A Practical Introduction to Quantum Computing

By

Shikha Mehrotra

What are the elements of a quantum circuit?

- Every computation has three elements: data, operations and results
- In quantum circuits:
  - Data = **qubits**
  - Operations = **quantum gates**
  - Results = **measurements**

What is a Qubit

- A classical bit can take two different values (0 or 1). It is discrete.
- A qubit can "take" infinitely many different values. It is continuous.
- Qubits live in a Hilbert vector space with a basis of two elements that we denote

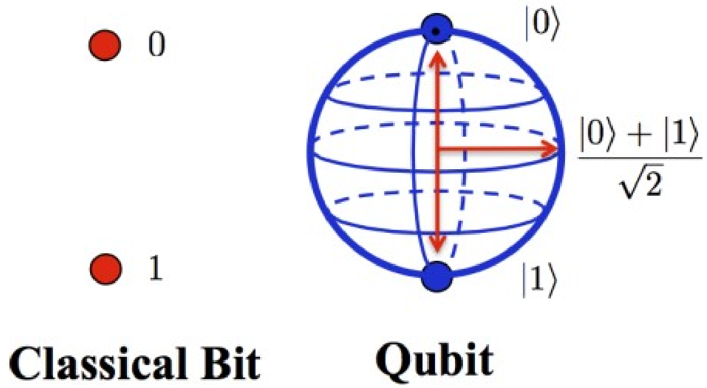
$$|0\rangle \quad \& \quad |1\rangle$$

- A generic qubit is in a superposition

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$$

where α and β are complex numbers such that

$$|\alpha|^2 + |\beta|^2 = 1$$



Measuring a qubit

- The way to know the value of a qubit is to perform a measurement. However
  - The result of the measurement is random
  - When we measure, we only obtain one (classical) bit of information

- If we measure the state $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$ we get 0 with probability $|\alpha|^2$ and 1 with probability $|\beta|^2$.

- Moreover, the new state after the measurement will be $|0\rangle$ or $|1\rangle$ depending of the result we have obtained (wavefunction colapse)

- We cannot perform several independent measurements of $|\psi\rangle$ because we cannot copy the state (no-cloning theorem)

What are quantum gates?

- Quantum mechanics tells us that the evolution of an isolated state is given by the Schrodinger equation

$$H(t)|\psi(t)\rangle = i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle$$

- In the case of quantum circuits, this implies that the operations that can be carried out are given by unitary matrices. That is, matrices U of complex numbers verifying

$$UU^\dagger = U^\dagger U = I$$

where $U^\dagger$ is the conjugate transpose of U.

- Each such matrix is a possible quantum gate in a quantum circuit

Reversible computing

- Reversible means given the operation and output value, you can find the input value
    - For Ax=b, given b and A, you can uniquely find x
- Operations which permute are reversible: operations which erase and overwrite are not
    - Identity and Negation are reversible
    - Onstatnt-0 and constant-1 are not reversible
- Quantum computers use only reversible operations
    - In fact, all quantum operators are their own inverses

Reversible computation

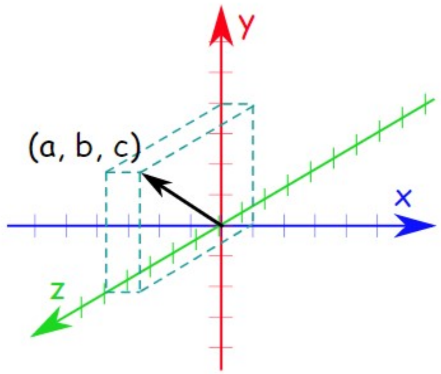
- As a consequence, all the operations have an inverse: reversible computing
- Every gate has the same number of inputs and outputs
- We cannot directly implement some classical gates such as or, and, nand, xor...
- But we can simulate any classical computation with small overhead

Writing convention

- Bra-Ket is a way of writing special [vectors]() used in Quantum Physics that looks like this:

<bra|ket>

Here is a vector in 3 dimensions:



We can write this as a column vector like this:

$$r = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

Or we can write it as a "ket":

$$|r\rangle = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

Note: the complex conjugate is written with a little star like this:

- $z = s + it$
- $z^* = s - it$

Example: This ket:

$$|a\rangle = \begin{bmatrix} 2-3i \\ 6+4i \\ 3-i \end{bmatrix}$$

Has this bra:

$$\langle a| = \begin{bmatrix} 2+3i & 6-4i & 3+i \end{bmatrix}$$

The values are now in a row, and we also **changed the sign** (+ to −, and − to +) in the middle of each element? That is all we have to do to get the conjugate.

But kets are special:

•The values (a, b and c above) are complex numbers (so they can be real numbers, imaginary numbers or a combination of both)

•Kets can have any number of dimensions, including infinite dimensions!

•The "bra" is similar, but the values are in a **row**, and each element is the complex **conjugate** of the ket's elements.

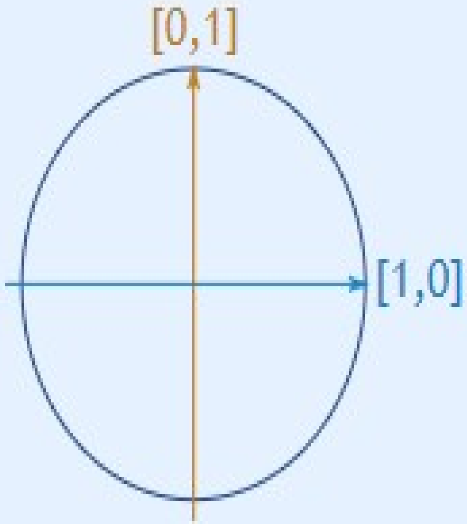
We can easily have many dimensions.

Imagine "Quantum Dice": any single die is a superposition of 1, 2, 3, 4, 5 and 6 until we measure it. Then it "collapses" into one of those states.

Its ket looks like:

$$|\text{die}\rangle = \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix}$$

For a fair die all elements (a, b, c, d, e, f) are equal, but **your** dice may be loaded!



$$|a\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \text{ and } |b\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

So:

$$\langle a|b\rangle = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 1\times0 + 0\times1 = 0$$

This can be a simple test to see if vectors are **orthogonal** (the more general concept of "at right angles")

Dirac Vector Notation
Representing classical bit as a vector

- |0>
  - One bit with the value 0

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

- |1>
  - One bit with the value 1

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

One qubit

- A single qubit state is

$$\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \alpha\begin{pmatrix}1\\ 0\end{pmatrix} + \beta\begin{pmatrix}0\\ 1\end{pmatrix} = \alpha\,|0\rangle + \beta\,|1\rangle$$

- We must not forget that

$$|\alpha|^2 + |\beta|^2 = 1$$

Then, a one-qubit gate can be identified with a matrix

$$U = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$ that satisfies

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \overline{a} & \overline{c} \\ \overline{b} & \overline{d} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Where $\overline{a}, \overline{b}, \overline{c}, \overline{d}$ are the conjugates of complex numbers a, b, c, d.

Mathematical Preliminaries

Recalling complex numbers

- A complex number is written as

$$z = x + iy$$

Where x, y are real numbers and $i^2 = -1$

- The conjugate of z is $\acute{z} = x - iy$

- The modulus of complex number is $|z|$

$$|z|^2 = z\acute{z} = x^2 + y^2$$

Matrix Multiplication

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}$$

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} ax + by + cz \\ dx + ey + fz \\ gx + hy + iz \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} w & x \\ y & z \end{pmatrix} = \begin{pmatrix} aw + by & ax + bz \\ cw + dy & cx + dz \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

Tensor product of vectors $\otimes$

- If we have two separated qubits, we can describe their collective state using the tensor product

$$\begin{pmatrix} x_0 \\ x_1 \end{pmatrix} \otimes \begin{pmatrix} y_0 \\ y_1 \end{pmatrix} = \begin{pmatrix} x_0 \begin{pmatrix} y_0 \\ y_1 \end{pmatrix} \\ x_1 \begin{pmatrix} y_0 \\ y_1 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} x_0 y_0 \\ x_0 y_1 \\ x_1 y_0 \\ x_1 y_1 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} \otimes \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 6 \\ 8 \end{pmatrix}$$

$$\begin{pmatrix} x_0 \\ x_1 \end{pmatrix} \otimes \begin{pmatrix} y_0 \\ y_1 \end{pmatrix} \otimes \begin{pmatrix} z_0 \\ z_1 \end{pmatrix} = \begin{pmatrix} x_0 y_0 z_0 \\ x_0 y_0 z_1 \\ x_0 y_1 z_0 \\ x_0 y_1 z_1 \\ x_1 y_0 z_0 \\ x_1 y_0 z_1 \\ x_1 y_1 z_0 \\ x_1 y_1 z_1 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

Quantum bits – Qubits

- Quantum bits – Qubits :

  A qubit is the fundamental unit of quantum information just as a bit is the fundamental unit of classical information.

- A bit can exist in two states: 0 and 1.

- A qubit is a vector having two complex components.

Consider the vector space $\mathbb{C}^2 = \mathbb{C} \times \mathbb{C} = \left\{ \begin{pmatrix} a \\ b \end{pmatrix} : a, b \in \mathbb{C} \right\}$

A vector of the form $a\ b$ defines a state of a qubit if and only if

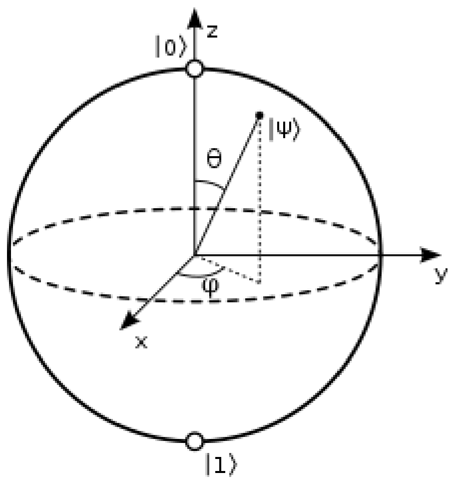
$$|a|^2 + |b|^2 = 1$$

Hilbert space

A Hilbert space is a vector space equipped with an inner product, an operation that allows defining lengths and angles.

Bloch sphere

the **Bloch sphere** is a geometrical representation of the pure state space of a two-level quantum mechanical system (qubit), named after the physicist Felix Bloch.



Basis of $\mathbb{C}^2$

- The set of vectors $\left\{\begin{pmatrix}1\\0\end{pmatrix},\begin{pmatrix}0\\1\end{pmatrix}\right\}$ is said to be a basis is $\mathbb{C}^2$ since any element in $\mathbb{C}^2$ can be written uniquely as a linear combination
$$\begin{pmatrix}a\\b\end{pmatrix} = a\begin{pmatrix}1\\0\end{pmatrix} + b\begin{pmatrix}0\\1\end{pmatrix}.$$

- Any set of vectors with this property is said to be a basis of $\mathbb{C}^2$.

  For example: $\left\{\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix},\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}\right\}$, $\left\{\frac{1}{\sqrt{2}}\begin{pmatrix}1\\\mathbf{i}\end{pmatrix},\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-\mathbf{i}\end{pmatrix}\right\}$

  where $\mathbf{i}^2 = -1$.

Inner product on $\mathbb{C}^2$

- Inner product of two vectors $\begin{pmatrix} a \\ b \end{pmatrix}, \begin{pmatrix} c \\ d \end{pmatrix} \in \mathbb{C}^2$ is

$$\begin{pmatrix} a \\ b \end{pmatrix}^\dagger \begin{pmatrix} c \\ d \end{pmatrix} = (\bar{a} \quad \bar{b}) \begin{pmatrix} c \\ d \end{pmatrix} = \bar{a}c + \bar{b}d.$$

- Two vector are said to be orthogonal if

$$\begin{pmatrix} a \\ b \end{pmatrix}^\dagger \begin{pmatrix} c \\ d \end{pmatrix} = (\bar{a} \quad \bar{b}) \begin{pmatrix} c \\ d \end{pmatrix} = \bar{a}c + \bar{b}d = 0.$$

† - conjugate transpose

Orthonormal basis of $\mathbb{C}^2$

- Suppose $\left\{\begin{pmatrix} a \\ b \end{pmatrix}, \begin{pmatrix} c \\ d \end{pmatrix}\right\}$ is a basis such that

$$\begin{pmatrix} a \\ b \end{pmatrix}^{\dagger} \begin{pmatrix} c \\ d \end{pmatrix} = (\bar{a} \quad \bar{b}) \begin{pmatrix} c \\ d \end{pmatrix} = \bar{a}c + \bar{b}d = 0$$

and

$$\begin{pmatrix} a \\ b \end{pmatrix}^{\dagger} \begin{pmatrix} a \\ b \end{pmatrix} = (\bar{a} \quad \bar{b}) \begin{pmatrix} a \\ b \end{pmatrix} = \bar{a}a + \bar{b}b = |a|^2 + |b|^2 = 1$$

$$\begin{pmatrix} c \\ d \end{pmatrix}^{\dagger} \begin{pmatrix} c \\ d \end{pmatrix} = (\bar{c} \quad \bar{d}) \begin{pmatrix} c \\ d \end{pmatrix} = \bar{c}c + \bar{d}d = |c|^2 + |d|^2 = 1$$

Orthonormal basis of $\mathbb{C}^2$

- Computational basis: $\mathcal{B} = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$:

  (Standard basis)

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}^\dagger \begin{pmatrix} 0 \\ 1 \end{pmatrix} = (\bar{1} \quad \bar{0}) \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \bar{1}0 + \bar{0}1 = 0$$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}^\dagger \begin{pmatrix} 1 \\ 0 \end{pmatrix} = (\bar{1} \quad \bar{0}) \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \bar{1}1 + \bar{0}0 = 1$$

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix}^\dagger \begin{pmatrix} 0 \\ 1 \end{pmatrix} = (\bar{0} \quad \bar{1}) \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \bar{0}0 + \bar{1}1 = 1$$

Orthonormal basis of $\mathbb{C}^2$: Examples

- Hadamard basis: $\mathcal{H} = \left\{\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}, \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}\right\}$.

- Nega-Hadamard basis: $\mathcal{N} = \left\{\frac{1}{\sqrt{2}}\begin{pmatrix}1\\\mathbf{i}\end{pmatrix}, \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-\mathbf{i}\end{pmatrix}\right\}$.

Dirac's bra/ket notation

- A vector $\begin{pmatrix} a \\ b \end{pmatrix} \in \mathbb{C}^2$ is written as $|\psi\rangle$ read as "*ket psi*".

- The vector $\begin{pmatrix} a \\ b \end{pmatrix}^\dagger = (\bar{a} \quad \bar{b})$ is written as $\langle\psi|$.

- Inner product of two vectors $|\phi\rangle = \begin{pmatrix} c \\ d \end{pmatrix}$, and $|\psi\rangle = \begin{pmatrix} a \\ b \end{pmatrix}$ is
$\langle\psi|\phi\rangle = \begin{pmatrix} a \\ b \end{pmatrix}^\dagger \begin{pmatrix} c \\ d \end{pmatrix} = (\bar{a} \quad \bar{b}) \begin{pmatrix} c \\ d \end{pmatrix} = \bar{a}c + \bar{b}d.$

  The order in the which $|\phi\rangle$ and $|\psi\rangle$ appear matters. This is the inner product of $|\phi\rangle$ and $|\psi\rangle$ and not $|\psi\rangle$ and $|\phi\rangle$.

Computational, Hadamard and Nega-Hadamard Bases in Dirac's notation

- Computational basis: $|0\rangle = \begin{pmatrix}1\\0\end{pmatrix}, |1\rangle = \begin{pmatrix}0\\1\end{pmatrix}.$
  (Z-bases)

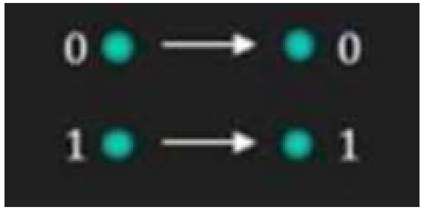
- Hadamard basis: $|+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} = \frac{|0\rangle+|1\rangle}{\sqrt{2}}, |-\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix} = \frac{|0\rangle-|1\rangle}{\sqrt{2}}$
  (X-bases)

- Nega-Hadamard basis: (Y-bases)
  $|\mathbf{i}\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\\mathbf{i}\end{pmatrix} = \frac{|0\rangle+\mathbf{i}|1\rangle}{\sqrt{2}}, |-\mathbf{i}\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-\mathbf{i}\end{pmatrix} = \frac{|0\rangle-\mathbf{i}|1\rangle}{\sqrt{2}}$

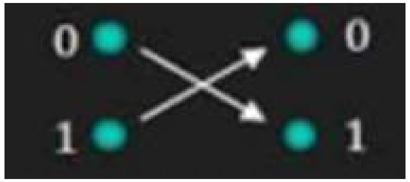
Single Qubit Gates

Operations on one classical bit

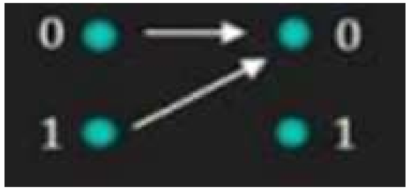
- Identity $f(x) = x$



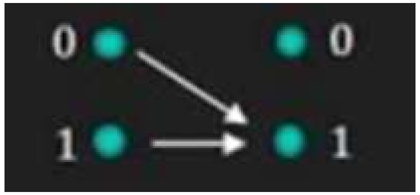
- Negation $f(x) = \neg x$



- Constant-0 $f(x) = 0$



- Constant-1 $f(x) = 1$



Operations on one classical bit…cont

- Identity $f(x) = x$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- Negation $f(x) = \neg x$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

- Constant-0 $f(x) = 0$

$$\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

- Constant-1 $f(x) = 1$

$$\begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

The I gate

- ‘Id-gate’ or ‘Identity gate’

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- a gate that does nothing

$$|0\rangle \;—\boxed{\mathrm{I}}—\; |0\rangle$$

$$|1\rangle \;—\boxed{\mathrm{I}}—\; |1\rangle$$

- Applying the identity gate anywhere in your circuit should have no effect on the qubit state

- Then why it is even considered a gate?
  - it is often used in calculations. for example: proving the X-gate is its own inverse:

$$I = XX$$

  - it is often useful when considering real hardware to specify a ‘do-nothing’ or ‘none’ operation.

The X or NOT gate

- The X gate is defined by the (unitary) matrix

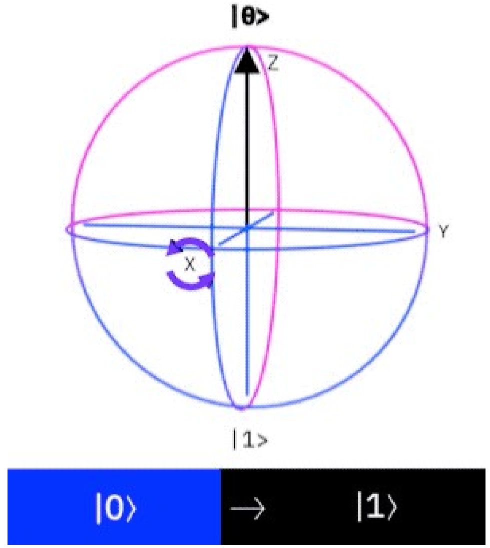
$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

X -> X, Z -> -Z

- Its action (in quantum circuit notation) is

$$|0\rangle \;-\!\boxed{X}\!-\; |1\rangle$$

$$|1\rangle \;-\!\boxed{X}\!-\; |0\rangle$$



that is, it acts like the classical NOT gate

- On a general qubit its action is

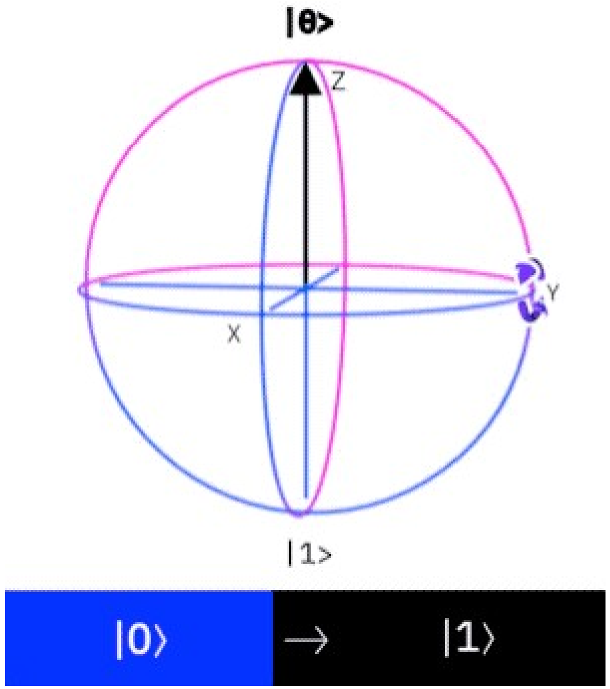
$$\alpha\,|0\rangle + \beta\,|1\rangle \;-\!\boxed{X}\!-\; \beta\,|0\rangle + \alpha\,|1\rangle$$

Y Gate

- The Pauli Y gate is equivalent to Ry for the angle π.

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

X -> -X, Z -> -Z



Z Gate

the Z-gate appears to have no effect on our qubit when it is in either of these two states. This is because the states $|0\rangle$ and $|1\rangle$ are the basis of the Z-gate. In fact, the *computational basis* is often called the Z-basis.

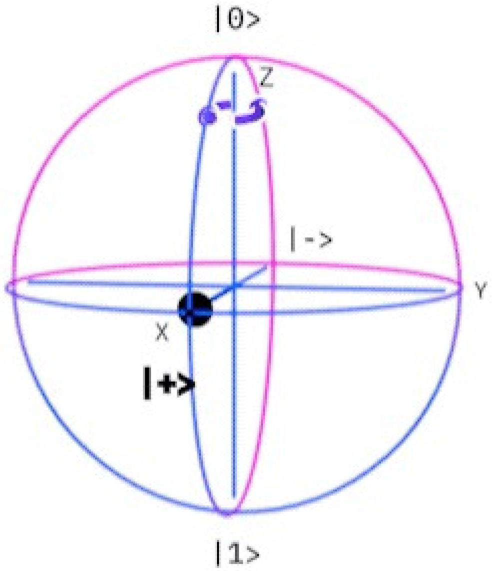
- The Pauli Z gate has the property of flipping the |+> to |-> , and vice versa. π.
- It is equivalent to Rz for the

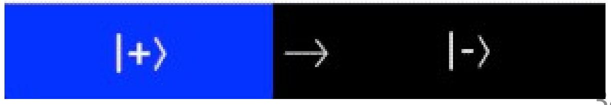
$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

X -> -X, Z -> Z

$|0\rangle -\boxed{Z}- |0\rangle$

$|1\rangle -\boxed{Z}- -|1\rangle$





Qbits and superposition

- The cbit vectors we've been using are just special cases of qbit vectors
- A qbit is represented by $\begin{pmatrix} a \\ b \end{pmatrix}$ where a and b are complex numbers and $\|a\|^2 + \|b\|^2 = 1$
- The cbit vectores $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ fit within this definition

$$\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \quad \begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix} \quad \begin{pmatrix} -1 \\ 0 \end{pmatrix} \quad \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix}$$

Qubits and superposition

- Superposition means the qbit is both 0 and 1 at the same time
- When we measure the qbit, it collapses to an actual vale of 0 or 1
- If a qbit has value $\begin{pmatrix} a \\ b \end{pmatrix}$ then it collapses to 0 with probatility a2 and 1 with probability b2

- For example, qbit $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$ has a $\left\| \frac{1}{\sqrt{2}} \right\|^2 = \frac{1}{2}$ chance of collapsing to 0 or 1 (coin flip)
- The qbit $\binom{1}{0}$ has a 100% chance of collapsing to 0, and $\binom{0}{1}$ has a 100% chance of collapsing to 1

The Hadamard Gate

H

- It is useful for making superposition

$$|0\rangle \;—\boxed{H}—\; \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$$|1\rangle \;—\boxed{H}—\; \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

- We usually denote

$$|+\rangle := \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

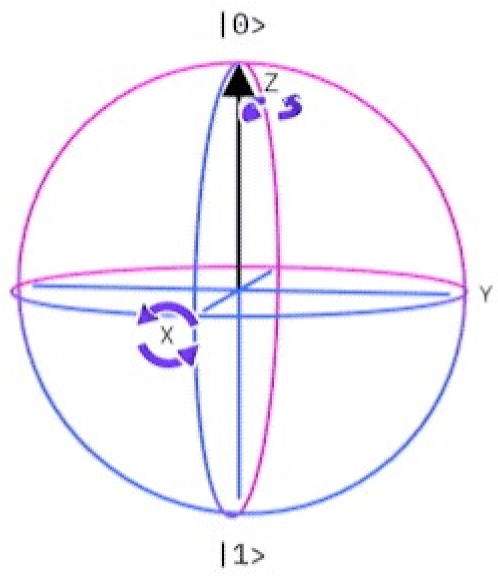
and

$$|-\rangle := \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

- it is useful for moving information between the x and z bases.

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

X -> Z, Z -> X



$|0\rangle \rightarrow |+\rangle$

The Hadamard gate

- The hadamard gate takes a 0 or 1-bit and puts it into exactly eqaul superposition

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{pmatrix}$$

$$H|0\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \qquad H|1\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix}$$