

Class 12

Chapter 10 - Vector Algebra

This is question 18 from exercise 10.5

1. The value of $\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{i} \times \hat{k}) + \hat{k} \cdot (\hat{i} \times \hat{j})$ is

- a) 0 b) -1 c) 1 d) 3

Solution: The Directional vectors of x, y and z axes are given respectively

$$\vec{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \vec{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \vec{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (1)$$

$$\text{Here, } \vec{e}_i = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1(i^{\text{th}} \text{ term}) \\ 0 \\ 0 \\ 0 \dots \end{pmatrix}, \vec{e}_j = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1(j^{\text{th}} \text{ term}) \\ 0 \\ 0 \\ 0 \dots \end{pmatrix}, \vec{e}_k = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1(k^{\text{th}} \text{ term}) \\ 0 \\ 0 \\ 0 \dots \end{pmatrix}$$

$$\begin{array}{l|l} \vec{e}_i \times \vec{e}_j = \vec{e}_k & \vec{e}_j \times \vec{e}_i = -\vec{e}_k \\ \vec{e}_j \times \vec{e}_k = \vec{e}_i & \vec{e}_k \times \vec{e}_j = -\vec{e}_i \\ \vec{e}_k \times \vec{e}_i = \vec{e}_j & \vec{e}_i \times \vec{e}_k = -\vec{e}_j \end{array}$$

$$\begin{aligned} \vec{e}_i \cdot \vec{e}_i &= |\vec{e}_i| |\vec{e}_i| \cos 0 \\ &= 1 \times 1 \times 1 \\ &= 1 \\ \text{similarly, } \vec{e}_j \cdot \vec{e}_j &= \vec{e}_k \cdot \vec{e}_k = 1 \end{aligned}$$

Now,

$$\begin{aligned} \vec{e}_1 \cdot (\vec{e}_2 \times \vec{e}_3) + \vec{e}_2 \cdot (\vec{e}_1 \times \vec{e}_3) + \vec{e}_3 \cdot (\vec{e}_1 \times \vec{e}_2) \\ = 1 - 1 + 1 \\ = 1 \end{aligned}$$

So, option (c) is correct.

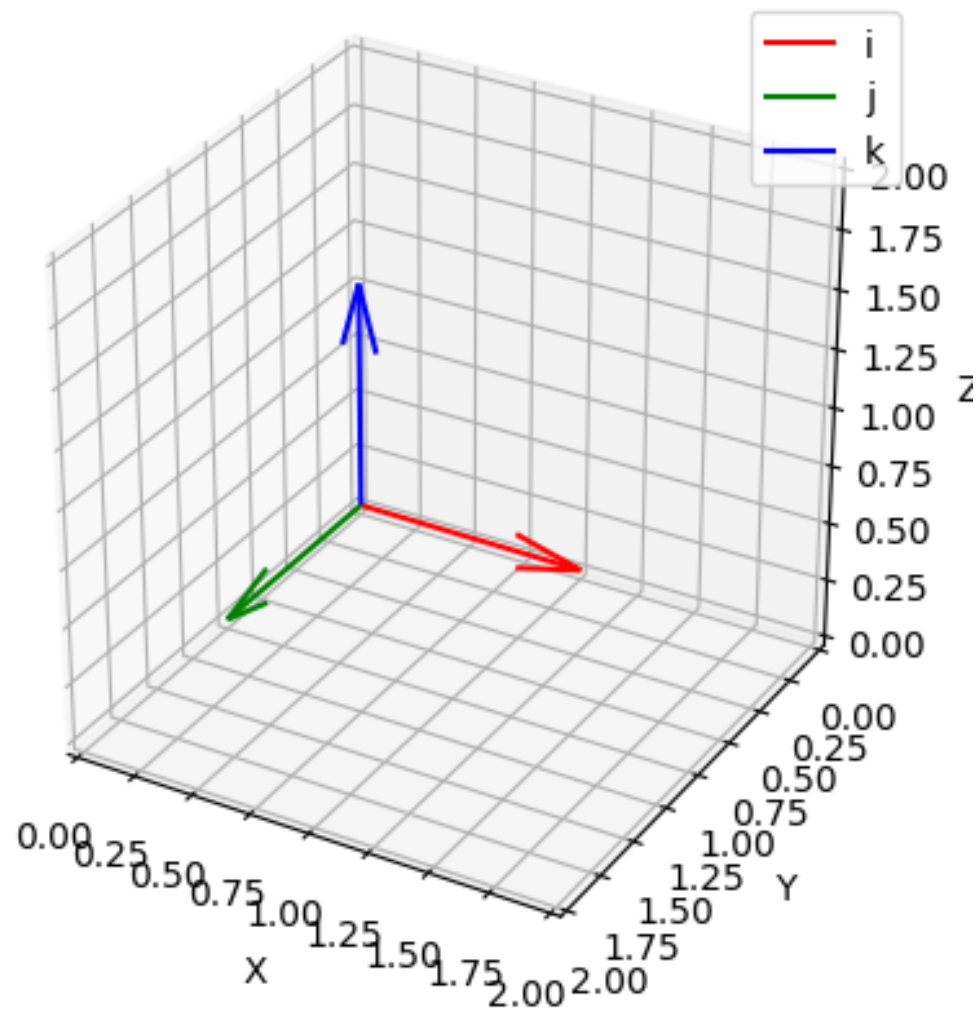


Figure 1: fig:1