Math 1232 Section A

Survey of Calculus

- Logarithmic Functions
 - 1. **Definition.** A **logarithm** is defined as follows:

$$\log_a x = y$$
 means $a^x = y$, $a > 0$, $a \neq 1$.

The number $\log_a x$ is the power of y to which we raise a to get x. The number a is called the **logarithmic base**.

- 2. Solve for x.
 - (a) $\log_5 125 = x$
 - (b) $\log_x 64 = 3$
 - (c) $\log_6 x = -1$
 - (d) $\log_4 1/16 = x$
- 3. Given $\log_a 2 = 0.27$ and $\log_a 3 = 0.43$. Find $\log_a 6$, $\log_a \frac{2}{3}$, $\log_a 81$, $\log_a \frac{1}{3}$, $\log_a \sqrt{a}$, $\log_a (2a)$, and $\frac{\log_a 3}{\log_a 2}$.
- 4. **Definition.** For any positive number x, the **common logarithm** of x is $\log_{10} x$ (abbreviated $\log x$), and the **natural logarithm** of x is $\log_e x$ (abbreviated $\ln x$).
- 5. Given $\ln 2 = 0.69$ and $\ln 3 = 1.10$. Find $\ln 6$, $\ln \frac{2}{3}$, $\ln 81$, $\ln \frac{1}{3}$, $\ln (3e^2)$, and $\log_2 3$.
- 6. Solve $e^{-0.25x} = 0.58$.
- 7. **Theorem.** The function $\ln x$ exists only for positive numbers x. The domain is $(0, \infty)$. When 0 < x < 1, $\ln x < 0$. When x = 1, $\ln x = 0$. When x > 1, $\ln x > 0$. The function $\ln x$ is an increasing function. The range is the entire real line $(-\infty, \infty)$.
- 8. **Theorem.** $\frac{d}{dx} \ln x = \frac{1}{x}$, $\frac{d}{dx} \ln |x| = \frac{1}{x}$.
- 9. Differentiate the following functions.
 - (a) $y = 8 \ln x$
 - (b) $y = 4 \ln |x|$
 - (c) $y = x^4 \ln x \frac{1}{2}x^2$
 - $(d) y = \frac{\ln x}{x^4 + 1}$
 - (e) $y = \ln(5x)$
 - (f) $y = \ln(6x^2 3x)$

10. A company begins a radio advertising campaign in a city to market its new energy drink. The percentage of the "target market" that buys a product is normally a function of the duration of the advertising campaign. The radio station estimates this percentage by using $f(t) = 1 - e^{-0.04t}$ for this type of product, where t is the number of days of the campaign. The target market is approximately 1,000,000 people, and the price per unit is \$1.50. If the campaign costs \$1000 per day, how long should it last in order to maximize profit?