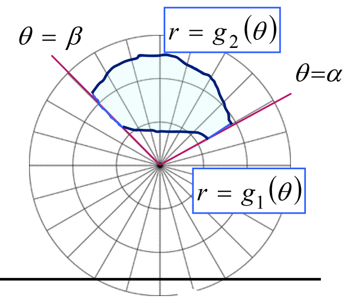
$$\iint_R f(r,\theta)\,dA = \int_{\theta=\alpha}^{\theta=\beta}\int_{r=g_1(\theta)}^{r=g_2(\theta)} f(r,\theta)\, r\, dr\, d\theta$$

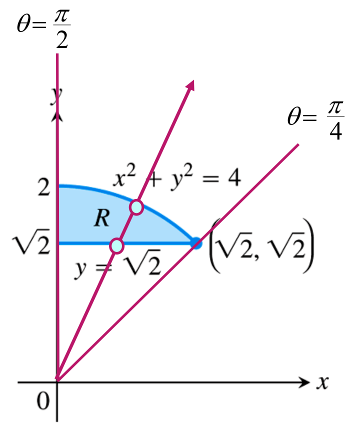
$$S_n = \sum_{k=1}^{n} f(r_k,\theta_k)\Delta A_k = \sum_{k=1}^{n} f(r_k,\theta_k)\, r_k\, \Delta r\, \Delta\theta \xrightarrow{n\to\infty}$$



Example

Find limits of integration as above for the region in the picture.

$$\iint_R f(r,\theta)\,dA = \boxed{\int_{\theta=\pi/4}^{\theta=\pi/2}\int_{r=\sqrt{2}\csc\theta}^{r=2} f(r,\theta)\, r\, dr\, d\theta}$$

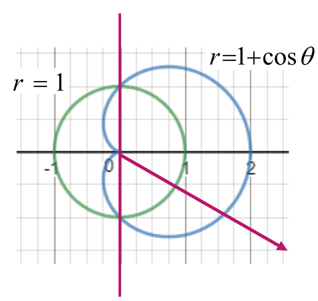


To find limits for r, convert to polar coordinates and solve for r.

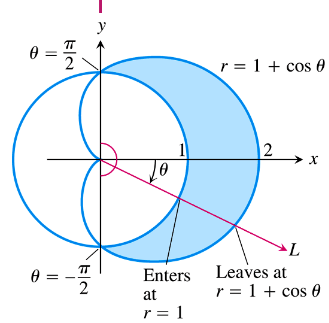
$y=\sqrt{2}$	$x^2+y^2=4$
$r\sin\theta=\sqrt{2}$	$r^2=4$
$r=\sqrt{2}\csc\theta=g_1(\theta)$	$r=2=g_2(\theta)$

Example

Find limits of integration for the region that lies inside the cardiod and outside the circle in the picture.



$$\iint_R f(r,\theta)\,dA = \boxed{\int_{\theta=-\pi/2}^{\theta=\pi/2}\int_{r=1}^{r=1+\cos\theta} f(r,\theta)\, r\, dr\, d\theta}$$



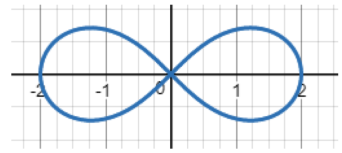
Area in Polar Coordinates

The area of a closed and bounded region R in the polar coordinate plane is

$$A = \iint_R r\,dr\,d\theta.$$

Example

Find the area enclosed by the lemniscate $r = \sqrt{4\cos 2\theta}$.



$$A = 4\int_0^{\pi/4}\int_0^{\sqrt{4\cos 2\theta}} r\,dr\,d\theta = 4\int_0^{\pi/4} 2\cos 2\theta\,d\theta = 4\left(\sin 2\theta\right)\Big|_0^{\pi/4} = 4$$

$$\left(\tfrac{1}{2}r^2\right)\Big|_0^{\sqrt{4\cos 2\theta}} = 2\cos 2\theta$$