Math 2065 Section 1
Review Exercises for Exam I

It will be necessary to show your work to receive credit.

1. Figure 1 (Page 2) is the direction field for the differential equation $y' = y(y-1)(y+1)$.
   (a) Draw on the direction field the solutions of the differential equation satisfying each of the following initial values.
      i. $y(0) = 0.0$
      ii. $y(0) = 0.5$
      iii. $y(0) = -1.5$
   (b) For the solution $y(t)$ with initial condition $y(0) = 0.5$, what is $\lim_{t\to\infty} y(t)$ and $\lim_{t\to-\infty} y(t)$?
   (c) For the solution $y(t)$ with initial condition $y(0) = -1.5$, what is $\lim_{t\to\infty} y(t)$ and $\lim_{t\to-\infty} y(t)$?

2. Figure 2 (Page 3) is the direction field for the differential equation $y' = y - t$.
   (a) Draw on the direction field the solutions of the differential equation satisfying each of the following initial values.
      i. $y(0) = 0.0$
      ii. $y(0) = 1.0$
      iii. $y(0) = -1.0$
      iv. $y(0) = 2.0$
   (b) Are there any constant solutions $y = c$ to this differential equation? If so, show them on the direction field.
   (c) Are there any straight line solutions $y = mt + b$? If so indicate them on the direction field.
   (d) There is a number c such that all solutions with initial condition $y(0) > c$ satisfy $\lim_{t\to\infty} = \infty$ and all solutions with initial condition $y(0) < c$ satisfy $\lim_{t\to\infty} = -\infty$. Find this number c by inspecting the direction field.

Figure 1: Direction Field for Exercise 1

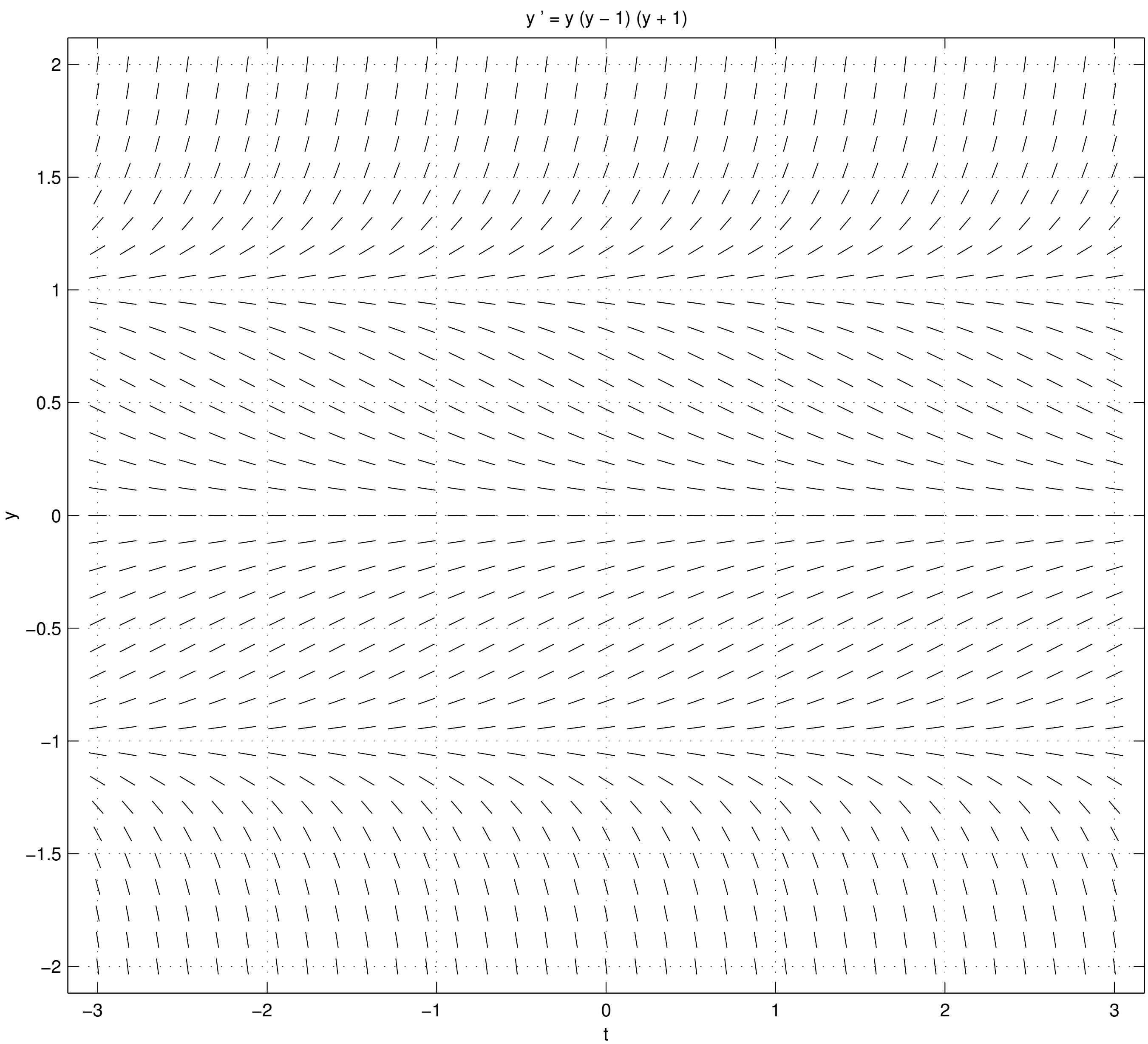
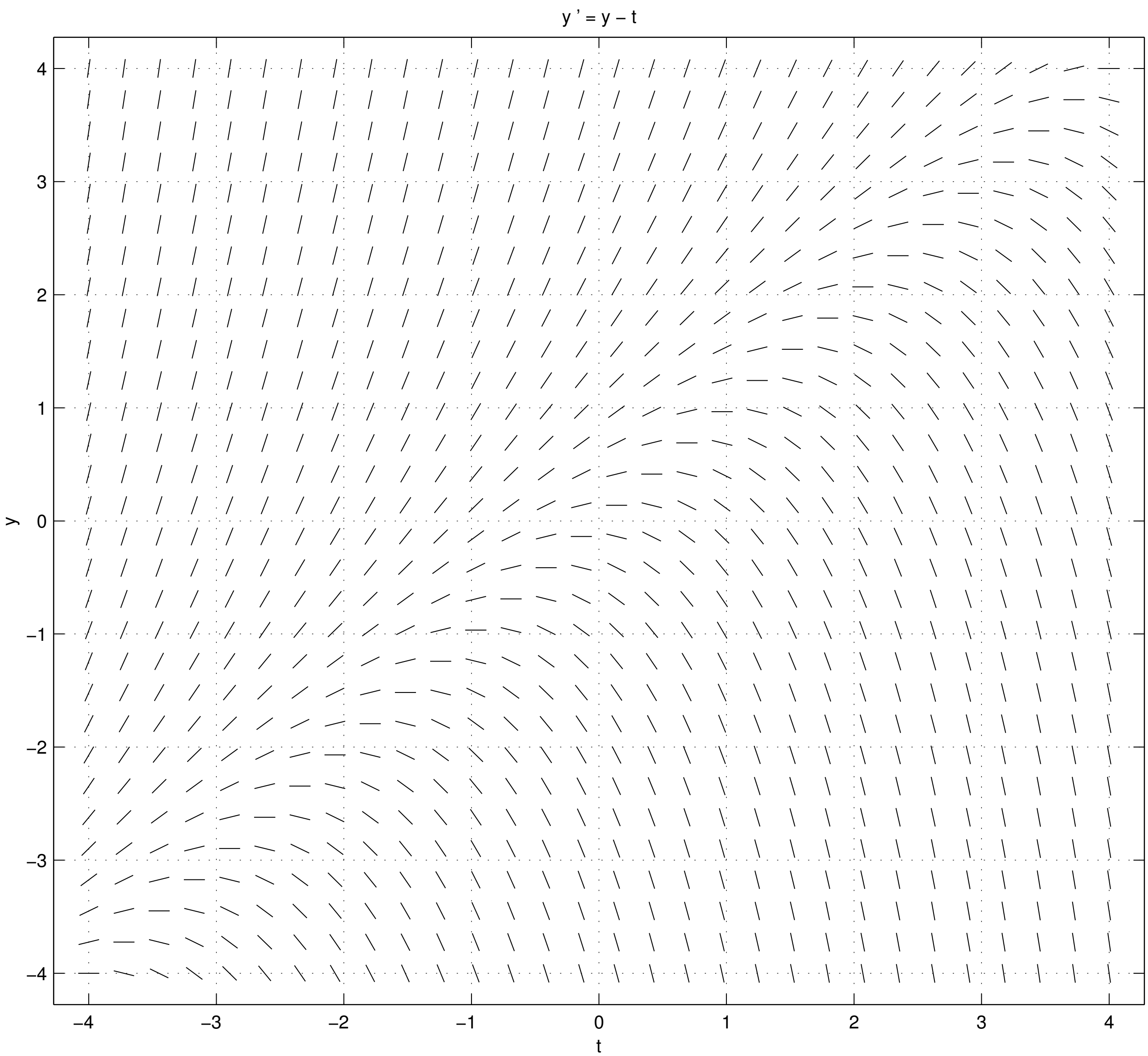


Figure 2: Direction Field for Exercise 1



3. Determine if each of the following equations is separable (yes or no), and/or linear (yes or no). Do **not** solve the equations!!

   (a) $y' = y^2 - t$
   (b) $t^2 y' = 1 - 2ty$
   (c) $yy' = 3 - 2y$
   (d) $\dfrac{y'}{y} = y - t$
   (e) $ty' = y - 2ty$
   (f) $(t^2 + 3y^2)y' = -2ty$
   (g) $y' = ty^2 - y^2 + t - 1$
   (h) $t + y' = y - 2ty$

4. Solve each of the following initial value problems. You **must** show your work.

   (a) $y' = 2y + 5e^{2t}, \quad y(0) = -1.$
   (b) $y' = y^2 t^3, \quad y(1) = -1.$
   (c) $y' + 3y = 4e^{-3t} \sin 2t, \quad y(0) = -1.$
   (d) $y' + \dfrac{3}{t} y = 7t^3, \quad y(1) = -1.$

5. Newton's law of cooling states that the rate at which a body cools (or heats up) is proportional to the difference between the temperature of the body and the temperature of the surrounding medium. A turkey which is initially at room temperature (70° F) is placed in a 350° F oven at time $t = 0$. Write an initial value problem which is satisfied by the temperature $T(t)$ of the turkey at time t.

6. A tank contains 100 gal of brine made by dissolving 80 lb of salt in water. Pure water runs into the tank at the rate of 4 gal/min, and the mixture, which is kept uniform by stirring, runs out at the same rate. Find the amount $y(t)$ of salt in the tank at any time t.

7. Apply Picard's method to compute the first two approximations $y_1(t)$ and $y_2(t)$ to the solution $y(t)$ of the initial value problem

$$y' = t^2 + y^2, \quad y(0) = 0.$$

8. (a) Complete the following definition: Suppose $f(t)$ is a continuous function of exponential type defined for all $t \geq 0$. The **Laplace transform** of f is the function $F(s)$ defined as follows

$$F(s) = \mathcal{L}(f(t))(s) = \boxed{}$$

   for all s sufficiently large.

(b) Using your definition compute the Laplace transform of the function $f(t) = 2t - 5$. You may need to review the integration by parts formula: $\int u\,dv = uv - \int v\,du$.

9. Compute the Laplace transform of each of the following functions using the table on Page 84. (This table will be provided to you on the exam.)

   (a) $f(t) = 3t^3 - 2t^2 + 7$

   (b) $g(t) = e^{-3t} + \sin\sqrt{2}t$

   (c) $h(t) = -8 + \cos(t/2)$