Review Test 1 **Name**
Math 2242 **Id**

Put your name and the question number on each page.
Put a box around the final answer to a question.
(use the back of the page if necessary).
You must show your work in order to get possible credits.

1. Show that the given function has an inverse and find the domain and range for the inverse function:
(i) $f(x) = 3x - \cos 2x,\ -\infty < x < \infty$
Evaluate $(f^{-1})'(-1)$.
(ii) $g(x) = (x+2)^3, -\infty < x < \infty$
Find g^{-1} and evaluate $(g^{-1})'(0)$.

2.Evaluate the integrals:
(i) $\int \tan^{-1}(1-x)\,dx$
(ii) $\int_{-\pi}^{\pi} \frac{\sin x}{\sqrt{3-\sin^2 x}}\,dx$
(iii)* $\int \frac{x^2+4x+4}{x^3+2x} dx$
(iv) $\displaystyle\int \frac{e^x}{\sqrt{1\ -\ e^{(2x)}}}\ dx\ =$
(v) $\displaystyle\int x^3\ 17^{(x^4)}\ dx\ =$

3. Find the following limits:

$$(i) \quad \lim_{x\to 0} \frac{\sin^{-1} x}{x}$$

$$(ii) \quad \lim_{x\to +\infty} x^2 e^{-(x+2)}$$

$$(iii) \quad \lim_{x\to 0^+} \tan^{-1}(\pi/x)$$

$$(iv) \quad \lim_{x\to\infty} \frac{x^5 - 2x^3 + 15x}{x^5 - 7x^3}$$

$$(v) \quad \lim_{x\to 0^+} (\ln x)^x$$

$$(vi) \quad \lim_{x\to\infty} (1+\frac{3}{x})^{5x}$$

$$(vii) \quad \lim_{x\to\infty} \frac{(\ln x)^2}{x+1}$$

4. Fill in the blanks (each worth 1 point).

a. $\int \frac{du}{u} =$ ________ $|u| + C$

b. If a is a constant and $a > 0$ but $a \neq 1$, then

$\int a^u\, du =$ ____ $+ C$

c. $\int \sec^2 u\, du =$ ____ $+ C$

d. $\int \sec u \; \tan u \; du =$ ____ $+ C$

e. $\int \sin u\, du =$ ____ $+ C$

f. $\int \cot u\, du =$ ____ $+ C$

g. $\int \sec u\, du =$ ____ $+ C$

h. If a is a contant and $a > 0$ then

$\int \frac{1}{\sqrt{a^2 - u^2}}\, du =$ ____ $+ C$

i. If a is a contant and $a > 0$ then

$\int \frac{1}{a^2 + u^2}\, du =$ ____ $+ C$

j. The integral of $y = f(x)$ <u>with respect to x</u> is denoted by $\int f(x)\, dx$.

The integral of $x = g(y)$ <u>with respect to y</u> is denoted by ______________________ .

5. Let R be the region enclosed by

$$y = x^2 \qquad \text{and} \qquad y = x \; + \; 2 \; .$$

Let A be the area of the region R.

a. The points of intersection of $y = x^2$ and $y = x + 2$ are $P =$ (____ , ____) and $Q =$ (____ , ____).

Make a rough sketch of the region R, labeling P and Q.

b. Express the area A as integral(s) with respect to x (so you want dx).
You do NOT have to evaluate the integral(s) nor do lots of algebra.

A =

c. Express the area A as integral(s) with respect to y (so you want dy).
You do NOT have to evaluate the integral(s) nor do lots of algebra.

A =

6. Evaluate the integrals.

a) $\displaystyle\int \frac{1}{x(\ln x)^{3/2}}\, dx$

b) $\displaystyle\int_0^1 \frac{1}{\sqrt{x^2+1}} dx$

c) $\displaystyle\int xe^{-2x} dx$

d) $\displaystyle\int_0^{\pi/4} e^x \sin(x) dx$

e*) $\displaystyle\int xe^x \cos(x) dx$

f*) $\displaystyle\int \sec^3 x dx$

g) $\displaystyle\int_{-\pi/12}^{\pi/12} \tan(3x) dx$

h) $\displaystyle\int \sin^2 x dx$

i) $\displaystyle\int \sin(\ln\theta) d\theta$