

§2.6 Limits Involving Infinity; Asymptotes of Graphs (Continued)

Example 2. Find (a) $\lim_{x \rightarrow \infty} \left(5 + \frac{1}{x}\right)$ (b) $\lim_{x \rightarrow -\infty} \frac{\pi\sqrt{3}}{x^2}$.

Example 3. Find (a) $\lim_{x \rightarrow \infty} \frac{5x^2 + 8x - 3}{3x^2 + 2}$.

Example 9. Find $\lim_{x \rightarrow \infty} \left(x - \sqrt{x^2 + 16}\right)$.

Example 11. Find $\lim_{x \rightarrow 1^+} \frac{1}{x-1}$ and $\lim_{x \rightarrow 1^-} \frac{1}{x-1}$.

Example 7. Find $\lim_{x \rightarrow 0^-} e^{1/x}$.

Example 13. Find (c) $\lim_{x \rightarrow 2^+} \frac{x-3}{x^2-4}$ (d) $\lim_{x \rightarrow 2^-} \frac{x-3}{x^2-4}$ (f) $\lim_{x \rightarrow 2} \frac{2-x}{(x-2)^3}$.

Example 14. Find $\lim_{x \rightarrow -\infty} \frac{2x^5 - 6x^4 + 1}{3x^2 + x - 7}$.

§3.1 Tangent Lines and the Derivative at a Point

Definition. The **slope of the curve** $y = f(x)$ at the point $P(x_0, f(x_0))$ is the number

$$\lim_{h \rightarrow 0} \frac{f(x_0 + h) - f(x_0)}{h} \quad (\text{provided the limit exists}).$$

The **tangent line** to the curve at P is the line through P with this slope.

Example 1. (a) Find the slope of the curve $y = 1/x$ at any point $x = a \neq 0$. What is the slope at the point $x = -1$?

(b) Where does the slope equal $-1/4$?

Definition. The **derivative of a function f at a point x_0** , denoted $f'(x_0)$, is

$$f'(x_0) = \lim_{h \rightarrow 0} \frac{f(x_0 + h) - f(x_0)}{h},$$

provided this limit exists.

§3.2 The Derivative as a Function

Definition. The **derivative** of the function $f(x)$ with respect to the variable x is the function f' whose value at x is

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h},$$

provided the limit exists.

If f' exists at a particular x , we say that f is **differentiable at x** . If f' exists at every point in the domain of f , we call f **differentiable**.

The process of calculating a derivative is called **differentiation**.

Example 1. Differentiate $f(x) = \frac{x}{x-1}$.

Example 2. (a) Find the derivative of $f(x) = \sqrt{x}$ for $x > 0$.

(b) Find the tangent line to the curve $y = \sqrt{x}$ at $x = 4$.

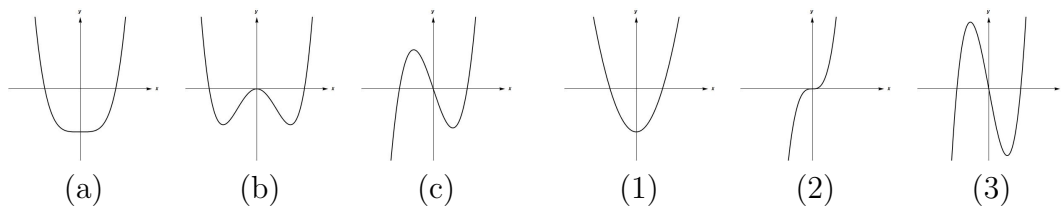
Notation. Some common alternative notations for the derivative include

$$f'(x) = y' = \frac{dy}{dx} = \frac{df}{dx} = \frac{d}{dx}f(x) = D(f)(x) = D_x f(x).$$

To indicate the value of a derivative at a specified number $x = a$, we use the notation

$$f'(a) = \left. \frac{dy}{dx} \right|_{x=a} = \left. \frac{df}{dx} \right|_{x=a} = \left. \frac{d}{dx}f(x) \right|_{x=a}.$$

Problem. Match the functions (a)–(c) with their derivatives (1)–(3).



Example 4. Show that the function $y = |x|$ is differentiable on $(-\infty, 0)$ and on $(0, \infty)$ but has no derivative at $x = 0$.

Theorem 1 (Differentiability Implies Continuity). If f has a derivative at $x = c$, then f is continuous at $x = c$.

§3.3 Differentiation Rules

Derivative of a Constant Function. If f has the constant value $f(x) = c$, then

$$\frac{df}{dx} = \frac{d}{dx}(c) = 0.$$

Power Rule. If n is any real number, then $\frac{d}{dx}x^n = nx^{n-1}$, for all x where the powers x^n and x^{n-1} are defined.

Example 1. Differentiate the following powers of x .

(a) x^3 (b) $x^{2/3}$ (d) $\frac{1}{x^4}$ (f) $\sqrt{x^{2+\pi}}$