Class 12

Chapter 10 - Vector Algebra

This is question 18 from exercise 10.5

1. The value of $\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{i} \times \hat{k}) + \hat{k} \cdot (\hat{i} \times \hat{j})$ is

a) 0 b) -1 c) 1 d) 3

Solution: The Directional vectors of x, y and z axes are given respectively

$$\mathbf{e_1} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \mathbf{e_2} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \mathbf{e_3} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{1}$$

Here, $\mathbf{e_i} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1(i^{\text{th}}term) \\ 0 \\ 0 \\ 0... \end{pmatrix}$, $\mathbf{e_j} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1(j^{\text{th}}term) \\ 0 \\ 0 \\ 0... \end{pmatrix}$, $\mathbf{e_k} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1(k^{\text{th}}term) \\ 0 \\ 0 \\ 0... \end{pmatrix}$

$\mathbf{e_i} \times \mathbf{e_j} = \mathbf{e_k}$	$\mathbf{e_j} \times \mathbf{e_i} = -\mathbf{e_k}$
$\mathbf{e_j} \times \mathbf{e_k} = \mathbf{e_i}$	$\mathbf{e_k} \times \mathbf{e_j} = -\mathbf{e_i}$
$\mathbf{e_k} \times \mathbf{e_i} = \mathbf{e_j}$	$\mathbf{e_i} \times \mathbf{e_k} = -\mathbf{e_j}$

$\mathbf{e_i}^\top \mathbf{e_i} = |\mathbf{e_i}||\mathbf{e_i}| \cos 0$
$= 1 \times 1 \times 1$
$= 1$
similarly, $\mathbf{e_j}^\top \mathbf{e_j} = \mathbf{e_k}^\top \mathbf{e_k} = 1$

Now,

$$\begin{aligned} &\mathbf{e_1}^\top(\mathbf{e_2} \times \mathbf{e_3}) + \mathbf{e_2}^\top(\mathbf{e_1} \times \mathbf{e_3}) + \mathbf{e_3}^\top(\mathbf{e_1} \times \mathbf{e_2}) \\ &= 1 - 1 + 1 \\ &= 1 \end{aligned}$$

So, option (c) is correct.

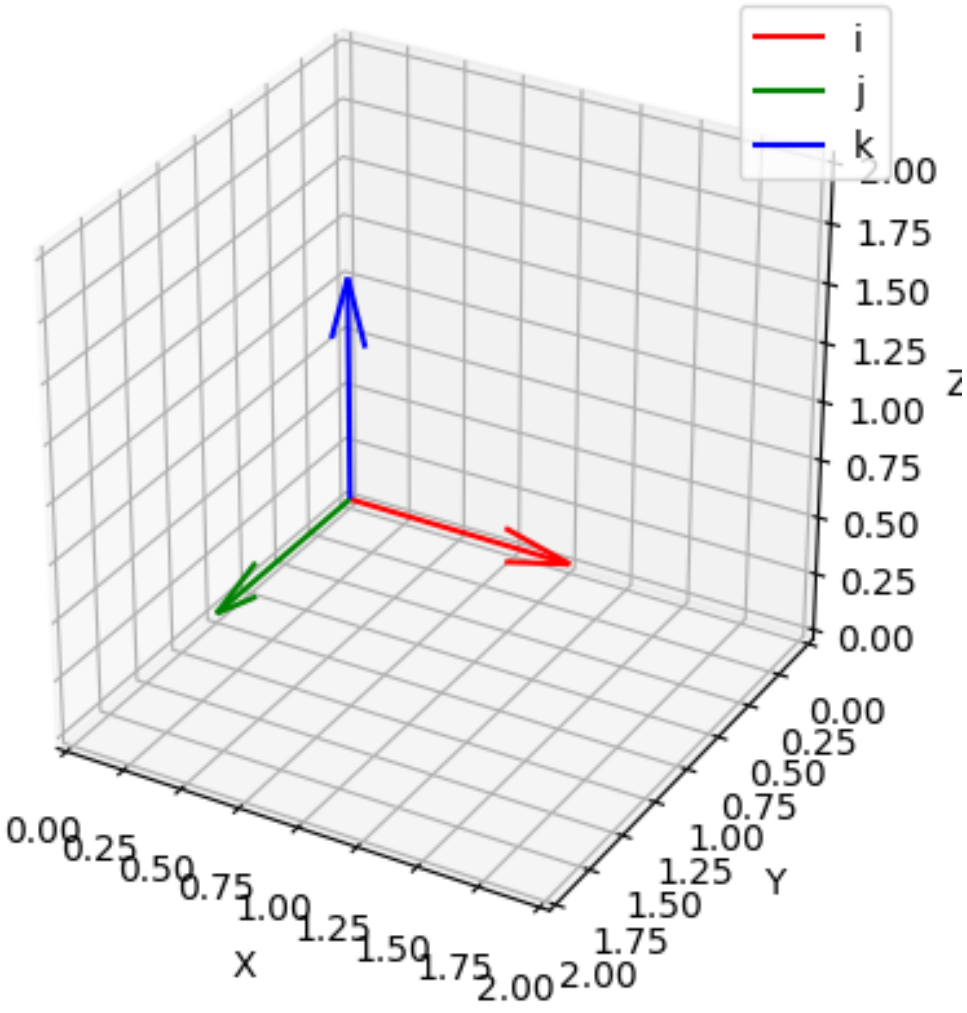

Figure 1: fig:1