Class 12

Chapter 10 - Vector Algebra

This is question 18 from exercise 10.5

1. The value of $\hat{i}\cdot(\hat{j}\times\hat{k})+\hat{j}\cdot(\hat{i}\times\hat{k})+\hat{k}\cdot(\hat{i}\times\hat{j})$ is

   a) 0 b) -1 c) 1 d) 3

Solution: The Directional vectors of x, y and z axes are given respectively

$$\mathbf{e_1}=\begin{pmatrix}1\\0\\0\end{pmatrix},\mathbf{e_2}=\begin{pmatrix}0\\1\\0\end{pmatrix},\mathbf{e_3}=\begin{pmatrix}0\\0\\1\end{pmatrix} \tag{1}$$

Now,

$$\begin{aligned}
&\begin{pmatrix}1\\0\\0\end{pmatrix}\cdot(\begin{pmatrix}0\\1\\0\end{pmatrix}\times\begin{pmatrix}0\\0\\1\end{pmatrix})+\begin{pmatrix}0\\1\\0\end{pmatrix}\cdot(\begin{pmatrix}1\\0\\0\end{pmatrix}\times\begin{pmatrix}0\\0\\1\end{pmatrix})+\begin{pmatrix}0\\0\\1\end{pmatrix}\cdot(\begin{pmatrix}1\\0\\0\end{pmatrix}\times\begin{pmatrix}0\\1\\0\end{pmatrix})\\
&=\begin{pmatrix}1\\0\\0\end{pmatrix}\cdot(\begin{pmatrix}1\\0\\0\end{pmatrix})+\begin{pmatrix}0\\1\\0\end{pmatrix}\cdot(-\begin{pmatrix}0\\1\\0\end{pmatrix})+\begin{pmatrix}0\\0\\1\end{pmatrix}\cdot(\begin{pmatrix}0\\0\\1\end{pmatrix})\\
&=\begin{pmatrix}1\\0\\0\end{pmatrix}\cdot\begin{pmatrix}1\\0\\0\end{pmatrix}-\begin{pmatrix}0\\1\\0\end{pmatrix}\cdot\begin{pmatrix}0\\1\\0\end{pmatrix}+\begin{pmatrix}0\\0\\1\end{pmatrix}\cdot\begin{pmatrix}1\\0\\0\end{pmatrix}
\end{aligned}$$

$$\boxed{\begin{array}{l}
\begin{pmatrix}1\\0\\0\end{pmatrix}\cdot\begin{pmatrix}1\\0\\0\end{pmatrix}=|\begin{pmatrix}1\\0\\0\end{pmatrix}||\begin{pmatrix}1\\0\\0\end{pmatrix}|\cos 0\\
=1\times1\times1\\
=1\\
\text{similarly,}\\
myvec010\cdot\begin{pmatrix}0\\1\\0\end{pmatrix}=\begin{pmatrix}0\\0\\1\end{pmatrix}\cdot\begin{pmatrix}0\\0\\1\end{pmatrix}=1
\end{array}}$$

$$=1-1+1$$
$$=1$$

So, option (c) is correct.

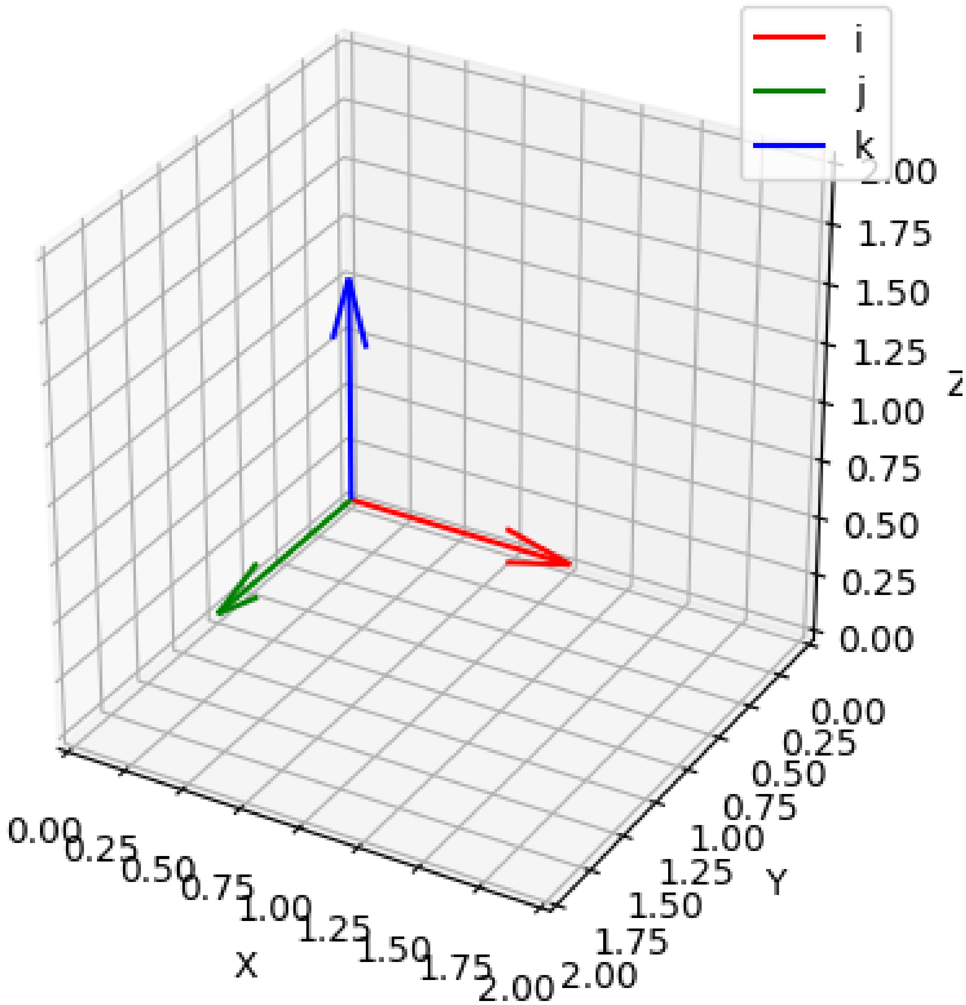

Figure 1: fig:1