

§5.1 Area and Estimating with Finite Sums

**A. Problem.** Estimate the area under the graph of  $f(x) = 20 - x^2$  on  $0 \leq x \leq 4$  using

- (a) left-endpoint values with two rectangles of equal width.
- (b) left-endpoint values with four rectangles of equal width.
- (c) right-endpoint values with two rectangles of equal width.
- (d) right-endpoint values with four rectangles of equal width.

**Exercise 10.** You are sitting on the bank of a tidal river watching the incoming tide carry a bottle upstream. You record the velocity of the flow every 5 minutes for half an hour, with the results shown in the accompanying table.

Time (min)	0	5	10	15	20	25	30
Velocity (m/sec)	1	1.2	1.7	2.0	1.8	1.6	1.4

About how far upstream did the bottle travel during those 30 minutes? Find an estimate using 6 subintervals of length 5 with

- (a) left-endpoint values.
- (b) right-endpoint values.

B. Recitation Session: Project on evaluate area under a curve using finite [Riemann sums](#)

[Ex. from [MML](#)]