Example

Use $\Delta y \approx \overset{dy}{f'(x)\Delta x}$ to estimate $\sqrt{5}$ and $\sqrt{11}$.

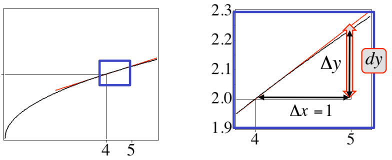
$f(x)=\sqrt{x}$

$f'(x)=\dfrac{1}{2\sqrt{x}}$

a) $\sqrt{5}=\sqrt{\underset{x}{4}+\underset{\Delta x}{1}}= 2+\Delta y \approx 2+dy = 2.25$ (***over***-estimation)

$dy = f'(4)\cdot 1 = 0.25$

$\dfrac{1}{2\sqrt{4}}$



$\dfrac{1}{2\sqrt{9}}$

$dy = f'(9)\cdot 2 = 0.\overline{3}$

b) $\sqrt{11}=\sqrt{\underset{x}{9}+\underset{\Delta x}{2}}= 3+\Delta y \approx 3+dy = 3.\overline{3}$ (***over***-estimation)

Note If $y=f(x)$ then $dy=f'(x)\overset{dx}{\Delta x}$. So if $y=x$ then $dy=dx$ but also

$dy=f'(x)dx$ $\quad dy=1\cdot\Delta x=\Delta x$ therefore $dx=\Delta x$.

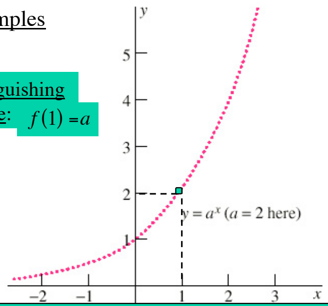
e.g. $y=\sqrt{x} \Rightarrow dy=\dfrac{1}{2\sqrt{x}}dx$

Exponential Functions (Review)

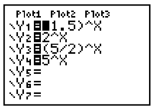
Definition
Suppose $a>0$ and $a\neq 1$. Then the function $f(x)=a^x$ is called the ***exponential function*** with base a.

Examples

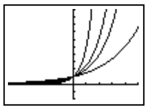
Distinguishing Feature: $f(1)=a$



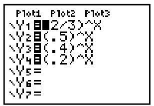
$a>1$ (increasing)



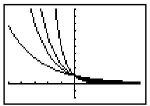
```
 Plot1 Plot2 Plot3
\Y1■█1.5)^X
\Y2■2^X
\Y3■(5/2)^X
\Y4■5^X
\Y5=
\Y6=
\Y7=
```



$0<a<1$ (decreasing)



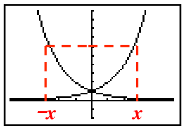
```
 Plot1 Plot2 Plot3
\Y1■█2/3)^X
\Y2■(.5)^X
\Y3■(.4)^X
\Y4■(.2)^X
\Y5=
\Y6=
\Y7=
```



Common features:
domain = $(-\infty,\infty)$ $\quad$ y-intercept $=1$ $\quad (a^0=1)$
range = $(0,\infty)$ $\quad$ horizontal asymptote : $y=0$

Compare the graphs of

$f(x)=2^x$ and $g(x)=\left(\frac{1}{2}\right)^x$



Symmetric about y-axis $\Rightarrow f(x)=g(-x)$

Reason: $g(-x)=\left(\frac{1}{2}\right)^{-x}=\left(\left(\frac{1}{2}\right)^{-1}\right)^x=(2)^x=f(x)$

In general
The graphs of $y=a^x$ and $y=\left(\frac{1}{a}\right)^x$ can be obtained from one another by the reflection in the y-axis.

Exponential Equations (Type 1)

Basic Principle:
$a^{x_1}=a^{x_2} \Rightarrow x_1=x_2 \quad (f(x)=a^x \text{ is one-to-one})$

Example
Solve the equation:

a) $2^x=8=2^{\boxed{3}}$ $\quad \boxed{x=3}$

b) $2^{2x+1}=32=2^{\boxed{5}}$ $\quad 2x+1=5$ $\quad \boxed{x=2}$

c) $\left(\frac{1}{9}\right)^x=27$

Hint : $\frac{1}{9}=a^{\boxed{-2}}$ and $27=a^{\boxed{3}}$ for some a. $(a=3)$

$\left(3^{-2}\right)^x=3^3 \quad 3^{-2x}=3^3 \quad -2x=3 \quad \boxed{x=-\frac{3}{2}}$

Logarithmic Functions (Review)

Definition
Suppose $a>0$ and $a\neq 1$. Then $y=\log_a x$ means that $a^y=x$.

E.g. $y=\log_2 8=3$ $\quad (2^y=8)$

$y=\log_3 81=4$ $\quad (3^y=81)$

$y=\log_{10}(.01)=-2$ $\quad (10^y=.01)$