- *Logarithmic Functions*

1. **Definition.** A **logarithm** is defined as follows:

$$\log_a x = y \quad \text{means} \quad a^x = y,\ a > 0,\ a \neq 1.$$

The number $\log_a x$ is the power of y to which we raise a to get x. The number a is called the **logarithmic base**.

2. Solve for x.
   (a) $\log_5 125 = x$
   (b) $\log_x 64 = 3$
   (c) $\log_6 x = -1$
   (d) $\log_4 1/16 = x$

3. Given $\log_a 2 = 0.27$ and $\log_a 3 = 0.43$. Find $\log_a 6$, $\log_a \frac{2}{3}$, $\log_a 81$, $\log_a \frac{1}{3}$, $\log_a \sqrt{a}$, $\log_a(2a)$, and $\frac{\log_a 3}{\log_a 2}$.

4. **Definition.** For any positive number x, the **common logarithm** of x is $\log_{10} x$ (abbreviated $\log x$), and the **natural logarithm** of x is $\log_e x$ (abbreviated $\ln x$).

5. Given $\ln 2 = 0.69$ and $\ln 3 = 1.10$. Find $\ln 6$, $\ln \frac{2}{3}$, $\ln 81$, $\ln \frac{1}{3}$, $\ln(3e^2)$, and $\log_2 3$.

6. Solve $e^{-0.25x} = 0.58$.

7. **Theorem.** *The function* $\ln x$ *exists only for positive numbers* x*. The domain is* $(0, \infty)$*. When* $0 < x < 1$*,* $\ln x < 0$*. When* $x = 1$*,* $\ln x = 0$*. When* $x > 1$*,* $\ln x > 0$*. The function* $\ln x$ *is an increasing function. The range is the entire real line* $(-\infty, \infty)$*.*

8. **Theorem.** $\dfrac{d}{dx} \ln x = \dfrac{1}{x}, \quad \dfrac{d}{dx} \ln |x| = \dfrac{1}{x}.$

9. Differentiate the following functions.
   (a) $y = 8 \ln x$
   (b) $y = 4 \ln |x|$
   (c) $y = x^4 \ln x - \frac{1}{2}x^2$
   (d) $y = \dfrac{\ln x}{x^4 + 1}$
   (e) $y = \ln(5x)$
   (f) $y = \ln(6x^2 - 3x)$

10. A company begins a radio advertising campaign in a city to market its new energy drink. The percentage of the "target market" that buys a product is normally a function of the duration of the advertising campaign. The radio station estimates this percentage by using $f(t) = 1 - e^{-0.04t}$ for this type of product, where t is the number of days of the campaign. The target market is approximately $1,000,000$ people, and the price per unit is \$1.50. If the campaign costs \$1000 per day, how long should it last in order to maximize profit?