**MATH 142 Review Test 1** **Name**

**Id**

Put the question number (including its sub-problem number, if any) for each problem on your answer sheet. Put a box around the final answer to a question.

For full credit you must *show your work*. You must have enough written work, including explanations when called for, to justify your answers. Incomplete solutions may receive partial credit if you have written down a reasonable partial solution.

**1** **[10P]** Evaluate the indefinite integrals

a. $\int (x+3)^{5/2} dx.$

b. $\int \frac{x}{4-x^2}\, dx$

c. $\int \frac{1+e^{2x}}{e^x}\, dx$

d. $\int \csc x \cot x dx$

**2.** Assume that $[x_{i-1}, x_i]$ denotes the $i^{th}$ subinterval of a subdivision of $[0,2]$, which is divided into $n$ subintervals having the same length $\Delta x = \frac{2}{n}$.

**a** **[5P]** Evaluate the Riemann sum $\sum_{i=1}^{n} (x_i - 0.5)\, \Delta x.$

**b** **[5P]** What is $\lim_{n\to\infty} \sum_{i=1}^{n} (x_i - 0.5)\, \Delta x.$

**c** **[5P]** Give an integral (but do not evaluate it) that is approximated by the Riemann sum in **a**.

**3** [**10P**] Evaluate the definite integrals

a. $\displaystyle\int_e^{100} \frac{dx}{x \ln x}$.

b. $\displaystyle\int_{-1/2}^{1/2} \frac{dt}{\sqrt{1-t^2}}$.

**4** [**10P**] Find the derivative. **a.** Use the Fundamental Theorem of Calculus to find:

$$\frac{d}{dx} \int_{17}^{x} e^{\sqrt{t+5}}\, dt \;=$$

**b.**

$$\frac{d}{dx}\, 7^{(\arctan x)^2} \;=$$

**5** [**10P**] Evaluate the integrals a. $\int \tan x \sec^5 x dx$.

b. $\int x^2 \tan^{-1} x dx$

c. $\int \frac{dx}{x(2+\ln x)^2}$

d. $\displaystyle\int (\sec x + \tan x) dx$

e. $\displaystyle\int_0^1 \sin^{-1} x dx$

**6** [**10P**] Find the total area of the bounded region given by the $x-$axis and the graph of the function $f(x) = x^3 - x$ on $[-1, 1]$.

**7** [**10P**] Find the work done by the force $F(t) = \frac{12}{2t^2+1}$ in moving a particle along the $t$-axis from $t = 0$ to $t = 1.5$

**8** [**10P**] Let $R$ be the region enclosed by

$$y = \sqrt{x} \quad \text{and} \quad x = 4 \quad \text{and} \quad x = 9 \quad \text{and} \quad y = 0\ .$$

Let $V$ be the volume of the solid obtained by revolving the region $R$ about the $y$-axis.

**a.** Make a rough sketch of the region $R$, labeling the important points.

**b.** Using the cylindrical shell method, express the volume $V$ as an integral (or maybe 2 integrals).

You do NOT have to evaluate the integral(s).

**9** **[5P]** Let $L$ be the arc length of the curve

$$x = \frac{y^4}{8} + \frac{y^{-2}}{4} \quad \text{from} \quad y = 1 \quad \text{to} \quad y = 4 \ .$$

Express $L$ as an integral. You do NOT have to evaluate the integral.

**10** **[10P]** Let $S$ be the area of the surface generated by revolving the curve below about the line $y = -0.5$

$$y = \frac{1}{x}\,, \qquad 2 \le x \le b,$$

where $b$ is a constant greater than 2. Set up an integral for the surface area $S = S(b)$ and find its value.