Chap. 4 Integration
4.1 Antidifferentiation
4.2 Antiderivatives as Areas
4.3 Area and Definite Integrals
4.4 Properties of Definite Integrals: Additive Property, Average Value and Moving Averages
4.5 Integration Techniques: Substitution
4.6* Integration Techniques: Integration by Parts
4.7* Numerical Integration

• *Antidifferentiation*

101. **Definition.** A function F is an **antiderivative** of f on an interval I if $F'(x) = f(x)$ for all x in I. The process of recovering a function F from its derivative f is called **antidifferentiation**.

102. Determine whether $F(x)$ is an antiderivative of $f(x) = e^{2x} + x$.

   (a) $F(x) = e^{2x} + \frac{x^2}{2}$ (b) $F(x) = \frac{1}{2}\left(e^{2x} + x^2\right)$ (c) $F(x) = \frac{1}{2}\left(e^{2x} + x^2\right) + 2016$

103. Antidifferentiate the following functions.

   (a) $f(x) = \dfrac{1}{\sqrt{x}} + 5x^3$

   (b) $f(x) = e^{3x}$

   (c) $f(x) = \dfrac{3x^4 - 2x^2 + x - 1}{x^2}$

104. Find an antiderivative $F(x)$ of $f(x) = 8x^3 - 2x^2$ that satisfies $F(-1) = 2$.

105. A ceramics company determines that the marginal revenue, R', in dollars per unit, from selling the xth vase is given by $R'(x) = x^2 - 1$. Find the total revenue after 3 units were sold.

• *Area and Definite Integrals*

106. **Theorem.** *Let f be a nonnegative continuous function on $[a, b]$, and let $A(x)$ be the area between the graph of f and the x-axis over $[a, x]$, with $a < x < b$. Then $A(x)$ is a differentiable function of x and $A'(x) = f(x)$.*

107. Find the area under the graph of $f(x) = 3x^2 + x$ over $[1, 4]$.

108. **Definition.** Let f be a continuous function on $[a, b]$ and F be any antiderivative of f. Then the **definite integral** of f from a to b is $\displaystyle\int_a^b f(x)\, dx = F(b) - F(a)$.

109. Evaluate the definite integrals.

(a) $\int_{-2}^{3} (x^2 - 2x + 3)\ dx$

(b) $\int_{0}^{3} e^{-3x}\ dx$

(c) $\int_{1}^{2} \frac{x^4 - x}{x^2}\ dx$

(d) $\int_{-5}^{-1} \frac{1}{x}\ dx$

110. Northeast Airlines determines that the marginal profit resulting from the sale of x seats on a jet traveling from Atlanta to Kansas City, in hundreds of dollars, is given by $P'(x) = \sqrt{x} - 6$. Find the total profit when 60 seats are sold.

- *Substitution*

111. Find the integrals.

(a) $\int (x^3 + 1)^4 x^2\ dx$

(b) $\int \frac{1}{5x + 7}\ dx$

(c) $\int x^3 e^{-x^4}\ dx$

(d) $\int_{1}^{e} \frac{(\ln x)^2}{x}\ dx$

(e) $\int_{0}^{1} \sqrt{8 - 3x}\ dx$

(f) $\int_{0}^{3} (x - 5)^2\ dx$

- *Consumer Surplus and Producer Surplus*

112. **Definition.** Let $p = D(x)$ be the demand function for a product. Then the **consumer surplus** for Q units of the product, at a price per unit P, is

$$\int_{0}^{Q} D(x)\ dx - QP.$$

Let $p = S(x)$ be the supply function for a product. Then the **producer surplus** for Q units of the product, at a price per unit P, is

$$QP - \int_{0}^{Q} S(x)\ dx.$$

The **equilibrium point** is the point at which the supply and demand curves intersect.

113. In the following problems, $D(x)$ is the price, in dollars per unit, that consumers will pay for x units of an item, and $S(x)$ is the price, in dollars per unit, that producers will accept for x units. Find the equilibrium point, the consumer surplus at the equilibrium point, and the producer surplus at the equilibrium point.

(a) $D(x) = -\frac{5}{6}x + 9$, $S(x) = \frac{1}{2}x + 1$

(b) $D(x) = (x - 4)^2$, $S(x) = x^2 + 2x + 6$