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No. 12 Gyroscope Teslameter

一个由**可导电、非铁磁性**材料制成的陀螺，放置在**磁场**中会出现**减速**的现象。研究相关参量如何影响减速的效果。

A **spinning gyroscope** made from a **conducting, but non-ferromagnetic** material **slows** down when placed in a **magnetic field**. Investigate how the deceleration depends on relevant parameters.



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- 实验现象
- 理论分析
- 实验对照
- 误差分析

1

2

报告纲要

第一部分

实验现象

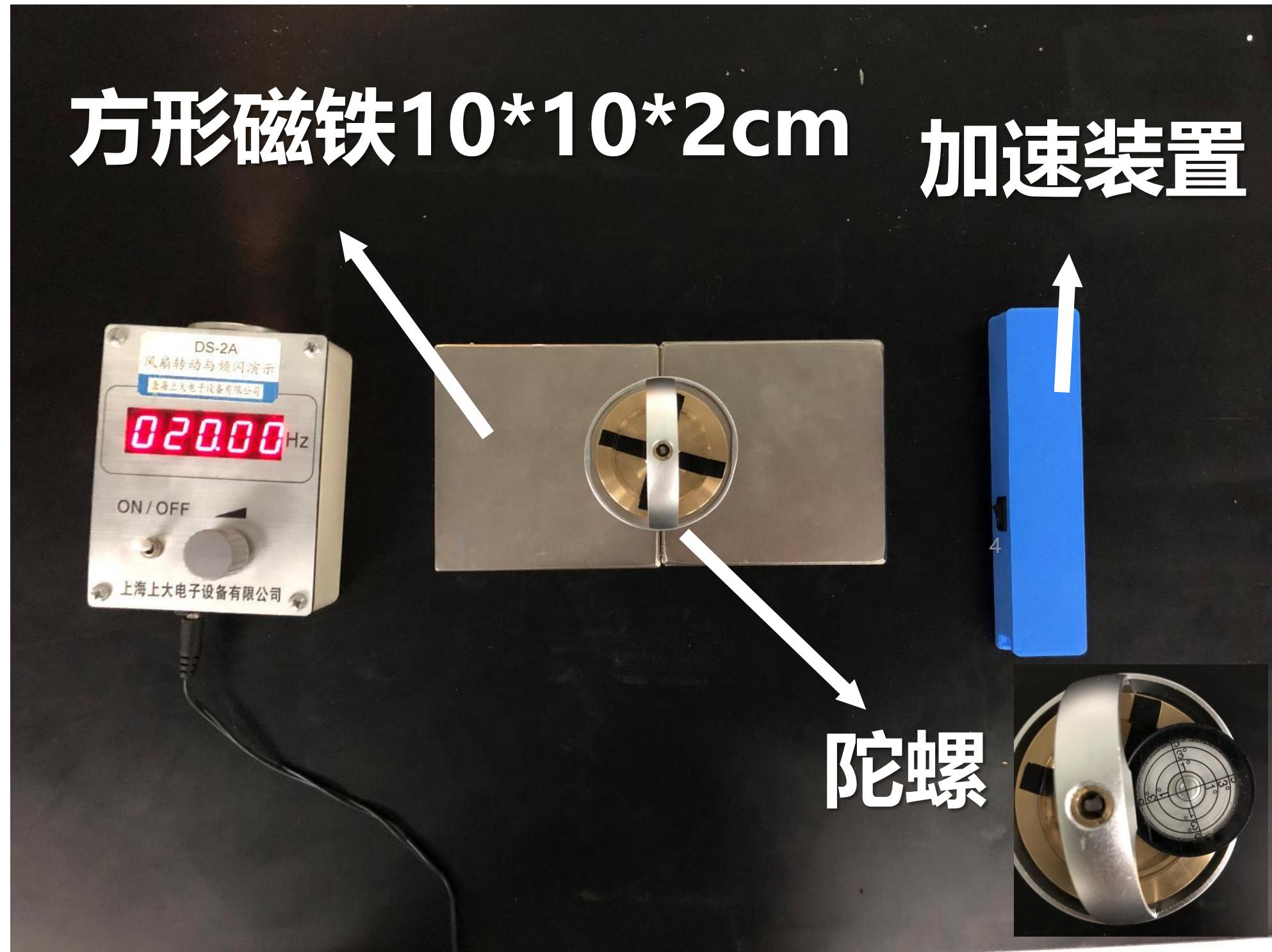
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3



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实验装置



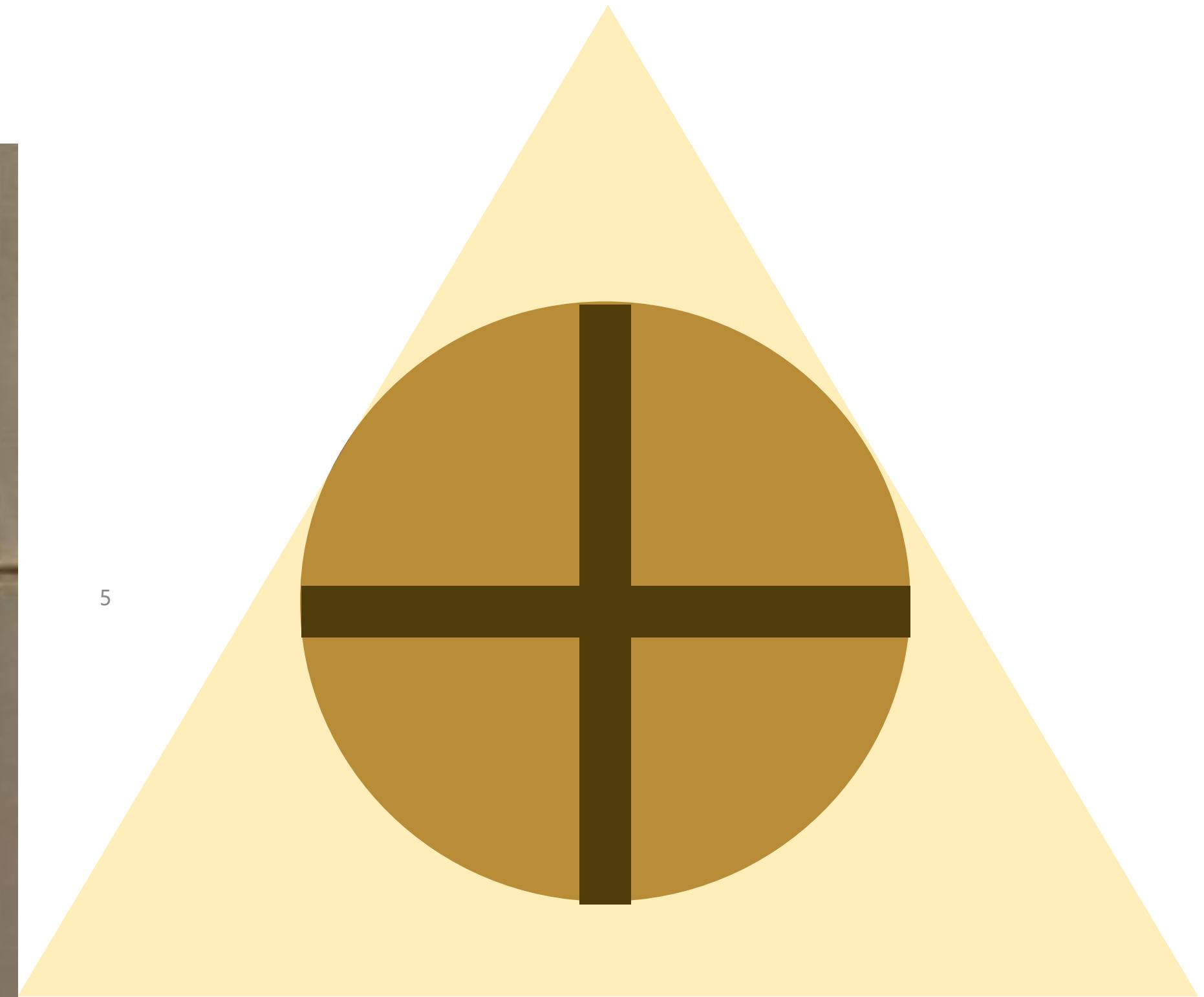
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加速装置?

15s后 7500r/min

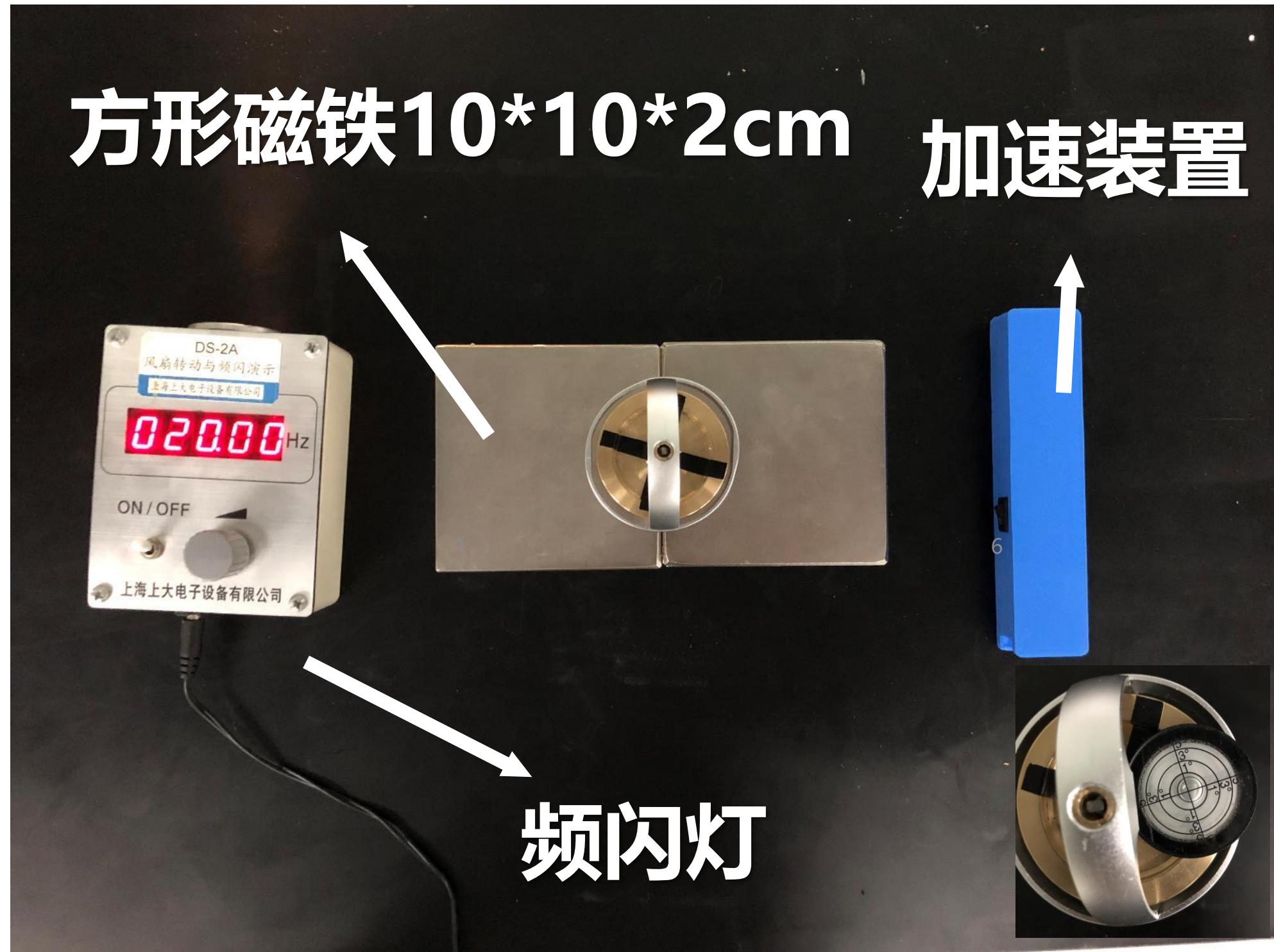
黑条纹?

实验装置



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实验装置



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加速装置?

最高 7500r/min

20Hz?

实验装置



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- >20Hz: 数据点过少
- <20Hz: 错过关键帧
- 20Hz: 较为合适

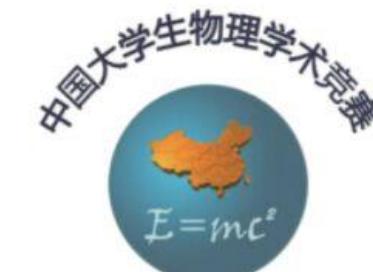
实验装置



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实验现象

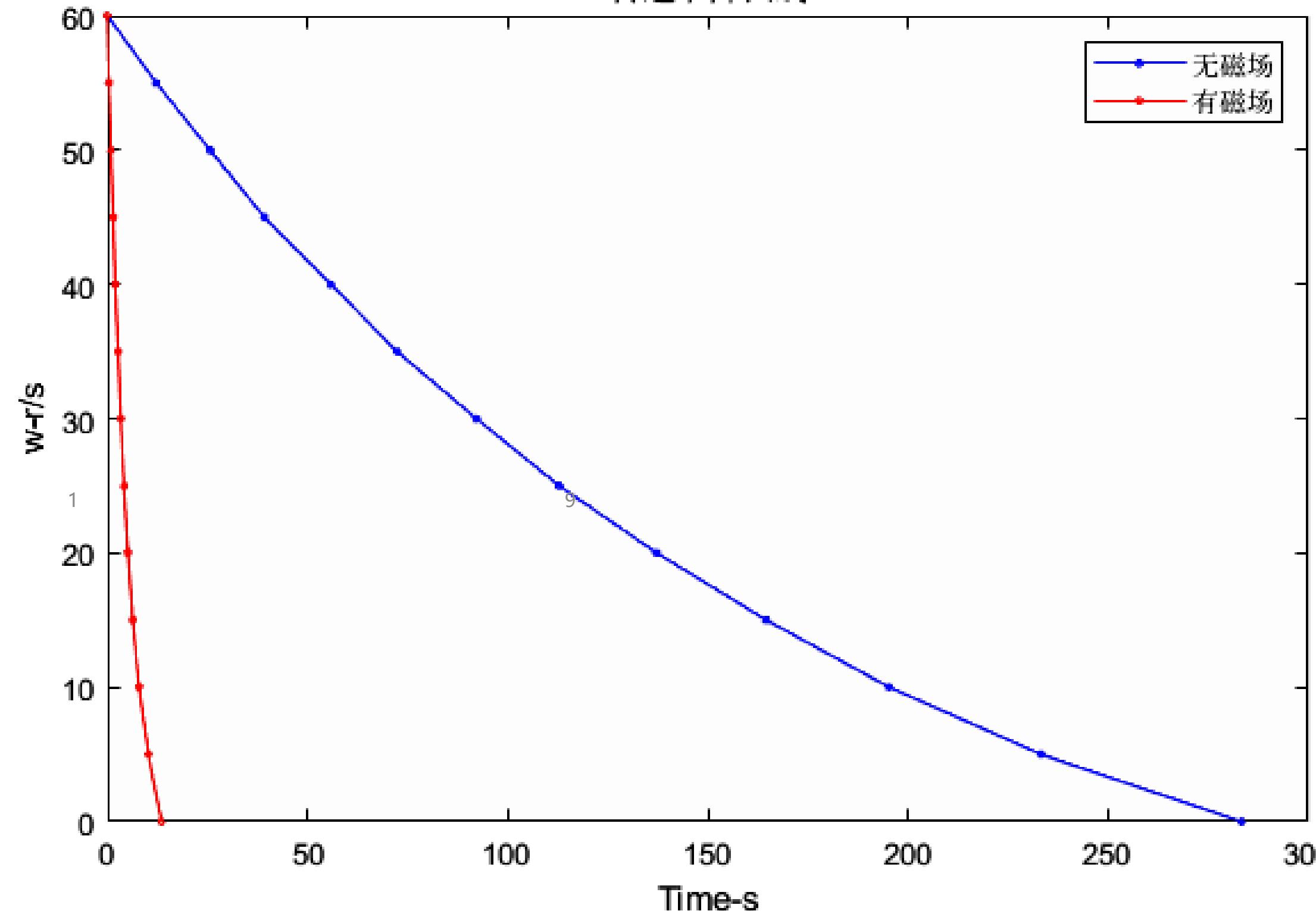


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转速下降曲线



第二部分

理论分析

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10



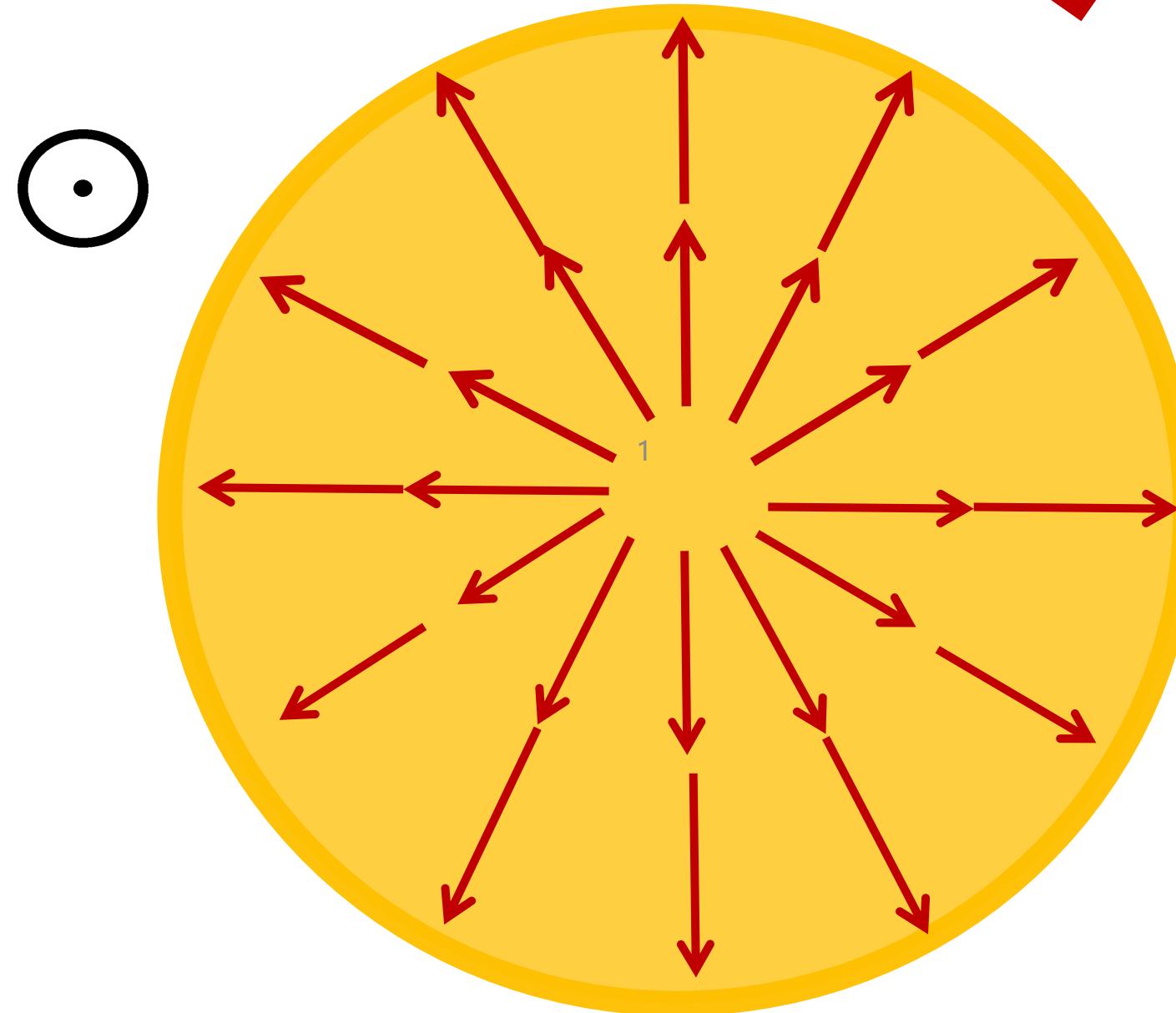
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定性分析



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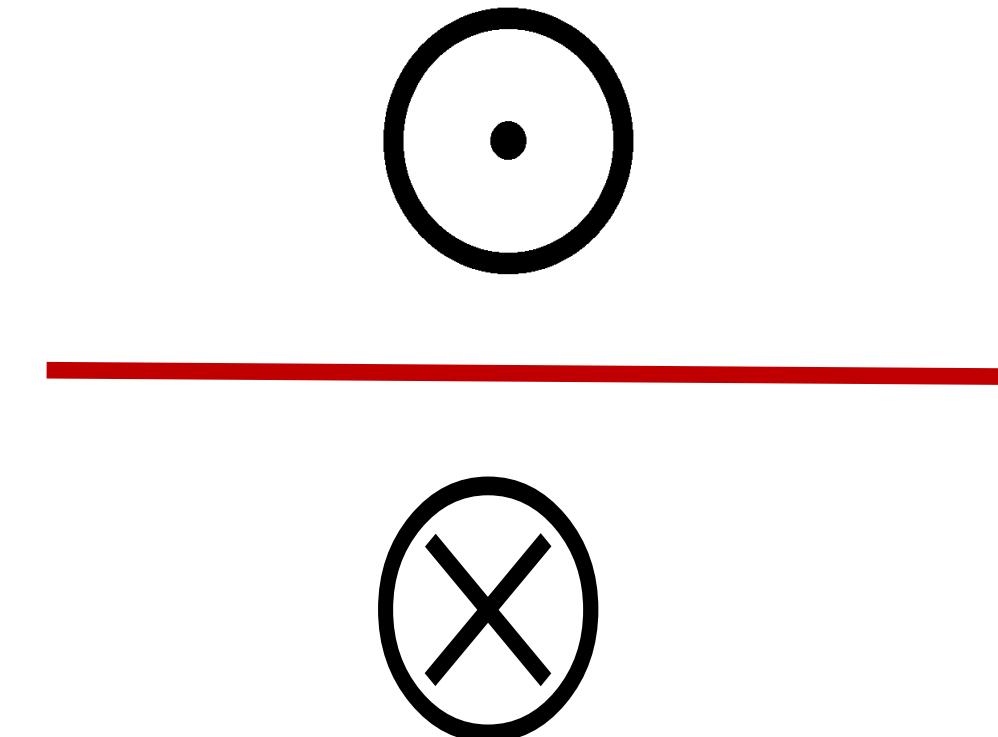
均匀磁场 ✗



非均匀磁场 ✓

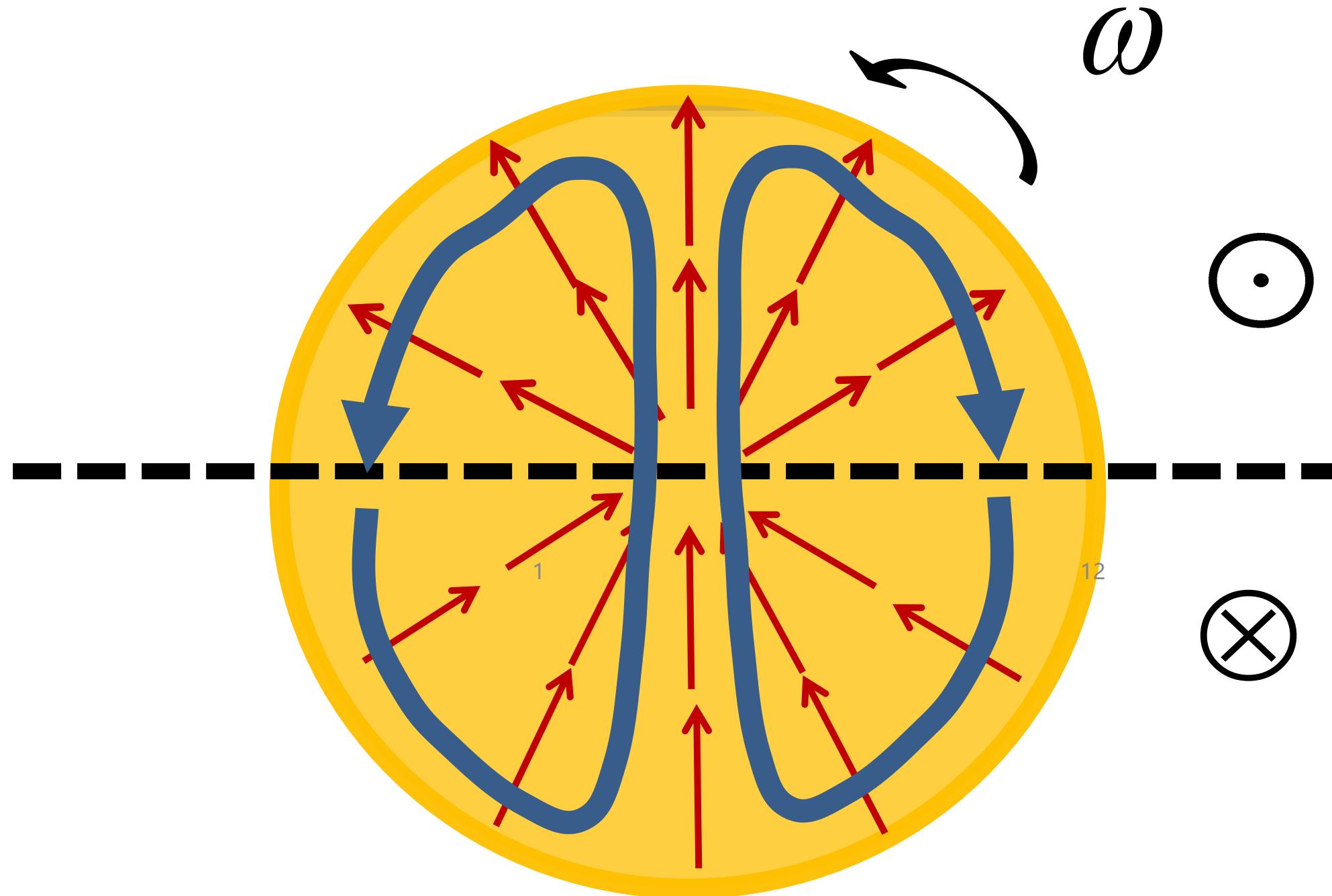


11



11

定性分析



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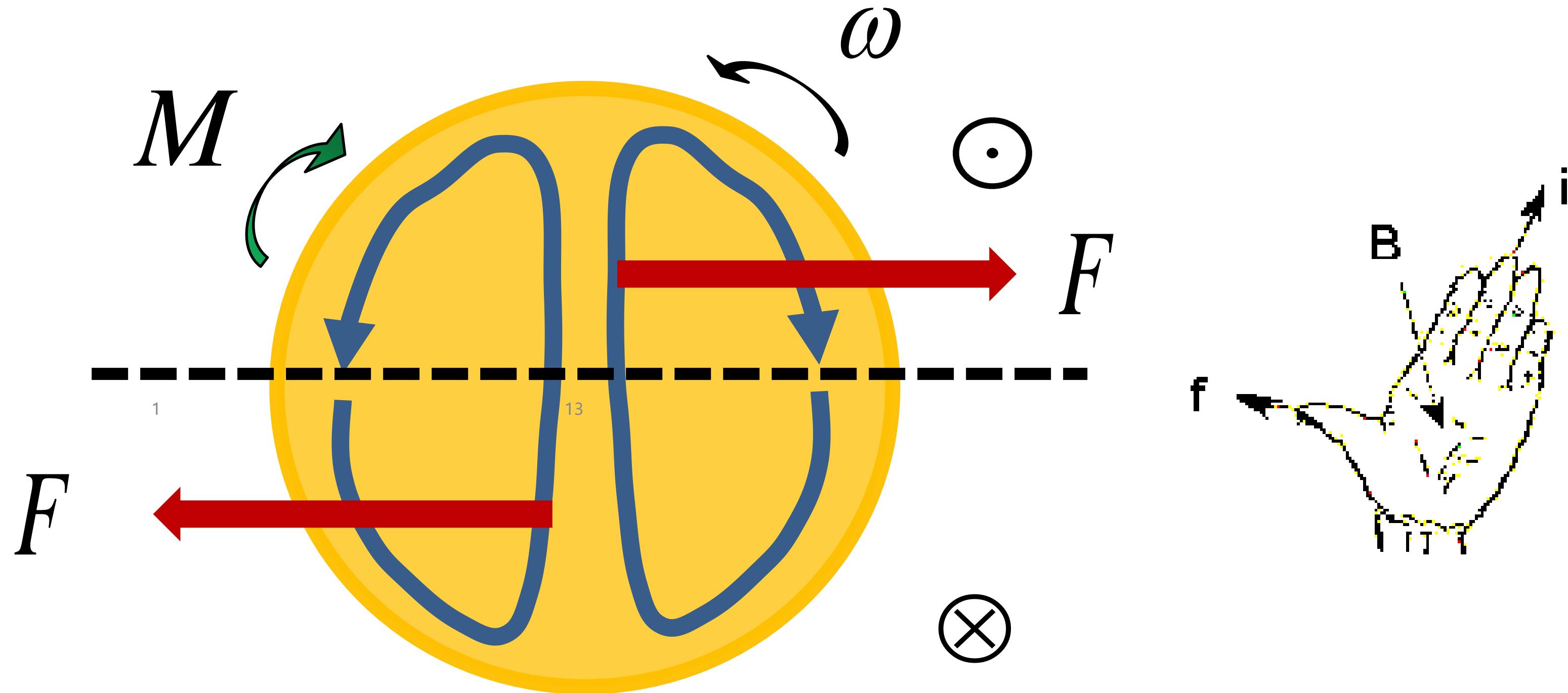
非静电场：

$$\vec{E}_N = \vec{v} \times \vec{B}$$

定性分析



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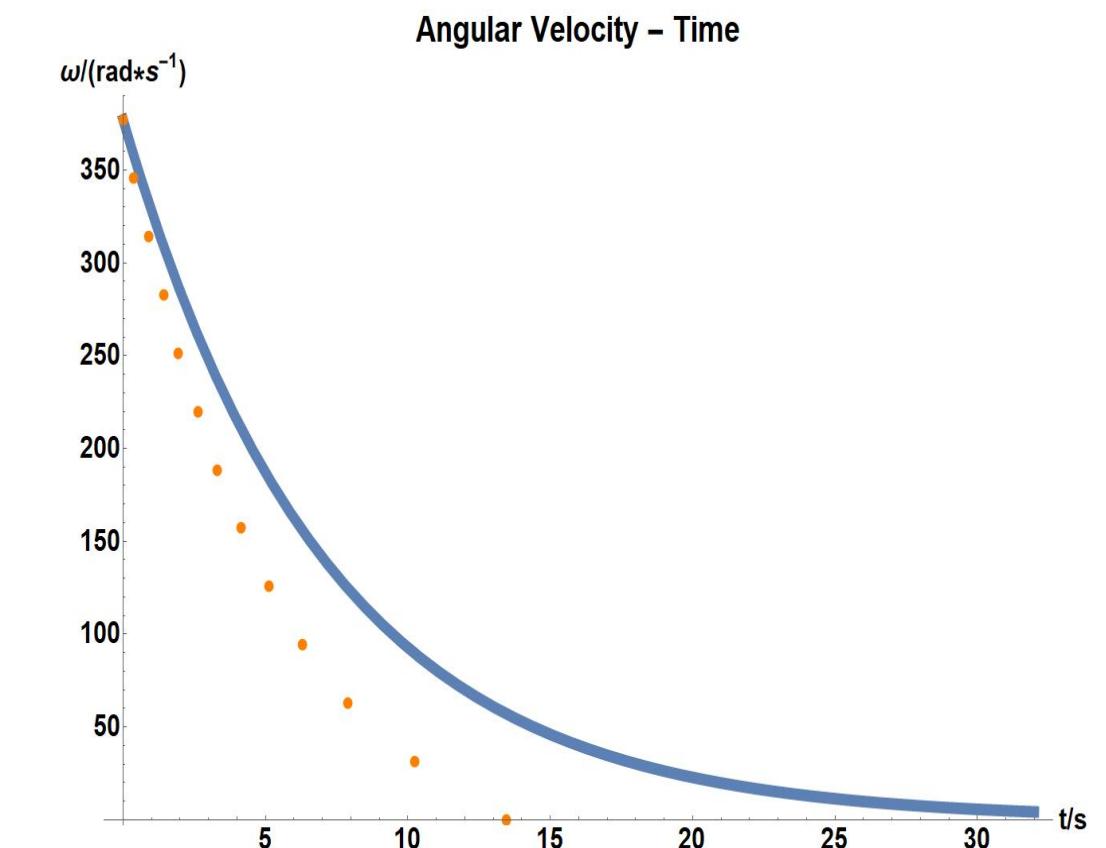
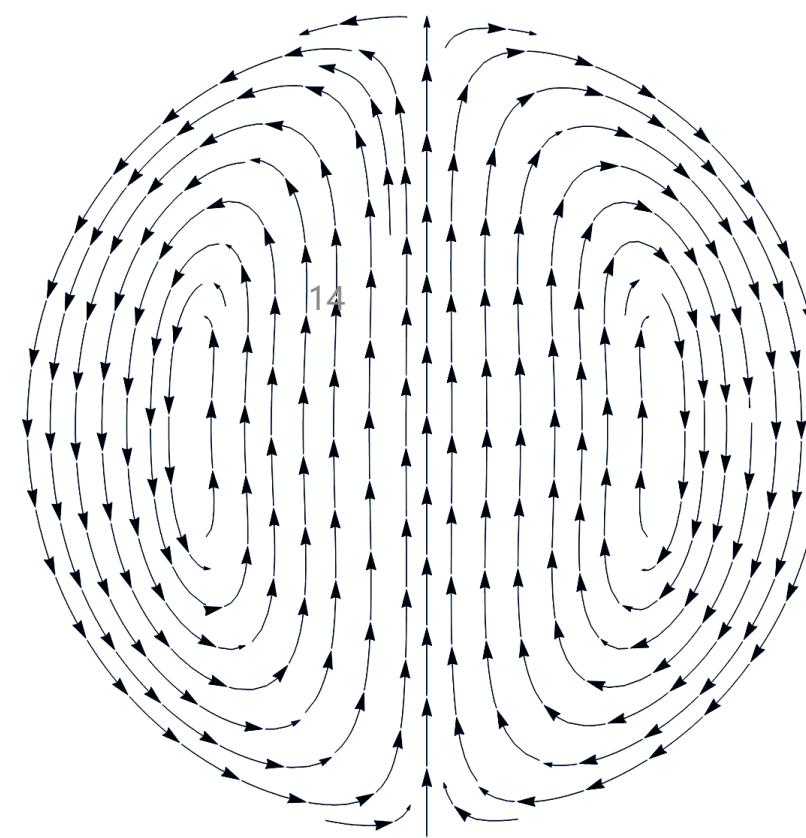
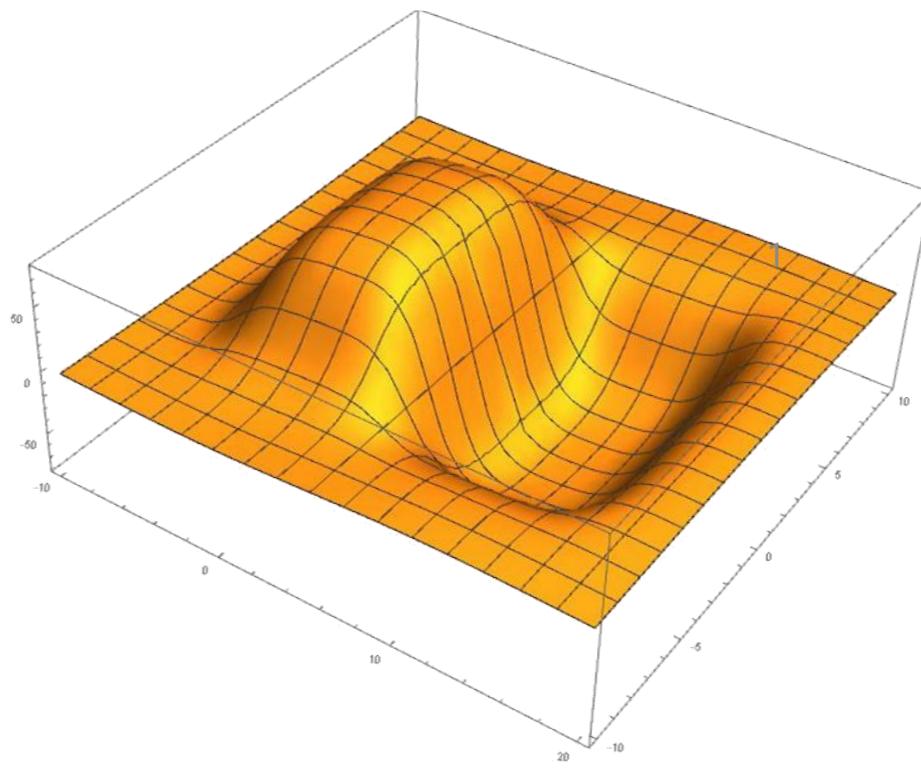


思路



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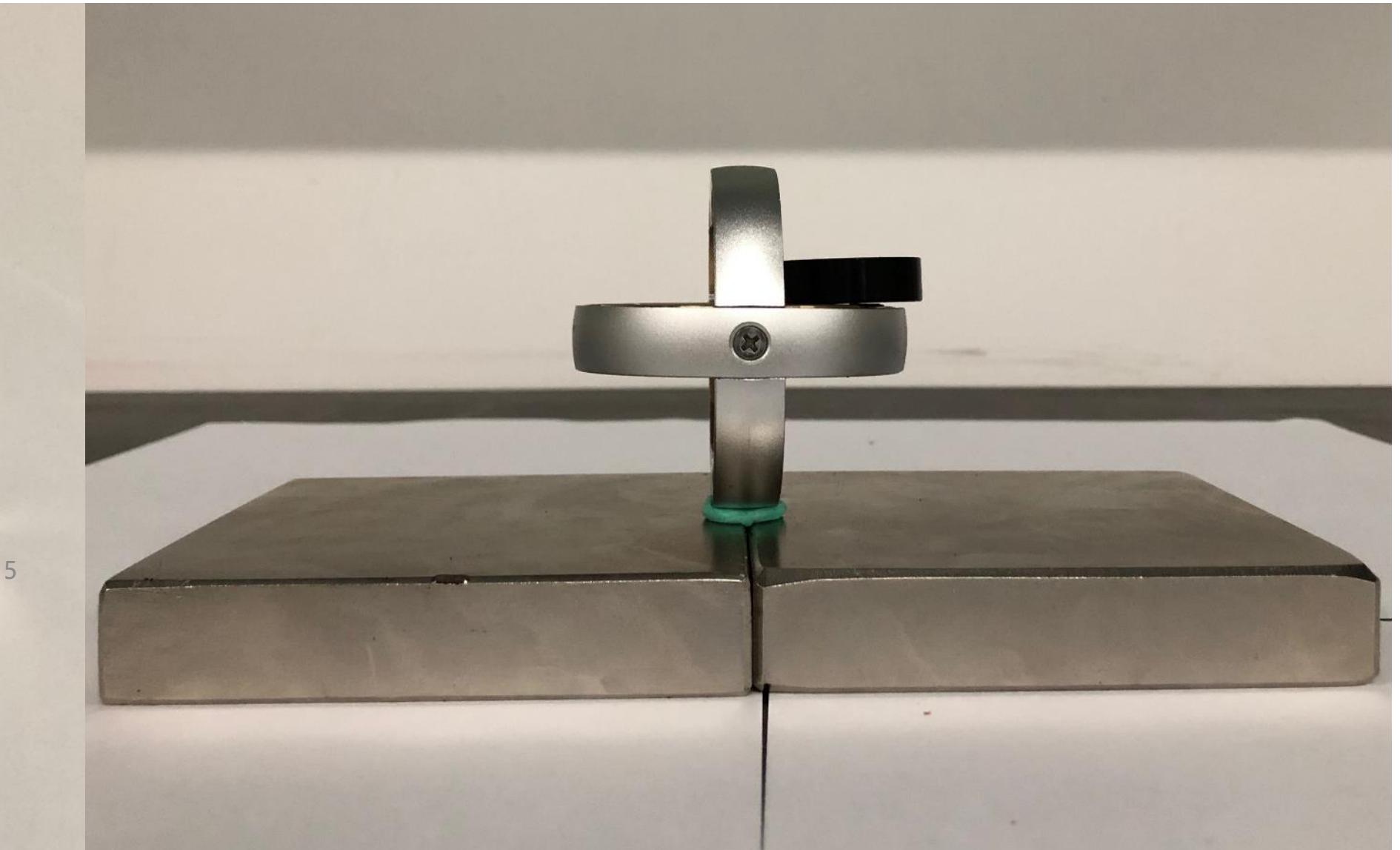
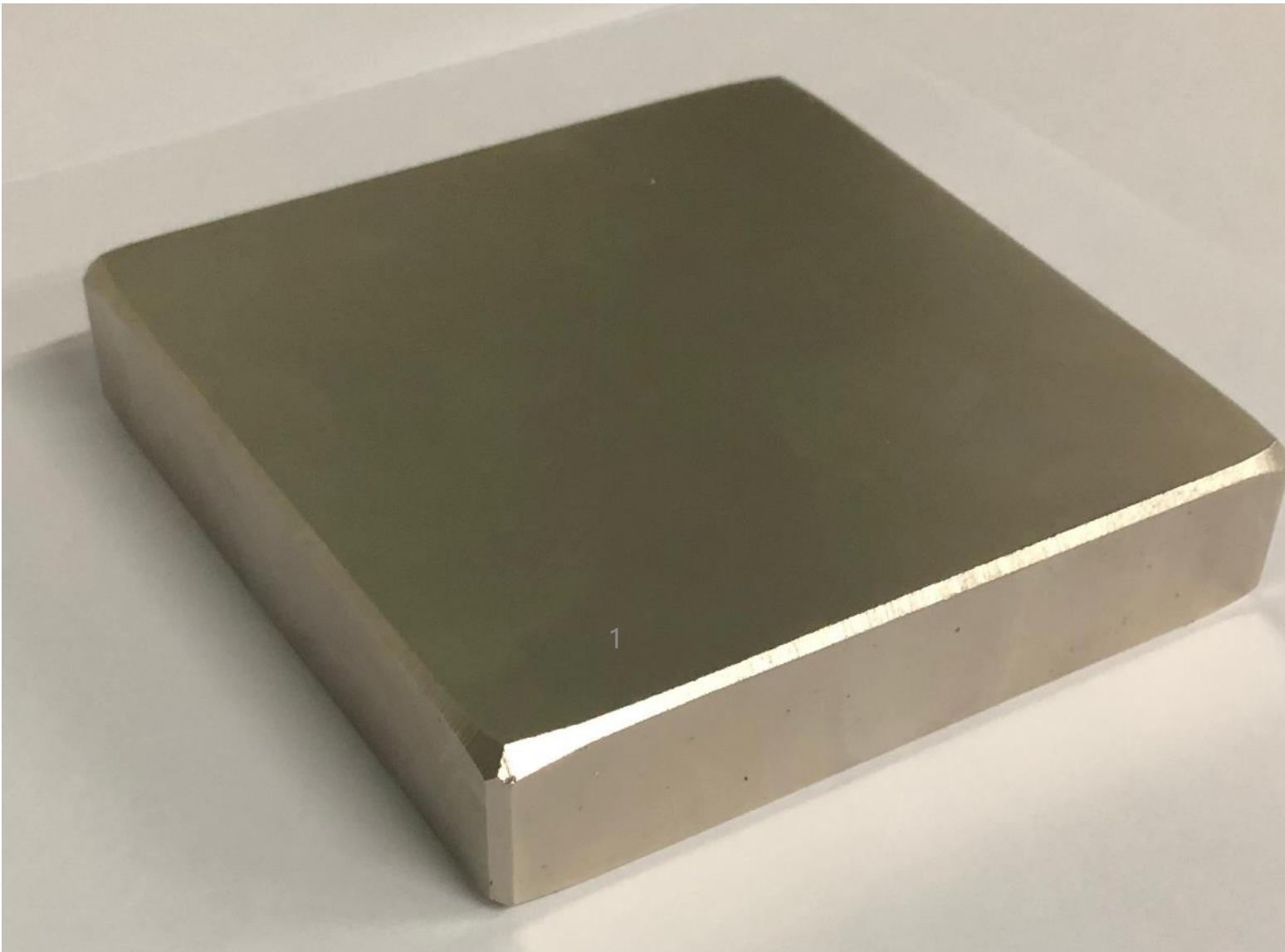
磁场分布 → 涡流分布 → 力矩作用



方形磁体



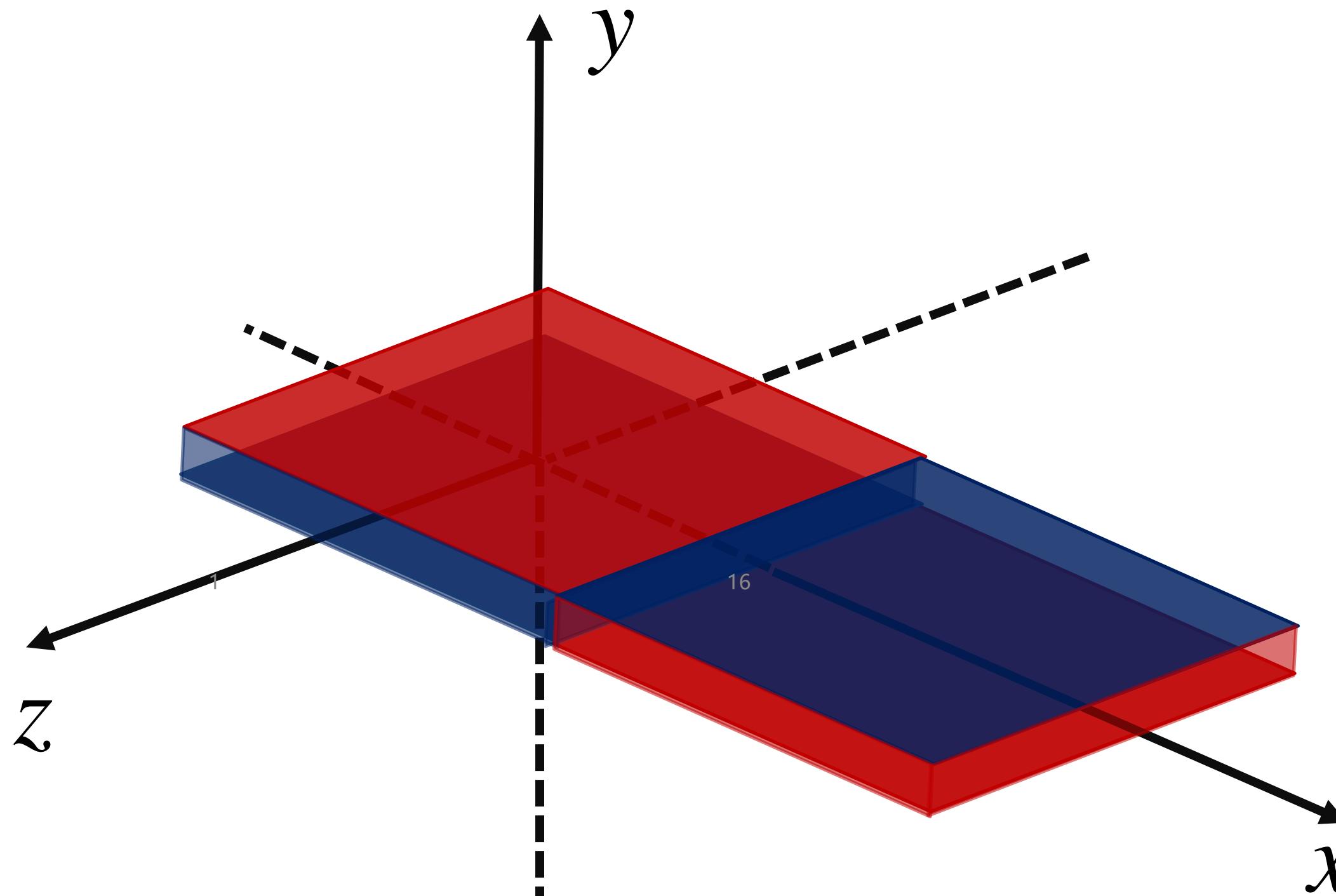
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磁场计算



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定量计算——磁场



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$$B_{1x} = \frac{B_r}{4\pi} \ln \frac{[z_2 - z + r(x_2, y_1, z_2)][z_1 - z + r(x_1, y_1, z_1)]}{[z_2 - z + r(x_1, y_1, z_2)][z_1 - z + r(x_2, y_1, z_1)]},$$

$$\begin{aligned} B_{1y} = & \frac{B_r}{4\pi} \left\{ \tan^{-1} \left[\frac{(z_2 - z)(x_1 - x)}{(y_1 - y)r(x_1, y_1, z_2)} \right] + \tan^{-1} \left[\frac{(z_1 - z)(x_2 - x)}{(y_1 - y)r(x_2, y_1, z_1)} \right] \right. \\ & \left. - \tan^{-1} \left[\frac{(z_1 - z)(x_1 - x)}{(y_1 - y)r(x_1, y_1, z_1)} \right] - \tan^{-1} \left[\frac{(z_2 - z)(x_2 - x)}{(y_1 - y)r(x_2, y_1, z_2)} \right] \right\}, \end{aligned}$$

$$^1 B_{1z} = \frac{B_r}{4\pi} \ln \frac{[x_2 - x + r(x_2, y_1, z_2)][x_1 - x + r(x_1, y_1, z_1)]}{[x_2 - x + r(x_1, y_1, z_1)][x_1 - x + r(x_1, y_1, z_2)]},$$

where

$$r(x_i, y_j, z_k) = \sqrt{(x_i - x)^2 + (y_j - y)^2 + (z_k - z)^2}.$$

定量计算——磁场



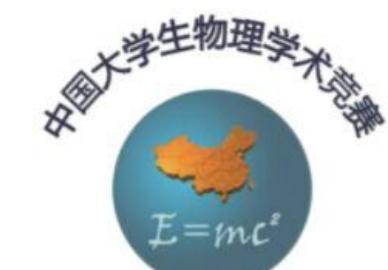
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$$B_{1x} = \frac{B_r}{4\pi} n \frac{[z_2 - z + r(x_2, y_1, z_2)][z_1 - z + r(x_1, y_1, z_1)]}{[z_2 - z + r(x_1, y_1, z_2)][z_1 - z + r(x_2, y_1, z_1)]},$$

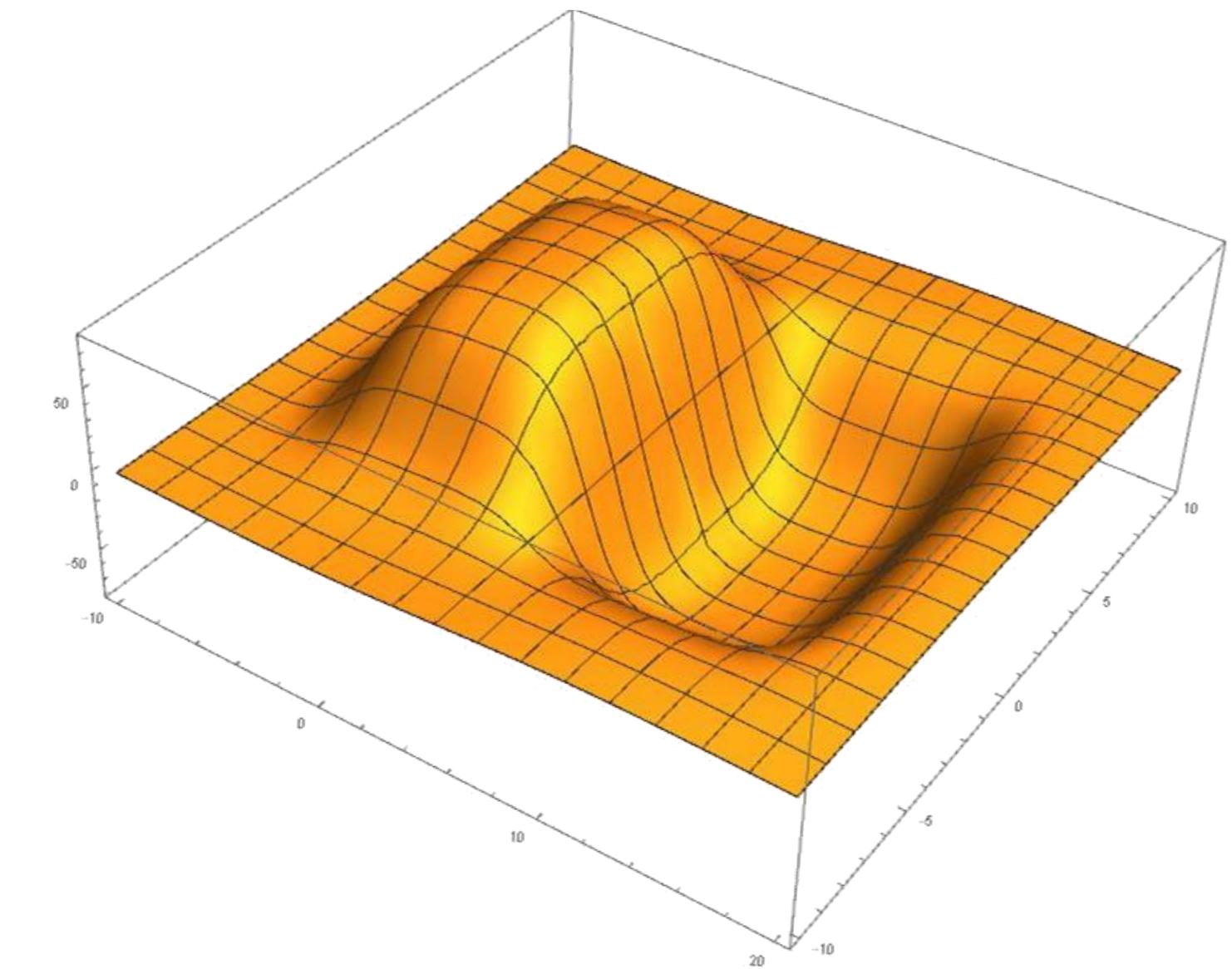
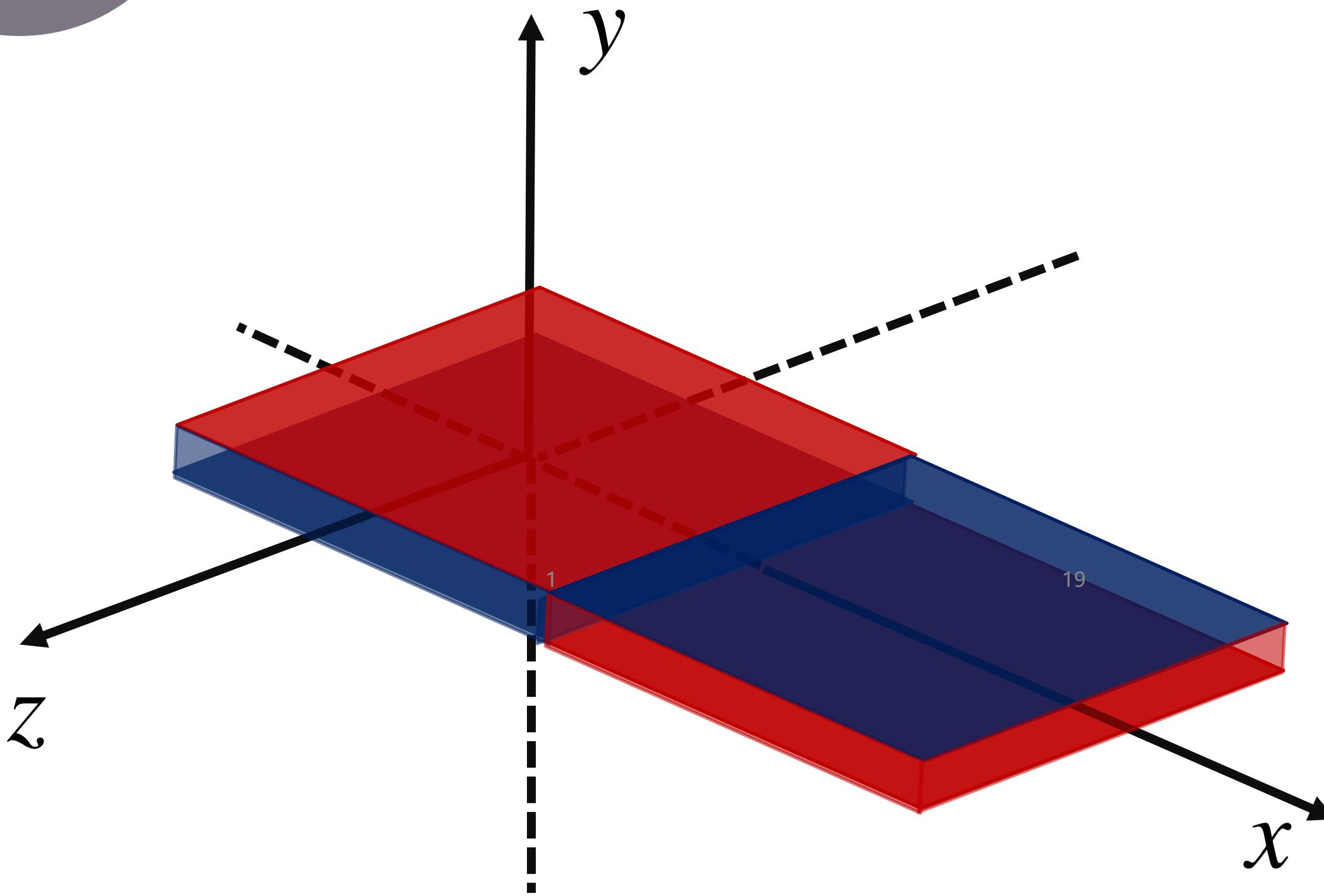
$$B_{1y} = \frac{B_r}{4\pi} \left\{ \tan^{-1} \left[\frac{(z_2 - z)(x_1 - x)}{(y_1 - y)r(x_1, y_1, z_2)} \right] + \tan^{-1} \left[\frac{(z_1 - z)(x_2 - x)}{(y_1 - y)r(x_2, y_1, z_1)} \right] \right. \\ \left. - \tan^{-1} \left[\frac{(z_1 - z)(x_1 - x)^{18}}{(y_1 - y)r(x_1, y_1, z_1)} \right] - \tan^{-1} \left[\frac{(z_2 - z)(x_2 - x)}{(y_1 - y)r(x_2, y_1, z_2)} \right] \right\},$$

$$B_{1z} = \frac{B_r}{4\pi} n \frac{[x_2 - x + r(x_2, y_1, z_2)][x_1 - x + r(x_1, y_1, z_1)]}{[x_2 - x + r(x_1, y_1, z_1)][x_1 - x + r(x_1, y_1, z_2)]},$$

磁场计算



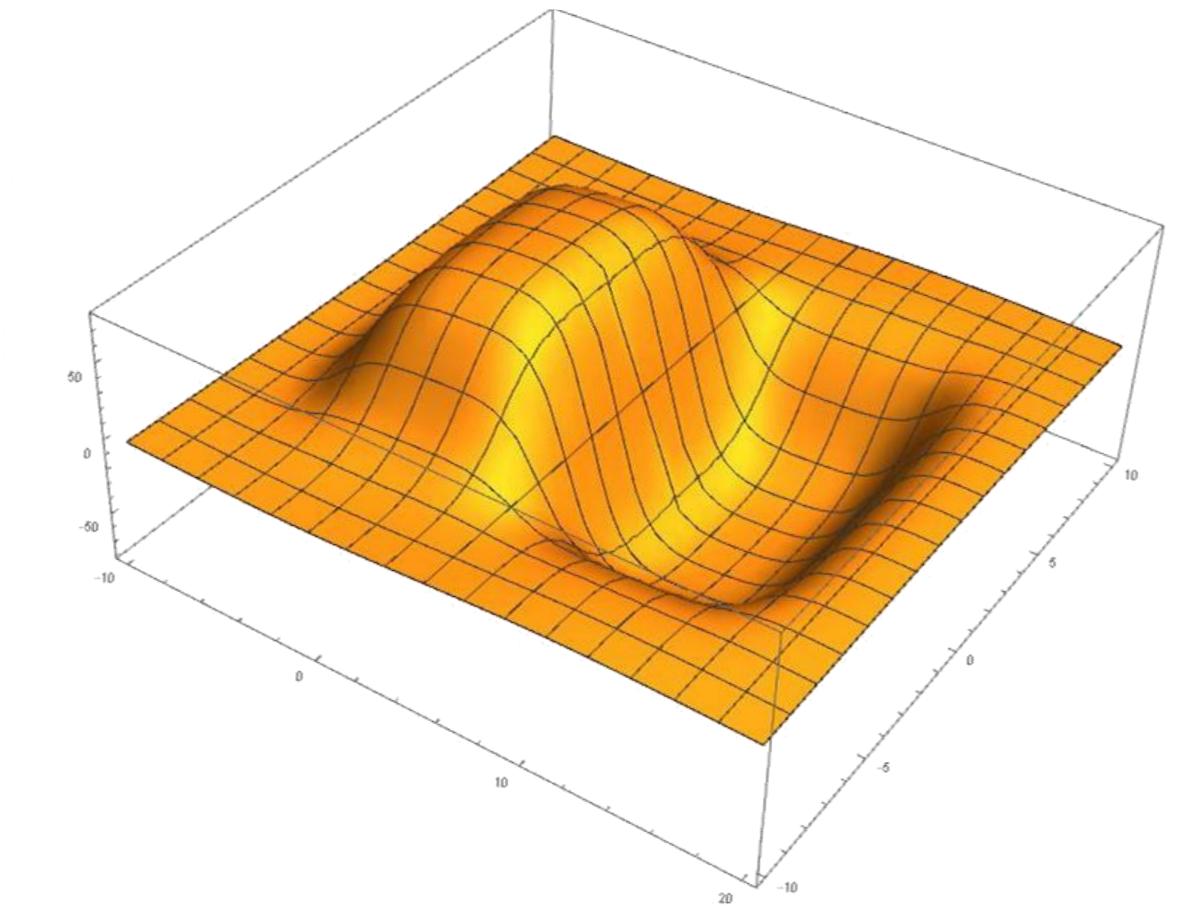
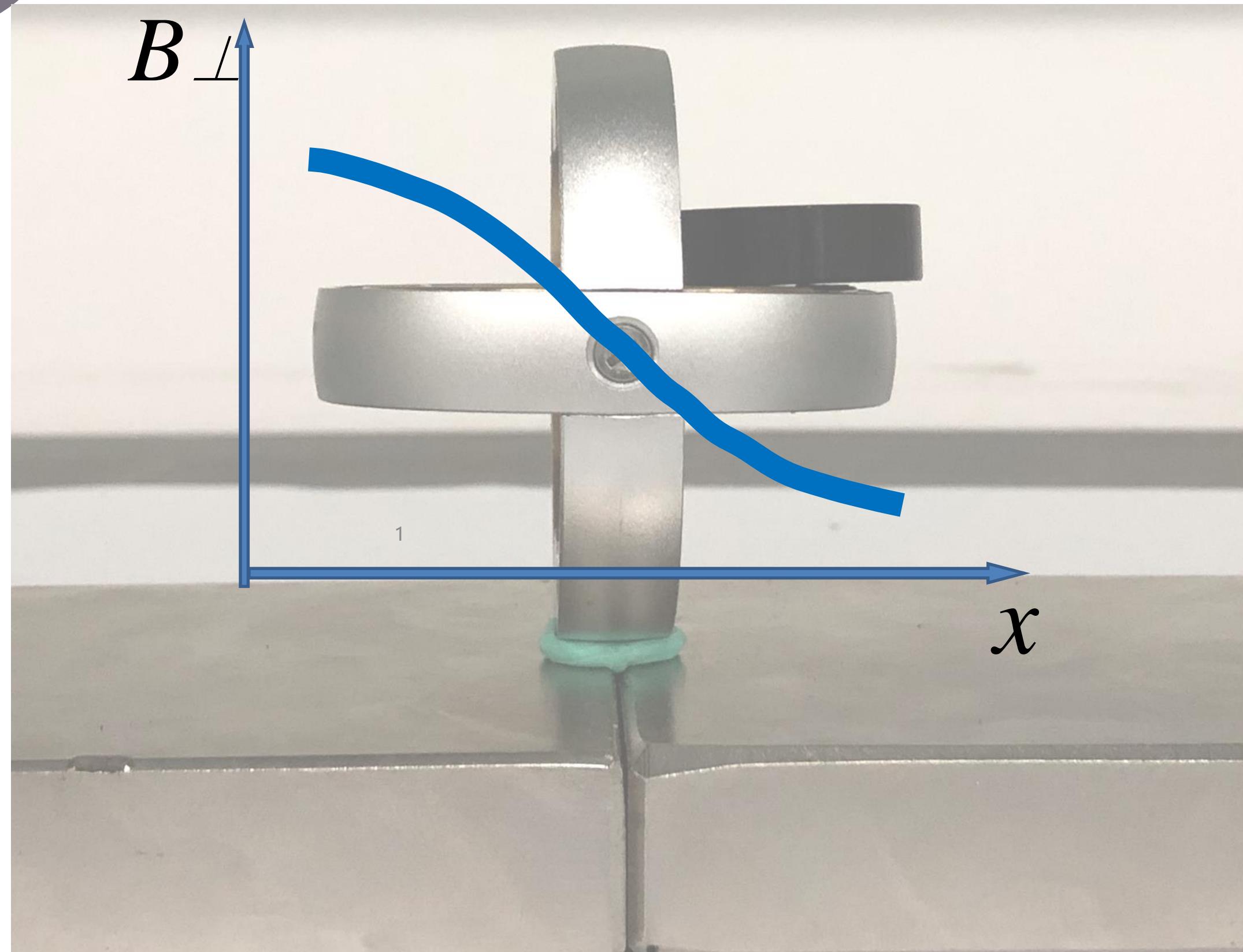
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磁场分布



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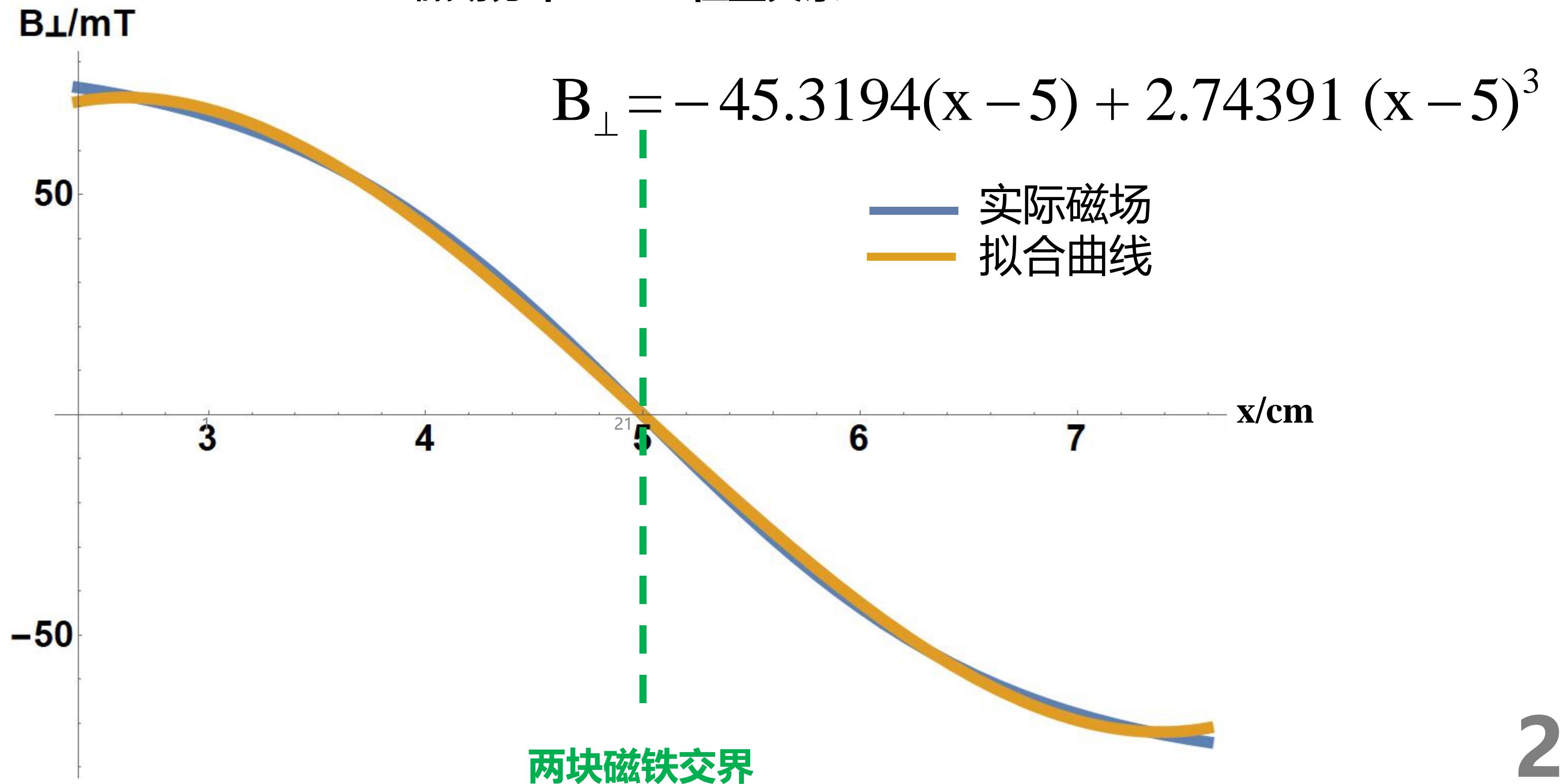


磁场拟合



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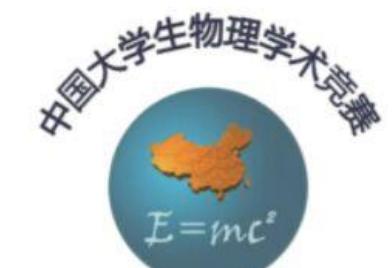
磁场分布： B_{\perp} – 位置关系



涡流



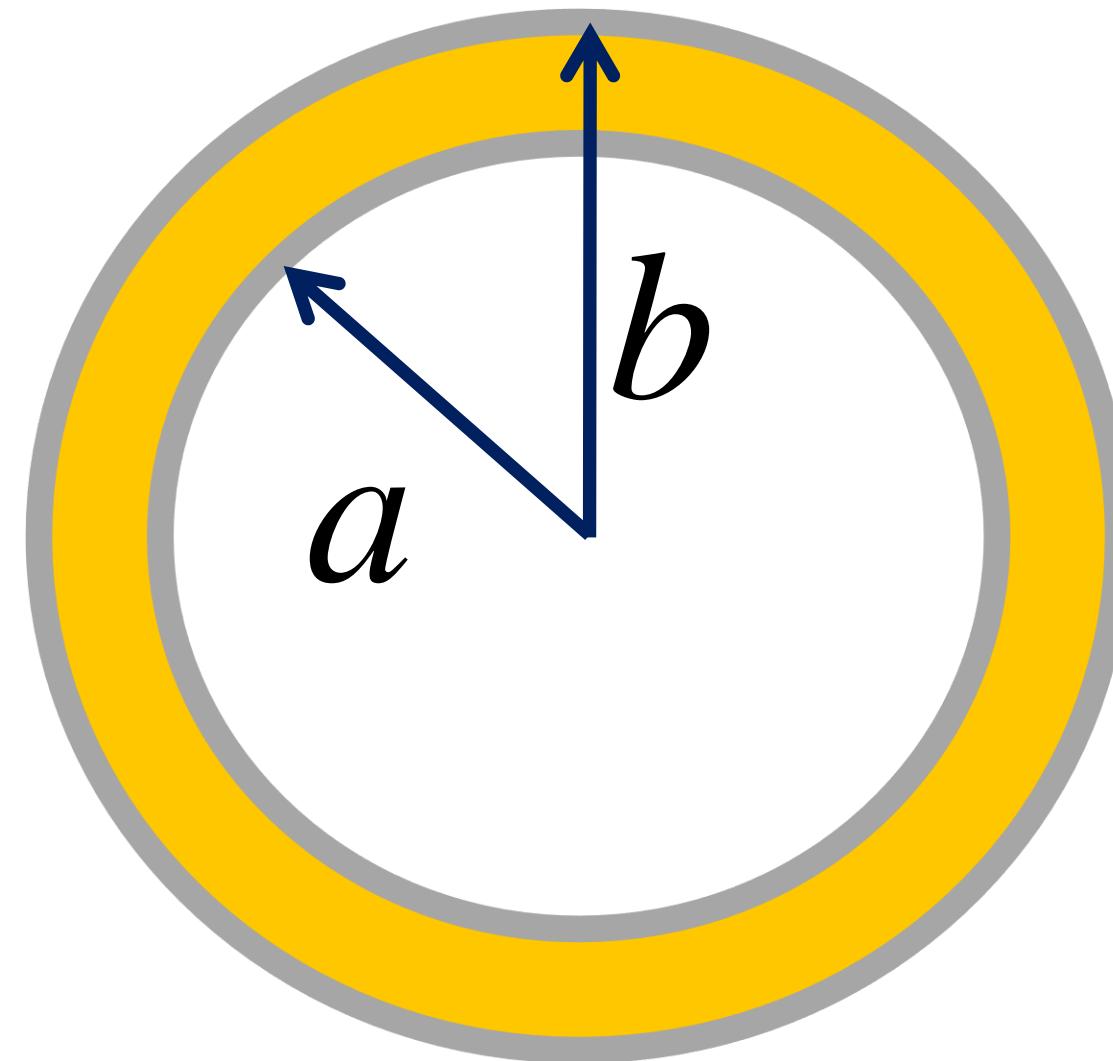
22



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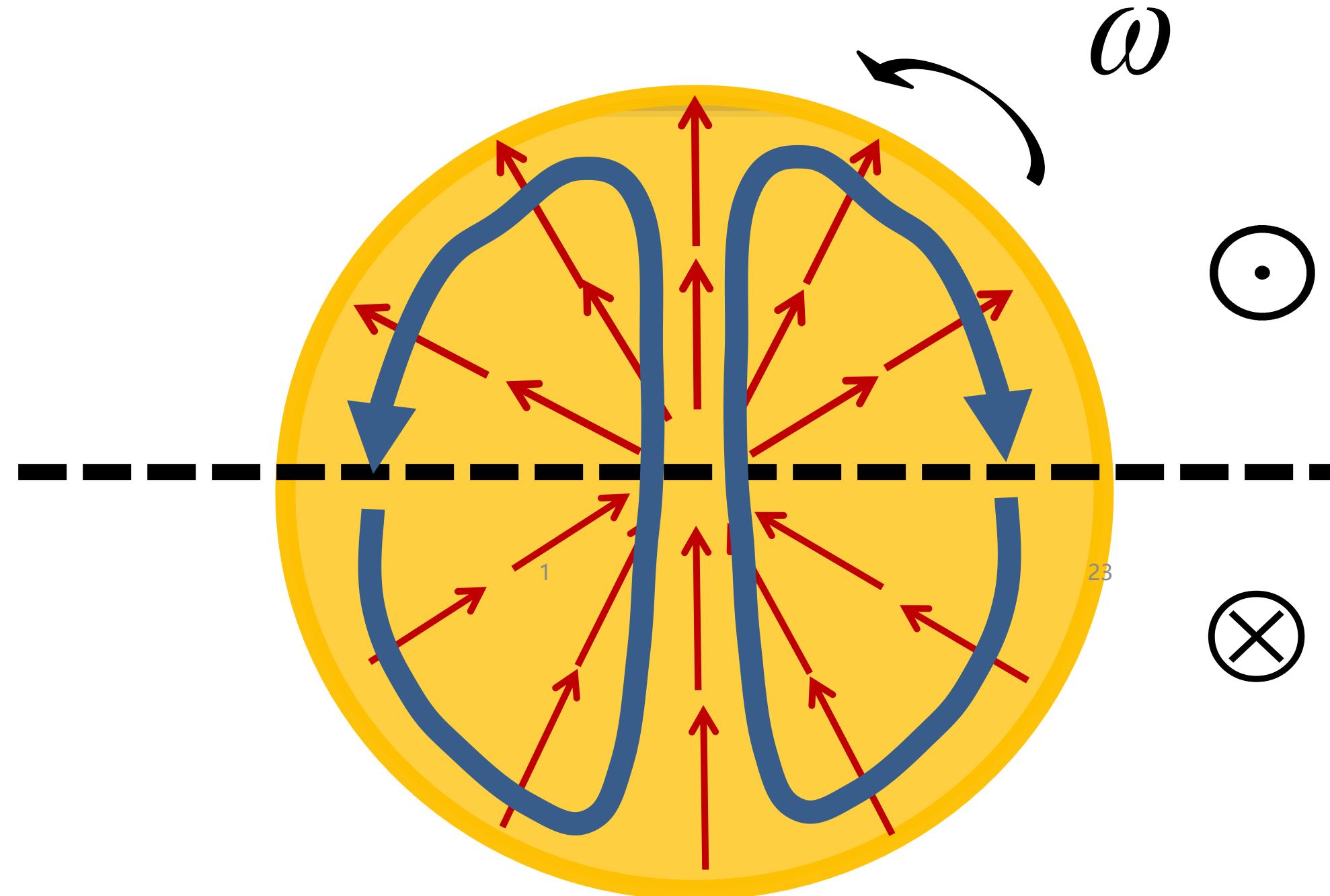


22

涡流计算



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$$\left\{ \begin{array}{l} \vec{E}_N = \vec{v} \times \vec{B} \\ \vec{j} = \sigma(\vec{E} + \vec{E}_N) \\ \nabla \cdot \vec{j} = 0 \end{array} \right.$$

电流分布的解析解



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偏微分方程

$$\left\{ \begin{array}{l} \vec{E}_N = \vec{v} \times \vec{B} \\ \vec{j} = \sigma(\vec{E} + \vec{E}_N) \\ \nabla \cdot \vec{j} = 0 \end{array} \right.$$

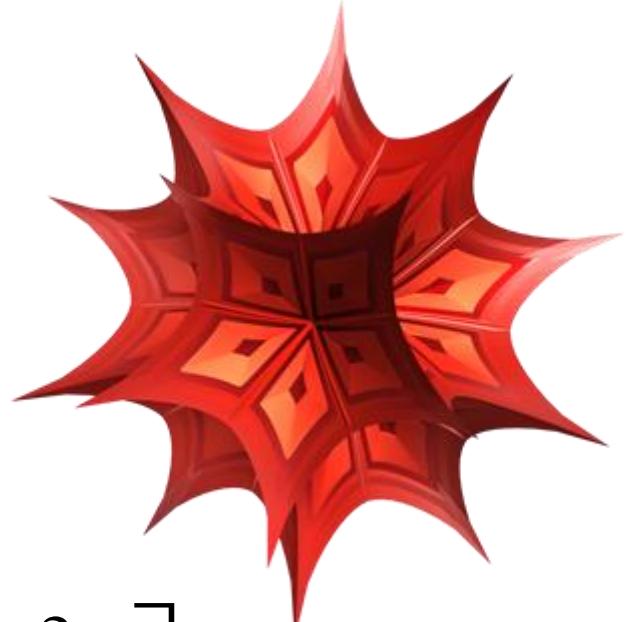
边界条件

$$\left\{ \begin{array}{l} \vec{j}|_{r=b} = 0 \\ \vec{j}|_{r=a} = 0 \end{array} \right. \Leftrightarrow \left\{ \begin{array}{l} (\vec{E} + \vec{E}_N)_r|_{r=b} = 0 \\ (\vec{E} + \vec{E}_N)_r|_{r=a} = 0 \end{array} \right. \Leftrightarrow \left\{ \begin{array}{l} \frac{\partial \phi}{\partial r}|_{r=b} = \vec{E}_{Nr}|_{r=b} \\ \frac{\partial \phi}{\partial r}|_{r=a} = \vec{E}_{Nr}|_{r=a} \end{array} \right.$$

电流分布的解析解



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电流密度的径向方向分量： $j_r(r, \theta)$

$$\begin{aligned} j_r(r, \theta) = & \sigma k_1 \omega \sin \theta \left[r^2 - \frac{1}{8} (-a^2 - b^2 + \frac{a^2 b^2}{r^2} + 9r^2) \right] \\ & + \sigma (\frac{1}{4} k_3 r^4 \omega (3 \sin \theta - \sin 3\theta) - k_3 \omega (\frac{3}{4} \sin \theta [a^4 + a^2 b^2 + b^4 - \frac{a^2 b^2 (a^2 + b^2)}{r^2}] \\ & - \frac{1}{12} \sin 3\theta [-\frac{3a^6 b^6}{(a^4 + a^2 b^2 + b^4)r^4} + \frac{3(a^8 - b^8)r^2}{a^6 - b^6}]) \\ & + \frac{5}{4} \{\frac{1}{8} \sin \theta [-5(a^4 + a^2 b^2 + b^4) + \frac{5a^2 b^2 (a^2 + b^2)}{r^2} + 5r^4] \\ & + \frac{1}{48} \sin 3\theta [-\frac{15a^6 b^6}{(a^4 + a^2 b^2 + b^4)r^4} + \frac{15(a^6 + a^4 b^2 + a^2 b^4 + b^6)r^2}{a^4 + a^2 b^2 + b^4} - 15r^4]\}) \end{aligned}$$

电流分布的解析解

电流密度的角向方向分量： $j_\theta(r, \theta)$

$$j_\theta(r, \theta) = \frac{k\omega \cos \theta}{8r} \left(\frac{\partial^2 B}{r} - (\hat{a} + \hat{B})r + 3r^3 \right) \frac{k_3 \omega^3}{r} \cos \theta$$
$$\left(\frac{\partial B(\hat{a} + \hat{B})}{r} + (\hat{a} + \hat{a}B + \hat{B}^2)r \right) \frac{1}{4} \cos^3 \theta \left(\frac{\partial B}{(\hat{a} + \hat{a}B + \hat{B}^2)r^3} + \frac{(\hat{a} - \hat{B})r^3}{\hat{a} - \hat{B}} \right) +$$
$$+ \frac{5}{48} \cos \theta \left(\frac{5\partial B(\hat{a} + \hat{B})}{r} - 5(\hat{a} + \hat{a}B + \hat{B}^2)r^5 \right)$$
$$+ \frac{1}{16} \cos^3 \theta \left(\frac{5\partial B}{(\hat{a} + \hat{a}B + \hat{B}^2)r^3} + \frac{5(\hat{a} + \hat{a}B + \hat{a}B^2 + \hat{B}^3)r^3}{\hat{a} + \hat{a}B + \hat{B}^2} - 3r^5 \right)$$



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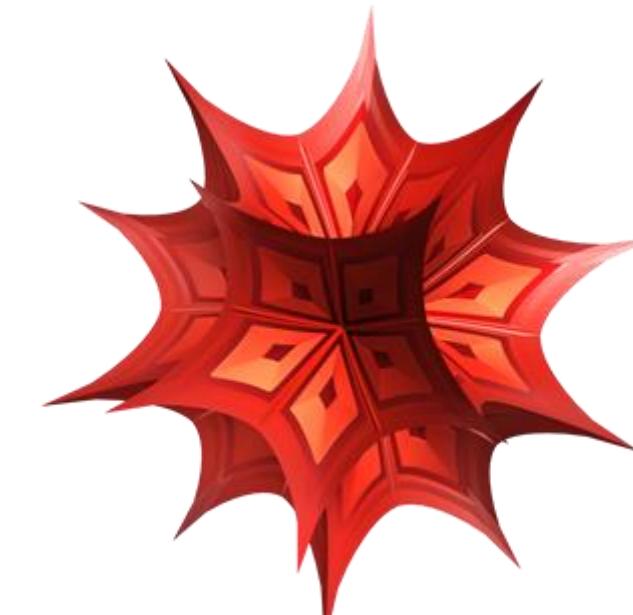


电流分布的解析解



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电流密度的角向方向分量： $j_\theta(r, \theta)$



$$j1R[r, \theta] [[2]] + j3R[r, \theta] [[2]] =$$

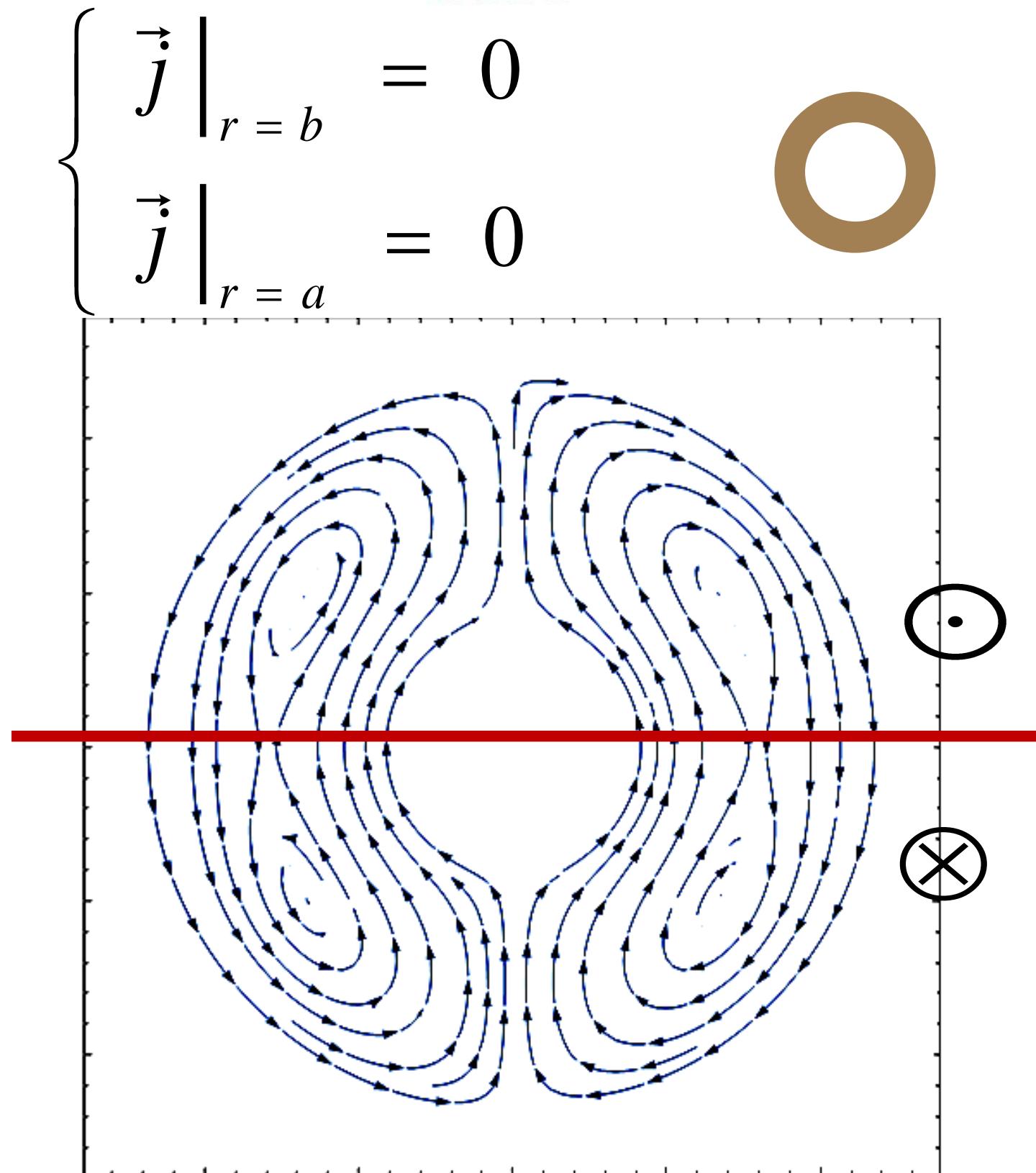
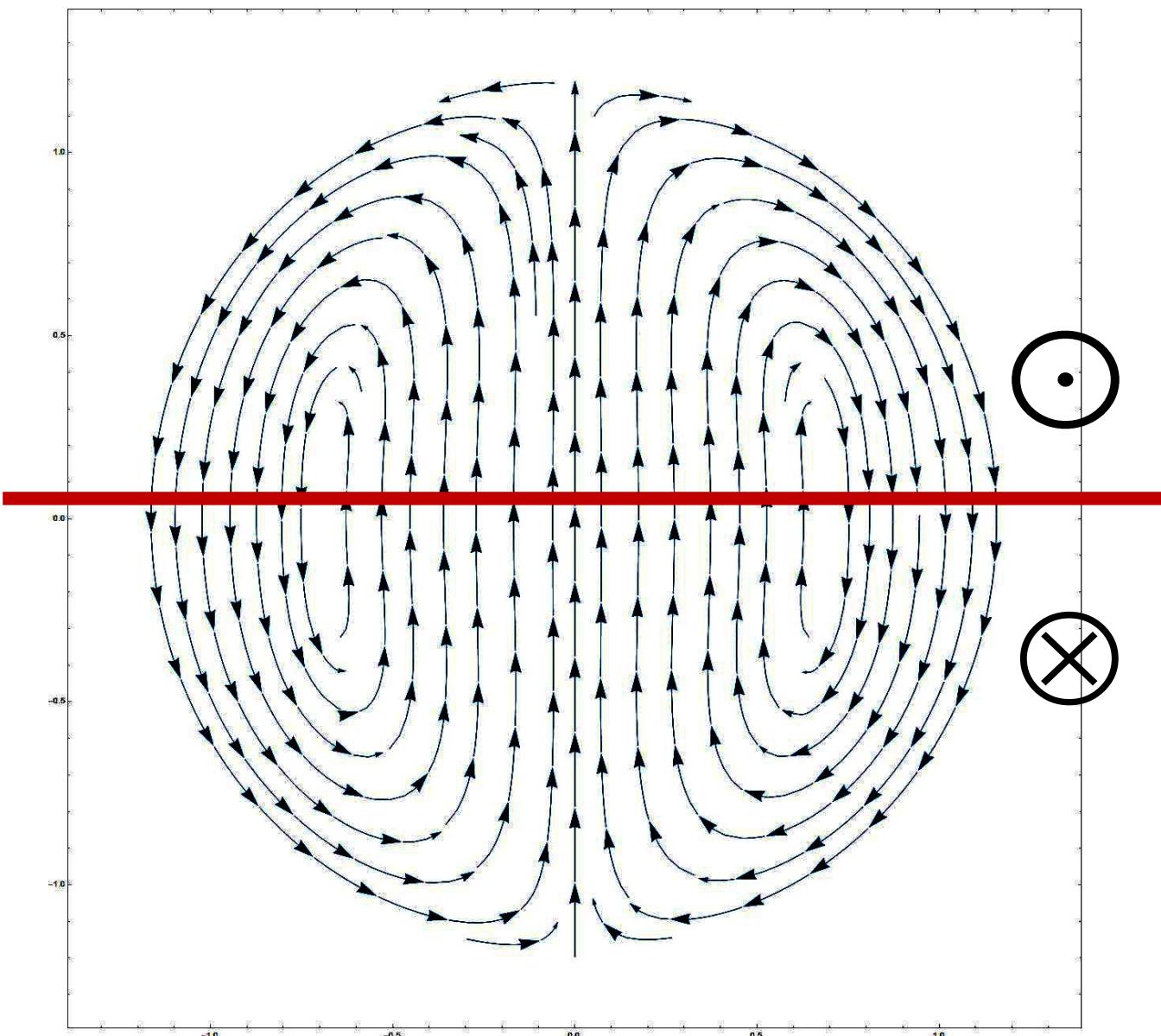
$$-\frac{k1 \left(-\frac{a^2 b^2}{r} - (a^2 + b^2) r + 3 r^3 \right) \sigma \omega \cos[\theta]}{8 r} - \frac{1}{r}$$

$$k3 \sigma \omega \left[\frac{3}{4} \left(\frac{a^2 b^2 (a^2 + b^2)}{r} + (a^4 + a^2 b^2 + b^4) r \right) \cos[\theta] - \frac{1}{4} \left(\frac{28 a^6 b^6}{(a^4 + a^2 b^2 + b^4) r^3} + \frac{(a^8 - b^8) r^3}{a^6 - b^6} \right) \cos[3 \theta] \right] \text{余弦}$$

$$\left. - \frac{5}{4} \left(\frac{1}{8} \left(-\frac{5 a^2 b^2 (a^2 + b^2)}{r} - 5 (a^4 + a^2 b^2 + b^4) r + r^5 \right) \cos[\theta] + \frac{1}{16} \left(\frac{5 a^6 b^6}{(a^4 + a^2 b^2 + b^4) r^3} + \frac{5 (a^6 + a^4 b^2 + a^2 b^4 + b^6) r^3}{a^4 + a^2 b^2 + b^4} - 3 r^5 \right) \cos[3 \theta] \right) \right] \text{余弦}$$

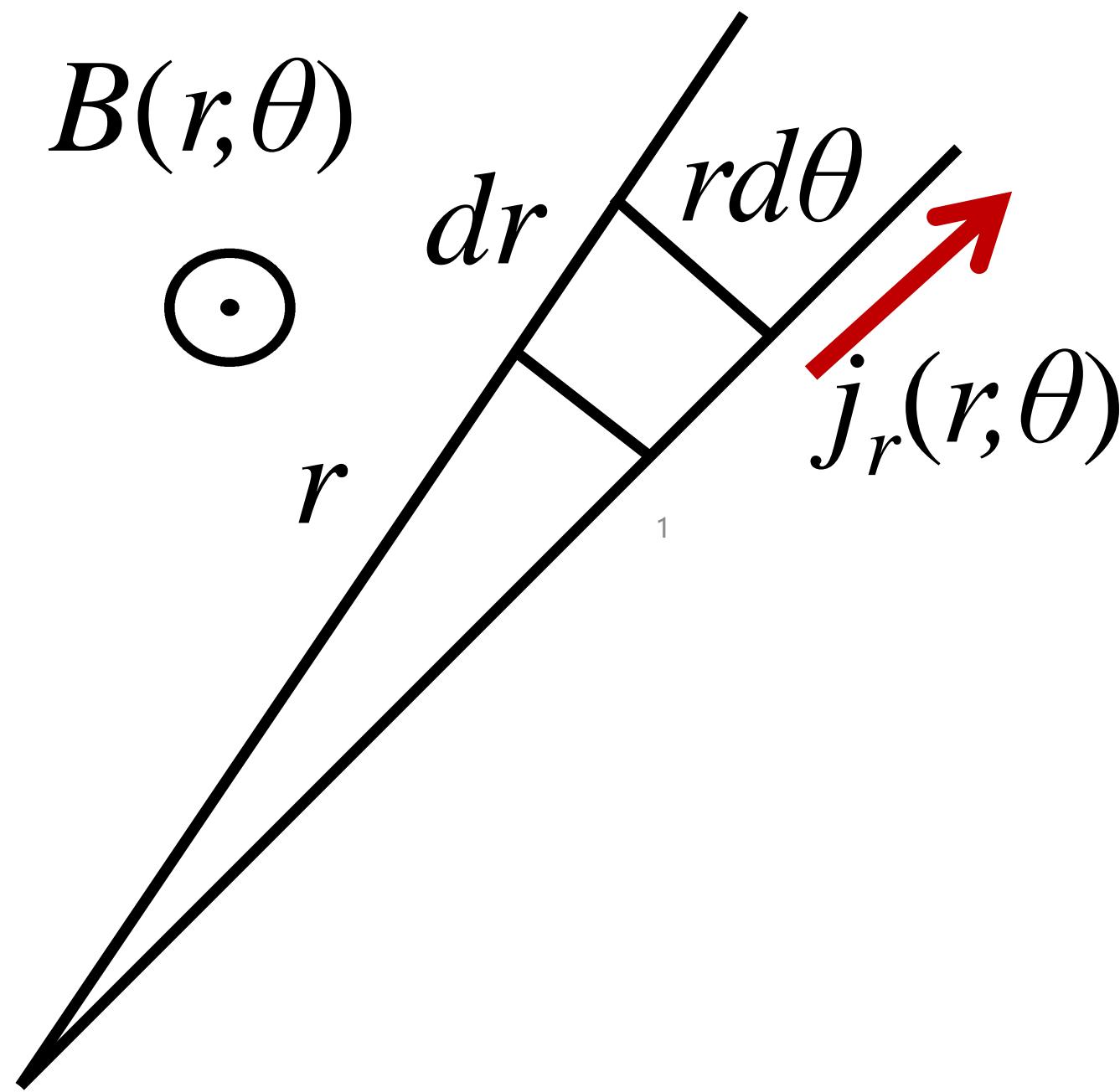
涡流分布($B=k_1x+k_3x^3$)

$$\vec{j} \Big|_{r=b} = 0$$



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力矩计算



$$I = j_r(r, \theta) r h d\theta$$

$$dM = B(r, \theta) I dr \cdot r$$

$$M = \iint_{\text{陀螺}} dM$$

$$M = \iint_{\text{陀螺}} B(r, \theta) j_r(r, \theta) (r d\theta) h r dr$$



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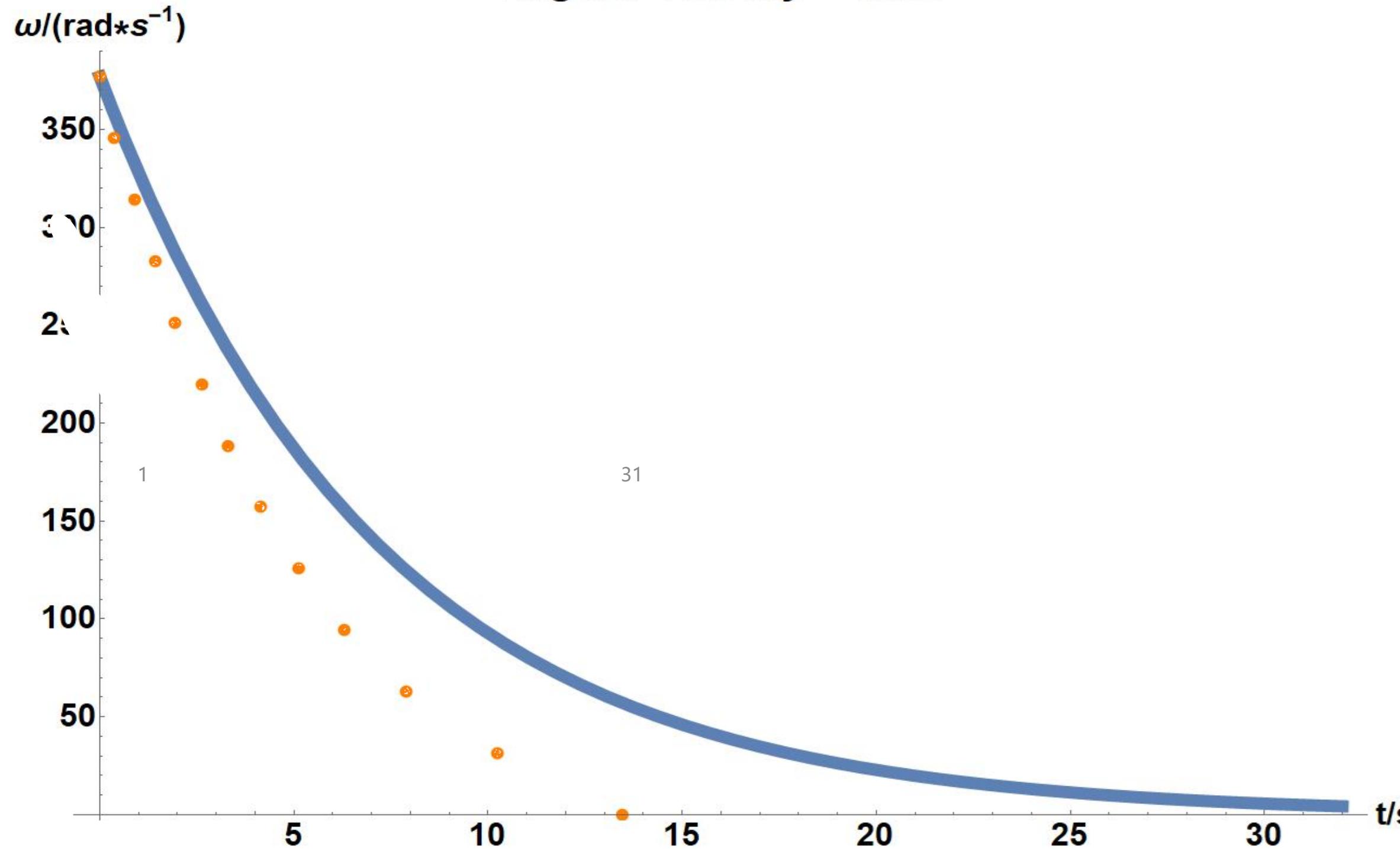
30

结果



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Angular Velocity – Time





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第三部分

1

32

理论与实验对照

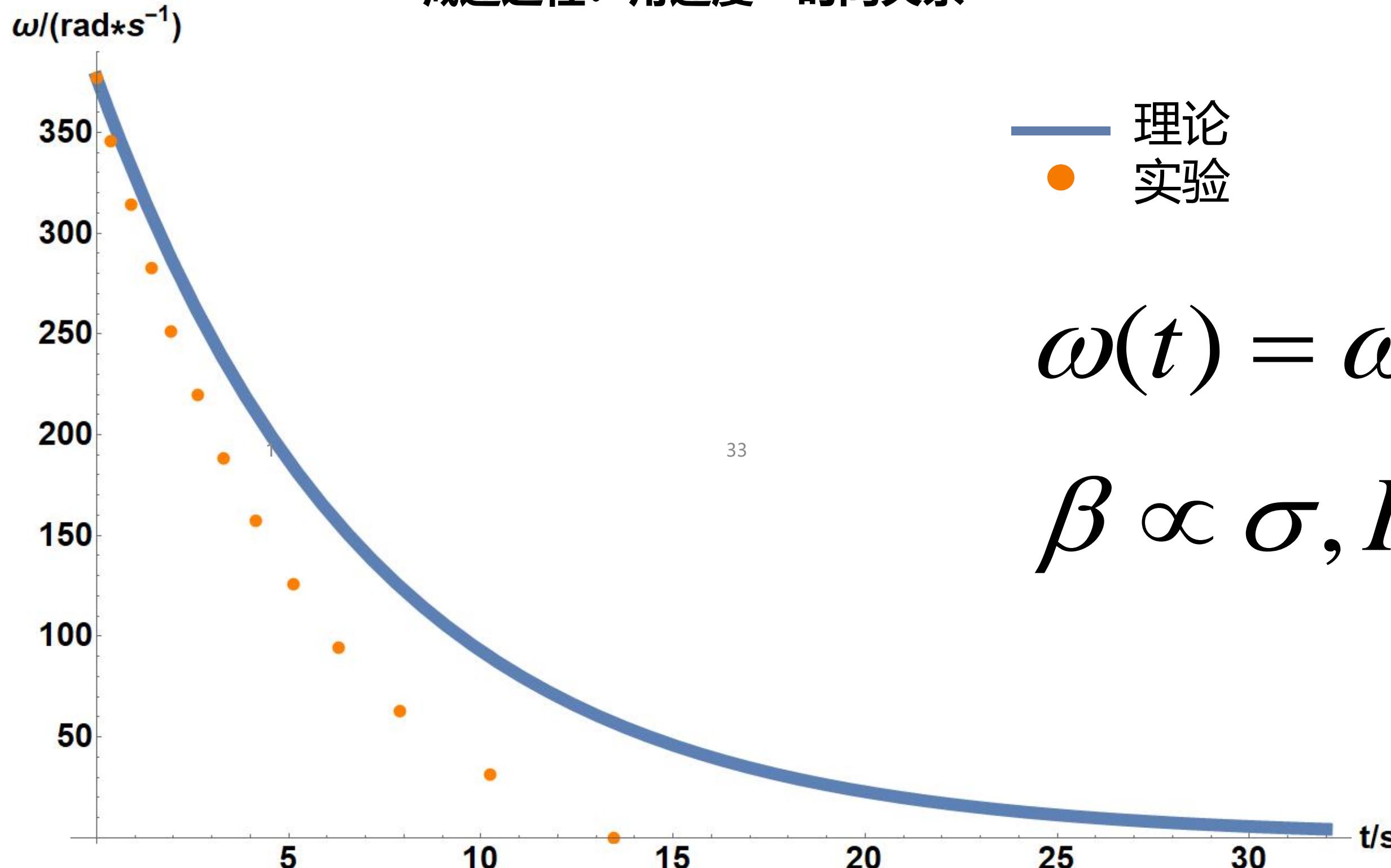
30

定量计算



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减速过程：角速度 – 时间关系



$$\omega(t) = \omega_0 e^{-\beta t}$$

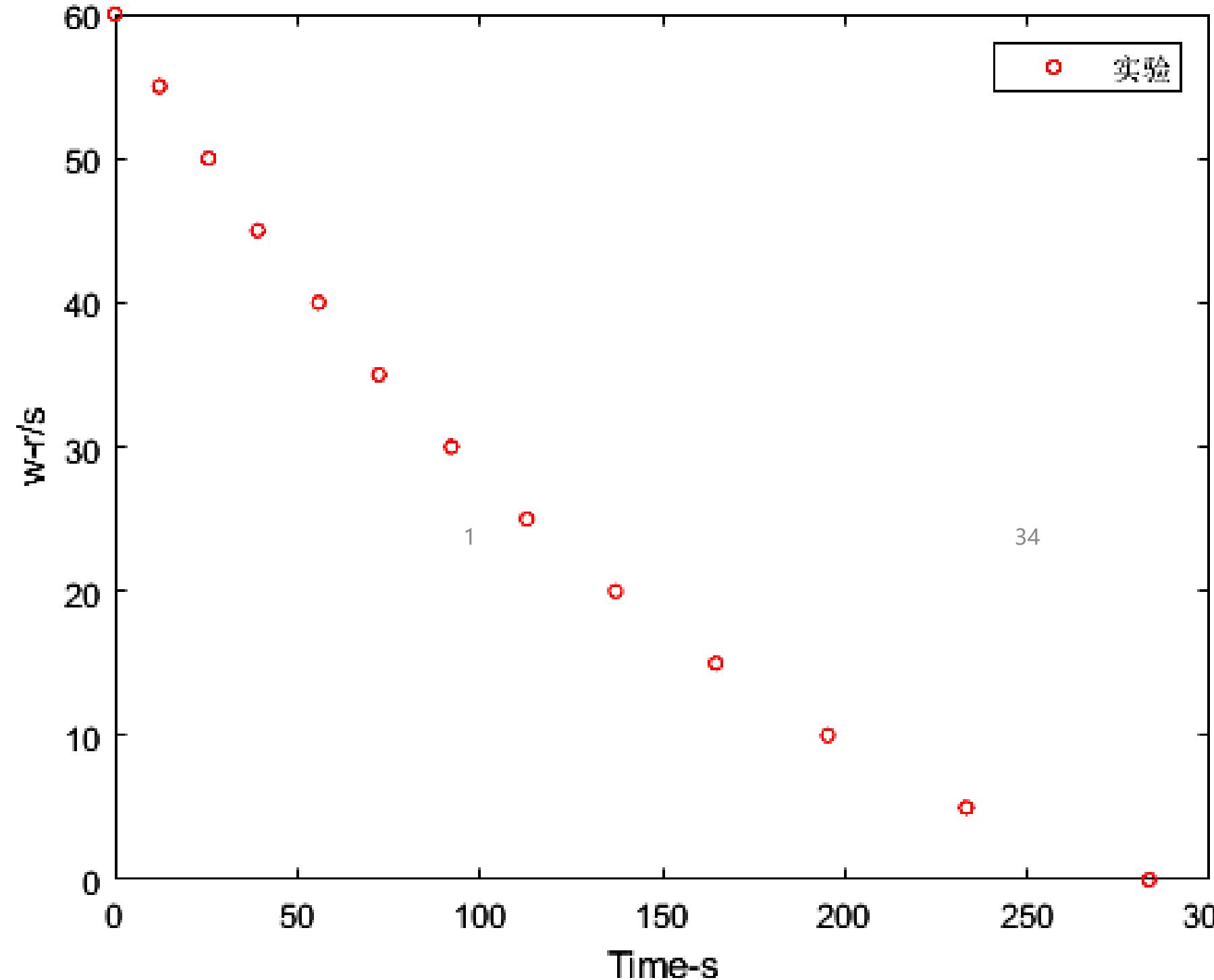
$$\beta \propto \sigma, I^{-1}$$

非电磁阻力作用



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仅有机械阻力作用：转速 – 时间关系



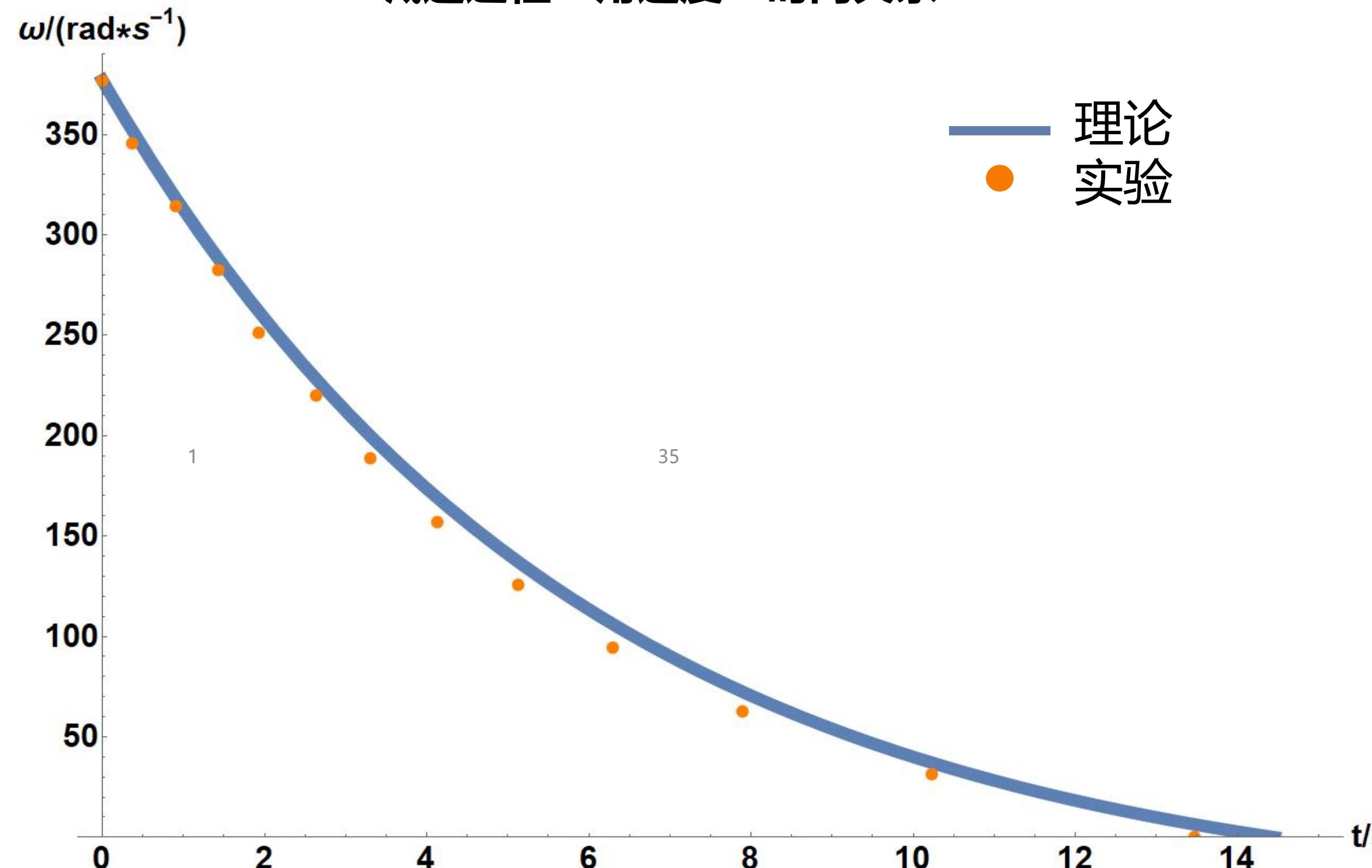
$$M_f = \alpha\omega + M_0$$

最终结果(7.5%误差)



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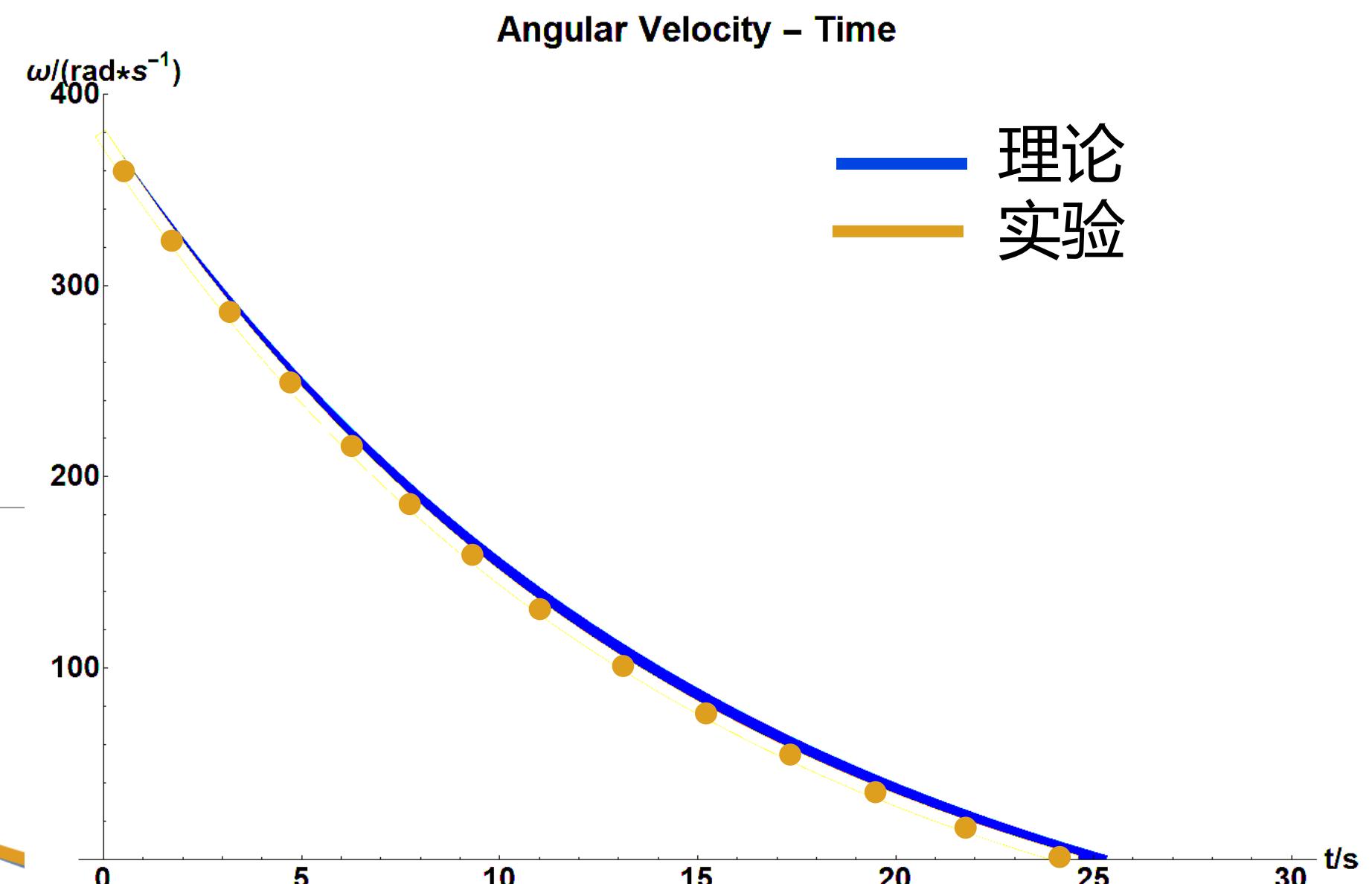
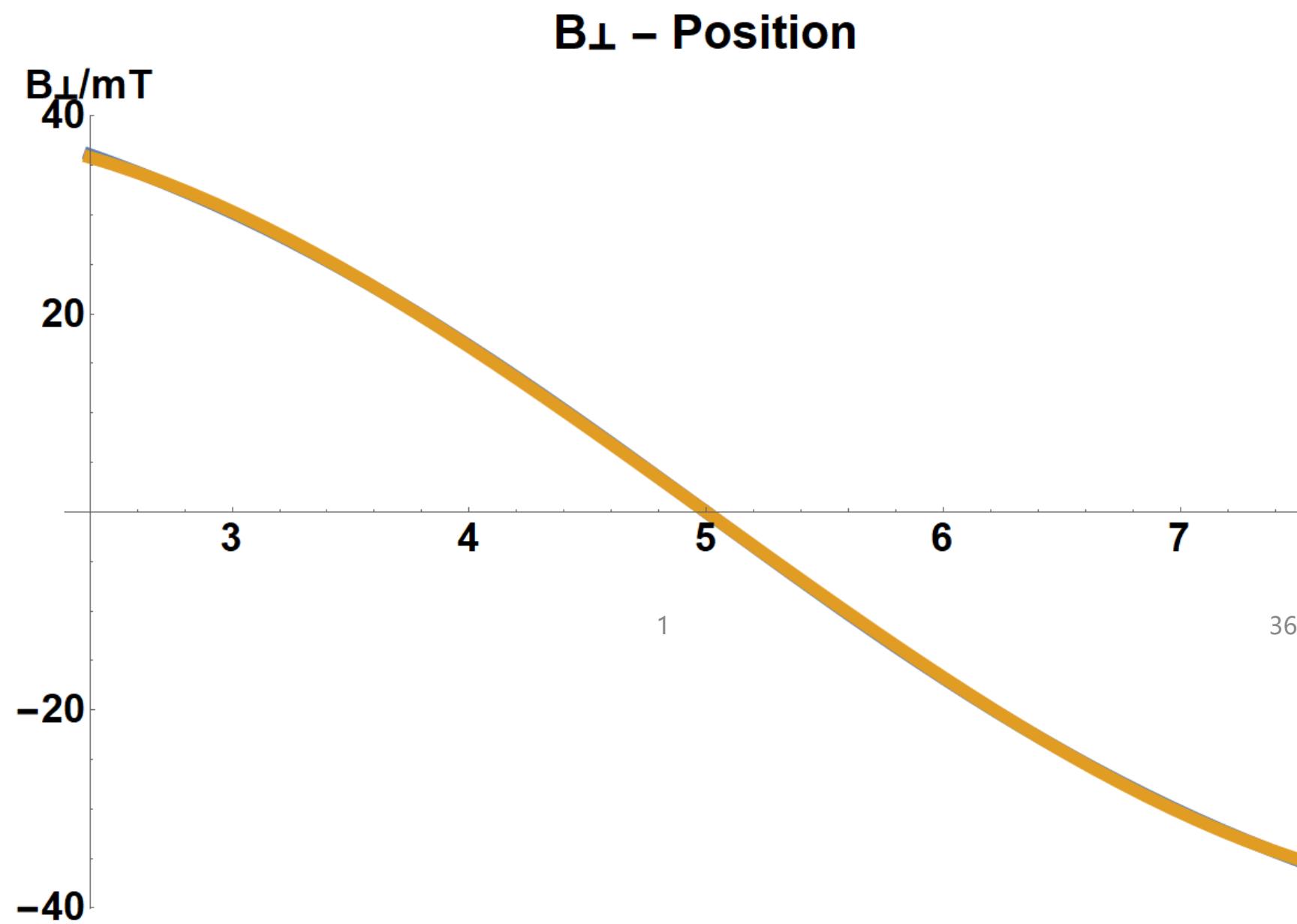
减速过程：角速度 – 时间关系



改变参量——磁场



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5cm高度处磁场

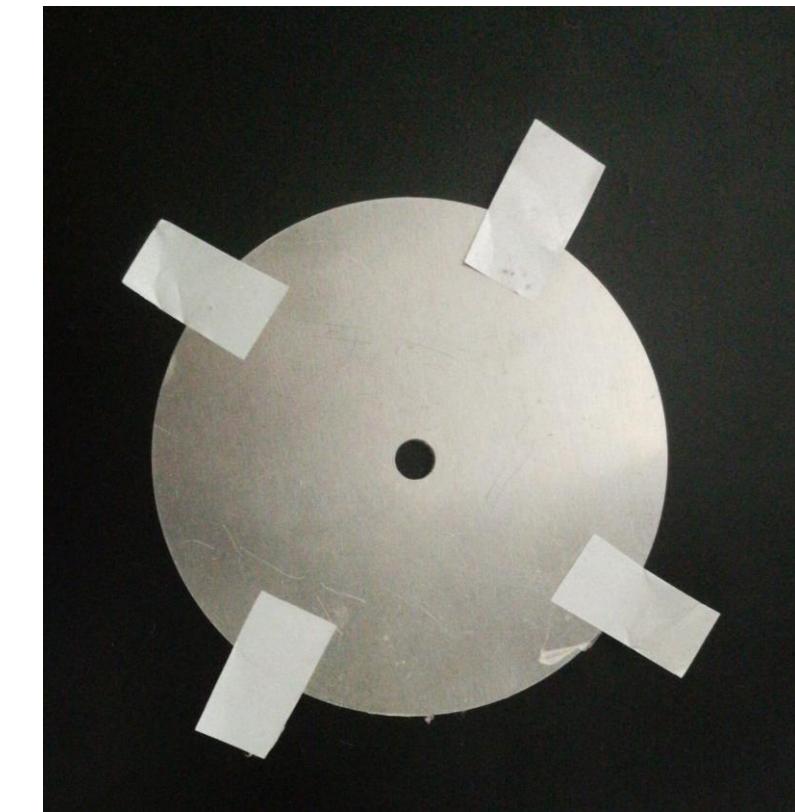
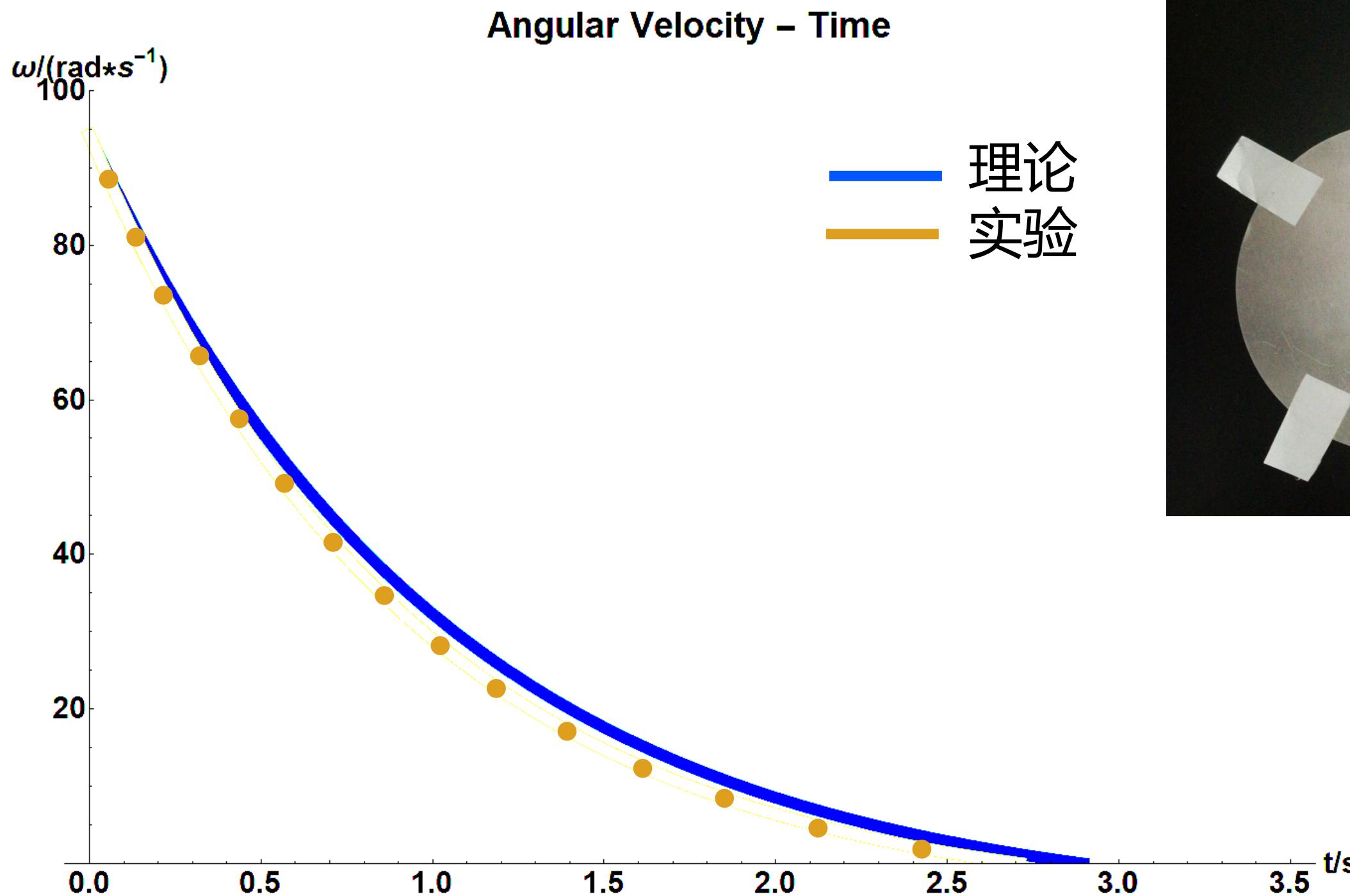
相对误差5.72%

34

改变参数——电导率



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相对误差9.45%

结论



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1. 磁场中旋转产生涡流，导致力矩，得出角速度与时间的关系
2. 拟合非电磁阻力力矩，对理论模型进行修正
3. 相关参量的讨论：
电导率，转动惯量，磁场分布

第四部分

误差分析

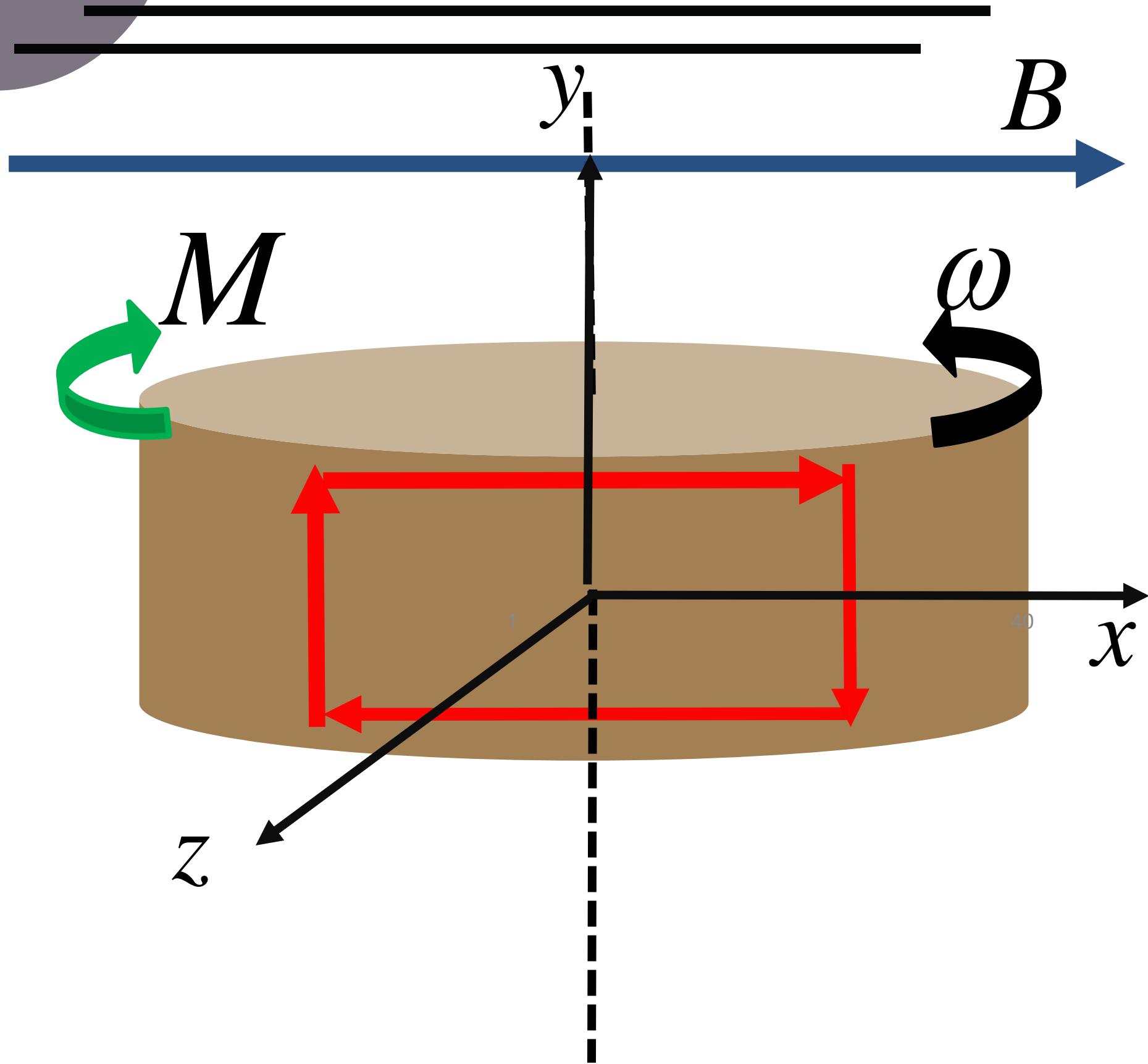
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39

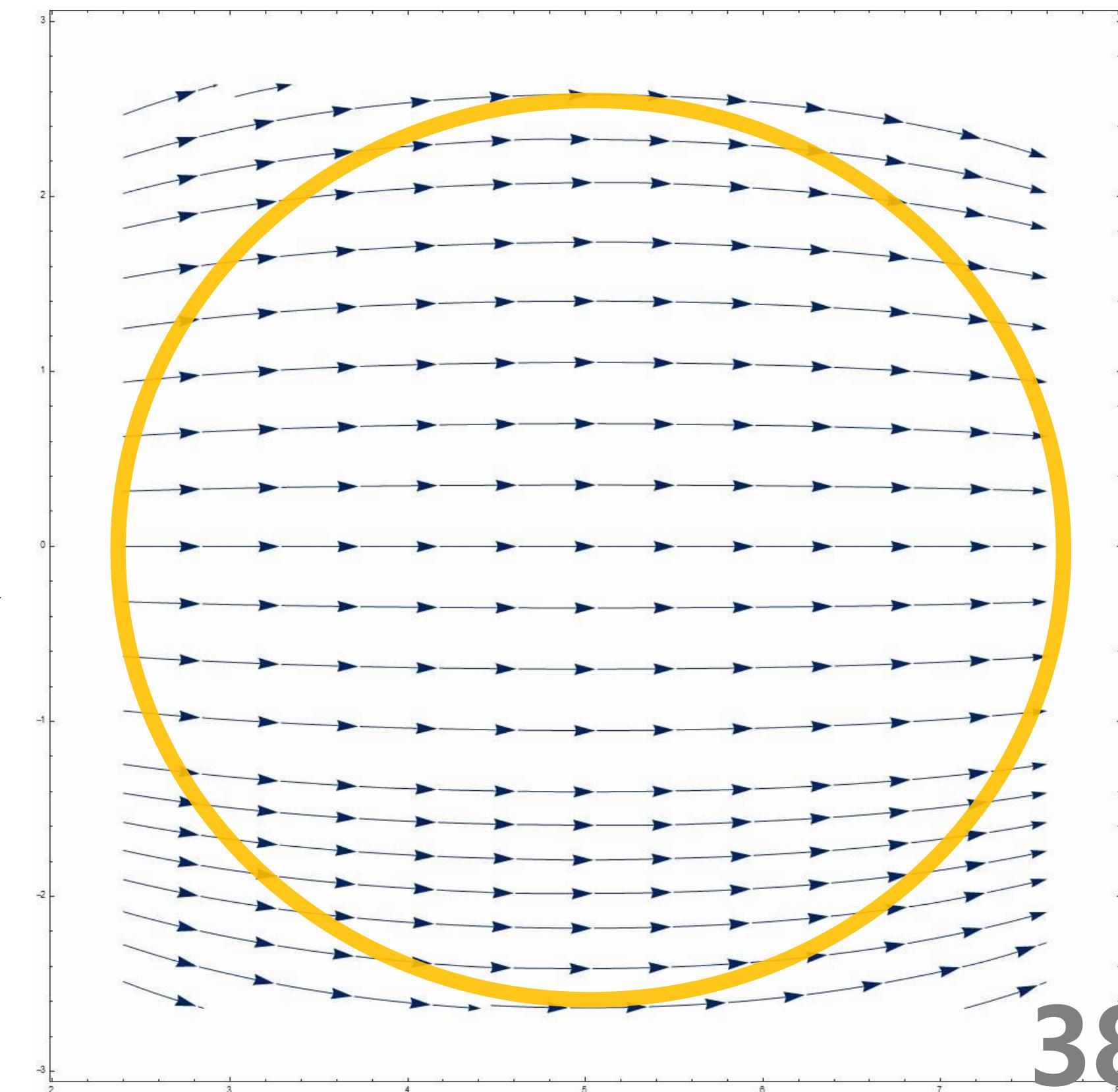


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理论偏差



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其它误差来源



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1. 趋肤效应，去磁效应

2. 磁场竖直方向分布的不均匀

3. 黄铜材料电阻率与标准值的微小偏差

遗留问题



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1.更多电阻率材料的实验

2.横向磁场所力矩计算

3.更复杂的磁场情况

总结



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1. 频闪灯测量转速获得实验数据
2. 涡流分布推导阻力矩，角速度关系
3. 阻力实测修正理论⁴³
4. 实验理论结果对比，相关误差分析

参考文献



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谢谢



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定量计算——磁场



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$$B_{1x} = \frac{B_r}{4\pi} \ln \frac{[z_2 - z + r(x_2, y_1, z_2)][z_1 - z + r(x_1, y_1, z_1)]}{[z_2 - z + r(x_1, y_1, z_2)][z_1 - z + r(x_2, y_1, z_1)]},$$

$$B_{1y} = \frac{B_r}{4\pi} \left\{ \tan^{-1} \left[\frac{(z_2 - z)(x_1 - x)}{(y_1 - y)r(x_1, y_1, z_2)} \right] + \tan^{-1} \left[\frac{(z_1 - z)(x_2 - x)}{(y_1 - y)r(x_2, y_1, z_1)} \right] \right. \\ \left. - \tan^{-1} \left[\frac{(z_1 - z)(x_1 - x)}{(y_1 - y)r(x_1, y_1, z_1)} \right] - \tan^{-1} \left[\frac{(z_2 - z)(x_2 - x)}{(y_1 - y)r(x_2, y_1, z_2)} \right] \right\},$$

$$B_{1z} = \frac{B_r}{4\pi} \ln \frac{[x_2 - x + r(x_2, y_1, z_2)][x_1 - x + r(x_1, y_1, z_1)]}{[x_2 - x + r(x_1, y_1, z_1)][x_1 - x + r(x_2, y_1, z_2)]},$$

where

$$r(x_i, y_j, z_k) = \sqrt{(x_i - x)^2 + (y_j - y)^2 + (z_k - z)^2}.$$

Magnetic Field Distribution of a Rectangular Permanent Magnet Block

Let us assume that the relative permeability is 1. The field can be described by uniform magnetic charge sheets. For the specific problem we have in mind, we only consider the case that magnification is along the y axis. As a result, the charge sheets exist only on the surfaces that are parallel to the x-z plane, i.e., the normal of the charge sheets is parallel to the y axis. The density of charge on the two charge sheets are of the same amplitude but opposite signs. For a single charge sheet, whose corners are (x_1, y_1, z_1) , (x_1, y_1, z_2) , (x_2, y_1, z_1) and (x_2, y_1, z_2) , the field components are

$$B_{1x} = \frac{B_r}{4\pi} \ln \frac{[z_2 - z + r(x_2, y_1, z_2)][z_1 - z + r(x_1, y_1, z_1)]}{[z_2 - z + r(x_1, y_1, z_2)][z_1 - z + r(x_2, y_1, z_1)]},$$

$$B_{1y} = \frac{B_r}{4\pi} \left\{ \tan^{-1} \left[\frac{(z_2 - z)(x_1 - x)}{(y_1 - y)r(x_1, y_1, z_2)} \right] + \tan^{-1} \left[\frac{(z_1 - z)(x_2 - x)}{(y_1 - y)r(x_2, y_1, z_1)} \right] \right. \\ \left. - \tan^{-1} \left[\frac{(z_1 - z)(x_1 - x)}{(y_1 - y)r(x_1, y_1, z_1)} \right] - \tan^{-1} \left[\frac{(z_2 - z)(x_2 - x)}{(y_1 - y)r(x_2, y_1, z_2)} \right] \right\},$$

$$B_{1z} = \frac{B_r}{4\pi} \ln \frac{[x_2 - x + r(x_2, y_1, z_2)][x_1 - x + r(x_1, y_1, z_1)]}{[x_2 - x + r(x_1, y_1, z_1)][x_1 - x + r(x_2, y_1, z_2)]},$$

where

$$r(x_i, y_j, z_k) = \sqrt{(x_i - x)^2 + (y_j - y)^2 + (z_k - z)^2}.$$

For the other single charge sheet of the block, whose corners are (x_1, y_2, z_1) , (x_1, y_2, z_2) , (x_2, y_2, z_1) and (x_2, y_2, z_2) , the field components are

$$B_{2x} = -\frac{B_r}{4\pi} \ln \frac{[z_2 - z + r(x_2, y_2, z_2)][z_1 - z + r(x_1, y_2, z_1)]}{[z_2 - z + r(x_1, y_2, z_2)][z_1 - z + r(x_2, y_2, z_1)]},$$

$$B_{2y} = -\frac{B_r}{4\pi} \left\{ \tan^{-1} \left[\frac{(z_2 - z)(x_1 - x)}{(y_2 - y)r(x_1, y_2, z_2)} \right] + \tan^{-1} \left[\frac{(z_1 - z)(x_2 - x)}{(y_2 - y)r(x_2, y_2, z_1)} \right] \right\}$$

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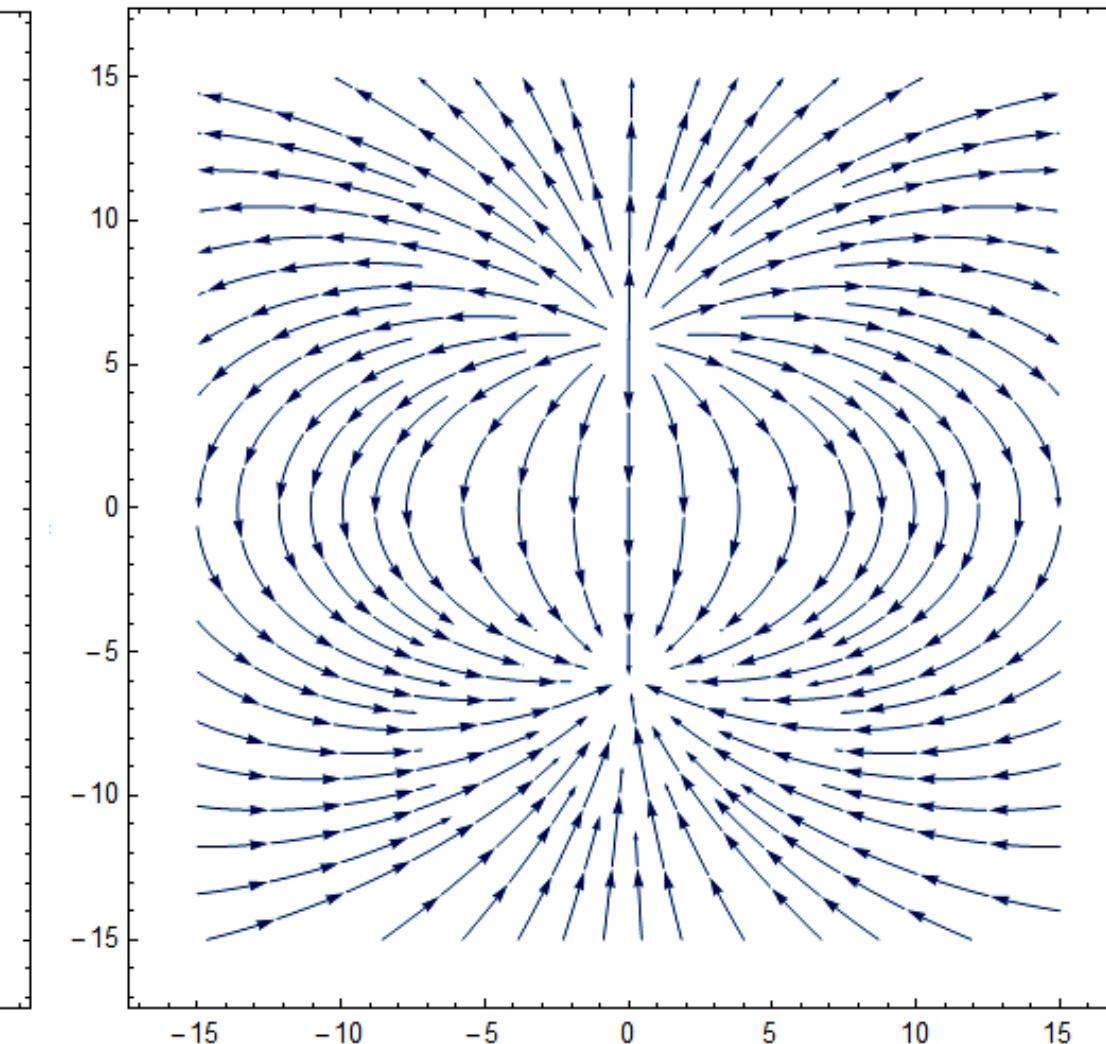
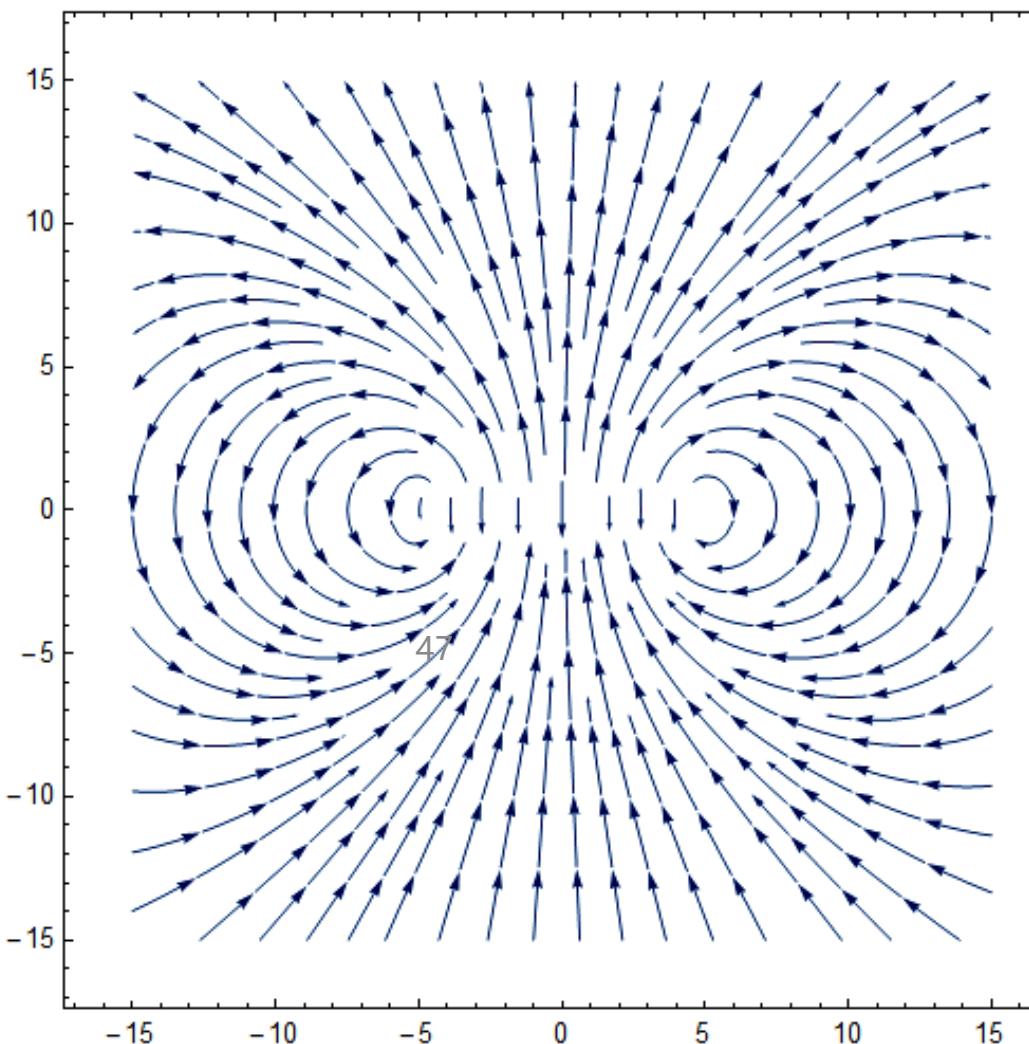
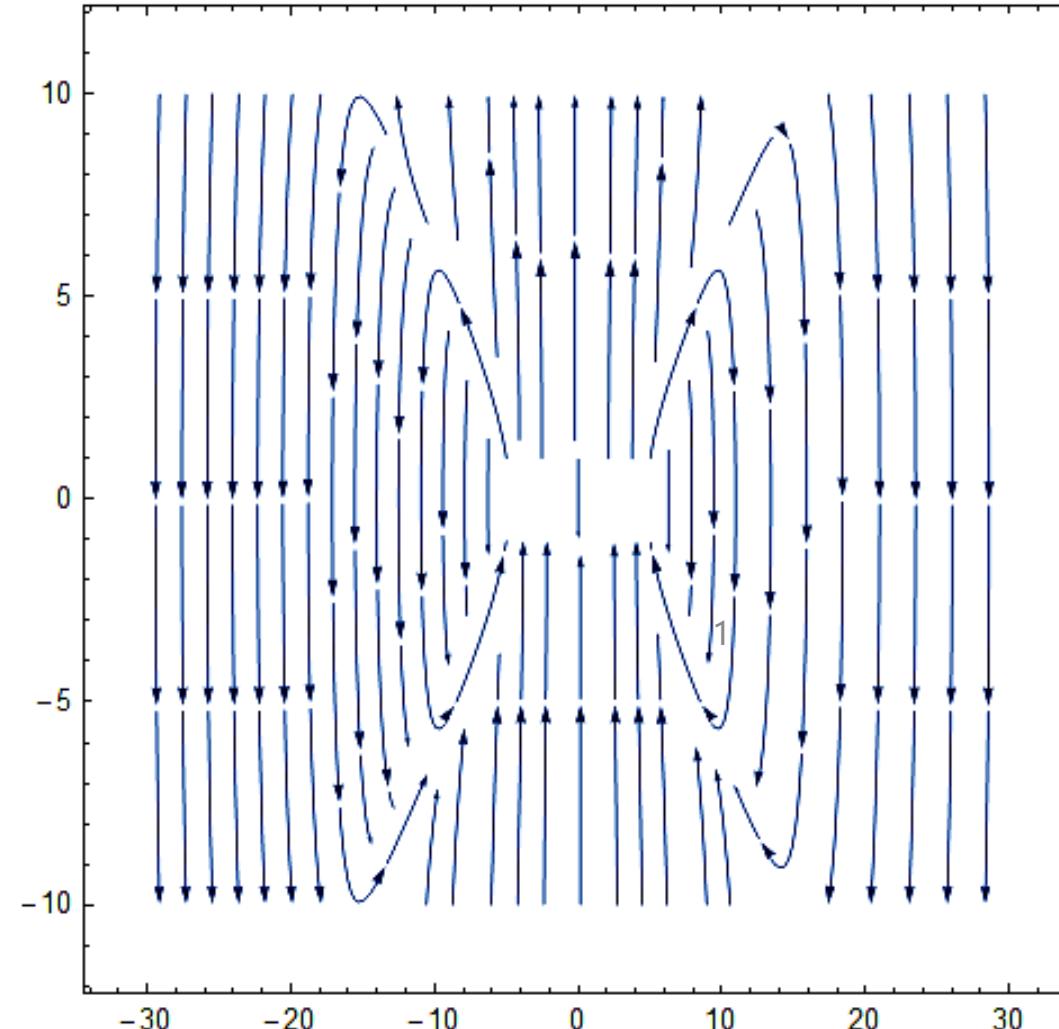
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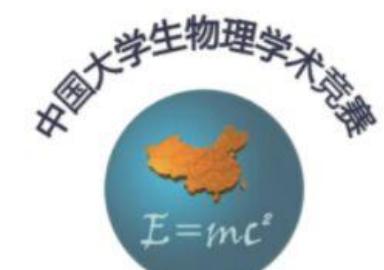
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StreamPlot[{B1x + B2x, B1y + B2y}, {x, -30, 30}, {y, -10, 10}]
StreamPlot[{B1x + B2x, B1y + B2y}, {x, -15, 15}, {y, -15, 15}]
StreamPlot[{B1x + B2x, B1y + B2y}, {x, -15, 15}, {y, -15, 15}]

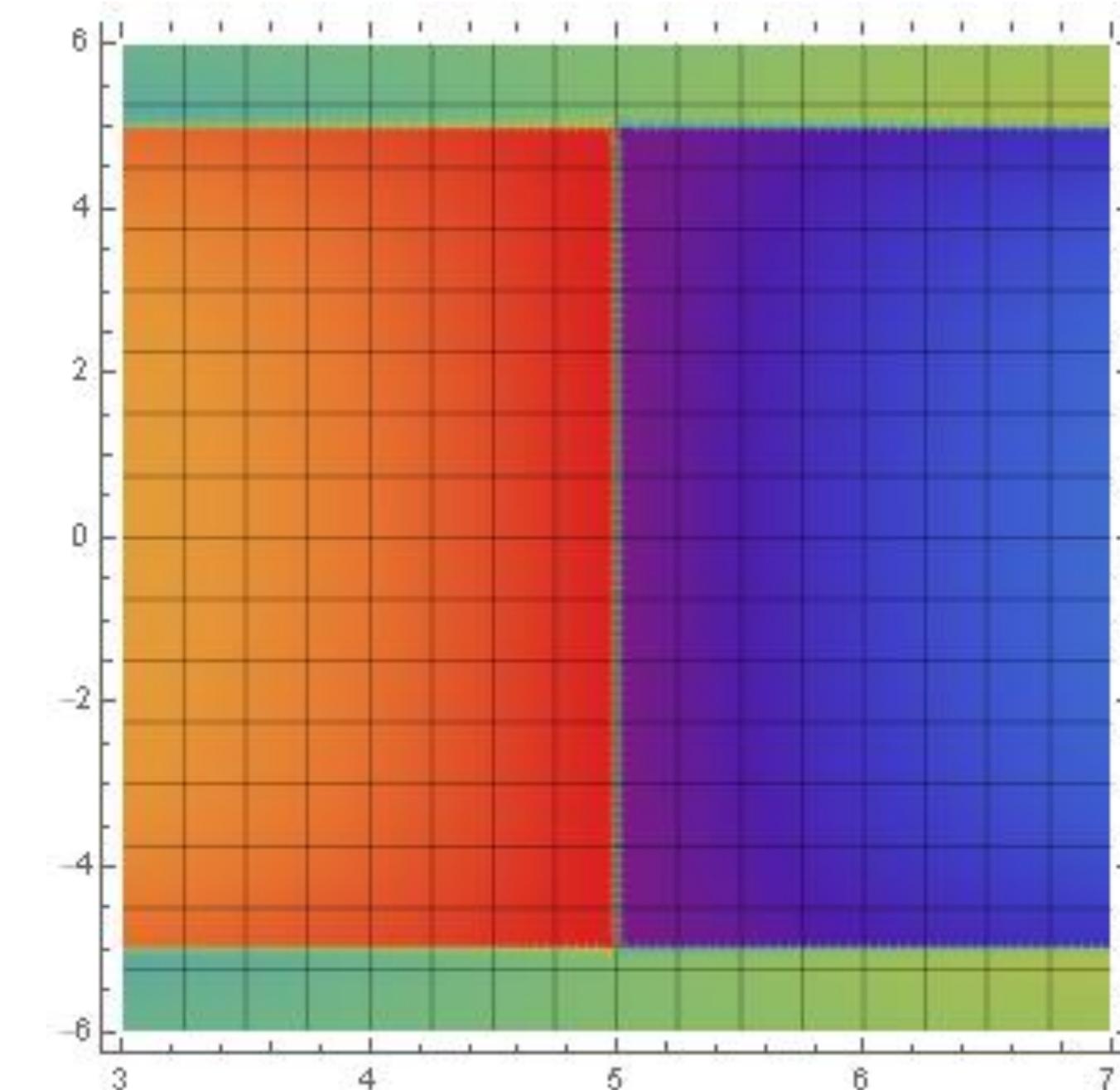
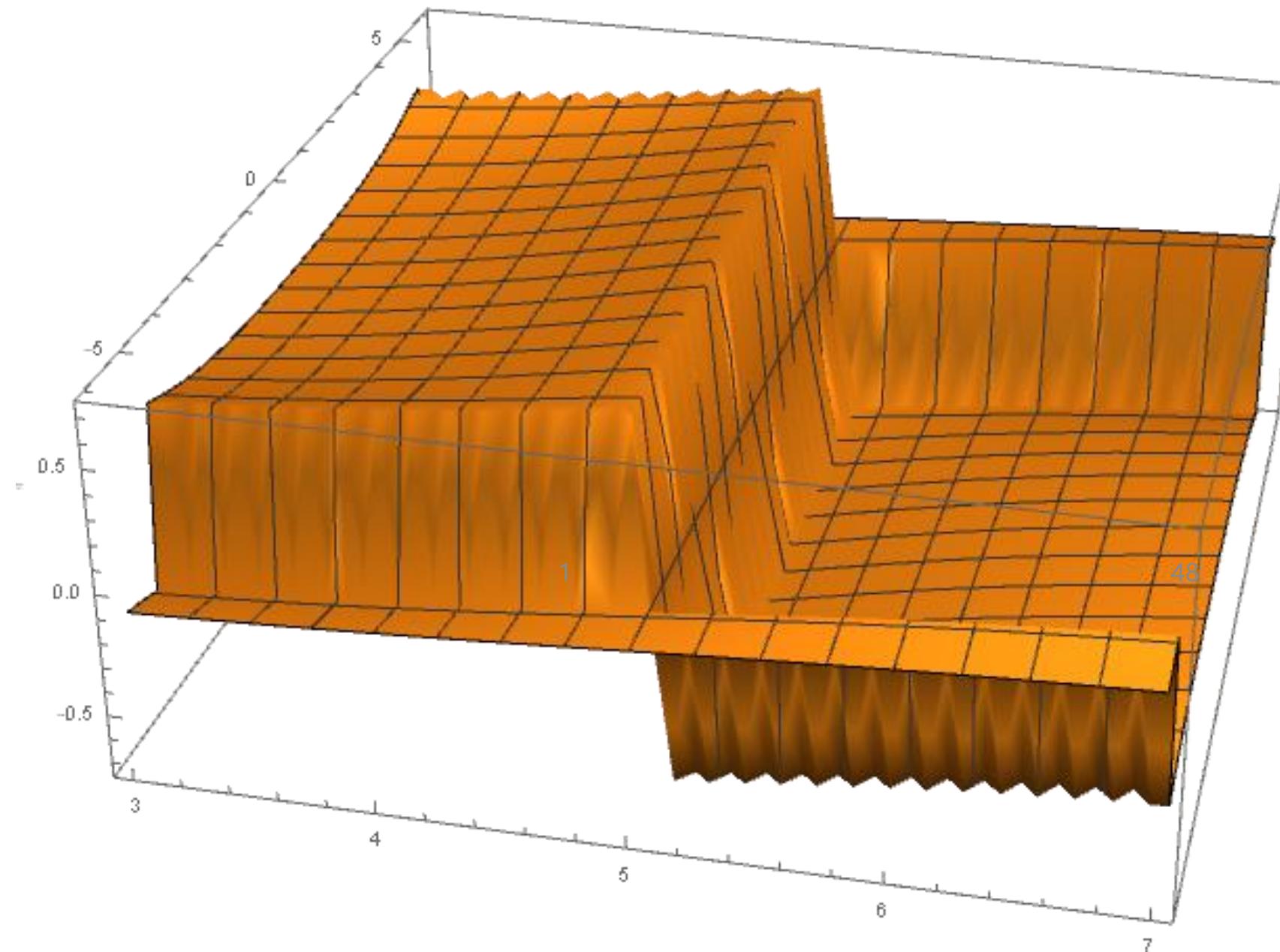
|流线图



磁场分布



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电流分布的解析解



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偏微分方程

$$\begin{cases} \vec{f} = \vec{v} \times \vec{B} \\ \vec{j} = \sigma(\vec{E} + \vec{f}) \\ \nabla \cdot \vec{j} = 0 \end{cases}$$

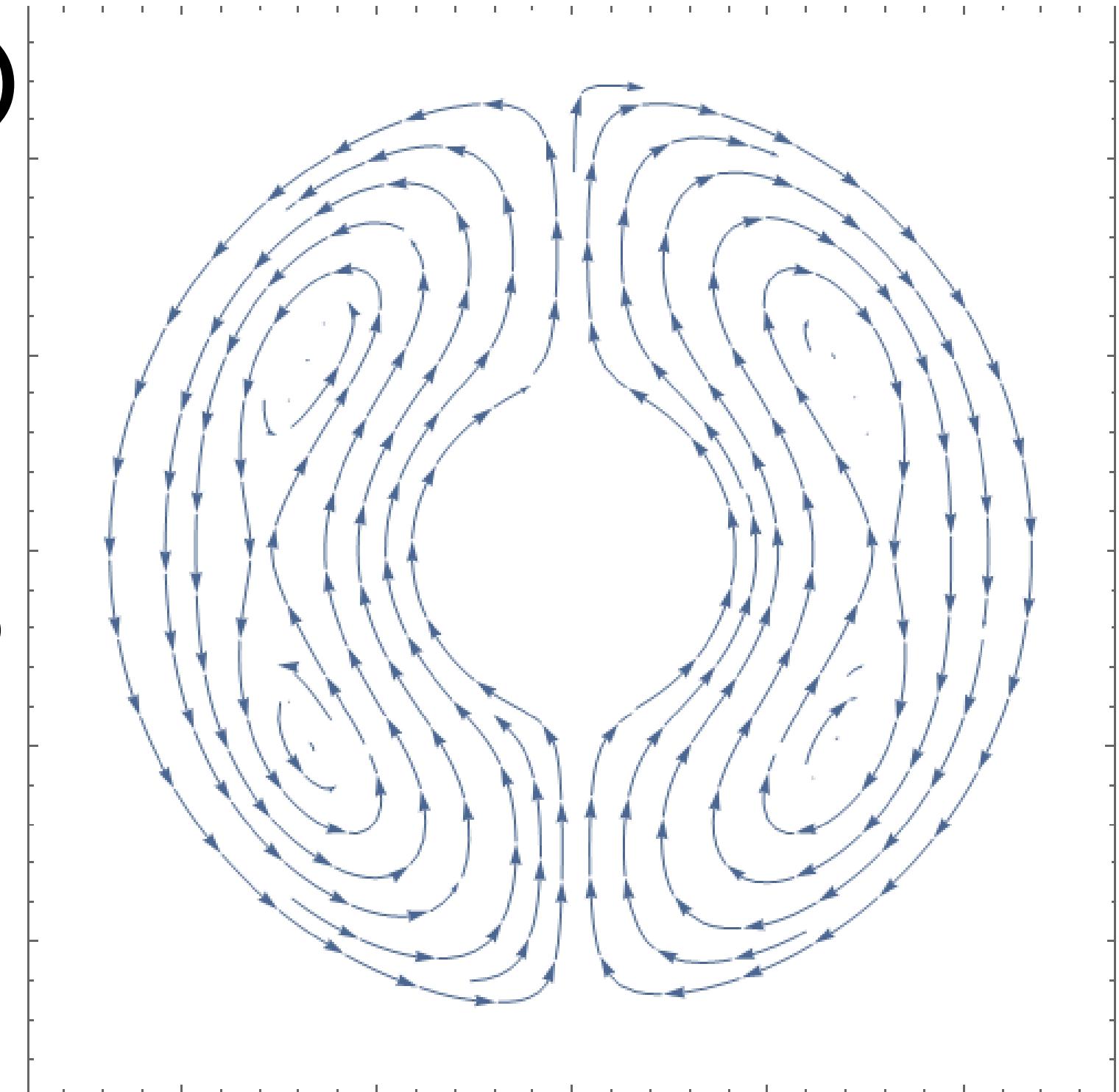
$$\Rightarrow \nabla \cdot \vec{E} = -\nabla \cdot \vec{f}$$

$$\Leftrightarrow \boxed{\nabla^2 \varphi = \nabla \cdot \vec{f}}$$

定解条件 (边界条件)

$$\Leftrightarrow \begin{cases} \vec{j} \Big|_{r=b} = 0 \\ \vec{j} \Big|_{r=a} = 0 \\ (\vec{E} + \vec{f})_r \Big|_{r=b} = 0 \\ (\vec{E} + \vec{f})_r \Big|_{r=a} = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} \frac{\partial \varphi}{\partial r} \Big|_{r=b} = f_r \Big|_{r=b} \\ \frac{\partial \varphi}{\partial r} \Big|_{r=a} = f_r \Big|_{r=a} \end{cases}$$



电流分布的解析解



020

From Page No./起始页码

I. $\nabla^2 \Psi = 3wkr^2 \sin\theta$ $\Psi = A_0 r \sin\theta$

$$\begin{cases} \frac{\partial \Psi}{\partial r}|_{r=a} = 0 \\ \frac{\partial \Psi}{\partial r}|_{r=b} = 0 \end{cases}$$

$$\Rightarrow \nabla^2 \Psi = [A_0'' + \frac{1}{r} A_0' - \frac{1}{r^2} A_0] \sin\theta = 3wkr^2 \sin\theta$$

$$\Rightarrow A_0'' + \frac{1}{r} A_0' - \frac{1}{r^2} A_0 = 3wkr^2$$

The particular solution is: $A_0 r^2 = A_0 r^3 = \frac{3wkr^2}{8} r^3$

The general solution is: $A_0 r^2 = A_0 r + \frac{A_1}{r}$

$$\Rightarrow A_0(r) = \frac{3wkr^2}{8} r^3 + A_0 r + \frac{A_1}{r}$$

To determine A_0 & A_1 , we plug in the boundary condition

$$\begin{cases} \frac{\partial \Psi}{\partial r}|_{r=a} = \frac{9wkr^2}{8} a^2 + A_0 - \frac{A_1}{a^2} = 0 \\ \frac{\partial \Psi}{\partial r}|_{r=b} = \frac{9wkr^2}{8} b^2 + A_0 - \frac{A_1}{b^2} = 0 \end{cases}$$

$$\Rightarrow \begin{cases} A_0 = -\frac{9wkr^2}{8} (a^2 + b^2) \\ A_1 = -\frac{9}{8} wkr^2 a^2 b^2 \end{cases}$$

$$\Rightarrow \Psi_1 = \frac{3wkr^2}{8} [r^3 - 3(a^2 + b^2)r - \frac{3a^2b^2}{r}] \sin\theta$$

Moreover, if we set $a=0$, we can

$$\Psi_1 = \left(\frac{3}{8} r^3 - \frac{1}{8} a^2 b^2 r \right) wkr \sin\theta$$

II. $\nabla^2 \Psi = 0$

$$\begin{cases} \frac{\partial \Psi}{\partial r}|_{r=a} = wkr^2 \sin\theta \\ \frac{\partial \Psi}{\partial r}|_{r=b} = 0 \end{cases}$$

To Page No./终止页码

DATE/日期	WITNESSED & UNDERSTOOD BY 见证人	DATE/日期

021

From Page No./起始页码

Similarly, we plug in the boundary condition to determine C_1 & C_2 :

$$\begin{cases} \frac{\partial \Psi}{\partial r}|_{r=a} = (\frac{C_1}{a} - \frac{C_2}{a}) \sin\theta = wkr^2 \sin\theta \\ \frac{\partial \Psi}{\partial r}|_{r=b} = (\frac{C_1}{a} - \frac{C_2}{b}) \sin\theta = 0 \end{cases}$$

$$\Rightarrow \begin{cases} C_1 = \frac{a^5}{a^5 - b^5} wkr^2 \\ C_2 = a^5 wkr^2 \end{cases} \Rightarrow \Psi_1 = wkr^2 \left[\frac{b^4}{b^5 - a^5} \right]$$

III. $\nabla^2 \Psi = 0$

$$\begin{cases} \frac{\partial \Psi}{\partial r}|_{r=a} = 0 \\ \frac{\partial \Psi}{\partial r}|_{r=b} = wkr^2 b^2 \sin\theta \end{cases}$$

In all,

$$\Psi_1 = \left[\frac{2}{3} r^3 + \left[\frac{a^2 - b^2}{a^5 - b^5} - \frac{9}{8} (a^2 + b^2) \right] r + \frac{9}{8} \right] wkr^2 \sin\theta$$

$$\Rightarrow \Psi_1 = \left[\frac{3}{8} r^3 + \frac{1}{8} (a^2 + b^2) r - \frac{1}{8} a^2 b^2 \right] wkr^2 \sin\theta$$

Moreover, if we set $a=0$, we can

$$\Psi_1 = \left(\frac{3}{8} r^3 - \frac{1}{8} b^2 r \right) wkr \sin\theta$$

To Page No./终止页码

SIGNATURE/签字	DATE/日期	WITNESSED & UNDERSTOOD BY 见证人

022

From Page No./起始页码

B. $\nabla^2 \Psi_2 = k_2 r^3$, $r = r \sin\theta$

$$\begin{cases} \frac{\partial \Psi_2}{\partial r}|_{r=a} = \frac{3}{4} wka^4 \sin\theta - \frac{1}{4} wka^4 \sin 3\theta \\ \frac{\partial \Psi_2}{\partial r}|_{r=b} = 0 \end{cases}$$

$$\Rightarrow \Psi_2 = (C_3 r^3 + d_3 r^3) \sin\theta + (C_4 \sin^3\theta - 3d_3 a^4) \sin 3\theta$$

$$\begin{cases} \frac{\partial \Psi_2}{\partial r}|_{r=a} = 0 \\ \frac{\partial \Psi_2}{\partial r}|_{r=b} = 0 \end{cases}$$

$$\Rightarrow \begin{cases} C_3 = \frac{wka^4}{4(a^5 - b^5)} \\ d_3 = \frac{3wka^4 b^5}{12(a^5 - b^5)} \end{cases}$$

$$\Rightarrow \begin{cases} \frac{\partial \Psi_2}{\partial r}|_{r=b} = 0 \\ \frac{\partial \Psi_2}{\partial r}|_{r=a} = 0 \end{cases}$$

$$\Rightarrow \begin{cases} C_3 = -\frac{wka^4 b^5}{12(a^5 - b^5)} \\ d_3 = -\frac{wka^4 b^5}{12(a^5 - b^5)} \end{cases}$$

As a result:

$$\begin{aligned} \Psi_2 &= \frac{3}{4} wkr^3 \left[\frac{1}{8} r^5 - 5(a^4 + a^2 b^2 + b^4) r - 5(a^2 + b^2) a^2 b^2 / r \right] \sin\theta \\ &\quad + \frac{1}{48} [-3r^5 + 5 \left(\frac{a^6 + a^4 b^2 + a^2 b^4 + b^6}{a^4 + a^2 b^2 + b^4} \right) r^3 + 5 \left(\frac{a^6 b^6}{a^4 + a^2 b^2 + b^4} \right) / r^5] \\ &\quad + wkr^3 \left[\frac{3}{4} \left[\frac{a^2 b^2}{a^5 - b^5} (a^2 + b^2) \right] r + \frac{a^6 b^6}{a^5 - b^5} \cdot r \right] \sin\theta \\ &\quad - \frac{1}{12} \left[\frac{a^2 - b^2}{a^5 - b^5} r^2 + \frac{a^6 b^6}{a^4 + a^2 b^2 + b^4} / r^2 \right] \sin 3\theta \end{aligned}$$

For a simpler plate ($a=0$):

$$\begin{aligned} \Psi_2 &= \frac{5}{4} wkr^3 \left[\frac{1}{8} [r^5 - 5b^4 r] \sin\theta + \frac{1}{48} [-3r^5 + 5b^2 r^3] \sin 3\theta \right] \\ &\quad + wkr^3 \left[\frac{3}{4} b^4 r \sin\theta - \frac{1}{12} b^2 r^3 \sin 3\theta \right] \end{aligned}$$

To Page No./终止页码

SIGNATURE/签字	DATE/日期	WITNESSED & UNDERSTOOD BY 见证人	DATE/日期

023

From Page No./起始页码

$\nabla^2 \Psi_3 = 0$

$$\begin{cases} \frac{\partial \Psi_3}{\partial r}|_{r=a} = \frac{3}{4} wka^4 \sin\theta - \frac{1}{4} wka^4 \sin 3\theta \\ \frac{\partial \Psi_3}{\partial r}|_{r=b} = 0 \end{cases}$$

$$\Rightarrow \Psi_3 = (C_5 r^3 + d_5 r^3) \sin\theta + (C_6 \sin^3\theta - 3d_5 a^4) \sin 3\theta$$

$$\begin{cases} \frac{\partial \Psi_3}{\partial r}|_{r=a} = 0 \\ \frac{\partial \Psi_3}{\partial r}|_{r=b} = 0 \end{cases}$$

$$\Rightarrow \begin{cases} C_5 = \frac{wka^4}{4(a^5 - b^5)} \\ d_5 = \frac{3wka^4 b^5}{12(a^5 - b^5)} \end{cases}$$

$$\Rightarrow \begin{cases} \frac{\partial \Psi_3}{\partial r}|_{r=b} = 0 \\ \frac{\partial \Psi_3}{\partial r}|_{r=a} = 0 \end{cases}$$

$$\Rightarrow \begin{cases} C_5 = -\frac{wka^4 b^5}{12(a^5 - b^5)} \\ d_5 = -\frac{wka^4 b^5}{12(a^5 - b^5)} \end{cases}$$

As a result:

$$\begin{aligned} \Psi_3 &= \frac{3}{4} wkr^3 \left[\frac{1}{8} r^5 - 5(a^4 + a^2 b^2 + b^4) r - 5(a^2 + b^2) a^2 b^2 / r \right] \sin\theta \\ &\quad + \frac{1}{48} [-3r^5 + 5 \left(\frac{a^6 + a^4 b^2 + a^2 b^4 + b^6}{a^4 + a^2 b^2 + b^4} \right) r^3 + 5 \left(\frac{a^6 b^6}{a^4 + a^2 b^2 + b^4} \right) / r^5] \\ &\quad + wkr^3 \left[\frac{3}{4} \left[\frac{a^2 b^2}{a^5 - b^5} (a^2 + b^2) \right] r + \frac{a^6 b^6}{a^5 - b^5} \cdot r \right] \sin\theta \\ &\quad - \frac{1}{12} \left[\frac{a^2 - b^2}{a^5 - b^5} r^2 + \frac{a^6 b^6}{a^4 + a^2 b^2 + b^4} / r^2 \right] \sin 3\theta \end{aligned}$$

To Page No./终止页码

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保密

电流分布的解析解



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电流密度的径向方向分量： $j_r(r, \theta)$

$$j_{1R}[r, \theta] [[1]] + j_{3R}[r, \theta] [[1]] =$$

$$\sigma \left(\frac{1}{8} k_1 r^2 \omega \sin[\theta] - \frac{1}{8} k_1 \left(-a^2 - b^2 + \frac{a^2 b^2}{r^2} + 9 r^2 \right) \omega \sin[\theta] \right) +$$

σ

$$\left(\frac{1}{4} k_3 r^4 \omega (3 \sin[\theta] - \sin[3\theta]) - \frac{1}{4} k_3 \omega \left(a^4 + a^2 b^2 + b^4 - \frac{a^2 b^2 (a^2 + b^2)}{r^2} \right) \sin[\theta] \right) -$$

51

$$k_3 \omega \left(\frac{3}{4} \left(a^4 + a^2 b^2 + b^4 - \frac{a^2 b^2 (a^2 + b^2)}{r^2} \right) \sin[\theta] - \frac{1}{12} \left(-\frac{3 a^6 b^6}{(a^4 + a^2 b^2 + b^4) r^4} + \frac{3 (a^8 - b^8) r^2}{a^6 - b^6} \right) \sin[3\theta] \right) +$$

$$\left. \left(\frac{5}{4} \left(\frac{1}{8} \left(-5 (a^4 + a^2 b^2 + b^4) + \frac{5 a^2 b^2 (a^2 + b^2)}{r^2} + 5 r^4 \right) \sin[\theta] + \frac{1}{48} \left(-\frac{15 a^6 b^6}{(a^4 + a^2 b^2 + b^4) r^4} + \frac{15 (a^6 + a^4 b^2 + a^2 b^4 + b^6) r^2}{a^4 + a^2 b^2 + b^4} - 15 r^4 \right) \sin[3\theta] \right) \right) \right)$$



电流分布的解析解



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电流密度的角向方向分量： $j_\theta(r, \theta)$



$$j1R[r, \theta] [[2]] + j3R[r, \theta] [[2]] =$$

$$-\frac{k1 \left(-\frac{a^2 b^2}{r} - (a^2 + b^2) r + 3 r^3 \right) \sigma \omega \cos[\theta]}{8 r} - \frac{1}{r}$$

$$k3 \sigma \omega \left[\frac{3}{4} \left(\frac{a^2 b^2 (a^2 + b^2)}{r} + (a^4 + a^2 b^2 + b^4) r \right) \cos[\theta] - \frac{1}{4} \left(\frac{52 a^6 b^6}{(a^4 + a^2 b^2 + b^4) r^3} + \frac{(a^8 - b^8) r^3}{a^6 - b^6} \right) \cos[3 \theta] \right] \text{余弦}$$

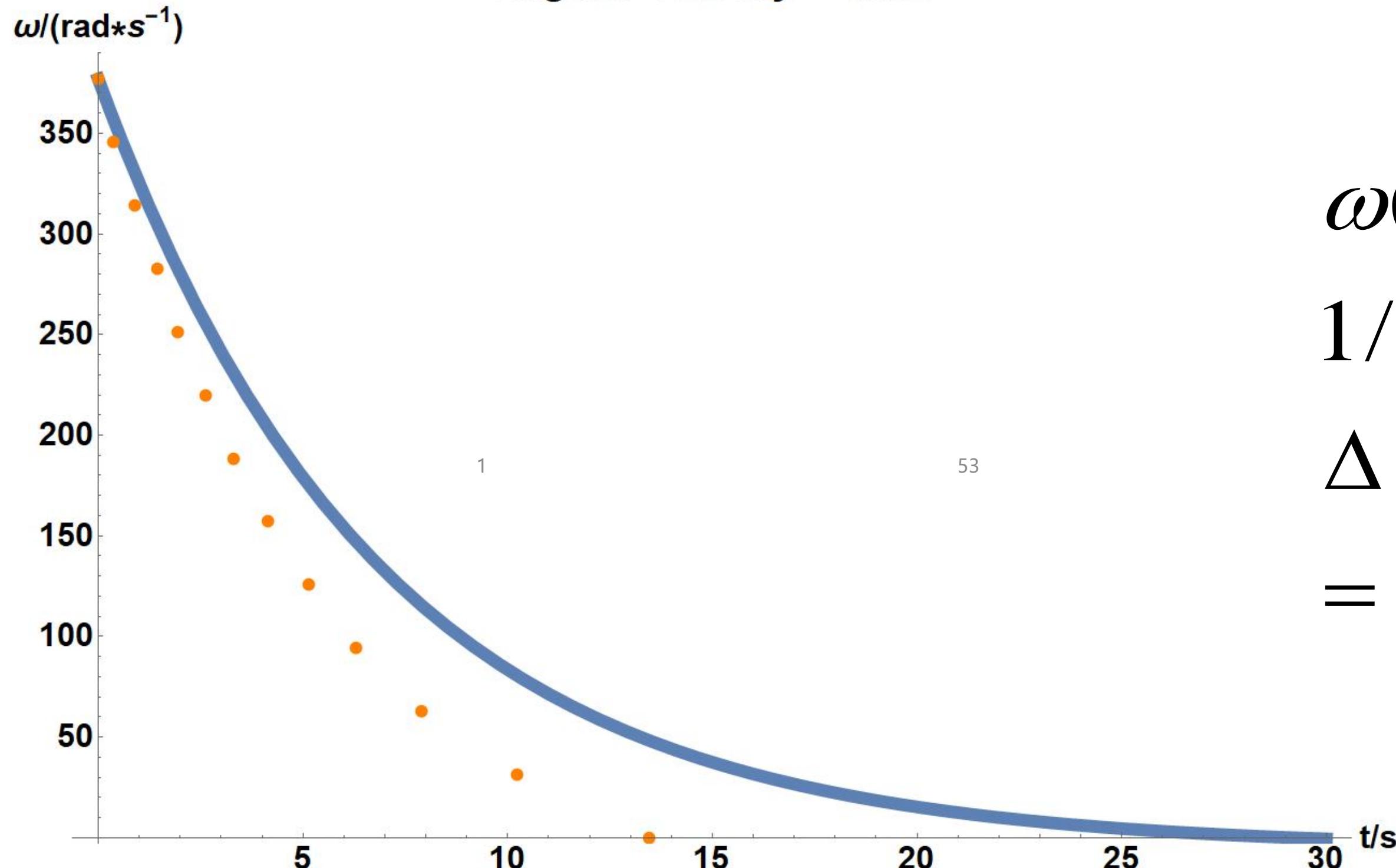
$$\left. - \frac{5}{4} \left(\frac{1}{8} \left(-\frac{5 a^2 b^2 (a^2 + b^2)}{r} - 5 (a^4 + a^2 b^2 + b^4) r + r^5 \right) \cos[\theta] + \frac{1}{16} \left(\frac{5 a^6 b^6}{(a^4 + a^2 b^2 + b^4) r^3} + \frac{5 (a^6 + a^4 b^2 + a^2 b^4 + b^6) r^3}{a^4 + a^2 b^2 + b^4} - 3 r^5 \right) \cos[3 \theta] \right) \right] \text{余弦}$$

定量计算——阻力



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Angular Velocity – Time



$$\omega(t) = \omega_0 e^{-t/\tau} - \Delta$$

$$1/\tau = 0.145857 / \text{s}$$

$$\Delta = 5.42186 \text{ rad} / \text{s}$$

$$= 0.862916 \text{ r} / \text{s}$$