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ShanghaiTech University

No. 12 Gyroscope Teslameter

一个由**可导电**、**非铁磁性**材料制成的陀螺，放置在**磁场**中会出现**减速**的现象。研究相关参量如何影响减速的效果。

A **spinning gyroscope** made from a **conducting**, but **non-ferromagnetic** material **slows** down when placed in a **magnetic field**. Investigate how the deceleration depends on relevant parameters.

报告纲要

- 实验现象
- 理论分析
- 实验对照
- 误差分析

1 2

中国大学生物理学术竞赛
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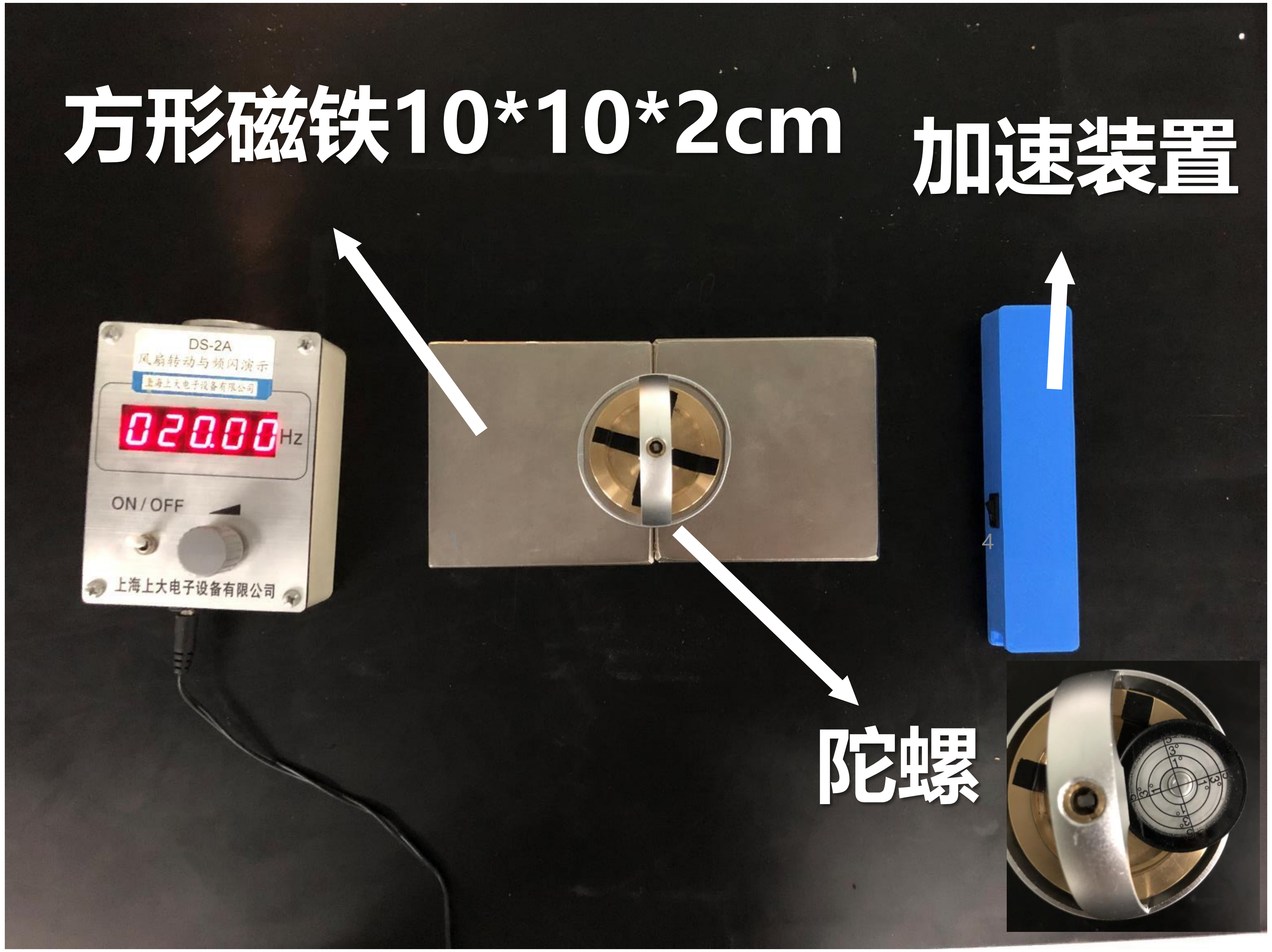
第一部分

实验现象

实验装置



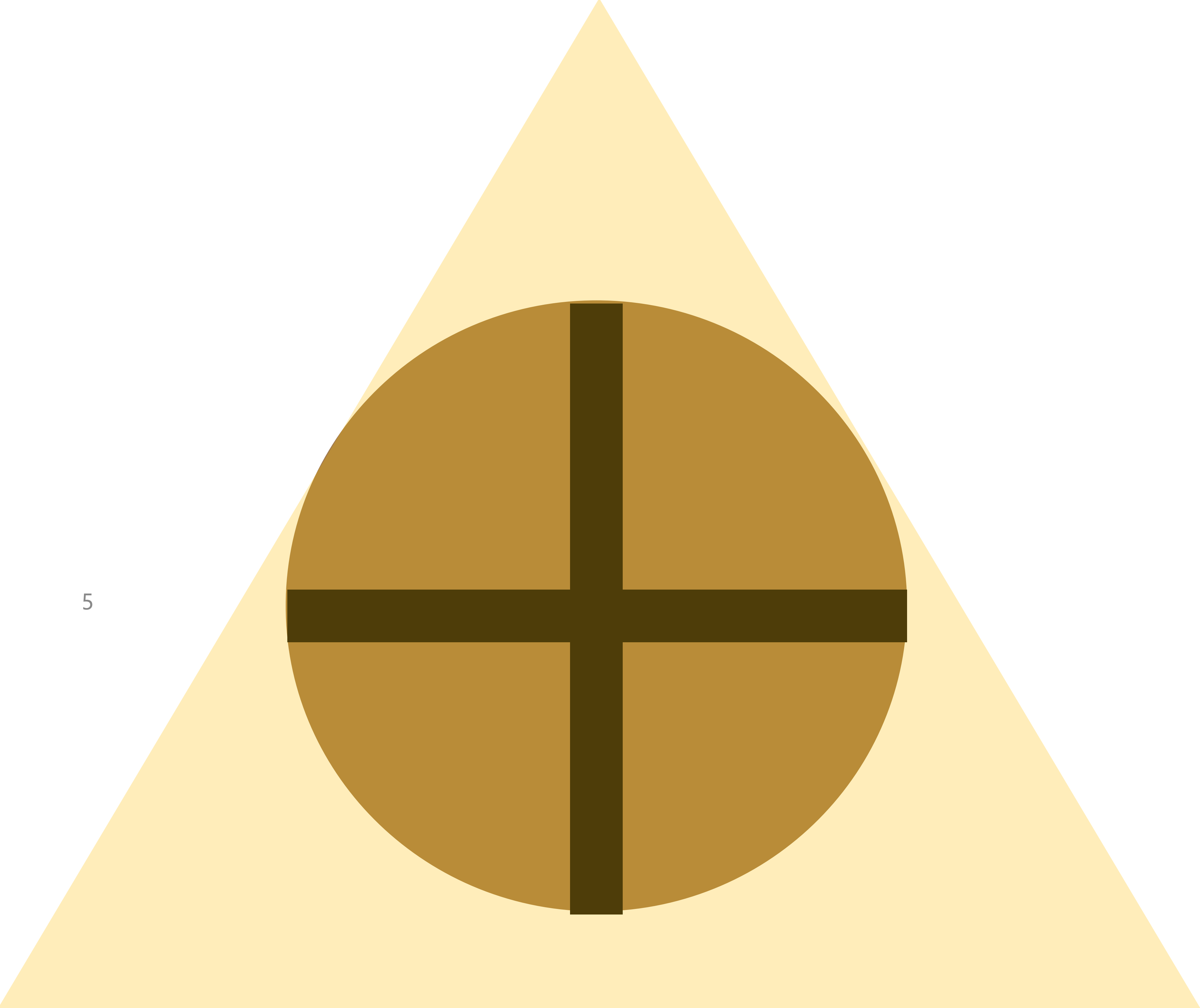
加速装置?

15s后 7500r/min

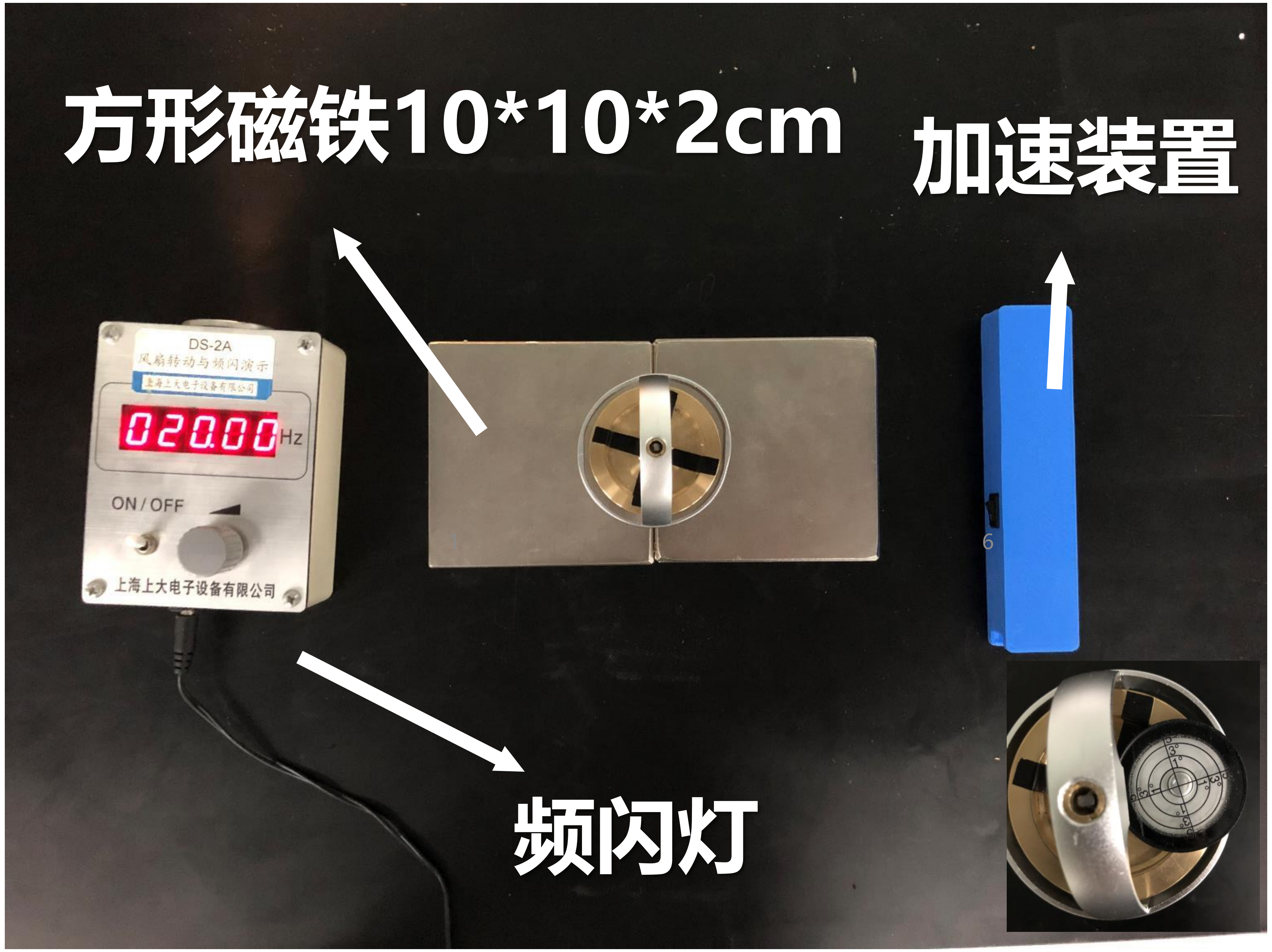
黑条纹?

实验装置

5



实验装置



加速装置?

最高 7500r/min

20Hz?

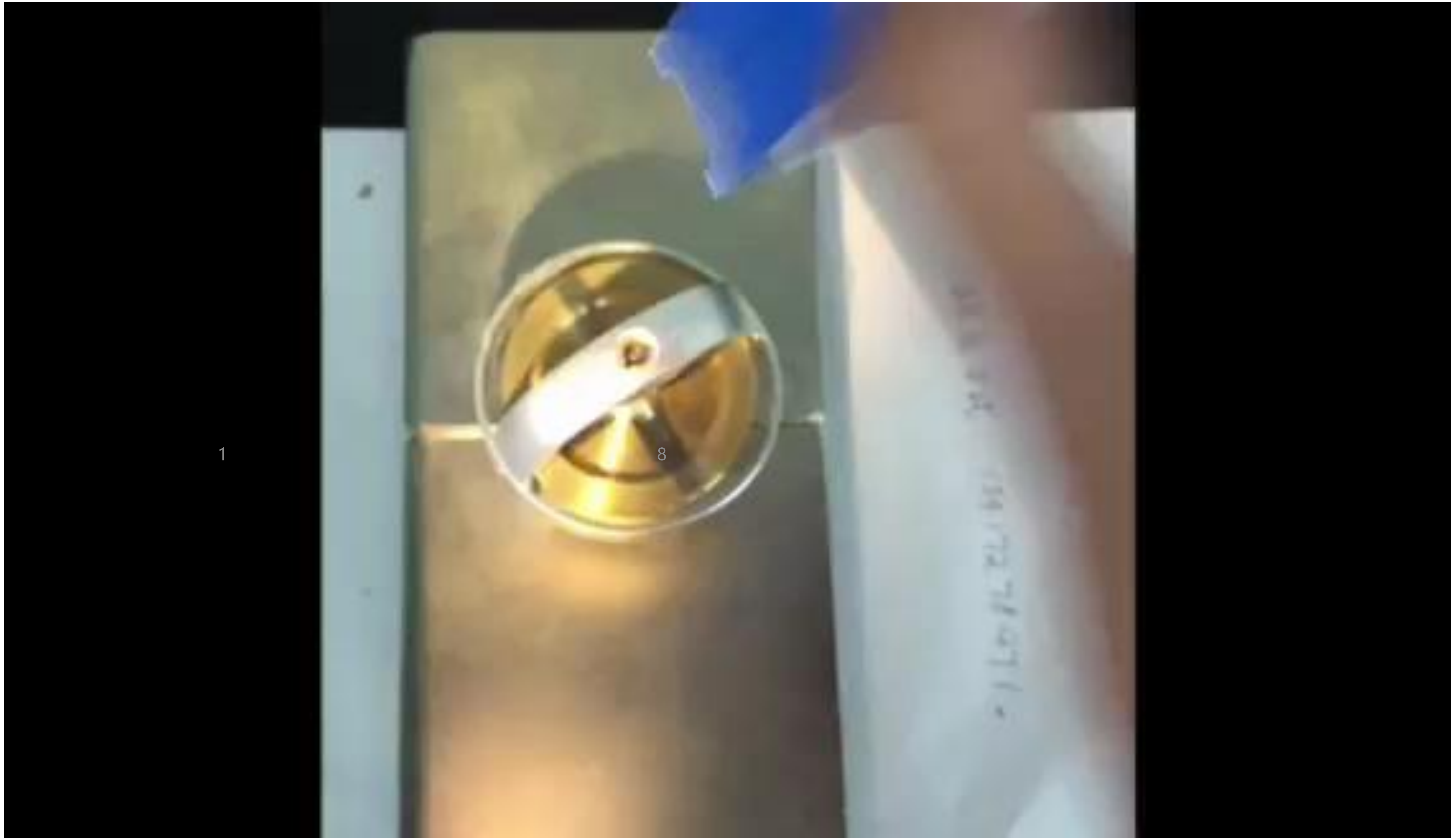
实验装置

>20Hz: 数据点过少

<20Hz: 错过关键帧

20Hz：较为合适

实验装置



实验现象

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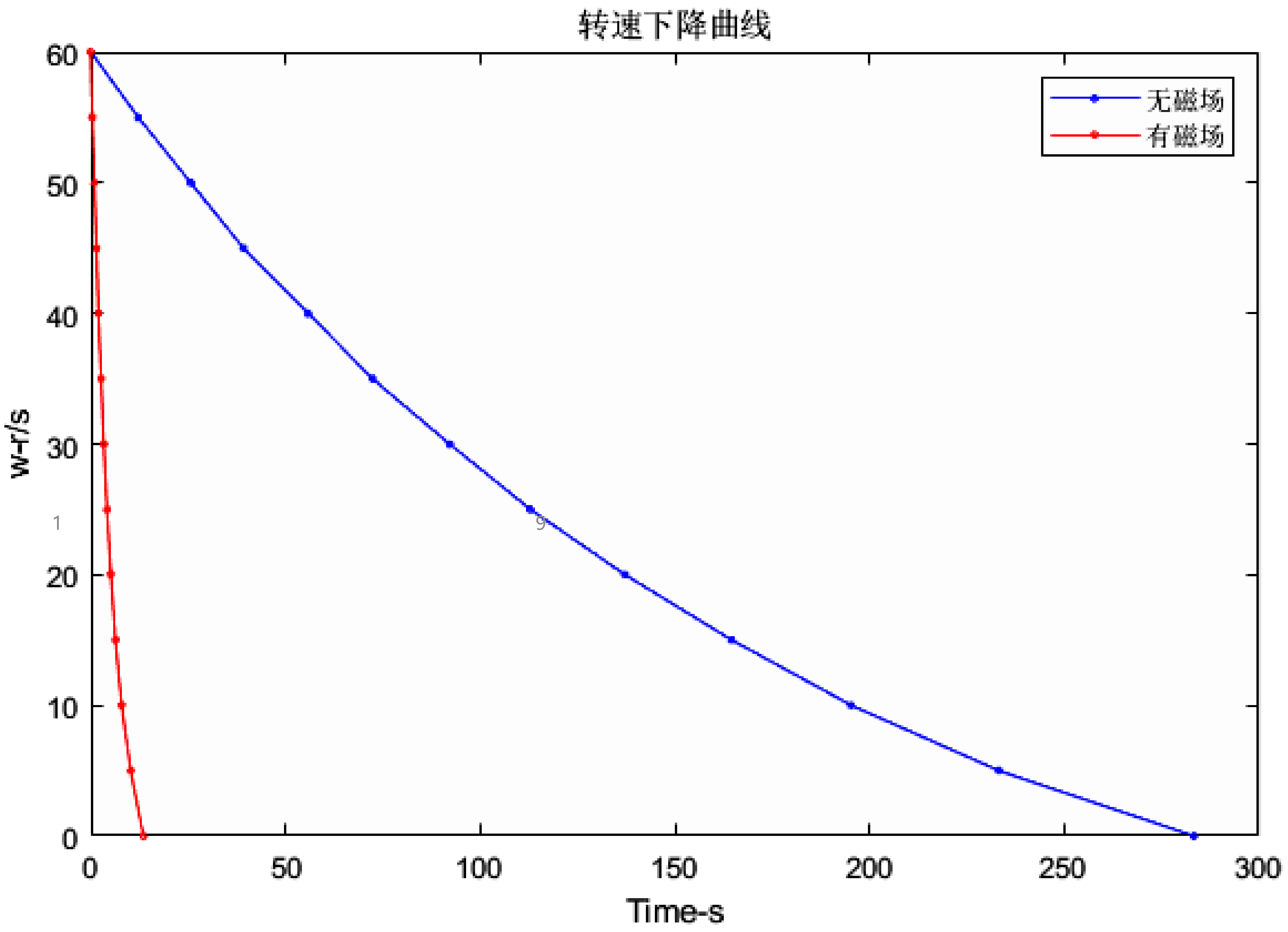

第二部分

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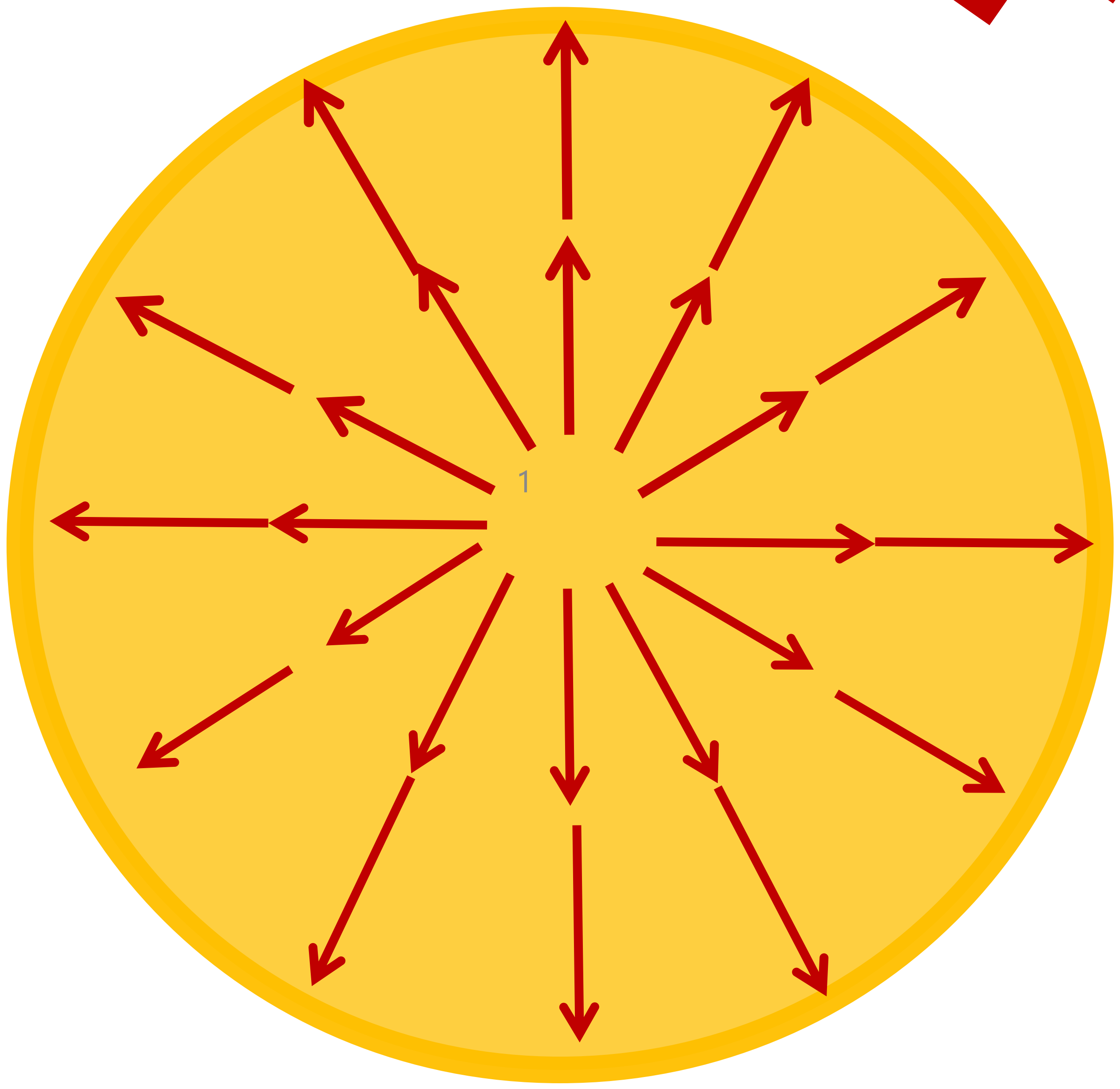
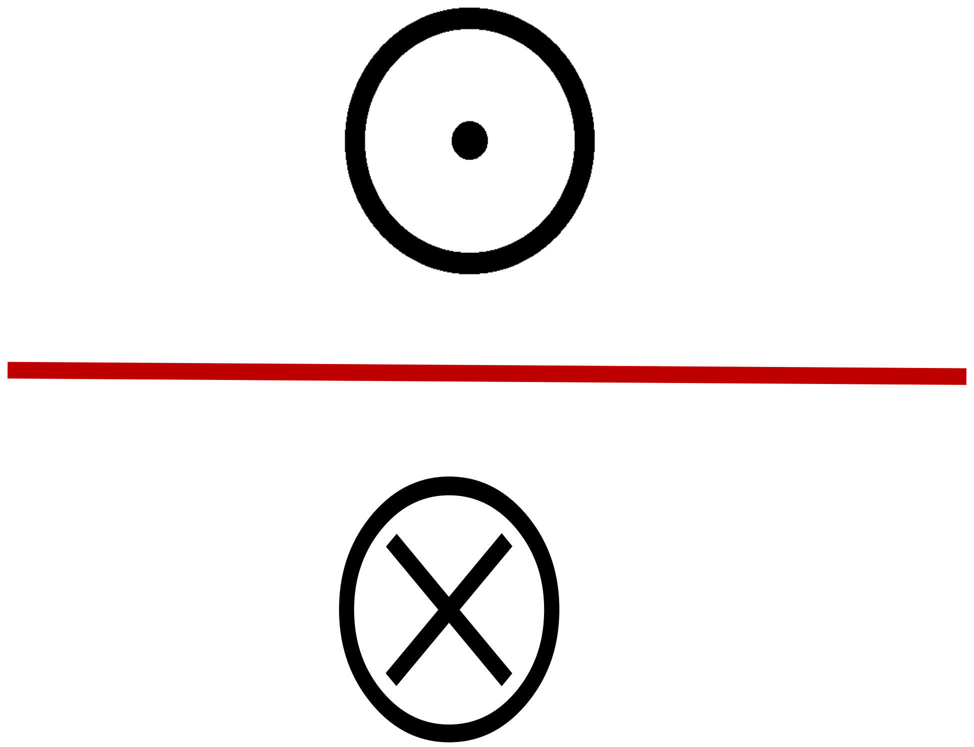
理论分析

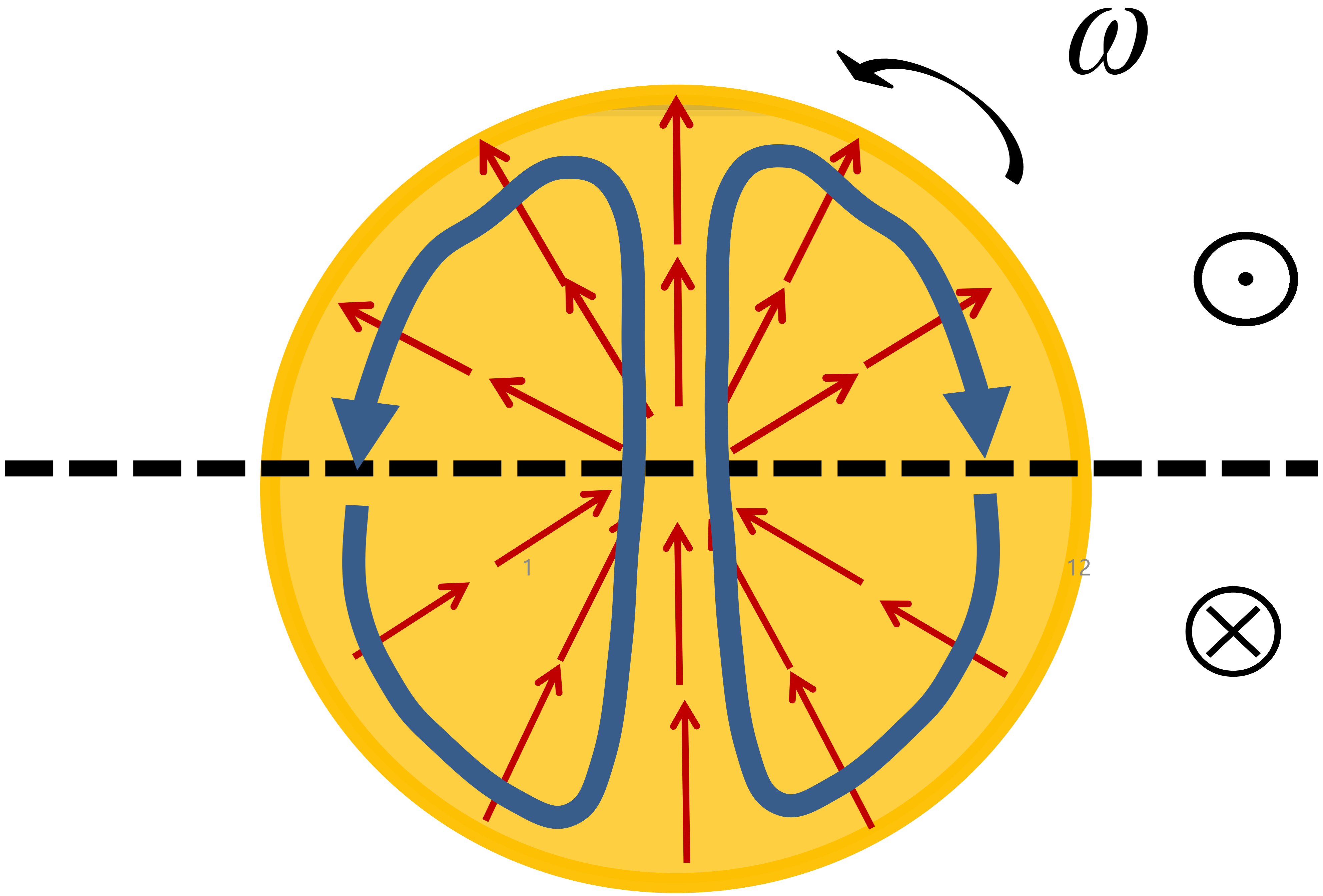
定性分析

均匀磁场 ✗　　非均匀磁场 ✓





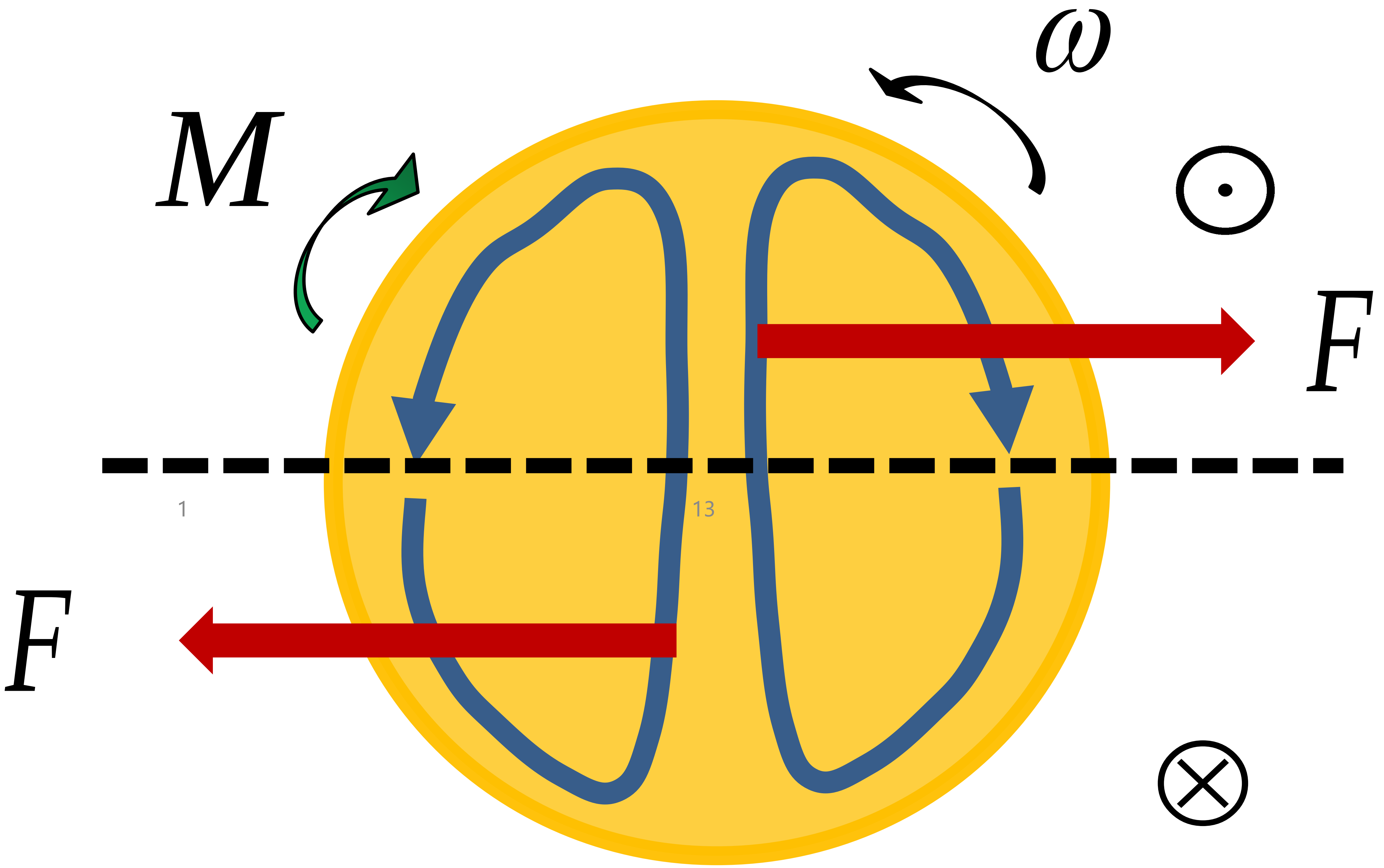
定性分析

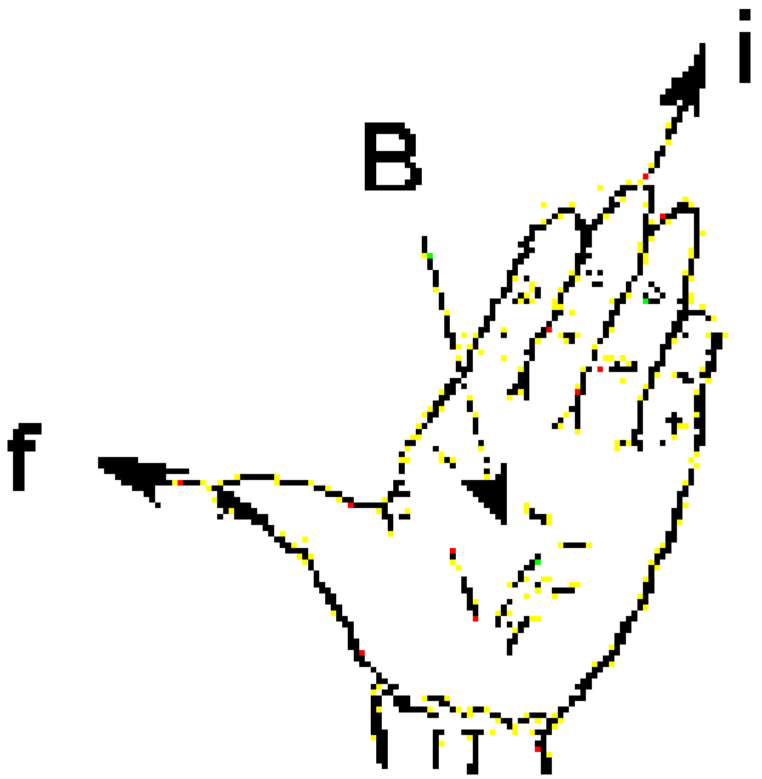


非静电场：

$$\vec{E}_N = \vec{v} \times \vec{B}$$

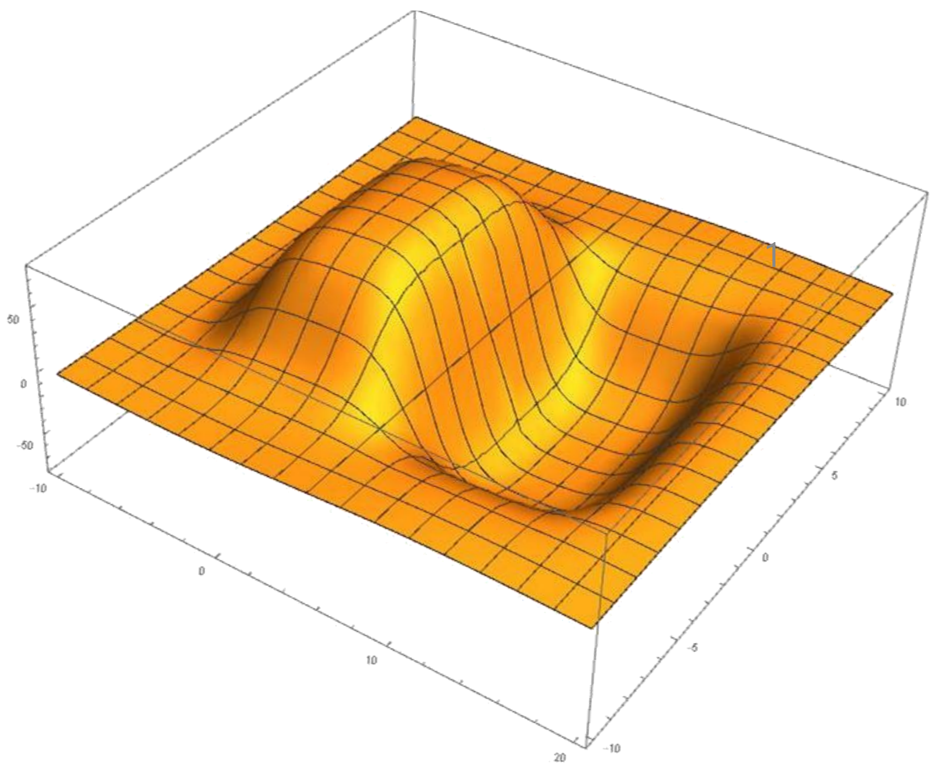
定性分析

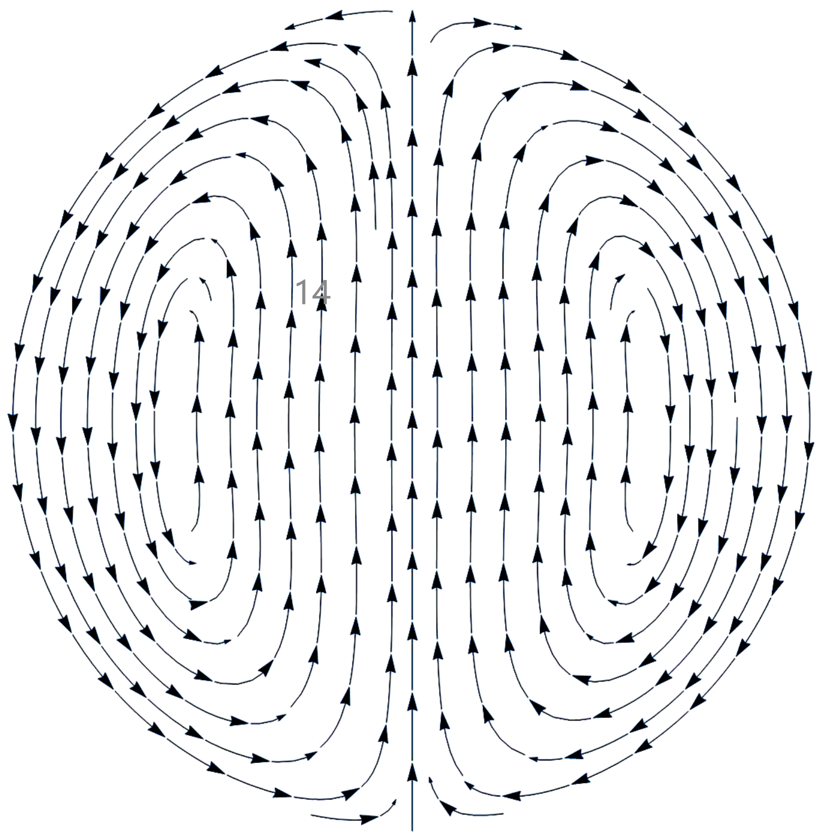


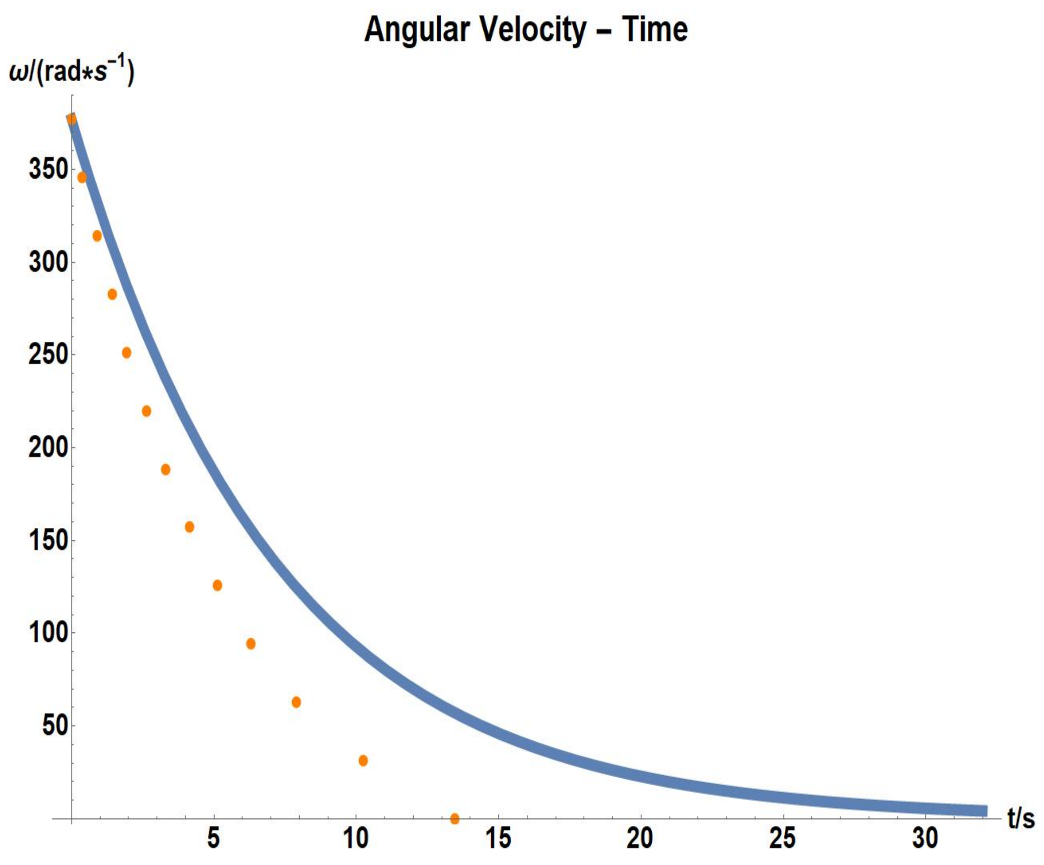


思路

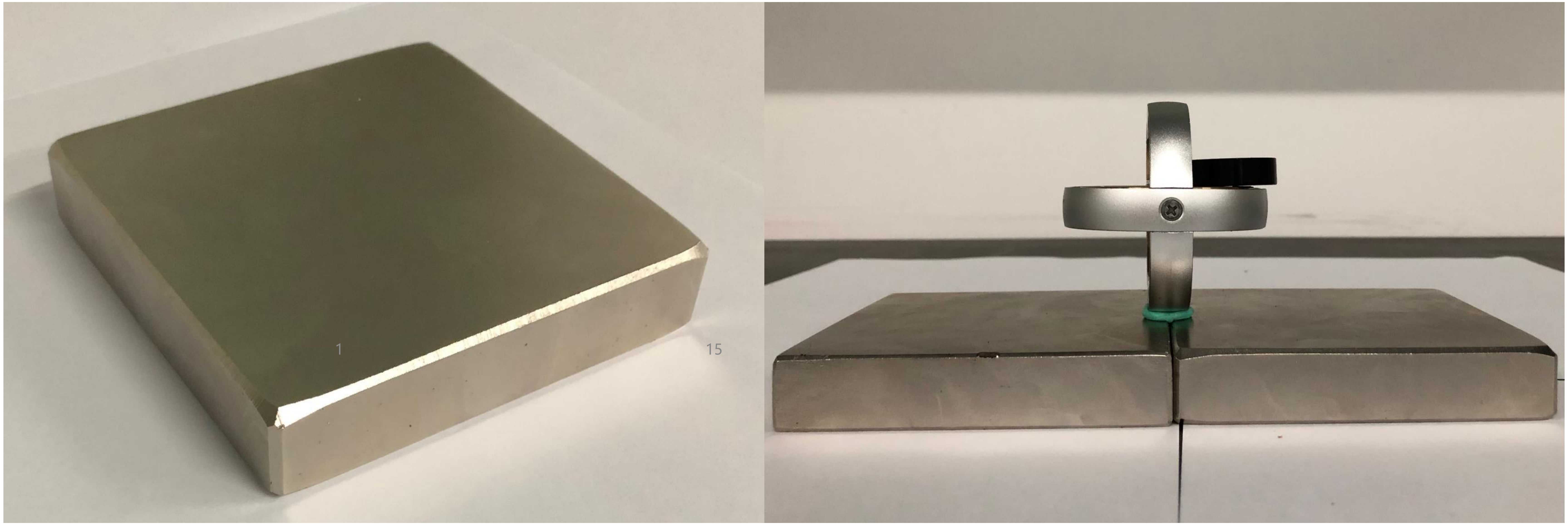
磁场分布→涡流分布→力矩作用



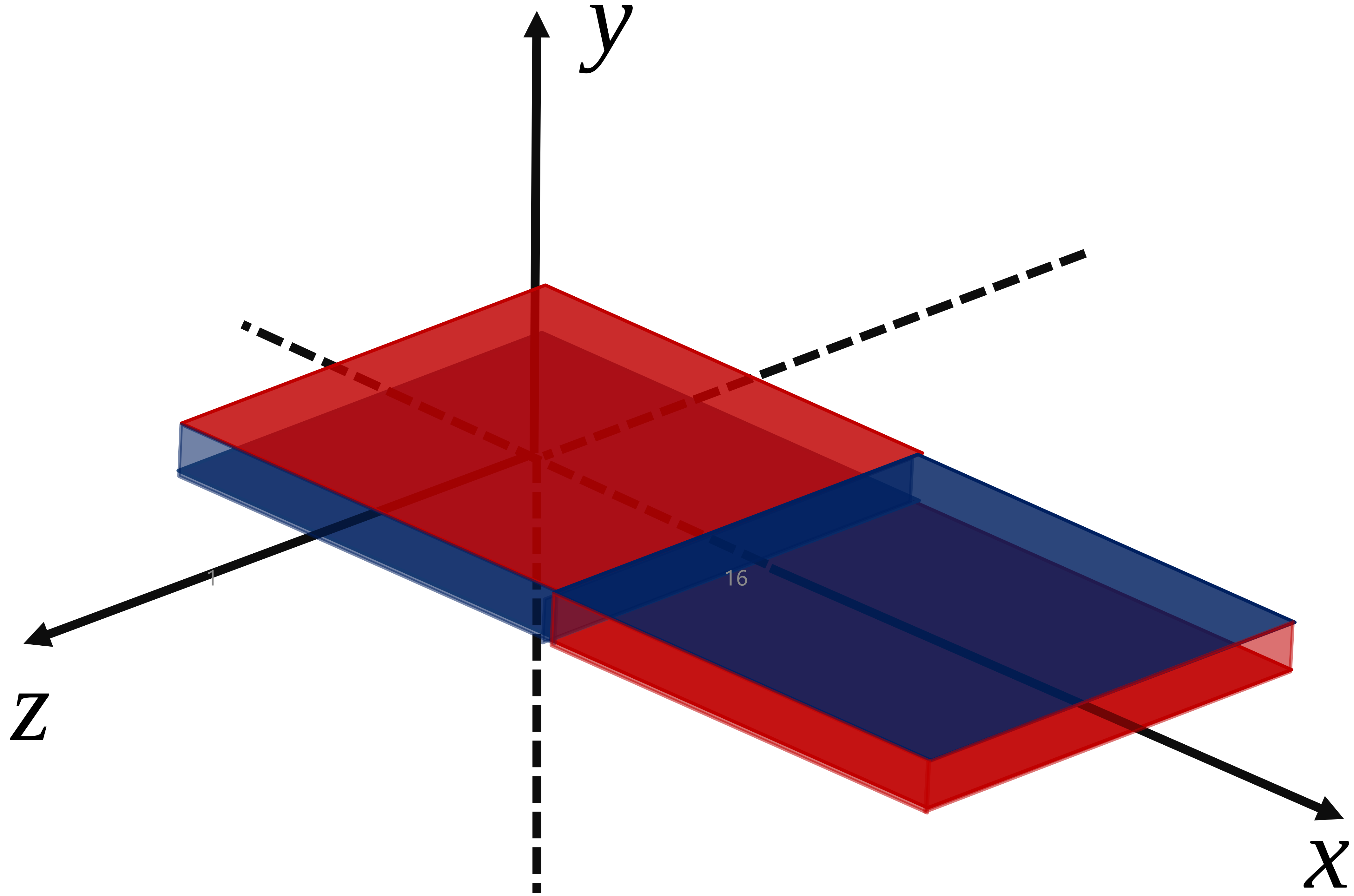




方形磁体



磁场计算



定量计算——磁场

$$B_{1x}=\frac{B_r}{4\pi}\ln\frac{[z_2-z+r(x_2,y_1,z_2)][z_1-z+r(x_1,y_1,z_1)]}{[z_2-z+r(x_1,y_1,z_2)][z_1-z+r(x_2,y_1,z_1)]},$$

$$B_{1y}=\frac{B_r}{4\pi}\left\{\tan^{-1}\left[\frac{(z_2-z)(x_1-x)}{(y_1-y)r(x_1,y_1,z_2)}\right]+\tan^{-1}\left[\frac{(z_1-z)(x_2-x)}{(y_1-y)r(x_2,y_1,z_1)}\right]\right.$$

$$\left.-\tan^{-1}\left[\frac{(z_1-z)(x_1-x)}{(y_1-y)r(x_1,y_1,z_1)}\right]-\tan^{-1}\left[\frac{(z_2-z)(x_2-x)}{(y_1-y)r(x_2,y_1,z_2)}\right]\right\},$$

$$B_{1z}=\frac{B_r}{4\pi}\ln\frac{[x_2-x+r(x_2,y_1,z_2)][x_1-x+r(x_1,y_1,z_1)]}{[x_2-x+r(x_2,y_1,z_1)][x_1-x+r(x_1,y_1,z_2)]},$$

where

$$r(x_i,y_j,z_k)=\sqrt{(x_i-x)^2+\left(y_j-y\right)^2+(z_k-z)^2}.$$

Weishi Wan.Magnetic Field Distribution of a Rectangular Permanent Magnet Block

定量计算——磁场

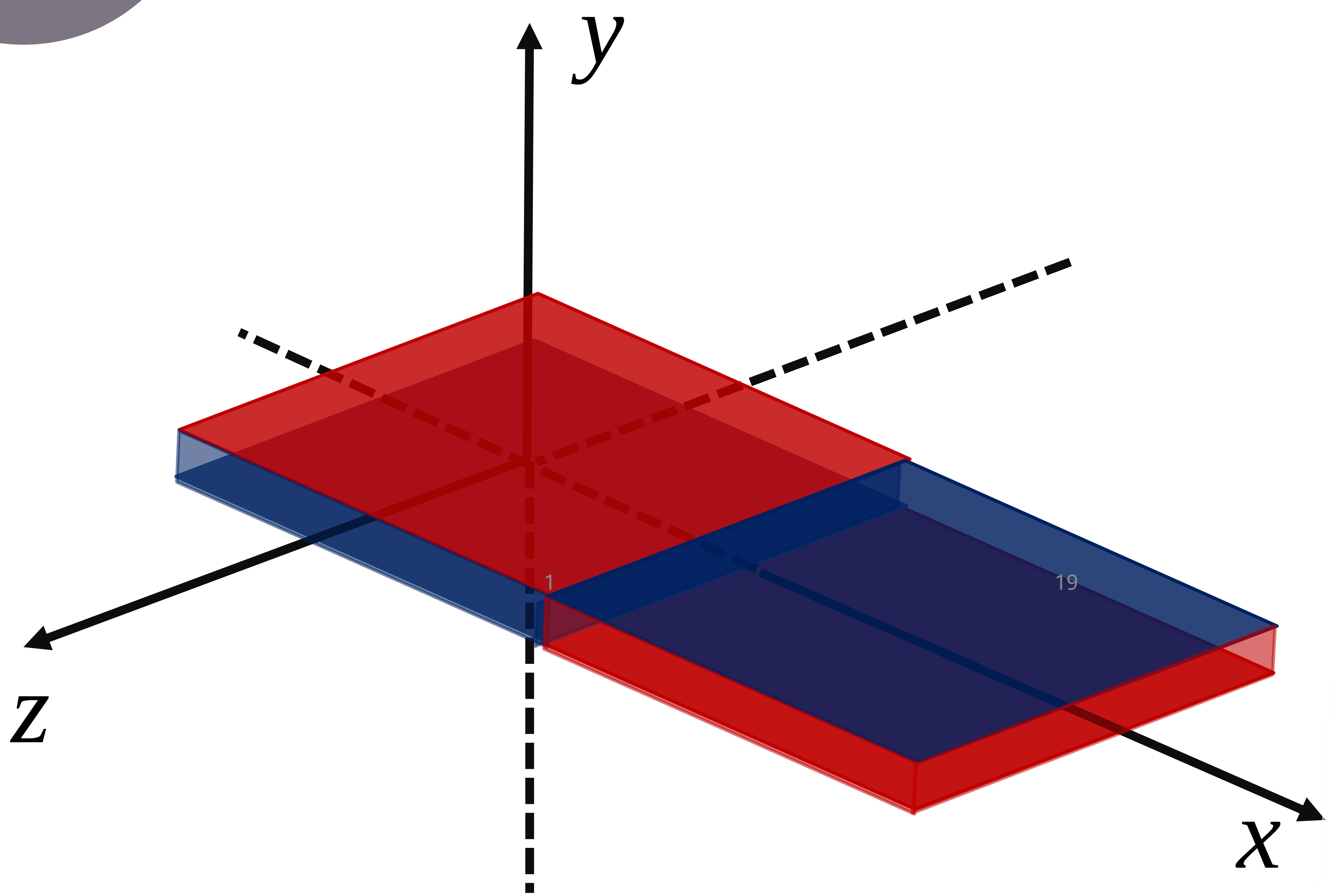
$$B_{1x}=\frac{\boxed{B_r}}{4\pi}\ln\frac{[z_2-z+r(x_2,y_1,z_2)][z_1-z+r(x_1,y_1,z_1)]}{[z_2-z+r(x_1,y_1,z_2)][z_1-z+r(x_2,y_1,z_1)]},$$

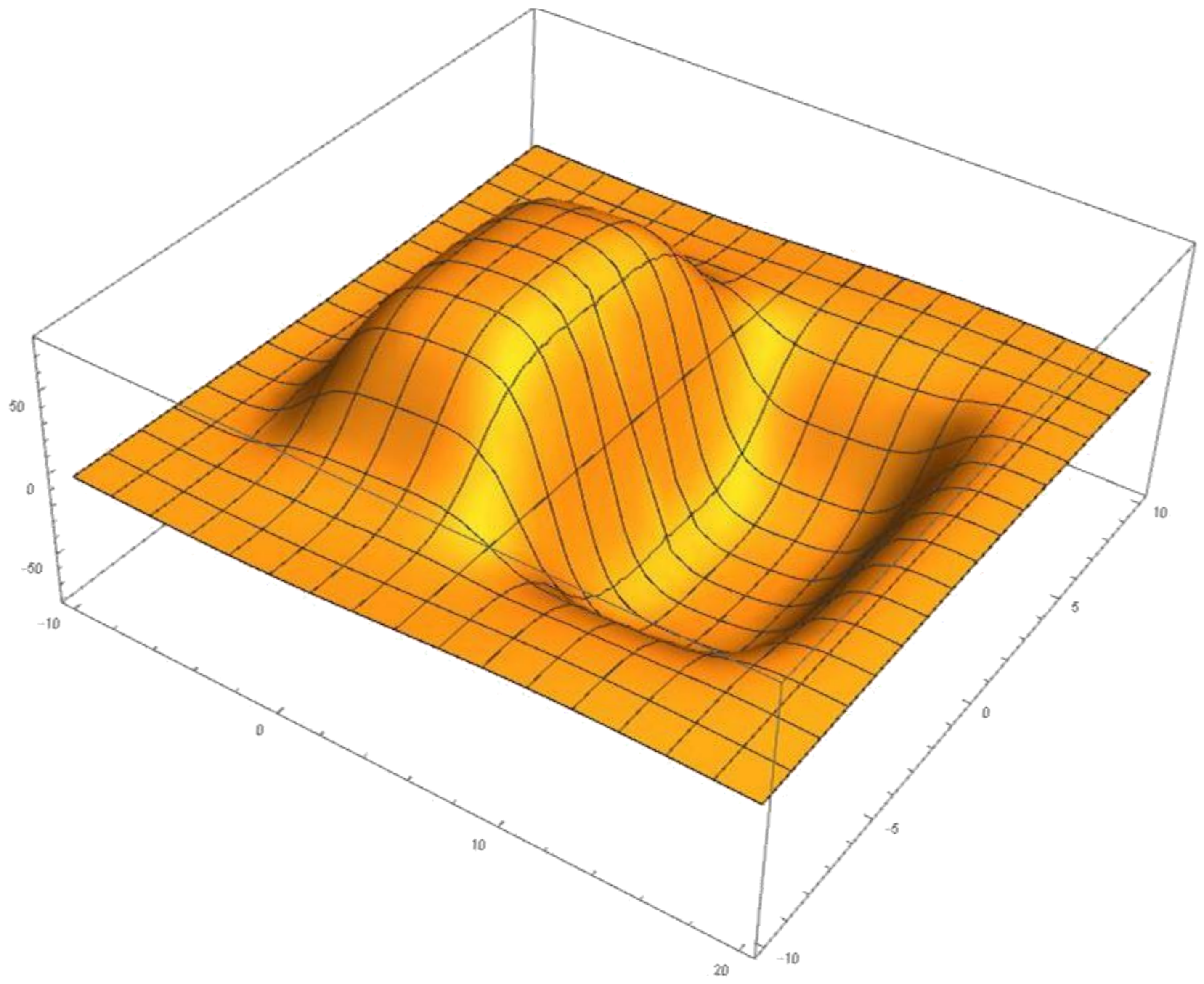
$$B_{1y}=\frac{\boxed{B_r}}{4\pi}\left\{\tan^{-1}\left[\frac{(z_2-z)(x_1-x)}{(y_1-y)r(x_1,y_1,z_2)}\right]+\tan^{-1}\left[\frac{(z_1-z)(x_2-x)}{(y_1-y)r(x_2,y_1,z_1)}\right]\right.$$

$$\left.-\tan^{-1}\left[\frac{(z_1-z)(x_1-x)}{(y_1-y)r(x_1,y_1,z_1)}\right]-\tan^{-1}\left[\frac{(z_2-z)(x_2-x)}{(y_1-y)r(x_2,y_1,z_2)}\right]\right\},$$

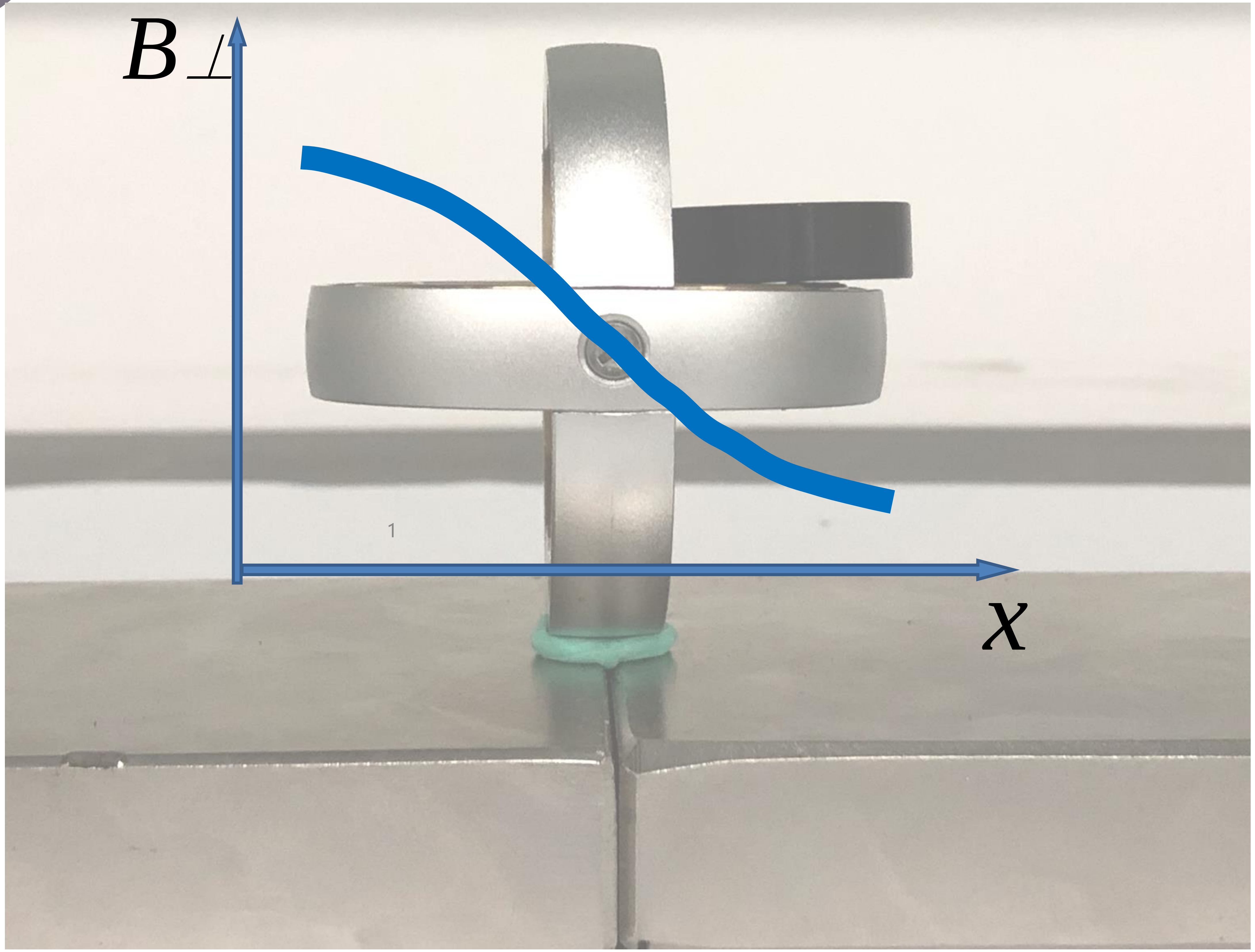
$$B_{1z}=\frac{\boxed{B_r}}{4\pi}\ln\frac{[x_2-x+r(x_2,y_1,z_2)][x_1-x+r(x_1,y_1,z_1)]}{[x_2-x+r(x_2,y_1,z_1)][x_1-x+r(x_1,y_1,z_2)]},$$

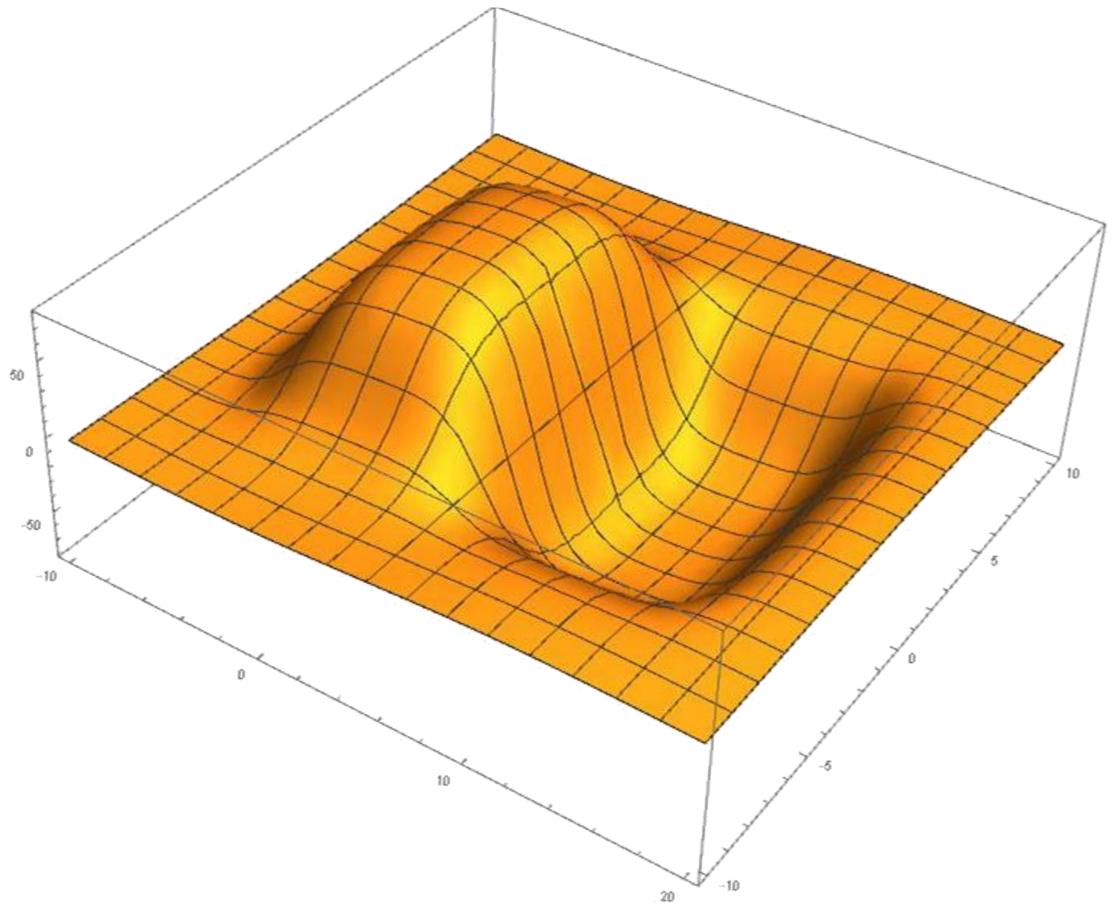
磁场计算





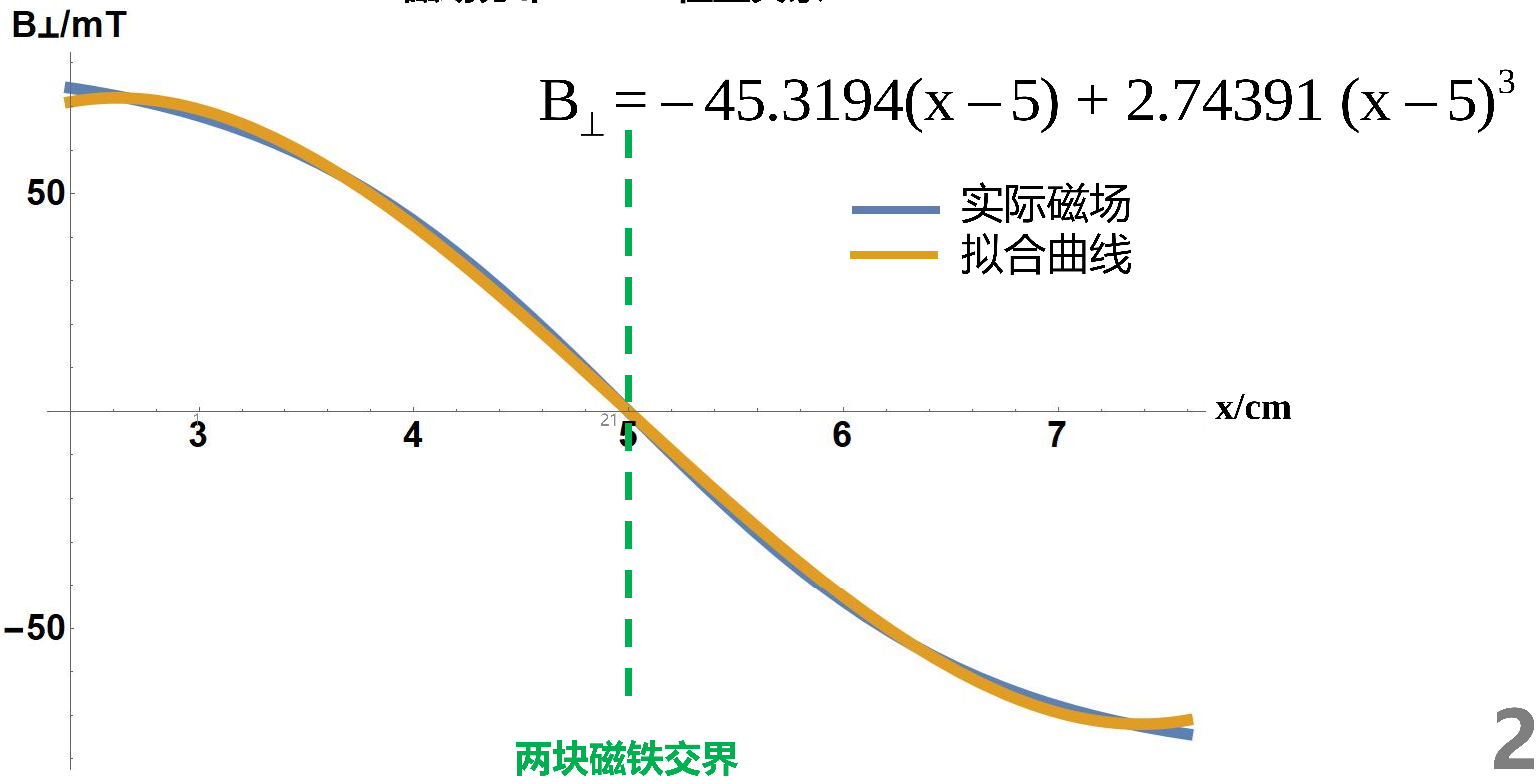
磁场分布



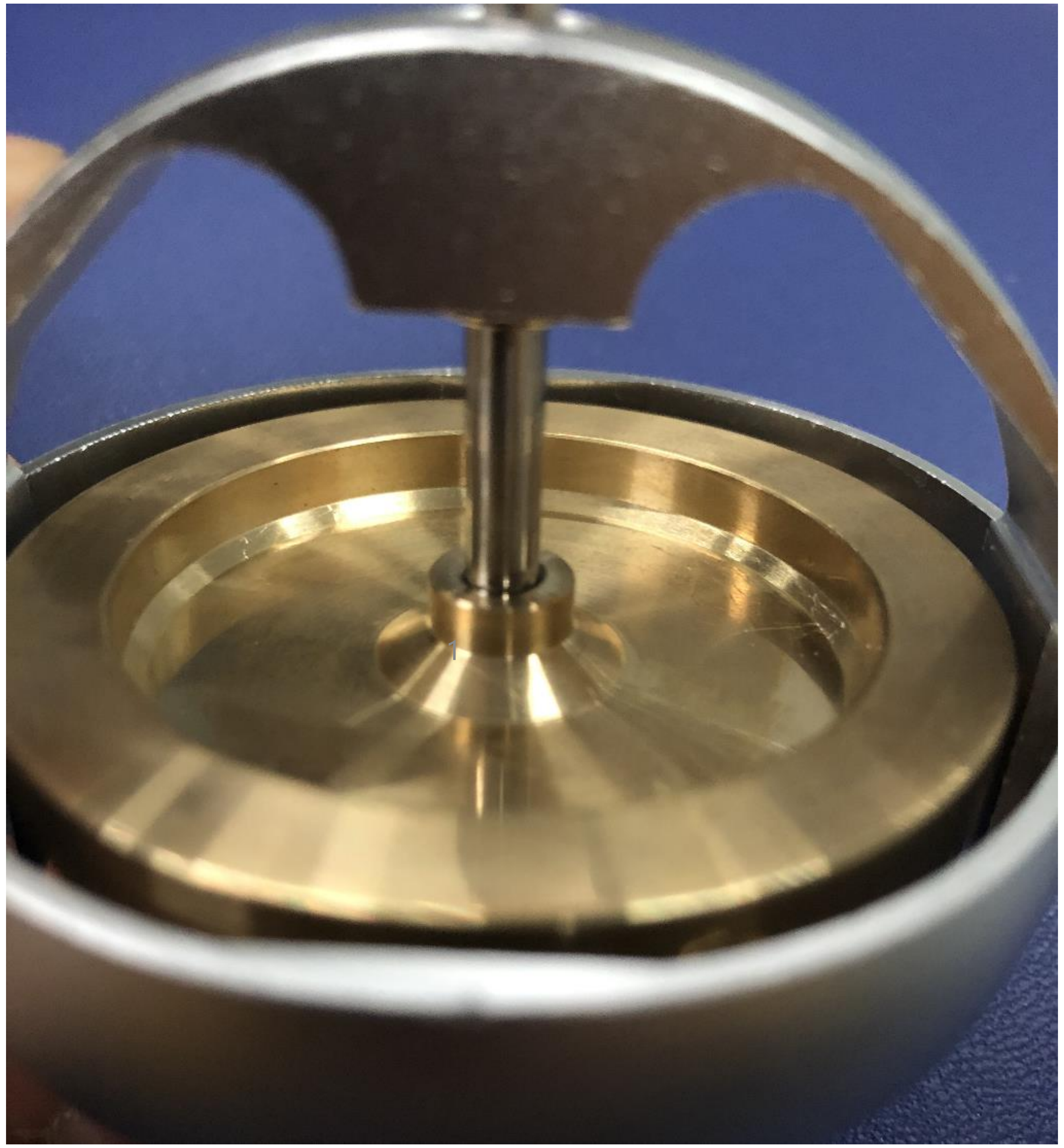


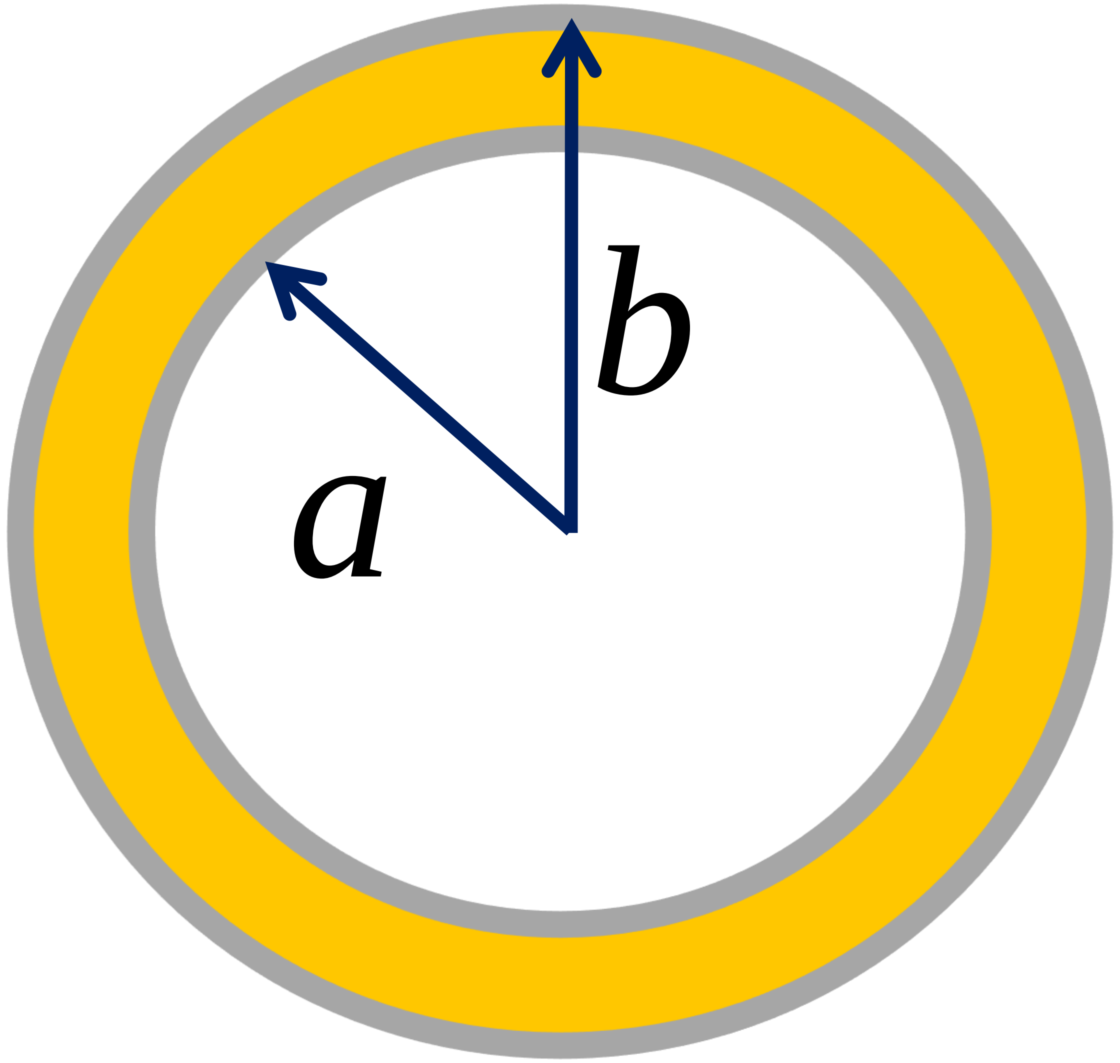
磁场拟合

磁场分布：B⊥ – 位置关系



涡流





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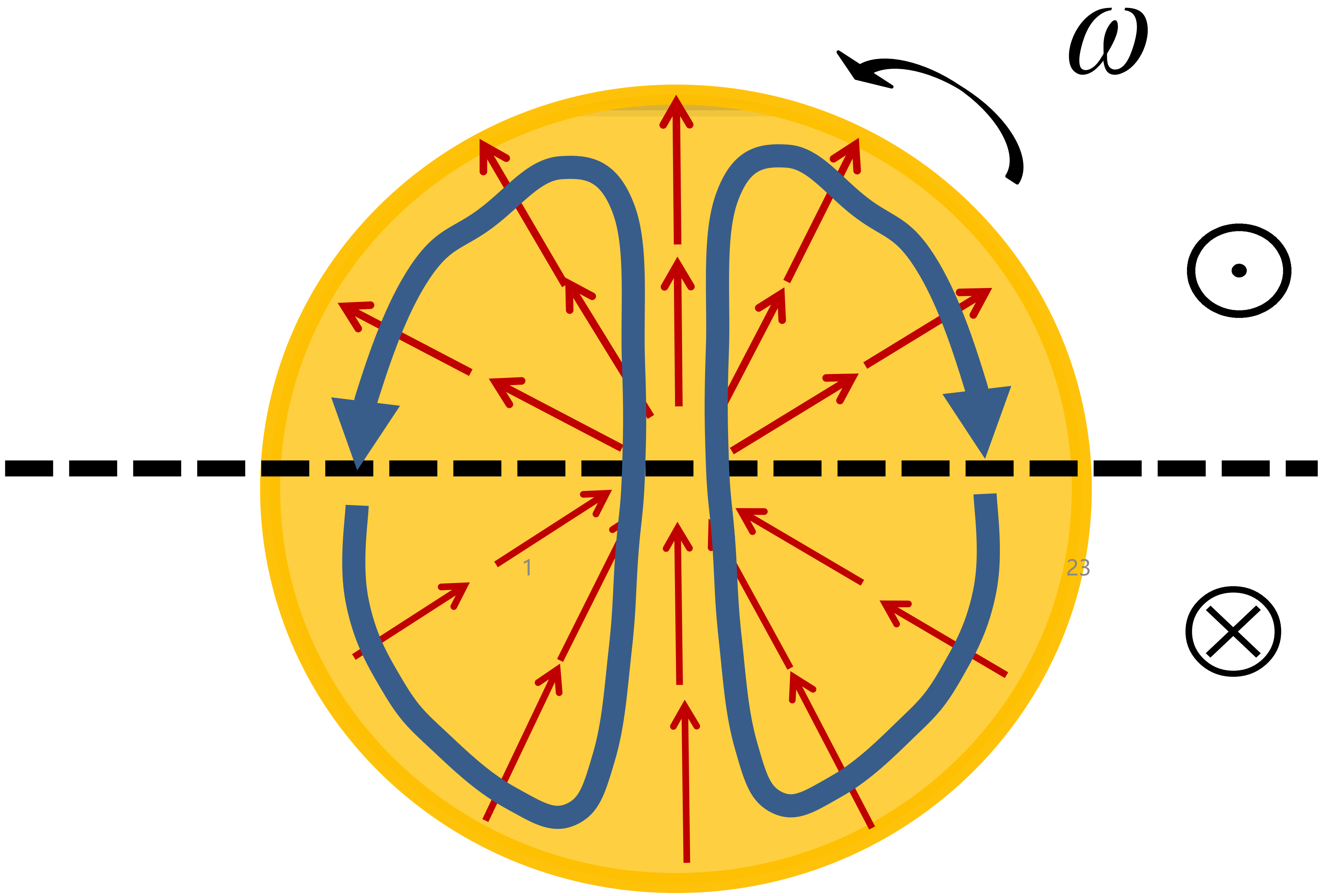
涡流计算



$$
\begin{cases}
\vec{E}_N = \vec{v} \times \vec{B} \\
\vec{j} = \sigma(\vec{E} + \vec{E}_N) \\
\nabla \cdot \vec{j} = 0
\end{cases}
$$

电流分布的解析解

偏微分方程

$$\begin{cases} \vec{E}_N = \vec{v}\times\vec{B} \\ \vec{j} = \sigma(\vec{E}+\vec{E}_N) \\ \nabla\cdot\vec{j} = 0 \end{cases}$$

$$\Longrightarrow \nabla\cdot\vec{E} = -\nabla\cdot\vec{E}_N \quad \boxed{\nabla^2\varphi = \nabla\cdot\vec{E}_N}$$

边界条件

$$\begin{cases} \vec{j}\Big|_{r=b} = 0 \\ \vec{j}\Big|_{r=a} = 0 \end{cases} \Leftrightarrow \begin{cases} (\vec{E}+\vec{E}_N)_r\Big|_{r=b} = 0 \\ (\vec{E}+\vec{E}_N)_r\Big|_{r=a} = 0 \end{cases} \Leftrightarrow \boxed{\begin{cases} \dfrac{\partial\varphi}{\partial r}\Big|_{r=b} = \vec{E}_{Nr}\Big|_{r=b} \\ \dfrac{\partial\varphi}{\partial r}\Big|_{r=a} = \vec{E}_{Nr}\Big|_{r=a} \end{cases}}$$

1

电流分布的解析解

电流密度的径向方向分量：$j_r(r,\theta)$

$$j_r(r,\theta)=\sigma k_1\omega\sin\theta\left[r^2-\frac{1}{8}(-a^2-b^2+\frac{a^2b^2}{r^2}+9r^2)\right]$$

$$+\sigma(\frac{1}{4}k_3r^4\omega(3\sin\theta-\sin3\theta)-k_3\omega(\frac{3}{4}\sin\theta\left[a^4+a^2b^2+b^4-\frac{a^2b^2(a^2+b^2)}{r^2}\right]$$

$$-\frac{1}{12}\sin3\theta\left[-\frac{3a^6b^6}{(a^4+a^2b^2+b^4)r^4}+\frac{3(a^8-b^8)r^2}{a^6-b^6}\right]$$

$$+\frac{5}{4}\{\frac{1}{8}\sin\theta\left[-5(a^4+a^2b^2+b^4)+\frac{5a^2b^2(a^2+b^2)}{r^2}+5r^4\right]$$

$$+\frac{1}{48}\sin3\theta\left[-\frac{15a^6b^6}{(a^4+a^2b^2+b^4)r^4}+\frac{15(a^6+a^4b^2+a^2b^4+b^6)r^2}{a^4+a^2b^2+b^4}-15r^4\right]\}))$$

电流分布的解析解

电流密度的角向方向分量：jθ(r,θ)

$$j_\theta(r,\theta)=-\frac{k_1\sigma\omega\cos\theta}{8r}\left(-\frac{a^2b^2}{r}-(a^2+b^2)r+3r^3\right)-\frac{k_3\sigma\omega}{r}\Bigg(\frac{3}{4}\cos\theta$$

$$\left(\frac{a^2b^2(a^2+b^2)}{r}+(a^4+a^2b^2+b^4)r\right)-\frac{1}{4}\cos 3\theta\left(\frac{a^6b^6}{(a^4+a^2b^2+b^4)r^3}+\frac{(a^8-b^8)r^3}{a^6-b^6}\right)+$$

$$+\frac{5}{48}\cos\theta\left(-\frac{5a^2b^2(a^2+b^2)}{r}-5r(a^4+a^2b^2+b^4)+r^5\right)$$

$$+\frac{1}{16}\cos 3\theta\left(\frac{5a^6b^6}{(a^4+a^2b^2+b^4)r^3}+\frac{5(a^6+a^4b^2+a^2b^4+b^6)r^3}{a^4+a^2b^2+b^4}-3r^5\right)\Bigg)$$

电流分布的解析解

电流密度的角向方向分量：jθ(r,θ)

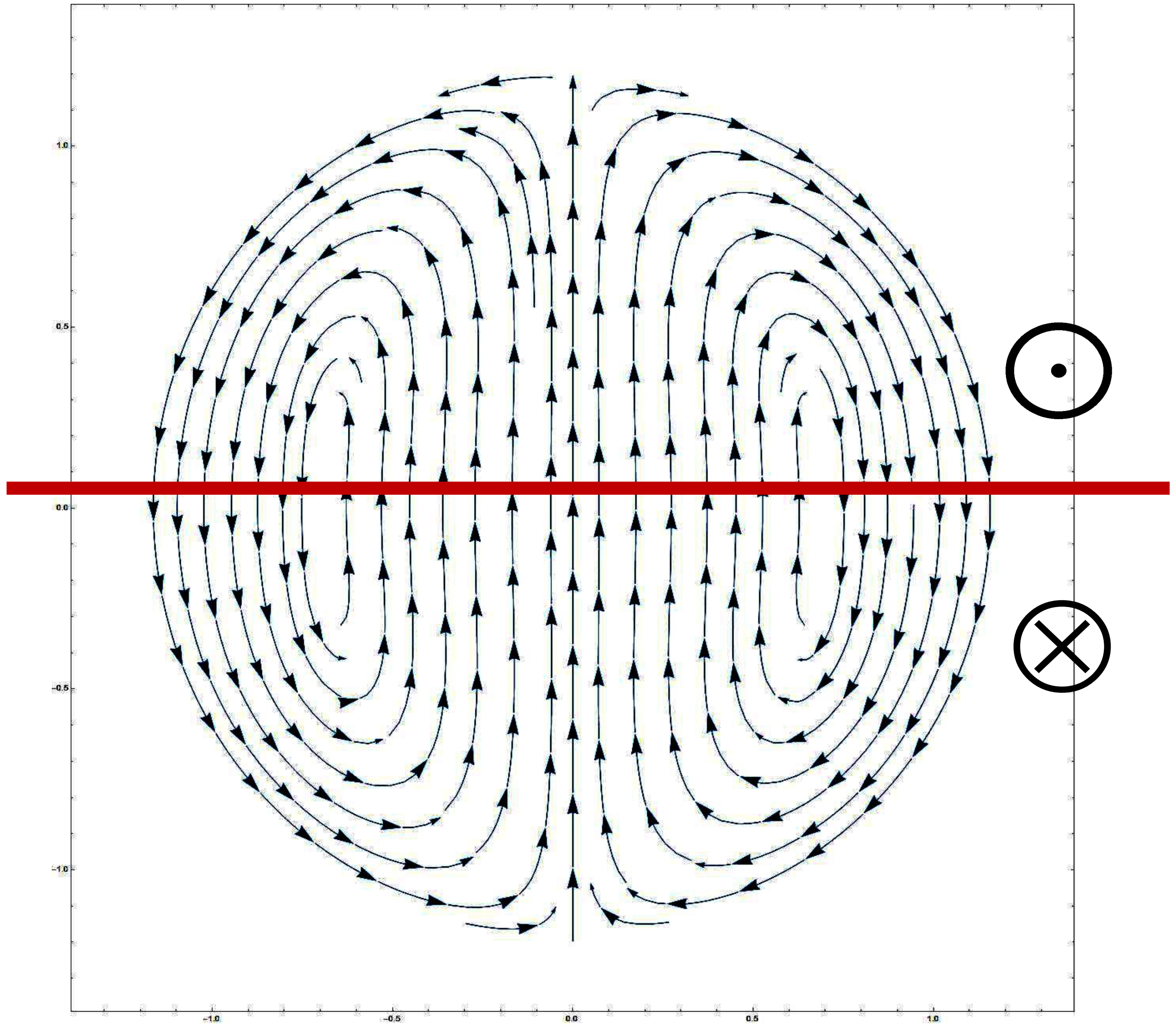
$$
\mathrm{j1R}[r,\theta][[2]]+\mathrm{j3R}[r,\theta][[2]]=
$$

$$
-\frac{\mathrm{k1}\left(-\frac{a^2 b^2}{r}-\left(a^2+b^2\right) r+3 r^3\right)\sigma\,\omega\,\mathrm{Cos}[\theta]}{8 r}-\frac{1}{r}\,\mathrm{k3}\,\sigma\,\omega\left(\frac{3}{4}\left(\frac{a^2 b^2\left(a^2+b^2\right)}{r}+\left(a^4+a^2 b^2+b^4\right) r\right)\underset{\text{余弦}}{\mathrm{Cos}[\theta]}-\frac{1}{4}\left(\frac{a^6 b^6}{\left(a^4+a^2 b^2+b^4\right) r^3}+\frac{\left(a^8-b^8\right) r^3}{a^6-b^6}\right)\underset{\text{余弦}}{\mathrm{Cos}[3\theta]}+\right.
$$

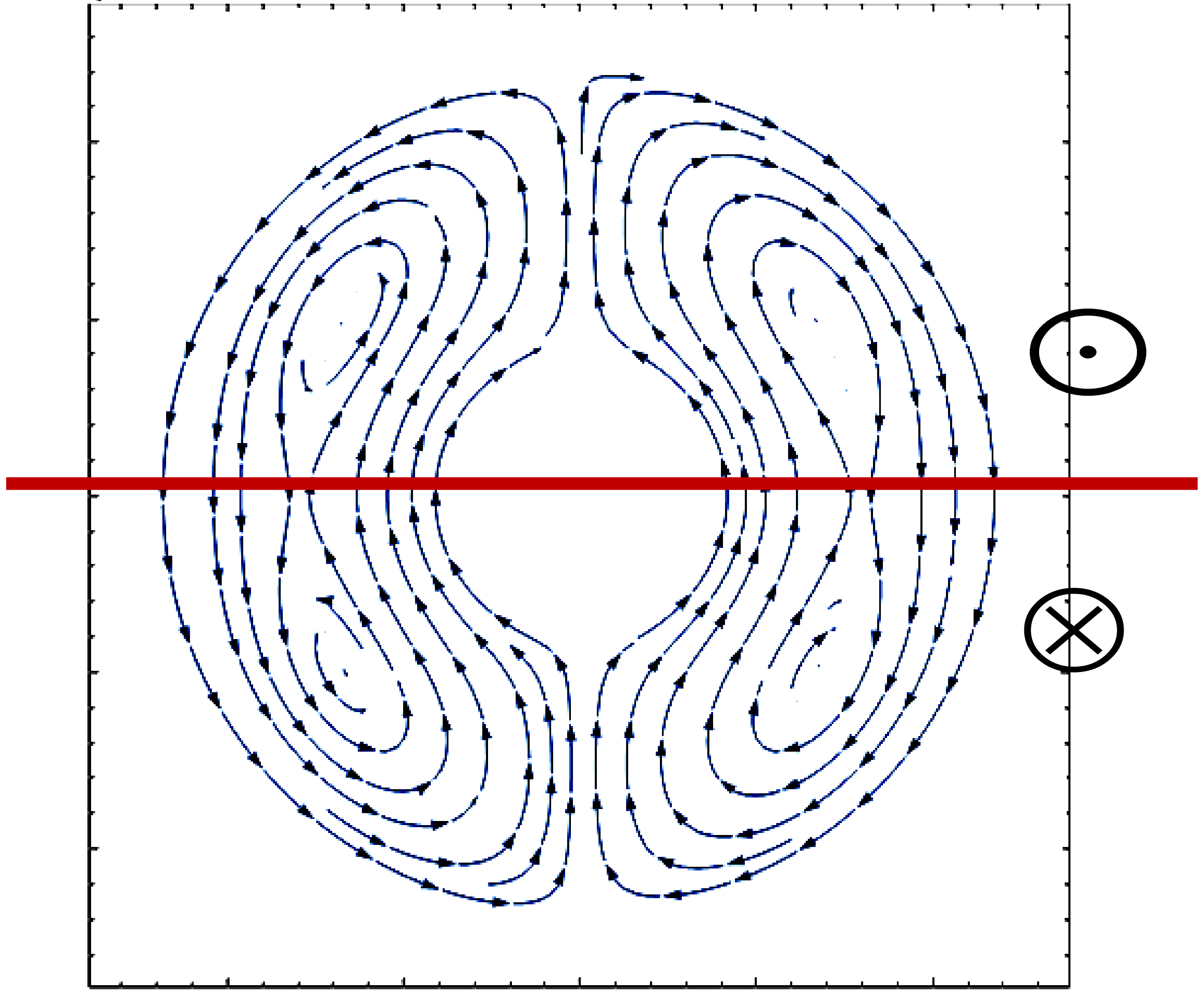
$$
\left.\frac{5}{4}\left(\frac{1}{8}\left(-\frac{5 a^2 b^2\left(a^2+b^2\right)}{r}-5\left(a^4+a^2 b^2+b^4\right) r+r^5\right)\underset{\text{余弦}}{\mathrm{Cos}[\theta]}+\frac{1}{16}\left(\frac{5 a^6 b^6}{\left(a^4+a^2 b^2+b^4\right) r^3}+\frac{5\left(a^6+a^4 b^2+a^2 b^4+b^6\right) r^3}{a^4+a^2 b^2+b^4}-3 r^5\right)\underset{\text{余弦}}{\mathrm{Cos}[3\theta]}\right)\right)
$$

涡流分布($B=k_1x+k_3x^3$)

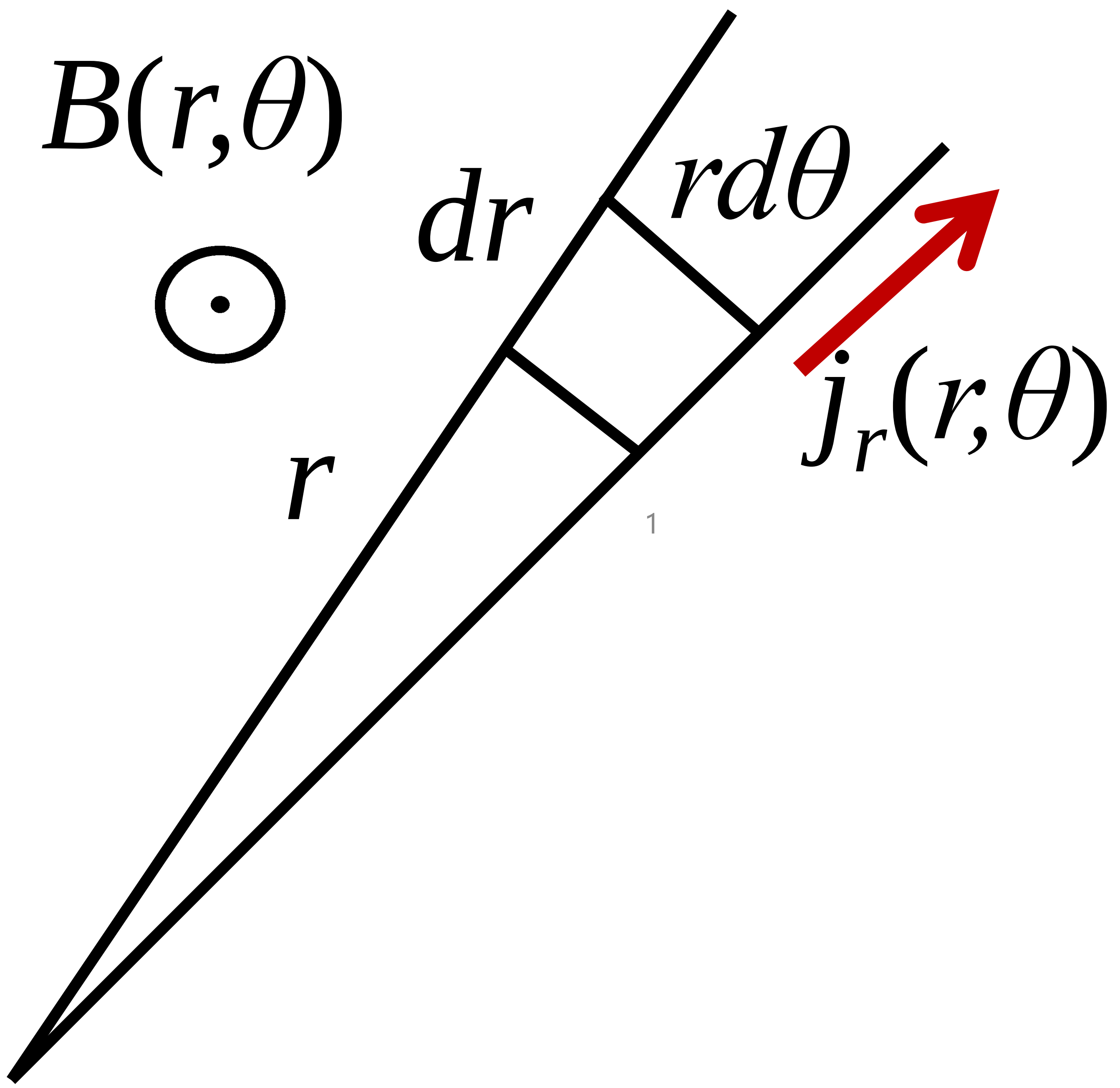
$$\vec{j}\Big|_{r=b}=0$$



$$\begin{cases} \vec{j}\Big|_{r=b} = 0 \\ \vec{j}\Big|_{r=a} = 0 \end{cases}$$



力矩计算



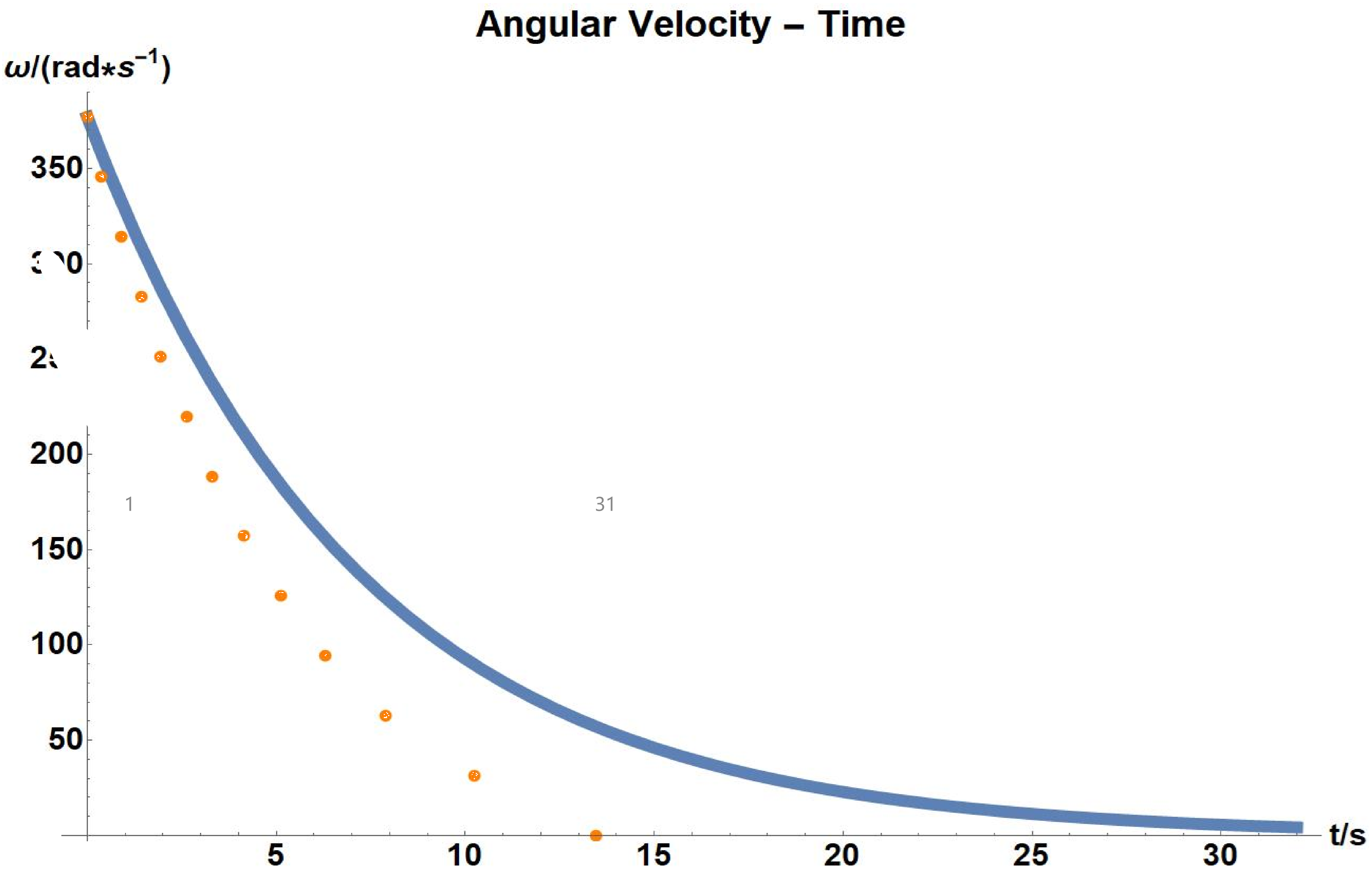
$$I = j_r(r,\theta) r h d\theta$$

$$dM = B(r,\theta) I dr \cdot r$$

$$M = \iint_{陀螺} dM$$

$$M = \iint_{陀螺} B(r,\theta) j_r(r,\theta)(rd\theta) hrdr$$

结果



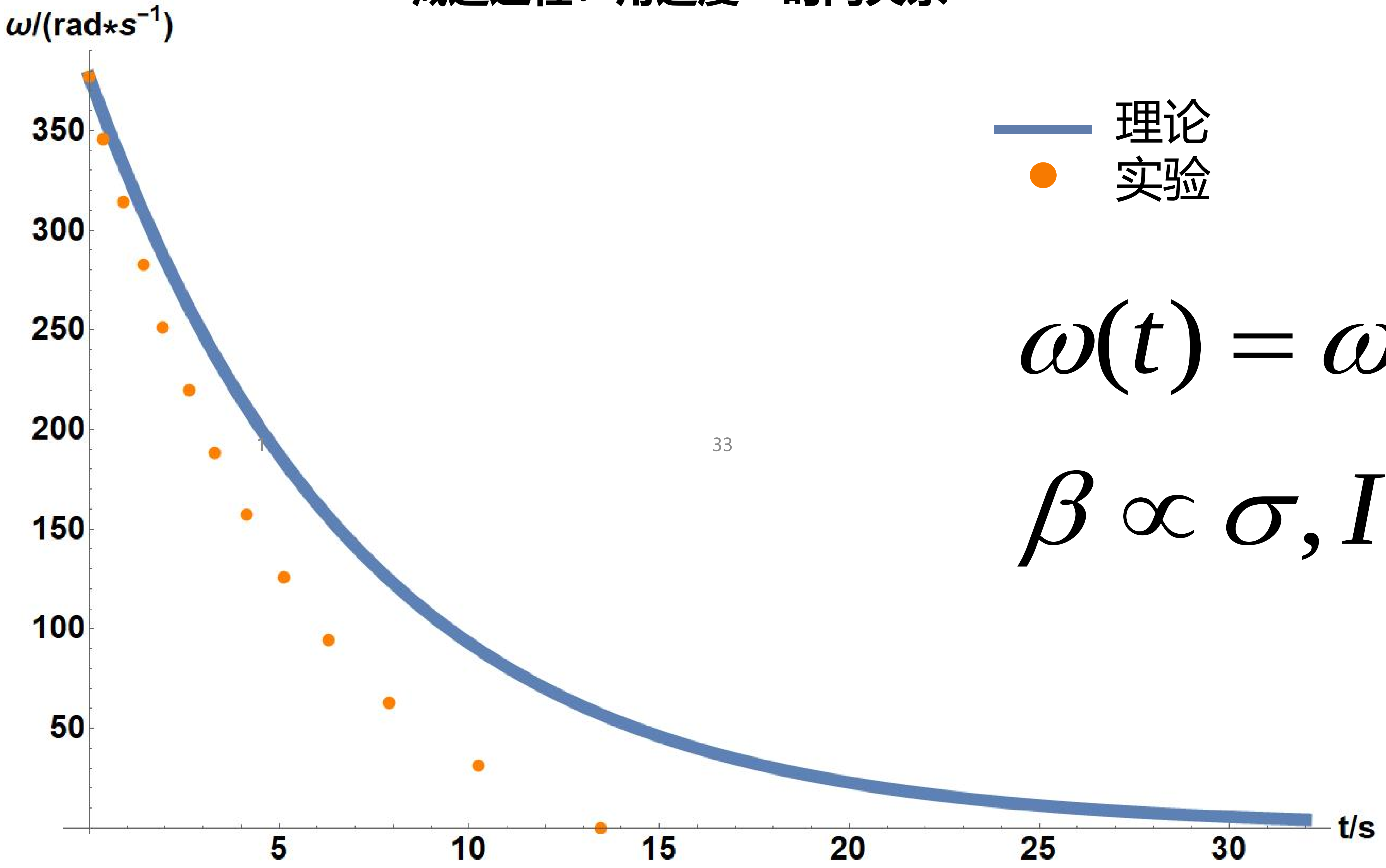
Angular Velocity – Time
ω/(rad*s⁻¹)
350
200
150
100
50
5
10
15
20
25
30
t/s

第三部分

理论与实验对照

定量计算

减速过程：角速度 – 时间关系

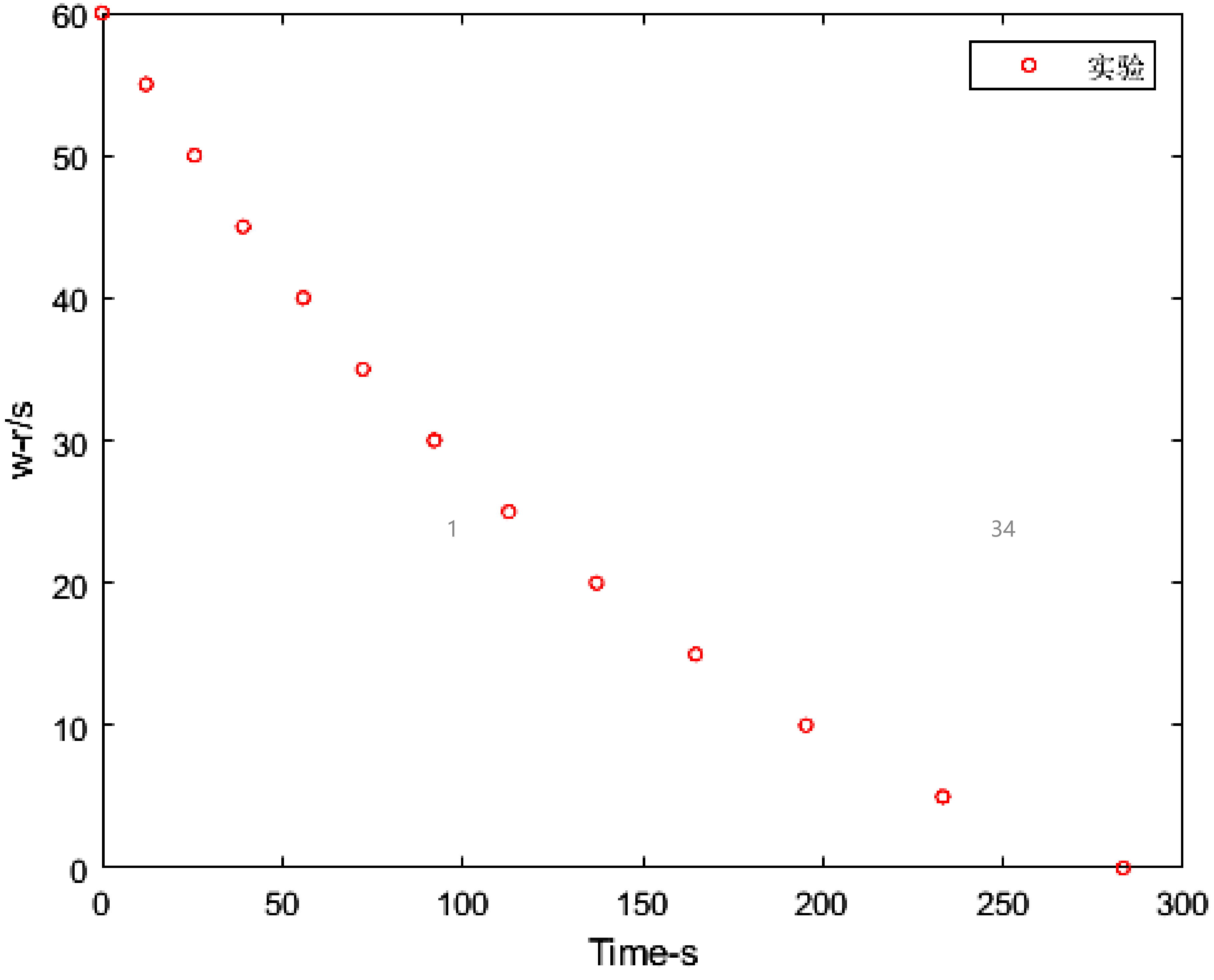


$$\omega(t) = \omega_0 e^{-\beta t}$$

$$\beta \propto \sigma, I^{-1}$$

非电磁阻力作用

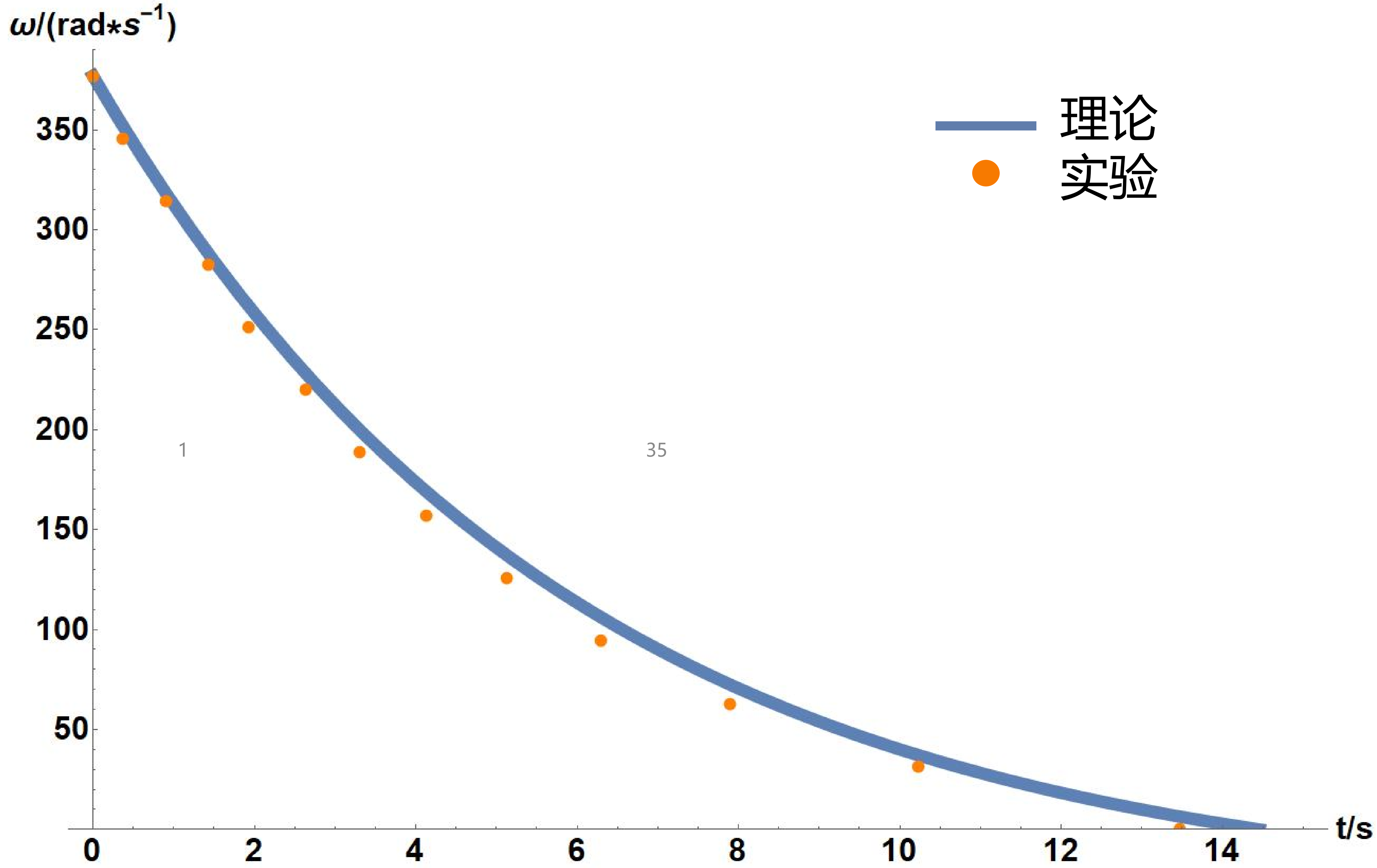
仅有机械阻力作用：转速 – 时间关系



$$M_f = \alpha\omega + M_0$$

最终结果(7.5%误差)

减速过程：角速度 – 时间关系



改变参量——磁场

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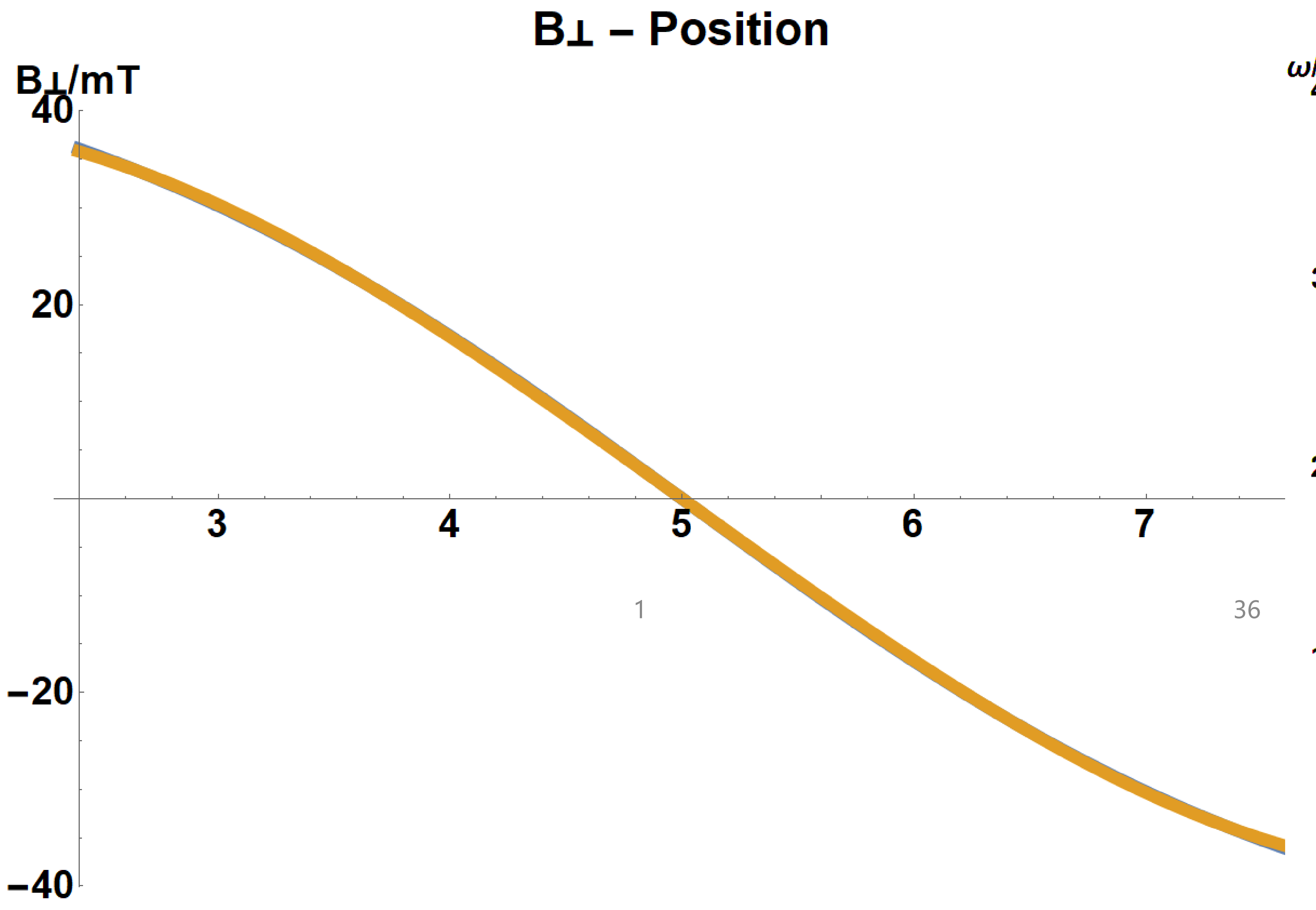
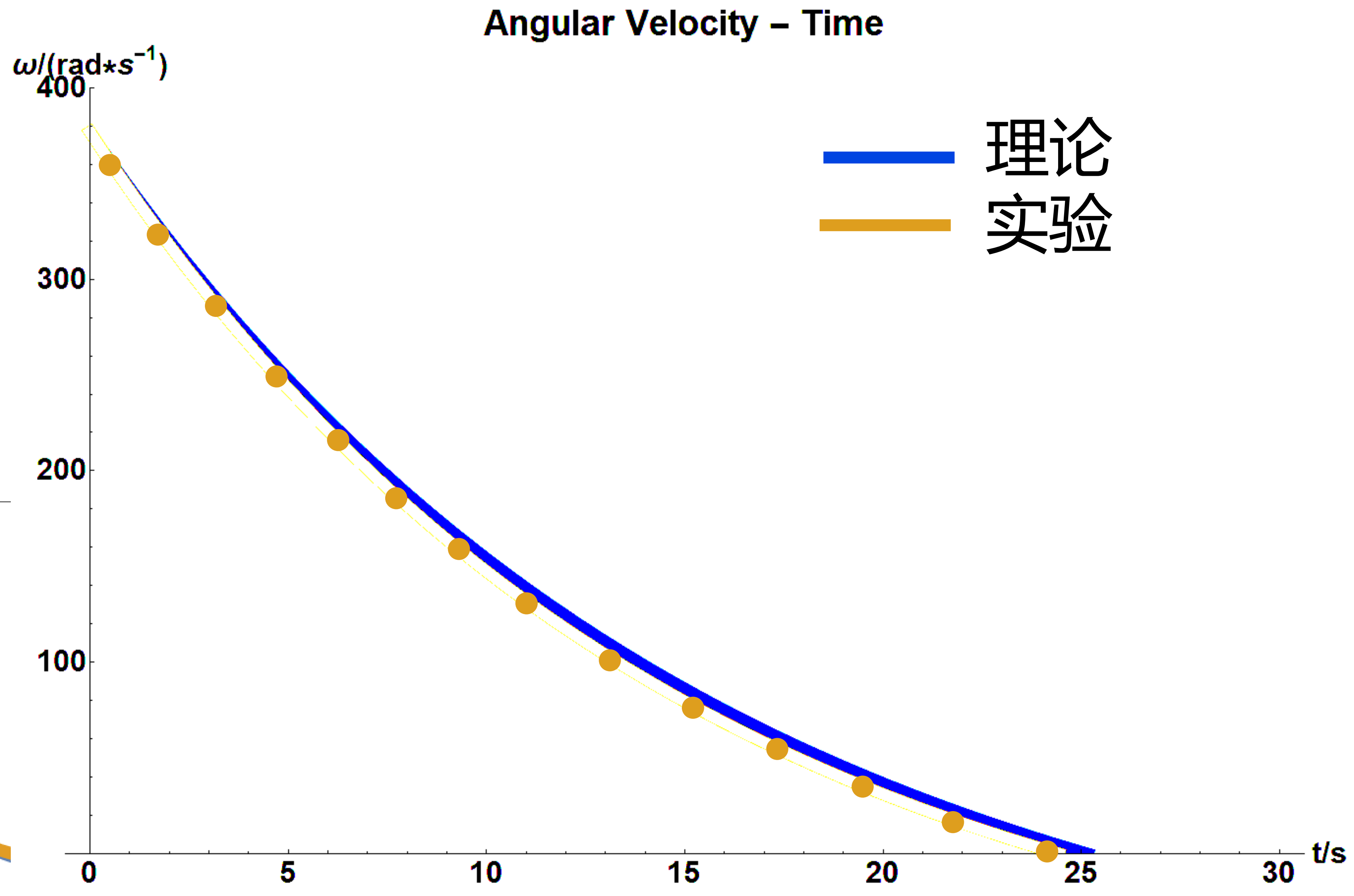
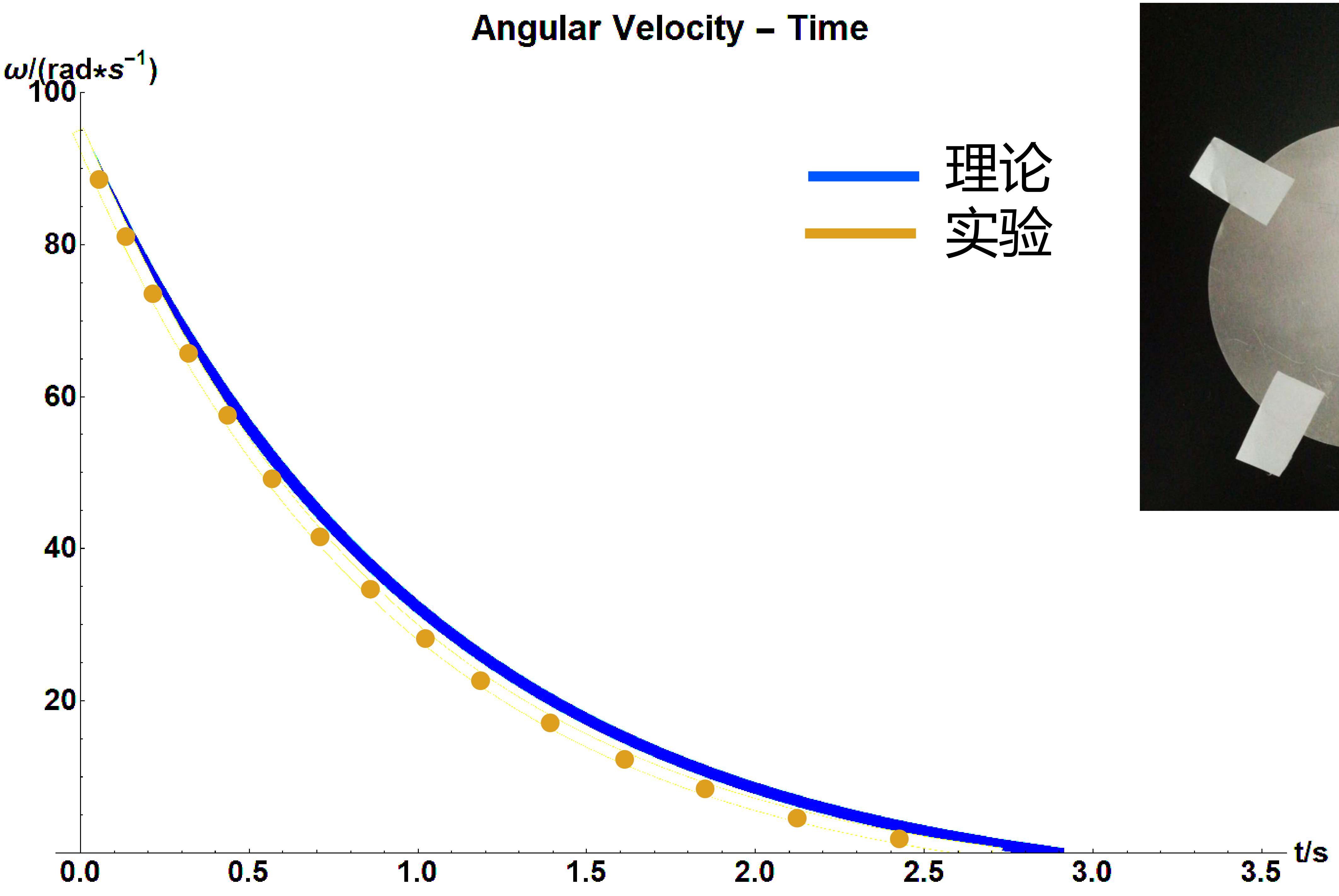


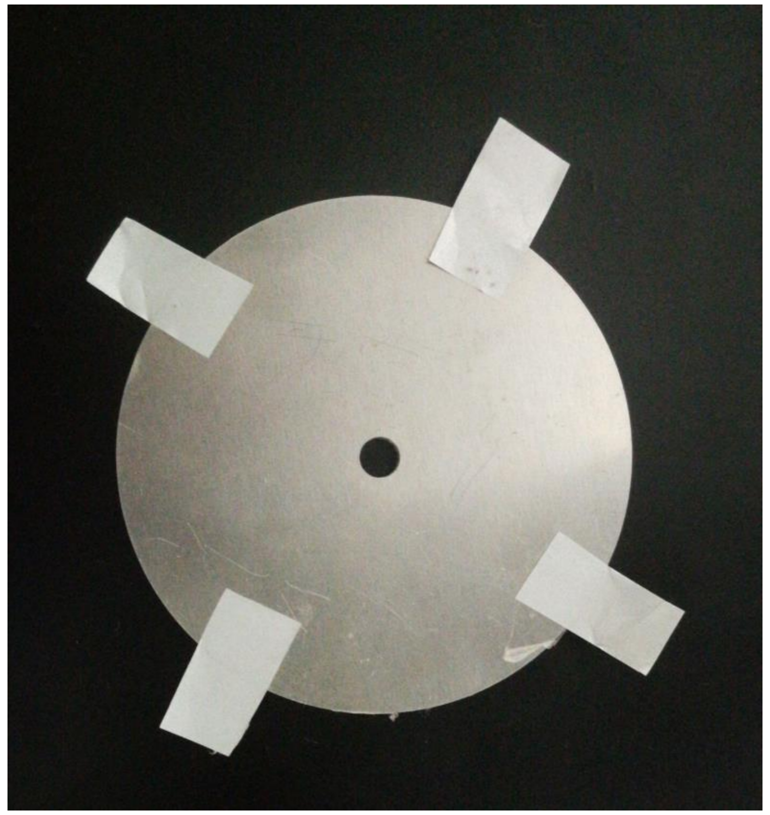


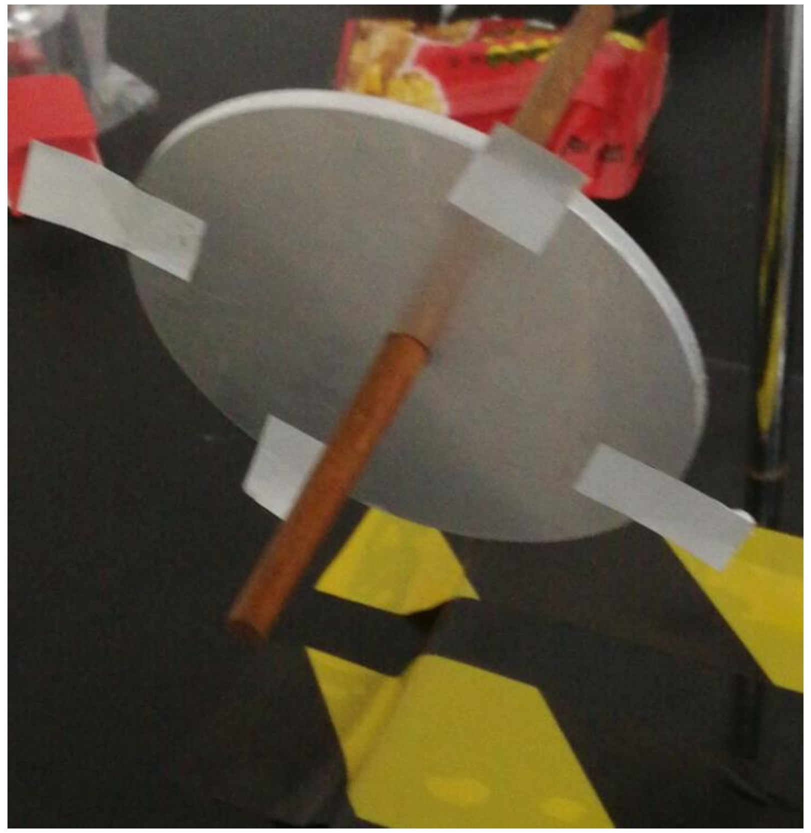
5cm高度处磁场

相对误差5.72%

改变参量——电导率







相对误差9.45%

结论

1.磁场中旋转产生涡流，导致力矩，得出角速度与时间的关系

2.拟合非电磁阻力力矩，对理论模型进行修正

3.相关参量的讨论：　38

电导率，转动惯量，磁场分布

第四部分

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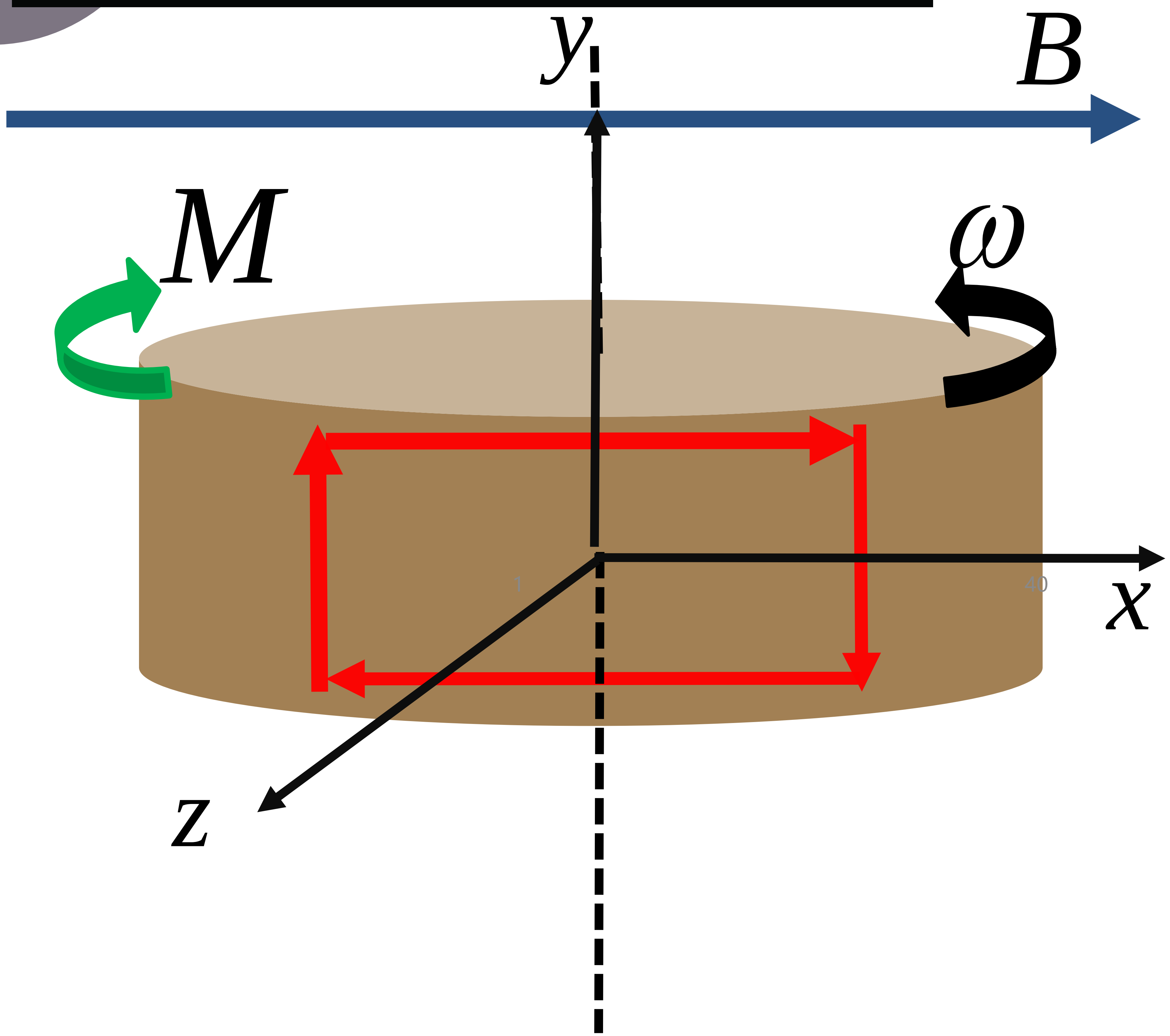
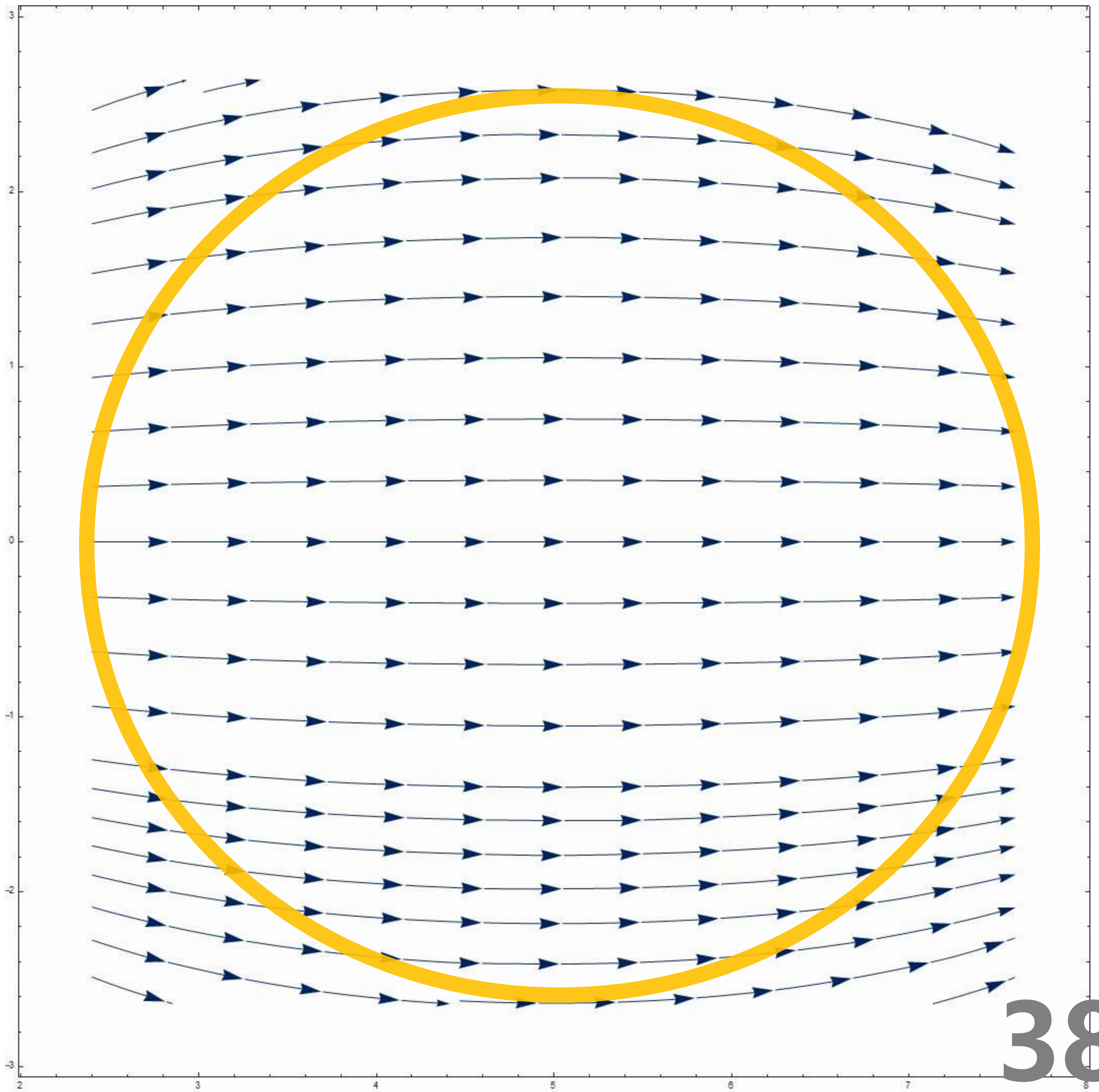
误差分析

理论偏差





其它误差来源

1.趋肤效应，去磁效应

2.磁场竖直方向分布的不均匀

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3.黄铜材料电阻率与标准值的微小偏差

遗留问题

1.更多电阻率材料的实验

2.横向磁场力矩计算

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3.更复杂的磁场情况

总结

1.频闪灯测量转速获得实验数据

2.涡流分布推导阻力力矩，角速度关系

3.阻力实测修正理论

4.实验理论结果对比，相关误差分析

参考文献

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谢谢

上海科技大学
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定量计算——磁场

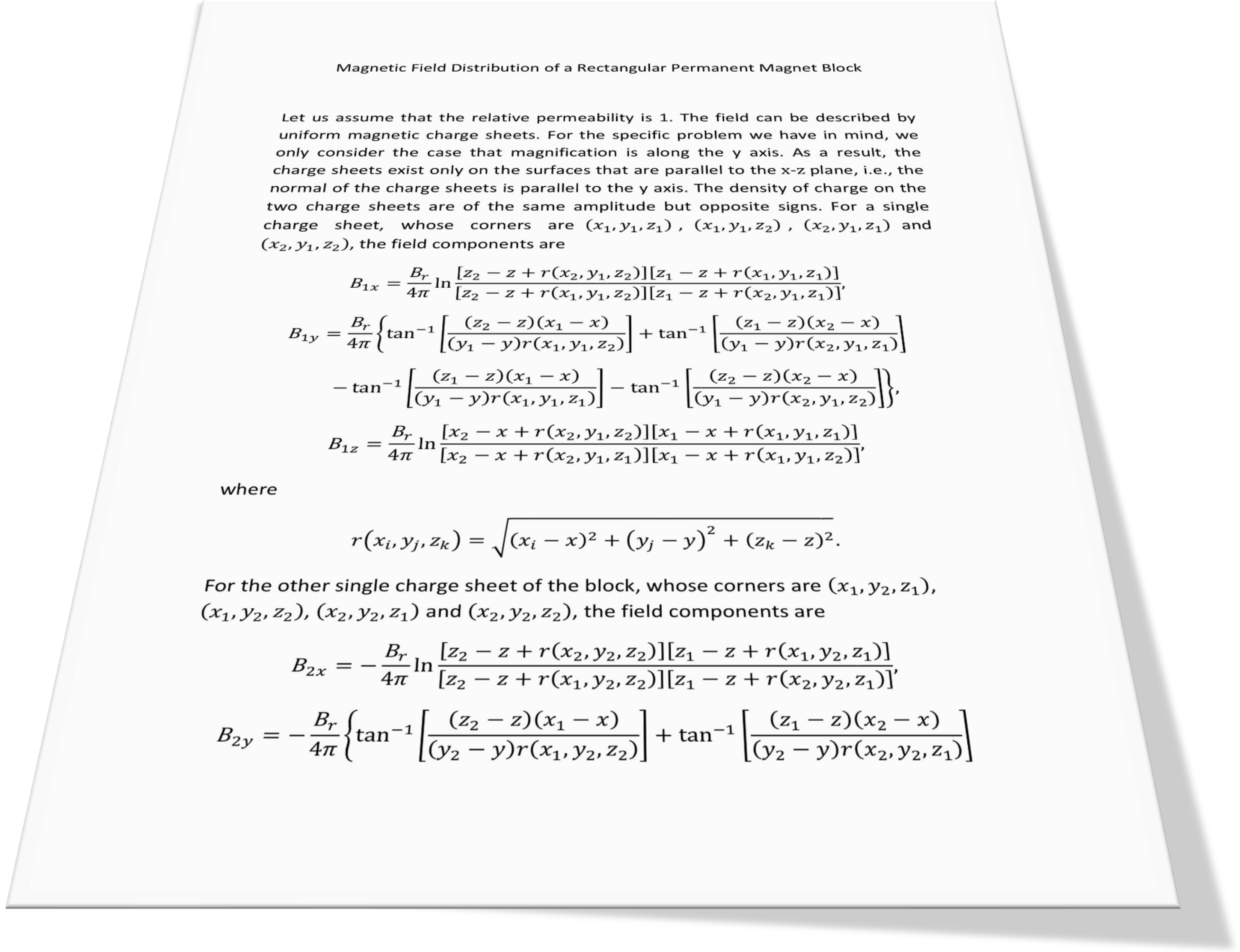
$$B_{1x} = \frac{B_r}{4\pi}\ln\frac{[z_2 - z + r(x_2, y_1, z_2)][z_1 - z + r(x_1, y_1, z_1)]}{[z_2 - z + r(x_1, y_1, z_2)][z_1 - z + r(x_2, y_1, z_1)]},$$

$$B_{1y} = \frac{B_r}{4\pi}\left\{\tan^{-1}\left[\frac{(z_2 - z)(x_1 - x)}{(y_1 - y)r(x_1, y_1, z_2)}\right] + \tan^{-1}\left[\frac{(z_1 - z)(x_2 - x)}{(y_1 - y)r(x_2, y_1, z_1)}\right]\right.$$
$$\left. - \tan^{-1}\left[\frac{(z_1 - z)(x_1 - x)}{(y_1 - y)r(x_1, y_1, z_1)}\right] - \tan^{-1}\left[\frac{(z_2 - z)(x_2 - x)}{(y_1 - y)r(x_2, y_1, z_2)}\right]\right\},$$

$$B_{1z} = \frac{B_r}{4\pi}\ln\frac{[x_2 - x + r(x_2, y_1, z_2)][x_1 - x + r(x_1, y_1, z_1)]}{[x_2 - x + r(x_2, y_1, z_1)][x_1 - x + r(x_1, y_1, z_2)]},$$

where

$$r(x_i, y_j, z_k) = \sqrt{(x_i - x)^2 + (y_j - y)^2 + (z_k - z)^2}.$$


Magnetic Field Distribution of a Rectangular Permanent Magnet Block

Let us assume that the relative permeability is 1. The field can be described by uniform magnetic charge sheets. For the specific problem we have in mind, we only consider the case that magnification is along the y axis. As a result, the charge sheets exist only on the surfaces that are parallel to the x-z plane, i.e., the normal of the charge sheets is parallel to the y axis. The density of charge on the two charge sheets are of the same amplitude but opposite signs. For a single charge sheet, whose corners are (x_1, y_1, z_1), (x_1, y_1, z_2), (x_2, y_1, z_1) and (x_2, y_1, z_2), the field components are

$$B_{1x} = \frac{B_r}{4\pi}\ln\frac{[z_2 - z + r(x_2, y_1, z_2)][z_1 - z + r(x_1, y_1, z_1)]}{[z_2 - z + r(x_1, y_1, z_2)][z_1 - z + r(x_2, y_1, z_1)]},$$

$$B_{1y} = \frac{B_r}{4\pi}\left\{\tan^{-1}\left[\frac{(z_2 - z)(x_1 - x)}{(y_1 - y)r(x_1, y_1, z_2)}\right] + \tan^{-1}\left[\frac{(z_1 - z)(x_2 - x)}{(y_1 - y)r(x_2, y_1, z_1)}\right]\right.$$
$$\left. - \tan^{-1}\left[\frac{(z_1 - z)(x_1 - x)}{(y_1 - y)r(x_1, y_1, z_1)}\right] - \tan^{-1}\left[\frac{(z_2 - z)(x_2 - x)}{(y_1 - y)r(x_2, y_1, z_2)}\right]\right\},$$

$$B_{1z} = \frac{B_r}{4\pi}\ln\frac{[x_2 - x + r(x_2, y_1, z_2)][x_1 - x + r(x_1, y_1, z_1)]}{[x_2 - x + r(x_2, y_1, z_1)][x_1 - x + r(x_1, y_1, z_2)]},$$

where

$$r(x_i, y_j, z_k) = \sqrt{(x_i - x)^2 + (y_j - y)^2 + (z_k - z)^2}.$$

For the other single charge sheet of the block, whose corners are (x_1, y_2, z_1), (x_1, y_2, z_2), (x_2, y_2, z_1) and (x_2, y_2, z_2), the field components are

$$B_{2x} = -\frac{B_r}{4\pi}\ln\frac{[z_2 - z + r(x_2, y_2, z_2)][z_1 - z + r(x_1, y_2, z_1)]}{[z_2 - z + r(x_1, y_2, z_2)][z_1 - z + r(x_2, y_2, z_1)]},$$

$$B_{2y} = -\frac{B_r}{4\pi}\left\{\tan^{-1}\left[\frac{(z_2 - z)(x_1 - x)}{(y_2 - y)r(x_1, y_2, z_2)}\right] + \tan^{-1}\left[\frac{(z_1 - z)(x_2 - x)}{(y_2 - y)r(x_2, y_2, z_1)}\right]\right.$$

From: 劳伦斯伯克利国家物理实验室加速器中心

定量计算——磁场

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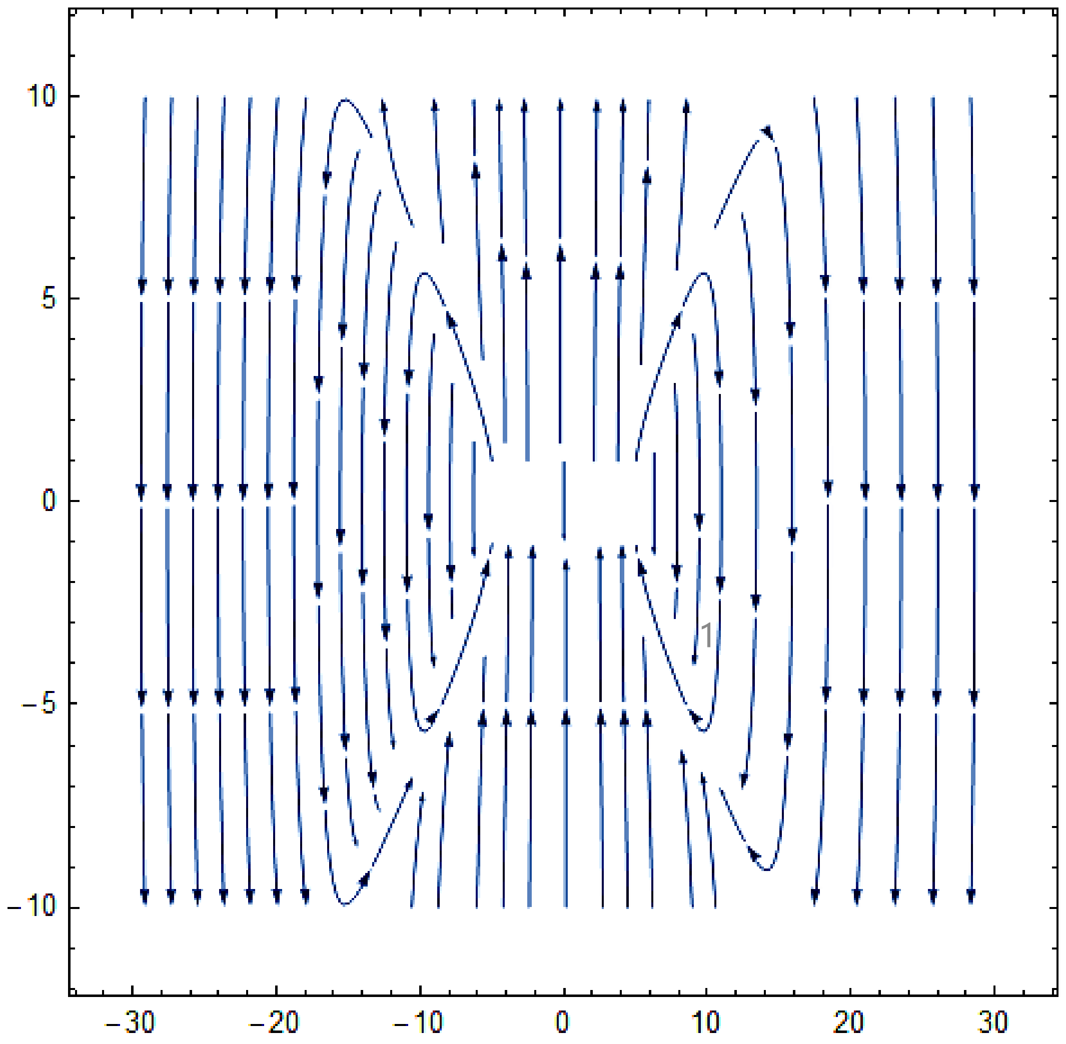
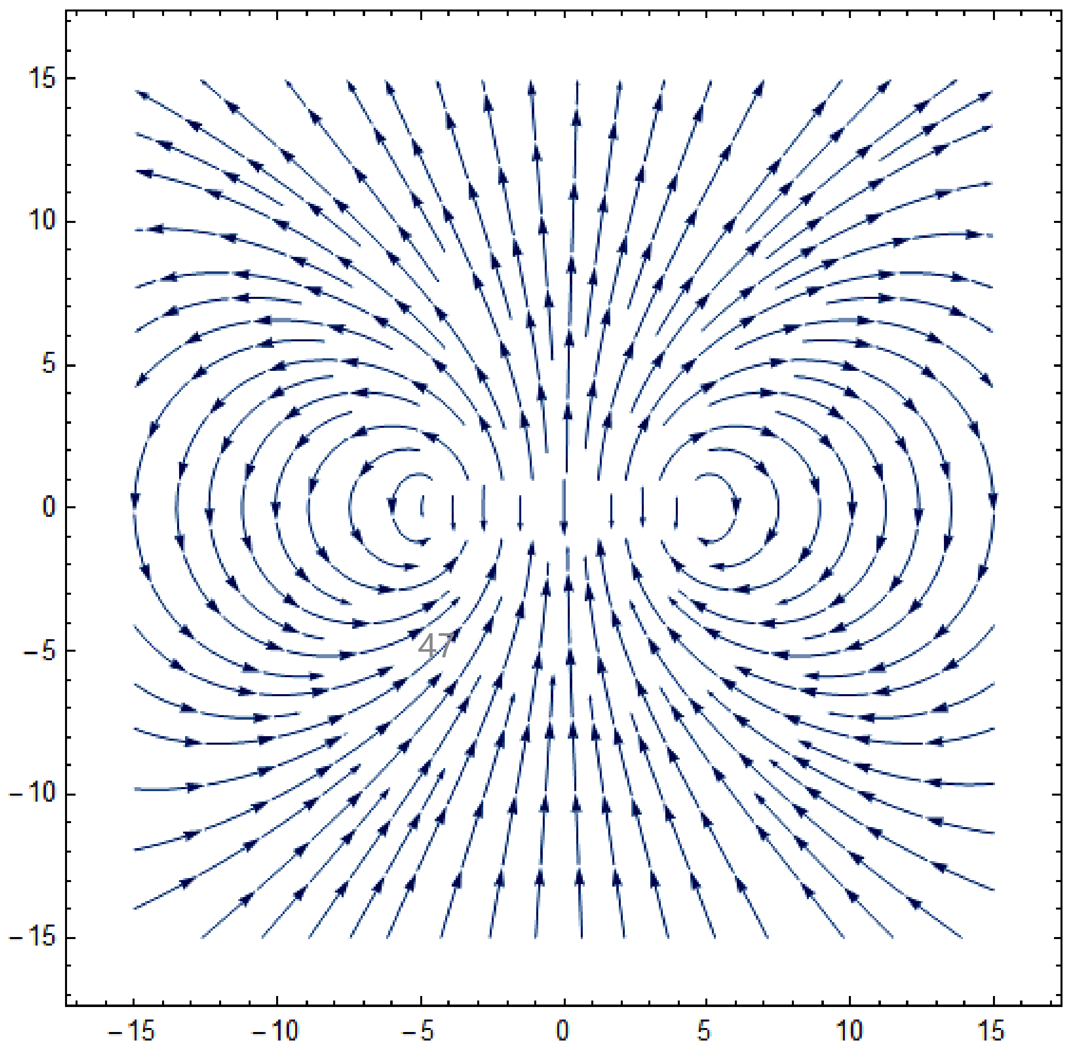
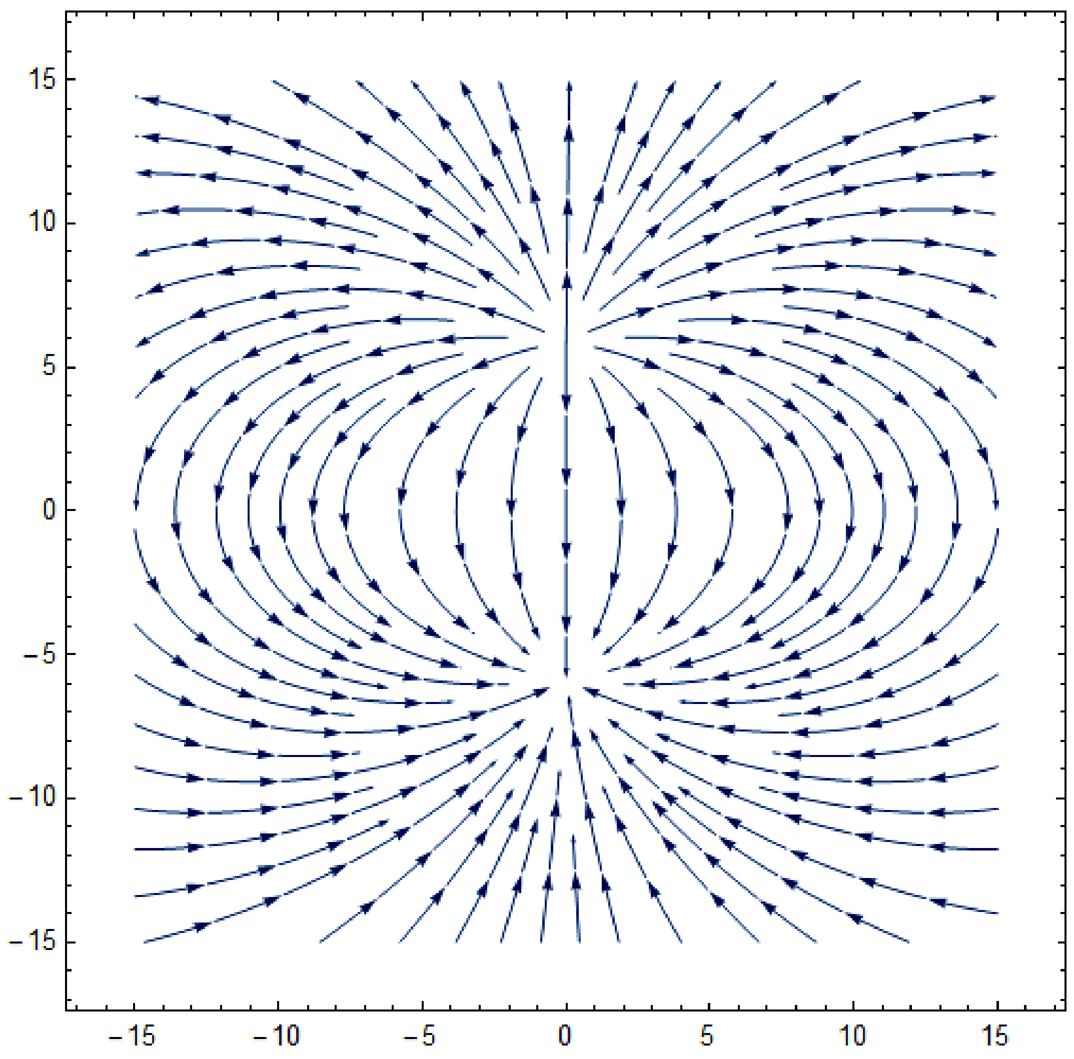
```
StreamPlot[{B1x + B2x, B1y + B2y}, {x, -30, 30}, {y, -10, 10}]
```
流线图



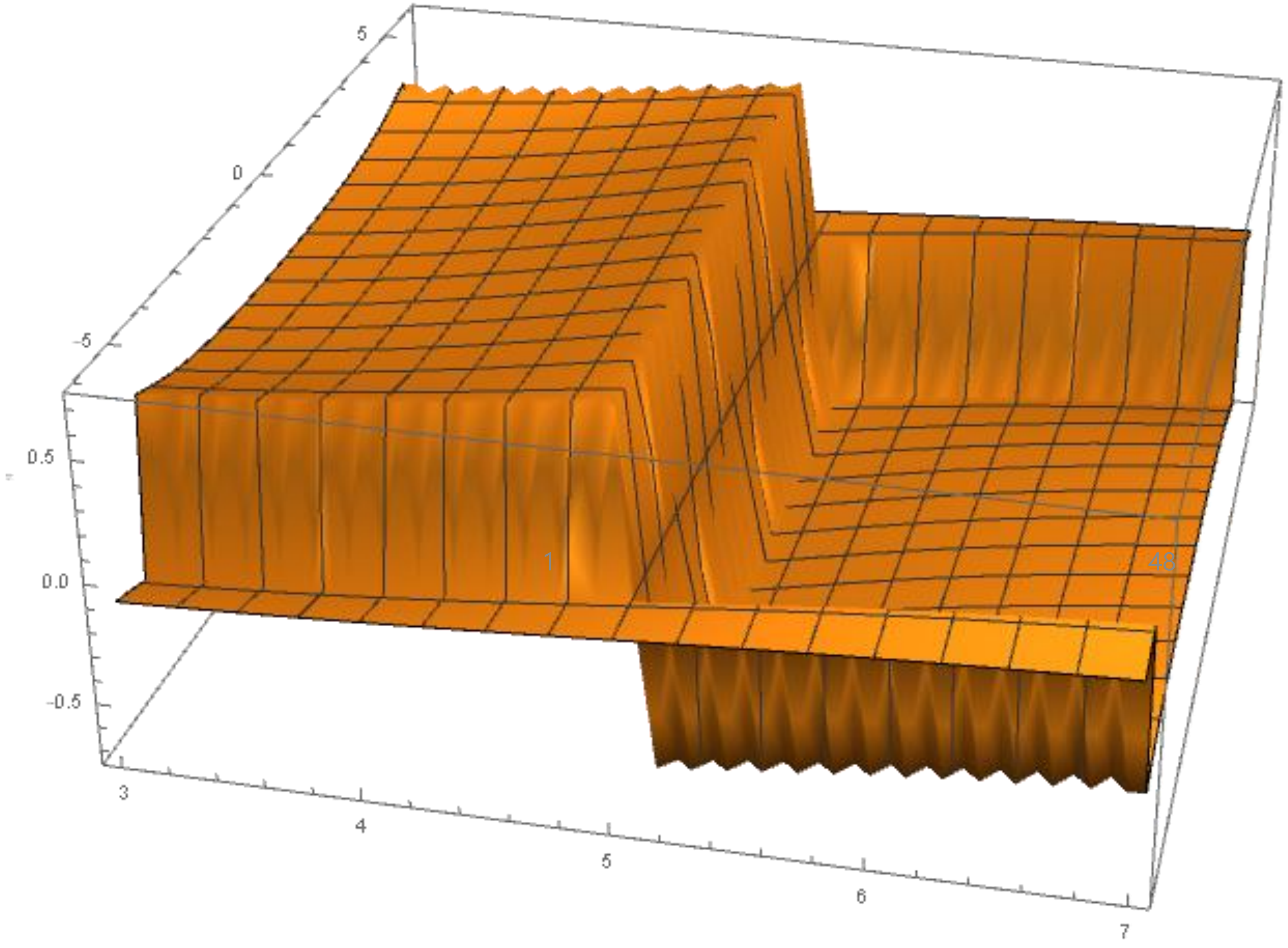

```
StreamPlot[{B1x + B2x, B1y + B2y}, {x, -15, 15}, {y, -15, 15
```
流线图

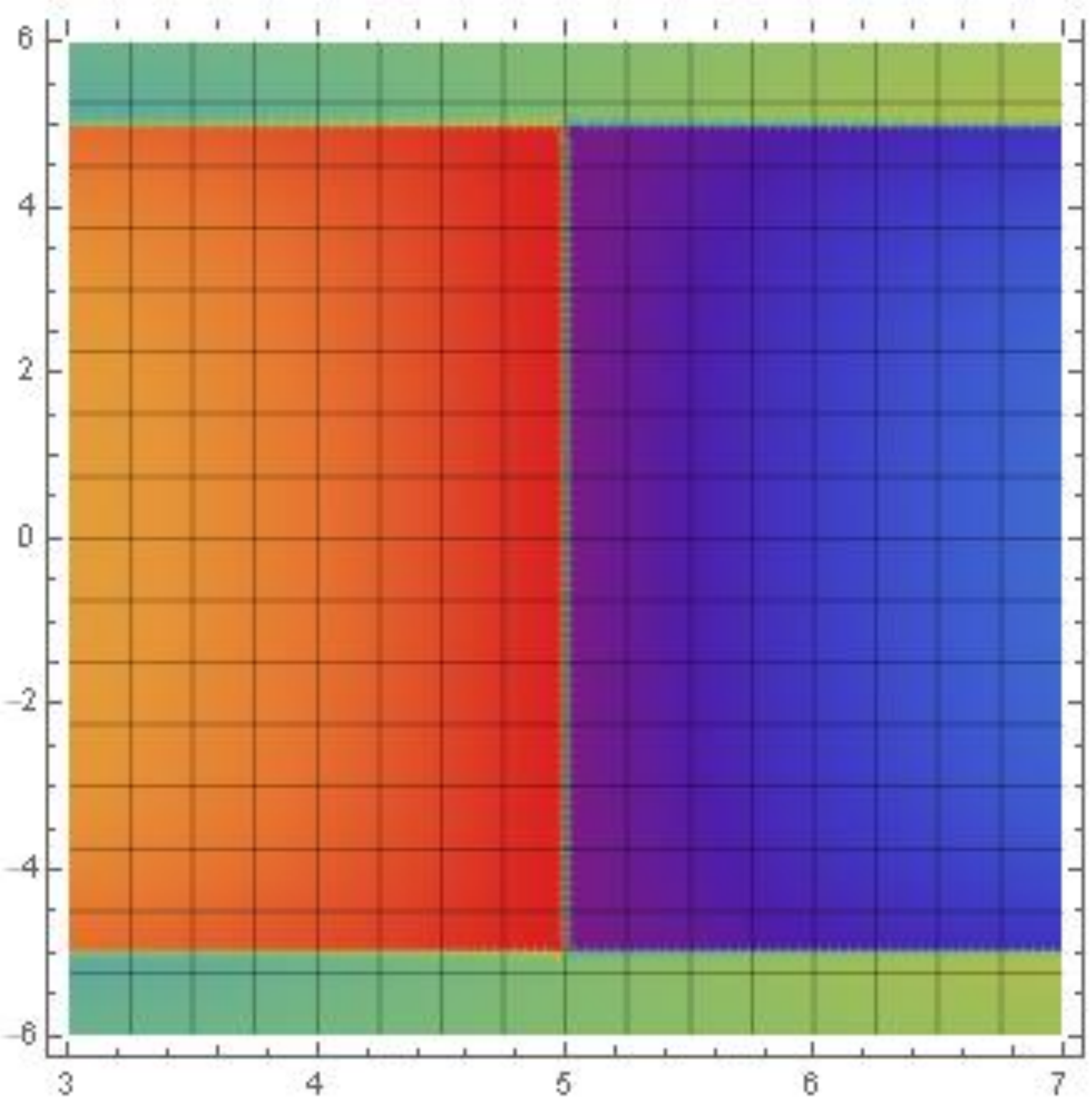



```
StreamPlot[{B1x + B2x, B1y + B2y}, {x, -15, 15}, {y, -15, 15}]
```
流线图



磁场分布





电流分布的解析解

CUPT 上海科技大学 ShanghaiTech University

偏微分方程

$$\begin{cases} \vec{f} = \vec{v} \times \vec{B} \\ \vec{j} = \sigma(\vec{E} + \vec{f}) \\ \nabla \cdot \vec{j} = 0 \end{cases}$$

$$\Rightarrow \nabla \cdot \vec{E} = -\nabla \cdot \vec{f}$$

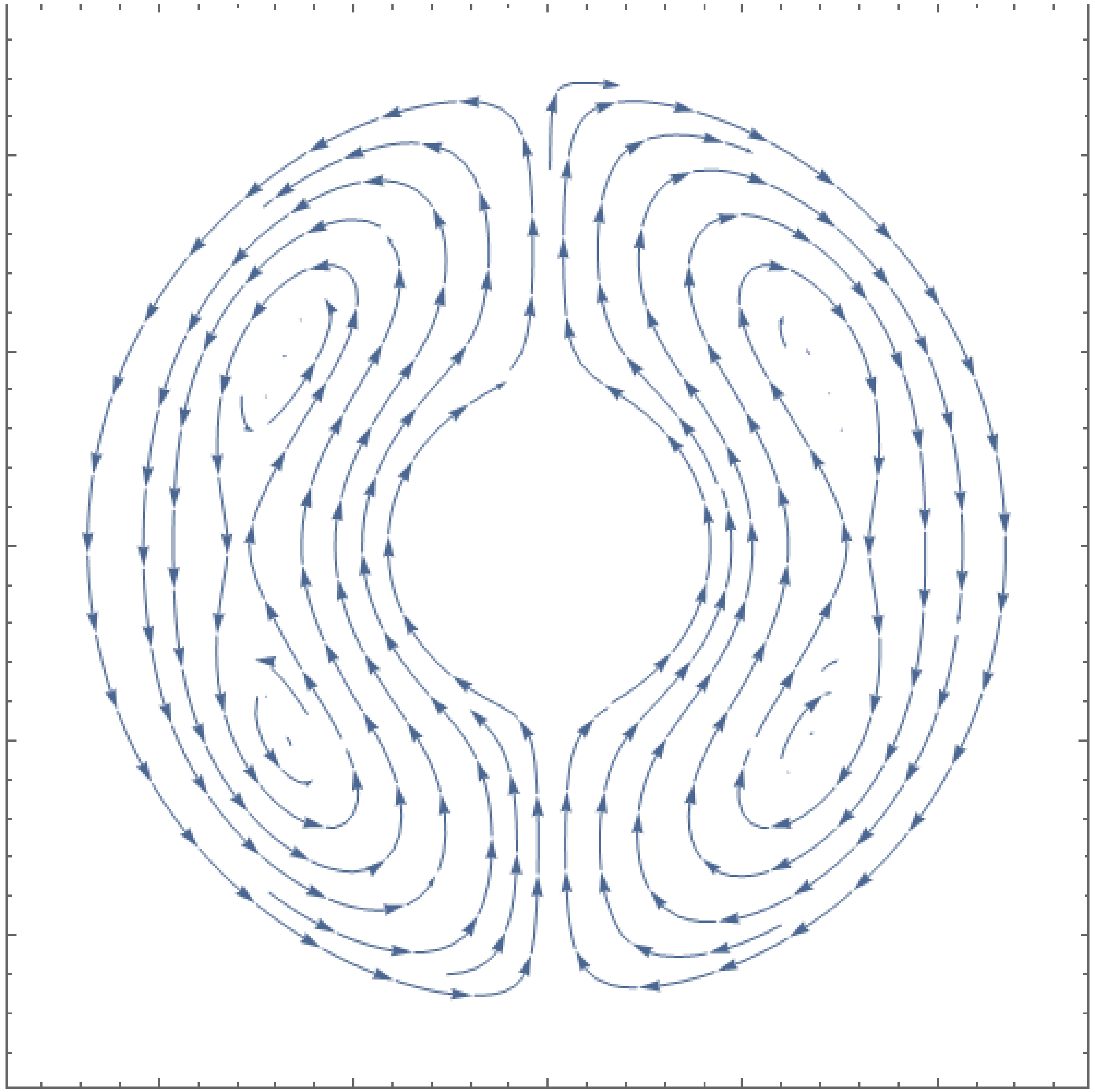
$$\Leftrightarrow \boxed{\nabla^2 \varphi = \nabla \cdot \vec{f}}$$

定解条件（边界条件）

$$\begin{cases} \vec{j}\big|_{r=b} = 0 \\ \vec{j}\big|_{r=a} = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} (\vec{E} + \vec{f})_r\big|_{r=b} = 0 \\ (\vec{E} + \vec{f})_r\big|_{r=a} = 0 \end{cases}$$

$$\Leftrightarrow \boxed{\begin{cases} \left.\dfrac{\partial \varphi}{\partial r}\right|_{r=b} = f_r\big|_{r=b} \\ \left.\dfrac{\partial \varphi}{\partial r}\right|_{r=a} = f_r\big|_{r=a} \end{cases}}$$



电流分布的解析解

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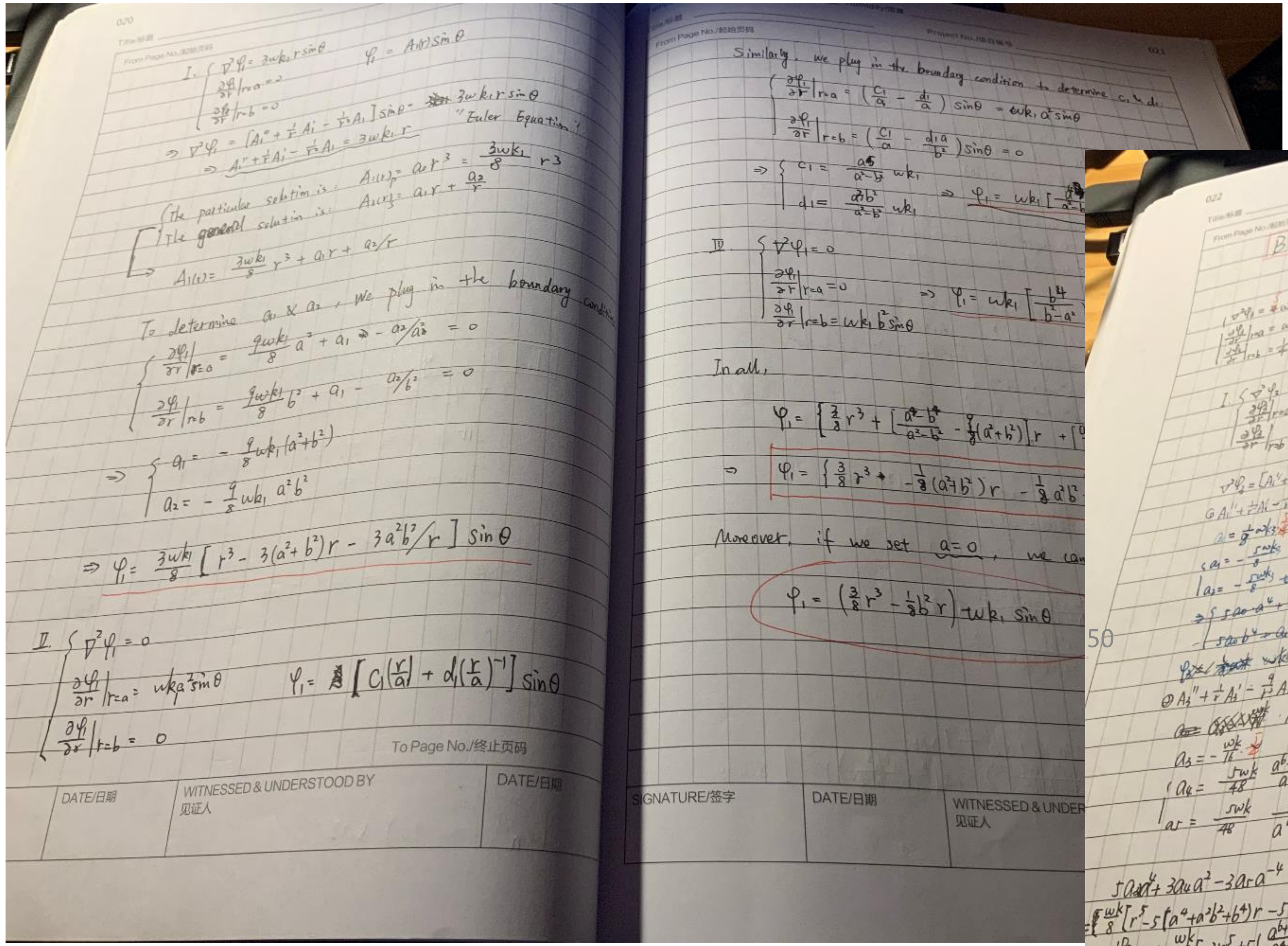
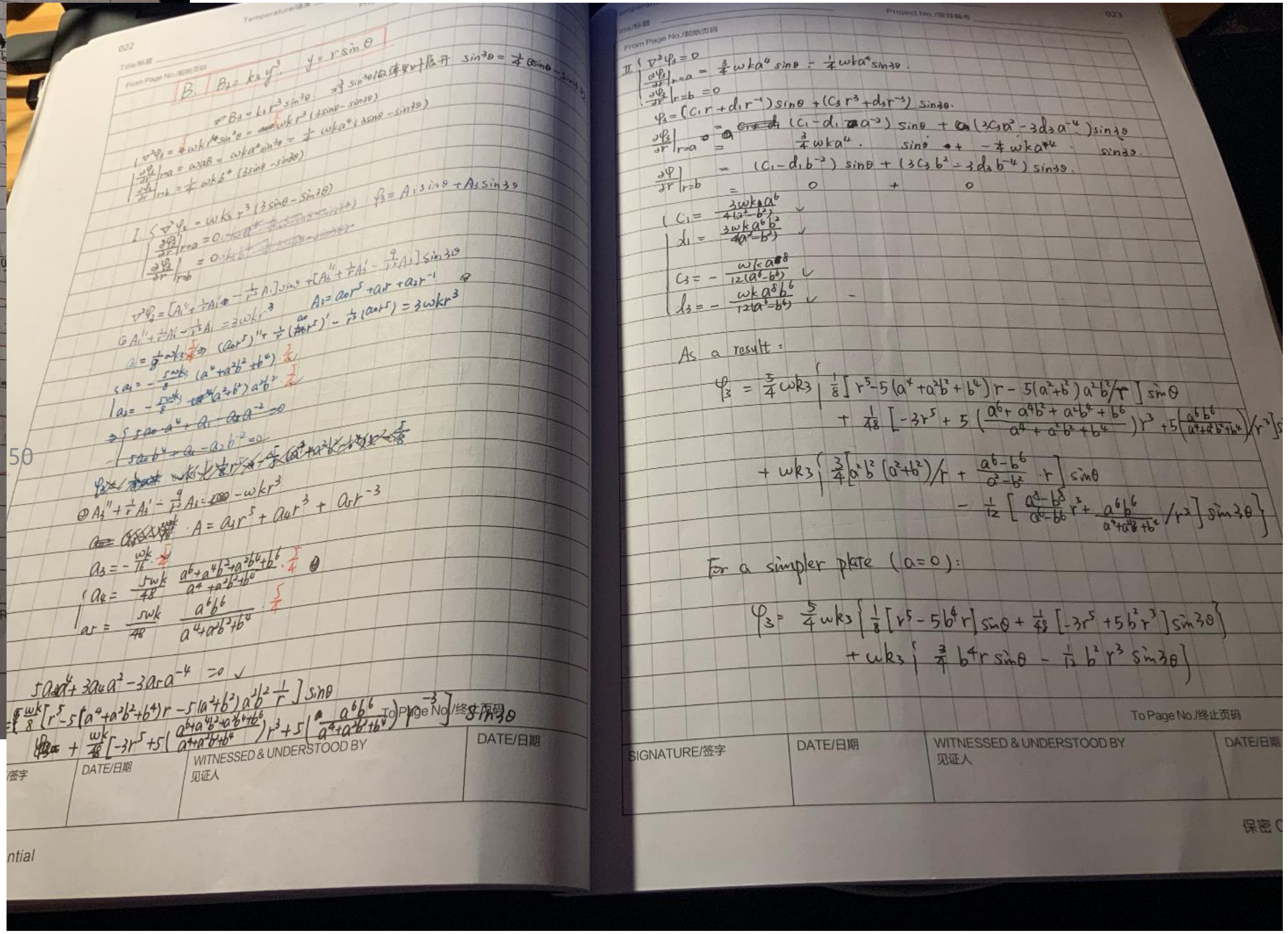
上海科技大学 ShanghaiTech University





电流分布的解析解

电流密度的径向方向分量：jr(r,θ)

```
j1R[r, θ][[1]] + j3R[r, θ][[1]] =
```

$$\sigma\left(k1\, r^2\, \omega\, \mathrm{Sin}[\theta]-\frac{1}{8}\,k1\left(-a^2-b^2+\frac{a^2 b^2}{r^2}+9\,r^2\right)\omega\,\mathrm{Sin}[\theta]\right)+$$

$$\sigma\left(\frac{1}{4}\,k3\, r^4\,\omega\,(3\,\mathrm{Sin}[\theta]-\mathrm{Sin}[3\,\theta])-\right.$$

$$k3\,\omega\left(\frac{3}{4}\left(a^4+a^2 b^2+b^4-\frac{a^2 b^2\,(a^2+b^2)}{r^2}\right)\mathrm{Sin}[\theta]-\frac{1}{12}\left(-\frac{3\,a^6 b^6}{(a^4+a^2 b^2+b^4)\,r^4}+\frac{3\,(a^8-b^8)\,r^2}{a^6-b^6}\right)\mathrm{Sin}[3\,\theta]+\right.$$

$$\left.\left.\frac{5}{4}\left(\frac{1}{8}\left(-5\,(a^4+a^2 b^2+b^4)+\frac{5\,a^2 b^2\,(a^2+b^2)}{r^2}+5\,r^4\right)\mathrm{Sin}[\theta]+\frac{1}{48}\left(-\frac{15\,a^6 b^6}{(a^4+a^2 b^2+b^4)\,r^4}+\frac{15\,(a^6+a^4 b^2+a^2 b^4+b^6)\,r^2}{a^4+a^2 b^2+b^4}-15\,r^4\right)\mathrm{Sin}[3\,\theta]\right)\right)\right)$$

1

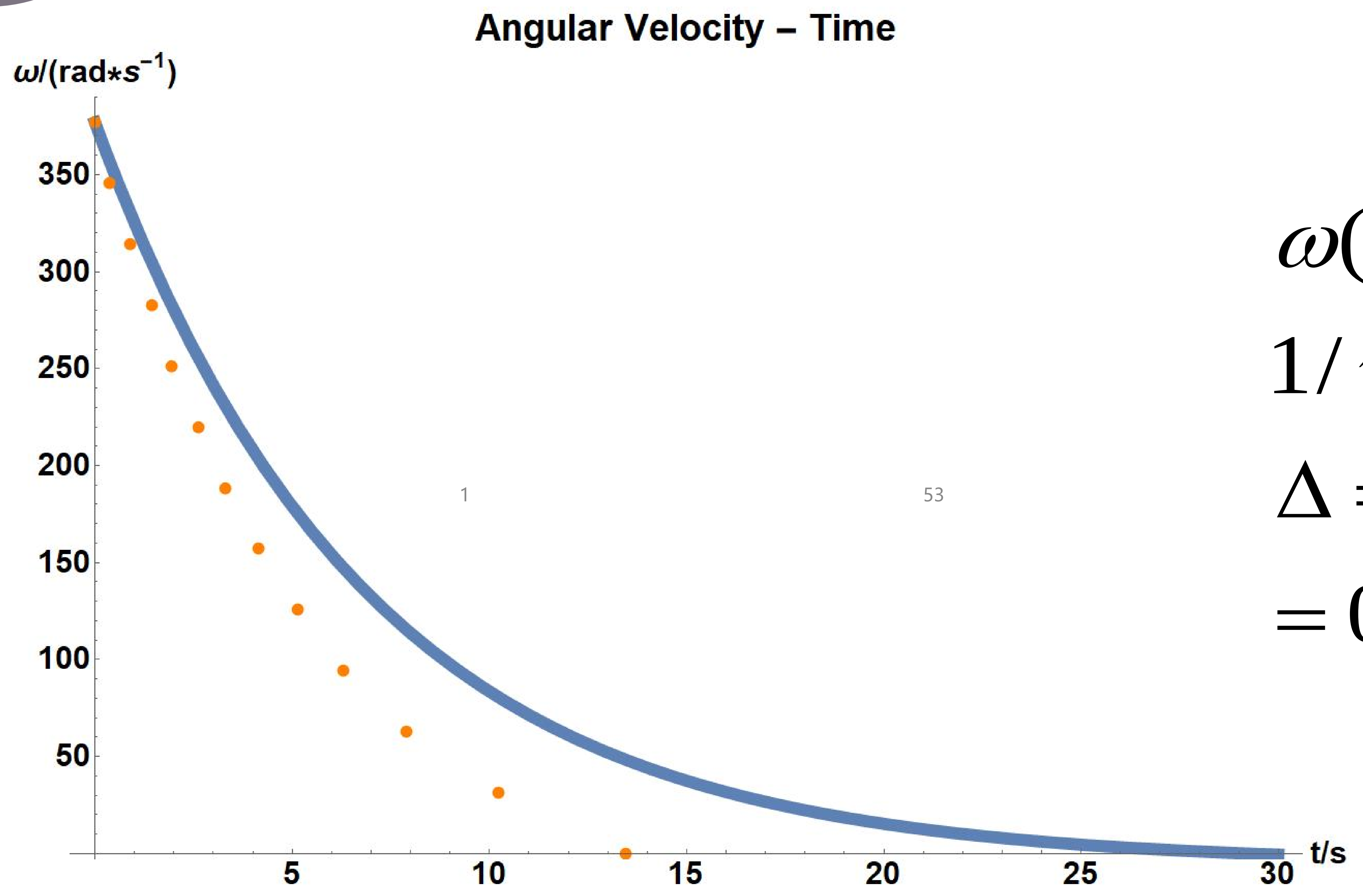
电流分布的解析解

电流密度的角向方向分量：jθ(r,θ)

```
j1R[r, θ][[2]] + j3R[r, θ][[2]] =
```

$$
-\frac{k1\left(-\frac{a^2 b^2}{r}-\left(a^2+b^2\right) r+3 r^3\right) \sigma\, \omega\, \mathrm{Cos}[\theta]}{8 r}-\frac{1}{r}
k3\, \sigma\, \omega \left(\frac{3}{4}\left(\frac{a^2 b^2\left(a^2+b^2\right)}{r}+\left(a^4+a^2 b^2+b^4\right) r\right) \underset{\text{余弦}}{\mathrm{Cos}[\theta]}-\frac{1}{4}\left(\frac{a^6 b^6}{\left(a^4+a^2 b^2+b^4\right) r^3}+\frac{\left(a^8-b^8\right) r^3}{a^6-b^6}\right) \underset{\text{余弦}}{\mathrm{Cos}[3 \theta]}+
\frac{5}{4}\left(\frac{1}{8}\left(-\frac{5 a^2 b^2\left(a^2+b^2\right)}{r}-5\left(a^4+a^2 b^2+b^4\right) r+r^5\right) \underset{\text{余弦}}{\mathrm{Cos}[\theta]}+\frac{1}{16}\left(\frac{5 a^6 b^6}{\left(a^4+a^2 b^2+b^4\right) r^3}+\frac{5\left(a^6+a^4 b^2+a^2 b^4+b^6\right) r^3}{a^4+a^2 b^2+b^4}-3 r^5\right) \underset{\text{余弦}}{\mathrm{Cos}[3 \theta]}\right)\right)
$$

定量计算——阻力



$$\omega(t)=\omega_0 e^{-t/\tau}-\Delta$$

$$1/\tau=0.145857/s$$

$$\Delta=5.42186rad/s$$

$$=0.862916r/s$$