§*3.3 Differentiation Rules* (Continued)

Derivative Constant Multiple Rule. If u is a differentiable function of x, and c is a constant, then $\frac{d}{dx}(cu) = c\frac{du}{dx}$.

Derivative Sum Rule. If u and v are differentiable functions of x, then their sum $u+v$ is differentiable at every point where u and v are both differentiable. At such points, $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$.

Example 3. Find the derivative of the polynomial $y = x^3 + \frac{4}{3}x^2 - 5x + 1$.

Example 4. Does the curve $y = x^4 - 2x^2 + 2$ have any horizontal tangent lines? If so, where?

Derivative of the Natural Exponential Function. $\frac{d}{dx}(e^x) = e^x$

Example 5. Find an equation for a line that is tangent to the graph of $y = e^x$ and goes through the origin.

Derivative Product Rule. If u and v are differentiable at x, then so is their product uv, and $\frac{d}{dx}(uv) = u\frac{dv}{dx} + \frac{du}{dx}v$.

Example 6. Find the derivative of (a) $y = \frac{1}{x}(x^2 + e^x)$.

Derivative Quotient Rule. If u and v are differentiable at x and if $v(x) \neq 0$, then the quotient u/v is differentiable at x, and $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$.

Example 7. Find the derivative of (a) $y = \frac{t^2-1}{t^3+1}$, (b) $y = e^{-x}$.

Definition. The **second derivative** of f is defined as $(f')'$, and is written in several ways:

$$f''(x) = \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{dy'}{dx} = y'' = D^2(f)(x) = D_x^2 f(x).$$

The ***n*th derivative** of y with respect to x for any positive integer n is denoted

$$y^{(n)} = \frac{d}{dx}y^{(n-1)} = \frac{d^ny}{dx^n} = D^n y.$$

Example 9. Find the first four derivative of $y = x^3 - 3x^2 + 2$.

Ex. From MML

§3.5 Derivatives of Trigonometric Functions

Derivative of Sine Function. $\frac{d}{dx}(\sin x) = \cos x.$

Example 1. Find the derivative: (b) $y = e^x \sin x$

Derivative of Cosine Function. $\frac{d}{dx}(\cos x) = -\sin x.$

Example 2. Find the derivative: (c) $y = \frac{\cos x}{1 - \sin x}$

Derivatives of Other Trigonometric Functions.

$$\frac{d}{dx}(\tan x) = \sec^2 x \qquad \frac{d}{dx}(\cot x) = -\csc^2 x$$
$$\frac{d}{dx}(\sec x) = \sec x \tan x \qquad \frac{d}{dx}(\csc x) = -\csc x \cot x$$

Example 5. Find $d(\tan x)/dx$.

Exercise 62. Derive the formula for the derivative with respect to x of: (a) $\sec x$

Example 6. Find y'' if $y = \sec x$.

§3.6 The Chain Rule

Example 1. The function $y = (3x^2+1)^2$ is obtained by composing the functions $y = f(u) = u^2$ and $u = g(x) = 3x^2 + 1$. Calculate dy/dx.

Theorem 2 (The Chain Rule). If $f(u)$ is differentiable at the point $u = g(x)$ and $g(x)$ is differentiable at x, then the composite function $(f \circ g)(x) = f(g(x))$ is differentiable at x, and $(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$. In Leibniz's notation, if $y = f(u)$ and $u = g(x)$, then $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$, where dy/du is evaluated at $u = g(x)$.

Exercise 82. Find the value of $(f \circ g)'$ at $x = -1$ for $f(u) = 1 - \frac{1}{u}$ and $u = g(x) = \frac{1}{1-x}$.

Example 3. Differentiate $\sin(x^2 + e^x)$ with respect to x.

Example 4. Differentiate $y = e^{\cos x}$.

Example 5. Find the derivative of $g(t) = \tan(5 - \sin 2t)$.

Example 8. Show that the slope of every line tangent to the curve $y = 1/(1-2x)^3$ is positive.

Ex. From MML