§4.1 Extreme Values of Functions on Closed Intervals

Definition. Let f be a function with domain D. Then f has an **absolute maximum** value on D at a point c if $f(x) \leq f(c)$ for all x in D, and an **absolute minimum** value on D at c if $f(x) \geq f(c)$ for all x in D. Absolute maxima or minima are also referred to as **global** maxima or minima.

Example 1. Consider the defining equation $y = x^2$ on various domains:
(a) $D = (-\infty, \infty)$ (b) $D = [0, 2]$ (c) $D = (0, 2]$ (d) $D = (0, 2)$

Theorem 1 (The Extreme Value Theorem). If f is continuous on a closed interval $[a, b]$, then f attains both an absolute maximum value M and an absolute minimum value m in $[a, b]$. That is, there are numbers x_1 and x_2 in $[a, b]$ with $f(x_1) = m$, $f(x_2) = M$, and $m \leq f(x) \leq M$ for every other x in $[a, b]$.

Definition. A function f has a **local maximum** value at a point c within its domain D if $f(x) \leq f(c)$ for all x in D lying in some open interval containing c. A function f has a **local minimum** value at a point c within its domain D if $f(x) \geq f(c)$ for all x in D lying in some open interval containing c. Local extrema are also called **relative extrema**.

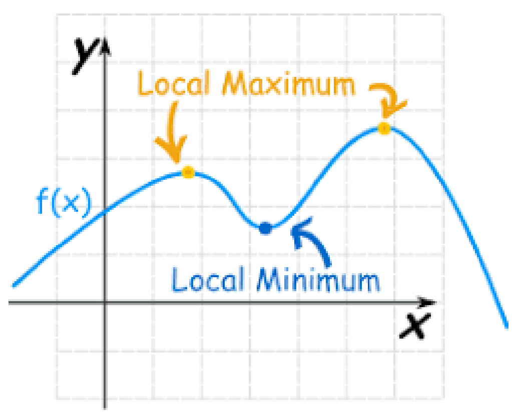

FIGURE 1. local extrema for $y = f(x)$ on its domain (courtesy: Math is fun)

Theorem 2 (The First Derivative Theorem for Local Extreme Values). If f has a local maximum or minimum value at an interior point c of its domain, and if f' is defined at c, then $f'(c) = 0$.

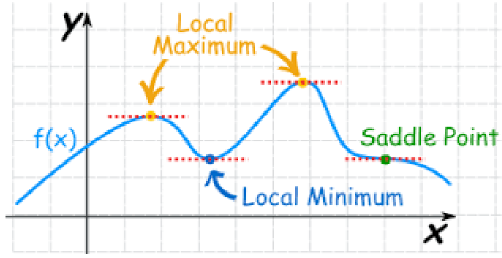

FIGURE 2. local extrema and saddle point for $f(x)$ on an interval (courtesy: Math is fun)

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