

MATH 141, Sample Test # 2

Show your work

1 [20P]) Find the derivative of the following functions:

a) $f(x) = \cos(x) + \frac{2}{x}$ $f'(x) =$

b) $f(x) = x^{\tan x}$ $f'(x) =$

c) $y = [1 + \ln(x)]^3$. $dy/dx =$

d) $y = e^x(x^2 + 2)$ $dy/dx =$

2 [10P]) Find the slope of the tangent line to the graph of the equation $xy^3 + x^2y = 10$ at the point $(1, 2)$. The slope is:

3 [5P]) Explain if the equation $x^3 + x + 1$ has a solution in the interval $[-1, 0]$ or not.

4 [10P]) Let $f(x) = x^4 - 2x^2 + 1$.

a) Find the intervals on the x -axis on which the function is increasing as well as those on which it is decreasing.

b) Determine the concavity of $y = f(x)$ on (possibly) different intervals.

c*) Find the absolute maximum and absolute minimum on the interval $[-2, 2]$.

5 [10P]) Calculate the first two derivatives of the function $f(x) = x \ln x$.

$$f'(x) =$$

$$f''(x) =$$

6* [20P]) Sketch the graph of the function $f(x) = \frac{x^2}{x+1}$. Identify and label all extrema ([4P]), inflection points ([4P]), intercepts ([4P]), and asymptotes ([4P]). Indicate the concave structure clearly ([4P]).

7 [15P]) Find the limits. Use the l'Hospital's Rule where applicable. If l'Hospital's Rule doesn't apply, explain why.

a) $\lim_{x \rightarrow -2} \frac{x+2}{x^2+3x+2}$

b) $\lim_{x \rightarrow (\pi/2)^+} \frac{1 - \sin x}{\cos x}$

c) $\lim_{x \rightarrow \infty} \frac{e^x}{x^3}$

d) $\lim_{x \rightarrow \infty} x \tan(1/x)$

e) $\lim_{x \rightarrow \infty} (x - \ln x)$

f) $\lim_{x \rightarrow 0} (\cos x)^{1/x^2}$

8 [10P] A boat leaves a dock at 2:00 p.m. and travels due south at a speed of 20 km/h. Another boat has been heading due east at 15 km/h and reaches the same dock at 3:00 p.m. At what time were the two boats closest together?