Contents of Chapter 1, Sections 1.5-1.8


- *Differentiation Rules*

(56) Differentiate the following functions.
  (a) $y = x^5 - 2x + 2\sqrt{x} - 7$
  (b) $y = 4x^3 + 3x^2 + \dfrac{9}{\sqrt[3]{x^2}}$
  (c) $y = \dfrac{4}{x^3} - x + 5$

(57) Find an equation of the tangent line to the graph of $f(x) = \sqrt{x} - x$ at the point $(4, -2)$.

(58) Differentiate the following functions.
  (a) $y = (x^2 + x - 1)(5x - \sqrt{x})$
  (b) $y = 4x^2(x^5 - x)$
  (c) $y = \dfrac{x^3 + 3x^2}{x - 1}$

(59) The population $P$ (in thousands) of a town is given by $P(t) = \dfrac{500t}{2t^2 + 9}$, where $t$ is the time (in years).
  (a) Find the population after 12 years.
  (b) Find the population growth rate at $t = 12$ years.

- *1.7. The Chain Rule*

(60) **Definition.** The **composed** function $f \circ g$, the **composition** of $f$ and $g$, is defined as $(f \circ g)(x) = f(g(x))$.

(61) For $f(x) = x^3$ and $g(x) = 3x + 4$, find $f \circ g$ and $g \circ f$.

Summary of C.R. in the following Table

power chain rule

$$\frac{d}{dx}\left(f(x)^n\right) = n(f(x))^{n-1} \cdot f'(x)$$

general chain rule

$$\frac{d}{dx}\left(f(g(x))\right) = f'(g(x)) \cdot g'(x)$$

gear demonstration

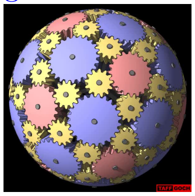

Figure: Chain of gears (courtesy of martahidegkuti)

*Proof of the Chain Rule.* Let $f(u)$ be differentiable at $u = g(c)$, and let $g(x)$ be differentiable at $x = c$. Then

$$\begin{aligned} \frac{d}{dx} f(g(x))\Big|_{x=c} &= \lim_{x\to c} \frac{f(g(x)) - f(g(c))}{x - c} \\ &= \lim_{x\to c} \frac{f(g(x)) - f(g(c))}{g(x) - g(c)} \cdot \frac{g(x) - g(c)}{x - c} \\ &= f'(g(c)) \cdot g'(c). \end{aligned}$$

□

Reference on Chain Rule

(62) Differentiate the following functions.
  (a) $y = (3x + 4)^3$
  (b) $y = \sqrt[3]{5 - x^3}$
  (c) $y = \dfrac{1}{(2x^2 - x)^4}$
  (d) $y = \sqrt{6x + 1}$
  (e) $y = \dfrac{1}{\sqrt{6x + 1}}$

(63) A new phone is released on the market. Its quantity sold $N$ is given as a function of time $t$, in weeks, by $N(t) = \dfrac{10,000t^2}{(2t + 3)^2}$. Find $N'(t)$. Then find $N'(20)$ and $N'(200)$.

- *Higher-Order Derivatives**

(64) Find the second derivative of the following functions.
  (a) $y = x^5 - 8x^7 + 9x$
  (b) $y = 1/x$

(65) **Definition.** The **velocity** $v(t)$ and **acceleration** $a(t)$ of an object that is $s(t)$ units from a starting point at time $t$ are given by $v(t) = s'(t)$ and $a(t) = v'(t) = s''(t)$.

(66) Suppose that a ball is dropped from the 86th floor observation deck of the Empire State Building, 320 meters above the ground. Let $t$ denote time (in seconds) and $s(t)$ denote the distance fallen after $t$ seconds (in meters). Then Galileo's law is expressed by $s(t) = 4.9t^2$. Find the velocity and acceleration of the ball at $t = 3$.

(67) Given $s(t) = -t^3 + 4t - 2$, where $s(t)$ is in meters and $t$ is in seconds, find the velocity and acceleration at $t = 1$.

Videos on Chapter 1:Differentiation from MLM Plus