The syllabus for Exam III is Sections 3.2, 3.3, 3.6 and 4.1-4.4

1. Solve each of the following homogeneous linear differential equations.

   (a) $y'' + 3y' + 2y = 0$

   (b) $y'' + 6y' + 13y = 0$

   (c) $8y'' + 4y' + y = 0$

   (d) $2y'' - 7y' + 5y = 0$

   (e) $y'' + .2y' + .01y = 0$

   (f) $y'' + 2y' + 2y = 0$

   (g) $y''' + 2y'' - 8y' = 0$

   (h) $y''' - 2y'' - 3y' = 0$

   (i) $y^{(4)} - 5y'' + 4y = 0$

   (j) $t^2y'' - 7ty' + 15y = 0$

   (k) $t^2y'' - 12y = 0$

2. Find the general solution of the constant coefficient homogeneous linear differential equation with the given characteristic polynomial $p(s)$.

   (a) $p(s) = (s-1)(s+3)(s-5)$

   (b) $p(s) = s^3 - 1$

   (c) $p(s) = s^3 - 3s^2 + s + 5$

   (d) $p(s) = (s^2+1)^3$

   (e) $p(s)$ has degree 4 and has roots $\sqrt{2}$ with multiplicity 2 and $2 \pm 3i$ with multiplicity 1.

   (f) $p(s)$ has degree 5 and roots 0 with multiplicity 3 and $1 \pm \sqrt{3}$ with multiplicity 1.

3. Solve each of the following initial value problems. You may (and should) use the work already done in exercise 1.

   (a) $y'' + 3y' + 2y = 0, \quad y(0) = 1, \quad y'(0) = -3.$

   (b) $y'' + 6y' + 13y = 0, \quad y(0) = 0, \quad y'(0) = -1.$

   (c) $y'' + 2y' + 2y = 0, \quad y(0) = 0, \quad y'(0) = 2$

   (d) $4t^2y'' - 7ty' + 6y = 0, \quad y(1) = 1, \quad y'(1) = 2$

4. Find a second order linear homogeneous differential equation with constant real coefficients that has the given function as a solution, or explain why there is no such equation.

   (a) $e^{-3t} + 2e^{-t}$

(b) $e^{-t}\cos 2t$

(c) $e^{t}t^{-2}$

5. Find the general solution to each of the following differential equations.

(a) $y'' - 2y' + y = t^2 - 1$

(b) $y'' - 2y' + y = 4\cos t$

(c) $y'' - 2y' + y = te^t$

6. Find a particular solution $y_p(t)$ of each of the following differential equations by using the method of variation of parameters. In each case, $\mathcal{S}$ denotes a fundamental set of solutions of the associated homogeneous equation.

(a) $y'' + 2y' + y = t^{-1}e^{-t}; \quad \mathcal{S} = \{e^{-t},\, te^{-t}\}$

(b) $y'' + 9y = 9\sec 3t; \quad \mathcal{S} = \{\cos 3t,\, \sin 3t\}$

(c) $t^2y'' + 2ty' - 6y = t^2; \quad \mathcal{S} = \{t^2,\, t^{-3}\}$

(d) $y'' + 4y' + 4y = t^{1/2}e^{-2t}; \quad \mathcal{S} = \{e^{-2t},\, te^{-2t}\}$

7. Find the Laplace transform of each of the following functions:

(a) $f(t) = t^2\chi_{[1,\,3)}$

(b) $g(t) = \begin{cases} 0 & \text{if } 0 \le t < 5, \\ t^2 - 5 & \text{if } t \ge 5. \end{cases}$

8. Find the inverse Laplace transform of each of the following functions

(a) $F(s) = \dfrac{se^{-2s}}{s^2 - 9}$

(b) $G(s) = \dfrac{e^{-s} - e^{-2s}}{s^4}$

9. Solve the following initial value problem:

(a)

$$y'' + 4y' + 5y = \begin{cases} 1 & \text{if } 0 \le t < \pi \\ 0 & \text{if } t \ge \pi \end{cases} \qquad y(0) = 0,\ y'(0) = 0.$$

(b)

$$y'' + 9y = h(t - 2\pi)\sin t, \quad y(0) = 1,\ y'(0) = 0.$$