A Study of α Factor Measurement in Lasers Based On Hakki-Paoli Method

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Semiconductor lasers are crucial for a wide range of applications in modern technology, from fiber optic communications to laser printing and medical equipment. The key parameters of semiconductor lasers include wavelength, output power, threshold current, external quantum efficiency, linewidth broadening factor(α factor) and so on. Measuring the α factor is essential for evaluating the impact of carrier-induced effects on the linewidth of semiconductor lasers. Hakki-Paoli method is a powerful technique for measuring the α factor in semiconductor lasers, which enables accurate characterization and optimization of their performance in various applications. This article provides an analysis of the measurement principle of the Hakki-Paoli method, as well as detailed experimental procedures, which can serve as a valuable reference for experimentalists.

I. INTRODUCTION

The α factor is a key parameter that characterizes the spectral purity of a semiconductor laser. It describes the degree to which the linewidth of a laser device is broadened due to carrier-induced effects, such as carrier heating and spectral hole burning. Physically, α factor represents the ratio of the change in the linewidth to the change in the optical power of the laser, and is related to the strength of the nonlinear gain and index coefficients. The measurement of α factor is important for assessing the performance of semiconductor lasers, particularly in applications where narrow linewidth and high spectral purity are required, such as in optical sensing, metrology, and spectroscopy. Accurate determination of α factor enables the optimization of laser design and operating conditions to minimize linewidth broadening effects and improve spectral purity.

In 1973, the Hakki-Paoli method[1] was introduced as a technique to measure the net optical gain of Fabry-Perot (F-P) cavity semiconductor lasers. This method has since undergone numerous improvements and has found widespread applications in various fields, such as optical communications, sensing, and spectroscopy. In addition to measuring the net optical gain, the Hakki-Paoli method can also be used to conveniently measure the alpha factor of a laser. In the following sections, this paper focuses on the definition of α factor, the measurement principles of the Hakki-Paoli method, and the experimental setup and steps for measuring the α factor.

II. α FACTOR

Semiconductor lasers experience spectral broadening during direct modulation operation, with increased modulation rates resulting in wider oscillation mode linewidths, especially at bias currents below the threshold. This broadening is caused by transient changes in injected carrier density near the threshold current, resulting in fluctuations in the spontaneous emission field phase and spectral broadening. Changes in the real and imaginary parts of the complex refractive index, caused by fluctuations in the optical field phase and intensity, represent changes in carrier density and medium gain, respectively. The band-filling effect is the primary reason for changes in refractive index with carrier density in semiconductor lasers, as injected carriers cause the Fermi levels of the conduction and valence bands to move in the high-energy direction, effectively increasing the bandgap width. The refractive index near the laser frequency depends on carrier density.

Suppose the material inside the laser has real refractive index n_r and imaginary refractive index n_i, induced carriers cause fluctuation on the imaginary part and further influence the real part by Kramers–Kronig relations, so the α factor could be difined as:

$$\alpha_H = \frac{\delta n_r}{\delta n_i} \tag{1}$$

The real part of the refractive index is related with the phase of the electric field transferred inside the laser, while the imaginary part is related with the amplitude of the electric field. By further deduction of equation (1)[2]:

$$\alpha_H = \frac{4\pi\Delta f(t)}{\frac{1}{p_{out}(t)}\frac{dp_{out}(t)}{dt}} \tag{2}$$

where f is the frequency of optical wave, p_{out} is the power output from the laser, Assume the gain g and the refractive index n have a linear dependence with the carrier density N, further written as:

$$\alpha_H = -\frac{4\pi}{\lambda}\frac{dn/dN}{dg/dN} = -\frac{4\pi}{\lambda}\frac{\Delta n}{\Delta g_{net}} \tag{3}$$

Considering the cavity length L, utilizing the relations:

$$m\lambda = 2nL \tag{4}$$

$$\frac{\Delta n}{n} = \frac{\Delta\lambda}{\lambda} \tag{5}$$

The mode spacing on wavelength:

$$D_\lambda = \frac{\lambda^2}{2nL} \tag{6}$$

The α factor could be written as:

$$\alpha_H = -\frac{2\pi}{LD_\lambda}\frac{\Delta\lambda}{\Delta g_{net}}, g_{net} = \alpha_T - g \tag{7}$$

g is the optical gain, and α_T is optical losses contain with inner losses and mirror losses:

$$\alpha_{\mathrm{T}} = \alpha_i + \frac{1}{2L}\ln\left(\frac{1}{R_1R_2}\right) \tag{8}$$

III. HAKKI-PAOLI METHOD

The Hakki-Paoli method is based on the relationship between laser Amplified Spontaneous Emission (ASE) gain proposed by Hakki and Paoli.[1] By measuring the changes in equivalent refractive index and gain, the linewidth broadening factor can be obtained.

Consider a plane wave with wave vector k and amplitude F, with optical gain and losses, the field along the laser cavity could be written as:

$$F(x) = F_0 e^{-\left(\frac{\alpha-g}{2}\right)x} e^{ikx} \tag{9}$$

The intensity:

$$|F|^2 = |F_0|^2 e^{-(\alpha-g)x} \tag{10}$$

The superposition of all possible waves after different round cycles of length 2L leads to the formula for the expected intensity spectrum from a resonating cavity[3]:

$$|F|^2 = \frac{|F_0|^2}{1+\rho^2-2\rho\cos(2kL)} \tag{11}$$

$$\rho = \exp\left(-\left(\alpha_{\mathrm{T}} - g\right)L\right) \tag{12}$$

From the spectrum of laser, the maximum and the minimum of $|F|^2$ could be obtained:

$$|F|^2_{MAX} = \frac{|F_0|^2}{(1-\rho)^2}, |F|^2_{\min} = \frac{|F_0|^2}{(1+\rho)^2} \tag{13}$$

Therefore the optical gain[4]:

$$g = \frac{1}{L}\ln\frac{S-1}{S+1} + \frac{1}{2L}\ln\left(\frac{1}{R_1R_2}\right) \tag{14}$$

$$S = \frac{|F|^2_{MAX}}{|F|^2_{\min}} \tag{15}$$

Cassidy has provided a integral methods, numerically calculating the integral of Equation (11) over one spectral range, which means a moving average over one period of the cosine function, defined as:

$$|F|^2_+ = \frac{|F_0|^2}{1-\rho^2}, \frac{|F|^2_+}{|F|^2_{\min}} = \frac{1+\rho}{1-\rho} \tag{16}$$

In this scenario, there is no longer a need for extremely high resolution when measuring gain, because the experimental equipment no longer causes smoothing of the peaks. Therefore, the benefit lies in the absence of concerns about the smoothing of the maxima.[3]

Altenatively, Vanzi raplaced $|F|^2_{\min}$ with $|F|^2_-$(the reciprocal of the moving average of $1/|F|^2$) to obtained the net gain:

$$|F|^2_- = \frac{|F_0|^2}{1+\rho^2}, \frac{|F|^2_+}{|F|^2_-} = \frac{1+\rho^2}{1-\rho^2} \tag{17}$$

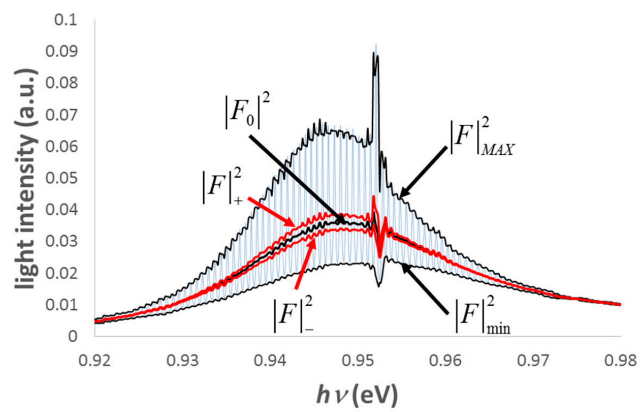

FIG. 1. Demonstration of Hakki-Paoli Method and it's extended Method proposed by Cassidy and Vanzi

IV. EXPERIMENT PROCEDURE

The laser is controlled by temperature and current controller, the output light pass through optical isolator to avoid backscattering light. A polarization controller is set to maintain a high extinction ratio light, then the light is captured by OSA to obtain a high wavelength resolution spectrum. At the end a personal computer is utilized to process the data.

The specific procedure of the measurement are shown below:

1. Measure L-I curve of the laser, obtaining the I_{th}.

2. Set the current controller from $I_{th} - I_{var}$ to $I_{th} - I_{var}$ by I_{var}/N, where $2I_{var}$ is the range of current control, N is the sample number which experimentalists desire, obtaining 2N+1 spectrum under different currents, measuring corresponding mode spacing D_λ.

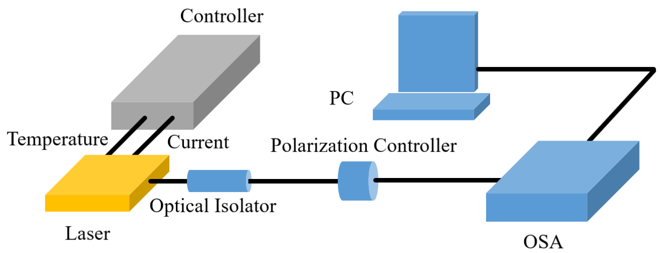

FIG. 2. Demonstration of α factor measurement experiment, including controller for temperature and current control, laser under test, optical isolator, polarization controller, optical spectrum analyser(OSA), personal computer(PC)

3. Use Hakki-Paoli method to calculate the relationship between net gain and wavelength, totally 2N+1 curves with different currents, find peak value of net gain g_{net} and corresponding wavelength.

4. For i = -N:N-1, calculate:

$$\alpha_{Hi} = -\frac{2\pi}{LD_\lambda}\frac{\lambda(I_{th}+(i+1)I_{var}/N)-\lambda(I_{th}+iI_{var}/N)}{g_{net}(I_{th}+(i+1)I_{var}/N)-g_{net}(I_{th}+iI_{var}/N)} \tag{18}$$

If only the peak values are concerned, there are totally 2N data points. 2N curves are obtained if all the wavelength are focused.

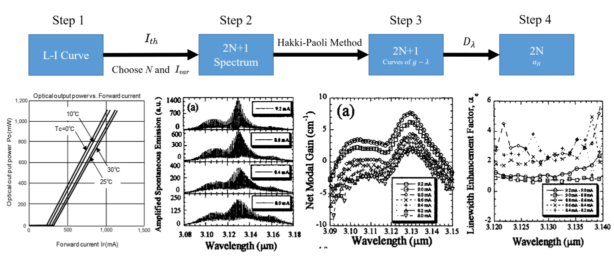

FIG. 3. Procedure of α factor measurement and example figures from [3]

V. EXPERIMENT RESULTS

A. L-I Curve Measurement

The device under study is a quantum well Fabry-Perot laser, lasing around 1550 nm or 1310 nm. The refractive index is 3.5, and the facet reflectivity is 0.32. From the L-I plot, extract the lasing threshold current. Fig.1. shows the experiment results of L-I curve measurements under temperature of 15 °C, 20 °C and 30 °C.According to the results, the threshold current of the laser is about 10 mA, as temperature goes up, the threshold current slightly increases. The higher temperature corresponds to the larger slope of the curves, which means the same current will generate higher output power.

Higher temperature increases laser threshold current and decreases maximum output power due to increased leakage current, enhanced nonradiative recombination, increased resistance, and reduced gain. These factors reduce the population inversion, efficiency of light generation, and gain, requiring higher current for laser emission and limiting output power. Dependencies may vary based on diode design, material, and operating conditions.

The experiment results of this laser did not obey the traditional trend of laser diode L-I curve characteristics with variation of temperature. There are several possible reasons for deviations from the typical trend of higher temperature leading to an increase in laser threshold current and a decrease in maximum output power:

1. Temperature-sensitive variations: Certain laser designs or materials may exhibit temperature responses that contradict the general trend. For example, some specific semiconductor materials or device structures may show a decrease in threshold current and an increase in output power with increasing temperature.

2. Temperature compensation mechanisms: Some lasers may incorporate temperature compensation mechanisms to counteract the effects of temperature on threshold current and output power. These mechanisms can be achieved by adjusting current injection or other activation parameters.

3. Temperature-related optical effects: In some cases, high temperature can induce changes in the optical properties of materials, such as variations in refractive index or optical absorption characteristics. These optical effects may interact with other factors, leading to non-typical temperature dependencies of threshold current and output power.

4. Design and manufacturing differences: Different laser designs and manufacturing processes can result in variations in temperature response. Differences can arise from material selection, structural design, process control, and other manufacturing factors, which may lead to threshold current and output power changes that deviate from the general trend.

In summary, the temperature dependence of lasers is a complex issue, and there may be specific cases where the trend differs from the norm. These factors depend on the specific laser design, materials, and manufacturing processes, and require specific experiments and analysis to understand their effects.

VI. OPTICAL SPECTRUM MEASUREMENT

With Matlab code, the peaks and their distance are found directly and a average mode spacing with 1.2914nm was obtained.

Utilizing n = 3.5 and equations below:

$$D_\lambda = \frac{\lambda^2}{2nL}, D_\nu = \frac{c}{2nL} \tag{19}$$

$$D_\nu = \frac{cD_\lambda}{\lambda^2} \approx 0.167THz, L = \frac{\lambda^2}{2nD_\lambda} =\approx 0.257mm \tag{20}$$

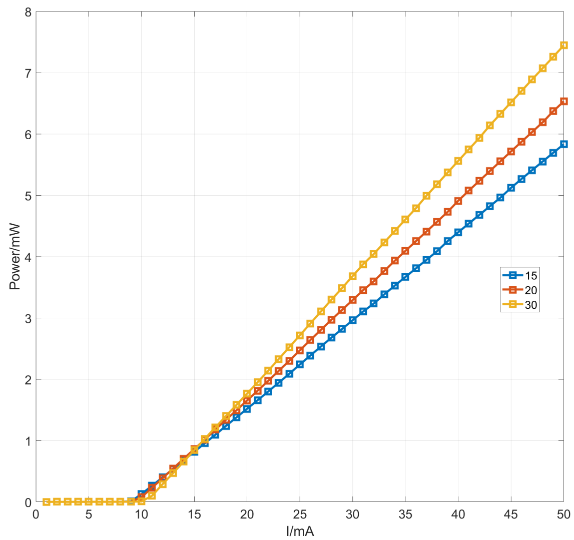

FIG. 4. L-I curve measurements under temperature of 15 °C, 20 °C and 30 °C.

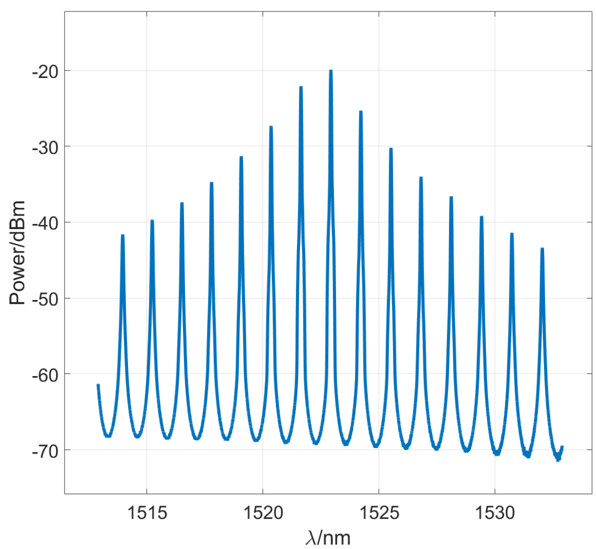

FIG. 5. Optical spectrum at threshold 10mA.

As the wavelength rises 20nm, the wavelength mode spacing slightly increases about 0.036nm, compared to the mode spacing, this quantity are small enough.

It could be seen that the peaks has a blue shift under threshold current, but a red shift above threshold current from a zoom in view of Fig.3. And The blue shift is more obvious than the red shift. As applied current increases, the electron-hole pairs are generated, thus causing plasma effect which decrease the refractive index of the cavity, so the peak wavelength decrease. Moreover, the band filling effect also plays a significant rule to the blue shift. In the contrary, above the threshold, the higher current induce higher temperature therefore increasing the refractive index which causes the red shift process. Increasing temperature causes the lattice expand thus reduce the band gap energy.

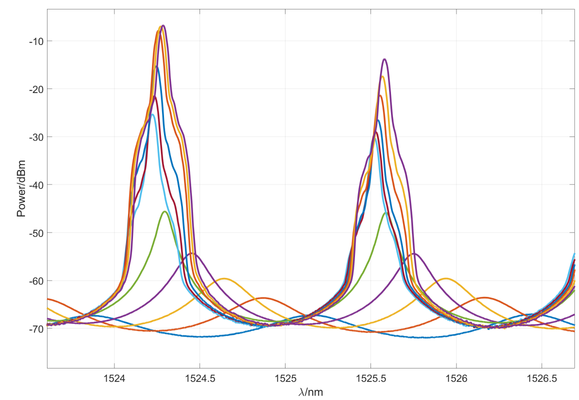

FIG. 6. Peak Wavelength Shift cross Applied Current

A. Gain Measurement

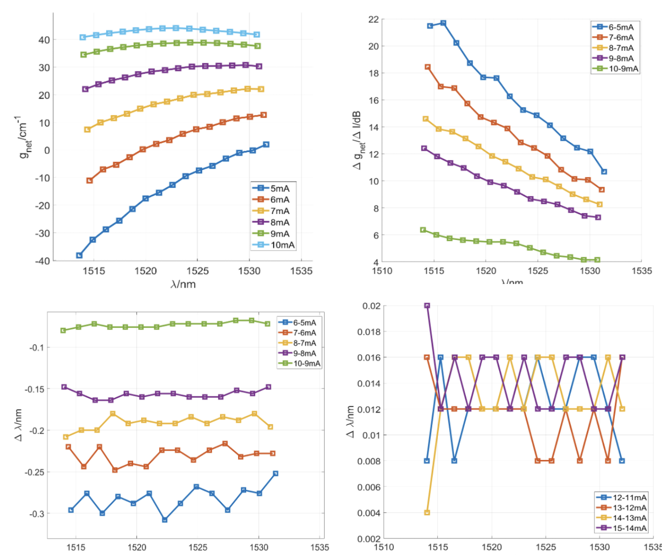

FIG. 7. Left Above: g_{net}-λ with different applied current below the threshold;Right Above:$\delta g_{net} - \lambda$ with current variation below the threshold; Left Below:$\delta\lambda$ with different applied current below the threshold; Right Below: $\delta\lambda$ with different applied current above the threshold;

The net modal gain of the laser can be obtained by using the relationship between the amplified spontaneous emission gain of the laser proposed by Hakki and Paoli:

$$g_{net} = L^{-1}(ln[(r^{1/2}-1)/(r^{1/2}+1)] - lnR) \qquad (21)$$

In the formula, r=Pmax/Pmin, where Pmax and Pmin are the intensities of adjacent peaks and valleys in the spontaneous emission spectrum, respectively; R is the power reflectivity of the laser resonant cavity surface.

With manually selection of each peak point and it's nearest larger valley value, the corresponding net gain at peak wavelength is obtained as Fig.4 left above panel shows. Each spectrum contains 15 peaks but due to the shift only 14 points in total is processed to obtain the trend in the same group of wavelength. As the current increase, the net gain also increase and above the threshold the gain profile is in symmetric around center wavelength. The left above of the panel shows the trend of net gain variation cross current variation. The points' wavelength value are attached to higher current value corresponding points' wavelength value. With increasing current and wavelength, the gain increases less. The same trend of applied current appears in the panel of lambda blue shift. However, above the threshold, the red shift has no obvious trend cross applied current and wavelength. The temperature's influence affects the red shift evenly on wavelength and linearly on current. The average variation is about 0.0128nm.

B. α Factor Measurement

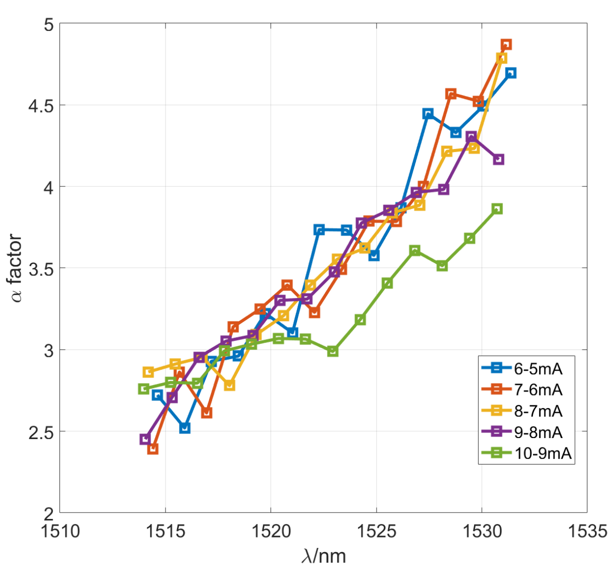

FIG. 8. α Factor Measurement Results

With utilizing the equation:

$$\alpha_H = -\frac{2\pi}{LD_\lambda}\frac{\delta g_{net}}{\delta\lambda} \tag{22}$$

Final results of α factor is calculated, from the figure illustrated it could be seen that all the data points roughly have a linear relationship with wavelength under the current threshold, in spite of current value.

VII. CONCLUSION

This article demonstrates the experimental procedure and calculation method for measuring the alpha factor, using a semiconductor laser as an example based on the Hakki-Paoli method. The L-I curves at different temperatures were first measured to obtain the laser's threshold current. After measuring the spectral data below and above the threshold current, the net gain at different driving currents was calculated. The wavelength and net gain variations caused by current changes were further calculated, and the trends and reasons behind the current and wavelength changes were discussed in the article. Finally, the measurement results of the alpha factor were presented, showing a linear correlation with the wavelength around the central wavelength within a 20nm range for different current ranges below the threshold current. The alpha factor measured in the experiment ranges from 2.3 to 4.8, and the alpha factor decreases with increasing current.

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