Parameter Estimation and Hypothesis Testing.

Question 1:

$\theta_1 = \mu, \ \theta_2 = \sigma^2$

MLE = ?

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$f(x, \theta_1, \theta_2) = \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x-\theta_1)}{2\theta_2}}$$

$$\theta_1 \in (-\infty, \infty), \ \theta_2 \in [0, \infty)$$

$$L(\theta_1, \theta_2) = \prod_{i=1}^{n} f(x_i; \theta_1, \theta_2) = 0$$

$$= \theta_2^{-n/2} (2\pi)^{-n/2} e^{-\frac{1}{2\theta_2} \sum_{i=1}^{n} (x_i - \theta_1)^2}$$

$$\log L(\theta_1, \theta_2) = \frac{-n}{2} \log \theta_2 - \frac{n}{2} \log(2\pi) - \frac{\sum_{i=1}^{n} (x_i - \theta_1)^2}{2\theta_2}$$

Partial Derivation both side W.R.T θ_1

$$\frac{\partial \log L(\theta_1, \theta_2)}{\partial \theta_1} = -\frac{2\Sigma(x_i - \theta_1)(-1)}{2\theta_2}$$

$$\Sigma(x_i - \theta_1) = 0$$

$$\hat{\theta}_1 = \mu = \frac{\Sigma x_i}{n} = \bar{x}$$

$$\frac{\partial \log L(\theta_1, \theta_2)}{\partial \theta_2} = -\frac{n}{2\theta_2} + \frac{\Sigma(x_i - \theta_1)^2}{2\theta_2^2} = 0$$

$$-n\theta_2 + \Sigma(x_i - \theta_1)^2 = 0$$

$$\boxed{\theta_2 \neq 0}$$

$$\hat{\theta}_2 = \hat{\sigma}_2 = \frac{\Sigma(x_i - \bar{x})^2}{n}$$

$$\hat{\mu} = \frac{\Sigma x_i}{n}$$

$$\hat{\sigma}^2 = \frac{\Sigma(x_i - \bar{x})^2}{n}$$

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Question 2:

$$L(\theta | x_1, x_2 \ldots x_n)$$

$$\prod_{i=1}^{n} \binom{m}{x_i} \theta^{x_i} (1-\theta)^{m-x_i}$$

Taking log-likelihood

$$L(\theta | x_1, x_2 \ldots x_n) =$$

$$\sum_{i=1}^{n} \left[\log \binom{m}{x_i} + x_i \log \theta + (m-x_i) \log (1-\theta)\right]$$

$$\frac{\partial L}{\partial \theta} = \frac{\partial}{\partial \theta}\left(\sum_{i=1}^{n} \left[\log \binom{m}{x_i} + x_i \log \theta + (m-x_i) \log (1-\theta)\right]\right)$$

$$\frac{\partial L}{\partial \theta} = \sum_{i=1}^{n} \left[\frac{x_i}{\theta} - \frac{m-x_i}{1-\theta}\right]$$

when $\frac{\partial L}{\partial \theta} = 0$

$$\sum_{i=1}^{n} \left[\frac{x_i}{\theta} - \frac{m-x_i}{1-\theta}\right] = 0$$

$$\sum_{i=1}^{n} \left[\frac{x_i - \theta m}{\theta(1-\theta)}\right] = 0$$

$$\sum_{i=1}^{n}\left[\frac{x_i}{\theta(1-\theta)}\right] - \sum_{i=1}^{n}\left[\frac{\theta m}{\theta(1-\theta)}\right] = 0$$

$$\sum_{i=1}^{n}\left[\frac{x_i}{\theta(1-\theta)}\right] = \frac{mn}{1-\theta}$$

$$\sum_{i=1}^{m} x_i = mn\theta$$

$$\theta = \frac{\sum_{i=1}^{n} x_i}{mn}$$