MATH 141 Sample Test 1

Show your work

1 [11P]) Write the equation for the tangent line and the normal line to the curve $f(x) = x^2 + 3x + 1$ at the point $P(1, 5)$:

Tangent line:

Normal line:

2 [32P]) Find the following limits or determine if they do not exist:

a) $\lim_{x\to 0} x \sin\left(\frac{1}{x^2}\right) =$

b) $\lim_{x\to 0} \sqrt{x^2 + 7x} - x =$

c) $\lim_{x\to 2} \frac{x^2 + 2x - 3}{x^2 + x - 6} =$

d) $\lim_{x\to 3} \frac{|x + 1|}{[[x + 1]]} =$

e) $\lim_{x\to 5^-} \frac{1.5}{10 - 2x} =$

3 [6P]) Explain if the equation $x^3 + x + 1$ has a solution in the interval $[-1, 0]$ or not.

4 [11P]) Prove using the $\varepsilon - \delta$ definition of limit that

$$\lim_{x\to 2} \quad x - 3 = -1$$

5 [11P]) Let $f(x) = \frac{1 - x}{1 - x^2}$. Where is $f(x)$ defined? Find the left hand side and the right hand side limits at the points where $f(x)$ is not defined. Is it possible to assign a value to $f(x)$ at those points such that $f(x)$ is continuous at the point?

6 [18P]) Find the derivative of the following functions:

a) $f(x) = \sqrt{x^2 - 1} + x^2$. $f'(x) =$

b) $f(x) = \frac{x^3 + 3x}{x + 1}$. $f'(x) =$

c) $f(x) = \frac{1}{3x - 5}$. $f'(x) =$

7 [11P]) A water bucket containing 10 gal of water develops a leak at time $t = 0$. The volume V of water in the bucket t seconds later is given by

$$V(t) = 10\left(1 - \frac{t}{100}\right)^2$$

until the bucket is empty at time $t = 100$. a) At what rate is water leaking from the bucket after exactly 1 min. b) What is the average rate of change of V from time $t = 0$ and $t = 50$ and from time $t = 0$ and $t = 100$?