§3.7 Implicit Differentiation

Example 2. Find the slope of the circle $x^2 + y^2 = 25$ at the point $(3, -4)$.

Example 3. Find dy/dx if $y^2 = x^2 + \sin xy$.

Example 4. Find d^2y/dx^2 if $2x^3 - 3y^2 = 8$.

Example 5. Show that the point $(2, 4)$ lies on the curve $x^3 + y^3 - 9xy = 0$. Then find the tangent and normal to the curve there.

§3.8 Derivatives of Inverse Functions and Logarithms

Theorem 3 (The Derivative Rule for Inverses). If f has an interval I as domain and $f'(x)$ exists and is never zero on I, then f^{-1} is differentiable at every point in its domain (the range of f). The value of $(f^{-1})'$ at a point b in the domain of f^{-1} is the reciprocal of the value of f' at the point $a = f^{-1}(b)$:

$$(f^{-1})'(b) = \frac{1}{f'(f^{-1}(b))} \qquad \text{or} \qquad \left.\frac{df^{-1}}{dx}\right|_{x=b} = \frac{1}{\left.\frac{df}{dx}\right|_{x=f^{-1}(b)}}.$$

Example 2. Let $f(x) = x^3 - 2$, $x > 0$. Find the value of df^{-1}/dx at $x = 6 = f(2)$ without finding a formula for $f^{-1}(x)$.

Derivative of Logarithm. $\dfrac{d}{dx}(\ln x) = \dfrac{1}{x}$, $x > 0$.

Example 3. Find (a) $\dfrac{d}{dx}\ln(2x)$ (b) $\dfrac{d}{dx}\ln(x^2 + 3)$.

Example 4. A line with slope m passes through the origin and is tangent to the graph $y = \ln x$. What is the value of m?

Example 6. Find dy/dx if $y = \dfrac{(x^2 + 1)(x + 3)^{1/2}}{x - 1}$, $x > 1$.

Example 7. Differentiate $f(x) = x^x$, $x > 0$.

§3.9 Inverse Trigonometric Functions

Definition. $\boldsymbol{y = \tan^{-1} x}$ is the number in $(-\pi/2, \pi/2)$ for which $\tan y = x$.
$\boldsymbol{y = \cot^{-1} x}$ is the number in $(0, \pi)$ for which $\cot y = x$.
$\boldsymbol{y = \sec^{-1} x}$ is the number in $[0, \pi/2) \cup (\pi/2, \pi]$ for which $\sec y = x$.
$\boldsymbol{y = \csc^{-1} x}$ is the number in $[-\pi/2, 0) \cup (0, \pi/2]$ for which $\csc y = x$.

Derivative of $\sin^{-1} x$. $\dfrac{d}{dx}(\sin^{-1} x) = \dfrac{1}{\sqrt{1-x^2}}$, $|x| < 1$.

Example 2. Find $\dfrac{d}{dx}(\sin^{-1} x^2)$.

Derivative of $\tan^{-1} x$. $\dfrac{d}{dx}(\tan^{-1} x) = \dfrac{1}{1+x^2}$.

Derivative of $\sec^{-1} x$. $\dfrac{d}{dx}(\sec^{-1} x) = \dfrac{1}{|x|\sqrt{x^2-1}}$, $|x| > 1$.

§3.10 Related Rates*

Exercise 6. If $x = y^3 - y$ and $dy/dt = 5$, then what is dx/dt when $y = 2$?

Exercise 10. If $r + s^2 + v^3 = 12$, $dr/dt = 4$, and $ds/dt = -3$, find dv/dt when $r = 3$ and $s = 1$.