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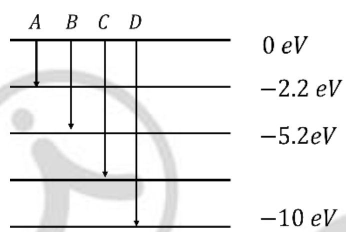
JEE Main 2023 (Memory based)

25 January 2023 - Shift 2

Answer & Solutions

PHYSICS

1. The diagram shown represents different transitions of electron (A, B, C, D) between the energy level with energies mentioned. Among the shown transitions which transition will generate photon of wavelength $124.1\ nm.(hc = 1241\ eVnm)$.



   A. A
   B. B
   C. C
   D. D

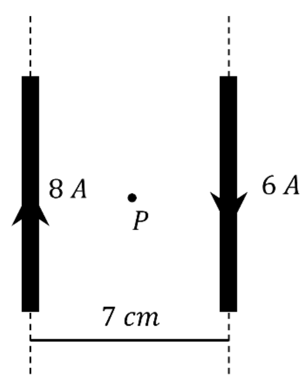
Answer (D)

Solution:

$$\Delta E = \frac{hc}{\lambda} = \frac{1241}{124.1} = 10eV$$

Only option D has energy to produce this wavelength.

2. Two straight wires placed parallel to each other are carrying currents as shown. P is equidistant from the wires. Find the magnetic field at point P.



   A. $8 \times 10^{-5}\ T$
   B. $8 \times 10^{-7}\ T$
   C. $16 \times 10^{-5}\ T$
   D. $2 \times 10^{-5}\ T$

Answer (A)

Solution:

$$B_{net} = \frac{\mu_0 i_1}{2\pi r_1} + \frac{\mu_0 i_2}{2\pi r_2}$$

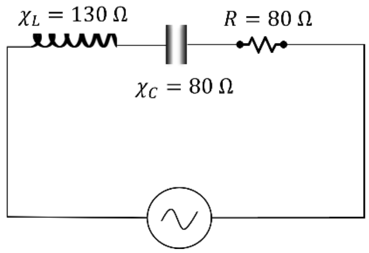
$$= \frac{2 \times 10^{-7}}{3.5 \times 10^{-2}}[8 + 6]\ T$$

$$= \frac{2 \times 10^{-7} \times 14}{3.5 \times 10^{-2}}\ T$$

$$= 8 \times 10^{-5}\ T$$

3. For a LCR series circuit $\chi_L = 130\ \Omega$, $\chi_C = 80\ \Omega$ and $R = 80\ \Omega$. The value of power factor of the circuit is equal to:

A. $\sqrt{54}/9$
B. $8/\sqrt{89}$
C. $8/13$
D. $7/9$

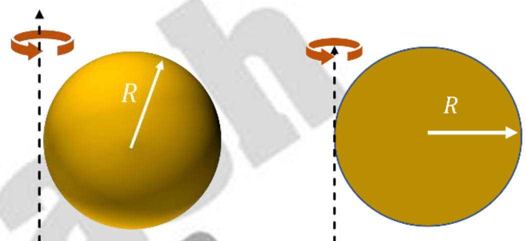


Answer (B)

Solution:

$$\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{(\chi_L - \chi_C)^2 + R^2}}$$
$$\cos\phi = \frac{80}{\sqrt{(130-80)^2 + 80^2}}$$
$$\cos\phi = \frac{80}{\sqrt{2500 + 6400}} = \frac{8}{\sqrt{89}}$$

4. A disk & a solid sphere of same radius are rotated as show in the figure. If masses of disk & solid sphere are $4\ kg$ & $5 kg$ respectively then $\frac{I_{disc}}{I_{solid\ sphere}} =$



A. $7/5$
B. $25/28$
C. $5/7$
D. $28/25$

Answer (C)

Solution:

Using parallel axis theorem,

$$I_{Solid\ sphere} = \left(\frac{2}{5}mR^2 + mR^2\right)$$
$$(m = 5\ Kg)$$
$$= \left(\frac{7}{5}mR^2\right) = 7R^2$$

$$I_{disc} = \left(\frac{1}{4}mR^2 + mR^2\right)$$
$$(m = 4\ Kg)$$
$$= \left(\frac{5}{4}mR^2\right) = 5R^2$$

5. Two projectiles are thrown at an angle of projection α and β with the horizontal. If $\alpha + \beta = 90°$ then ratio of range of two projectiles on horizontal plane is equal to

A. $1:1$
B. $2:1$
C. $1:2$
D. $1:3$

Answer (A)

Solution:

Range of the first projectile

$$R_1 = \frac{u^2 \sin 2\alpha}{g}$$

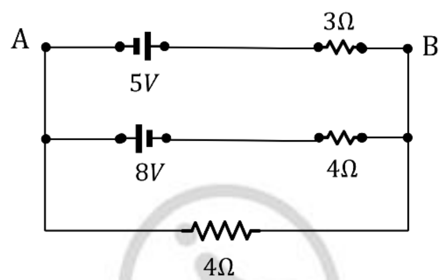
Range of the Second projectile

$$R_2 = \frac{u^2 \sin 2\beta}{g} = \frac{u^2 \sin 2(90-\alpha)}{g} = \frac{u^2\, sin\, 2\alpha}{g}$$

So, $R_1 = R_2$

$R_1 : R_2 = 1 : 1$

6. In the circuit shown , the current $(in\ A)$ through the 4Ω resistor connected across A & B is $\frac{1}{n}$ Amperes. Find n



Answer (10)

Solution:

For the equivalent cell of the combination

$$r_{eq} = \frac{3 \times 4}{3+4} = \frac{12}{7}\ \Omega$$

$$E_{eq} = \left(\frac{8}{4} - \frac{5}{3}\right) \times \frac{12}{7}\ V = \frac{4}{7}\ V$$

Current in the external $4\ \Omega$ resistor

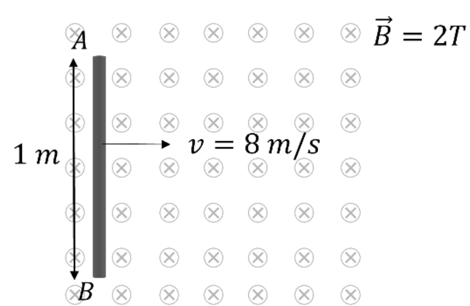
$$I = \frac{E_{eq}}{r_{eq}+4} = \frac{4}{7\left(\frac{12}{7}+4\right)} = \frac{1}{10}\ A$$

$n = 10$

7. A metal rod of length $1\ m$ is moving perpendicular to its length with $8\ m/s$ velocity along positive x –axis. If a magnetic field $B = 2T$ exist perpendicular to the plane of motion. Find the emf induced between the 2 end of rod.

Answer (16)

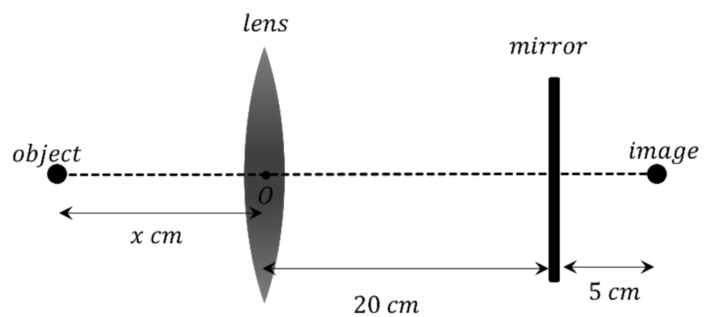
Solution:



$$|V_A - V_B| = Bvl$$
$$= 2 \times 8 \times 1$$
$$= 16\ V$$

8. In the arrangement shown. The image shown is formed after refraction from lens and reflection from mirror. If the

focal length of lens is $10\ cm$, find x.



Answer (30)

Solution:

Image formed by mirror I is $5\ cm$ behind the mirror

$$r_{eq} = \frac{3 \times 4}{3+4} = \frac{12}{7}\ \Omega$$

Image formed by lens I_{lens} must be $5\ cm$ in front of the mirror

For the lens,

$u = -x\ cm\ v = 15\ cm\ f = 10\ cm$

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\Rightarrow \frac{1}{15} - \frac{1}{-x} = \frac{1}{10}$

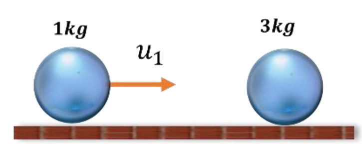
$\Rightarrow x = 30$

9. A particle of mass $1\ kg$ is moving with a velocity towards a stationary particle of mass $3\ kg$. After collision, the lighter particle returns along same path with speed $2\ m/s$. If the collision was elastic, then speed of $1\ kg$ particle before collision is ______________ m/s.

Answer (4)

Solution:

$m_1 = 1\ kg$ and $m_2 = 3kg$



Before collision

Conserving linear momentum

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

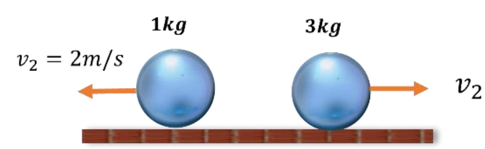
$u_1 + 0 = 2 + 3v_2$ -------------(1)

For elastic collision

$e = \frac{velocity\ of\ separation}{velocity\ of\ approach} = 1$

$\Rightarrow \frac{v_1 + v_2}{u_1} = 1$

$\Rightarrow 2 + v_2 = u_1$ ------------(2)



After collision

From (1) and (2)

$u_1 = 4\ m/s$

10. A wire with resistance $5\ \Omega$ is redrawn to increase its length 5 times. What is the final resistance of the wire?

A. $25\ \Omega$
B. $16\ \Omega$
C. $125\ \Omega$
D. $32\ \Omega$

Answer (C)

Solution:

Resistance of wire can be given as:
$R = \frac{\rho l}{A} = 5\ \Omega$
Volume is constant So,
$l_0 A_0 = 5 l_0 A$
$A = \frac{A_0}{5}$
$R' = \rho \frac{5 l_0}{A_0/5} = \frac{25 \rho l_0}{A_0} = 25\ R = 125\ \Omega$

11. Find the velocity of the particle if the position of the particle is given by $x = 2t^2$ at $t = 2\ sec$.

A. $8\ m/s$
B. $4\ m/s$
C. $16\ m/s$
D. $32\ m/s$

Answer (A)

Solution:
Given: $x = 2t^2$
$\frac{dx}{dt} = 4t$
$v = 4t$
v (at $t = 2\ s$) $= 8\ m/s$

12. A particle performing SHM with amplitude A starts from $x = 0$ and reaches $x = A/2$ in $2\ s$. Find the time required for the particle to go from $x = A/2$ to $x = A$.

A. $1.5\ s$
B. $4\ s$
C. $6\ s$
D. $1\ s$

Answer (B)

Solution:

Let equation of SHM be: $x = A \sin\left(\frac{2\pi}{T} t\right)$
Time to go from $x = 0\ to\ \ x = A/2$
$t_1 = \frac{T}{12}$
Time to go from $x = A/2\ to\ \ x = A$
$t_2 = \frac{T}{4} - \frac{T}{12} = \frac{T}{6}$
$\frac{t_2}{t_1} = 2 \Rightarrow t_2 = 2 \times 2\ s = 4\ s$

13. An object of mass m is placed at a height R_e from the surface of the earth. Find the increase in potential energy of the object if the height of the object is increased to $2R_e$ from the surface. (R_e: $Radius\ of\ the\ earth$)

A. $\frac{1}{3}mgR_e$

B. $\frac{1}{6}mgR_e$

C. $\frac{1}{2}mgR_e$

D. $\frac{1}{4}mgR_e$

Answer (B)

Solution:

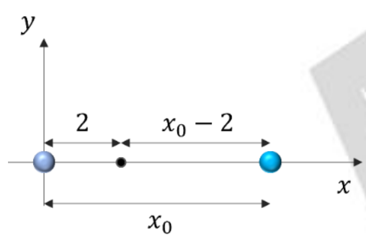
$$U_i = -\frac{GM_e m}{R_e + R_e}$$
$$U_f = -\frac{GM_e m}{R_e + 2R_e}$$
$$\Delta U = U_f - U_i = \frac{GM_e m}{6R_e} = \frac{mgR_e}{6}$$

14. A charge of $10\ \mu C$ is placed at origin. Where should a charge of $40\ \mu C$ be placed on $x-$axis such that electric field is zero at $x = 2$.

A. $x = -2$
B. $x = 4$
C. $x = 6$
D. $x = 2$

Answer (C)

Solution:



For electric field to be zero:

$$\frac{1}{4\pi\epsilon_0} \times \frac{10}{2^2} = \frac{1}{4\pi\epsilon_0} \times \frac{40}{(x_0 - 2)^2}$$
$$x_0 - 2 = 4$$
$$x_0 = 6$$

15. What will be the molar specific heat capacity of an isochoric process of a diatomic gas if it has additional vibrational mode?

A. $\frac{5}{2}R$
B. $\frac{3}{2}R$
C. $\frac{7}{2}R$
D. $\frac{9}{2}R$

Answer (C)

Solution:

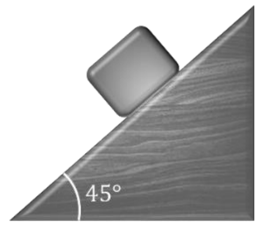
For each additional vibrational mode degree of freedom is increased by 2 so new degree of freedom

$f = 3 + 2 + 2 = 7$

$C_V = \frac{f}{2} \times R = \frac{7}{2} R$

Volume is constant in isochoric process.

16. A block is placed on a rough inclined plane with $45°$ inclination. If minimum force required to push the block up the incline is equal to 2 times the minimum force required to slide the block down the inclined plane, then find the value of coefficient of friction between block and incline.



A. 0.25
B. 1
C. 2
D. 3

Answer (B)

Solution:

$F_{up} = mg \sin\theta + \mu mg \cos\theta$

$F_{down} = \mu mg \cos\theta - mg \sin\theta$

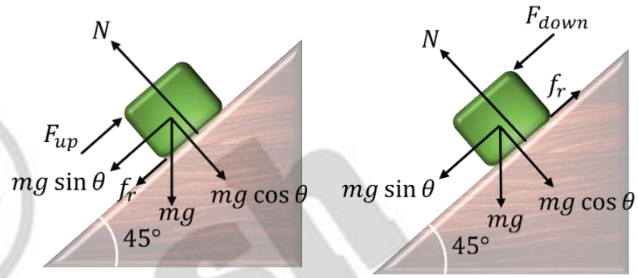
$F_{up} = 2F_{down}$

$mg \sin\theta + \mu mg \cos\theta = 2(\mu mg \cos\theta - mg \sin\theta)$

$3 \sin\theta = \mu \cos\theta$

$\mu = 3 \tan\theta$

$\mu = 3 \tan 45° = 1$



17. Correctly match the two lists:

List I	List II
Physical Quantity	**Dimensions**
P. Young's Modulus	A. $[ML^2T^{-1}]$
Q. Planck's Constant	B. $[ML^{-1}T^{-2}]$
R. Work function	C. $[ML^{-1}T^{-1}]$
S. Co-efficient of viscosity	D. $[ML^2T^{-2}]$

A. $P \to A,\ Q \to B,\ R \to C, S \to D$
B. $P \to B,\ Q \to A,\ R \to D, S \to C$
C. $P \to D,\ Q \to A,\ R \to C, S \to B$
D. $P \to D,\ Q \to A,\ R \to B, S \to C$

Answer (B)

Solution:

$$[Young's\ Modulus] = \frac{\left[\frac{F}{A}\right]}{\left[\frac{\Delta L}{L}\right]} = [ML^{-1}T^{-2}]$$

$$[Planck's\ Constant] = \frac{[E]}{[f]} = [ML^2T^{-1}]$$

$$[Work\ function] = [ML^2T^{-2}]$$

$$[Co-efficient\ of\ Viscosity] = [ML^{-1}T^{-1}]$$

18. A big drop is divided into 1000 identical droplets. If the big drop had surface energy U_i and all small droplets together had a surface energy U_f, then $\frac{U_i}{U_f}$ is equal to

A. $1/100$
B. 10
C. $1/10$
D. 1000

Answer (C)

Solution:

Volume will remain constant in the process.

$\frac{4}{3}\pi R^3 = 1000 \times \frac{4}{3}\pi r^3 \Rightarrow R = 10r$

Surface energy of big drop,

$U_i = 4\pi R^2 T$

Surface energy of all the small drops,

$U_f = 1000 \times 4\pi r^2 T = 40\pi R^2 T$

Taking the ratio, we get,

$\frac{U_i}{U_f} = \frac{4\pi R^2 T}{40\pi R^2 T} = \frac{1}{10}$

19. Correctly match the two lists

List I	List II
a. Gauss law (electrostatics)	$P. \oint \vec{B}.d\vec{A} = 0$
b. Amperes circuital law	$Q. \oint \vec{B}.d\vec{l} = \mu_o i_{inclosed}$
c. Gauss law (Magnetism)	$R. \oint \vec{E}.d\vec{A} = \frac{q_{in}}{\epsilon_o}$
d. Faraday's law	$S. \epsilon = -\frac{d\phi_B}{dt}$

A. $a \to R,\ b \to Q,\ c \to S, d \to P$
B. $a \to R,\ b \to Q,\ c \to P, d \to S$
C. $a \to R,\ b \to S,\ c \to Q, d \to P$
D. $a \to R,\ b \to S,\ c \to P, d \to Q$

Answer (B)

Solution:

Gauss law (electrostatics)= $\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\epsilon_o}$

Amperes circuital law= $\oint \vec{B}.d\vec{l} = \mu_o i_{inclosed}$

Gauss law (Magnetism)= $\oint \vec{B}.d\vec{A} = 0$

Faraday's law, $\epsilon = -\frac{d\phi_B}{dt}$

20. A stationary nucleus breaks into two daughter nuclei having velocities in the ration $3:2$. find the radius of their nuclear sizes.

A. $\left(\frac{2}{3}\right)^{1/2}$

B. $\left(\frac{2}{3}\right)^{1/3}$

C. $\left(\frac{4}{9}\right)^{1/3}$

D. $\left(\frac{9}{4}\right)^{1/2}$

Answer (B)

Solution:

Applying momentum conservation:

$m_1 v_1 = m_2 v_2$

$$\frac{m_1}{m_2} = \frac{v_2}{v_1} = \frac{2}{3} \dots\dots\dots (1)$$

As nuclear density is constant:

$$\frac{m_1}{m_2} = \frac{V_1}{V_2} = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \left(\frac{r_1}{r_2}\right)^3 \dots\dots (2)$$

From (1) and (2):

$$\frac{r_1}{r_2} = \left(\frac{2}{3}\right)^{1/3}$$

21. Match the two lists:

List I	List II
P. Adiabatic process	A. No work done by or on gas.
Q. Isochoric process	B. Some amount of heat given is converted into internal energy.
R. Isobaric process	C. No heat exchange.
S. Isothermal process	D. No change in internal energy.

A. $P - A,\ Q - B,\ R - C,\ S - D$

B. $P - A,\ Q - C,\ R - D,\ S - B$

C. $P - C,\ Q - A,\ R - B,\ S - D$

D. $P - B,\ Q - D,\ R - C,\ S - A$

Answer (C)

Solution:

Adiabatic $\Rightarrow \Delta Q = 0$

Isochoric$\Rightarrow W = 0$

Isothermal$\Rightarrow \Delta U = 0$

Isobaric$\Rightarrow \Delta Q = \Delta U + W$ (Both ΔU and W are non-zero

1. If H^+ion concentration is increased by a factor of 1000 then pH?

A. Decreased by 3
B. Increased by 3
C. There is no change in pH
D. Decreased by 1

Answer (A)

Solution:

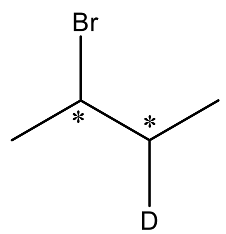
If H^+ion concentration is increased by a factor of 1000 then pH will be decreased by 3.

2. Which of the following has two chiral centres?

A. 2-Bromo-3-deuterobutane
B. 1-Bromo-2-deuterobutane
C. 1-Bromo-3-deuterobutane
D. 1-Bromo-4-deuterobutane

Answer (A)

Solution:



2-Bromo-3-deuterobutane has two chiral centres.

3. Match List – I with List – II

Amine (List – I)	pK_b (aqueous medium) List - II
A. Aniline	1. 9.0
B. Ethanamine	2. 3.29
C. N-Ethylethanamine	3. 3.25
D. N,N-diethylethanamine	4. 3.0

A. A – 1, B – 2, C – 4, D - 3
B. A – 1, B – 4, C – 3, D – 2
C. A – 1, B – 2, C – 3, D – 4
D. A – 2, B – 3, C – 4, D - 1

Answer (A)

Solution:

The order of basicity is C > D > B > A
Therefore pK_b order is C < D < B < A

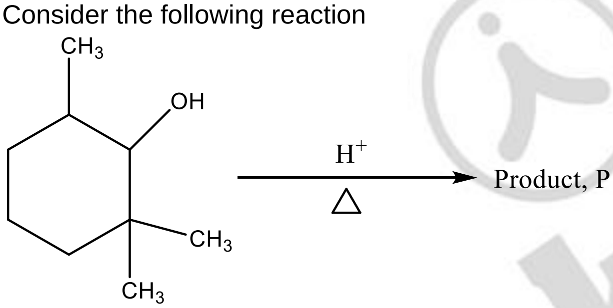
4. Consider the following cell
$Pt/\,H_2\,/\,H^+\,//\,M^{3+}/\,M^+$
(1 bar) (1M)
Then value of $\frac{[M^{3+}]}{[M^+]}$ is 10^x, then find the value of 'X' ?
(Given: $E^o_{M^{3+}/M^+} = 2V$ and $E_{cell} = 1.1V$)

Answer (30)

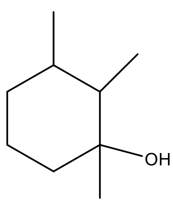
Solution:

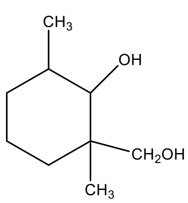
$1.1 = 2 - \frac{0.06}{2} log \frac{[M^{3+}]}{[M^+]}$
$0.9 = 0.03 log \frac{[M^{3+}]}{[M^+]}$
$\frac{[M^{3+}]}{[M^+]} = 10^{30}$
$X = 30$

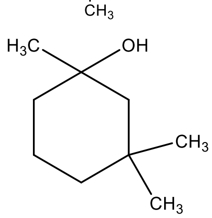
5. Consider the following reaction

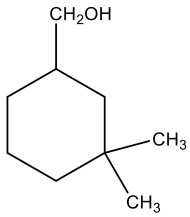


The correct product 'P' is

A. 

B. 

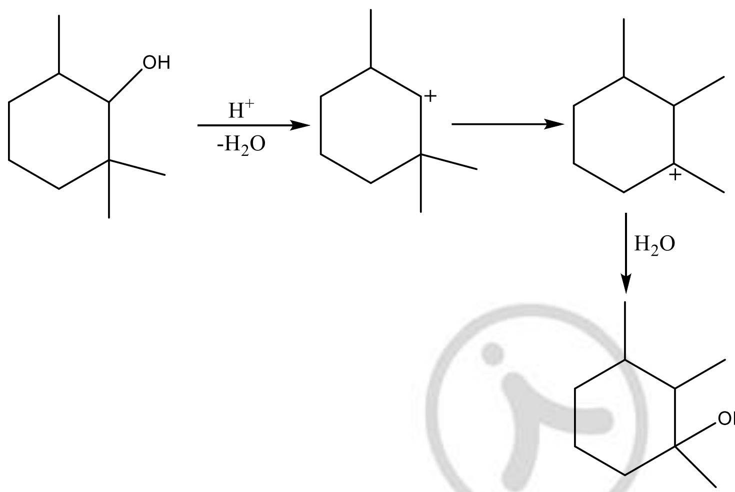
C. 



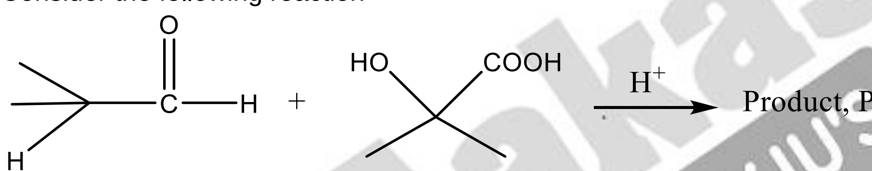
D.

Answer (A)

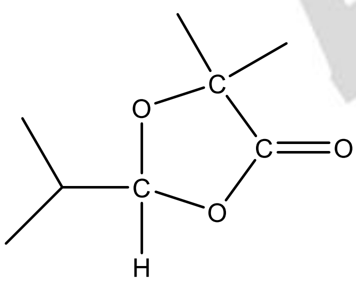
Solution:



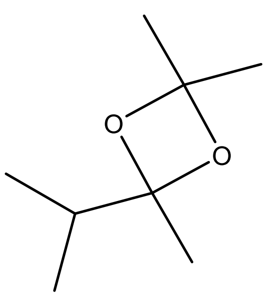
6. Consider the following reaction



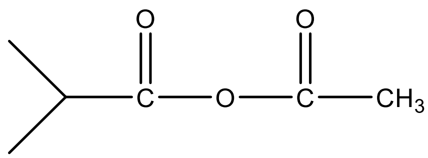
The Product 'P' is?

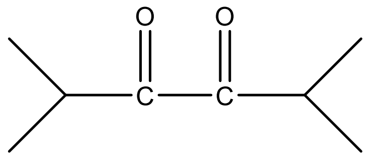


A.



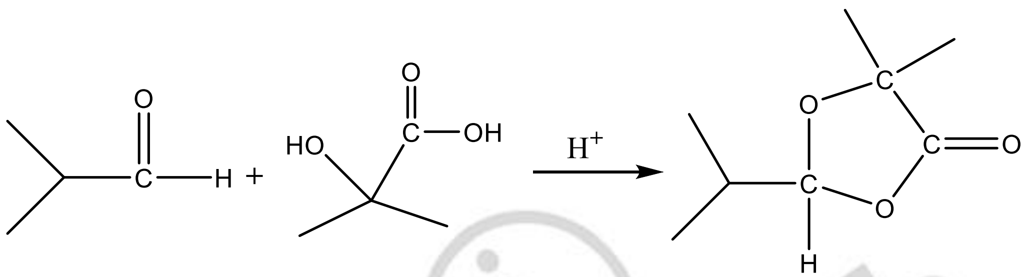
B.

C. 

D. 

Answer (A)

Solution:



7. Match the following

I.	Neoprene	A.	Synthetic wool
II.	Acrolein	B.	Paint
III.	LDP	C.	Flexible pipes
IV.	Glyptal	D.	Gaskets

A. II - D, IV - B III - A, I - C
B. II – C, IV – D, III – A, I - C
C. II – A, IV – B, III – C, I - D
D. II – B, IV – C, III – D, I – A

Answer (C)

Solution:

Neoprene is a synthetic rubber. It is used for manufacturing of Gaskets.
Acrolein is used for making synthetic wool.
LDP is used for making flexible pipes
Glyptal is used for making paints.

8. A hydrocarbon is having molar mass 84 $gmol^{-1}$ and 85.8% C by mass. Calculate the number of H-atomS in one molecule?

A. 8
B. 10
C. 12
D. 14

Answer (C)

Solution:

C	85.8	$\frac{85.8}{12}$	7	1
H	14.2	$\frac{14.2}{1}$	14	2

Empirical formula = .CH_2
Molecular formula = n x empirical formula
$n = molar\frac{mass}{empirical\ formula} = \frac{84}{14} = 6$
Therefore, Molecular formula = C_6H_{12}

9. Find out mass ratio of ethylene glycol (62g) required to make 500 ml, 0.25 M and 250 ml , 0.25 M solution

A. 1:1
B. 1:2
C. 2:1
D. 4:1

Answer (C)

Solution:

Milli moles of ethylene glycol in 1st case = 500 X 0.25
Milli moles of ethylene glycol in 2nd case = 250 X 0.25
Molar ratio = $\frac{50}{25} = \frac{10}{5} = \frac{2}{1}$

10. Assertion [A]: Alkali metals shows characteristic colour in reducing flame
Reason [R]: They can be identified by flame test

A. [A] is correct, while [R] is incorrect
B. [A] is incorrect while [R] is correct
C. [A] and [R] both are correct and [R] is the correct explanation of [A]
D. [A] and [R] both are correct and [R] is not the correct explanation of [A]

Answer (B)

Solution:

Alkali metal show characteristic color in oxidizing flame.

11. How many of the following orbitals is/are considered as axial orbital(s)
$p_x, p_y, p_z, d_{xy}, d_{yz}, d_{zx}, d_{x^2-y^2},\ d_{z^2}$

Answer (5)

Solution:
$p_x, p_y, p_z, d_{x^2-y^2}\ and\ d_{z^2}$ orbitals are called axial orbitals.

12. Consider a mixture of CH_4 and C_2H_4 having volume 16.8 L at 273 K and 1 atm. It undergoes combustion to form CO_2 with total volume 28 L at the same temperature and pressure. If the enthalpy of combustion of CH_4 is -900 KJ/mol and enthalpy of combustion of C_2H_4 is -1400 KJ/mol then find the magnitude of heat released on combustion of given mixture in KJ

Answer (925)

Solution:

$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$
x x

$C_2H_4 + 3O_2 \rightarrow 2CO_2 + 2H_2O$

16.8 – x 2(16.8 – x)

x + 2(16.8 – x) = 28
x = 5.6 L

Heat released = $\frac{1}{4} \times 900 + \frac{1}{2} \times 1400 = 225 + 700$
= 925 KJ

13. Arrange the following elements in increasing order of the metallic character: Si, K, Mg and Be

A. $Si < Mg < Be < K$
B. $Be < Mg < Si < K$
C. $Si < Be < Mg < K$
D. $K < Mg < Si < Be$

Answer (C)

Solution:

Based on the electronegativity of the given elements, the correct increasing order of metallic character is $Si < Be < Mg < K$

14. Which of the following elements is the weakest reducing agent in aqueous solution?

A. Na
B. K
C. Li
D. Rb

Answer (A)

Solution:

As per the standard reduction potential values, Na is the weakest reducing agent.

15. Assertion: Carbon forms two oxides CO and CO_2 where CO is neutral while CO_2 is acidic.
Reason: CO_2 will combine with water to give carbonic acid and CO is soluble in water

A. Assertion and Reason both are correct and Reason is the correct explanation of Assertion.

B. Assertion and Reason both are correct and Reason is not the correct explanation of Assertion.
C. Assertion is true while Reason is false.
D. Assertion is false while Reason is true.

Answer (B)

Solution:

CO_2 will form carbonic acid with water and it is acidic in nature while CO is neutral but there is no relation of neutrality with solubility.

16. Select the correct match.

A. $Hexane-2-one$ and $Hexane-3-one$: Position isomers
B. $Pentane-3-one$ and $Pentane-2-one$: Functional isomers
C. $2-Pentene$ and $1-Pentene$: Metamers
D. $Pentanoic\ acid$ and $Hexanoic\ acid$: Functional isomers

Answer (A)

Solution:

$Hexane-2-one$ and $Hexane-3-one$ are position isomers.

17. Chloride salt of M is treated with excess of $AgNO_3$. It forms curdly white precipitate 'A'. When 'A' is treated with NH_4OH, it forms a soluble salt 'B'. The A and B respectively are

A. $AgCl,\ [Ag(NH_3)_2]^+$
B. $AgBr,\ [Ag(OH)_2]^-$
C. $AgCl,\ [Ag(OH)_4]^{2-}$
D. $AgBr,\ [Ag(OH)_4]^{2-}$

Answer (A)

Solution:

$AgCl$ forms white precipitate which is soluble in NH_4OH.

18. Final oxidation number of Cr when $K_2Cr_2O_7$ is used in acidic medium during titration will be

A. $+6$
B. $+2$
C. $+3$
D. $+4$

Answer (C)

Solution:

$$Cr_2O_7^{2-}+14H^+ +6e^- \rightarrow 2Cr^{3+}+7H_2O$$

19. Assertion: BHA is added to butter to increase shelf life.
Reason: BHA reacts with oxygen more than butter.

A. Assertion and Reason both are correct.
B. Assertion is correct but Reason is not correct.
C. Assertion is incorrect but Reason is correct.
D. Assertion and Reason both are incorrect.

Answer (A)

Solution:

Butyrated hydroxy anisole (BHA) is an antioxidant. It is added to butter to increase its shelf life from months to years. BHA reacts with oxygen present in air in preference to butter. So, both the assertion and reason are correct.

20. Which of the following options contains the correct match?

List I	List II
(A) Adiabatic	(P) $\Delta T = 0$
(B) Isothermal	(Q) Heat exchange is zero
(C) Isochoric	(R) $\Delta P = 0$
(D) Isobaric	(S) Work done is zero

A. $A \rightarrow Q, B \rightarrow P, C \rightarrow S, D \rightarrow R$
B. $A \rightarrow P, B \rightarrow Q, C \rightarrow R, D \rightarrow S$
C. $A \rightarrow S, B \rightarrow R, C \rightarrow Q, D \rightarrow P$
D. $A \rightarrow P, B \rightarrow R, C \rightarrow S, D \rightarrow Q$

Answer (A)

Solution:

Adiabatic → Heat exchange is zero

Isothermal→ $\Delta T = 0$

Isobaric → $\Delta P = 0$

Isochoric → Work done is zero

21. Which of the following options contains the correct match?

List I (Complex)	List II (λ(absorbed))
(A) $[Co(CN)_6]^{3-}$	(P) $535\ nm$
(B) $[Co(NH_3)_6]^{3+}$	(Q) $375\ nm$
(C) $[Co(NH_3)_5Cl]^{2+}$	(R) $600\ nm$

A. $A \rightarrow S, B \rightarrow P, C \rightarrow Q$
B. $A \rightarrow P, B \rightarrow Q, C \rightarrow S$
C. $A \rightarrow Q, B \rightarrow P, C \rightarrow S$
D. $A \rightarrow S, B \rightarrow Q, C \rightarrow P$

Answer (C)

Solution:

Order of CFSE values for the given complexes are:

$[Co(CN)_6]^{3-}$ > $[Co(NH_3)_6]^{3+}$ > $[Co(NH_3)_5Cl]^{2+}$

Hence, λ(absorbed)) will be in the reverse order.

22. For a reaction $A \rightarrow B,$ $k = 2 \times 10^{-3}\, s^{-1}$ Consider the following statements for the above reaction

S1: The reaction is complete in 100 s.
S2: Half-life of the reaction is 500 s.
S3: Unit of rate constant is same as unit of rate
S4: Degree of dissociation is $(1 - e^{-kT})$
S5: It is a zero-order reaction

How many statements are incorrect?

Answer (4)

Solution:

Except S4, all statements are incorrect. As, $[B] = a(1 - e^{-kT})$

$$2 = \frac{[B]}{a} = (1 - e^{-kT})$$

1. The value of $\sum_{k=0}^{6} {}^{51-k}C_3$ is

A. ${}^{52}C_4 - {}^{46}C_4$
B. ${}^{52}C_4 - {}^{45}C_4$
C. ${}^{51}C_4 - {}^{45}C_4$
D. ${}^{51}C_4 - {}^{46}C_4$

Answer (B)

Solution:

$$\sum_{k=0}^{6} {}^{51-k}C_3 = {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + \cdots + {}^{45}C_3 + {}^{45}C_4 - {}^{45}C_4$$

As we know that ${}^{45}C_3 + {}^{45}C_4 = {}^{46}C_4$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{46}C_3 + {}^{46}C_4 - {}^{45}C_4$

As we know that ${}^{46}C_3 + {}^{46}C_4 = {}^{47}C_4$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{47}C_4 - {}^{45}C_4$

Continuing the same process, we have

$= {}^{52}C_4 - {}^{45}C_4$

2. If $f(x) = 2x^n + \lambda$ and $f(4) = 133,\ f(5) = 255$, then sum of positive integral divisors of $f(3) - f(2)$ is :

A. 60
B. 22
C. 40
D. 6

Answer (A)

Solution:

As $f(4) = 133$ and $f(x) = 2x^n + \lambda$

$\Rightarrow 2 \cdot 4^n + \lambda = 133 \cdots (i)$

As $f(5) = 255$

$\Rightarrow 2 \cdot 5^n + \lambda = 255 \cdots (ii)$

Subtracting Equation (i) from Equation (ii)

$2 \cdot (5^n - 4^n) = 122$

$\Rightarrow n = 3$

$f(x) = 2x^3 + \lambda$

$f(3) - f(2) = 2(3^3 - 2^3) = 38$

Divisors are: $1,2,19,38$

Sum= $1 + 2 + 19 + 38 = 60$

3. If $\left|\frac{z+2i}{z-i}\right| = 2$ is a circle, then centre of circle is:

A. $(0,\ 0)$
B. $(0,\ 2)$
C. $(2,\ 0)$
D. $(-2,\ 0)$

Answer (B)

Solution:

$(z+2i)(\overline{z}-2i\ ) = 4(z-i)(\overline{z}+i\ )$

$\Rightarrow\ z\overline{z} + 2i\overline{z} - 2iz + 4 = 4z\overline{z} - 4i\overline{z} + 4iz + 4$

$\Rightarrow\ 3z\overline{z} - 6i\overline{z} + 6iz = 0$

$\Rightarrow\ z\overline{z} - 2i\overline{z} + 2iz = 0$

$\therefore$ Centre $\equiv 2i$

$i.e. (0, 2)$

4. If $\frac{dy}{dt} + \alpha y = \gamma \cdot e^{-\beta t}$, then $\lim\limits_{t\to\infty} y(t)$, where $\alpha > 0,\ \beta > 0,\ \gamma > 0,\ \alpha \neq \beta$, is equal to:

A. 0
B. 1
C. Does not exist
D. αβ

Answer (A)

Solution:

$\frac{dy}{dt} + \alpha y = \gamma e^{-\beta t}$
Integrating factor (I. F.) = $e^{\alpha t}$
Solution of L.D.E.
$ye^{\alpha t} = \gamma \int e^{(\alpha-\beta)t} dt$
$\Rightarrow ye^{\alpha t} = \frac{\gamma}{\alpha-\beta} \cdot e^{(\alpha-\beta)t} + C$
$\Rightarrow y(t) = \frac{\gamma}{\alpha-\beta} \cdot e^{-\beta t} + C \cdot e^{-\alpha t}$
$\Rightarrow \lim\limits_{t\to\infty} y(t) = 0$

5. If $(p \to q)\nabla(p\Delta q)$ is tautology, then operator ∇, Δ denotes:

A. $\Delta \to OR$ and $\nabla \to AND$
B. $\Delta \to AND$ and $\nabla \to OR$
C. $\Delta \to AND$ and $\nabla \to AND$
D. $\Delta \to OR$ and $\nabla \to OR$

Answer (D)

Solution:
$(p \to q)\nabla(p\Delta q) \equiv T$
Only if ∇ is OR and Δ is OR

6. The number of numbers between 5000 & 10000 formed by using the digits $1,3,5,7,9$ without repetition is equal to:

A. 120
B. 72
C. 12
D. 6

Answer (B)

Solution:

The leftmost digit can be chosen in 3 ways i.e. $5,7,9$

Now, the digits can be chosen from remaining digits for remaining places in 4 ways, 3 ways, 2 ways and 1 way.

Total numbers= $3 \times 4 \times 3 \times 2 \times 1 = 72$

7. If $f(x) = \log_{\sqrt{m}}\left(\sqrt{2}(\sin x - \cos x) + m - 2\right)$, the range of $f(x)$ is $[0,\ 2]$, then the value of m is:

A. 3
B. 4
C. 5
D. None

Answer (C)

Solution:

We know that $\sin x - \cos x \in \left[-\sqrt{2},\ \sqrt{2}\right]$

$\log_{\sqrt{m}}\left((\sin x - \cos x) + m - 2\right) \in \left[\log_{\sqrt{m}}(m-4), \log_{\sqrt{m}} m\right]$

$\log_{\sqrt{m}}(m-4) = 0$ & $\log_{\sqrt{m}} m = 2$

$\Rightarrow m = 5$

8. If A be a symmetric matrix and B & C are skew symmetric matrices of same order, then:

A. $A^{13} \cdot B^{26} - B^{26} \cdot A^{13}$ is symmetric.
B. $AC - A$ is symmetric.
C. $A^{13} \cdot B^{26} - B^{26} \cdot A^{13}$ is symmetric.
D. $AC - A$ is skew symmetric.

Answer (C)

Solution:

A is symmetric $\Rightarrow A^{13}$ is symmetric.

B is skew-symmetric $\Rightarrow B^{26}$ is skew-symmetric.

Now, let $A^{13} = P$ and $B^{26} = Q$

$A^{13} \cdot B^{26} - B^{26} \cdot A^{13}$

$= PQ - QP$

Now, $(PQ - QP)^T = (PQ)^T - (QP)^T = Q^T \cdot P^T - P^T \cdot Q^T$

$\Rightarrow QP - PQ = -(PQ - QP)$

$\Rightarrow (A^{13}B^{26} - B^{26}A^{13})^T = -(A^{13}B^{26} - B^{26}A^{13})$

$\therefore (A^{13}B^{26} - B^{26}A^{13})$ is skew-symmetric matrix.

9. Consider the function $f(x) = \begin{cases} (1+|\cos x|)^{\frac{\lambda}{|\cos x|}}, & x < \frac{\pi}{2} \\ \mu, & x = \frac{\pi}{2} \\ e^{\frac{\cot 6x}{\cot 4x}}, & x > \frac{\pi}{2} \end{cases}$

A. $\lambda = \frac{2}{3}$, $\mu = e^{\frac{2}{3}}$

B. $\lambda = e^{\frac{2}{3}}$, $\mu = \frac{2}{3}$

C. $\lambda = \frac{3}{2}$, $\mu = e^{\frac{3}{2}}$

D. $\lambda = e^{\frac{3}{2}}$, $\mu = \frac{3}{2}$

Answer (A)

Solution:

$$\lim_{x\to\frac{\pi}{2}^-} f(x) = e^{\lim_{x\to\frac{\pi}{2}^-} |\cos x|\cdot\frac{\lambda}{|\cos x|}} = e^{\lambda}$$

$\Rightarrow \mu = e^{\lambda}$

$$\lim_{x\to\frac{\pi}{2}^+} f(x) = e^{\lim_{x\to\frac{\pi}{2}^+}\frac{\cot 6x}{\cot 4x}} = e^{\lim_{h\to 0}\frac{\cot 6h}{\cot 4h}} = e^{\frac{2}{3}}$$

$\Rightarrow \mu = e^{\frac{2}{3}}$, $\lambda = \frac{2}{3}$

10. Two dice are rolled. If the probability the sum of the numbers on dice is n, where $n-2$, $\sqrt{3n}$, $n+2$ are in geometric progression, is $\frac{x}{48}$, then the value of x is:

A. 4

B. 12

C. 7

D. 3

Answer (A)

Solution:

As given, $\left(\sqrt{3n}\right)^2 = (n-2)(n+2)$

$\Rightarrow 3n = n^2 - 4$

$\Rightarrow n = 4$, $n = -1$ (Not possible)

Favourable outcomes $(1,2)$, $(2,1)$, $(2,2)$

Total outcomes $6 \times 6 = 36$

Given $\frac{3}{36} = \frac{x}{48}$

$\Rightarrow x = 4$

11. Let $\vec{a} = -\hat{\imath} - \hat{\jmath} + \hat{k}$ such that $\vec{a}\cdot\vec{b} = 1$ and $\vec{a}\times\vec{b} = \hat{\imath} - \hat{\jmath}$ then $\vec{a} - 6\vec{b}$ equals:

A. $3(\hat{\imath} + \hat{\jmath} + \hat{k})$

B. $\hat{\imath} + \hat{\jmath} + \hat{k}$

C. $2(\hat{\imath}+\hat{\jmath}+\hat{k})$
D. $4(\hat{\imath}+\hat{\jmath}+\hat{k})$

Answer (A)

Solution:

$\vec{a}\times\vec{b}=\hat{\imath}-\hat{\jmath}$

$\Rightarrow \vec{a}\times(\vec{a}\times\vec{b})=(-\hat{\imath}-\hat{\jmath}+\hat{k})\times(\hat{\imath}-\hat{\jmath})$

$\Rightarrow (\vec{a}\cdot\vec{b})\vec{a}-(\vec{a}\cdot\vec{a})\vec{b}=\hat{\imath}+\hat{\jmath}+2\hat{k}$

$\Rightarrow 3\vec{b}=-2\hat{\imath}-2\hat{\jmath}-\hat{k}$

$\therefore \vec{a}-6\vec{b}=-\hat{\imath}-\hat{\jmath}+\hat{k}-(-4\hat{\imath}-4\hat{\jmath}-2\hat{k})$

$\Rightarrow \vec{a}-6\vec{b}=3\hat{\imath}+3\hat{\jmath}+3\hat{k}=3(\hat{\imath}+\hat{\jmath}+\hat{k})$

12. $16\int_1^2 \frac{dx}{x^3(x^2+2)^2}$ is equal to:

A. $\frac{11}{12}+\ln 4$

B. $\frac{11}{12}-\ln 4$

C. $\frac{11}{6}-\ln 4$

D. $\frac{11}{6}+\ln 4$

Answer (C)

Solution:

$I=\int \frac{dx}{x^3(x^2+2)^2}$

$=\frac{1}{4}\int\frac{x}{x^2+2}dx+\frac{1}{4}\int\frac{x}{(x^2+2)^2}dx-\frac{1}{4}\int\frac{1}{x}dx+\frac{1}{4}\int\frac{1}{x^3}dx$

$I=\frac{\ln(x^2+2)}{8}-\frac{1}{8(x^2+2)}-\frac{\ln x}{4}-\frac{1}{8x^2}$

$16\int_1^2\frac{dx}{x^3(x^2+2)^2}=2\ln 6-2\ln 3-4\ln 2+\frac{11}{6}$

$=\frac{11}{6}-\ln 4$

13. If $A=\begin{bmatrix}\frac{3}{\sqrt{10}} & \frac{1}{\sqrt{10}}\\ -\frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}}\end{bmatrix}$ and $B=\begin{bmatrix}1 & -i\\ 0 & 1\end{bmatrix}$. If $M=A^TBA$, then the matrix $AM^{2023}A^T$ is:

A. $\begin{bmatrix}1 & -2023i\\ 0 & 1\end{bmatrix}$

B. $\begin{bmatrix}1 & 2023i\\ 0 & 1\end{bmatrix}$

C. $\begin{bmatrix}1 & -2023i\\ 0 & -1\end{bmatrix}$

D. $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

Answer (A)

Solution:

$AA^T=I$

$M=A^TBA$

$$AM^{2023}A^T = A\underbrace{(A^TBA)(A^TBA)(A^TBA)\cdots(A^TBA)}_{2023\ times}A^T$$

$$= B^{2023}$$

$$AM^{2023}A^T = B^{2023}$$

$$B^2 = \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2i \\ 0 & 1 \end{bmatrix}$$

$$B^3 = \begin{bmatrix} 1 & -2i \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -3i \\ 0 & 1 \end{bmatrix}$$

$$B^{2023} = \begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$$

14. The remainder when $(2023)^{2023}$ is divided by 35 is __________.

Answer (7)

Solution:

$2023 \equiv -7(35)$

$(2023)^2 \equiv 14(35)$

$(2023)^4 \equiv -14(35)$

$(2023)^{16} \equiv -14(35)$

.

.

.

$(2023)^{2020} \equiv -14(35)$

and $(2023)^3 \equiv 7(35)$

$\therefore (2023)^{2023} \equiv 7(35)$

$\therefore$ remainder $= 7$

15. If $\int_{\frac{1}{3}}^{3} |\ln x| dx = \frac{m}{n} \ln\left(\frac{n^2}{e}\right)$, then value of $m^2 + n^2 - 5$ is equal to _______.

Answer (20)

Solution:

$$\int_{\frac{1}{3}}^{3} |\ln x| dx = \int_{\frac{1}{3}}^{1} -\ln x\, dx + \int_{1}^{3} \ln x\, dx$$

$$= -[(x\ln x - x)]_{\frac{1}{3}}^{1} + [(x\ln x - x)]_{1}^{3}$$

$$= \frac{2}{3} + \frac{1}{3}\ln\frac{1}{3} + 3\ln 3 - 2$$

$$= \frac{4}{3}(\ln 9 - \ln e)$$

$$= \frac{4}{3}\ln\left(\frac{3^2}{e}\right)$$

$\therefore m = 4, n = 3$

$m^2 + n^2 - 5 = 16 + 9 - 5 = 20$

16. A triangle is formed with x –axis, y –axis & line $3x + 4y = 60$. A point $P(a, b)$ lies strictly inside the triangle such that a is a positive integer and b is a multiple of $'a'$. The numbers of such points (a, b) is ______.

Answer (31)

Solution:

x	y	Points	No. of points
1	$\frac{57}{4}$	$(1,1),(1,2),\ldots,(1,14)$	14
2	$\frac{27}{2}$	$(2,2),(2,4),\ldots,(2,12)$	6
3	$\frac{51}{4}$	$(3,3),\ (3,6),\ldots,(3,12)$	4
4	12	$(4,4),(4,8)$	2
5	$\frac{45}{4}$	$(5,5),\ (5,10)$	2
6	$\frac{21}{2}$	$(6,6)$	2
7	$\frac{39}{4}$	$(7,7)$	1
8	9	$(8,8)$	1
9	$\frac{33}{4}$	0	0
10	$\frac{15}{2}$	0	0

Total points $= 14+6+4+2+2+1+1+1$

$= 31$

17. If $a,\ b, \frac{1}{18}$ are in G.P and $\frac{1}{10}, \frac{1}{a}, \frac{1}{b}$ are in A.P, then the value of $a + 180b$ is __________.

Answer (20)

Solution:

$b^2 = \frac{a}{18}, \frac{2}{a} = \frac{1}{10} + \frac{1}{b}$

$\Rightarrow a = \frac{20b}{10+b}$ or $18b^2 = \frac{20}{10+b}$

$b = 0$ or $9b = \frac{10}{10+b}$

$90b + 9b^2 = 10$

$\Rightarrow 9b^2 + 90b - 10 = 0$

$\therefore a + 180b = 18b^2 + 180b = 20$

18. In a city, 25% of the population is smoker, and a smoker has 27 times more chance of being diagnosed with lung cancer. A person is selected at random and found to be diagnosed with lung cancer. If the probability of him being smoker is, $\frac{k}{40}$. Then the value of k is __________.

Answer (36)

Solution:

Probability of a person being a smoker $= \frac{1}{4}$

Probability of a person being nonsmoker $= \frac{3}{4}$

$P\left(\frac{S}{SC}\right) = \frac{\frac{1}{4} \cdot 27P}{\frac{1}{4} \cdot 27P + \frac{3}{4}P} = \frac{27}{30} = \frac{9}{10} = \frac{36}{40}$

$\Rightarrow k = 36$