A

Chapter 6.3 补充

① Example 4.

书上有代子 $e^{At} = e^{It}e^{(A-I)t}$

可是我之前说 in general

BUT! $e^{A+B} \neq e^{A} \cdot e^{B}$.

$\boxed{\text{If } AB=BA \quad e^{A} \cdot e^{B} = e^{B}e^{A} = e^{A+B}}$

Proof:

- We want to show that $e^{A}e^{B} = e^{A+B} = e^{B}e^{A}$ if $AB=BA$.
- take derivative:

$$\frac{d\left(e^{t(A+B)}e^{-tA}e^{-tB}\right)}{dt}$$

$$= (A+B)e^{t(A+B)}e^{-tA}e^{-tB}$$

$$- e^{t(A+B)}Ae^{-tA}e^{-tB}$$

$$- e^{t(A+B)}e^{-tA}Be^{-tB}$$

$$= (A+B-A-B)e^{t(A+B)} \times e^{-tA} \times e^{-tB}$$

$$= 0$$

If $AB=BA$.

$e^{t(A+B)}A = Ae^{t(A+B)}$?

① 首先: If $AB=BA$ then $(A+B)^{n}A = A(A+B)^{n}$

② $e^{t(A+B)}A$

$$= \sum_{k=0}^{n}\left[\frac{1}{k!}((A+B)t)^{k}A\right]$$

$$= \sum_{k=0}^{n}\left[A\frac{1}{k!}(A+B)t^{k}\right]$$

$$= Ae^{t(A+B)}$$

$\Rightarrow e^{t(A+B)}$ and A also commute.

Since Identity matrix commutes with every matrix, the equation at the beginning is always valid.

B 此外, 对于 2×2 A, 我们可以 easily prove that if $\lambda_1=\lambda_2=\lambda$,

then $(A-\lambda I)^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

写出 $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

尝试一下!

Thus we can actually write out a simplified general formula for the case where we have shared lambda's

C

$$e^{\Lambda t} = \begin{bmatrix} e^{\lambda_1 t} & & \\ & \ddots & \\ & & e^{\lambda_n t} \end{bmatrix}.$$

It does not look true is what I meant

— ok. this looks too simple.

But before you attempt to show that it is/not the case.

think about quality.

Is left side a matrix? Yes!

OK. Now you proceed.

— $e^{\Lambda t} = I + \Lambda t + \frac{1}{2}(\Lambda t)^2 + \frac{1}{6}(\Lambda t)^3 + \dots$

- You should see that they are all diagonal matrices.
- for each entry on the diagonal.

$$a_{ii} = 1 + \lambda_i t + \frac{1}{2}(\lambda_i t)^2 + \frac{1}{6}(\lambda_i t)^3 \dots$$

$$= e^{\lambda_i t}$$

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