Math 2065 Section 1
Review Exercises for Exam II

The syllabus for Exam II is Chap.2 and Chap.3, section 1. Table C.1 (Laplace Transform Rules), Table C.2 (Table of Laplace Transforms), and Table C.3 (Table of Convolutions) will be provided with the test.

1. Compute the Laplace transform of each of the following functions.

   (a) $f(t) = 3t^3 - 2t^2 + 7$

   (b) $g(t) = e^{-3t} + \sin\sqrt{2}\,t$

   (c) $h(t) = -8 + \cos(t/2)$

2. Compute the Laplace transform of each of the following functions.

   (a) $f(t) = 7e^{2t}\cos 3t - 2e^{7t}\sin 5t$

   (b) $g(t) = 3t\sin 2t$

   (c) $h(t) = (2 - t^2)e^{-5t}$

3. Find the inverse Laplace transform of each of the following functions.

   (a) $F(s) = \dfrac{7}{(s+3)^3}$

   (b) $G(s) = \dfrac{s+2}{s^2 - 3s - 4}$

   (c) $H(s) = \dfrac{s}{(s+4)^2 + 4}$

4. Find the Laplace transform of each of the following functions.

   (a) $t^2 e^{-9t}$

   (b) $e^{2t} - t^3 + t^2 - \sin 5t$

   (c) $t\cos 6t$

   (d) $2\sin t + 3\cos 2t$

   (e) $e^{-5t}\sin 6t$

   (f) $t^2\cos at$ where a is a constant

5. Find the inverse Laplace transform of each of the following functions.

   (a) $\dfrac{1}{s^2 - 10s + 9}$

   (b) $\dfrac{2s - 18}{s^2 + 9}$

   (c) $\dfrac{2s + 18}{s^2 + 25}$

   (d) $\dfrac{s + 3}{s^2 + 5}$

   (e) $\dfrac{s - 3}{s^2 - 6s + 25}$

   (f) $\dfrac{1}{s(s^2 + 4)}$

   (g) $\dfrac{1}{s^2(s + 1)^2}$

6. Solve the following differential equations by means of the Laplace transform:

   (a) $y' + 3y = t^2 e^{-3t} + te^{-2t} + t, \qquad y(0) = 1$

   (b) $y'' - 3y' + 2y = 4, \qquad y(0) = 2, \quad y'(0) = 3$

   (c) $y'' + 4y = 6 \sin t, \qquad y(0) = 6, \quad y'(0) = 0$

   (d) $y''' - y' = 2, \qquad y(0) = y'(0) = y''(0) = 4$

   (e) $y''' - y' = 6 - 3t^2, \qquad y(0) = y'(0) = y''(0) = 1$

7. Using the Laplace transform, find the solution of the following differential equations with initial conditions $y(0) = 0$, $y'(0) = 0$:

   (a) $y'' - y = 2 \sin t$

   (b) $y'' + 2y' = 5y$

   (c) $y'' + y = \sin 4t$

   (d) $y'' + y' = 1 + 2t$

   (e) $y'' + 4y' + 3y = 6$

   (f) $y'' - 2y' = 3(t + e^{2t})$

   (g) $y'' - 2y' = 20e^{-t} \cos t$

   (h) $y'' + y = 2 + 2 \cos t$

(i) $y'' - y' = 30\cos 3t$

8. Compute the convolution $t * t^3$ directly from the definition.

9. Using the table of convolutions, compute the following convolutions:

   (a) $(1+3t) * e^{5t}$
   (b) $(1/2 + 2t^2) * \cos\sqrt{2}t$
   (c) $(e^{2t} - 3e^{4t}) * (e^{2t} + 4e^{3t})$

10. Answer the following questions concerning the differential equation

$$(*) \qquad t^2y'' + ty' - y = 6t^2.$$

   (a) Verify that $\varphi_1(t) = t$ and $\varphi_2(t) = \dfrac{1}{t}$ are solutions of the associated homogeneous differential equation $t^2y'' + ty' - y = 0$.
   (b) Verify that $y_p(t) = 2t^2$ is a solution to equation $(*)$.
   (c) What is the general solution of equation $(*)$?
   (d) What is the solution of the initial value problem

$$(**) \qquad t^2y'' + ty' - y = 6t^2, \quad y(1) = -1, \quad y'(1) = 1?$$

   (e) What is the largest interval on which the initial value problem $(**)$ is guaranteed to have a solution by the existence and uniqueness theorem? Is this answer consistent with the solution that you found in the previous part of this exercise?