Instructions. Answer each of the questions on your own paper. Be sure to show your work so that partial credit can be adequately assessed. Credit may not be given for answers (even correct ones) without supporting work. Put your name on each page of your paper.

1. [20 Points] Answer the following questions about the differential equation

$$(*) \qquad y' = -ty^2.$$

   (a) Find the general solution of Equation $(*)$.

   (b) Find all the constant solutions of $(*)$, if any.

   (c) Find the solution of $(*)$ with initial condition $y(0) = 4$.

   (d) Draw (and clearly label) the solution found in part (c) on the attached direction field.

   (e) Draw on the direction field (and clearly label) the solution of Equation $(*)$ with initial value $y(0) = -0.5$. Do *not* try to solve it.

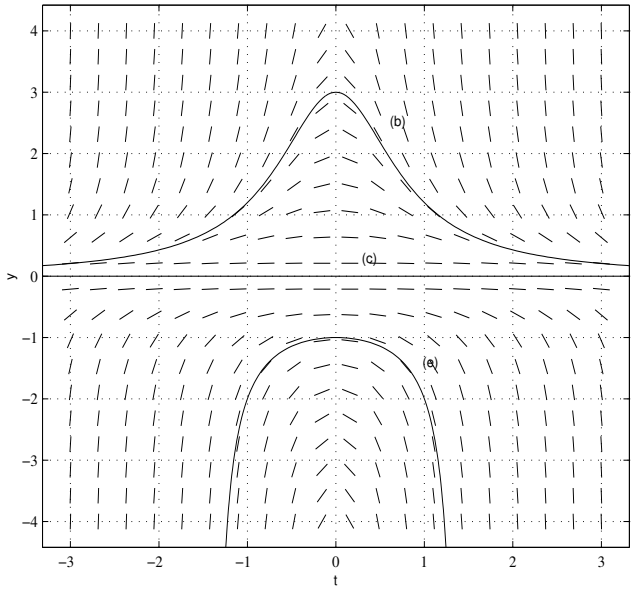

Figure 1: Direction Field for $y' = -ty^2$

2. [20 Points] Solve the following initial value problem. Be sure to show all of your work.

$$y' + 2y = te^{-5t} + e^t, \quad y(0) = -2$$

3. [20 Points]

   (a) Complete the following definition: Suppose $f(t)$ is a continuous function defined for all $t \geq 0$. The **Laplace transform** of f is the function $F(s)$ defined as follows:

   $$F(s) = \mathcal{L}\{f(t)\}(s) = \boxed{}$$

   for all s sufficiently large.

   (b) Using your definition compute the Laplace transform of $f(t) = e^{-3t} + 2$.

4. [20 Points] Compute the Laplace transform of each of the following functions.

   (a) $f_1(t) = 10t^3e^t$

   (b) $f_2(t) = e^{5t}f(t)$ where $f(t)$ is the function with Laplace transform

   $$F(s) = \frac{s^3}{s^4 - s + 2}.$$

5. [20 Points] Apply Picard's method to compute the first two approximations $y_1(t)$ and $y_2(t)$ to the solution of the initial value problem

   $$y' = t^2 + y^2, \quad y(0) = 0.$$