Exercise 5: Observe: 只有一个完全平方: Not full rank.

Write in LDL^T rather than $Q\Lambda Q^T$.

$X_0 = L^T\vec{x}$.

$$\begin{bmatrix} l_{11} & l_{21} & l_{31} \\ & l_{22} & l_{32} \\ & & l_{33} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} l_{11}x_1 + l_{21}x_2 + l_{31}x_3 \\ l_{22}x_2 + l_{32}x_3 \\ l_{33}x_3 \end{bmatrix}$$

$$\Rightarrow d_1(l_{11}x_1 + l_{21}x_2 + l_{31}x_3)^2 + d_2(l_{22}x_2 + l_{32}x_3)^2 + d_3(l_{33}x_3)^2.$$

4. 2. 1

Exercise 6: If $R^TR = A$

then, $x^TAx = x^TR^TRx = (Rx)^T(Rx) = \|Rx\|$.

If x^TAx alway >0, then Rx cannot be zero vector when $x \neq 0$.

$\Rightarrow null(R) = Z$.

R has independent columns.

$\Rightarrow$ 若 $null(R) \neq Z$ Then $x^TAx \geq 0$ positive semidefinite

R always have dependent columns.

$$\begin{bmatrix} 4 & -4 & 8 \\ -4 & 4 & -8 \\ 8 & -8 & 16 \end{bmatrix}$$

rank=1.

det=0.

eigenvalue 24

2 nullspace eigenvectors with $\lambda = 0$. $\Rightarrow \{ 0, 0$.

trace = 24. So the third is 24

Exercise 7. $L\sqrt{D}$ or $Q\sqrt{\Lambda}$ $\quad Q\sqrt{\Lambda}Q^T$

Exercise 8. $D_1 = c \quad c > 0$

Determinant test.

$D_2 = c^2 - 1 > 0 \Rightarrow c > 1$. (with $c > 0$)

$D_3 = (c-1)^2(c+2) > 0 \quad c > 1$

$\Rightarrow c > 1$.

$$= c(c^2-1) - (c-1) + (1-c)$$
$$= c(c+1)(c-1) - 2(c-1)$$
$$= (c-1)(c^2+c-2)$$
$$= (c-1)(c+2)(c-1)$$
$$= (c+2)(c-1)^2$$

Ex9. $BABB^{-1} = BA$ $\quad$ AB's eigenvector x

Bx 为 BA's eigenvector.

Ex10. PAP^T.

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

More

Ex1 ① If $a_i < \lambda$'s.

eigenvalues of $A - a_{ii}I$ will be $\lambda_i - a_i > 0$ for all i $\Rightarrow$ positive def

② zero. $\Rightarrow$ A cannot be positive def. see $\boxed{6.5\text{-}16}$ $\Rightarrow$ Not positive def

Contradiction $\Rightarrow$ cannot

Ex2. ① Same λ's same x's Same $S\Lambda S^{-1} \Rightarrow A = B$.

② $A\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} B\begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$