M 1232 **Dr. J.S. Zheng**
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Chapter 1. Differentiation.
1.3* Average Rates of Change
1.4 Differentiation Using Limits and Difference Quotients

- *Algebraic Limits*

(46) Find the limits.
(a) $\lim_{x\to -1}(2x^2-4x+4)$
(b) $\lim_{x\to -3}\dfrac{x^2+2x+2}{x-x^3}$
(c) $\lim_{x\to -3}\sqrt{\dfrac{x^2+2x+2}{x-x^3}}$

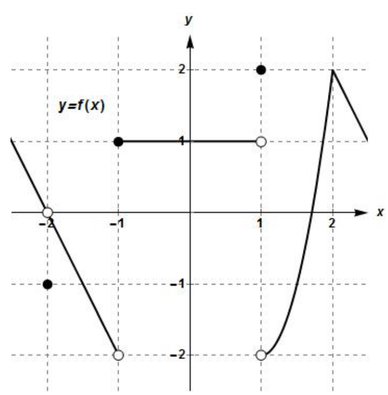
(47) Find the limits.
(a) $\lim_{x\to -1}\dfrac{x^2-x-2}{x^2-1}$
(b) $\lim_{x\to -3}\dfrac{x+3}{x^2+4x+3}$
(c) $\lim_{x\to 4}\dfrac{x^2-3x-4}{x^2-5x+4}$

- *Continuity*

(48) **Definition.** A function f is **continuous** at $x=a$ if: (a) $f(a)$ exists, (b) $\lim_{x\to a}f(x)$ exists, and (c) $\lim_{x\to a}f(x)=f(a)$.

A function is **continuous over an interval *I*** if it is continuous at each point a in I. If f is not continuous at $x=a$, we say that f is **discontinuous**, or has a **discontinuity**, at $x=a$.

(49) The graph of the function $y=f(x)$ is given as follows. At what points of x is f **not** continuous?



(50) Determine whether the function is continuous at the given point.
(a) $f(x)=x^3+2x, \quad$ at $x=2$

(b) $f(x) = \begin{cases} 2x - 3, & \text{for } x \geq 3, \\ 12 - x^2, & \text{for } x < 3, \end{cases}$ at $x = 3$

(c) $f(x) = \begin{cases} \dfrac{x^2 - 1}{x - 1}, & \text{for } x \neq 1, \\ 2, & \text{for } x = 1, \end{cases}$ at $x = 1$

- *Difference Quotient and Derivatives*

(51) The following table shows total production of suits at a company during one morning work. What was the average number of suits produced per hour from 9 am to 11 am?

Time (number of hours since 8 am)	0	1	2	3	4
Total number of suits produced	0	20	55	64	100

(52) **Definition.** The average rate of change of $f(x)$ with respect to x is also called the **difference quotient**. It is given by

$$\frac{\Delta y}{\Delta x} = \frac{\Delta f(x)}{\Delta x} := \frac{f(x+h) - f(x)}{h}, \qquad \text{where } h \neq 0.$$

(53) For $f(x) = x^2$, find the difference quotient for $x = 1$ and

(a) $h = 1$, $h = 0.5$, $h = 0.2$, and $h = 0.1$ (h approaches 0 from the right)

(b) $h = -1$, $h = -0.5$, $h = -0.2$, and $h = -0.1$ (h approaches 0 from the left)

(54) **Definition.** For a function $y = f(x)$, its **derivative** at x is the function f' (also written as $\dfrac{dy}{dx}$ or $\dfrac{df(x)}{dx}$) defined by

$$f'(x) = \frac{df(x)}{dx} := \lim_{h \to 0} \frac{f(x+h) - f(x)}{h},$$

provided the limit exists. If $f'(x)$ exists, then we say that f is **differentiable** at x. The process of finding a derivative is called **differentiation**.

Geometrically, the derivative of f at x is the slope of the tangent line at $(x, f(x))$. This limit is also called the **instantaneous rate of change** of f at x.

(55) For each function $f(x)$, first find $f'(x)$, then find $f'(-3)$ and $f'(2)$.

(a) $f(x) = 4x - 2$

(b) $f(x) = \dfrac{1}{x}$

(c) $f(x) = x^2$

Videos on Chapter 1:Differentiation from MLM Plus