Final Report

TITLE: Smoky Face

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Introduction

In this project, we have implemented smoke formation and transformation that can be applied to complicated target, such as human face. More specifically, we added driven pressure term and momentum term to the common Navier-Stoke equation. We modified density according to highfields algorithm. In addition to this, we utilized image segmentation algorithm to refine smoke transformation between two complicated geometry.

General Equation

Smoke motion in our setting is described by the following equations:
(a) $U_t = -U * \nabla U - \nabla P + V_f * U$; (b) $\nabla U = 0$; (c) $U = U * c$;
U stand for velocity, P for pressure, V_f is a constant we use to adjust momentum, c is parameter we extract from input image. Equation (b) is the same as Navier-Stoke equation. Being different from common Navier-Stoke equation, our velocity in the equation (a) was updated by a driven pressure term which drives smoke to move to the direction with weaker density and a momentum term which allows smoke to somehow moves out shape for better visual effect. On the basis of this update, we limit the smoke velocity by a parameter which we extract from input image in the equation (c).

Staggered (MAC) grid

I use the traditional staggered arrangement for the computational grid. Signed distance function samples are stored at the cell centers, while the velocity components are sampled at the facet centers. Everywhere else, quantities can be computed by linear interpolation.

Simulation

My simulator solves the fluid equations frame-by-frame based on the global timestep. Solution to smoke equations is obtained by splitting: each part is solved separately. I.e., for each substep the following procedures are performed:

- update source will only update initial source for 20 frames;
- advect signed distances and velocity components in the velocity field;
- apply driven pressure and momentum;
- restrain the velocity by a corresponding image parameter;
- interpolate position and density according to image segmentation;
- modify density according to their depth in z direction;
- advect density.

Advection

For advection, we use the unconditionally stable semi-Lagrangian method. To find the quantity value $\varphi(\mathbf{x}, t)$ at the point **x** and the time *t*, we compute $\varphi(\mathbf{x}', t - \Delta t)$, where **x'** is obtained by integrating the point **x** for the interval $-\Delta t$ (back in time). We use the Euler integrator for integration.

Driving Pressure

Instead of using conjugate gradient. We use our own driving pressure projection. Driving pressure drives smoke to move to the direction with weaker density. Algorithm is described as follow:

if(Density(i+/-1, j+/-1, k+/-1)< Density(i, j,k))

$$\nabla P = c * \mathrm{Density}(i+/-1, j+/-1, k+/-1)/\Delta X^2$$

Then velocity is updated as:

$$U = U' + \nabla P$$

Momentum

We momentum term to keep the inertial of smoke, so that smoke will not completely limited by target shape all the time. The smoke can somehow move out of the shape which will produce better visual effect. The extra momentum will be eliminated in the velocity restrain step.

Velocity Restrain

When we go through each grid, we figure out the corresponding point in the input image and then extract the color of pixel. We use this color to calculate a limit, so that we can cut down all the extra velocities. For those velocity higher than limit, they will be gradually reduced until they are less than the limit. The followings pictures show that velocity has been completely restrained by image color:

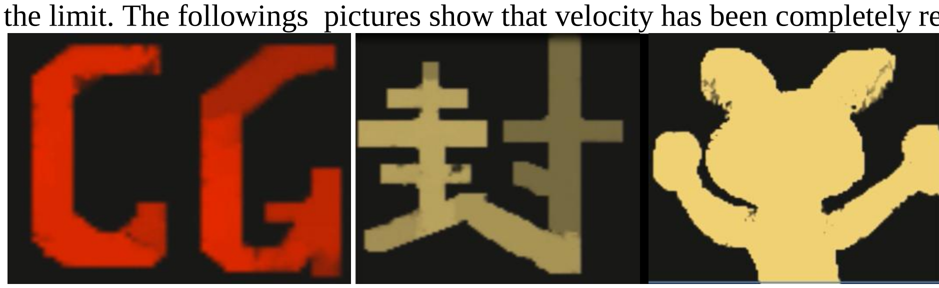
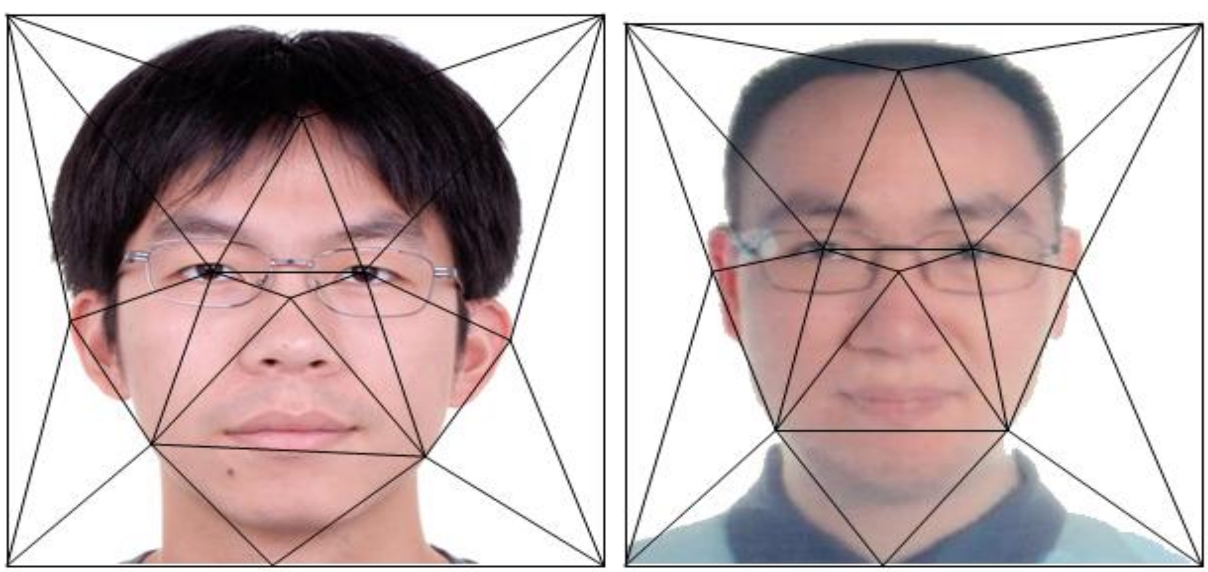


Image Segmentation

In order to produce better smoke transformation between two human faces, we segment each face sample pictures into 19 small triangles.



For each grid in the source, we figure out which triangle it belongs to and the its corresponding position in the target triangle, using the following formula:

$$P = \lambda_1 * A_1 + \lambda_2 * B_1 + \lambda_3 * C_1 \quad , \text{where } \lambda_1 + \lambda_2 + \lambda_3 = 1 \text{ and } \lambda_i >= 0$$

$$Q = T(P) = \lambda_1 * A_2 + \lambda_2 * B_2 + \lambda_3 * C_2$$

Then we interpolate the source position with the target position and the source density with the target density to obtain an intermediate position and density, using the formula:

$$C = (1-t)*P + t*Q, \text{ where } 0<t<1$$

$$\text{Density}(C) = (1-t)*\text{Density}(P) + t*\text{Density}(Q)$$

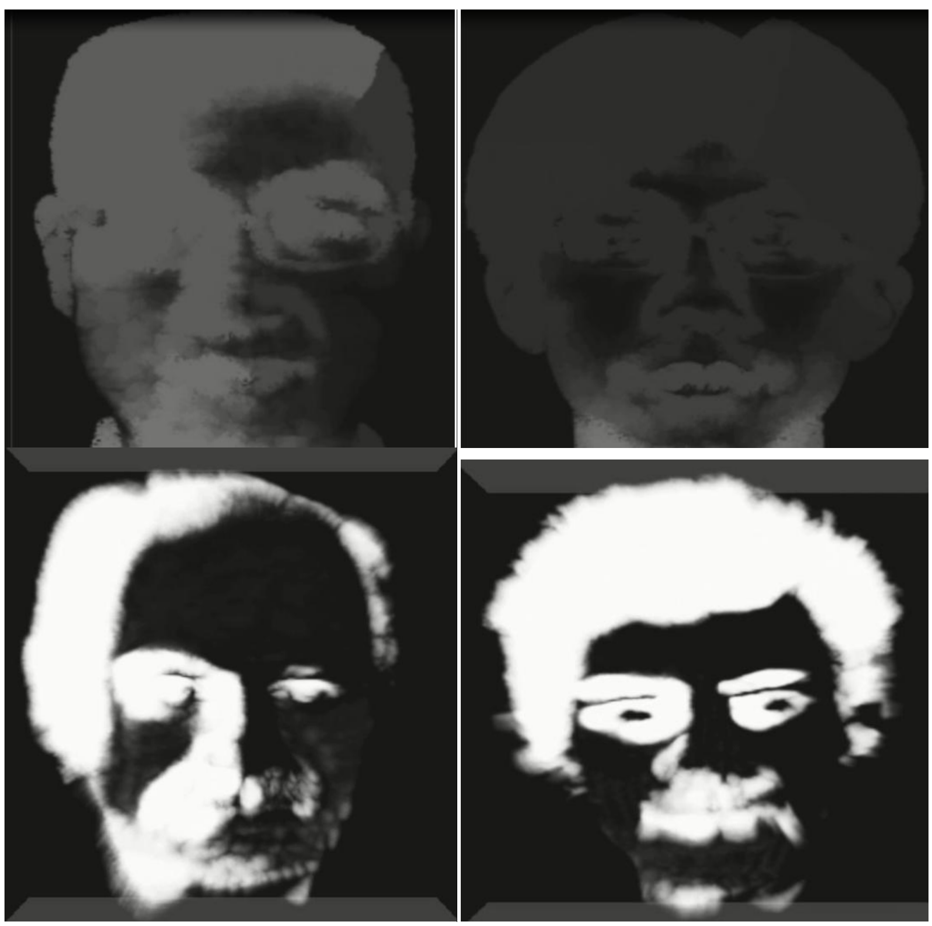
Finally, we assign this intermediate density to the intermediate position. The image segmentation is expressed in the following picture.

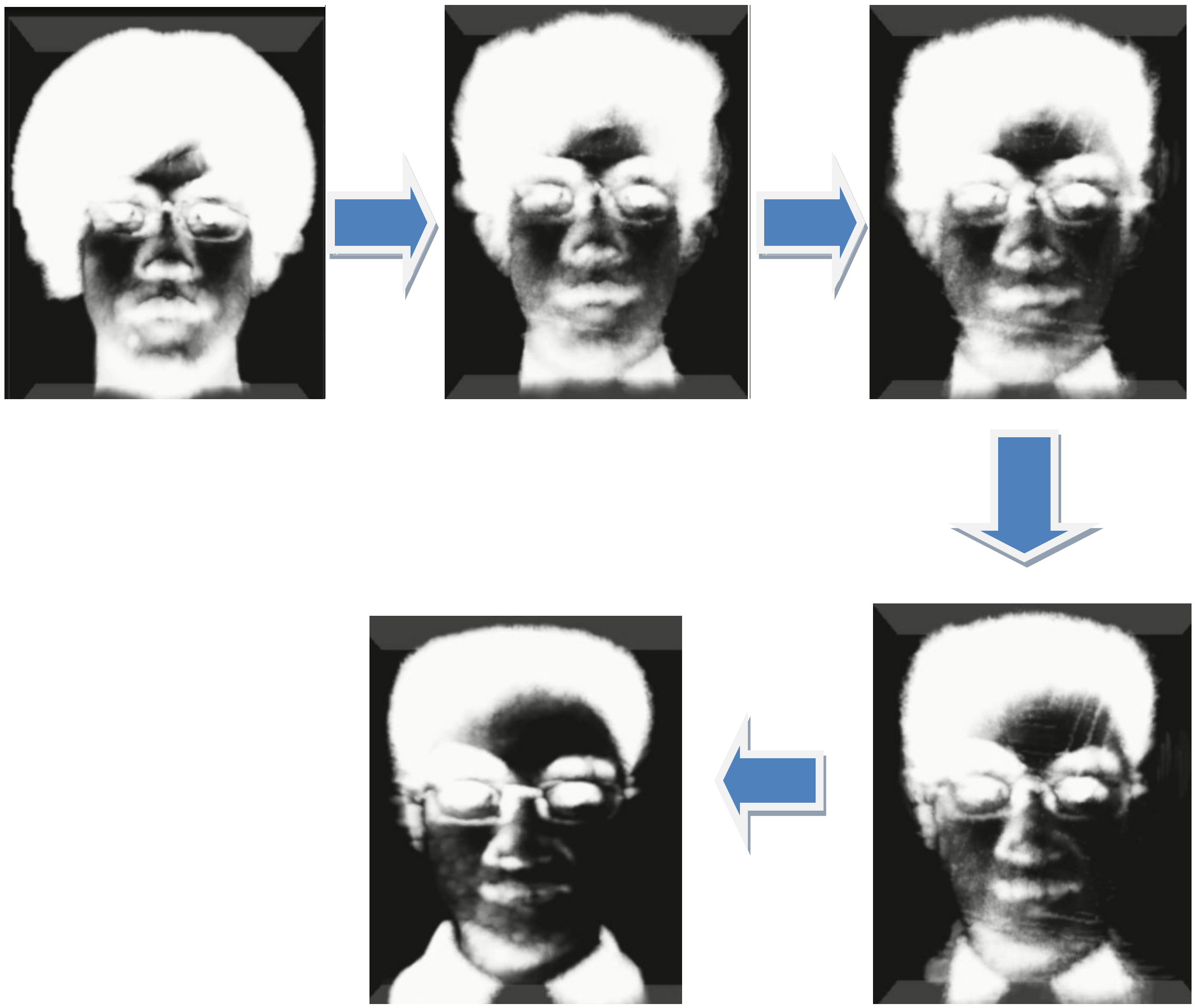
Density Modification

Our density is also modified in a way that the density of closer grids will be restrained by a smaller limit, whereas the density of farther grids will be restrained by a bigger limit. The density greater than limit will be gradually reduced until it’s less than the limit.

Results

Please look at our final video for dynamic smoke result. Here only shows some final images.





Reference

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