Advanced Partial Differential Equations Syllabus (Math 7332 A)

Instructor: Dr. Shijun Zheng
Lecture Hours & Location: TR 9:30–10:45 MP 3031
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Text: *Partial Differential Equations, An Introduction*, by Walter Strauss, 2nd Ed.
Main Reference: *Applied Partial Differential Equations*, with Fourier Series and Boundary Value Problems, Fourth Edition, by R. Haberman

Other references: *Introduction to Fourier Analysis and Differential Equatoins*, Lecture Notes by the Instructor
Partial Differential Equations, by Lawrence C. Evans
Introduction to Partial Differential Equations, Second Edition, Gerald Folland

Course Description: This course will study some fundamental theory of linear PDEs. Topics include: Classical theory of elliptic, parabolic and hyperbolic PDEs and their solutions, maximum principle, existence of weak solutions, regularity of solutions, Duhamel's principle and Cauchy's problem, which arise in particular in physics and engineering sciences. Methods include separation of variables, eigenfunction expansions, Fourier transforms. Time permitting I will give introductory examples of nonlinear ODEs and PDEs, and numerical methods. Graduate students will be given some extra assignments which are not required for undergraduate students.

We will mainly use [Strauss], together with part of [Haberman] and the Instructor's Lecture Notes.

Some of the topics include in-depth analysis and case study that admit both theoretical and numerical components.

Prerequisites: Grade of C or better in M2243 (Calculus III), M3230 (ODE), Math 5339 (PDE) or by the Instructor's consent.

Grading: Your course grade will be based on the percentage of total points you have earned divided by the number of total points available. An approximation of the number of points available is: 60 points for homework/projects and class participation, two tests worth 100 points each, and a final exam worth 90 points. The approximate total points available is then 350.

The grading scale is: A 85% or above; B 75–84 %; C 65-74%; D 55-64%; and F 54% or below.

Homework: *Working on homework problems or projects is strongly recommended.* The written work you submit for grading should be neat so that it is easy-to-read by anyone. We will discuss the tentative course structure during the course.

Group Discussions and Term Project. During the course of the semester, several sets of problems will be given, either in-class or take-home. Students will form several groups to discuss and work on the assigned problems for better understanding the materials.

Course Materials: [**Strauss**]: PDEs from Physics (Chapter 1)
Fourier series (Chapter 5)
Laplace equation (Chapter 6)
*Greens functions (Chapter 7)
and, Selections from
**Chapters 8 (Computation of Solutions),
Chapter 9 (Waves in Space) —9.4 (The diffusion and Schrödinger equations)
**Chapter 10 (Boundaries in the plane and in space)
Chapter 12 (Distributions and Transforms),
Chapter 13 (PDE Problems from Physics) 13.1. Electromagnetism, 13.3 Scattering.
Chapter 14 (Nonlinear PDEs) **14.2 Solitons. **14.5 Water waves

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[**Haberman**]: Separation of variables: Heat equation 2.1–2.4
Laplace equation Fourier series 2.5.1–2.5.2, 3.2–3.6, 8.2–8.3
Wave equation 4.1–4.5
**Maximum principle, Uniqueness
Sturm-Liouville problem, Self-adjoint operator 5.2–5.5
Approximation properties: Large eigenvalues 5.9–5.10
PDE in higher dimensions. **Bessel functions 7.5–7.8.2
Laplace equation, **Legendre polynomials 7.8.4–7.10

Some tentative topics: **[Strauss/Lecture Notes]** Introduction, classification of PDEs (Strauss 1.1; the List in Lecture Notes),
First order PDEs: characteristics (Strauss 1.2, lecture notes),
Classification of second order equations; wave equation on R (lecture notes, Strauss 1.5, 2.1)
Energy conservation for the wave equation; the maximum principle for Laplace equation (Strauss 2.2-2.3)
*Inhomogeneous PDE: Duhamels principle (lecture notes, Strauss 3.3-3.4)
Separation of variables, eigenvalue problems (lecture notes, Strauss 4.1-4.3)
Inner product spaces and symmetric boundary conditions; Fourier series (Lecture notes, Strauss 5.1-5.3)
Fourier series (Strauss 5.1-5.5)
*Convergence of Fourier series (Strauss 5.4-5.5)
General Fourier Series (10.1, 10.2, 10.5, lecture notes)
The Fourier transform and solutions of PDEs (lecture notes, Strauss 12.3-12.4)
Convolutions, solutions of PDEs by Fourier transform (lecture notes, Strauss 12.3-12.4)
*The maximum principle and energy decay for the heat equation; energy estimates for Laplaces equation; the Fourier transform (Strauss 2.3, 12.3, lecture notes)
**Numerical Methods 8.1, 8.2, 8.3

**Calculus of Variations 14.3
*Greens Functions 7.1-7.4
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[**Haberman**] with Worked Examples
Chapter 2. Method of Separation of Variables.
Chapter 3. Fourier Series (Solve PDE on an interval)
Chapter 4. Wave Equations: Vibrating Strings and Membranes
**Chapter 5. Sturm-Liouville Eigenvalue Problems; Self-adjoint Operators
**Chapter 6. Finite Difference. Numerical Methods for PDEs; Finite Element Method (FEM)
Chapter 7. PDE in Higher Dimensions
Chapter 8. Inhomogeneous Problems.
*Chapter 9. Green's Functions for Time-Independent Problems.
Chapter 10. Infinite Domain Problems: Fourier Transform Solutions of PDEs
Chapter 11. Green's Functions for Wave and Heat Equations.
**Chapter 12. The Method of Characteristics for Linear and Quasi-linear Wave Equations
**Chapter 14. Dispersive Waves: Slow Variations, Stability, Nonlinearity, and Perturbation Methods.

Study Philosophy: In addition to solving problems and reviewing class notes, you should also seriously and carefully learn and relearn, read (and reread) and study (and restudy) the text on a regular basis as the Text is especially well-written for a course like this.

Attendance Policy: Students are expected to attend each class meeting and pay attention. A student who misses class is responsible to find out what was discussed and learn the material that was covered on the missed day.

Make-up Policy: Late homework or project will not be accepted unless a reasonable excuse is given. No make-up exams will be given. When a student misses an exam the score from the final exam will be substituted for the missing exam score.

Academic honesty: Unless otherwise indicated, it is taken for granted that all work is your own. http://students.georgiasouthern.edu/sta/guide/

Civility Statement: See the Student Conduct Code at
http://students.georgiasouthern.edu/judicial/SCC08-09.pdf

Final Exam: Tuesday May 10, 10:00-12:00 pm.
http://students.georgiasouthern.edu/registrar/finalSchedule.htm

Homework [Strauss]

5.1 2*, 3, 4, 5, 7*, 8.9 10, 11
5.2 2, 5, 6, 9, 11, 12,14
5.3 2,3,5,8,9,10,12*,13,14* 15
5.4 3,4,6,8,11,14,15,16*,18
5.5 Gibbs phenomenon 1,4,13,14,15

5.6 2, 4, 6, 7,13
6.1 3, 5,7,9,10,12,13
6.2 1, 3,5,6,7
6.3 1,2,3
6.4* 1,4,5,6,10,11,12*,14
7.1 2,3,5,8*,10*
7.2 1,2,3
7.3 2,3
7.4 1,2,3,5,7,8,11,14,17,19,20,22,23,25*
8.1 1,2,3,4,10,11,12,15*
8.2 1,2, 3,4,8,11
8.4* 2,3,4,10
8.5* 1,2,3,4,5
9.1 3,5,6
9.2* 3,6,12,16,17,18,19*,20
9.3* 1,3,4,6,7*
9.4* Schrdinger 3,6,7
9.5* Hydrogen 1,2
10.1 1,2,3,6
10.2 Bessel 2,3,5,7
10.3 Legendre 2,4,5,6,10,12
10.4 1, 2,3,4*
10.5* Bessel 3,4,8,12,13,14,18,19
10.6* Legendre 2,3,4,6,8
10.7* Angular 1,3,5
11.1 1,3,4*,5
11.2 1,2,6,7,8,9
11.3 1,2,3
11.4* Sturm-Liouville 1,2,5,6
11.5* 1,2,3
12.1 Distributions and Transforms 1,2,4,9,12
12.2 1,3,5,8,13
12.3 2,3,5,6,9
12.4 1,2,3,5
12.5 3,4,6,7
13.1 PDE from Physics 3,4,8
13.2 1,2
13.3 3,4,6
13.4 1,3,4,7,11
14.1 Nonlinear PDEs 2 5 7 11
14.2 1, 3, 5 *, 9, 10, 13
14.3 1, 3, 5, 9, 11
14.4 2, 4
14.5 2, 3, 7