Matrices

RANK OF MATRIX

① Let $A=[a_{ij}]$ of matrix of $m \times n$, then on deleting some rows & columns is known as sub matrix

② The determinant of a squares of matrix is called minor of A

③ The order of the highest non-zero minor of A is called rank of A & denoted by $\rho A \leq \min(m,n)$

RANK

Row transformation	Reducing into normal form
↓	↓
transform into ~~vertice~~ upper triangle	use both row & column transformation
(the number of non zero rows is rank)	(The no of diagonals with one is rank)
	⇓
any row with all zeros is zero row	the no of 1's at the diagonal position is the rank of that matrix

Q Find the rank of matrix.

1) $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$

By Row operation

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - 2R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & -1 \\ 0 & 2 & -1 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

So rank is 2.

It can also be done by 2^{nd} method

(2) $A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$

by Row operation:

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & -1 & -3 \\ 0 & 1 & 1 & 3 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$$\begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & -1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank = 2

By normal form

$$\begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & -1 & -3 \\ 0 & 1 & 1 & 3 \end{bmatrix}$$

$C_2 \to C_2 - 2C_1$ $\quad C_4 \to C_4 - 2C_1$
$C_3 \to C_3 - 3C_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & -3 \\ 0 & 1 & 1 & 3 \end{bmatrix}$$

~~Rank is 2~~.

$R_3 \to R_3 + R_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_3 \to C_3 - C_2 \quad C_4 \to C_4 - 3C_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank is 2

* Consistent and inconsistent system of non-homogenous equation.

$$a_{11}x_1 + a_{12}x_2 + \cdots a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots a_{2n}x_n = b_2$$
$$\vdots \qquad \vdots \qquad \vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots a_{mn}x_n = b_n.$$

[AB] = augmented matrix

$$[AB] = \left[\begin{array}{cccc|c} a_{11} & a_{11} & \cdots & a_{1n} & b_1 \\ \vdots & & & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} & b_m \end{array}\right]$$

→ Coefficient matrix.

→ By row operation convert it into upper triangle matrix
See the rank of augmented & coefficient matrix.

① If the rank of coefficient = rank of augmented matrix = no of unknowns the system is consistant & unique solution

② $\rho_A = \rho_{AB} <$ no of ~~rows~~ unknowns, the system is consistant, and has infinite solutions

③ $\rho_A \neq \rho_{AB}$, the system is consitent and has no solution.

$\rho_A \rightarrow$ rank of coefficient matrix
$\rho_{AB} \rightarrow$ rank of augmented matrix

Q For what value of λ, μ.

$x+y+z=6$.
$x+2y+3z=10$
$x+2y+\lambda z=\mu$. have

① Unique solution
② no solution
③ infinite solutions.

$$[AB] = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{bmatrix}$$

$R_2 \rightarrow R_2 - R_1$
$R_3 \rightarrow R_3 - R_1$

$$[AB] = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda - 1 & \mu - 6 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{bmatrix}$$

(1) for unique → $\lambda-3 \neq 0$, $\lambda \neq 3$, μ can have any value

(2) for no solution → $\lambda-3=0$, $\lambda=3$, and $\mu-10 \neq 0$, $\mu \neq 10$

(3) for infinite solution → $\lambda-3=0$, $\mu-10=0$ or $\lambda=3$, $\mu=10$

Q Solve with the help of matrices the system of equations.

$x+y+z=3$
$x+2y+3z=4$
$x+4y+9z=6$

$$[AB] = \begin{bmatrix} 1 & 1 & 1 & 3 \\ 1 & 2 & 3 & 4 \\ 1 & 4 & 9 & 6 \end{bmatrix}$$

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 3 & 8 & 3 \end{bmatrix}$$

$R_3 \to R_3 - 3R_2$

$$\begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 2 & 0 \end{bmatrix}$$

$$2z = 0$$
$$z = 0$$
$$y + 2z = 1$$
$$y = 1$$
$$x + y + z = 3$$
$$x + 1 + 0 = 3$$
$$x = 2$$

$$\boxed{x = 2,\ y = 1,\ z = 0}$$

Homogenous equations are always consistent
→ hence either
1) ~~equ~~ unique solution or
2) infinite solution.

Q Solve the following using matrices:

$$4x - 2y + 6z = 8$$
$$x + y - 3z = -1$$
$$15x - 3y + 9z = 21$$

$$[A\,B] = \begin{bmatrix} 4 & -2 & 6 & 8 \\ 1 & 1 & -3 & -1 \\ 15 & -3 & 9 & 21 \end{bmatrix}$$

$\cancel{R_1 \to R_2 -}$ $\quad R_1 \to R_1 - 3R_2$

$$\begin{bmatrix} 1 & -5 & 15 & 11 \\ 1 & 1 & -3 & -1 \\ 15 & -3 & 9 & 21 \end{bmatrix}$$

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - 15R_1$

$$\begin{bmatrix} 1 & -5 & 15 & 11 \\ 0 & 6 & -18 & -12 \\ 0 & 72 & 216 & -144 \end{bmatrix}.$$

$R_3 \to R_3 - 12R_2$

$$\begin{bmatrix} 1 & -5 & 15 & 11 \\ 0 & 6 & -18 & -12 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

$6y - 18z = -12$

$z = k$

$y - 3k = -2$

$y = 3k - 2$

$x - 5y + 15z = 11$

$x - 5(3k - 2) + 15k = 11$

$x - \cancel{15k} + 10 + \cancel{15k} = 11$

$x = 1.$

$\boxed{x = 1,\ y = 3k - 2,\ z = k}$

Eigen values & Eigen vector of matrix

① $A = [a_{ij}]$ of order $n \times n$.

② $(A - \lambda I)$ is called characteristic matrix of A

③ $|A - \lambda I|$ is called characteristic polynomial of A

④ $|A - \lambda I| = 0$ is called characteristic equation of A

⑤ roots of this equation is called characteristic roots, or Eigen values or latent root or proper root

⑥ Corresponding to each value of λ, there is non-zero value such that

$$[A - \lambda I] x = 0.$$

When x is characteristic value vector or Eigen vector of A

Eigen values and Eigen vector can only be for a square matrix not rectangular matrix

Find:

Q Eigen values & Eigen vector.

$$\begin{bmatrix} -2 & 2 & 3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{vmatrix} = 0.$$

$(-2-\lambda)\left[(1-\lambda)(-\lambda) - 12\right] - 2\left[-2\lambda - 6\right]$
$-3\left[-4 + 1 - \lambda\right]\Big] = 0$

$(2+\lambda)\left[-\lambda + \lambda^2 - 12\right] + \cancel{2\lambda} - 4\lambda - 12$
$-12 + 3 - 3\lambda = 0.$

$-2\lambda + 2\lambda^2 - 24 - \lambda^2 + \lambda^3 - 12\lambda - 7\lambda - 21 = 0$

$\lambda^3 + \lambda^2 - 21\lambda - 45 = 0.$

$$\begin{array}{r l}
 & \lambda^2 - 2\lambda - 15 \\
\lambda + 3 \,\big) & \lambda^3 + \lambda^2 - 21\lambda - 45 \,\big(\\
 & \lambda^3 + 3\lambda^2 \\
 & (-)\quad (-) \\
\hline
 & -2\lambda^2 - 21\lambda \\
 & -2\lambda^2 - 6\lambda \\
 & (+)\quad (+) \\
\hline
 & -15\lambda - 45 \\
 & -15\lambda - 45
\end{array}$$

27
9
63

$(\lambda+3)(\lambda^2-2\lambda-15)=0$

$(\lambda+3)(\lambda^2-5\lambda+3\lambda-15)=0$

$(\lambda+3)(\lambda-5)(\lambda+3)=0$

Eigen values $= 5, -3, -3$

for $= 5$

$$[A-\lambda I] = \begin{bmatrix} -7 & 2 & -3 \\ 2 & -4 & -6 \\ -1 & -2 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

Q Find Eigen values and Eigen vector of the matrix.

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

$$|A-\lambda I| = \begin{vmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{vmatrix} = 0$$

$(8-\lambda)[(7-\lambda)(3-\lambda)-16]+6[6(\lambda-3)+8]+2[24+2(\lambda-7)] = 0$

$(8-\lambda)[21-10\lambda+\lambda^2-16]+6[6\lambda-10]+2[2\lambda+10] = 0$

$(8-\lambda)[\lambda^2-10\lambda+5]+40\lambda-40=0$

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$$8\lambda^2 - 80\lambda + 40 - \lambda^3 + 10\lambda^2 - 5\lambda + 40\lambda - 40 = 0$$

$$-\lambda^3 + 18\lambda^2 + \cancel{35\lambda} - 45\lambda = 0$$

$$\lambda^3 - 18\lambda^2 + 45\lambda = 0$$

$$\lambda[\lambda^2 - 18\lambda + 45] = 0$$

$$\lambda = 0, 3, 15$$

for $\lambda = 0$

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$\cancel{R_1 \leftrightarrow R_1 + R_2 - R_3}$

$R_2 \rightarrow R_1/2$

$R_1 \leftrightarrow R_3$

$$\begin{bmatrix} 2 & -4 & 3 \\ 8 & -6 & 2 \\ 8 & -6 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$R_1 \rightarrow R_1/2$

~~6 7~~

$$\begin{bmatrix} 2 & -4 & 3 \\ -6 & 7 & -4 \\ 8 & -6 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$R_2 \to R_2 + 3R_1$
$R_3 \to R_3 - 4R_1$

$$\begin{bmatrix} 2 & -4 & 3 \\ 0 & -5 & -13 \\ 0 & 10 & -10 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$R_3 \to R_3 + 2R_2$

$$\begin{bmatrix} 2 & -4 & 3 \\ 0 & -5 & -13 \\ 0 & 0 & 16 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$16z = 0$ $\quad z = 0.$
$-5y - 13z = 0$
$2x - 4y + 3z = 0$

If a, b, c are Eigen vectors correspon to 1st Eigen value & so on

then

$$E = \begin{bmatrix} a & a_1 & a_2 \\ b & b_1 & b_2 \\ c & c_1 & c_2 \end{bmatrix}$$ is modal matrix

* If modal matrix / Eigen vector and Eigen values are given, Find matrix.

$E^{-1} A E = D$ (holds only when Eigen vector are linearly independent)

Where D is Diagonal matrix

$$\begin{bmatrix} d_1 & 0 & - & 0 & 0 \\ 0 & d_2 & & - & \\ 0 & 0 & - & & d_n \end{bmatrix}$$

$AE = ED$

$$\boxed{A = EDE^{-1}}$$

Q Find Eigen values & Eigen vector

$$A \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$|A-\lambda I| = \begin{vmatrix} 2-\lambda & 1 & 1 \\ 1 & 2-\lambda & 1 \\ 0 & 0 & 1-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)[(2-\lambda)(1-\lambda)] - [1-\lambda] + 1[0] = 0$$

$$(2-\lambda)[2+\lambda^2-3\lambda] + \lambda - 1 = 0$$

$$4+2\lambda^2-6\lambda-2\lambda-\lambda^3+3\lambda^2+\lambda-1=0$$

$$-\lambda^3+5\lambda^2-7\lambda+3=0$$

$$\lambda^3-5\lambda^2+7\lambda-3=0$$

$$\lambda-1 \overline{\big)\ \lambda^3-5\lambda^2+7\lambda-3\ } \big(\ \lambda^2-4\lambda+3$$

$$\lambda^3-\lambda^2$$
$$(-)\quad(+)$$
$$\overline{\quad -4\lambda^2+7\lambda \quad}$$
$$-4\lambda^2+4\lambda$$
$$(+)\quad(-)$$
$$\overline{\quad 3\lambda-3 \quad}$$

$$(\lambda-1)(\lambda^2-4\lambda+3)=0$$

$$(\lambda-1)(\lambda-1)(\lambda-3)=0$$

Eign value $\underline{1, 1, 3}$

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Eign vector

for $\lambda = 1$,

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0.$$

$x + y + z = 0.$

$z = 0, \; y = k.$

$x = -k$

$E = [-k, k, 0] = [-1, 1, 0]$

for $\lambda = 3$,

$$\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$-2z = 0$

$z = 0$

$x - y + z = 0$

$-x + y + z = 0$

$z = 0$

$x - y = 0$

$y - x = 0$

$x = y$

$E = [1, 1, 0]$

Caley Hamiltan theorem

Every square matrix must satisfy it's own characteristic equation

A is matrix of order $n \times n$

$$|A - \lambda I| = 0$$

To prove

$$\lambda^n + a_1 \lambda^{n-1} + a_2 \lambda^{n-2} + \cdots a_n = 0$$

$$adj(A - \lambda I)$$

$$= B_0 \lambda^{n-1} + B_1 \lambda^{n-2} + B_2 \lambda^{n-3} + \cdots B_{n-1}$$

$B_0, B_1, \cdots B_{n-1}$ are matrices of order $n \times n$

$\Rightarrow$ $A \, adj(A) = |A| I$

$$(A - \lambda I) \, adj(A - \lambda I) = |A - \lambda I| I$$

$$(A - \lambda I) \cdot (B_0 \lambda^{n-1} + B_1 \lambda^{n-2} + \cdots B_{n-1}) = \lambda^n + a_1 \lambda^{n-1} + a_2 \lambda^{n-2} + \cdots a_n$$

Comparing Coefficient.

$$-B_0 = I \qquad \times A^n$$

$$AB_0 - B_1 = a_1 I \qquad \times A^{n-1}$$

$$AB_1 - B_2 = a_2 I$$

$$\vdots$$

$+$ $$AB_{n-1} = a_n I.$$

$A^n + a_1 A^{n-1} + a_2 A^{n-2} \ldots + a_n = 0$

$A \Leftrightarrow \lambda$

$\lambda^n + a_1 \lambda^{n-1} + a_2 \lambda^{n-2} - - + a_n = 0$

Hence proved.

Q Verify Calley-Hamilton theorem and hence find A^{-1}.

$$A = \begin{bmatrix} 7 & -1 & 3 \\ 6 & 1 & 4 \\ 2 & 4 & 8 \end{bmatrix}$$

$|A - \lambda I| = 0$

$$\begin{vmatrix} 7-\lambda & -1 & 3 \\ 6 & 1-\lambda & 4 \\ 2 & 4 & 8-\lambda \end{vmatrix} = 0$$

$(7-\lambda)[(1-\lambda)(8-\lambda) - 16] + [6(8-\lambda) - 8]$

$+ 3[24 + 2(\lambda - 1)] = 0$

$(7-\lambda)[\lambda^2 - 9\lambda - 8] + [-6\lambda + 40] + 66 + 6\lambda = 0$

$7\lambda^2 - 63\lambda - 56 - \lambda^3 + 9\lambda^2 + 8\lambda + 106 = 0$

$-\lambda^3 + 16\lambda^2 - 55\lambda + 50 = 0$

$\lambda^3 - 16\lambda^2 + 55\lambda - 50 = 0$

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$$A^2 = \begin{bmatrix} 7 & -1 & 3 \\ 6 & 1 & 4 \\ 2 & 4 & 8 \end{bmatrix} \begin{bmatrix} 7 & -1 & 3 \\ 6 & 1 & 4 \\ 2 & 4 & 7 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 49 & 4 & 41 \\ 56 & 11 & 54 \\ 54 & 34 & 86 \end{bmatrix}$$

$$A^2 + \cancel{8A + 5I + 6A^{-1}}$$

$$A^2 - 16A + 55I - 50A^{-1} = 0$$

$$A^{-1} = \frac{1}{50}[A^2 - 16A + 55I]$$

$$A^{-1} = \frac{1}{50} \begin{bmatrix} 49-112+55 & 4+16 & 41-48 \\ 56-96 & 11-16+55 & 54-64 \\ 54-32 & 34 & 86-128+55 \end{bmatrix}$$

Q Find Characteristic equation of

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

2 + 3 + 2

4 − 1

and hence find matrix,

$$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$$

$$\boxed{A^3 - 5A^2 + 7A - 3I = 0}$$

A

$$\begin{array}{r l}
 & A^5 + A \\
A^3 - 5A^2 + 7A - 3I \,\big)\!\! & \cancel{A^8} - \cancel{5A^7} + \cancel{7A^6} - \cancel{3A^5} + A^4 - 5A^3 + 8A^2 - 2A + I \\
 & A^8 - 5A^7 + 7A^6 - 3A^5 \\
 & (-)\quad (+)\quad (-)\quad (+) \\
\hline
 & \cancel{A^4} - \cancel{5A^3} + 8A^2 - 2A + I \\
 & A^4 - 5A^3 + 7A^2 - 3A \\
 & (-)\quad (+)\quad (-)\quad (+) \\
\hline
 & A^2 + A + I \\
\hline
\end{array}$$

$$A^2 = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 4 & 4 \\ 0 & 1 & 0 \\ 4 & 4 & 5 \end{bmatrix}$$

$$A^2 + A + I = \begin{bmatrix} 5+2+1 & 4+1 & 4+1 \\ 0 & 1+1+1 & 0+ \\ 4+1 & 4+1 & 5+2+1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 & 5 \\ 0 & 3 & 0 \\ 5 & 5 & 8 \end{bmatrix}.$$

Q Show that if $\lambda_1, \lambda_2 \ldots \lambda_n$ are latent roots of matrix A then

(1) A^{-1} has latent roots $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \ldots \frac{1}{\lambda_n}$

(2) A^3 has latent root $\lambda_1^3, \lambda_2^3 \ldots \lambda_n^3$

$(A-\lambda I)X = 0$

$AX = \lambda X$

$\frac{1}{\lambda}X = A^{-1}X$ multiply by A^{-1}

$AX = \lambda X$

$A^2 X = \lambda AX$ $AX = \lambda X$

$A^2 X = \lambda^2 X$

$A^3 X = \lambda^2 AX$

$A^3 X = \lambda^3 X$.

Hence proved

Linear Combination of vector
(Dependence & Independence)

1. Let $\alpha_1, \alpha_2 - - - \alpha_n$ be any n scaler then vector $\alpha_1 x_1 + \alpha_2 x_2 - - \alpha_n x_n$ is called linear combination of vector

2. set of vector $x_1 x_2 - - x_n$ is said to be linearly dependent if and only if there exit scalers $\alpha_1, \alpha_2 - - \alpha_n$ not all of which are equal to zero, such that $\alpha_1 x_1 + \alpha_2 x_2 + - - - \alpha_n x_n = 0$

3. If set of vectors $x_1, x_2 - - x_n$ is not linearly dependent is said to be independent.

Q Show that the vector $x_1 = (1, 2, 4)$, $x_2 = (2, -1, 3)$, $x_3 = (0, 1, 2)$, $x_4 = (-3, 7, 2)$ are linearly dependent.

Ans $x_2 = x_2 - 2x_1 = (0, -5, -5)$

$x_4 = x_4 + 3x_1 = (0, 13, 14)$

$x_2 - 2x_1 + 5x_3 = (0, 0, 5)$ × 12

(+)

$x_4 + 3x_1 - 13x_3 = (0, 0, -12)$ × 5

as both are zero, linearily dependent.

Q Are the following vector linearly dependent? If so, find the relation between them

$x_1 = (1, 2, 1)$
$x_2 = (2, 1, 4)$
$x_3 = (4, 5, 6)$
$x_4 = (1, 8, -3)$

$x_1 = (1, 2, 1)$
$x_2 = x_2 - 2x_1 = (0, -3, 2)$
$x_3 = x_4 - 4x_1 = (0, -3, 2)$
$x_4 = x_4 - x_1 = (0, 6, -4)$.

$x_3 = x_3 - x_2 = (0, 0, 0)$
$x_4 = x_4 + 2x_2 = (0, 0, 0)$.

Hence linearly dependent

Q $x_1 = (2, -1, 4)$, $x_2 = (0, 1, 2)$.
$x_3 = (6, 1, 16)$, $x_4 = (4, 0, 12)$

$x_1 = (2, -1, 4)$
$x_2 = (0, 1, 2)$
$x_3 = x_3 - 3x_1 = (0, 4, 4)$
$x_4 = x_4 - 2x_1 = (0, 2, 4)$

$x_3 = x_3 - 4x_2 = (0, 0, -4)$] as it is non zero.
$x_4 = x_4 - 2x_2 = (0, 0, 0)$.

It is independent

Linear Transformation

The relation $Y = AX$

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \quad A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & & & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

defines a linear transformation which carries any vector X to any vector Y over the matrix A which is called linear operator of the transformation.
If $|A| \neq 0$ then transformation is called non-singular transformation.
If $|A| = 0$, then transformation is called singular transformation.

$$X = A^{-1}Y$$

A^{-1} is called inverse operator.

Q A transformation from the variables x_1, x_2, x_3 to y_1, y_2, y_3 is given by $Y = AX$
and another transformation is from y_1, y_2, y_3 to z_1, z_2, z_3 is given by $Z = BY$
where

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 2 \\ -1 & 2 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 1 & 3 & 5 \end{bmatrix}$$

Obtain transformation from x_1, x_2, x_3 to (z_1, z_2, z_3)

Ans $Z = BY$

$Y = AX$

$Z = \underline{BAX}$

Calculate

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \underset{BA}{\begin{bmatrix} \ \end{bmatrix}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Q For what values of λ the equation

$x+y+z=1$

$x+2y+4z=\lambda$

$x+4y+10z=\lambda^2$

have a Solution & solve them in each case.

Ans $x+y+z=1$

$x+2y+4z=\lambda$

$x+4y+10z=\lambda^2$

$$[AB] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & \lambda \\ 1 & 4 & 10 & \lambda^2 \end{bmatrix}$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$[A|B] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 3 & 9 & \lambda^2-1 \end{bmatrix}$$

$R_3 \to R_3 - 3R_2$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 0 & 0 & \lambda^2-3\lambda+2 \end{bmatrix}$$

Rank of coefficient matrix is 2
For To have solution rank of augment matrix should also be zero

here $\lambda^2 - 3\lambda + 2 = 0$
hence $\lambda = 1, \lambda = 2$

Solve for both $\lambda = 1, \lambda = 2$
(homogenous equations)

Q Are the following vectors linearly dependent?

$x_1 = (2, -1, 3, 2)$
$x_2 = (1, 3, 4, 2)$
$x_3 = (3, -5, 2, 2)$

$x_2 = (1, 3, 4, 2)$
$X_1 = x_1 - 2x_2 = (0, -7, -5, -2)$
$X_3 = x_3 - 3x_2 = (0, -14, -10, -4)$
$2X_1 + X_3 = (0, 0, 0, 0)$
linearly dependent.

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Q If A is given singular matrix, find P & Q (non-singular matrix) such that

$$I = PAQ.$$

Ans $A = IAI$

$$\begin{bmatrix} \text{Row/} \\ \text{Column} \end{bmatrix} = \begin{bmatrix} \text{Row} \\ \text{Only} \end{bmatrix} A \begin{bmatrix} \text{Column} \\ \text{Only} \end{bmatrix}$$

Convert to I ($m \times n$)

$\Downarrow$ $m \times m$ (acc to no of rows in A)

$\Downarrow$ $n \times n$ (acc to no of columns in A)