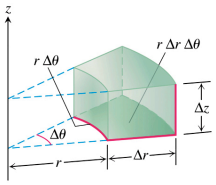
The Definite Integral in Cylindrical Coordinates

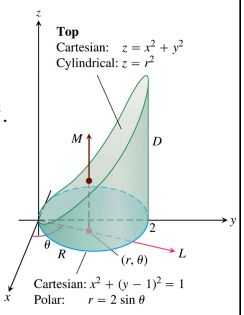
$$\iiint_D f\,dV = \int_{\cdots}\int_{\cdots}\int_{\cdots} f(r,\theta,z)\,dz\,r\,dr\,d\theta$$

$$\Delta V = \Delta z \cdot r\,\Delta r\,\Delta\theta$$



Example

Find the limits of integration in cylindrical coordinates for integrating a function $f(r,\theta,z)$ over the region D bounded below by the plane $z=0$, laterally by the cylinder $x^2+(y-1)^2=1$ and above by the paraboloid $z=x^2+y^2$.



$$\iiint_D f(r,\theta,z)\,dV = \int_0^{\pi}\int_0^{2\sin\theta}\int_0^{r^2} f(r,\theta,z)\,dz\,r\,dr\,d\theta$$

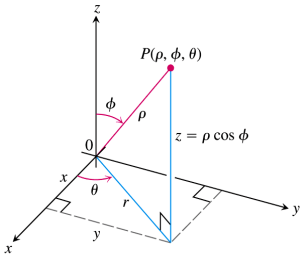
Spherical Coordinates

$$P=(\rho,\phi,\theta)$$

ρ = distance from the origin $(\rho \ge 0)$

ϕ = angle between $\vec{OP}$ and positive z-axis $(0 \le \phi \le \pi)$

θ as in cylindrical coordinates $(0 \le \theta \le 2\pi)$



Conversion Formulas

$r = \rho\sin\phi$ $\quad x = r\cos\theta = \rho\sin\phi\cos\theta$

$z = \rho\cos\phi$ $\quad y = r\sin\theta = \rho\sin\phi\sin\theta$

$$\rho = \sqrt{x^2+y^2+z^2} = \sqrt{r^2+z^2}$$

Example

Describe the set given by the equation:

a) $\rho = a \quad (a \ge 0)$

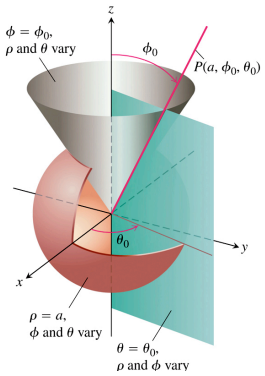
The sphere of radius a centered at the origin

b) $\phi = \phi_0$

A cone with vertex at the origin and the axis along the z-axis (the xy-plane, if $\phi_0 = \pi/2$)

c) $\theta = \theta_0$

The half-plane that contains the z-axis and makes an angle θ_0 with the positive x-axis



Example

Convert to a spherical coordinate equation

a) $x^2+y^2+(z-1)^2=1$ (a sphere)

$x^2+y^2+z^2-2z+1=1$

$\rho^2-2\rho\cos\phi=0$

$\rho(\rho-2\cos\phi)=0$

$\rho=0$ or $\boxed{\rho=2\cos\phi}$ (includes $\rho=0$ $(\phi=\pi/2)$)