Instructor: Dr. Shijun Zheng

Review Test 2	**Name**
Math 2243	**Id**

Read each question carefully. Avoid simple mistakes. Put a box around the final answer to a question. (Use the back of the page if necessary).

You must show your work to support your answer.

1. Find the differential of $u = \sqrt{x^2 + y^2 + z^2}$.

2. Use the Chain rule (plot a tree diagram) to find the indicated partial derivatives $u = x^2 + \sqrt{y^2 + z^2}$, $x = \sin(r)\cos(s)$, $y = \sin(r)\sin(s)$, $z = 3$. Find $\frac{\partial u}{\partial r}$ and $\frac{\partial u}{\partial s}$. Your final answer should be given in terms of r and s only.

3. Find the absolute maximum and absolute minimum values of $f(x, y) = xy - x + y$ on the closed set D which is bounded by the parabola $y = x^2$ and the line $y = 4$.

4. Find the directional derivative of the function at the given point in the direction of $\mathbf{v}$:
$$f(x, y) = x - 2x\sqrt{y}, \quad (2, 9), \qquad \mathbf{v} = \langle 1, -1 \rangle.$$

5.
   - Let $f(x, y) = 5xy^2/(x^2 + y^2)$.
     a) Find an equation for the tangent plane to the graph $z = f(x, y)$ at the point $(1, 2, 4)$.
     b) In which direction is f increasing most rapidly at the point $(1, 2)$?
   - Find the tangent plane to the surface $z = y \ln x$ at $(1, 4, 0)$.

6. a) Evaluate the following limit
$$\lim_{x \to \infty} \left(\frac{x}{\sqrt{x^2 + 1}}\mathbf{i} + \frac{\sin x}{x}\mathbf{j} - \tan^{-1}(x)\mathbf{k} \right)$$
   b) Find $\mathbf{r}(t)$ if $\mathbf{r}'(t) = \mathbf{i} + \sin t\mathbf{j} - \sqrt{t}\mathbf{k}$ and $\mathbf{r}(1) = \mathbf{i} - \mathbf{j}$.

7. Find and sketch the domain of the function
$$f(x, y) = \frac{\sqrt{y - x^2}}{1 - x^2}$$

8. Use the definition of continuity to explain whether or not the function $f(x, y)$ is continuous at $(0, 0)$
$$f(x, y) = \begin{cases} \frac{3xy^2}{x^2 + y^4} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

9.
   - Use the limit definition to find the partial derivative $\frac{\partial f(x,y)}{\partial y}$ where $f(x,y) = xy^2$.
   - Find the indicated partial derivatives: f_{xyy}, where

$$f(x,y) = ye^{\frac{x}{y}}$$

10. Sketch the region of integration, and evaluate the integral:

   - [Ex 15.1# 18]

$$\int_0^1 \int_0^{\sqrt{1-s^2}} 8tdtds$$

   - [Ex. 15.1 # 5]

$$\int_0^{\pi} \int_0^{x} x\sin(y)dydx$$

   - $\iint_R x\sin(xy)dA$ where $R = \left\{(x,y)|0 \le x \le 1,\ \ 0 \le y \le \frac{\pi}{4}\right\}$

11. Evaluate the integral $\iint_D y^2 dA$, where D is the region bounded by the upper half of the circles $x^2 + y^2 = 1$, $x^2 + y^2 = 4$, $y = x$, and $y = -x$.

12. (optional*) Given a rectangular coordinates $(-1, -2, 3)$, convert into cylindrical coordinates and spherical coordinates respectively.

13. Sketch the region of integration, reverse the order of integration and evaluate the integral:

   - [Ex 15.1# 31]

$$\int_0^{\pi} \int_x^{\pi} \frac{\sin(y)}{y} dydx$$

   - [Ex. 15.1 #32]

$$\int_0^2 \int_x^2 2y^2 \sin(xy)dydx$$

14. [Ex 15.2 #5] Sketch the region bounded by the given lines and curves. Then find the area using double integral:
    The curve $y = e^x$ and the lines $y = 0, x = 0$, and $x = \ln 2$.