

MATH 141, Sample Test

Show your work

NAME :_____

1 Write the equation for the tangent line and the normal line to the graph of

a) $f(x) = x^2 + 4x + 2$ at the Point $P(1, 7)$.

Tangent line:_____

Normal line:_____

b) $y = x \cos(x)$ at the point $P(\frac{\pi}{2}, 0)$.

Tangent line:_____

Normal line:_____

2 Find the limits

a) $\lim_{x \rightarrow 2} \frac{x^3 - 7x + 6}{x^2 - x - 2} =$ _____

b) $\lim_{x \rightarrow 0} \frac{\sin 3x}{5x} =$ _____

c) $\lim_{x \rightarrow \infty} \frac{x + \ln x}{x} =$ _____

3 Find the maximum and minimum values of the function

a) $f(x) = x^2 - \frac{4}{x^2}$ on the closed interval $[1, 3]$. The maximum is:_____

The minimum value is: _____

b) $f(x) = |x^2 - 1| + \frac{1}{2}x^2$ on the closed interval $[0, 2]$.

The maximum is:_____

The minimum value is: _____

4 Find the derivative of the following functions

a) $f(x) = \frac{1}{x} + xe^{x^2}$. $f'(x) =$

b) $f(x) = 3^x + \log_{10}(x)$. $f'(x) =$

c) $h(x) = (x + 1)^{-x}$. $h'(x) =$

5 Evaluate the following antiderivatives:

a) $\int \left(4^x + x^{3/2} - \frac{1}{x^4} \right) dx =$

b) $\int \frac{1 + \ln(x)}{x} dx =$

6 Evaluate the following integrals:

a) $\int_0^3 (1 - 3x)^5 dx =$

b) $\int_0^{\pi/2} (\sin x)^2 \cos x dx =$

c) $\int \left(x^5 + x^{3/2} - \frac{1}{x^4} \right) dx =$

d) $\int \tan x \ln |\cos x| dx =$

7 Evaluate the sum $\sum_{j=1}^{20} (j^2 - 2j + 1) =$ _____

8 Sketch the graph of the function $f(x) = \frac{x^3}{x^2 - 1}$. Determine the domain and range. If there are any, then identify and label all extrema ([4P]), inflection points ([4P]), intercepts ([4P]), and asymptotes ([4P]). Indicate the concave structure clearly ([4P]).

9 Use linear approximation to estimate $28^{2/3} \approx$ _____

10 Sketch the region bounded by the curves $y = x^3$ and $y = 2x$ and find its area.

The area is : _____

11 State the Fundamental Theorem of Calculus (two parts). Give an example for each part to illustrate your statement.

12 Find the derivative of the functions in x .

a)

$$\ln \sqrt{x^2 + 15}$$

b)

$$\int_2^x \ln |t| dt$$

c)

$$\int_1^{x^3} \cos(y^2) dy$$

13 Suppose that the fish population $P(t)$ in a lake is given by the differential equation $\frac{dP}{dt} = -k\sqrt{P(t)}$. If there were 576 fish in the lake at $t = 0$ and 4 weeks later only 144, how many are there after 5 weeks and how long will it take all the fish in the lake to die?

14 Lynda shoots an arrow straight upwards from the ground with initial velocity 320 ft/s.

a) How high is the arrow after 3 s?

b) At what time is the arrow exactly 1200 ft above the ground?

c) How many seconds after its release does the arrow strike the ground?

Hint: For a few of the problems you might want to read the Text or talk to your TA, SI or Instructor. But most of the problems you should try to work out independently first.