Instructions. Answer each of the questions on your own paper. Put your name on each page of your paper. Be sure to show your work so that partial credit can be adequately assessed. *Credit may not be given for answers (even correct ones) without supporting work.*

You can bring a copy of the Table of Laplace transforms. Also you could use the following trigonometric identities if necessary.

$$\begin{array}{|rcl|} e^{i\theta} & = & \cos\theta + i\sin\theta \\ \sin(\theta+\varphi) & = & \sin\theta\cos\varphi + \sin\varphi\cos\theta \\ \cos(\theta+\varphi) & = & \cos\theta\cos\varphi - \sin\theta\sin\varphi \end{array}$$

1. [**25 Points**] Find the general solution for each of the following homogeneous linear differential equations.

   (a) $y'' - 6y' + 5y = 0$

   (b) $y'' - 6y' + 9y = 0$

   (c) $y'' - 6y' + 13y = 0$

   (d) $y'' + 6y = 0$

   (e) $y''' - y'' + 4y' - 4y = 0$

2. [**25 Points**] You may assume that $\mathcal{S} = \{e^{-2t},\, te^{-2t}\}$ is a fundamental set of solutions for the homogeneous equation

$$y'' + 4y' + 4y = 0.$$

   Using the method of *variation of parameters*, find a particular solution of the nonhomogeneous equation

$$y'' + 4y' + 4y = t^{3/2}e^{-2t}.$$

3. [**25 Points**] Find the Laplace transform of each of the following functions:

   (a) $f(t) = (t-1)^2\chi_{[1,\,4)}(t)$

   (b) $g(t) = \begin{cases} 0 & \text{if } 0 \le t < \pi/2, \\ \sin 2t & \text{if } t \ge \pi/2. \end{cases}$

4. [**25 Points**] Solve the following initial value problem:

$$y'' + 4y = \begin{cases} t & \text{if } 0 \le t < 2 \\ 2 & \text{if } t \ge 2 \end{cases} \qquad y(0) = 0,\ y'(0) = 0.$$