Review Test 2 **Name**
Math 142 **Section Id**

Use exactly one page for each of the five numbered questions (use the back of the page if necessary).
Put your name and the question number on each page.
Put a box around the final answer to a question.
You must *show* your work in order to get possible credits.

1. Evaluate the integrals.
a) $\int x^2 \cos x dx$
b) $\int x^{\frac{1}{2}} \ln x \, dx$
c) $\int \sqrt{\frac{1-x}{1+x}} \, dx$ (hint: let $x = \cos 2u$ and recall $1 + \cos 2u = 2\cos^2 u$ and $1 - \cos 2u = 2\sin^2 u$)
d) $\int \frac{e^x}{\sqrt{e^{2x}-1}} dx$

2. Evaluate the definite integrals:
a) $\int_0^2 \tan^{-1}(1-x) \, dx$
b) $\int_{-\pi}^{\pi} \frac{\sin x}{\sqrt{3-\sin^2 x}} \, dx$
c) $\int_0^1 \frac{1}{\sqrt{x^2+1}} dx$
d) $\int_{-\pi/8}^{\pi/8} \sec(2x) \tan^3(2x) dx$
e) $\int_{-\pi}^{\pi} \sin^2 x \cos^2 x dx$

3. Find the following limits:

$$(a) \quad \lim_{x \to 0} \frac{\sin^{-1} x}{x}$$

$$(b) \quad \lim_{x \to +\infty} x^2 e^{-(x+2)}$$

4. Use partial fractions to decompose the rational function $P(x)/Q(x)$ as is the integrand, then evaluate the integral.

a)

$$\int \frac{x^2+4x+4}{x^3+2x}dx$$

b)

$$\int \frac{7s+4}{(s-2)(s+4)} \quad ds$$

c)

$$\int_0^1 \frac{u^3}{(u+1)^2}du$$

d)

$$\int \frac{5x^3-3x^2+2x-1}{x^4+x^2}dx$$

Hint: $\frac{P(x)}{Q(x)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+1}$

e)

$$\int \frac{x}{x^4+4x^2+8}dx$$

Hint: complete square of the denominator $Q(x)$ first.

5. Integration by Parts. Evaluate the following integral.

$$a) \int x\cos 4x \quad dx$$

$$b) \int \sin^{-1}(2x) \quad dx$$

6. L'Hopital's Rule. Determine whether the limit exists, if so, find the limit.

$$a) \lim_{t\to 0} \frac{t-\sin t}{\tan t}$$

$$b) \lim_{y\to 2} \frac{y^2+6}{y-2}$$

$$c) \lim_{z\to 1} \frac{z^2+4z-5}{z^3-1}$$

$$d)\ \lim_{t\to\infty}\frac{\ln(t^2+5t)}{\ln t}$$

$$e)\ \lim_{r\to 0}(1+3r)^{\frac{1}{r}}$$

7. Determine whether the improper integral converges. If it does, find the value of the integral:

$$(a)\quad \int_0^1 \frac{1}{1-x}\,dx$$

$$(b)\quad \int_2^\infty \frac{1}{x\ln x}\,dx$$

$$(c)\quad \int_0^\infty 5^{-x}\,dx$$

8.Determine whether the improper integral converges. If it does, find the value of the integral.

$$a)\quad \int_0^2 \ln x\,dx$$

$$b)\quad \int_1^\infty \frac{1}{x^{0.99}}\,dx$$

$$c)\quad \int_0^1 \ln(1-x)\,dx$$

$$c)\quad \int_0^{\pi/2} \frac{\cos x}{\sqrt{\sin x}}\,dx$$

9. Give the integral expressions of both the surface area S and the volume V of the object (called *Gabriel cone*) generated by revolving the curve from 1 to ∞

$$y = 1/x, \qquad 1 \le x < \infty$$

around the x axis. Do you think the cone has a finite volume or a finite surface area ?

[Hint] Show that V is finite and $S = \infty$!!