Math 141 Test 1

Name
Id

Put the question number (including its sub-problem number, if any) for each problem on your answer sheet. Put a box around the final answer to a question.

For full credit you must *show your work*. You must have enough written work, including explanations when called for, to justify your answers. Incomplete solutions may receive partial credit if you have written down a reasonable partial solution.

1 [15 pts] Write the equation for the tangent line to the curve $f(x) = x^2 - 2x + 1$ at the point $P(2, 1)$.

2 [30 pts] Find the following limits or determine if they do not exist.

a) $\lim_{x\to 2} \dfrac{x^3 - 8}{x - 2} =$

b) $\lim_{x\to\infty} \sqrt{x^2 + 11} - x =$

c) $\lim_{x\to 1.5^-} \dfrac{5}{3 - 2x} =$

d) $\lim_{x\to 0} x^2 \left(\cos \dfrac{1}{x}\right)^2 =$

e) $\lim_{x\to 0} \dfrac{|x|}{x} =$

3 [10 pts] Prove using the $\varepsilon - \delta$ definition of limit that

$$\lim_{x\to -1} \quad 8x + 7 = -1$$

4 [15 pts] Let

$$f(x) = \begin{cases} 3x - 1 & x < 0 \\ \sqrt{3} & x = 0 \\ x^4 + x + \sqrt{3} & x > 0 \end{cases}$$

a) Explain why f has a discontinuity at $x = 0$.

b) What type of discontinuity does f have at $x = 0$?

5 [15 pts] Find the derivative of the following function and determine the domain of the derivative function:
a) $f(x) = \sqrt{x-5}$. $f'(x) =$
b) $g(x) = \dfrac{x}{x+1}$. $g'(x) =$
c) $p(t) = (t-4)^3$. $p'(t) =$

6 [15 pts] A ball is thrown vertically upward so that its height (in feet) at time t (in seconds) is given by

$$h(t) = -16t^2 + 64t$$

a) Find the velocity $v(t)$ at time t.
b) What is the average velocity of the ball from time $t = 1$ to $t = 2$? What is the instantaneous velocity at t=1?