Review Test 3 **Name**
Math 142 **Section** **Id**

Use exactly one page for each of the five numbered questions (use the back of the page if necessary).
Put your name and the question number on each page.
Put a box around the final answer to a question.
You must *show* your work in order to get possible credits.

1. Find the limit of the sequence

a) $\lim_{n\to\infty} n\tan\frac{1}{n}$

b) $\lim_{n\to\infty} \frac{\ln(3n+5)}{n}$

c) $\lim_{n\to\infty} \frac{2n^2}{(n+1)^2}$

d) $\lim_{n\to\infty} \frac{e^n}{e^{2n}-1}$

e) $\lim_{n\to-\infty} \frac{e^n}{e^{2n}-1}$

2. Determine whether the limit of the following sequence exists as $n \to +\infty$, if so, find the limit:

a) $\frac{(-1)^n}{n!}$

b) $\frac{\sin(3n)}{\sqrt{n\pi}}$

c) $(-1)^n + 100$

d^*) (optional) $\tan(n-\pi)$

3. Evaluate the limit

a. $\lim_{n\to\infty} \dfrac{n^{5/2} + 7n^2 + 9}{-n^{5/2} + 3n^2 - 3n - 11} \quad =$

b. $\lim_{n\to\infty} \dfrac{5n^5 - 7n^3 + 10}{5n^4 + 6n^2 + 9} \quad =$

Fill in the blanks or parenthesis in Problems **4** to **8**.

4. n^{th}**-term test:** Let $\{a_n\}$ be an arbitrary sequence.

(a) If $\lim_{n\to\infty} a_n \neq 0$ or $\lim_{n\to\infty} a_n$ does not exist, then $\sum a_n$ _________
(b) If $\sum_n a_n$ converges, then $\lim a_n =$ _________

5. **Integral Test:** $a_n > 0$.   Let $f : [1, \infty) \to \mathbf{R}$ be so that

- $a_n = f(\ \ )$ for each $n \in \mathbf{N}$
- f is a ________________________________ function
- f is a ________________________________ function
- f is a ________________________________ function .

Then $\sum a_n$ converges if and only if ________________________________ converges.

6. (a) **Comparison Test:** $a_n > 0$

- If $0 \leq a_n \leq b_n$ for all $n \in \mathbf{N}$ and $\sum b_n$ __________________, then $\sum a_n$ __________________.
- If $0 \leq b_n \leq a_n$ for all $n \in \mathbf{N}$ and $\sum b_n$ __________________, then $\sum a_n$ __________________.

(b). **Limit Comparison Test:** $a_n > 0$
Let $b_n > 0$ and $\lim_{n\to\infty} \frac{a_n}{b_n} = L$.
If _________ $< L <$ _________, then $\sum a_n$ converges if and only if $\sum b_n$ __________________ .

7. (a) **Ratio Test:** $a_n > 0$
Let $\rho = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$.

- If $\rho <$ _________ then $\sum a_n$ converges.
- If $\rho >$ _________ then $\sum a_n$ diverges.
- If $\rho =$ _________ then the test is inconclusive.

(b) **Root Test:** $a_n > 0$

Let $\rho = \lim_{n\to\infty} (a_n)^{\frac{1}{n}}$.

- If $\rho <$ _________ then $\sum a_n$ converges.
- If $\rho >$ _________ then $\sum a_n$ diverges.
- If $\rho =$ _________ then the test is inconclusive.

8*. (Optional) **Alternating Series Test:** $a_n > 0$. If

- a_n _________ a_{n+1} for each $n \in \mathbf{N}$
- $\lim_{n\to\infty} a_n =$ _________

then $\sum (-1)^n a_n$ _________

9. Determine whether the series converges. If it does, find the value of the sum.

$$(a) \quad \sum_{n=1}^{\infty} \left(\frac{-2}{3}\right)^n$$

$$(b) \quad \sum_{k=1}^{\infty} \frac{1}{k^2 + 2k - 3}$$

10. Determine the convergence/divergence of the series below. A correctly checked box without appropriate explanation will receive 0 or 1 point.

$$\sum_{n=1}^{\infty} (-1)^n \frac{\ln\ n}{n}$$

☐ absolutely convergent

☐ conditionally convergent

☐ divergent

(Hint: Conditional convergent means $\sum a_n$ is convergent but $\sum |a_n|$ divergent)

11. Let $a_n = \frac{n^3\ (n!)}{(2n)!}$ Find a_{n+1}/a_n. Simplify your answer so that no factorial sign (i.e., !) appears.

answer: $\dfrac{a_{n+1}}{a_n} \ =$

$$\sum_{n=2}^{\infty} (-1)^n \ \frac{n^3\ (n!)}{(2n)!}$$

- ☐ absolutely convergent
- ☐ conditionally convergent
- ☐ divergent

12. Consider the formal power series

$$\sum_{n=1}^{\infty} \frac{(x+7)^n}{n^2} \ .$$

In the box below draw a diagram indicating for which x's this series is: absolutely convergent, conditionally convergent, and divergent. Of course, indicate your reasoning.

(Hint: use ratio/root test to determine the radius of convergence, interval of convergence)

13. Find the Taylor or Maclaurin series of $y = f(x)$
(a)

$$f(x) = e^{2x}$$

about $x = 1$
(b)

$$f(x) = \frac{1}{1+x}$$

about $x = 0$.

14. **Geometric Series**
a. If $|r| < 1$, then

$$\sum_{n=0}^{\infty} r^n \quad = \boxed{}$$

(Hint for part (b), if $|r| < 1$, then $\sum_{\mathbf{n=0}}^{\infty} r^n \;=\; 1 + r + r^2 + r^3 + r^4 + \ldots\,.)$

b. Find the sum of the below series. (Note that the sum begins at $n = 10$ instead of $n = 0$.)

$$\sum_{n=10}^{\infty} 2\left(\frac{1}{5}\right)^{n-2} \quad = \boxed{}$$

c. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \left(\frac{\pi}{e}\right)^n$

- ☐ absolutely convergent
- ☐ conditionally convergent
- ☐ divergent

Hint: $\frac{\pi}{e} \approx \frac{3.14159}{2.71828} \approx 1.16$.