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JEE Main 2023 (Memory based)

31 January 2023 - Shift 1

Answer & Solutions

PHYSICS

1. The ratio of molar specific heat capacity at constant pressure (C_p) to that at constant volume (C_v) varies with temperature (T) as:
[Assume temperature to be low]

A. T^0
B. $T^{1/2}$
C. T^1
D. $T^{3/2}$

Answer (A)

Solution:

We know that:

$$\frac{C_p}{C_v} = \frac{f+2}{f} = \gamma = 1 + \frac{2}{f} = constant$$

We take f to be constant for molecule at low temperature (Independent of temperature)

$$\frac{C_p}{C_v} \propto T^0$$

2. A drop of water of $10\ mm$ radius is divided into 1000 droplets. If surface tension of water surface is equal to $0.073\ J/m^2$ then increment in surface energy while breaking down the bigger drop in small droplets as mentioned is equal to

A. $8.25 \times 10^{-5}\ J$
B. $9.17 \times 10^{-4}\ J$
C. $9.17 \times 10^{-5}\ J$
D. $8.25 \times 10^{-4}\ J$

Answer (D)

Solution:

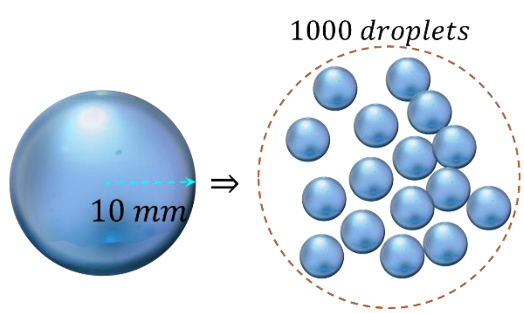
Let the radius of one small droplet is r then:

$$1000 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(10)^3$$

$$\Rightarrow r = 1\ mm$$

$$v_f = 1000 \times 4\pi r^2 T = 1000 \times 4\pi \times 10^{-6} \times 0.073$$

$$v_f = 9.17 \times 10^{-4}\ J$$



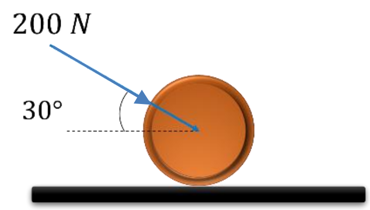
$v_i = 4 \times \pi \times (10^{-2})^2 T = 9.17 \times 10^{-5}\, J$

So,

$\Delta U = 8.25 \times 10^{-4}\, J$

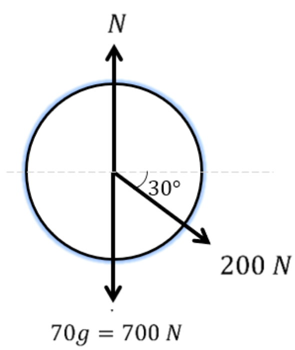
3. A force $200\, N$ is exerted on a disc of mass $70\, kg$ as shown. Find the normal reaction given by ground on the disc.

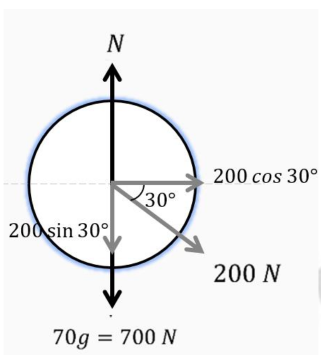
A. $200\, N$
B. $600\, N$
C. $800\, N$
D. $200/\sqrt{3}\, N$



Answer (C)

Solution:





For Equilibrium condition, $\Sigma F_y = 0$
$N = mg + F_{\perp} = 700 + 100 = 800\, N$

4. At depth d from surface of earth acceleration due to gravity is same as its value at height d above the surface of earth. If earth is a sphere of radius $6400\, km$, then value of d is equal to

A. $2975\, km$
B. $3955\, km$
C. $2525\, km$
D. $4915\, km$

Answer (B)

Solution:

Given $g_h = g_d$

We know that:

$$g_0\left(1-\frac{d}{R}\right)=\frac{g_0}{\left(1+\frac{d}{R}\right)^2}$$

$$\left(1+\frac{d}{R}\right)^2\left(1-\frac{d}{R}\right)=1$$

On solving:

$$\frac{d}{R}=0,-\left(\frac{\sqrt{5}+1}{2}\right),\left(\frac{\sqrt{5}-1}{2}\right)$$

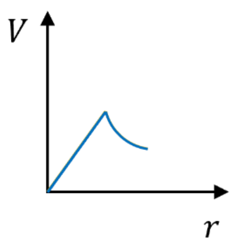
So,

$$d=\left(\frac{\sqrt{5}-1}{2}\right)R$$

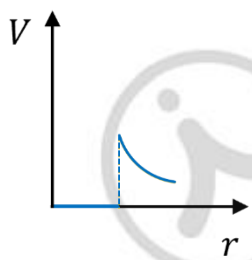
$$d=3955\ km$$

5. Which of the following graphs depicts the variation of electric potential with respect to radial distance from center of a conducting sphere charged with positive charge.

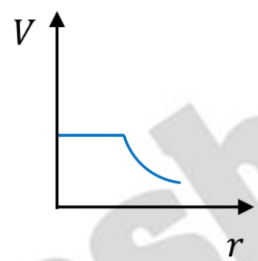
A.



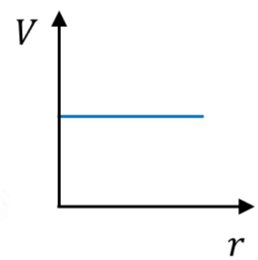
B.



C.



D.

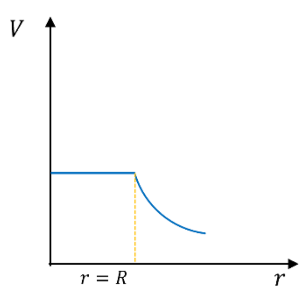


Answer (C)

Solution:

$$V(r)=\begin{cases}\dfrac{q}{4\pi\varepsilon_0 R} & if\ r<R\\ \dfrac{q}{4\pi\varepsilon_0 r} & if\ r>R\end{cases}$$

Where r is the radial distance and R is radius of sphere,
As charge will be on the surface because the sphere is conducting so, graph will be:



6. In a sample of $Hydrogen$ atoms, one atom goes through a transition $n=3\rightarrow ground\ state$ with emitted wavelength λ_1. Another atom goes through a transition $n=2\rightarrow ground\ state$ with emitted wavelength λ_2. Find $\frac{\lambda_1}{\lambda_2}$.

A. $6/5$
B. $5/6$
C. $27/32$
D. $32/27$

Answer (C)

Solution:

Wavelength for transition from $3 \rightarrow Ground\ state$

$$\frac{1}{\lambda_1} = RZ^2\left[1 - \frac{1}{3^2}\right]$$

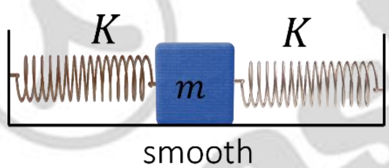
Wavelength for transition from $2 \rightarrow Ground\ state$

$$\frac{1}{\lambda_2} = RZ^2\left[1 - \frac{1}{2^2}\right]$$

Dividing both equations:

$$\frac{\lambda_1}{\lambda_2} = \frac{\left(\frac{3}{4}\right)}{\left(\frac{8}{9}\right)} = \frac{27}{32}$$

7. A block of mass m is connected to two identical springs of force constant K as shown. Find the total number of oscillations of block per unit time.



A. $2\pi\sqrt{\left[\frac{2m}{K}\right]}$

B. $\frac{1}{2\pi}\sqrt{\left[\frac{K}{m}\right]}$

C. $2\pi\sqrt{\left[\frac{m}{2K}\right]}$

D. $\frac{1}{2\pi}\sqrt{\left[\frac{2K}{m}\right]}$

Answer (D)

Solution:

For series combination of springs:

$K_{eq} = K + K = 2K$

$$\omega = \sqrt{\frac{K_{eq}}{m}} = \sqrt{\frac{2K}{m}}$$

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{2K}{m}}\ \text{Oscillation per second}$$

8. Consider the two statements:
Assertion: The beam of electrons shows wave nature and exhibits interference and diffraction.
Reason: Davisson - Germer experiment verified the wave nature of electrons.

A. Both are correct. Reason correctly explains assertion.
B. Both are incorrect.
C. Assertion is correct but reason is incorrect.
D. Both are correct. Reason does not explain assertion.

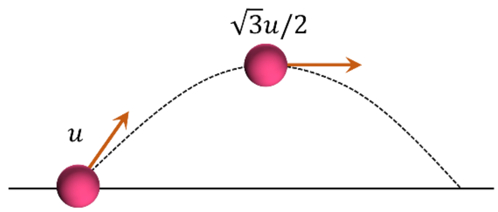
Answer (A)

Solution:

Davisson - Germer experiment verified the wave nature of electrons.

9. A projectile is launched on horizontal surface such that if thrown with initial velocity of u, it has velocity of $\frac{\sqrt{3}u}{2}$ at maximum height. Then time of flight of the projectile is equal to:

A. $\sqrt{3}u/g$
B. $2u/g$
C. u/g
D. $u/2g$



Answer (C)

Solution:

Velocity of ball at maximum height:

$$u\cos\theta = \frac{\sqrt{3}u}{2}$$

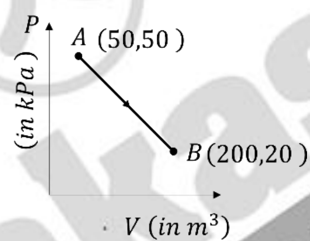
$$\theta = \frac{\pi}{6} \rightarrow angle\ of\ projection$$

Time of flight can be given as:

$$T = \frac{2u\sin\theta}{g} = \frac{u}{g}$$

10. A diatomic gas is taken from point A to point B in a thermodynamic process as described in the pressure−volume graph shown. The change in internal energy is equal to

A. $3.75 \times 10^6\ J$
B. $2.25 \times 10^6\ J$
C. $7.5 \times 10^6\ J$
D. $4.5 \times 10^6\ J$



Answer (A)

Solution:

Change in internal energy
$\Delta U = nC_v\Delta T$

Assuming as to be ideal, $PV = nRT$

$$= \frac{5}{2}\left(P_fV_f - P_iV_i\right) \ldots\ldots\ldots. for\ diatomic\ gas, C_v = \frac{5}{2}R$$

$$= \frac{5}{2}(200 \times 20 \times 10^3 - 50 \times 50 \times 10^3)\ J$$

$$= \frac{5}{2} \times 1500 \times 10^3\ J$$

$$= 3.75 \times 10^6\ J$$

11. Unpolarized light of intensity I_o is incident on a polariser A and subsequently on polariser B whose pass axis is perpendicular to that of A . Now a polariser C is introduced between A and B such that pass axis of C is at $45°$ with the pass axis of A. find the intensity of that comes out of B.

A. $\frac{I_o}{8}$

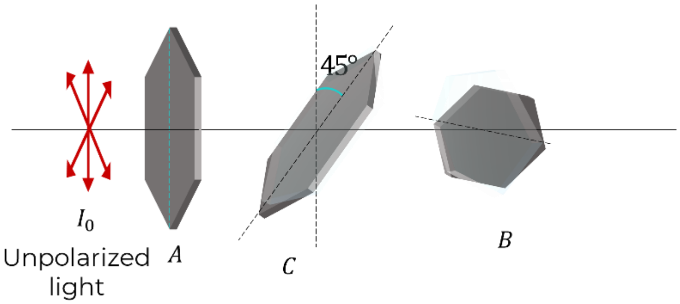
B. $\frac{I_o}{4}$

C. $Zero$

D. $\frac{3I_o}{8}$

Answer (A)

Solution:



Intensity of light passing through A is $I_0/2$

Resultant Intensity can be calculated as:

$$I_{net} = I_0 \times \frac{1}{2} \times \cos^2 45° \times \cos^2 45°$$

12. A bar magnet with magnetic moment of $5\ Am^2$ is lying at stable equilibrium in external uniform magnetic field of strength $0.4\ T$. Work done in slowly rotating the bar magnet to the position of unstable equilibrium is equal to

A. $1\ J$
B. $2\ J$
C. $3\ J$
D. $4\ J$

Answer (D)

Solution:

$$U_i = -MB\cos 0°$$

$$U_f = -MB\cos 180°$$

So,

$$W = \Delta U$$

$$= 2MB = 2 \times 5 \times 0.4$$

$$W = 4\ J$$

13. If n: number density of charge carriers.
A: cross sectional area of conductor
q: charge on each charge carrier
I: current through the conductor
Then the expression of drift velocity is

A. $\frac{nAq}{I}$
B. $\frac{I}{nAq}$
C. $nAqI$
D. $\frac{IA}{nq}$

Answer (B)

Solution:

$$I = nqAv_d$$

$$v_d = \frac{I}{nAq}$$

14. If R, X_L and X_C denote resistance, inductive reactance, and capacitive reactance respectively. Then which of the following options shows the dimensionless physical quantity.

A. $\frac{X_L X_C}{R}$
B. $\frac{R}{\sqrt{X_L X_C}}$

C. $\frac{R}{X_L X_C}$
D. $\frac{R}{(X_L X_C)^2}$

Answer (B)

Solution:

$X_L = Inductive\ reactance = [R] = dimension\ of\ Resistance$
$X_C = Reactive\ reactance = [R] = dimension\ of\ Resistance$

So, option B, $\frac{R}{\sqrt{X_L X_C}}$ is dimensionless.

15. A conductor of length l and cross-sectional are A has drift velocity v_d when used across a potential difference V. When another conductor of same material and length l but double cross-sectional area than first, is used across same potential difference then drift velocity is equal to

A. $v_d/2$
B. v_d
C. $2v_d$
D. $4v_d$

Answer (B)

Solution:

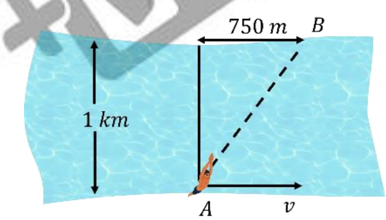
$$I = \frac{V}{R} = \frac{V}{\frac{\rho l}{A}} = \frac{VA}{\rho l}$$

$$neAv_d = \frac{VA}{\rho l}$$

All the parameters remain same except cross sectional area and v_d is independent of cross-sectional area when compared in two different conductors so, v_d remains same.

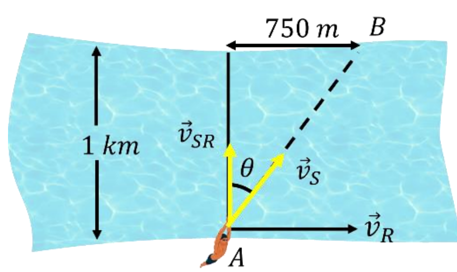
16. A swimmer swims perpendicular to river flow and reaches point B. If velocity of swimmer in still water is $4\ km/h$, find velocity of river flow.

A. $3\ km/hr$
B. $5\ km/hr$
C. $2\ km/hr$
D. $6\ km/hr$



Answer (A)

Solution:



$$\frac{|\vec{v}_R|}{|\vec{v}_{SR}|} = \frac{|\vec{v}_R|}{4} = \tan\theta = \frac{750}{1000}$$

$$|\vec{v}_R| = 3\ km/hr$$

17. A solid sphere is rolling on a smooth surface with kinetic energy $= 7 \times 10^{-3}\,J$. If mass of the sphere is $1\,kg$, then find the speed of the centre of mass in cm/s. (Consider pure rolling)

Answer (10)

Solution:

$$KE = \frac{1}{2}mV_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$$

For pure rolling,

$KE = \frac{1}{2}mV_{cm}^2 + \frac{1}{2} \times \frac{2}{5}mR^2\left(\frac{V_{cm}}{R}\right)^2 \ldots\ldots\ldots\ldots\ldots\ldots \omega = \frac{V}{R}$ and for solid sphere $I = \frac{2}{5}mR^2$

$$7 \times 10^{-3} = \frac{7}{10}mV_{cm}^2$$

$$V_{cm} = \sqrt{10^{-2}} = 10^{-1}\ m/s = 10\ \ cm/s$$

18. A lift of mass $500\ kg$ starts moving downwards with initial speed $2\ m/s$ and accelerates at $2\ m/s^2$. The kinetic energy of the lift when it has moved $6\ m$ down is__________ kJ.

Answer (7)

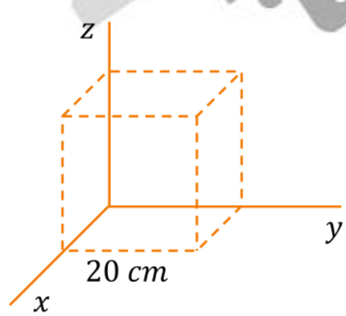
Solution:

$u = 2\ m/s$
$a = 2\ m/s^2$
$s = 6\ m$

For uniform acceleration, $v^2 - u^2 = 2as$
$\Rightarrow v^2 = 2as + u^2 = 2 \times 2 \times 6 + 4 = 28$

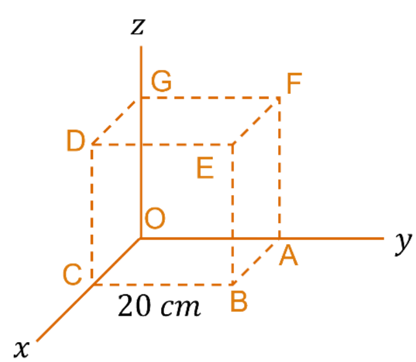
So,
$$\text{K. E.} = \frac{1}{2}Mv^2 = \frac{1}{2} \times 500 \times 28 = 7000\,J = 7\,kJ$$

19. Electric field in a region is $4000x^2\ \hat{\imath}\ N/C$. The flux through the cube is $\frac{x}{5}\ Nm^2/C$. Find x.



Answer (32)

Solution:



$$\vec{E} = 4000\ x^2\ \hat{\imath}$$

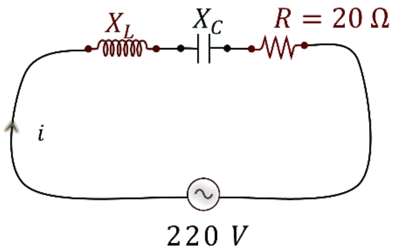
As E_y and E_z are zero,
So, flux through face ABEF, OCDG, EDGF and OABC is zero.

At face OAFG, $x = 0$ so $\phi = 0$
At face EBCD,$x = 0.2\ m$
So,
$\vec{E} = 4000\ x^2\ \hat{\imath} = 4000 \times (0.2)^2\hat{\imath} = 160\ N/C\ \hat{\imath}$
$\phi_{BEDC} = EA = 160 \times (0.2)^2 = 6.4\ Nm^2/C$
$x = 6.4 \times 5\ Nm^2/C = 32\ Nm^2/C$

20. For a series LCR circuit across an AC source, current and voltage are in same phase. Given the resistance is of $20\ \Omega$ and voltage of the source is $220\ V$. Find current (in A) in the circuit.

Answer (11)

Solution:



The given circuit is in resonance. So,
$i = \frac{V}{R} = \frac{220}{20} = 11\ A$

21. For a particle performing SHM, maximum potential energy is $25\ J$. The kinetic energy (in J) at half the amplitude is $x/4$. find x.

Answer (75)

Solution:

$\text{Maximum potential energy} = \frac{1}{2}kA^2 = 25\ J$

$\text{K. E.} = \frac{1}{2}kA^2 - \frac{1}{2}k\left(\frac{A}{2}\right)^2$

$= \frac{1}{2}kA^2\left(\frac{3}{4}\right)$

$= \frac{3}{4} \times 25\ J$

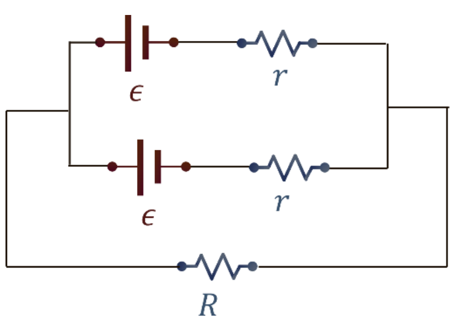
$= \frac{75}{4}\ J$

22. The current through a 5Ω resistance remains same, irrespective of its connection across series or parallel combination of two identical cells. Find the internal resistance $(in\ \Omega)$ of the cell.

Answer (5)

Solution:

When connected in parallel (A):



$\frac{\varepsilon_{eq}}{\frac{r}{2}} = \frac{\varepsilon}{r} + \frac{\varepsilon}{r}$

$\varepsilon_{eq} = \varepsilon$ and $r_{eq} = \frac{r}{2}$

$current, i_A = \frac{\varepsilon}{R + \frac{r}{2}}$

When connected in series (B):

$\varepsilon_{eq} = 2\varepsilon$

$r_{eq} = 2r$

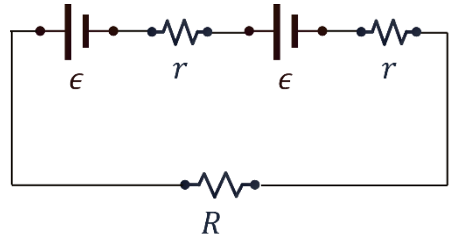
$current, i_B = \frac{2\mathcal{E}}{R+2r}$

As, $i_A = i_B$

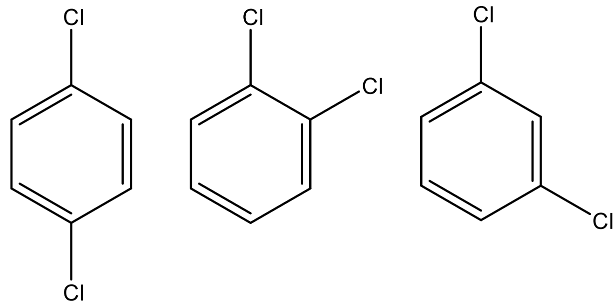
$\Rightarrow \frac{\mathcal{E}}{R+\frac{r}{2}} = \frac{2\mathcal{E}}{R+2r}$

$\Rightarrow R + 2r = 2R + r$

$\Rightarrow R = r = 5\Omega$



1. Melting point order of



A B C

A. A > B > C

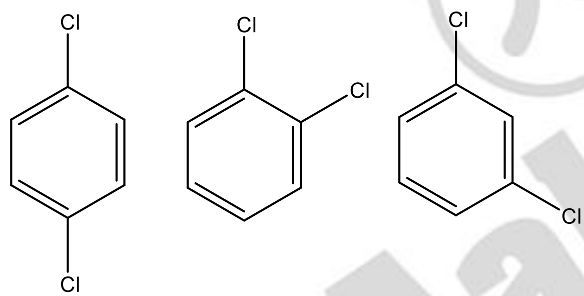
B. C > A > B

C. B > A > C

D. A > C > B

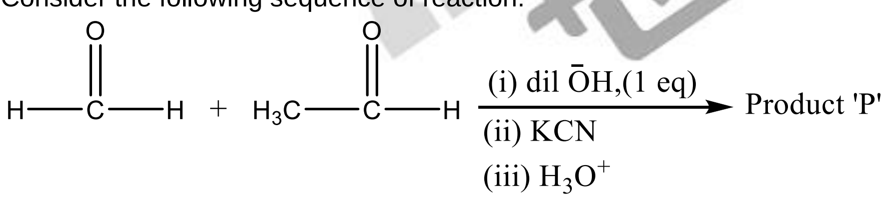
Answer (A)

Solution:



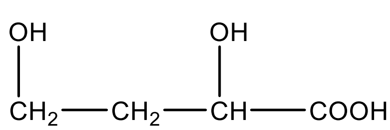
M.P = 323 K M.P = 256 K M.P = 244 K

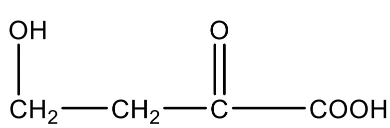
2. Consider the following sequence of reaction:

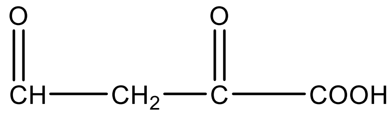


The Product ‘P’ is ?

A. CH_3-CH_2-COOH

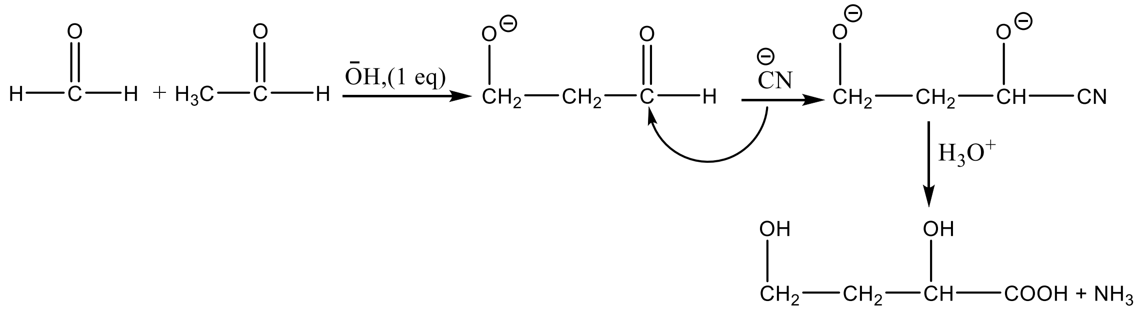
B. 

C. 

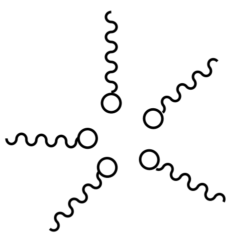
D. 

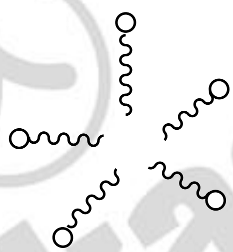
Answer (B)

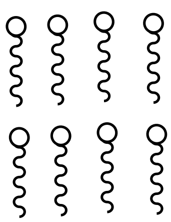
Solution:

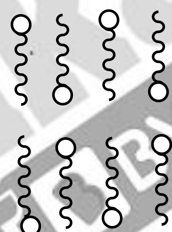


3. A detergent is dissolved in non-polar solvent. The structure of micelle in non-polar solvent is

A. 

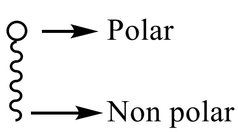
B. 

C. 

D. 

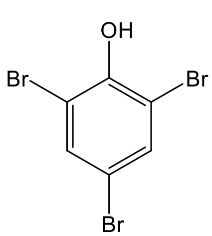
Answer (A)

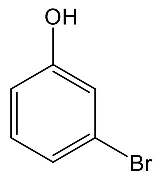
Solution:

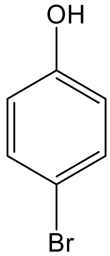


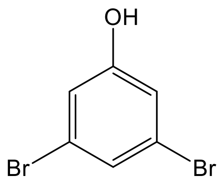
In non-polar solvent, the non-polar part will be outside.

4. When phenol reacts with Br_2 in low polarity solvent, it produces a major product ______?

A. 

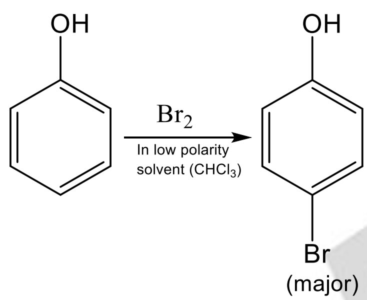
B. 

C. 

D. 

Answer (C)

Solution:

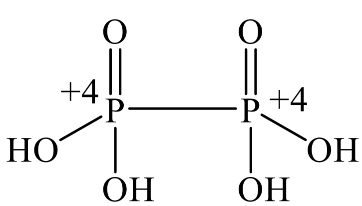


5. The oxidation state of Phosphorus atom in hypophosporic acid is ______?

Answer (4)

Solution:

The hypophosphoric acid is:



6. Electronic configuration of Nd^{2+} is

A. $4f^2$
B. $4f^3$

C. $4f^4$
D. $4f^5$

Answer (C)

Solution:

Nd^{2+} = [Xe] $4f^4$

7. Following values of K (Rate constants) are given at different temperatures. Find out the activation energy (E_a).
Given:
T = 200K → K_1 = 0.03
T = 300K → K_2 = 0.05

A. 2.548 KJ
B. 11.488 KJ
C. 1.106 KJ
D. 51.437 KJ

Answer (A)

Solution:

$$\log\frac{0.05}{0.03} = \frac{E_a}{2.303 \times 8.314} \times \left(\frac{1}{200} - \frac{1}{300}\right)$$
$$= \frac{E_a}{2.303 \times 8.314} \times \left(\frac{1}{600}\right)$$
$$E_a = 2.548\ KJ$$

8. Basic strength of oxides of V:
V_2O_3 V_2O_5 V_2O_4

A. $V_2O_3 < V_2O_5 < V_2O_4$
B. $V_2O_3 < V_2O_4 < V_2O_5$
C. $V_2O_3 > V_2O_4 > V_2O_5$
D. $V_2O_3 = V_2O_5 = V_2O_4$

Answer (C)

Solution:

As oxidation state of V increases then its acidic nature increases. So, the correct basic order is
$V_2O_3 > V_2O_4 > V_2O_5$

9. Choose the correct information regarding the products obtained on electrolysis of brine solution.

A. Cl_2 at cathode
B. O_2 at cathode
C. H_2 at cathode
D. OH^- at cathode

Answer (C)

Solution:

At anode

$2Cl^- \rightarrow Cl_2 + 2e^-$

At cathode

$2e^- + 2H_2O \rightarrow H_2 + 2OH^-$

Net reaction

$2Cl^- + 2H_2O \rightarrow Cl_2 + H_2 + 2OH^-$

10. Consider the following reaction

$SO_2(g) + \frac{1}{2}O_2(g) \rightleftharpoons SO_3(g)$

If $K_P = 2 \times 10^{12}$ and $K_C = x \times 10^{13}$, the value of x in terms of RT will be

A. $\frac{\sqrt{RT}}{4}$

B. $\frac{\sqrt{RT}}{5}$

C. $\frac{\sqrt{RT}}{10}$

D. $10\sqrt{RT}$

Answer (B)

Solution:

$K_P = K_C \times (RT)^{\frac{-1}{2}}$

$2 \times 10^{12} = x \times 10^{13} \times (RT)^{\frac{-1}{2}}$

$x = \frac{2 \times 10^{12}}{10^{13} \times (RT)^{\frac{-1}{2}}} = \frac{2\sqrt{RT}}{10} = \frac{\sqrt{RT}}{5}$

11. Arrange the following ions in the increasing order of their ionic radii.

S^{2-}, Cl^-, K^+ and Ca^{2+}

A. $S^{2-} < Cl^- < K^+ < Ca^{2+}$

B. $Cl^- < S^{2-} < K^+ < Ca^{2+}$

C. $K^+ < Ca^{2+} < Cl^- < S^{2-}$

D. $Ca^{2+} < K^+ < Cl^- < S^{2-}$

Answer (D)

Solution:

The given ionic species are isoelectronic species. The radii of isoelectronic ionic species increases as the atomic charge of ion decreases. Therefore, the correct increasing order of radii of ionic species is

$Ca^{2+} < K^+ < Cl^- < S^{2-}$

12. Which of the following options contains the compound which has highest sweetening value?

A. Aspartame

B. Saccharin

C. Sucralose

D. Alitame

Answer (D)

Solution:

Sweetener	Sweetening Value
Aspartame	100
Saccharin	550
Sucralose	600
Alitame	2000

Alitame has the highest sweetening value.

13. Which of the following method is not a concentration of ore?

A. Electrolysis
B. Leaching
C. Froth flotation
D. Hydraulic washing

Answer (A)

Solution:

The following methods are commonly used for concentration of ore

1. Hydraulic washing
2. Leaching
3. Froth floatation
4. Magnetic separation

But Electrolysis is used for refining of the crude metal.

14. A complex compound of CO(X) is pink colour in water. On reaction with conc. HCl forms (Y) of deep blue colour and has geometry (Z). Identify (X), (Y) and (Z).

A. $[Co(H_2O)_6]^{2+}$, $[CoCl_6]^{3-}$, Octahedral
B. $[Co(H_2O)_6]^{3+}$, $[CoCl_4]^{2-}$, Tetrahedral
C. $[Co(H_2O)_6]^{2+}$, $[CoCl_4]^{2-}$, Tetrahedral
D. $[Co(H_2O)_6]^{3+}$, $[CoCl_6]^{3-}$, Octahedral

Answer (C)

Solution:

Co^{2+} ions in aqueous medium are pink in colour. On addition of conc. HCl, the solution becomes blue due to formation of $[CoCl_4]^{2-}$ which is tetrahedral.

$Co^{2+} + 6H_2O \rightarrow [Co(H_2O)_6]^{2+}$

Here, X is $[Co(H_2O)_6]^{2+}$ which is pink in colour.

$[Co(H_2O)_6]^{2+} + 4HCl \rightarrow [CoCl_4]^{2-} + 4H^+ + 6H_2O$

Here, Y is $[CoCl_4]^{2-}$ which is blue in colour and tetrahedral in structure.

15. Consider the following reaction.

$ClO(g) + NO_2 \rightarrow A \xrightarrow{H_2O(g)} B + C$

A, B and C are respectively.

A. $ClONO_2(g)$; $HOCl(g)$; $HNO_3(g)$
B. $ClONO_2(g)$; $HOCl(g)$; $NO_2(g)$
C. $ClNO_2(g)$; HCl; Cl_2
D. $ClNO_2(g)$; HCl; $HNO_3(g)$

Answer (A)

Solution:

$ClO(g) + NO_2(g) \rightarrow ClONO_2\ (g)$

$ClONO_2(g) + H_2O(g) \rightarrow HOCl(g) + HNO_3(g)$

Hence, the correct answer is option (A).

16. $Cu^{2+} + I^- \rightarrow A \rightarrow B + C$
Find B and C
A. I_2, Cu_2I_2
B. Cu_2I_4
C. CuI_3^-
D. I^-, CuI_2

Answer (A)

Solution:

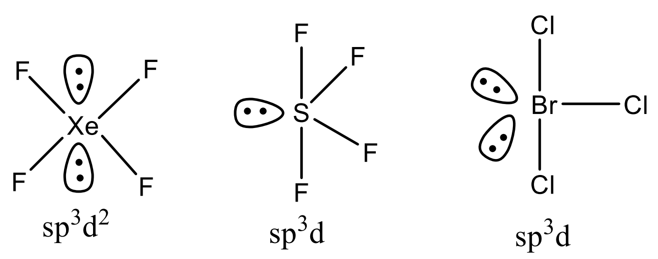
$$\underset{}{Cu^{2+} + I^-} \rightarrow \underset{(A)}{[CuI_2]} \rightarrow \underset{(B)}{\frac{1}{2}Cu_2I_2} + \underset{(C)}{\frac{1}{2}I_2}$$

Products (B) and (C) are Cu_2I_2 and I_2 respectively

17. XeF_4, SF_4 and $BrCl_3$ show hybridisations respectively
A. sp^3, sp^3, sp^3
B. dsp^2, sp^3, sp^3
C. sp^3d^2, sp^3d, sp^3d
D. d^2sp^2, sp^3d, sp^3d

Answer (C)

Solution:



18. In which of the following reaction, H_2O_2 acts as a reducing agent.
A. $H_2O_2 + Mn^{2+} \rightarrow MnO_2 + H_2O$
B. $NaOCl + H_2O_2 \rightarrow NaCl + O_2$
C. $Fe^{2+} + H_2O_2 \rightarrow Fe^{3+} + H_2O$
D. $PbS + H_2O_2 \rightarrow PbSO_4 + H_2O$

Answer (B)

Solution:

Oxidation

$$NaOCl + H_2O_2 \longrightarrow NaCl + O_2$$

$$\begin{array}{cccc} +1 & -1 & -1 & 0 \end{array}$$

Reduction

In option (B), oxidation of H_2O_2 is taking place and hence H_2O_2 acts as a reducing agent.

19. Which of the following transition emits the same wavelength as that for (n = 4 → n = 2) for He^+ ion

A. H (n = 3 → n = 1)

B. H (n = 2 → n = 1)

C. H^{2+} (n = 4 → n = 3)

D. He^+ (n = 6 → n = 3)

Answer (B)

Solution:

$$\frac{1}{\lambda} = \frac{RZ^2}{hc}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

For He^+ ion, (n = 4 → n = 2)

$$\frac{1}{\lambda_{He^+}} = \frac{R2^2}{hc}\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{R\ X\ 4}{hc}\left(\frac{1}{4} - \frac{1}{16}\right) = \frac{3R}{4hc}$$

For H ion, (n = 2 → n = 1)

$$\frac{1}{\lambda_H} = \frac{R1^2}{hc}\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{R}{hc}\left(\frac{1}{1} - \frac{1}{4}\right) = \frac{3R}{4hc}$$

20. Which of the following option contains the correct match?

List - I	List - II
A. XeF_4	(P) T- shape
B. SF_4	(Q) See-saw
C. NH_4^+	(R) Square planar
D. BrF_3	(S) Tetrahedral

A. A → P, B →Q, C→R, D→S

B. A → R, B →Q, C→S, D→P

C. A → Q, B →P, C→S, D→R

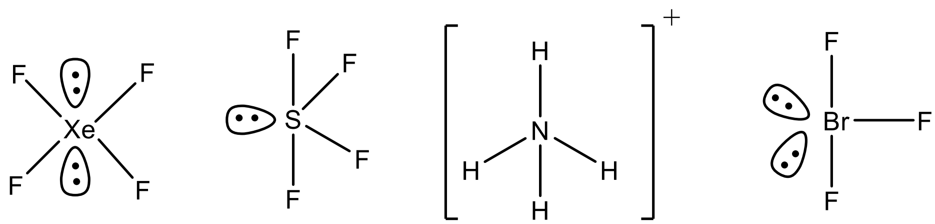
D. A → S, B →R, C→P, D→Q

Answer (B)

Solution:

Molecule	Number of lone pairs	Number of sigma bonds	Shape
XeF_4	2	4	Square Planar

SF_4	1	4	See - saw
NH_4^+	0	4	Tetrahedral
BrF_3	2	3	T- shape



21. 2.56 g of a non-electrolyte solute is dissolved in one litre of a solution, it has osmotic pressure equal to 4 bar at 300 K temperature. Then, find the molar mass of the compound.
Given, R = 0.083 bar, round off to the nearest integer

Answer (16)

Solution:

$\prod$ = CRT

4 = $\frac{2.56}{M} X\ 0.083\ X\ 300$

M = $\frac{2.56}{4} X\ 0.083\ X\ 300$

= 16 g

22. Weight of an organic compound is 0.492 g. When the hydrocarbon undergoes combustion, it produces 0.792 g of CO_2. Find the % of carbon in the given hydrocarbon. (Round off to nearest integer)

Answer (44)

Solution:

%Carbon= $\frac{MW_C}{MW_{CO_2}} X \frac{W_{CO_2}}{W} X\ 100$

MW_C – Molecular weight of Carbon

MW_{CO_2} – Molecular weight of CO_2

W_{CO_2}– Weight of CO_2 produced

W – Weight of the organic compound

= $\frac{12}{44}\ X \frac{0.792}{0.492}\ X\ 100$

= 43.90%

23. What is the volume of hydrogen gas produced (lit) when 11.2 g of Zn metal reacts with excess of dil. HCl. (Closest integer)

Given, Molar volume of H_2 = 22.7 L/mol, Molar mass of Zn = 65 g/mol

Answer (4)

Solution:

$Zn + 2HCl \rightarrow ZnCl_2 + H_2$

Weight of the Zn = 11.2 g

From the equation, one mole of Zn i.e 65 g produces one mole of H_2 i.e 22.7 L

Therefore, volume of H_2 produced by 11.2 g of Zn

$= \frac{11.2}{65} X\ 22.7\ L$

= 3.911 L ≈ 4L

24. The value of logarithms of the equilibrium constant of the following reaction is $\frac{X}{3}$. Then X is ?

$Pd^{2+} + 4Cl^- \leftrightarrow PdCl_4^{2-}$

Given : [$Pd^{2+} + 2e^- \rightarrow Pd \quad E^o = 0.83\ V$

$PdCl_4^{2-} + 2e^- \rightarrow Pd + 4Cl^- \quad E^o = 0.63\ V\ and\ 2.303\frac{RT}{F} = 0.06]$

Answer (20)

Solution:

From Nernst Equation,

$$\boldsymbol{E_{cell} = E^o_{cell} - 2.303\frac{RT}{nF}\ log\ Q}$$

Q – Reaction quotient

At equilibrium,

$$\boldsymbol{E_{cell} = 0\ and\ Q = K_{eq}}$$

$$\Rightarrow, \boldsymbol{E^o_{cell} = 2.303\frac{RT}{nF} log\ K_{eq}} ------(\mathbf{1})$$

Given, $Pd^{2+} + 2e^- \rightarrow Pd \quad E^o = 0.83\ V$

$PdCl_4^{2-} + 2e^- \rightarrow Pd + 4Cl^- \quad E^o = 0.63\ V$

Net reaction is,

$Pd^{2+} + 4Cl^- \leftrightarrow PdCl_4^{2-}$

From the above reactions,

$\boldsymbol{E^o_{cell} = E^o_{Pd^{2+}/Pd} + E^o_{Pd/PdCl_4^{2-}}}$

= 0.83 – 0.63 = 0.20 V

Putting values in eqn (1)

$$0.20 = \frac{0.06}{2}\frac{x}{3}$$

$x = \frac{0.20\ X\ 6}{0.06}$ = 20

25. Find the value $|\Delta H|$ in KJ for

$\frac{1}{2}Cl_2(g) \rightarrow Cl^-(aq)$

Given : [$\Delta H_{diss}\ Cl_2(g) \rightarrow 2Cl(g) \quad 240\ kJ/mol^{-1}$

$\Delta H_{eg}\ Cl(g) + e^- \rightarrow Cl^-(g) \quad -320\ kJ/mol^{-1}$

$\Delta H_{hydration}\ Cl^-(g) + aq \rightarrow Cl^-(aq) \quad -340\ kJ/mol^{-1}]$

Answer (540)

Solution:

$\frac{1}{2}Cl_2(g) \rightarrow Cl(g) \qquad \Delta H_1 = \frac{240}{2} = 120\ kJ$

$Cl(g) + e^- \rightarrow Cl^-(g) \qquad \Delta H_2 = -320\ kJ$

$Cl^-(g) + aq \rightarrow Cl^-(aq) \quad \Delta H_3 = -340\ KJ$

$$\frac{1}{2}Cl_2(g) + e^- + aq \rightarrow Cl^-(aq)$$

$\Delta H = \Delta H_1 + \Delta H_2 + \Delta H_3$

= 120 – 320 -340

= - 540 kJ

$|\Delta H| = 540\ kJ$

1. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{2+3\sin x}{\sin x(1+\cos x)} dx$ is equal to:

A. $\ln(\sqrt{3}+2) - \frac{\ln 3}{2} + 6\sqrt{3} - \frac{28}{3}$

B. $\ln(\sqrt{3}+2) - \frac{\ln 3}{2}$

C. $\ln(\sqrt{3}+2) - \frac{\ln 3}{2} - \frac{28}{3}$

D. $6\sqrt{3} - \frac{28}{3}$

Answer (A)

Solution:

$I = \int \frac{2}{\sin x(1+\cos x)} dx + \int \frac{3}{(1+\cos x)} dx$

$= \int \frac{2\sin x}{\sin^2 x(1+\cos x)} dx + \int \frac{3}{2\cos^2\frac{x}{2}} dx$

Let I_1 and I_2 be the first and second integral respectively.

Let $\cos x = t$

$I_1 = \int \frac{-2dt}{(1-t^2)(1+t)}$

$I_1 = -2\left(\frac{\ln(t+1)}{4} + \frac{1}{2t+2} - \frac{\ln|t-1|}{4}\right) + C$

$I_1 = -2\left(\frac{\ln(t+1)}{4} + \frac{1}{2t+2} - \frac{\ln|t-1|}{4}\right) + C$

$I_1 = -2\left(\frac{\ln(\cos x+1)}{4} + \frac{1}{2\cos x+2} - \frac{\ln|\cos x-1|}{4}\right) + C$

$I_2 = \frac{3}{2}\left(2\tan\frac{x}{2}\right) + C$

So, $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{2+3\sin x}{\sin x(1+\cos x)} dx = \ln(\sqrt{3}+2) - \frac{\ln 3}{2} + 6\sqrt{3} - \frac{28}{3}$

2. The product and sum of first four terms of G.P. is 1296 and 126 respectively, then sum of the possible values of common difference is:

A. 14

B. $\frac{10}{3}$

C. $\frac{7}{2}$

D. 3

Answer (D)

Solution:

$\frac{a}{r^3}.\frac{a}{r}.ar.ar^3 = 1296$

$\Rightarrow a = 6$

$\frac{a}{r^3} + \frac{a}{r} + ar + ar^3 = 126$

$\Rightarrow \frac{1}{r^3} + \frac{1}{r} + r + r^3 = 21$

$\Rightarrow \left(r+\frac{1}{r}\right)\left(\left(r+\frac{1}{r}\right)^2-3\right)+\left(r+\frac{1}{r}\right)=21$

Let $r+\frac{1}{r}=t$

$\Rightarrow t^3-3t+t=21$

$\Rightarrow t^3-2t-21=0$

$\Rightarrow t=3$

$\Rightarrow r+\frac{1}{r}=3$

$\Rightarrow r^2-3r+1=0$

$\Rightarrow r_1+r_2=3$

Sum of possible values of r is 3.

3. If $B=\ln(1-a)$ and $P(a)=\left(a+\frac{a^2}{2}+\frac{a^3}{3}+\cdots+\frac{a^{50}}{50}\right)$, then $\int_0^a \frac{t^{50}}{1-t}dt$ equals:

A. $-(B+P(a))$
B. $-B+P(a)$
C. $B-P(a)$
D. $B+P(a)$

Answer (A)

Solution:

$\int_0^a \frac{t^{50}}{1-t}dt=\int_0^a \frac{t^{50}-1+1}{1-t}dt$

$\Rightarrow \int_0^a\left(\frac{t^{50}-1}{1-t}+\frac{1}{1-t}\right)dt$

Since $(1+t+t^2+\cdots+t^{49})$ constitute as a G.P. with sum $=\frac{t^{50}-1}{t-1}$

$\Rightarrow \int_0^a\left(-(1+t+t^2+\cdots+t^{49})+\frac{1}{1-t}\right)dt$

$\Rightarrow [-\ln(1-t)]_0^a-\left[\left(t+\frac{t^2}{2}+\frac{t^3}{3}+\cdots+\frac{t^{50}}{50}\right)\right]_0^a$

$\Rightarrow -\ln(1-a)-\left(a+\frac{a^2}{2}+\frac{a^3}{3}+\cdots+\frac{a^{50}}{50}\right)\Rightarrow -(B+P(a))$

4. $\sin^{-1}\left(\frac{a}{17}\right)+\cos^{-1}\left(\frac{4}{5}\right)-\tan^{-1}\left(\frac{77}{36}\right)=0$, then the value of $\sin^{-1}(\sin a)+\cos^{-1}(\cos a)$ is :

A. 0
B. $16-2\pi$
C. π
D. 5

Answer (C)

Solution:

$\sin^{-1}\left(\frac{a}{17}\right)=-\cos^{-1}\left(\frac{4}{5}\right)+\tan^{-1}\left(\frac{77}{36}\right)$

Let $\cos^{-1}\left(\frac{4}{5}\right)=\beta$ & $\tan^{-1}\left(\frac{77}{36}\right)=\alpha$

$\Rightarrow \sin\left(\sin^{-1}\frac{a}{17}\right)=\sin(\alpha-\beta)$

$\Rightarrow \sin\left(\sin^{-1}\frac{a}{17}\right)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$

$\Rightarrow \frac{a}{17} = \frac{77}{85} \times \frac{4}{5} - \frac{36}{85} \times \frac{3}{5}$

$\Rightarrow a = \frac{200}{25} = 8$

$\Rightarrow \sin^{-1} \sin 8 + \cos^{-1} \cos 8 = 3\pi - 8 + 8 - 2\pi$

$= \pi$

5. If maximum distance of a normal to the ellipse $\frac{x^2}{4} + \frac{y^2}{b^2} = 1$ from $(0,0)$ is 1, then the eccentricity of the ellipse is:

A. $\frac{\sqrt{3}}{4}$
B. $\frac{1}{\sqrt{2}}$
C. $\frac{1}{2}$
D. $\frac{\sqrt{3}}{2}$

Answer (D)

Solution:

Equation of normal is

$(2\sec\theta)x - (b\ \text{cosec}\,\theta)y = 4 - b^2$

Perpendicular distance from $(0,0)$ is

$D = \left|\frac{4-b^2}{\sqrt{4\sec^2\theta + b^2\text{cosec}^2\theta}}\right|$

$= \frac{4-b^2}{\sqrt{(4+b^2)+4\tan^2\theta+b^2\cot^2\theta}} \leq \frac{4-b^2}{\sqrt{b^2+4+4b}}$ (using AM $\geq$ GM for $4\tan^2\theta + b^2\cot^2\theta$)

$= \frac{4-b^2}{(2+b)}$

$= 2 - b$

$D_{max} = 2 - b = 1$

$\Rightarrow b = 1$

$\Rightarrow e = \sqrt{1 - \frac{1}{4}}$

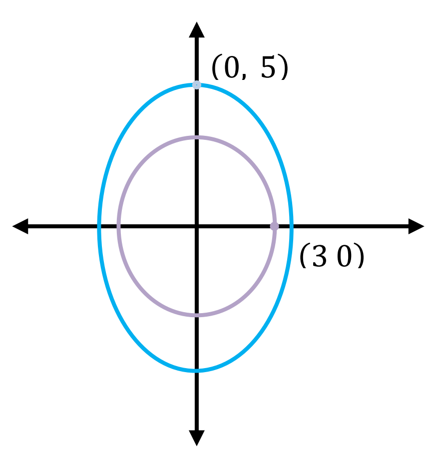
$\Rightarrow e = \frac{\sqrt{3}}{2}$

6. Let the curve C_1 be represented by $|z| = 2$ and C_2 by $\left|z + \frac{\bar{z}}{4}\right| = \frac{15}{4}$, then:

A. C_1 lies inside C_2
B. C_2 lies inside C_1
C. C_1 & C_2 has 2 points of intersections.
D. C_1 & C_2 has 4 points of intersections.

Answer (A)

Solution:



Let $z = x + iy$

$C_1 \Rightarrow x^2 + y^2 = 4 \Rightarrow$ circle

$C_2 \Rightarrow \left|x + iy + \frac{x-iy}{4}\right| = \frac{15}{4}$

$\Rightarrow \left(\frac{5x}{4}\right)^2 + \left(\frac{3y}{4}\right)^2 = \frac{225}{16}$

$\Rightarrow \frac{x^2}{9} + \frac{y^2}{25} = 1 \Rightarrow$ ellipse

$\Rightarrow C_1$ lies inside C_2

7. Find the number of real solutions of $\sqrt{x^2-4x+3} + \sqrt{x^2-9} = \sqrt{4x^2-14x+6}$ is:

A. 1

B. 2

C. 3

D. 4

Answer (A)

Solution:

Effectively, the network is

$x^2 - 4x + 3 \geq 0$

$\Rightarrow (x-1)(x-3) \geq 0$

$\Rightarrow x \in (-\infty,\ 1] \cup [3,\ \infty) \cdots (1)$

$x^2 - 9 \geq 0$

$\Rightarrow x \in (-\infty,\ -3] \cup [3,\ \infty) \cdots (2)$

$4x^2 - 14x + 6 \geq 0$

$\Rightarrow (2x-1)(x-3) \geq 0$

$\Rightarrow x \in \left(-\infty, \frac{1}{2}\right] \cup [3,\ \infty) \cdots (3)$

$(1) \cap (2) \cap (3)$, we get

$x \in (-\infty,\ -3] \cup [3,\ \infty)$

Now squaring both sides of given equation.

$(x^2-4x+3) + (x^2-9) + 2\sqrt{(x^2-4x+3)(x^2-9)} = 4x^2 - 14x + 6$

$\Rightarrow 2\sqrt{(x^2-4x+3)(x-3)(x+3)} = 2(x^2-5x+6)$

$\Rightarrow (x^2-4x+3)(x-3)(x+3) = (x-3)^2(x-2)^2$

$x = 3$ is one solution

$\Rightarrow (x^2-4x+3)(x+3) = (x^2-4x+4)(x-3)$

$\Rightarrow x^3 - 4x^2 + 3x + 3x^2 - 12x + 9 = x^3 - 4x^2 + 4x - 3x^2 + 12x - 12$

$\Rightarrow 6x^2 - 25x + 21 = 0$

$\Rightarrow x = 3, \frac{7}{6}$

$x = \frac{7}{6}$ is not in domain. So, only one solution.

8. If $f(x) = \sin^3\left(\frac{\pi}{3}\cos\left(\frac{\pi}{3\sqrt{2}}(-4x^3+5x^2+1)^{\frac{3}{2}}\right)\right)$, then $f'(1)$ is :

A. $\frac{3\pi^2}{8}$

B. $\frac{3\pi^2}{4}$

C. $\frac{3\pi^2}{16}$

D. $\frac{\pi^2}{2}$

Answer (C)

Solution:

$$f(x) = \sin^3\left(\frac{\pi}{3}\cos\left(\frac{\pi}{3\sqrt{2}}(-4x^3+5x^2+1)^{\frac{3}{2}}\right)\right)$$

$$f'(x) = 3\sin^2\left(\frac{\pi}{3}\cos\left(\frac{\pi}{3\sqrt{2}}(-4x^3+5x^2+1)^{\frac{3}{2}}\right)\right)\times\cos\left(\frac{\pi}{3}\cos\left(\frac{\pi}{3\sqrt{2}}(-4x^3+5x^2+1)^{\frac{3}{2}}\right)\right)\times$$
$$\frac{\pi}{3}\left(-\sin\left(\frac{\pi}{3\sqrt{2}}(-4x^3+5x^2+1)^{\frac{3}{2}}\right)\right)\times\frac{\pi}{3\sqrt{2}}\times\frac{3}{2}(-4x^3+5x^2+1)^{\frac{1}{2}}\times(-12x^2+10x)$$

$$f'(1) = 3\sin^2\left(\frac{\pi}{3}\cos\left(\frac{2\pi}{3}\right)\right)\times\cos\left(\frac{\pi}{3}\cos\left(\frac{2\pi}{3}\right)\right)\times\frac{\pi}{3}\left(-\sin\frac{2\pi}{3}\right)\times\frac{\pi}{2\sqrt{2}}(\sqrt{2})(-2)$$

$$f'(1) = 3\sin^2\left(-\frac{\pi}{6}\right)\times\cos\left(-\frac{\pi}{6}\right)\times\frac{\pi}{3}\left(-\frac{\sqrt{3}}{2}\right)\times(-\pi)$$

$$f'(1) = \frac{3}{4}\times\frac{\sqrt{3}}{2}\times\frac{\pi}{3}\times\frac{\sqrt{3}}{2}\times\pi = \frac{3\pi^2}{16}$$

9. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-zero vectors such that $|\vec{a}+\vec{b}+\vec{c}| = |\vec{a}+\vec{b}-\vec{c}|$ and $\vec{b}\cdot\vec{c} = 0$ then:

S-I: $|\vec{a}+\lambda\vec{c}| \geq 0$ for all $\lambda \in \mathbb{R}$

S-II: $\vec{a}$ is always parallel to $\vec{c}$

A. S-I is True, S-II is False.
B. S-I is True, S-II is True.
C. S-I is False, S-II is True.
D. S-I is False, S-II is False.

Answer (A)

Solution:

$|\vec{a}+\vec{b}+\vec{c}| = |\vec{a}+\vec{b}-\vec{c}|$

$\Rightarrow |\vec{a}+\vec{b}+\vec{c}|^2 = |\vec{a}+\vec{b}-\vec{c}|^2$

$\Rightarrow \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a} = \vec{a}\cdot\vec{b}-\vec{b}\cdot\vec{c}-\vec{c}\cdot\vec{a}$

$\Rightarrow \vec{c}\cdot\vec{a} = 0$

$\therefore$ $\vec{a}$ is perpendicular to $\vec{c}$

$\Rightarrow$ S-II is False.

$\Rightarrow |\vec{a}+\lambda\vec{c}| \geq 0$ (However, it is always true)

$\Rightarrow$ S-I is True.

10. $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then find sum of diagonal elements of $(A-I)^{11}$.

A. 4096
B. 4097
C. 2048
D. 2049

Answer (D)

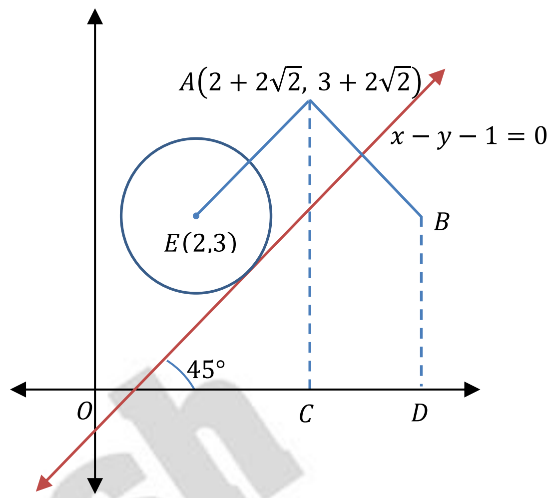
Solution:

$$A - I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$(A-I)^{11}=\begin{bmatrix}1^{11} & 0 & 0\\ 0 & 2^{11} & 0\\ 0 & 0 & 0^{11}\end{bmatrix}$$
$trace\ (A-I)^{11}=2^{11}+1^{11}+0$
$trace\ (A-I)^{11}=2049$

11. Circle $x^2+y^2-4x-6y+11=0$ is rolled up by 4 units along a tangent to it at the point $(3,\ 2)$. Let this be circle C_1. C_2 is the mirror image of circle C_1 about the tangent. A and B are centres of circles C_1 and C_2. C and D are the feet of perpendiculars from A and B respectively upon X-axis. The area of the trapezium $ABCD$ equals to:

A. $4(1+\sqrt{2})$
B. $2(1+\sqrt{2})$
C. $3(1+\sqrt{2})$
D. $(1+\sqrt{2})$



Answer (A)

Solution:

Given circle is $x^2+y^2-4x-6y+11=0$, Centre $E(2,3)$

Tangent at $(3,2)$ is $x-y-1=0$

After rolling up by 4 units, centre of C_1 is A

Where $A\equiv\left(2+4\times\frac{1}{\sqrt{2}},3+4\times\frac{1}{\sqrt{2}}\right)\equiv(2+2\sqrt{2},3+2\sqrt{2})$

B is image of A about $x-y-1=0$

$\frac{x-(2+2\sqrt{2})}{1}=\frac{y-(3+2\sqrt{2})}{-1}=-2\times\left(\frac{-2}{2}\right)=2$

$B\equiv(4+2\sqrt{2},1+2\sqrt{2})$

Area of $ABCD=\frac{1}{2}\times(4+4\sqrt{2})\times\left((4+2\sqrt{2})-(2+2\sqrt{2})\right)$

Area of $ABCD=4(1+\sqrt{2})$

12. Let the solution R, $(a,\ b)\ R(c,\ d)$ be such that $ab(d-c)=cd(a-b)$ then R is:

A. Reflexive only
B. Symmetric only
C. Transitive but not symmetric
D. Reflexive and symmetric but not transitive

Answer (B)

Solution:

Checking for Reflexive

$\therefore (a,b)\ R\ (a,b)$

$\Rightarrow ab(b-a)=ab(a-b)$

$\Rightarrow b-a=a-b$ $\therefore$ Not reflexive

Checking for $(a,b)\ R\ (c,d)$ then $(c,d)\ R\ (a,b)$

$\Rightarrow cd(b-a)=ab(c-d)$

$\Rightarrow ab(d-c)=cd(a-b)$

$\therefore R$ is symmetric.

$(a, b)\ R\ (c, d) \equiv ab(d-c) = cd(a-b)$

$\Rightarrow \frac{ab}{a-b} = \frac{cd}{d-c} \quad \cdots (1)$

$(c, d)\ R\ (e, f) \equiv \frac{cd}{c-d} = \frac{ef}{f-e} \quad \cdots (2)$

For relation to be transitive, we need to check whether $(a, b)\ R\ (e, f)$ or not.

$i.e. \frac{ab}{a-b} = \frac{ef}{f-e}$

But, by (1) and (2) we get,

$\frac{ab}{a-b} = -\frac{ef}{f-e}$

$\therefore R$ is not transitive.

13. Find the number of 5-digit numbers formed using the digits $0, 3, 4, 7, 9$ when repetition of digits is allowed:

Answer (2500)

Solution:

↓	↓	↓	↓	↓
3	0	0	0	0
4	3	3	3	3
7	4	4	4	4
9	7	7	7	7
	9	9	9	9

Total number of 5-digit numbers $= 4 \times 5 \times 5 \times 5 \times 5 = 2500$

14. Reminder when 5^{99} is divided by 11 is __________.

Answer (9)

Solution:

We have,

$5^{99} = (5^5)^{19} \cdot 5^4$

$= (3125)^{19} \cdot 5^4$

$= (11\lambda + 1)^{19} \cdot 5^4$

$= (11k + 1)\ \cdot 5^4$

$= 11k_1 + 5^4$

When 5^4 is divisible by 11 we get remainder $= 9$

15. If $f(x) + \int_3^x \frac{f(t)}{t} dt = \sqrt{x+1}$ then value of $12f(8)$ equals _________.

Answer (17)

Solution:

$f(x) + \int_3^x \frac{f(t)}{t} dt = \sqrt{x+1}$

Differentiating on both sides,

$f'(x) + \frac{f(x)}{x} = \frac{1}{2\sqrt{x+1}}$

since $y = f(x)$ we get $\frac{dy}{dx} = f'(x)$

$\Rightarrow \frac{dy}{dx} + \frac{y}{x} = \frac{1}{2\sqrt{x+1}}$

Integrating factor(I.F.) $= e^{\int \frac{1}{x} dx} = e^{\ln x} = x$

$\Rightarrow xy = \frac{1}{2}\int \frac{x}{\sqrt{x+1}}$
$\Rightarrow xy = \frac{1}{2}\int \sqrt{x+1} - \frac{1}{\sqrt{x+1}}$
$\Rightarrow xy = \frac{1}{2}\left(\frac{2}{3}(x+1)^{\frac{3}{2}} - 2\sqrt{x+1}\right) + c \quad \cdots (1)$
If we put $x = 3$ in $f(x) + \int_3^x \frac{f(t)}{t} dt = \sqrt{x+1}$ we get,
$\Rightarrow f(3) + \int_3^3 \frac{f(t)}{t} dt = \sqrt{4}$
$\Rightarrow f(3) = 2$
By substituting $f(3) = 2$ in eq.(1)
$\Rightarrow 3 \times 2 = \frac{1}{2}\left(\frac{2}{3}(4)^{\frac{3}{2}} - 2\sqrt{4}\right) + c$
$\Rightarrow c = \frac{16}{3}$
$\therefore 8f(8) = \frac{1}{2}\left(\frac{2}{3}(9)^{\frac{3}{2}} - 2\sqrt{9}\right) + \frac{16}{3}$
$\Rightarrow 8f(8) = \frac{27}{3} - 3 + \frac{16}{3}$
$\Rightarrow 8f(8) = \frac{34}{3}$
$\Rightarrow 12f(8) = 17$

16. $y = f(x)$ is a parabola with focus $\left(-\frac{1}{2}, 0\right)$ and directrix $y = -\frac{1}{2}$. Given that $\tan^{-1}\sqrt{f(x)} + \sin^{-1}\sqrt{f(x)+1} = \frac{\pi}{2}$. Then number of real solutions for x is _____.

Answer (2)

Solution:

$SP = SQ$
$\left(x + \frac{1}{2}\right)^2 + y^2 = \left(y + \frac{1}{2}\right)^2$
$x^2 + \frac{1}{4} + x + y^2 = y^2 + \frac{1}{4} + y$
Equation of parabola: $y = x^2 + x$
$\Rightarrow f(x) = x^2 + x$
$\Rightarrow \tan^{-1}\sqrt{x^2+x} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$
$\Rightarrow \tan^{-1}\sqrt{x^2+x} = \cos^{-1}\sqrt{x^2+x+1}$
$\Rightarrow \cos^{-1}\left(\frac{1}{\sqrt{x^2+x}}\right) = \cos^{-1}\sqrt{x^2+x+1}$
$\Rightarrow \left(\frac{1}{\sqrt{x^2+x}}\right) = \sqrt{x^2+x+1}$
$\Rightarrow x^2 + x + 1 = 1$
$\Rightarrow x^2 + x = 0$
$\Rightarrow x(x+1) = 0$
$\Rightarrow x = 0$ or -1
$\therefore$ Number of real solutions for x are 2.

17. The direction ratio's of two lines which are parallel are given by $< 2, 1, -1 >$ and $< \alpha+\beta, 1+\beta, 2 >$. Then the value of $|2\alpha + 3\beta|$ is __________.

Answer (11)

Solution:

Since, the lines are parallel.
$\therefore \frac{\alpha+\beta}{2} = \frac{1+\beta}{2} = \frac{2}{-1}$
$\Rightarrow \alpha + \beta = -4$ and $1 + \beta = -2$
$\Rightarrow \beta = -3$
$\Rightarrow \alpha = -1$
So, $|2\alpha + 3\beta| = |2(-1) + 3(-3)| = |-11| = 11$

18. Given $|\vec{a}| = \sqrt{14}$, $|\vec{b}| = \sqrt{6}$, $|\vec{a} \times \vec{b}| = \sqrt{48}$. Then the value of $(\vec{a} \cdot \vec{b})^2$ is ______.

Answer (36)

Solution:

$$(\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2|\vec{b}|^2 - |\vec{a} \times \vec{b}|^2$$
$$= 14 \times 6 - 48$$
$$= 36$$