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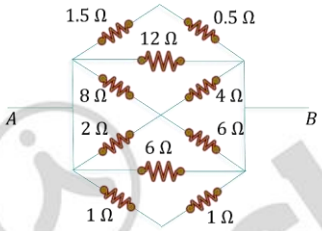
JEE Main 2023 (Memory based)

30 January 2023 - Shift 2

Answer & Solutions

PHYSICS

1. In the given circuit the resistance between terminals A and B is equal to

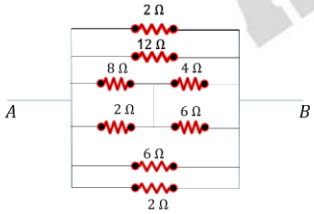


A. $2\ \Omega$
B. $\frac{3}{2}\ \Omega$
C. $\frac{2}{3}\ \Omega$
D. $6\ \Omega$

Answer (C)

Solution:

The circuit can be redrawn as:



So, the net resistance across A and B is:

$$\frac{1}{R_{net}} = \frac{1}{2} + \frac{1}{12} + \frac{1}{4} + \frac{1}{6} + \frac{1}{2}$$

$$\frac{1}{R_{net}} = \frac{18}{12}$$

$$R_{net} = \frac{2}{3}\ \Omega$$

2. A car travels $4\ km$ distance with a speed of $3\ km/hr$ and next $4\ km$ with a speed of $5\ km/h$. Find average speed of car.

A. $\frac{15}{2}\ km/hr$
B. $\frac{15}{4}\ km/hr$
C. $15\ km/hr$
D. $10\ km/hr$

Answer (B)

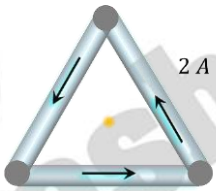
Solution:

$$\text{Velocity} = \frac{\text{Total Distance}}{\text{Total time}}$$

$$v = \frac{4+4}{\frac{4}{3}+\frac{4}{5}}\ km/h$$

$$v = \frac{15}{4}\ km/h$$

3. A current $2\ A$ if flowing through the sides of an equilateral triangular loop of side $4\sqrt{3}\ m$ as shown. Find the magnetic field induction at the centroid of the triangle.



A. $3\sqrt{3} \times 10^{-7}\ T$
B. $\sqrt{3} \times 10^{-7}\ T$
C. $2\sqrt{3} \times 10^{-7}\ T$
D. $5\sqrt{3} \times 10^{-7}\ T$

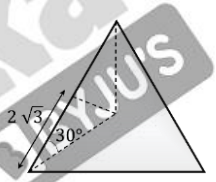
Answer (A)

Solution:

$\frac{r}{2\sqrt{3}} = \tan 30°$

$r = 2\ m$

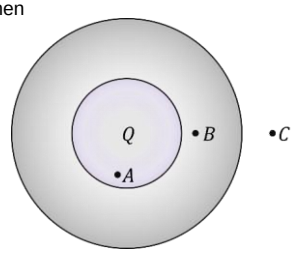
Magnetic field at centroid



$$B = 3 \times \frac{\mu_0 I}{4\pi r}(\sin 60° + \sin 60°)$$

$$= 3 \times \frac{\mu_0 \times 2}{4\pi \times 2}\left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right) = 3\sqrt{3} \times 10^{-7}\ T$$

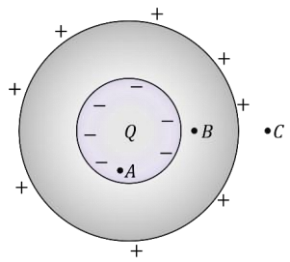
4. A point charge Q is placed inside the cavity made in uniform conducting solid sphere as shown. E_A, E_B *and* E_C are electric field magnitudes at points A, B and C respectively. Then



A. $E_A = 0$, $E_B = 0$ and $E_C \neq 0$
B. $E_A \neq 0$, $E_B = 0$ and $E_C \neq 0$
C. $E_A \neq 0$, $E_B = 0$ and $E_C = 0$
D. $E_A \neq 0$, $E_B \neq 0$ and $E_C = 0$

Answer (B)

Solution:



$E_A \neq 0$ (Electric field due to both Q and induced charges on the inner surface of cavity)

$E_B = 0$ (No field line inside conductor)

$E_C \neq 0$ (Electric field due to charge induced on outer surface of conductor)

5. In the shown mass-spring system, when it is set into oscillations along the spring, it has angular frequency ω_1 if $m = 1\ kg$ and ω_2 if $m = 2\ kg$. Then value of ω_1/ω_2 is equal to

A. 1
B. $\sqrt{2}$
C. $1/\sqrt{2}$
D. 2

Answer (B)

Solution:

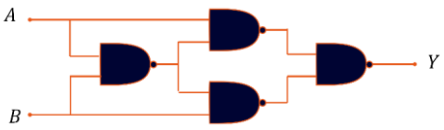
$$\omega_1 = \sqrt{\frac{k}{m}} = \sqrt{\frac{k}{1}}$$

$$\omega_2 = \sqrt{\frac{k}{m}} = \sqrt{\frac{k}{2}}$$

So,

$$\frac{\omega_1}{\omega_2} = \sqrt{2}$$

6. For the given logic circuit, which of the following truth table is correct?



A.

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

B.

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

C.

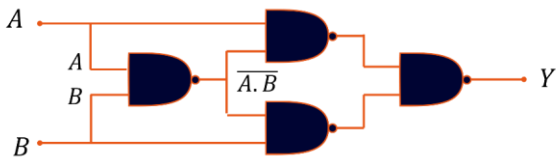
A	B	Y
0	0	1
0	1	1
1	0	1
1	0	0

D.

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

Answer (A)

Solution:



$$X_1 = \overline{\left(A.\overline{(A.B)}.B.\overline{(A.B)}\right)}$$
$$= A.\overline{(A.B)} + B.\overline{(A.B)}$$
$$= A.\bar{B} + B.\bar{A}$$
$$= XOR\ gate$$

7. A particle of mass m is moving under a force whose delivered power P is constant. Initial velocity of particle is $zero$. Find the position of particle at $t = 4\ s$

A. $x = \frac{16}{3}\sqrt{\left[\frac{2P}{m}\right]}$

B. $x = \frac{4}{3}\sqrt{\left[\frac{2P}{m}\right]}$

C. $x = \frac{2}{3}\sqrt{\left[\frac{P}{m}\right]}$

D. $x = \frac{3}{10}\sqrt{\left[\frac{P}{m}\right]}$

Answer (A)

Solution:

We know that:

$$P = \frac{W}{t}$$
$$\frac{1}{2}mv^2 = P \times t$$
$$v = \sqrt{\frac{2Pt}{m}} = \frac{dx}{dt}$$
$$x = \frac{16}{3}\sqrt{\frac{2P}{m}}$$

8. Column-I list few physical quantities and column-II lists their dimensions. Choose the correct option matching two lists correctly.

column I	column II
(P) Pressure gradient	(A) $[M^1L^2T^{-2}]$
(Q) Energy density	(B) $[M^1L^1T^{-1}]$
(R) Torque	(C) $[M^1L^{-2}T^{-2}]$
(S) Impulse	(D) $[M^1L^{-1}T^{-2}]$

A. $P-C,\ Q-A,\ R-B,\ S-D$
B. $P-C,\ Q-D,\ R-A,\ S-B$
C. $P-A,\ Q-D,\ R-B,\ S-C$
D. $P-A,\ Q-C,\ R-B,\ S-D$

Answer (B)

Solution:

$[Pressure\ gradient] \Rightarrow \left[\frac{dP}{dz}\right] = \left[\frac{ML^{-1}T^{-2}}{L}\right] = [ML^{-2}T^{-2}]$

$[Energy\ density] \Rightarrow \left[\frac{dU}{dV}\right] = \left[\frac{ML^2T^{-2}}{L^3}\right] = [ML^{-1}T^{-2}]$

$[Torque] \Rightarrow [F] \times [r] = [MLT^{-2}] \times [L] = [ML^2T^{-2}]$

$[Impulse] \Rightarrow [F][t] = [MLT^{-2}][T] = [MLT^{-1}]$

So, $P-C,\ Q-D,\ R-A,\ S-B$ is the correct match.

9. Consider the following assertion and reason:
Assertion (A): At sink temperature of $-273°C$, the efficiency of a Carnot engine will be 1.
Reason (R): Efficiency of a Carnot engine is given by $\eta = 1 - \frac{T_{sink}}{T_{source}}$

A. A is correct. R is correct and correctly explains A.
B. A is not correct. R is correct.
C. Both A and R are incorrect.
D. Both A and R are correct. R doesn't explain A.

Answer (A)

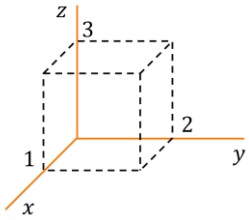
Solution:

We know that Carnot efficiency is:

$\eta = 1 - \frac{T_{Sink}}{T_{Source}}$

For $T_{sink} = -273°\ C = 0\ K \Rightarrow \eta = 1$

10. Electric field in a region is $\vec{E} = 2x^2\hat{\imath} - 4y\hat{\jmath} + 6z\hat{k}$
Find the charge inside the cuboid shown:



A. $-8\varepsilon_o$
B. $36\varepsilon_o$
C. $12\varepsilon_o$
D. $24\varepsilon_o$

Answer (D)

Solution:

Total flux can be calculated as:

$\phi_{total} = 2(1)^2[2 \times 3] - 4(2)[1 \times 3] + 6(3)[1 \times 2]$
$\phi_{total} = 12 - 24 + 36$
$\phi_{total} = 24$
From Gauss's Law:

$\Rightarrow \frac{q}{\varepsilon_o} = 24$
$\Rightarrow q = 24\varepsilon_o$

11. Find the ratio of de-Broglie wavelength of proton when it is accelerated across V and $3V$ potential difference.

A. $3:1$
B. $1:\sqrt{3}$
C. $1:3$
D. $\sqrt{3}:1$

Answer (D)

Solution:

When proton is accelerated by potential difference V, then linear momentum of proton
$\frac{P^2}{2m} = eV$
$P = \sqrt{2meV} \Rightarrow \lambda_1 = \frac{h}{\sqrt{2meV}}$

When proton is accelerated by potential difference $3\,V$, then linear momentum of proton

$\frac{P^2}{2m} = 3eV$
$P = \sqrt{6meV} \Rightarrow \lambda_2 = \frac{h}{\sqrt{6meV}} \Rightarrow \frac{\lambda_1}{\lambda_2} = \sqrt{3}$

12. A faulty scale reads $5°C$ at melting point and $95°C$ at steam point. Find original temperature if this faulty scale reads $41°C$.

A. $40°C$
B. $41°C$
C. $36°C$
D. $45°C$

Answer (A)

Solution:

Suppose X is the original temperature.
So,
$\frac{41-5}{95-5} = \frac{X-0}{100-0}$
$9X = 360$
$X = 40$

13. A particle is released at a height equal to radius of earth above the surface of the earth. Its velocity when it hits the surface of earth is equal to
M_e=mass of earth, R_e= radius of earth

A. $v = \sqrt{\left[\frac{2GM_e}{R_e}\right]}$
B. $v = \sqrt{\left[\frac{GM_e}{2R_e}\right]}$
C. $v = \sqrt{\left[\frac{GM_e}{R_e}\right]}$
D. $v = \sqrt{\left[\frac{2GM_e}{3R_e}\right]}$

Answer (C)

Solution:

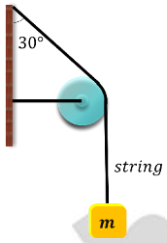
Applying law of conservation of mechanical energy,

$$-\frac{GM_em}{2R_e} = -\frac{GM_em}{R_e} + \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{GM_e}{R_e}}$$

14. A block stays in equilibrium as shown. Find the tension in the string if $m = \sqrt{3}\,kg$.

A. $\sqrt{3}g\ N$
B. $3g\ N$
C. $g/2\ N$
D. $g/\sqrt{3}\ N$



Answer (A)

Solution:

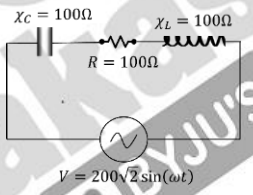
Since block is in equilibrium,

$\Rightarrow T = mg$

$\Rightarrow T = \sqrt{3}\,g$

15. In the AC circuit shown in the figure, the value of I_{rms} is equal to

A. $2A$
B. $2\sqrt{2}A$
C. $4A$
D. $\sqrt{2}A$



Answer (A)

Solution

$$Z = \sqrt{R^2 + (\chi_L - \chi_C)^2}$$

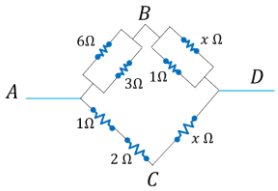
$$Z = \sqrt{100^2 + (100 - 100)^2} = 100\Omega$$

So,

$$i_o = \frac{V_o}{Z} = \frac{200\sqrt{2}}{100} = 2\sqrt{2}$$

$$i_{rms} = \frac{i_o}{\sqrt{2}} = 2\ A$$

16. For the given electrical circuit, the potential difference between point B and C is zero. The value of $2x$ is _____.



Answer (1)

Solution:

Since, $V_B = V_C$
Then,

$$\frac{2}{3} = \frac{x}{\frac{x+1}{x}}$$

$$x + 1 = \frac{3}{2} \Rightarrow x = \frac{1}{2}\ \Omega$$

17. Two waves of same intensity from sources in phase are made to superimpose at a point. If path difference between these two coherent waves is zero, then resultant intensity is I_o. If this path difference is $\frac{\lambda}{2}$ where λ is wavelength of these waves, then resultant intensity is I_1 and if the difference is $\frac{\lambda}{4}$ then resultant Intensity is I_2. Value of $\frac{I_1+I_2}{I_o}$ is equal to.

Answer (0.5)

Solution:

Let individual intensity from source is I thus,

$$I_0 = I + I + 2\sqrt{I \times I}\cos\left(0 \times \frac{2\pi}{\lambda}\right) \Rightarrow I_0 = 4I$$

$$I_1 = I + I + 2\sqrt{I \times I}\cos\left(\frac{\lambda}{2} \times \frac{2\pi}{\lambda}\right) \Rightarrow I_1 = 0$$

$$I_2 = I + I + 2\sqrt{I \times I}\cos\left(\frac{\lambda}{4} \times \frac{2\pi}{\lambda}\right) \Rightarrow I_2 = 2I$$

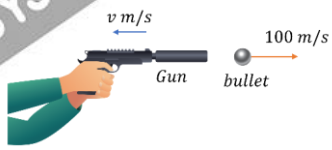
So,

$$\frac{I_1 + I_2}{I_0} = \frac{1}{2}\ or\ 0.5$$

18. A bullet of mass $10\ grams$ is fired from a gun (mass of $10\ kg$ without bullet) with a speed of $100\ m/s$. The recoil speed of gun is $\frac{x}{10}\ m/s$. Find x.

Answer (1)

Solution:



Applying Conservation of linear momentum

$$10 \times 10^{-3} \times v = 10 \times 100$$

$$v = \frac{1}{10}\ m/s$$

19. The relation between velocity (v) and position (x) of a particle moving along x-axis is given by $4v^2 = 50 - x^2$. The time period of oscillatory motion of the particle is $\frac{88}{n}\ seconds$. Find n. Use $\pi = \frac{22}{7}$

Answer (7)

Solution:

$$4v^2 = 50 - x^2$$

$$v^2 = \frac{1}{4}(50 - x^2)$$

$$v = \frac{1}{2}\sqrt{[(50 - x^2)]}$$

comparing with equation of SHM

$$v = \omega\sqrt{A^2 - x^2}$$

$A^2 = 50 \Rightarrow A = 5\sqrt{2}$
$\omega = \frac{1}{2} = 0.5\ rad/s$
$T = \frac{2\pi}{\omega} = \frac{2\pi}{0.5} = 4\pi\ second$
$T = 4\left(\frac{22}{7}\right) = \frac{88}{7}\ \ second$
$So,\ n = 7$

20. The ratio of temperature in K of hydrogen and oxygen is $2:1$. The ratio of their average kinetic energy per molecule is

Answer (2)

Solution:

$\text{Average kinetic energy} = \frac{f}{2}K_B T$
As $f\ (both\ are\ diatomic)$ and K_B are same for hydrogen and oxygen.

$$\frac{(\text{Average kinetic energy per molecule})_{H_2}}{(\text{Average kinetic energy per molecule})_{O_2}} = \frac{T_{H_2}}{T_{O_2}} = \frac{2}{1}$$

21. Prism A has angle of prism equal to 6^0 and its material has refractive index 1.5. It is used in combination with prism B of refractive index 1.8 to produce dispersion without deviation. Angle of prism B is equal to _____degrees.

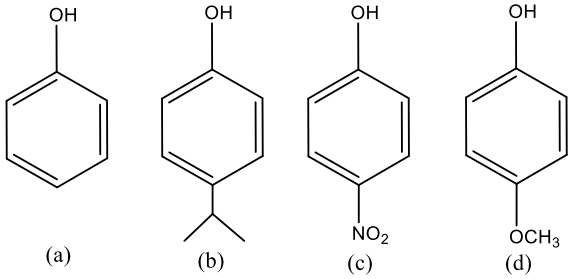
Answer (3.75)

Solution:

For dispersion without deviation
$A(\mu - 1) + A'(\mu' - 1) = 0$
$6°(1.5 - 1) + A'(1.8 - 1) = 0$
$A' = -3.75°$
Negative sign indicate that prison are inverted with respect to each other
$|A'| = 3.75°$

CHEMISTRY

1. The correct order of acidic strength of the following compounds



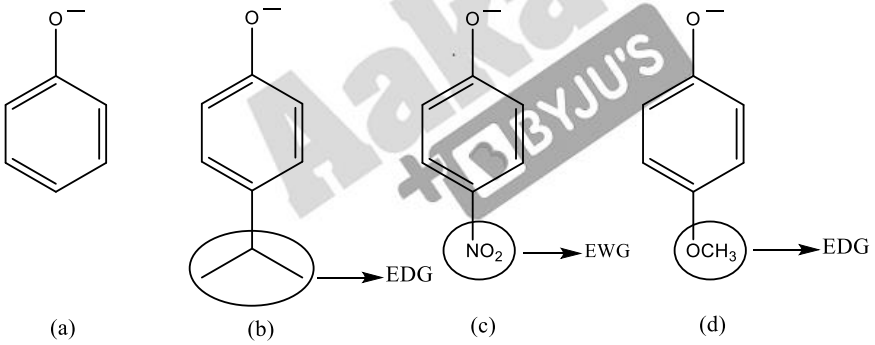
A. a > b > c > d
B. c > a > b > d
C. d > c > b > a
D. c > b > a > d

Answer (B)

Solution:

Acidic character stability of conjugate base,

Let's see conjugate base the given molecules.



Stability of conjugate base increases due to the presence of electron withdrawing group (EWG) on the aromatic ring. Similarly, stability of conjugate base decreases in the presence of electron donating group (EDG).

The presence of $-NO_2$ (EWG) in (c) makes it most acidic.

Comparing (a) (b) and (c), absence of EDG in (a) makes it more acidic than others.

Although (b) and (d) have electron donating group, since +M effect in (d) is more powerful than the hyperconjugation effect in (b), makes (b) more acidic than (d).

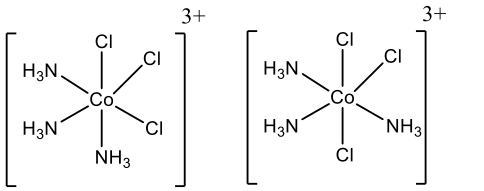
The correct acidic order is: c > a > b > d

2. What is the $Cl - Co - Cl$ bond angle in $[Co(NH_3)_3Cl_3]$

A. $20^o \; and \; 90^o$
B. $90^o and \; 180^o$
C. 90^o
D. 180^o

Answer (B)

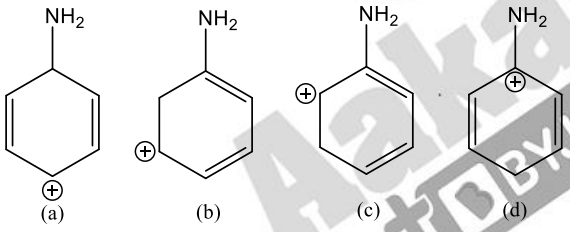
Solution: Since $[Co(NH_3)_3Cl_3]$ has 6 ligands, the structure octahedral structure. The molecule is of type $[MA_3B_3]$, thus, it has two geometrical isomers namely facial and meridional isomers.



Bond angle = 90^o (Facial isomer)

Bond angle = 90^o and 180^o (Meridional isomer)

3. The correct order of decreasing stability of the following compounds is

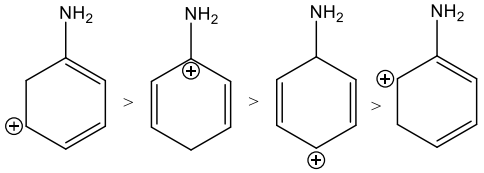


A. a > b > c > d
B. d > b > c > a
C. b > d > a > c
D. b > a > d > c

Answer (C)

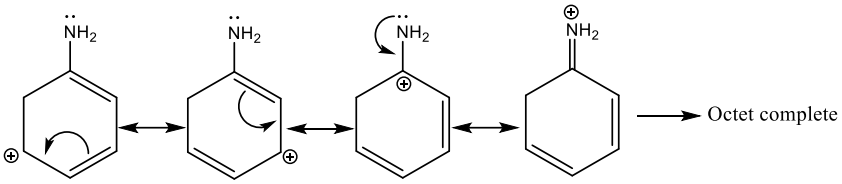
Solution:

The correct acidic order is

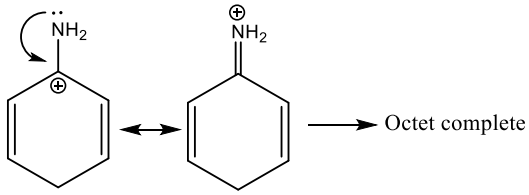


In (b) conjugated alkenes as well as +ve charge will be next to nitrogen in one of its resonating structure due to Which completion of octet of every atom (Duplet of hydrogen) is seen.

(b)



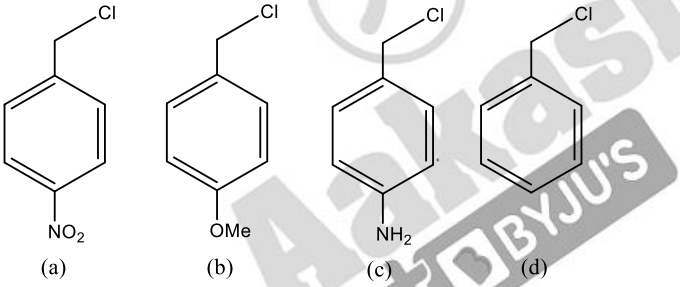
(d)



But in (a) and (c) only resonance is seen and no completion of octet.

In (c) +ve charge is next to NH_2 group which shows -I effect.

4. Which of the following is correct order of S_N1 reaction



A. a > b > c > d

B. c > b > d > a

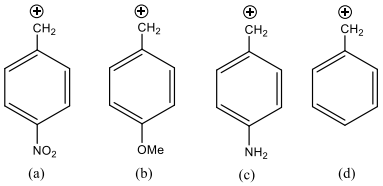
C. c > a > b > d

D. d > a > b > c

Answer (B)

Solution:

The reactivity order of the given aryl halides towards S_N1 reaction will be decided by the stability of their corresponding carbocations.



The benzyl carbocation is stabilized by resonance. The presence of $-NH_2$ group at the p – position promotes the resonance due to +R effect. The -OMe group also promotes but to a lesser extent due to higher electronegativity of O – atom than N – atom. The $-NO_2$ group opposes the resonance stabilization due to its -R effect.

Therefore, the correct order is c > b > d > a.

5. Statement 1: Antihistamines prevent the secretion of acid in the stomach
Statement 2: Antiallergic and antacid work on same receptors

A. Statement 1 is correct and Statement 2 is incorrect
B. Statement 1 and Statement 2 both are correct
C. Statement 1 is incorrect and Statement 2 is correct
D. Statement 1 and Statement 2 both are incorrect

Answer (D)

Solution:

Antihistamines do not affect the secretion of acid in stomach. Antiallergic and antacid drugs work on different receptors.

Therefore, both the statements are incorrect.

6. Statement 1: During Hall-Heroult process, mixing of CaF_2 and Na_3AlF_6 decreases the M.P. of Al_2O_3
Statement 2: During electrolytic refining, anode is pure and cathode is impure.

A. Statement 1 is correct and Statement 2 is incorrect
B. Statement 1 is correct and Statement 2 is incorrect
C. Statement 1 and Statement 2 both are incorrect
D. Statement 1 is incorrect and Statement 2 is correct

Answer (B)

Solution:

Mixture of CaF_2 and Na_3AlF_6 decreases the melting point of Al_2O_3. During electrolytic refining of alumina (Al_2O_3) impure metal is taken in anode and pure metal in cathode. At the end of the process, impurities get deposited below anode as anode mud.

7. 1 mole of gas undergoes adiabatic process, given that
$C_v = 20\ JKmol^{-1}, W = 3\ KJ, T_1 = 27^oC, T_2 = ?$

Answer (177)

Solution:

$W = nC_v(T_2 - T_1)$

$3000 = 1 \times 20 \times (T_2 - 300)$

$150 = (T_2 - 300)$

$T_2 = 450K$

$T_2 = 177^oC$

8. Volume strength of H_2O_2 solution is 60. Strength of solution in ____ g/L (round of to the nearest integer)

Answer (182)

Solution:

Volume strength of H_2O_2 = 60 Volume

Molarity of H_2O_2 solution = $\frac{60}{11.2}M$

Strength of H_2O_2 solution = $\frac{60\times34}{11.2}$

$= 182.14\ g/L$

$\approx 182\ g/L$

9. For 1st order reaction, 540 s takes for 60% completion, and the time taken for 90% completion is 1.35 x 10^x s. Find x. (log4 = 0.6)

Answer (3)

Solution:

$\frac{t_{90}}{t_{60}} = \frac{\log\frac{100}{100-90}}{\log\frac{100}{100-60}} = \frac{1}{\frac{\log10}{4}} = \frac{1}{1-0.6} = \frac{1}{0.4}$

$t_{90} = \frac{540}{0.4} = 1350\ sec$

$1350 = 1.35 \times 10^x$

x = 3

10. Find the number of formula units of FeO per unit cell. Given that: density = 4 g/cm^3 , $a = 5A^o$, $N_A = 6$ x 10^{23}

Answer (4)

Solution:

Density = $\frac{ZM}{N_A\ X\ a^3} \Rightarrow Z = \frac{density\times N_A\times a^3}{M}$

$= \frac{4\times6.0\times10^{23}\times(5\times10^{-8})^3}{(56+16)}$

$= \frac{4\times6\times125\times10^{-1}}{72}$

= 4.16

11. Maximum no. of e^- in n = 4 shell is

A. 72
B. 50
C. 16
D. 32

Answer (D)

Solution:

Maximum number of e^- in a shell = $2n^2$

$= 2(4)^2$

= 32

12. BOD value of a water sample is 3 ppm. Select the correct option about the given sample of water.

A. It is highly polluted water
B. It is clear water

C. Concentration of oxygen in the given sample is very less
D. None of these

Answer (B)

Solution:

The given sample of water is clean water as BOD value of clean water ranges between 3 to 5.

13. Which of the following chloride is more soluble in organic solvent?

A. Be
B. K
C. Ca
D. Mg

Answer (A)

Solution:

Out of the given elements, the chlorides of K and Ca are largely ionic. So, they will be more soluble in water and less soluble in organic solvents. $BeCl_2$ has higher covalent character than $MgCl_2$. Therefore, $BeCl_2$ is more soluble in organic solvents than $MgCl_2$.

14. The correct order of bond strength of H_2O, H_2S, H_2Se, H_2Te

A. $H_2O > H_2S > H_2Se > H_2Te$
B. $H_2S > H_2O > H_2Se > H_2Te$
C. $H_2Te > H_2Se > H_2S > H_2O$
D. $H_2Te > H_2S > H_2O > H_2Se$

Answer (A)

Solution:

As moving down in the hydrides of 16^{th} group elements, the bond length between central atom and hydrogen increases and the bond strength decreases. Therefore, the correct order of bond strength is $H_2O > H_2S > H_2Se > H_2Te$

15. Lead storage battery have 38% (w/w) H_2SO_4. Find the temperature at which the liquid of battery will freeze (i = 2.67) ; K_f of water = 1.86 $\frac{K.kg}{mol}$

A. -3.1°C
B. -31°C
C. -0.31°C
D. -0.031°C

Answer (B)

Solution:

$\Delta T_f = i \times K_f \times m$

$= 2.67 \times 1.86 \times m$

$m = \frac{38 \times 1000}{98 \times 62} = 6.25$

$\Delta T_f = 2.67 \times 1.86 \times 6.25$

$= 31.06℃$

Freezing point $= 31.06℃$

$\approx -31℃$

16. $KMnO_4$ oxidises I^- in acidic and neutral medium in which form respectively.

A. IO_3^-/IO^-
B. IO_3^-/IO_3^-
C. IO_3^-/I_3^-
D. I_2/IO_3^-

Answer (D)

Solution:

In acidic medium:

$$10I^- + 2MnO_4^- + 16H^+ \rightarrow 2Mn^{2+} + 8H_2O + 5I_2$$

In neutral medium:

$$2MnO_4^- + H_2O + I^- \rightarrow 2MnO_2 + 2OH^- + IO_3^-$$

I^- Converts to I_2 in acidic medium and converts to IO_3^- in neutral medium.

17. Which of the following equation is correct?

1) $LiNO_3 \rightarrow Li + NO_2 + O_2$
2) $LiNO_3 \rightarrow LiNO_2 + O_2$
3) $LiNO_3 \rightarrow Li_2O + NO_2 + O_2$
4) $LiNO_3 \rightarrow Li_2O + N_2O_4 + O_2$

A. 1
B. 2
C. 3
D. 4

Answer (C)

Solution:

$LiNO_3$ is thermally unstable. It decomposes to give lithium oxide, nitrogen dioxide and oxygen.

$$2LiNO_3 \xrightarrow{\Delta} Li_2O + 2NO_2 + \frac{1}{2}O_2$$

=

18. The option containing the correct match is given as :

	List - I	List - II
A.	$Ni(CO)_4$	i. sp^3
B.	$[Ni(CN)_4]^{2-}$	ii. sp^3d^2
C.	$[Cu(H_2O)_6]^{2+}$	iii. d^2sp^3
D.	$[Fe(CN)_6]^{4-}$	iv. dsp^2

A. (A)-(i) ; (B)-(iv) ; (C)-(ii) ; (D)-(iii)
B. (A)-(iii) ; (B)-(ii) ; (C)-(iv) ; (D)-(i)
C. (A)-(ii) ; (B)-(iii) ; (C)-(iv) ; (D)-(i)
D. (A)-(iv) ; (B)-(ii) ; (C)-(i) ; (D)-(iii)

Answer (A)

Solution:

	Coordination complex	Number of ligands	Oxidation state of central metal	Type of ligand	Hybridisation
A.	$Ni(CO)_4$	4	0	Strong field	sp^3
B.	$[Ni(CN)_4]^{2-}$	4	+2	Strong field	dsp^2
C.	$[Cu(H_2O)_6]^{2+}$	6	+2	Weak field	sp^3d^2
D.	$[Fe(CN)_6]^{4-}$	6	+2	Strong field	d^2sp^3

19. Nessler's Reagent is

A. $K_2[HgI_4]$
B. $K_3[HgI_4]$
C. Hg_2I_2
D. HgI_2

Answer (A)

Solution:

Nessler's Reagent is $K_2[HgI_4]$.

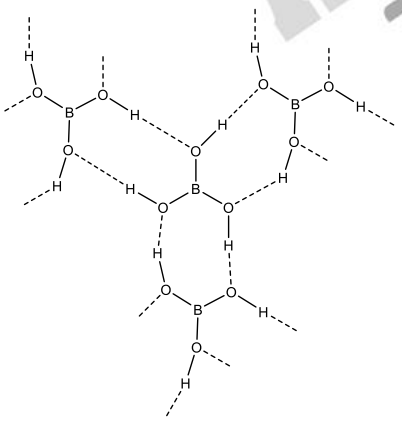
20. Boric acid is present in solid state while BF_3 is gas at room temperature, because of?

A. Hydrogen bonding is present in boric acid
B. Boric acid has more molar mass as compared to BF_3
C. BF_3 is polymeric in nature
D. Both (B) and (C)

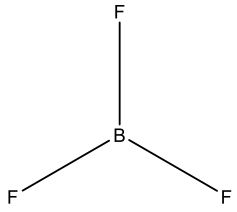
Answer (A)

Solution:

Due to H-bonding boric acid is solid at room temperature.



(Boric acid)



(Boran trifluoride)

21. For the given electrochemical cell,

$X|X^{2+}(0.001m) \parallel Y^{2+}(0.01m)|Y$

at $298k$

$E^{o}_{X^{2+}/X} = -0.76$

$E^{o}_{Y^{2+}/Y} = +0.34$

$\frac{2.303RT}{F} = 0.06$

If $E_{cell} = t$, find $5t$ (closest Integer)

Answer (6)

Solution:

$E_{cell} = E^{o}_{cell} - \frac{0.06}{2} \times \log\frac{10^{-3}}{10^{-2}}$

$= 1.10 - 0.03 \times (-1)$

$= 1.10 + 0.03$

$t = 1.13\,V$

$5t = 5.65$

The nearest integer is 6.

22. Common tangent is drawn to $y^2 = 16x$ and $x^2 + y^2 = 8$. The square of distance between point of contact of common tangent on both the curves is:

A. 78
B. 72
C. 42
D. 76

Answer (B)

Solution:

Given equation of parabola is $y^2 = 16x$

$\Rightarrow a = 4$

General equation of tangent to a parabola is $y = mx + \frac{a}{m}$

Given equation of circle is $x^2 + y^2 = 8$

$\Rightarrow r = \sqrt{8}$ and Centre of circle $(0,0)$

$y = mx + \frac{4}{m}$ is tangent to circle

$\therefore$ Perpendicular distance from $(0,0)$ to $y = mx + \frac{4}{m}$ is equal to radius of circle.

$\Rightarrow \left|\frac{\frac{4}{m}}{\sqrt{m^2+1}}\right| = \sqrt{8}$

$\Rightarrow \frac{16}{m^2} = 8m^2 + 8$

$\Rightarrow 8m^4 + 8m^2 - 16 = 0$

$\Rightarrow 8m^4 + 16m^2 - 8m^2 - 16 = 0$

$\Rightarrow 8m^2(m^2 + 2) - 8(m^2 + 2) = 0$

$\Rightarrow m = \pm 1$

Point of contact on parabola $= \left(\frac{a}{m^2}, \frac{2a}{m}\right)$

$= (4, \pm 8)$

Point of contact on circle $= \left(\pm\frac{am}{\sqrt{1+m^2}}, \mp\frac{a}{\sqrt{1+m^2}}\right)$

$= (-2, +2)$ or $(-2, -2)$

Distance between $(4,8)$ and $(-2, 2) = \sqrt{6^2 + 6^2} = \sqrt{72}$

Also, Distance between $(4, -8)$ and $(-2, -2) = \sqrt{6^2 + 6^2} = \sqrt{72}$

$\therefore$ Square of distance between point of contact of common tangent on both the curves $= 72$

23. Let $f(x) = \begin{cases} \frac{x}{|x|}, & x \neq 0 \\ 1, & x = 0 \end{cases}, g(x) = \begin{cases} \frac{\sin(x+1)}{x+1}, & x \neq -1 \\ 1, & x = -1 \end{cases}, h(x) = 2[x] + f(x)$ ([.] denotes greatest integer function).Then $\lim_{x \to 1} g(h(x-1))$ is :

A. $\frac{\sin 1}{1}$
B. $\frac{\sin 2}{2}$

Commented [GU1]: make it this (-2,-2)
Commented [N12R1]: Done
Commented [GU3]: show distance between (4,-8) and (-2,-2) also. It will come sqrt(72)
Commented [N14R3]: Done
Commented [GU5]: show distance between (4,-8) and (-2,-2) also. It will come sqrt(72)
Commented [N16R5]: Done

C. -1

D. 2

Answer (B)

Solution:

$h(x-1) = 2[x-1] + f(x-1)$

$\lim_{x\to 1^+} h(x-1) = 2\cdot 0 + f(0^+) = 1$

$\lim_{x\to 1^-} h(x-1) = 2\cdot(-1) + f(0^-) = 2\cdot(-1) + (-1) = -3$

R.H.L.: $\lim_{x\to 1^+} g(h(x-1)) = g(1) = \frac{\sin(1+1)}{1+1} = \frac{\sin 2}{2}$

L.H.L.: $\lim_{x\to 1^-} g(h(x-1)) = g(-3) = \frac{\sin(-3+1)}{(-3+1)} = \frac{\sin 2}{2}$

$\Rightarrow$ L.H.L.= R.H.L.

$\therefore \lim_{x\to 1} g(h(x-1)) = \frac{\sin 2}{2}$

Commented [GU7]: write it 2(-1)+(-1) = -3. Do not write -3 directly

Commented [N18R7]: Done

Commented [GU9]: wrong steps. pls discuss

Commented [N110R9]: Done

24. If $|\vec{a}| = 1$, $|\vec{b}| = 2$, $\vec{a}\cdot\vec{b} = 4$, $\vec{c} = 2(\vec{a}\times\vec{b}) - 3\vec{b}$. Then $\vec{b}\cdot\vec{c}$ equals:

A. -48

B. -12

C. 12

D. 48

Answer (B)

Solution:

$\vec{c} = 2(\vec{a}\times\vec{b}) - 3\vec{b}$

$\vec{b}\cdot\vec{c} = 2\vec{b}\cdot(\vec{a}\times\vec{b}) - 3|\vec{b}|^2$

$\vec{b}\cdot\vec{c} = -3|\vec{b}|^2$ $\cdots$ (since $(\vec{a}\times\vec{b})\cdot\vec{b} = 0$)

$\vec{b}\cdot\vec{c} = -12$

25. $\lim_{n\to\infty} \frac{3}{n}\left[4 + \left(2+\frac{1}{n}\right)^2 + \left(2+\frac{2}{n}\right)^2 + \cdots + \left(3-\frac{1}{n}\right)^2\right]$ is:

A. 19

B. 21

C. -19

D. 0

Answer (A)

Solution:

$\lim_{n\to\infty} \frac{3}{n}\left[4 + \left(2+\frac{1}{n}\right)^2 + \left(2+\frac{2}{n}\right)^2 + \cdots + \left(3-\frac{1}{n}\right)^2\right]$ $\cdots$ (given)

we can rewrite the above equation as

$\lim_{n\to\infty}\frac{3}{n}\left[\left(2+\frac{0}{n}\right)^2+\left(2+\frac{1}{n}\right)^2+\left(2+\frac{2}{n}\right)^2+\cdots+\left(2+\left(\frac{n-1}{n}\right)\right)^2\right]$

$\Rightarrow \lim_{n\to\infty}\frac{3}{n}\sum_{r=0}^{n-1}\left(2+\frac{r}{n}\right)^2$

$\frac{r}{n}\to x$

$\frac{1}{n}\to dx$

$\frac{0}{n}<\frac{r}{n}<\frac{n-1}{n}$

$\Rightarrow 0<\frac{r}{n}<1-\frac{1}{n}$

$\Rightarrow \lim_{n\to\infty}0<\lim_{n\to\infty}\frac{r}{n}<\lim_{n\to\infty}1-\frac{1}{n}$

$\Rightarrow 0<\lim_{n\to\infty}\frac{r}{n}<1$

$\Rightarrow \lim_{n\to\infty}\frac{3}{n}\sum_{r=0}^{n-1}\left(2+\frac{r}{n}\right)^2=3\int_0^1(2+x)^2\,dx$

$=3\cdot\left[\frac{(2+x)^3}{3}\right]_0^1$

$=27-8$

$\Rightarrow \lim_{n\to\infty}\frac{3}{n}\sum_{r=0}^{n-1}\left(2+\frac{r}{n}\right)^2=19$

Commented [GU11]: here, write limit n tends to infinity (r/n)

Commented [N112R11]: Done

26. Let $f(x)=\sqrt{3-x}+\sqrt{x+2}$. The range of $f(x)$ is:

A. $(2\sqrt{2},\sqrt{10})$

B. $(\sqrt{5},\sqrt{10})$

C. $(\sqrt{2},\sqrt{7})$

D. $(\sqrt{7},\sqrt{10})$

Answer (B)

Solution:

$y=\sqrt{3-x}+\sqrt{x+2}$

$y'=\frac{1}{2\sqrt{3-x}}(-1)+\frac{1}{2\sqrt{x+2}}=0$

$\Rightarrow \sqrt{3-x}=\sqrt{x+2}$

$\Rightarrow x=\frac{1}{2}$

$y\left(\frac{1}{2}\right)=\sqrt{\frac{5}{2}}+\sqrt{\frac{5}{2}}$

$y_{\max}=\sqrt{10}$

$y_{\min}$ at $x=-2$ or $x=3=\sqrt{5}$

$\therefore y\in[\sqrt{5},\sqrt{10}]$

27. The value of $\tan^{-1}\left(\frac{1}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{1}{1+a_2a_3}\right)+\cdots+\tan^{-1}\left(\frac{1}{1+a_{2021}a_{2022}}\right)$. If $a_1=1$ and a_i are consecutive natural numbers

A. $\frac{\pi}{4} - \cot^{-1} 2021$

B. $\frac{\pi}{4} - \cot^{-1} 2022$

C. $\frac{\pi}{4} - \tan^{-1} 2021$

D. $\frac{\pi}{4} - \tan^{-1} 2022$

Answer (B)

Solution:

$\tan^{-1}\left(\frac{a_2-a_1}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{a_3-a_2}{1+a_2a_3}\right) + \cdots + \tan^{-1}\left(\frac{a_{2022}-a_{2021}}{1+a_{2021}a_{2022}}\right)$.

$= (\tan^{-1} a_2 - \tan^{-1} a_1) + (\tan^{-1} a_3 - \tan^{-1} a_2) + \cdots + (\tan^{-1} a_{2022} - \tan^{-1} a_{2021})$

$= \tan^{-1}(a_{2022}) - \tan^{-1}(a_1))$

As $a_1 = 1, a_2 = 2 \cdots, a_{2022} = 2022$

$= \tan^{-1}(2022) - \tan^{-1}(1))$

$= \tan^{-1}(2022) - \frac{\pi}{4}$

$= \frac{\pi}{2} - \cot^{-1}(2022) - \frac{\pi}{4}$

$= \frac{\pi}{4} - \cot^{-1}(2022)$

28. Let $P = \left(8\sqrt{3} + 13\right)^{13}$, $Q = \left(6\sqrt{2} + 9\right)^{9}$ then : (where [.] represents G.I.F.)

A. $[P]$ = odd, $[Q]$ = even

B. $[P]$ = even, $[Q]$ = odd

C. $[P]$ = odd, $[Q]$ = odd

D. $[P] + [Q]$ = even

Answer (B)

Solution:

Let $P = I + f_1 = \left(8\sqrt{3} + 13\right)^{13}$ such that $(0 < f_1 < 1)$ and *Let* $f_1' = \left(8\sqrt{3} - 13\right)^{13}$ such that $(0 < f_1' < 1)$.

$I + f_1 - f_1' = \left(8\sqrt{3} + 13\right)^{13} - \left(8\sqrt{3} - 13\right)^{13}$

$= 2\left({}^{13}C_1\left(8\sqrt{3}\right)^{12}(13)^1 + {}^{13}C_3\left(8\sqrt{3}\right)^{10}(13)^3 + {}^{13}C_5\left(8\sqrt{3}\right)^{8}(13)^5 + \cdots + {}^{13}C_{13}\left(8\sqrt{3}\right)^{0}(13)^{13}\right)$

$I + f_1 - f_1' = 2p$

Since $-1 < f_1 - f_1' < 1 \Rightarrow f_1 - f_1' = 0$

$\Rightarrow I_1 = 2p$

So, I_1 is even.

Let $Q = I_2 + f_2$ such that $(0 < f_2 < 1)$

Also, let $f_2' = \left(9 - 6\sqrt{2}\right)^{9}$

$I_2 + f_2 + f_2' = \left(9 + 6\sqrt{2}\right)^{9} + \left(9 - 6\sqrt{2}\right)^{9}$

$= 2\left({}^{9}C_0(9)^9\left(6\sqrt{2}\right)^{0} + {}^{9}C_2(9)^7\left(6\sqrt{2}\right)^{2} + {}^{9}C_4(9)^5\left(6\sqrt{2}\right)^{4} + \cdots + {}^{9}C_8(9)^1\left(6\sqrt{2}\right)^{8}\right)$

$I_2 + f_2 + f_2' = 2p$ where $p \in Z$

$0 < f_2 + f_2' < 2$

Commented [GU13]: it will be (tan^(-1)a2 - tan^(-1)a1) + (tan^(-1)a3 - tan^(-1)a2) + (tan^(-1)a4 - tan^(-1)a3+..... (tan^(-1)a2022 - tan^(-1)a2021)

Commented [N114R13]: Done

Commented [GU15]: here write, P = I + f1 = (8sqrt3+13)^13 and f1' = (8sqrt3-13)^13 such that f1 anf f1' is between 0 and 1

Commented [N116R15]: Done

Commented [GU17]: it should be I + f1 - f1'

Commented [GU18R17]: Correct R.H.S also accordingly

Commented [N119R17]: Done

$f_2 + f_2' = 1$

$I_2 + 1 = 2p$

$\Rightarrow I_2 = 2p - 1$

$\Rightarrow [Q] = \text{odd number}$

Commented [GU20]: small small mistakes in whole solution. Pplease discuss to do correction

29. Let p: I am well.,

q: I will not take rest.

r: I will not sleep properly,

then "If I am not well then I will not take rest and I will not sleep properly" is logically equivalent to:

A. $(\sim p \to q) \vee r$

B. $\sim p \to (q \wedge r)$

C. $(\sim p \wedge q) \to r$

D. $(\sim p \vee q) \to r$

Answer (B)

Solution:

$\sim p$: I am not well

q: I will not take rest.

r: I will not sleep properly

I will not take rest and I will not sleep properly $\equiv q \wedge r$

If I am not well then I will not take rest and I will not sleep properly $\equiv \sim p \to (q \wedge r)$

30. q is maximum value of P lying in interval $[0, 10]$, roots of $x^2 - Px + \frac{5P}{4} = 0$ are having rational roots. Find area of region $S: \{0 \le y \le (x - q)^2\}$

A. 243

B. 723

C. 81

D. 3

Answer (A)

Solution:

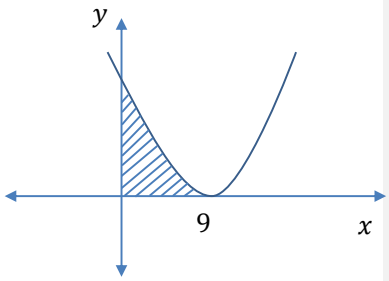
$D = P^2 - 5P$ must be perfect square i.e possible when $P = 9$

Region for $0 \le y \le (x - 9)^2$ is in 1st quadrant

$A = \int_0^9 (x - 9)^2 dx$

$A = \left[\frac{(x-9)^3}{3}\right]_0^9$

$A = 0 + \frac{9^3}{3} = 243$ sq. unit



31. If $\frac{dy}{dx} = -\frac{3x^2+y^2}{3y^2+x^2}$, $y(1) = 0$ then $f(x)$ is:

A. $\log(x+y)+\frac{2xy}{(x+y)^2}=0$

B. $\log(x+y)-\frac{2xy}{(x+y)^2}=0$

C. $3=(3y^2-2xy+3x^2)(x+y)^2$

D. $3=(3y^2-2xy+3x^2)(x+y)$

Answer (A)

Solution:

$\frac{dy}{dx}=-\frac{3x^2+y^2}{3y^2+x^2}=-\frac{3+\left(\frac{y}{x}\right)^2}{3\left(\frac{y}{x}\right)^2+1}$

Let $\frac{y}{x}=u$

$\frac{dy}{dx}=u+x\frac{du}{dx}$

$u+x\frac{du}{dx}=\frac{-(3+u^2)}{3u^2+1}$

$x\frac{du}{dx}=\frac{-(3+u^2)-u(3u^2+1)}{3u^2+1}$

$x\frac{du}{dx}=\frac{-[3u^3+u^2+u+3]}{(3u^2+1)}$

$x\frac{du}{dx}=\frac{-(u+1)(3u^2-2u+3)}{(3u^2+1)}$

$\int\frac{(3u^2+1)}{(u+1)(3u^2-2u+3)}du=-\int\frac{dx}{x}$

$\int\left(\frac{\frac{1}{2}}{u+1}+\frac{\frac{1}{4}(6u-2)}{3u^2-2u+3}\right)du=-\int\frac{dx}{x}$

$\frac{1}{2}\ln(x+y)-\frac{1}{2}\ln x+\frac{1}{4}\ln(3y^2-2xy+3x^2)-\frac{1}{4}\times 2\ln x=-\ln x+C$

$\ln(x+y)^2+\ln(3y^2-2xy+3x^2)=C$

$(x+y)^2(3x^2-2xy+3y^2)=C$

$y(1)=0\Rightarrow C=3$

$(x+y)^2(3x^2-2xy+3y^2)=3$

Commented [T21]: This is zero

Commented [N122R21]: Done

32. Two A.P's are given as under 3, 7, 11, ⋯ and 1, 6, 11, 16, ⋯Then 8^{th} common term that is appearing in both the series is _____.

Answer (151)

Solution:

First common term is 11 and common terms will appear in an A.P having common difference as LCM of $(4,\ 5)=20$

$T_8=11+(8-1)20$

$T_8=151$

33. Using the digits 1, 2, 2, 2, 3, 3, 5 number of 7-digit odd numbers that can be formed is _____.

Answer (240)

Solution:

We need 7-digit odd numbers,

Hence the unit digit will any one of $\{1,3\ 5\}$

$\_\_\_\_\_1$

Total numbers with unit digit $1=\frac{6!}{2!3!}=60$

$\_\_\_\_\_3$

Total numbers with unit digit $3=\frac{6!}{3!}=120$

$\_\_\_\_\_5$

Total numbers with unit digit $5 = \frac{6!}{3!2!} = 60$
Total 7- digit odd numbers $= 60 + 120 + 60 = 240$

34. 50^{th} Root of x is 12.
50^{th} Root of y is 18.
Reminder when $x + y$ is divided by 25 is _____.

Answer (23)

Solution:

$x + y = 12^{50} + 18^{50} = 144^{25} + 324^{25}$
$= (25k_1 - 6)^{25} + (25k_2 - 1)^{25}$
$= 25\lambda - 6^{25} - 1$
$6^{25} + 1 = (6^5)^5 + 1$
$= (7776)^5 + 1$
$= (25\lambda_1 + 1)^5 + 1$
$= 25p + 2$
$\Rightarrow 12^{50} + 18^{50} = 25\lambda - (25p + 2) = 25\lambda - 25p - 2 = 25\lambda - 25p - 25 + 23 = 25n + 23 \; where \; n = \lambda - p - 1$
$\Rightarrow$ Reminder$= 23$

35. Let $a = \{1,3,5,\ldots,99\}$ & $b = \{2,4,6,\ldots,100\}$ The number of ordered pairs (a, b) such that $a + b$ when divided by 23 leaves remainder by 23 leaves remainder 2 is _____.

Answer (109)

Solution:

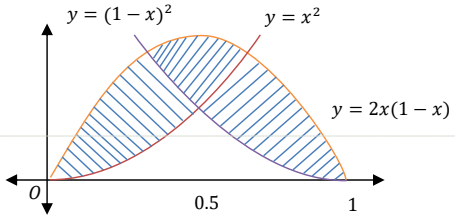
$a + b = 23\lambda + 2$ where $\lambda = 0,1,2,\ldots$
but λ can't be even. So, if
if $\lambda = 1$ $(a, b) \rightarrow 12$ pairs
if $\lambda = 3$ $(a, b) \rightarrow 35$ pairs
if $\lambda = 5$ $(a, b) \rightarrow 42$ pairs
if $\lambda = 7$ $(a, b) \rightarrow 19$ pairs
if $\lambda = 9$ $(a, b) \rightarrow 0$ pairs
Total $= 12 + 35 + 42 + 19 = 108$ ordered pairs

36. If area of the region bounded by the curves $y = x^2$, $y = (1 - x)^2$ and $y = 2x(1 - x)$ is A, then the value of $540A$ is _____.

Answer (135)

Solution:

$A = \int_0^1 2x(1-x)dx - \int_0^{\frac{1}{2}} x^2 dx - \int_{\frac{1}{2}}^1 (1-x)^2 dx$
$= \left[x^2 - \frac{2x^2}{3}\right]_0^1 - \left|\left[\frac{x^3}{3}\right]_0^{\frac{1}{2}}\right| + \left[\frac{(1-x)^3}{3}\right]_{\frac{1}{2}}^1$
$= \frac{1}{4}$
$\Rightarrow 540A = 540 \times \frac{1}{4} = 135$



37. $A = \{2,4,6,8,10\}$ Then the total no of functions defined on A such that $F(m \cdot n) = F(m) \cdot F(n)$, $m, n \in A$ are _____.

Answer (25)

Commented [T23]: Here, mention x+y = 12^50 + 18^50
Commented [N124R23]: Done
Commented [T25]: It will be 2
Commented [N126R25]: Done
Commented [T27]: After this step. Add one step. Please discuss that
Commented [T28]: In figure, we need to share more area. Pls discuss what to shade
Commented [N129R28]: Done
Commented [T30]: Limits are wrong
Commented [N131R30]: Done

Solution:

$f(\mathrm{m} \cdot n) = f(m) \cdot f(n),\ m, n \in A$
$f(x) = x^k, k \in R$
$f(2) = 2^k$ can be connected to 5 objects
$f(4) = 4^k$ can be connected to 5 objects
$f(6) = 6^k$ can be connected to 5 objects
$f(8) = 8^k$ can be connected to 5 objects
$f(10) = 10^k$ can be connected to 5 objects
Total functions$= 5 \times 5 = 25$