & 7679341.383 \\ 5 & -1250 & 312500 & -78125000 \\ 0 & 0 & 0 & -122.35 \end{bmatrix}$$

The controllability matrix's rank is 4. Therefore it's full rank and controllable.

4.2. Obtain K and K_e

From Part 4.1. we obtain $a(s)$ based on $\bar{A}$:
$a(s) = \det(sI - \bar{A}) = s^4 + 250.08s^3 - 224.65s^2 - 61250s$
$\rightarrow [250.08 \quad -224.65 \quad -61250 \quad 0]$

Assume that we have three groups of desired poles:
1- -3, -4, -5, -7
2- -30, -35, -45, -50

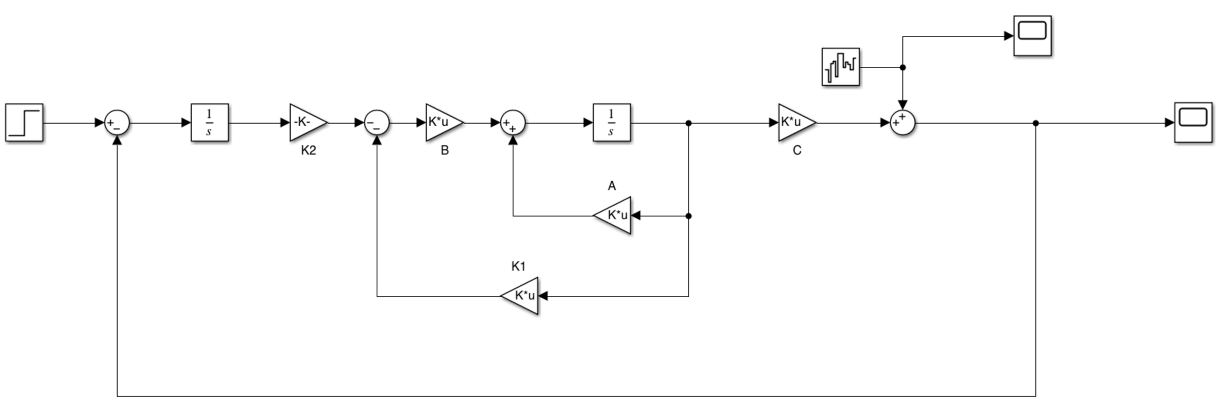

Fig 14

We used the White Noise as is shown in Fig. 14.

The response using slow poles are shown in Fig . 15.

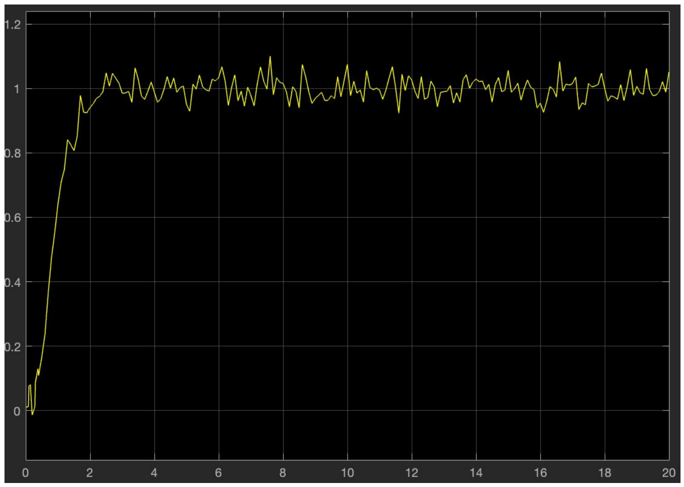

Fig. 15

The response using fast poles are shown in Fig. 16 .

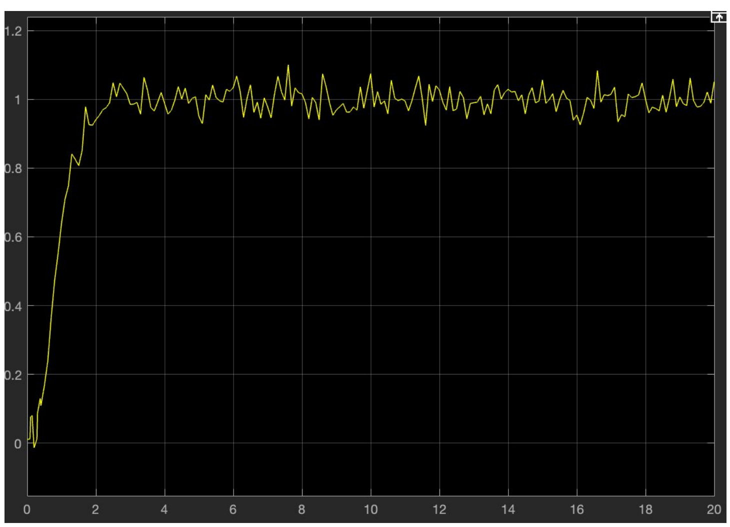

Figure. 16

The results show that the system is robust against disturbance and noise.

5. Observer

When we can't measure all state variables $\mathbf{x}$ (often the case in practice), we can build an **observer** to estimate them, while measuring only the output $y = C\mathbf{x}$. For the magnetic ball example, we will add three new, estimated state variables ($\hat{\mathbf{x}}$) to the system. The schematic is as follows:

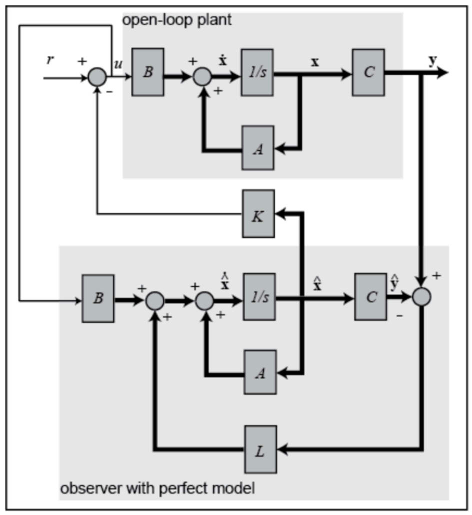

Figure. 15.

We know that system is observable as was mentioned in project's phase 1.

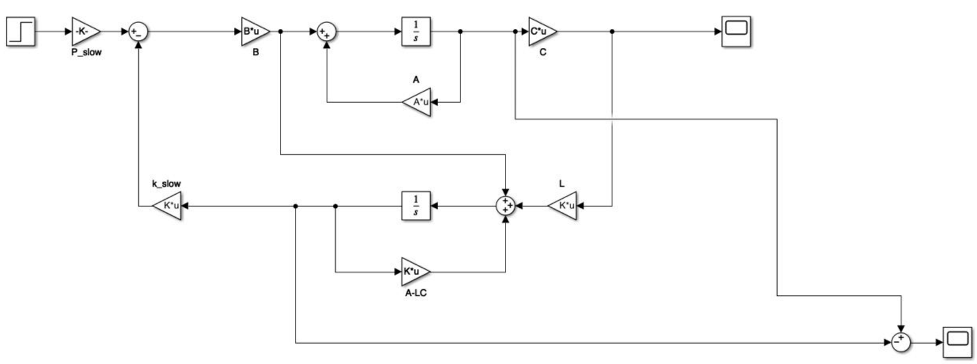

Figure. 16

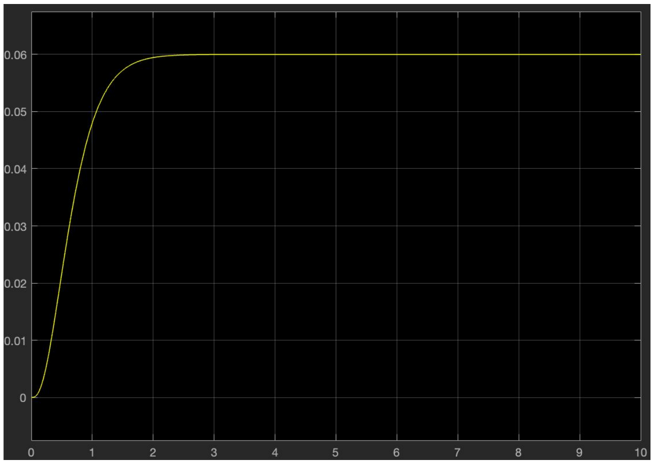

Figure. 17

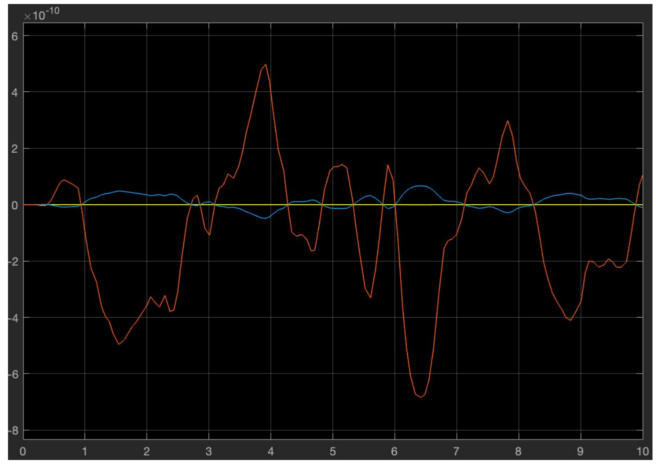

Figure. 18. $x - \hat{x}$

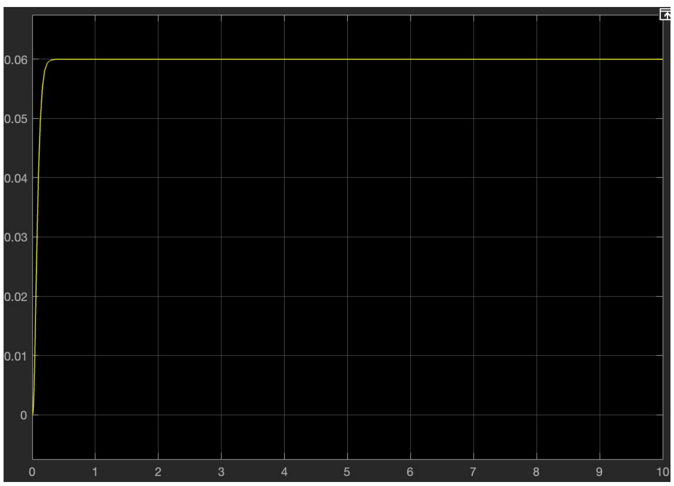

Figure. 19

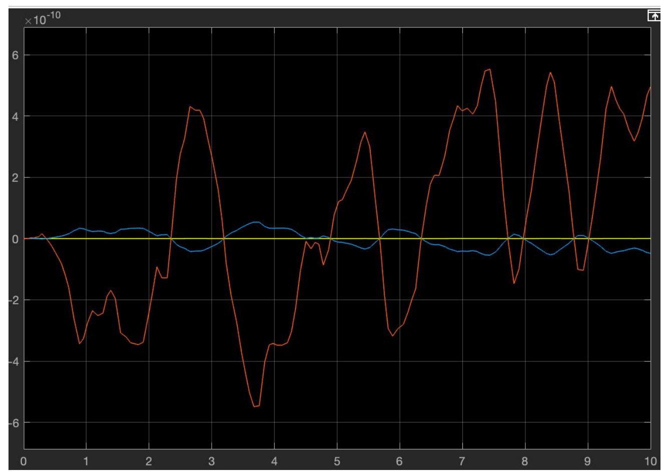

Figure. 20. $x - \hat{x}$

Also, in order to check whether the estimator has worked well or not, we also check the difference between the main states and the estimated states, whose graph is as follows, as it can be seen that they match.

6. Reduced Order Observer

Using the Lyapunov equation, we design a reduced order estimator:

$$F = \begin{bmatrix} -8 & 0 \\ 0 & -10 \end{bmatrix}, L = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$\Phi_{(F,L)} = \begin{bmatrix} 1 & -8 \\ 1 & -10 \end{bmatrix} \to rank = 2$, then it's controllable.

Therefore, the matrix T is obtained by the Lyapunov equation method as follows:

$$TA - FT = LC \to T = \begin{bmatrix} -0.0038 & 0.0041 & 0.0004 \\ -0.0080 & 0.0041 & 0.0004 \end{bmatrix}$$

$$\dot{z} = Fz + TBu + Ly, \hat{x} = \begin{bmatrix} C \\ T \end{bmatrix}^{-1} \begin{bmatrix} y \\ z \end{bmatrix}$$

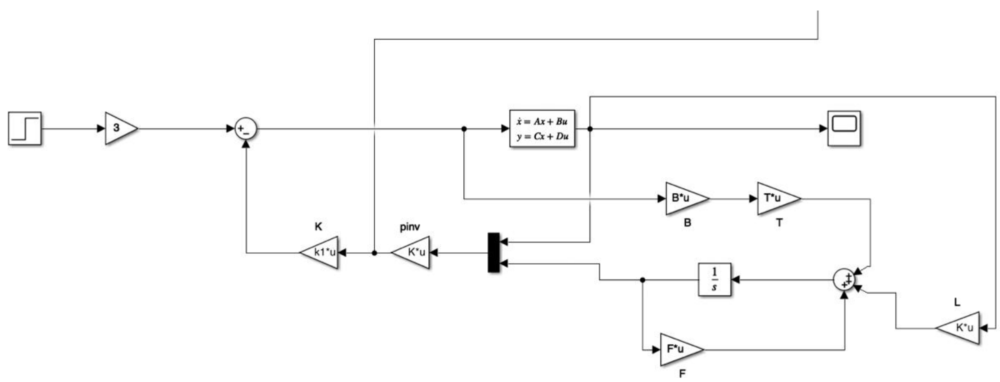

Figure. 21

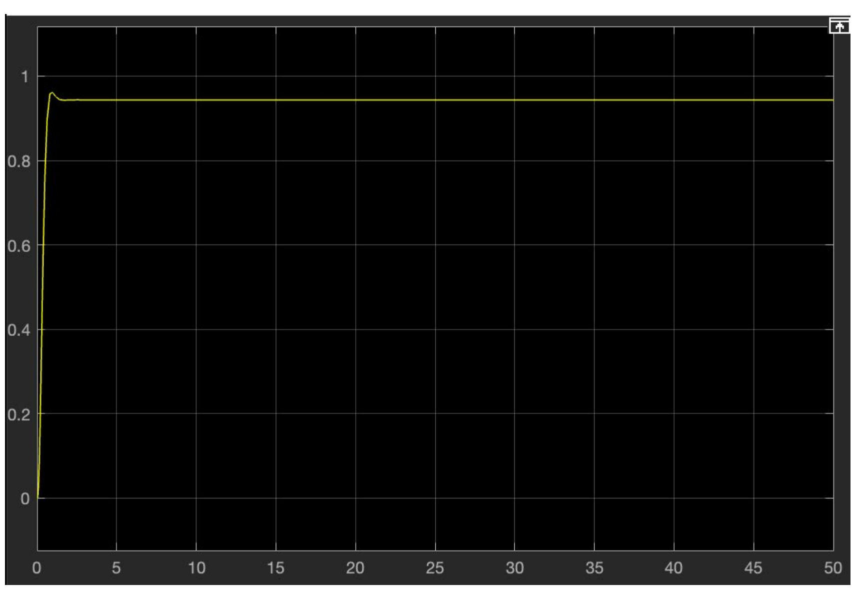

Figure. 22

7. Non-Linear Model Control

A magnetic levitation system is a system that uses magnetic forces to levitate an object. The magnetic levitation system is a non-linear system because the magnetic forces are proportional to the magnetic field intensity and the inverse of the distance between the magnetic source and the levitated object. This results in a non-linear relationship between the input variables and the output variables (position of the levitated object).

We will see the results of applying one of the controllers and an observer to a non-linear system in this section.