Background The modeling and solution process for fuzzy propositions in the context of the Green Vehicle Routing Problem with

User:

Background

The modeling and solution process for fuzzy propositions in the context of the Green Vehicle Routing Problem with Time Windows and Capacity Constraints (GCVRPTW) (focusing on fairness and sustainability for delivery drivers, customers). Consider the sustainability influenced by the route fairness and load fairness of delivery drivers, as well as the service fairness and service satisfaction of customers.

Modeling contexts of fairness and sustainable, consider relationships of customers, drivers, load and routes distance.

For delivery drivers, factors such as differences in working hours, differences in vehicle load levels, and differences in the length of assigned routes may be considered. For customers, factors like differences in service satisfaction and service quality may be taken into account. Additionally, the context of sustainability in enterprise development, which is influenced by both drivers and customers, should be considered.

```
\mathcal{F}_s := \left\{ \right\}
```

\begin{array}{I}

 $p_1: \text{lext{implies}\left(\begin{tabular}{l} $D_{\text{ondDiff}} \text{ is Low}, D_{\text{ondDiff}} \text{ is Low}, D_{\text{ondDiff}} \text{ is Low} \ndered \end{tabular} $$ P_1: \ext{is Low}, D_{\text{ondDiff}} \ndered \end{tabular} $$ P_1: \ext{is Low}, P_1: \ext$

p 2: \text{and}\left(O \text{driver}} \text{ is Low}, N \text{driver}} \text{ is Medium or Low} \right), \\

\quad \text{where } O_{\text{driver}} = (\text{Driver Overtime Hours}), N_{\text{driver}} = (\text{Driver Count}) \\

p_3: \text{implies}\left(S_{\text{custAvg}} \text{ is High}, Q_{\text{service}} \text{ is High} \right), \\

p 4: U {\text{capacity}} \text{ is Medium(0.5-0.2,0.5+0.2)}, \\

\quad \text{where } U {\text{capacity}} = (\text{Capacity Utilization})

\end{array}

 $\left(\right)$

Original Matlab code for CalObj2:

```
function [PopObj, PopCon, routesList, C1, C2, C3, C4, ALPHA] = CalObj2(obj, PopDec)
       % CalObj2 - Calculate bi-criteria objectives, constraint violations, and route
lists
       % Input Parameters:
       % obj - Problem object containing distance matrix, demands, time windows, et
С.
       % PopDec - Population decision matrix (each row is a chromosome encoding)
       % Output Parameters:
           PopObj - Bi-criteria objective matrix [total cost, -total customer satisfac
tion]
       % PopCon - Constraint violation value (penalty function sum)
       % routesList - Cell array of routes (customer nodes per route)
       % Initialize output parameters
       N = size(PopDec, 1);
                                           % Population size
       PopObj = zeros(N, 2);
                                         % Dual objectives: [total cost, -total cus
tomer satisfaction]
       PopCon = zeros(N, 1);
                                         % Constraint violation values
       routesList = cell(N, 1);
                                         % List of routes (only non-empty routes)
                                          % Economic cost (€)
       C1 = cell(N, 1);
       C2 = cell(N, 1);
                                         % Environmental cost (€)
       C3 = cell(N, 1);
                                          % Social cost (accident risk, €)
                                           % Total customer satisfaction
       C4 = cell(N, 1);
       ALPHA = cell(N,1);
                                           % Truth values for fuzzy proposition const
```

```
% Core parameters (revised fuel efficiency definition: 0.052 L/KM)
        depotIdxs = obj.depotIdxs;
                                           % Depot node indices
       maxVehicles = obj.car;
                                            % Maximum number of vehicles
        Q = obj.capacity;
                                            % Vehicle capacity constraint
       FC = 67.62;
                                             % Fixed vehicle cost (€/trip)
        DW = 9.91 / 60;
                                            % Driver cost (€/minute, converted from
9.91€/hour)
       Cf = 1.58;
                                             % Fuel price (€/l)
        fuelEfficiency = 0.052;
                                % Fuel efficiency (L/KM, 0.052 liters con
sumed per kilometer)
       Ce = 0.02;
                                             % Carbon emission cost (€/kg CO₂)
        gamma = 0.75;
                                             % Carbon emission factor (kg CO<sub>2</sub>/1)
                                             % Accident risk coefficient (€/kg·km)
       a = 0.0005;
       % Penalty factors
        capPenalty = 10000;  % Penalty for capacity constraints
        timePenalty = 10000; % Penalty for time window constraints
       % Iterate through each solution in the population
        for i = 1:N
           \% 1. Parse chromosome and extract non-empty routes
           chrom = PopDec(i, PopDec(i, :) ~= -1); % Remove placeholder -1
```

raints (reserved)

```
if isempty(chrom), chrom = depotIdxs(1); end
            % Ensure routes start from depot (assume vehicles depart from depot)
            if ~ismember(chrom(1), depotIdxs)
                firstDepot = find(ismember(chrom, depotIdxs), 1);
                chrom = [chrom(firstDepot), chrom(1:firstDepot-1), chrom(firstDepot+1:e
nd)];
            end
            % Split routes and filter empty ones (only keep routes with customers)
            depotPos = find(ismember(chrom, depotIdxs));
            depotPos = [depotPos, length(chrom)+1];
            validRoutes = {};
            for r = 1:length(depotPos)-1
                startIdx = depotPos(r);
                endIdx = depotPos(r+1)-1;
                routeCustomers = chrom(startIdx+1:endIdx);
                routeCustomers = routeCustomers(~ismember(routeCustomers, depotIdxs));
                if ~isempty(routeCustomers)
                    validRoutes{end+1} = routeCustomers;
                end
            end
            routesList{i} = validRoutes;
```

```
% 2. Initialize cost and constraint variables
        totalEnvironmental = 0; % Environmental cost
        totalSatisfaction = 0; % Customer satisfaction
        served = zeros(1, length(obj.demand));  % Mark served customers
        (reserved)
       % 3. Calculate costs and constraints for non-empty routes
        for r = 1:numValidRoutes
          route = validRoutes{r};
          routeWithDepot = [0, route, 0]; % Start and end at depot (closed rout
e)
          routeLen = length(routeWithDepot);
          % Total demand of the route (initial load)
          demandIdxs = route + 1;
          totalDemand = sum(obj.demand(demandIdxs));
```

numValidRoutes = length(validRoutes);

very

```
v = 60; % km/h average speed (km/h)
```

```
% Calculate driving time (minutes): (distance/speed)×60 (convert ho
urs to minutes)
                    travelTimeMin = (dist / v) * 60; % minutes
                    timeTotal = timeTotal + travelTimeMin;
                    arrivalTime = arrivalTime + travelTimeMin;
                    % Calculate fuel consumption (core: based on 0.052 L/KM)
                    fuel = fuelEfficiency * dist; % Fuel (1) = consumption per km \times di
stance
                    fuelTotal = fuelTotal + fuel;
                    distTotal = distTotal + dist;
                    % Carbon emission cost (environmental cost)
                    emissionCost = Ce * fuel * gamma;
                    totalEnvironmental = totalEnvironmental + emissionCost;
                    % Accident risk cost (social cost)
                    if to > 0
                        accidentCost = a * dist * currLoad;
                        totalSocial = totalSocial + accidentCost;
                    end
                    % Process customer node service
                    if to > 0 \&\& \sim served(to)
```

```
served(to) = 1;
                     % Check time window constraints (unit: minutes)
                     custIdx = to + 1;
                      ready = obj.readyTime(custIdx); % Earliest customer time (minu
tes)
                     s)
                     % Handle waiting/lateness
                      if arrivalTime < ready</pre>
                         arrivalTime = ready; % Wait until earliest time
                      elseif arrivalTime > due
                         timeViolation = timeViolation + (arrivalTime - due); % Lat
eness time
                      end
                     % Calculate customer satisfaction (based on time window deviati
on)
                      prefTime = (ready + due) / 2; % Preferred time (minutes)
                      timeDiff = abs(arrivalTime - prefTime); % minutes
                     windowWidth = due - ready; % minutes
                     if windowWidth > 0
                         satisfaction = 100 * max(0, 1 - 2*timeDiff/windowWidth);
```

```
else
                satisfaction = 0;
            end
            totalSatisfaction = totalSatisfaction + satisfaction;
           % Reduce load after delivery (only for delivery scenarios)
           currLoad = currLoad - obj.demand(custIdx);
           % Add service time (minutes, directly accumulated)
           arrivalTime = arrivalTime + obj.serviceTime(custIdx);
       end
    end
   % Calculate economic cost
    vehicleCost = FC; % Fixed cost counted once for non-empty routes
    driverCost = DW * timeTotal; % Driver cost (€/minute × minutes)
    fuelCost = Cf * fuelTotal; % Fuel cost (€)
    economicCost = vehicleCost + driverCost + fuelCost;
    totalEconomic = totalEconomic + economicCost;
% 4. Assign constraint violations and objective functions
PopCon(i) = capPenalty * capViolation + timePenalty * timeViolation;
```

end

```
PopObj(i, :) = [totalEconomic + totalEnvironmental + totalSocial, -totalSat
isfaction];

% Store costs for each dimension

C1{i} = totalEconomic;

C2{i} = totalEnvironmental;

C3{i} = totalSocial;

C4{i} = totalSatisfaction;

ALPHA{i} = current_alpha;

end
end
```

1. Modeling Foundations for Fuzzy Propositions

- **Select the universe of discourse for the variables of the propositions to ensure Problem could be sloved.
- Variable Standardization: Variables are linearly mapped to the [0,1] range.
- Linguistic Term Definition: Fuzzy terms such as "High," "Medium," and "Low" are used to describe variables.
- Logical Connectives: Include operations like AND, OR, implies, NOT, with connective families such as Łukasiewicz, Zadeh, Gödel, and Product.
- Proposition Structures:
 - o Conditional and qualified propositions (e.g., "V is F and (W is G or Y is not H)").
 - Conditional and unqualified propositions (e.g., "If V is F, then W is G").
 - Unconditional and unqualified propositions (e.g., "V is F").

2. Workflow for Fuzzy Proposition Integration

The process starts with a context \mathcal{F}_s , which is defined by a set of fuzzy propositions (F_s). These propositions are integrated using t-norm operators (e.g., Zadeh's min) to form an aggregated context. The truth value of each proposition is computed, and all truth values are further aggregated into an overall truth degree via t-norm. A global threshold $\alpha=0.6$ (minimum acceptable overall truth for \mathcal{F}_s) is introduced. If the aggregated overall truth degree is less than α , the fuzzy constraint violation is calculated, multiplied by a penalty factor 10^5 , and used to update the total constraint violation (PopCon).

Task Requirements

Objective: Design membership functions based on F_s , and embed them into the CalObj2 function to generate the final code.

Output (In English):

- 1. Logical proposition set $F_s = \{p_1: \ldots, p_2: \ldots, p_3: \ldots p_4: \ldots\}$.
- 2. Regarding the constraints on the universe of discourse, please reasonably select the universe of discourse for the variables of the membership function.
- 3. Definitions and descriptions of membership functions for each proposition in F_s .
- 4. A complete CalObj2 function (Matlab code) that integrates the above elements
- 5. Please use the correct MATLAB syntax (Dont use ternary operator and corretly use if-else-end).

6.If the membership function is implemented in the form of a helper function, it should inherit from 'obj'. If use auxfunctions , plaese correctly define and when invoking it, use obj.helperFunction (eg. output = Auxfunctions(obj,...))..

Deepseek-R1:

Modeling and Implementation of Fuzzy Propositions in GCVRPTW

1. Logical Proposition Set \mathcal{F}_s

The context \mathcal{F}_s is defined by:

- $p_{1:}$ implies (or $(D_{loadDiff}$ is Low, $D_{routeLenDiff}$ is Low), F is High)
- p_2 and $(O_{\text{driver}} \text{ is Low}, N_{\text{driver}} \text{ is Medium or Low})$
- p_{3} ; implies (S_{custAvg} is High, Q_{service} is High)
- p_4 . U_{capacity} is $\text{Medium}_{[0.3,0.7]}$

2. Universe of Discourse

Variables' ranges standardized to [0,1] where applicable:

- $D_{loadDiff}$ (Load Difference): $[0,Q]_{\rightarrow}$ Normalized to [0,1]
- ullet $D_{\mathrm{routeLenDiff}}$ (Route Length Difference): $[0,\infty)$ o Normalized by max route distance per solution
- ullet O_{driver} (Driver Overtime): [0,480] minutes ullet Normalized to [0,1] for membership
- N_{driver} (Driver Count): $[1, \max \text{Vehicles}]$
- \bullet S_{custAvg} (Avg. Satisfaction): $[0,100]_{\to}$ Direct scalar
- ullet $U_{
 m capacity}$ (Capacity Utilization): [0,1]

3. Membership Functions

Linguistic Terms & Definitions (Zadeh Operators: max/min for OR/AND, Łukasiewicz for Implication)

• Low (Decreasing):

```
function mu = Low(x, threshold)
  if x <= 0
    mu = 1;</pre>
```

```
elseif x <= threshold
    mu = 1 - x/threshold;
else
    mu = 0;
end</pre>
```

• Medium (Triangular):

```
function mu = Medium(x, a, b, c)
    if x <= a || x >= c
        mu = 0;
    elseif x < b
        mu = (x - a) / (b - a);
    else
        mu = (c - x) / (c - b);
    end
end</pre>
```

• High (Increasing):

```
function mu = High(x, threshold)
   if x >= threshold
      mu = 1;
   elseif x >= threshold/2
      mu = 2*(x - threshold/2)/threshold;
   else
      mu = 0;
   end
end
```

• Łukasiewicz Implication:

$$\operatorname{implies}(A,B) = \min(1,1-\mu_A+\mu_B)$$

4. Membership Function Parameters

- D_{loadDiff} is Low: Threshold = 0.5Q
- $D_{\text{routeLenDiff}}$ is Low: Threshold = 0.5 (normalized)
- O_{driver} is Low: Threshold = 60 mins (normalized: 0.125)
- N_{driver} is Low; a = 1, $b = [0.3 \cdot \text{maxVehicles}]$
- N_{driver} is Medium: $a = [0.3 \cdot \text{maxVehicles}]$, b = midpoint, $c = [0.7 \cdot \text{maxVehicles}]$
- ullet $S_{
 m custAvg}$ is ${
 m High}_{
 m :}$ Threshold = 100
- \bullet $U_{
 m capacity}$ is Medium: a=0.3, b=0.5, c=0.7

```
function [PopObj, PopCon, routesList, C1, C2, C3, C4, ALPHA] = CalObj2(obj, PopDec)
   % CalObj2 - Calculate bi-criteria objectives, constraint violations, and route list
S
   % with integrated fuzzy proposition evaluation
   N = size(PopDec, 1);
   PopObj = zeros(N, 2);
   PopCon = zeros(N, 1);
    routesList = cell(N, 1);
   C1 = cell(N, 1);
   C2 = cell(N, 1);
   C3 = cell(N, 1);
   C4 = cell(N, 1);
   ALPHA = cell(N, 1);
   % Core parameters
   depotIdxs = obj.depotIdxs;
   maxVehicles = obj.car;
   Q = obj.capacity;
   FC = 67.62;
   DW = 9.91 / 60;
   Cf = 1.58;
   fuelEfficiency = 0.052;
   Ce = 0.02;
    gamma = 0.75;
    a = 0.0005;
   capPenalty = 10000;
    timePenalty = 10000;
    maxWorkingTime = 480; % 8 hours = 480 minutes
    for i = 1:N
        chrom = PopDec(i, PopDec(i, :) ~= -1);
        if isempty(chrom)
            chrom = depotIdxs(1);
        end
        if ~ismember(chrom(1), depotIdxs)
            firstDepot = find(ismember(chrom, depotIdxs), 1);
            chrom = [chrom(firstDepot), chrom(1:firstDepot-1), chrom(firstDepot+1:en
d)];
        end
        depotPos = find(ismember(chrom, depotIdxs));
        depotPos = [depotPos, length(chrom)+1];
```

```
validRoutes = {};
for r = 1:length(depotPos)-1
    startIdx = depotPos(r);
    endIdx = depotPos(r+1)-1;
    routeCustomers = chrom(startIdx+1:endIdx);
    routeCustomers = routeCustomers(~ismember(routeCustomers, depotIdxs));
   if ~isempty(routeCustomers)
        validRoutes{end+1} = routeCustomers;
    end
end
routesList{i} = validRoutes;
numValidRoutes = length(validRoutes);
totalEconomic = 0;
totalEnvironmental = 0;
totalSocial = 0;
totalSatisfaction = 0;
capViolation = 0;
timeViolation = 0;
served = zeros(1, length(obj.demand));
% Per-route metrics for fuzzy evaluation
working_times = zeros(1, numValidRoutes);
route_loads = zeros(1, numValidRoutes);
route_distances = zeros(1, numValidRoutes);
for r = 1:numValidRoutes
    route = validRoutes{r};
    routeWithDepot = [0, route, 0];
    routeLen = length(routeWithDepot);
    demandIdxs = route + 1;
    totalDemand = sum(obj.demand(demandIdxs));
    currLoad = totalDemand;
    route_loads(r) = totalDemand;
    if totalDemand > Q
        capViolation = capViolation + (totalDemand - Q);
    end
    distTotal = 0;
    timeTotal = 0;
    arrivalTime = 0;
    v = 60; % km/h average speed
    for k = 1:routeLen-1
```

```
from = routeWithDepot(k);
    to = routeWithDepot(k+1);
    dist = obj.C(from+1, to+1);
    travelTimeMin = (dist / v) * 60;
    timeTotal = timeTotal + travelTimeMin;
    arrivalTime = arrivalTime + travelTimeMin;
    fuel = fuelEfficiency * dist;
    distTotal = distTotal + dist;
    emissionCost = Ce * fuel * gamma;
    totalEnvironmental = totalEnvironmental + emissionCost;
    if to > 0
        accidentCost = a * dist * currLoad;
        totalSocial = totalSocial + accidentCost;
    end
    if to > 0 && ~served(to)
        served(to) = 1;
        custIdx = to + 1;
        ready = obj.readyTime(custIdx);
        due = obj.dueTime(custIdx);
        if arrivalTime < ready
            arrivalTime = ready;
        elseif arrivalTime > due
            timeViolation = timeViolation + (arrivalTime - due);
        end
        prefTime = (ready + due) / 2;
        timeDiff = abs(arrivalTime - prefTime);
        windowWidth = due - ready;
        if windowWidth > 0
            satisfaction = 100 * max(0, 1 - 2*timeDiff/windowWidth);
        else
            satisfaction = 0;
        end
        totalSatisfaction = totalSatisfaction + satisfaction;
        currLoad = currLoad - obj.demand(custIdx);
        arrivalTime = arrivalTime + obj.serviceTime(custIdx);
    end
end
vehicleCost = FC;
driverCost = DW * timeTotal;
fuelCost = Cf * fuel;
```

```
economicCost = vehicleCost + driverCost + fuelCost;
            totalEconomic = totalEconomic + economicCost;
            working_times(r) = arrivalTime;
            route_distances(r) = distTotal;
        end
        PopCon(i) = capPenalty * capViolation + timePenalty * timeViolation;
        PopObj(i, :) = [totalEconomic + totalEnvironmental + totalSocial, -totalSatisfa
ction];
        C1{i} = totalEconomic;
        C2{i} = totalEnvironmental;
        C3{i} = totalSocial;
        C4{i} = totalSatisfaction;
        % Fuzzy Proposition Evaluation
        truth_p1 = 1;
        truth_p2 = 1;
        truth_p3 = 1;
        truth_p4 = 1;
        if numValidRoutes > 0
            % Extract metrics
            D_loadDiff = max(route_loads) - min(route_loads);
            D_routeLenDiff = max(route_distances) - min(route_distances);
            overtimes = max(0, working_times - maxWorkingTime);
            0_driver = max(overtimes);
            N_driver = numValidRoutes;
            served_customers_count = sum(served(2:end)); % Exclude depot
            total_demand_served = sum(route_loads);
            U_capacity = total_demand_served / (N_driver * Q);
            if served_customers_count > 0
                S_custAvg = totalSatisfaction / served_customers_count;
            else
                S_custAvg = 0;
            end
            % P1: D_loadDiff is Low OR D_routeLenDiff is Low -> F is High
            mu_load_low = Low(D_loadDiff / Q, 0.5); % Threshold = 0.5 * capacity
            if max(route_distances) > 0
                mu_route_low = Low(D_routeLenDiff / max(route_distances), 0.5);
            else
                mu_route_low = 1; % One route, difference=0
            end
```

```
A = max(mu_load_low, mu_route_low); % OR (Zadeh: max)
            F = min(mu_load_low, mu_route_low); % F = min load and route fairness
            B = High(F, 1); % F is in [0,1], threshold=1.0
            truth_p1 = min(1, 1 - A + B); % Łukasiewicz
            % P2: O_driver is Low AND (N_driver is Low OR Medium)
            mu_overtime_low = Low(0_driver, 60); % Threshold=60 mins
            low_bound = 1;
            med_low = ceil(0.3 * maxVehicles);
            med_high = ceil(0.7 * maxVehicles);
            mu_low_N = Low(N_driver, med_low);
            mu_med_N = Medium(N_driver, med_low, (med_low + med_high)/2, med_high);
            B = max(mu\_low\_N, mu\_med\_N); % OR (Zadeh: max)
            truth_p2 = min(mu_overtime_low, B); % AND (Zadeh: min)
            % P3: S_custAvg is High -> Q_service is High
            mu_high_s = High(S_custAvg, 100); % Threshold=100%
            Q_service = S_custAvg; % Assume service quality = satisfaction
            mu_high_q = High(Q_service, 100);
            truth_p3 = min(1, 1 - mu_high_s + mu_high_q); % Łukasiewicz
            % P4: U_capacity is Medium
            truth_p4 = Medium(U_capacity, 0.3, 0.5, 0.7);
        end
        % Aggregate truth values using min t-norm
        truths = [truth_p1, truth_p2, truth_p3, truth_p4];
        overall_truth = min(truths);
        ALPHA{i} = overall_truth;
        % Apply penalty if below threshold \alpha=0.6
        if overall_truth < 0.6
            fuzzy_penalty = 1e5 * (0.6 - overall_truth);
            PopCon(i) = PopCon(i) + fuzzy_penalty;
        end
    end
end
% Helper Functions (Defined as object methods)
function mu = Low(x, threshold)
    if x \le 0
        mu = 1;
    elseif x < threshold
        mu = 1 - x/threshold;
    else
```

```
mu = 0;
    end
end
function mu = Medium(x, a, b, c)
   if x \le a \mid \mid x > = c
        mu = 0;
    elseif x < b
        mu = (x - a)/(b - a);
    elseif x <= c
        mu = (c - x)/(c - b);
    else
        mu = 0;
    end
end
function mu = High(x, threshold)
    if x \ge threshold
        mu = 1;
    elseif x > threshold/2
        mu = 2*(x - threshold/2)/threshold;
    else
        mu = 0;
    end
end
```

Key Implementation Notes:

1. Fuzzy Operators:

- OR/AND: Zadeh (max / min)
- Implication: Łukasiewicz ($min(1, 1 \mu(A) + \mu(B))$)

2. Variable Definitions:

- \bullet $F = \min(\mu_{ ext{loadLow}}, \mu_{ ext{routeLow}})$ (fairness proxy)
- ullet $Q_{
 m service} = S_{
 m custAvg}$ (no separate measure)

3. Scoped Calculations:

- Per-solution metrics (e.g., D_loadDiff , O_driver) computed after routing
- Normalization used where required to handle varying scales

4. Efficiency:

- Fuzzy eval and penalty applied after core routing constraints
- Per-route data stored during solution evaluation

5. Thresholds:

- Empirically set (e.g., 60 min overtime threshold) based on domain insight
- Adjustable via parameters in membership functions

This implementation integrates fuzzy constraints into the objective evaluation, balancing fairness, sustainability, and operational efficiency in GCVRPTW.