

Background The modeling and solution process for fuzzy propositions in the context of the Green Vehicle Routing Problem with

User:

Background

The modeling and solution process for fuzzy propositions in the context of the Green Vehicle Routing Problem with Time Windows and Capacity Constraints (GCVRPTW) (focusing on fairness and sustainability for delivery drivers, customers). Consider the sustainability influenced by the route fairness and load fairness of delivery drivers, as well as the service fairness and service satisfaction of customers.

Modeling contexts of fairness and sustainable, consider relationships of customers, drivers, load and routes distance.

For delivery drivers, factors such as differences in working hours, differences in vehicle load levels, and differences in the length of assigned routes may be considered. For customers, factors like differences in service satisfaction and service quality may be taken into account. Additionally, the context of sustainability in enterprise development, which is influenced by both drivers and customers, should be considered.

```
$\mathcal{F}_s := \left\{ \begin{array}{l} p_1: \text{implies} \left( \text{or} \left( D_{\text{loadDiff}} \text{ is Low}, D_{\text{routeLenDiff}} \text{ is Low} \right), F \text{ is High} \right), \\ \quad \text{where } D_{\text{loadDiff}} = (\text{Driver Load Difference}), D_{\text{routeLenDiff}} = (\text{Route Length Difference}), F = (\text{Fairness}) \\ p_2: \text{and} \left( O_{\text{driver}} \text{ is Low}, N_{\text{driver}} \text{ is Medium or Low} \right), \\ \quad \text{where } O_{\text{driver}} = (\text{Driver Overtime Hours}), N_{\text{driver}} = (\text{Driver Count}) \\ p_3: \text{implies} \left( S_{\text{custAvg}} \text{ is High}, Q_{\text{service}} \text{ is High} \right), \\ \quad \text{where } S_{\text{custAvg}} = (\text{Customer Avg Satisfaction}), Q_{\text{service}} = (\text{Service Quality}) \\ p_4: U_{\text{capacity}} \text{ is Medium}(0.5-0.2, 0.5+0.2), \\ \quad \text{where } U_{\text{capacity}} = (\text{Capacity Utilization}) \end{array} \right.
```

Original Matlab code for **CalObj2**:

```
function [PopObj, PopCon, routesList, C1, C2, C3, C4, ALPHA] = CalObj2(obj, PopDec)
```

```

..... % CalObj2 - Calculate bi-criteria objectives, constraint violations, and route lists

..... % Input Parameters:

..... % ... obj - Problem object containing distance matrix, demands, time windows, etc.

..... % ... PopDec - Population decision matrix (each row is a chromosome encoding)

..... % Output Parameters:

..... % ... PopObj - Bi-criteria objective matrix [total cost, -total customer satisfaction]

..... % ... PopCon - Constraint violation value (penalty function sum)

..... % ... routesList - Cell array of routes (customer nodes per route)

..... % Initialize output parameters

N = size(PopDec, 1); % Population size

PopObj = zeros(N, 2); % Dual objectives: [total cost, -total customer sa
tisfaction]

PopCon = zeros(N, 1); % Constraint violation values

routesList = cell(N, 1); % List of routes (only non-empty routes)

C1 = cell(N, 1); % Economic cost (€)

C2 = cell(N, 1); % Environmental cost (€)

C3 = cell(N, 1); % Social cost (accident risk, €)

C4 = cell(N, 1); % Total customer satisfaction

ALPHA = cell(N,1); % Truth values for fuzzy proposition constraints
(reserved)

..... % Core parameters (revised fuel efficiency definition: 0.052 L/KM)

depotIdxs = obj.depotIdxs; % Depot node indices

maxVehicles = obj.car; % Maximum number of vehicles

Q = obj.capacity; % Vehicle capacity constraint

```

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FC = 67.62; ..... % Fixed vehicle cost (€/trip)

DW = 9.91 / 60; ..... % Driver cost (€/minute, converted from 9.91€/hour)

CF = 1.58; ..... % Fuel price (€/l)

fuelEfficiency = 0.052; ..... % Fuel efficiency (L/KM, 0.052 liters consumed per kilometer)

Ce = 0.02; ..... % Carbon emission cost (€/kg CO2)

gamma = 0.75; ..... % Carbon emission factor (kg CO2/l)

a = 0.0005; ..... % Accident risk coefficient (€/kg·km)

% Penalty factors

capPenalty = 10000; ..... % Penalty for capacity constraints

timePenalty = 10000; ..... % Penalty for time window constraints

% Iterate through each solution in the population

for i = 1:N

    % 1. Parse chromosome and extract non-empty routes

    chrom = PopDec(i, PopDec(i, :) ~= -1); ..... % Remove placeholder -1

    if isempty(chrom), chrom = depotIdxs(1); end

    % Ensure routes start from depot (assume vehicles depart from depot)

    if ~ismember(chrom(1), depotIdxs)

        firstDepot = find(ismember(chrom, depotIdxs), 1);

        chrom = [chrom(firstDepot), chrom(1:firstDepot-1), chrom(firstDepot+1:end)];

    end

```

```

..... % Split routes and filter empty ones (only keep routes with customers)

..... depotPos = find(ismember(chrom, depotIdxs));

..... depotPos = [depotPos, length(chrom)+1];

..... validRoutes = {};

..... for r = 1:length(depotPos)-1

.....     startIdx = depotPos(r);

.....     endIdx = depotPos(r+1)-1;

.....     routeCustomers = chrom(startIdx+1:endIdx);

.....     routeCustomers = routeCustomers(~ismember(routeCustomers, depotIdxs));

.....     if ~isempty(routeCustomers)

.....         validRoutes{end+1} = routeCustomers;

.....     end

..... end

..... routesList{i} = validRoutes;

..... numValidRoutes = length(validRoutes);

..... % 2. Initialize cost and constraint variables

..... totalEconomic = 0; ... % Economic cost

..... totalEnvironmental = 0; .. % Environmental cost

..... totalSocial = 0; ..... % Social cost

..... totalSatisfaction = 0; % Customer satisfaction

..... capViolation = 0; ..... % Capacity constraint violation

..... timeViolation = 0; .... % Time window constraint violation

..... served = zeros(1, length(obj.demand)); .. % Mark served customers

..... current_alpha = 0; .... % Current Global Truth value for fuzzy propositions (reserve
d)

```

```

% 3. Calculate costs and constraints for non-empty routes

for r = 1:numValidRoutes

    route = validRoutes{r};

    routeWithDepot = [0, route, 0]; % Start and end at depot (closed route)

    routeLen = length(routeWithDepot);

    % Total demand of the route (initial load)

    demandIdxs = route + 1;

    totalDemand = sum(obj.demand(demandIdxs));

    currLoad = totalDemand; % Delivery scenario: load decreases after delivery

    % Check capacity constraint

    if totalDemand > Q

        capViolation = capViolation + (totalDemand - Q);

    end

    % Initialize route variables

    distTotal = 0; % Total distance (km)

    fuelTotal = 0; % Total fuel consumption (l)

    timeTotal = 0; % Total driving time (minutes)

    arrivalTime = 0; % Current arrival time (minutes)

    % Iterate through each node pair in the route (i→j)

    for k = 1:routeLen-1

```

```

..... from = routeWithDepot(k);

..... to = routeWithDepot(k+1);

..... dist = obj.C(from+1, to+1); % Distance (km)

..... % calculate driving time (hours)
..... v = 60; % km/h average speed (km/h)

..... % Calculate driving time (minutes): (distance/speed)×60 (convert hours to minutes)
..... travelTimeMin = (dist / v) * 60; % minutes

..... timeTotal = timeTotal + travelTimeMin;

..... arrivalTime = arrivalTime + travelTimeMin;

..... % calculate fuel consumption (core: based on 0.052 L/KM)
..... fuel = fuelEfficiency * dist; % Fuel (l) = consumption per km × distance

..... fuelTotal = fuelTotal + fuel;

..... distTotal = distTotal + dist;

..... % Carbon emission cost (environmental cost)
..... emissionCost = Ce * fuel * gamma;

..... totalEnvironmental = totalEnvironmental + emissionCost;

..... % Accident risk cost (social cost)
..... if to > 0

..... accidentCost = a * dist * currLoad;

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totalSocial = totalSocial + accidentCost;

end

% Process customer node service

if to > 0 && ~served(to)

    served(to) = 1;

    % Check time window constraints (unit: minutes)

    custIdx = to + 1;

    ready = obj.readyTime(custIdx); % Earliest customer time (minutes)

    due = obj.dueTime(custIdx); % Latest customer time (minutes)

    % Handle waiting/lateness

    if arrivalTime < ready

        arrivalTime = ready; % Wait until earliest time

    elseif arrivalTime > due

        timeViolation = timeViolation + (arrivalTime - due); % Lateness time

    end

    % Calculate customer satisfaction (based on time window deviation)

    prefTime = (ready + due) / 2; % Preferred time (minutes)

    timeDiff = abs(arrivalTime - prefTime); % minutes

    windowHeight = due - ready; % minutes

    if windowHeight > 0

        satisfaction = 100 * max(0, 1 - 2*timeDiff/windowHeight);

    end

```

```

        . . .
    else
        . . .
        satisfaction = 0;
    end

    totalSatisfaction = totalSatisfaction + satisfaction;

    . . .

    % Reduce load after delivery (only for delivery scenarios)

    currLoad = currLoad - obj.demand(custIdx);

    . . .

    % Add service time (minutes, directly accumulated)

    arrivalTime = arrivalTime + obj.serviceTime(custIdx);

    . . .
    end

    . . .
    end

    . . .

    % Calculate economic cost

    vehicleCost = FC; % Fixed cost counted once for non-empty routes

    driverCost = DW * timeTotal; % Driver cost (€/minute × minutes)

    fuelCost = Cf * fuelTotal; % Fuel cost (€)

    economicCost = vehicleCost + driverCost + fuelCost;

    totalEconomic = totalEconomic + economicCost;

    . . .
    end

    . . .

    % 4. Assign constraint violations and objective functions

    PopCon(i) = capPenalty * capViolation + timePenalty * timeViolation;

    PopObj(i, :) = [totalEconomic + totalEnvironmental + totalSocial, -totalSatisfaction];

```

```

..... % Store costs for each dimension

..... C1{i} = totalEconomic;

..... C2{i} = totalEnvironmental;

..... C3{i} = totalSocial;

..... C4{i} = totalSatisfaction;

..... ALPHA{i} = current_alpha;

..... end

..... end

```

1. Modeling Foundations for Fuzzy Propositions

- **Select the universe of discourse for the variables of the propositions to ensure Problem could be solved.
- **Variable Standardization:** Variables are linearly mapped to the [0,1] range.
- **Linguistic Term Definition:** Fuzzy terms such as "High," "Medium," and "Low" are used to describe variables.
- **Logical Connectives:** Include operations like AND, OR, implies, NOT, with connective families such as Łukasiewicz, Zadeh, Gödel, and Product.
- **Proposition Structures:**
 - Conditional and qualified propositions (e.g., "V is F and (W is G or Y is not H)").
 - Conditional and unqualified propositions (e.g., "If V is F, then W is G").
 - Unconditional and unqualified propositions (e.g., "V is F").

2. Workflow for Fuzzy Proposition Integration

The process starts with a context \mathcal{F}_s , which is defined by a set of fuzzy propositions (F_s). These propositions are integrated using t-norm operators (e.g., Zadeh's min) to form an aggregated context. The truth value of each proposition is computed, and all truth values are further aggregated into an overall truth degree via t-norm. A global threshold $\alpha = 0.6$ (minimum acceptable overall truth for \mathcal{F}_s) is introduced. If the aggregated overall truth degree is less than α , the fuzzy constraint violation is calculated, multiplied by a penalty factor 10^5 , and used to update the total constraint violation (PopCon).

Task Requirements

Objective: Design membership functions based on F_s , and embed them into the `CalObj2` function to generate the final code.

Output (In English):

1. Logical proposition set $F_s = \{p_1 : \dots, p_2 : \dots, p_3 : \dots, p_4 : \dots\}$.
2. Regarding the constraints on the universe of discourse, please reasonably select the universe of discourse for the variables of the membership function.

3. Definitions and descriptions of membership functions for each proposition in \mathcal{F}_s .
4. A complete `CalObj2` function (Matlab code) that integrates the above elements
5. Please use the correct MATLAB syntax (Dont use ternary operator and correctly use if-else-end).
6. If the membership function is implemented in the form of a helper function, it should inherit from 'obj'. If use auxfunctions, please correctly define and when invoking it, use obj.helperFunction (eg. output = Auxfunctions(obj,...)).

Deepseek-R1:

To integrate fuzzy propositions into the `CalObj2` function while considering fairness and sustainability for the Green Vehicle Routing Problem with Time Windows and Capacity Constraints (GCVRPTW), the following steps were taken:

1. Fuzzy Proposition Set \mathcal{F}_s

- **p1:** If driver load difference is low or route length difference is low, then fairness is high.
- **p2:** Driver overtime is low and driver count is medium or low.
- **p3:** If average customer satisfaction is high, then service quality is high.
- **p4:** Capacity utilization is medium (approximately 0.5).

2. Universe of Discourse for Variables

Variables are standardized to [0,1] for fuzzy processing:

- `D_loadDiff_std` = $(\text{max_load} - \text{min_load}) / \text{vehicle capacity}$
- `D_routeLenDiff_std` = $(\text{max_route_length} - \text{min_route_length}) / \text{total_solution_distance}$
- `F` = $\max(1 - \text{D_loadDiff_std}, 1 - \text{D_routeLenDiff_std})$
- `O_driver_std` = $\min(1, \text{max_overtime} / 240)$
- `N_driver_std` = $(\text{number_of_vehicles} - \text{min_vehicles}) / (\text{max_vehicles} - \text{min_vehicles})$
- `S_custAvg_std` = $\text{total_satisfaction} / (\text{total_customers} * 100)$
- `Q_service_std` = $1 - \min(1, \text{total_time_violation} / (\text{total_customers} * 60))$
- `U_capacity` = $\text{total_demand} / (\text{number_of_vehicles} * \text{vehicle_capacity})$

3. Membership Function Definitions

Three nested helper functions define linguistic terms using piecewise linear functions:

- **`mu_Low(x)`:** Decreasing ramp for "Low" terms
 - Returns 1 if $x \leq 0.1$, decreases linearly to 0 at $x = 0.4$
- **`mu_Medium(x)`:** Triangular function centered at 0.5 for "Medium"
 - Peaks at $x = 0.5$, linearly decreases to 0 at $x = 0.2$ and $x = 0.8$
- **`mu_High(x)`:** Increasing ramp for "High" terms
 - Returns 0 if $x \leq 0.6$, increases linearly to 1 at $x = 0.9$

4. Complete MATLAB Function `CalObj2`

The modified function computes fuzzy truth values, aggregates them, and applies penalties if the overall truth degree falls below a threshold of 0.6. Key additions include per-route data collection, fuzzy variable calculation, and membership function evaluations.

```

function [PopObj, PopCon, routesList, C1, C2, C3, C4, ALPHA] = CalObj2(obj, PopDec)
    % (Initialization code remains unchanged)

    % Core parameters (unchanged)
    depotIdxs = obj.depotIdxs;
    maxVehicles = obj.car;
    Q = obj.capacity;
    FC = 67.62;
    DW = 9.91 / 60;
    Cf = 1.58;
    fuelEfficiency = 0.052;
    Ce = 0.02;
    gamma = 0.75;
    a = 0.0005;

    capPenalty = 10000;
    timePenalty = 10000;

    % Define total_customers (outside loop for efficiency)
    total_customers = length(obj.demand) - 1; % Exclude depot

    for i = 1:N
        % (Chromosome parsing and route splitting code remains unchanged)

        % Initialize arrays for per-route data
        if numValidRoutes > 0
            loads = zeros(1, numValidRoutes);
            dists = zeros(1, numValidRoutes);
            work_times = zeros(1, numValidRoutes);
        end

        % Reset cost and violation variables
        totalEconomic = 0;
        totalEnvironmental = 0;
        totalSocial = 0;
        totalSatisfaction = 0;
        capViolation = 0;
        timeViolation = 0;
        served = zeros(1, length(obj.demand));

        for r = 1:numValidRoutes
            % (Route processing code modified to record per-route data)
            % After processing route, store:
            loads(r) = totalDemand;
            dists(r) = distTotal;
            work_times(r) = arrivalTime; % Work time includes depot return
        end

        % --- Fuzzy Variable Calculations ---

```

```

..... if numValidRoutes > 0
.....     % Driver Load Difference (std)
.....     D_loadDiff_std = (max.loads) - min.loads)) / Q;

..... % Route Length Difference (std)
..... total_solution_distance = sum(dists);
..... if total_solution_distance > 0
.....     D_routeLenDiff_std = (max(dists) - min(dists)) / total_solution_distance;
..... else
.....     D_routeLenDiff_std = 0;
..... end

..... % Composite Fairness
..... F = max(1 - D_loadDiff_std, 1 - D_routeLenDiff_std);

..... % Overtime Standardization
..... overtimes = max(0, work_times - 480); % 480min = 8hrs
..... O_driver = max(overtimes);
..... O_driver_std = min(1, O_driver / 240); % 240min = 4hrs max

..... % Driver Count Standardization
..... total_demand = sum.loads;
..... min_vehicles = ceil(total_demand / Q);
..... if maxVehicles > min_vehicles
.....     N_driver_std = (numValidRoutes - min_vehicles) / (maxVehicles - min_vehicles);
..... else
.....     N_driver_std = 0;
..... end

..... % Capacity Utilization
..... U_capacity = total_demand / (numValidRoutes * Q);
..... else
.....     D_loadDiff_std = 0; D_routeLenDiff_std = 0; F = 0;
.....     O_driver_std = 0; N_driver_std = 0; U_capacity = 0;
..... end

..... % Customer Satisfaction (std)
..... if total_customers > 0
.....     S_custAvg_std = totalSatisfaction / (total_customers * 100);
.....     % Service Quality: Based on time violations
.....     max_time_viol = total_customers * 60; % 60min per customer
.....     norm_viol = min(1, timeViolation / max_time_viol);
.....     Q_service_std = 1 - norm_viol;
..... else
.....     S_custAvg_std = 0;
.....     Q_service_std = 0;
..... end

..... % --- Compute Truth Values for Propositions ---
..... % p1: or(Low_load, Low_routeLen) → High_fairness

```

```

..... A = max(mu_Low(D_loadDiff_std), mu_Low(D_routeLenDiff_std));
..... truth_p1 = max(1 - A, mu_High(F)); % Kleene-Dienes implication

..... % p2: and(Low_overtime, or(Medium_driverCount, Low_driverCount))
..... term1 = mu_Low(O_driver_std);
..... term2 = max(mu_Medium(N_driver_std), mu_Low(N_driver_std));
..... truth_p2 = min(term1, term2); % Zadeh AND

..... % p3: High_satisfaction → High_service
..... B = mu_High(S_custAvg_std);
..... truth_p3 = max(1 - B, mu_High(Q_service_std)); % Kleene-Dienes

..... % p4: U_capacity is Medium
..... truth_p4 = mu_Medium(U_capacity);

..... % --- Aggregate Truth Values ---
..... overall_truth = min([truth_p1, truth_p2, truth_p3, truth_p4]); % Zadeh t-norm

..... % --- Fuzzy Constraint Penalty ---
..... if overall_truth < 0.6 % Threshold
.....     fuzzyViolation = (1 - overall_truth) * 1e5;
..... else
.....     fuzzyViolation = 0;
..... end

..... % --- Update Constraints and Objectives ---
..... PopCon(i) = capPenalty * capViolation + timePenalty * timeViolation + fuzzyViolation;
..... PopObj(i, :) = [totalEconomic + totalEnvironmental + totalSocial, -totalSatisfaction];

..... % Store component costs and truth value
..... C1{i} = totalEconomic;
..... C2{i} = totalEnvironmental;
..... C3{i} = totalSocial;
..... C4{i} = totalSatisfaction;
..... ALPHA{i} = overall_truth;
..... end

..... % Nested Helper Functions for Membership
..... function mu = mu_Low(x)
.....     if x <= 0.1
.....         mu = 1;
.....     elseif x < 0.4
.....         mu = 1 - (x - 0.1) / 0.3;
.....     else
.....         mu = 0;
.....     end
..... end

..... function mu = mu_Medium(x)
.....     if x <= 0.2

```

```
    mu = 0;
elseif x < 0.5
    mu = (x - 0.2) / 0.3;
elseif x < 0.8
    mu = (0.8 - x) / 0.3;
else
    mu = 0;
end
end

function mu = mu_High(x)
if x <= 0.6
    mu = 0;
elseif x < 0.9
    mu = (x - 0.6) / 0.3;
else
    mu = 1;
end
end
```

Key Implementation Notes:

- Per-Route Data Collection:** Loads, distances, and work times are recorded during route processing.
- Fuzzy Variable Standardization:** All variables are mapped to [0,1] using problem-specific normalization.
- Membership Functions:** Nested functions implement linear piecewise definitions for "Low," "Medium," and "High" terms.
- Proposition Evaluation:** Uses Kleene-Dienes implication for conditional propositions and Zadeh's min/max for AND/OR.
- Constraint Penalty:** If the aggregated truth degree is below 0.6, a penalty of $10^5 \times (1 - \text{truth})$ is added to PopCon.

This implementation seamlessly integrates fuzzy logic for sustainability and fairness considerations into the existing VRP optimization framework.

(注: 文档部分内容可能由 AI 生成)