Instructions. Answer each of the questions on your own paper. Put your name on each page of your paper. Be sure to show your work so that partial credit can be adequately assessed. *Credit will not be given for answers (even correct ones) without supporting work.* A copy of the table of Laplace transforms and convolution products from the text will be supplied, and these tables can be used for all problems.

1. [10 Points]

   (a) Complete the following definition: Suppose f and g are continuous functions defined for all $t \geq 0$. The **convolution** of f and g is the function $f * g$ defined as follows:
   $$f * g(t) = \boxed{}$$

   (b) Compute the convolution of t^2 and t^2 (you can use either the formula in part (a) or the table).

2. [30 Points] Compute the Laplace transform of each of the following functions.

   (a) $f_1(t) = te^t$

   (b) $f_2(t) = te^{-t}\sin 3t$

   (c) The solution $y(t)$ of the initial value problem
   $$2y'' - 3y' + 2y = te^t, \qquad y(0) = -1,\ y'(0) = 2.$$
   Note that you are only asked to find the Laplace transform $Y(s)$ of $y(t)$, not $y(t)$ itself.

3. [30 Points] Compute the inverse Laplace transform of each of the following functions.

   (a) $F(s) = \dfrac{2s^2 - 3s + 1}{s^3}$

   (b) $G(s) = \dfrac{s^2 + 4}{(s+1)(s^2 - 4)}$

   (c) $H(s) = \dfrac{10}{s^2 + 2s + 5}$

4. [20 Points] Solve the following initial value problem using Laplace transforms:
   $$y'' - 3y' + 2y = 6, \quad y(0) = 1,\ y'(0) = 0.$$

5. [10 Points]

   (a) Verify that $y_p(t) = 2t - 3$ is a particular solution to the nonhomogeneous equation
   $$y'' + 3y' + 2y = 4t. \qquad (*)$$

(b) Verify that e^{-t} and e^{-2t} are two solutions to the associated homogeneous equation $y'' + 3y' + 2y = 0$.

(c) What is the general solution of the equation $(*)$?

(d) Find the solution of the initial value problem

$$y'' + 3y' + 2y = 4t, \quad y(0) = y'(0) = 0.$$