

SMAI Class 8 - SVM

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Maximum Margin Classification

- **Margin:** Width of a band around decision boundary without any training samples. Also the Radius of region around each training sample through which the decision boundary cannot pass.

Margin varies with position and origin of the separating hyperplane. As margin increases, the feasible region (for the decision boundary) decreases.

The circles touching the margin are the corresponding samples that *control* the boundary. Such samples are called **Support Vectors** and the resulting linear classifier is called the SVM.

Radius = R Margin = $\rho = 2R$

$w^T x + b = 0$: Decision Boundary $w^T x + b = \pm \epsilon$: Margins

We want to maximize ϵ .

Some constraints

- Normalize w ($\|w\| = 1$).

Problem

$$g(x) = w^T x + b$$

Minimize $1/2 w^T w$ subject to $y_i(w^T x_i + b) - 1 \geq 0 \forall i$.

Pegasos Algorithm

[Pegasos paper ref.](#)

INPUT: Training Data: S , λ , Max iterations: T

INITIALIZE: w_1

FOR $t = 1, 2, 3 \dots T$

- Choose $i = \{1, 2, \dots |S|\}$ at random and pick sample (x_i, y_i)
- Set $\eta_t = 1 / \lambda$
- IF($y_i (w_t^T x_i) < 1$):
 - Update $w_{t+1} \leftarrow w_t - \eta_t \lambda w_t + \eta_t y_i x_i$
- ELSE

- Update $w_{t+1} \leftarrow w_t - \eta_t \lambda w_t$

OUTPUT w_{T+1}

SVM Theory (A high level view)

Concept called VC-dimension(h) that measures the complexity of classifiers.

R test (chance of classifier making error during testing). Sort of expected loss.

The R-test is R-train + some function.

$$R_{test}(\alpha) = R_{train}(\alpha) + f(h, N)$$

$$h \leq \min\{d, \lceil 1/m^2 \rceil\} + 1$$

where d is dimension of the feature space. m is defined as the relative margin.

$$m = \rho/D$$

ρ : Margin, D : Data diameter