Review Final **Name**

Math 142 **Section** **Id**

Use exactly one page for each of the numbered questions (use the back of the page if necessary).
Put your name and the question number on each page.
Put a box around the final answer to a question.
You must *show* your work in order to get possible credits.

1. Find the limit of the sequence

a) $\lim_{n\to\infty} \frac{\cos n}{\ln n}$

b) $\lim_{n\to\infty} \frac{n^5-4n^3+7}{5n^4+n^2+100}$

c) $\lim_{n\to\infty}(1-\frac{2}{3n})^n$

d) $\lim_{n\to-\infty} \frac{e^n-1}{e^n+1}$

2. Determine whether the limit of the following function/sequence exists, if so, find the limit:

a) $\lim_{x\to 0+} \frac{\sin^{-1}\sqrt{x}}{\sqrt{x}}$

b) $\lim_{x\to+\infty}(x\ln x)^2 e^{-x}$

c) $\lim_{x\to\infty} x\tan(\pi/x)$

d) $\lim_{n\to\infty} \frac{n!}{n^n}$

e) $\lim_{n\to\infty} 2^{-n}$

f) $\lim_{n\to\infty}(-2)^n$

Fill in the blanks or parenthesis in Problems **3** to **8**.

3. Let $a > 0$ be a constant. (a) $\int \frac{dx}{a^2+x^2} =$ ________ $+C$

(b) $\int \frac{dx}{a^2-x^2} =$ ________ $+C$

c) $\int \frac{dx}{\sqrt{a^2+x^2}} =$ ________ $+C$

d) $\int \frac{dx}{\sqrt{a^2-x^2}} =$ ________ $+C$

4 (a) If $a > 0$ but $a \neq 1$, then $D_x\,(a^x) =$ ________________________

Hint: $a^x = e^{\ln(a^x)} = e^{x\ln(a)}$.

(b) $D_x\,(x^x) =$ ________________________

5. Trig substitution: (recall that the *integrand* is the function you are integrating)

a) if the integrand involves $a^2 - u^2$, then one makes the substitution $u =$ ________________________

b) if the integrand involves $a^2 + u^2$, then one makes the substitution $u =$ ________________________

6. Partial Fraction Decomposition. If one wants to integrate $\frac{f(x)}{g(x)}$ where f and g are polyonomials and [degree of f] $\geq$ [degree of g], then one must first do ____________________________

7 (a) A series $\sum a_n$ is said to *converge absolutely* if $\sum |a_n|$ _______________

Give an example of an absolutely convergent series: _______________

(b) A series $\sum a_n$ is said to *converge conditionally* if $\sum a_n$ is _______________ but $\sum |a_n|$ _______________. Give an example of a conditionally convergent series: _______________

8 (a) Consider the interval $I = (a - R, a + R)$ center about $x = a$ and of radius R.

Let $y = f(x)$ be a function that can be differentiated N times $x = a$. Then the N^{th}-order Taylor polynomial $y = P_N(x)$ of f about a is (your answer should have a summation sign $\sum$ in it)

$P_N(x) \ =$

(b) Consider the interval $I = (a - R, a + R)$ center about $x = a$ and of radius R.
Let $y = f(x)$ be a function that can be differentieated $N + 1$ times for each $x \in I$.
Consider the the N^{th}-order Taylor Reminder term $R_N(x)$, where $f(x) = P_N(x) + R_N(x)$.
Then an upper bound for $|R_N(x)|$ for an $x \in I$ is:

$|R_N(x)| \leq$

9. Use chain rule or logarithm derivative method to find the derivative. a) $D_x \ \ (\cos(\ln x)) \ =$

b) $D_x \ (7^{(x-2)^2}) \ =$

10. Evaluate the integrals. a) $\displaystyle\int (\tan x) \ \left(\sec^7 x\right) \ dx \ =$

b) $\displaystyle\int x^2 \arctan x \, dx \ =$

c) $\displaystyle\int \frac{x^2}{\sqrt{4 - x^2}} \ dx \ =$

d) $\int x^{\frac{1}{2}} \ln x \, dx$

e) $\int_0^1 \frac{u^3}{(u+1)^2} du$

f) $\int \frac{1-x}{1+x+x^2}\, dx$

g) $\int x(\sqrt{x}+1)^{\frac{1}{3}} dx$

11. Let R be the region enclosed by $y = x^2, \quad x = 2 \quad \text{and} \quad y = 0$. Let V be the volume of the solid obtained by revolving the region R about the line $x = 3$.

(a) Make a rough sketch below of the region R, labeling the important points.

(b) Using the disk/washer method, express the volume V as an integral (or maybe 2 integrals). You do NOT have to evaluate the integral(s).

V =

12. Determine whether the improper integral converges.

$$(a) \quad \int_0^1 \ln(1-x)\, dx$$

$$(b) \quad \int_0^{\pi/2} \frac{\cos x}{\sqrt{x}}\, dx$$

$$(c) \int_0^{\infty} \frac{dx}{5000+x}$$

13. Does the following series converge or diverge? Explain your answer by stating which kind of test you are using and how it works.

$$a. \quad \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+\sqrt{n+1}}$$

$$b^*(optional) \quad \sum_{n=0}^{\infty}(-1)^n \frac{1}{(n+1)^{0.6}}$$

$$c) \quad \sum_{n=2}^{\infty} n e^{-\sqrt{n}}$$

14. Determine the radius and interval of convergence of the power series

$$a) \quad \sum_{n=1}^{\infty} \frac{x^n}{n}$$

$$b) \quad \sum_{n=2}^{\infty} (\ln n) x^{2n+1}$$

15. Let $a_n = \frac{e^n \cdot n!}{(2n)!}$ Find $\frac{a_{n+1}}{a_n}$. Simplify your answer so that no factorial sign (i.e., !) appears.

answer: $\frac{a_{n+1}}{a_n} =$

$$\sum_{n=1}^{\infty} (-1)^n \frac{e^n\,(n!)}{(2n)!}$$

☐ absolutely convergent

☐ conditionally convergent

☐ divergent

16. Consider the formal power series

$$\sum_{n=1}^{\infty} \frac{(2-x)^{n-1}}{5^n} .$$

In the box below draw a diagram indicating for which x's this series is: absolutely convergent, conditionally convergent, and divergent. Of course, indicate your reasoning.

17. Let

$$f(x) = (1+x)^{3/2}$$

Find the 3^{rd}-order Taylor polynomial of $y = f(x)$ about $x = 0$.

$P_3(x) \ =$

18. Find the Taylor or Maclaurin series of $y = f(x)$

(a)

$$f(x) = e^{2x+1}$$

about $x = 1$

(b)

$$f(x) = \frac{1}{1+x^2}$$

about $x = 0$.

19. The equation of the ellips is given by

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

Express the length of the ellips as an definite integral. Do not evaluate the integral.