

10/04/2023

Evening



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## Memory Based Answers & Solutions for

Time : 3 hrs.

M.M. : 300

## JEE (Main)-2023 (Online) Phase-2 (Physics, Chemistry and Mathematics)

### IMPORTANT INSTRUCTIONS:

- (1) The test is of **3 hours** duration.
- (2) The Test Booklet consists of 90 questions. The maximum marks are 300.
- (3) There are **three** parts in the question paper consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.
  - (i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **-1 mark** for wrong answer.
  - (ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10**. The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **-1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

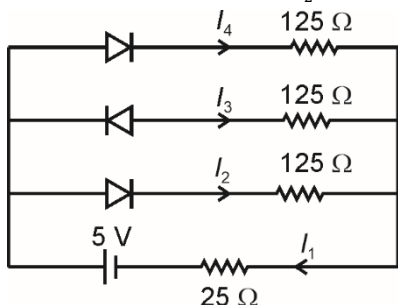
# PHYSICS

## SECTION - A

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

**Choose the correct answer:**

1. Following circuit contains diodes with forward bias having resistance  $25 \Omega$  and reverse bias having infinite resistance. The ratio of  $\frac{I_1}{I_2}$  is equal to



- (1) 1 (2) 2  
(3) 3 (4) 4

**Answer (2)**

**Sol.**  $I_2 = I_4$

$$I_3 = 0$$

$$\text{and } I_2 + I_4 + I_3 = I_1$$

$$\Rightarrow I_2 = \frac{I_1}{2}$$

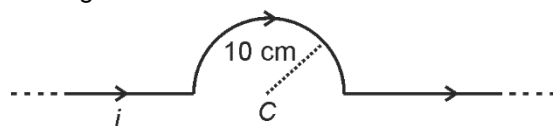
2. An object moves  $x$  distance with speed  $v_1$  and next  $x$  distance with speed  $v_2$ . The average velocity  $v$  is related to  $v_1$  and  $v_2$  as

- (1)  $v = \left( \frac{v_1 + v_2}{2} \right)$  (2)  $\frac{1}{v} = \frac{1}{v_1} + \frac{1}{v_2}$   
(3)  $v = \left( \frac{2v_1v_2}{v_1 + v_2} \right)$  (4)  $v = \left( \frac{v_1 - v_2}{2} \right)$

**Answer (3)**

**Sol.**  $v = \frac{2x}{\left( \frac{x}{v_1} + \frac{x}{v_2} \right)} = \left( \frac{2v_1v_2}{v_1 + v_2} \right)$

3. An infinitely-long conductor has a current 14 A flowing as shown:



Find magnetic field at centre C.

- (1)  $88 \mu\text{T}$  (2)  $44 \mu\text{T}$

- (3)  $10 \mu\text{T}$

- (4)  $120 \mu\text{T}$

**Answer (2)**

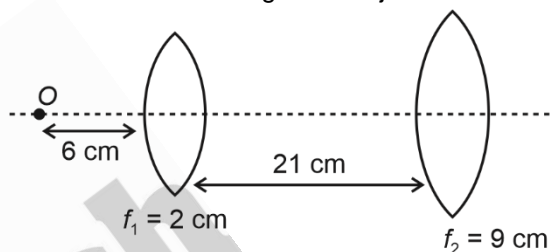
**Sol.**  $B = \frac{\mu_0 I}{4R}$   

$$= \frac{4\pi \times 10^{-7} \times 14}{4 \times 0.1} \text{ T}$$
  

$$= \frac{22}{7} \times 10^{-7} \times 140 \text{ T}$$
  

$$= 44 \mu\text{T}$$

4. A point object (O) is placed on the principal axis of a system of two lenses as shown. Find the distance between the final image and object.



- (1) 45 cm (2) 40 cm  
(3) 55 cm (4) 50 cm

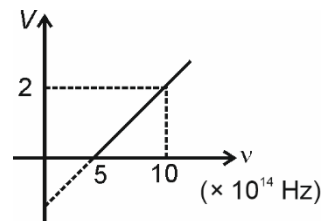
**Answer (1)**

**Sol.** From lens (1)  $\frac{1}{v} - \frac{1}{(-6)} = \frac{1}{2} \Rightarrow v = 3 \text{ cm}$

From lens (2)  $\frac{1}{v'} - \frac{1}{(-18)} = \frac{1}{9} \Rightarrow v' = +18 \text{ cm}$

Distance between O and I =  $6 + 21 + 18 = 45 \text{ cm}$

5. Based on given graph between stopping potential and frequency of irradiation, work function of metal is equal to



- (1) 1 eV (2) 3 eV  
(3) 2 eV (4) 4 eV

**Answer (3)**

**Sol.**  $eV_s = h\nu - \phi$

On extrapolating the graph, the graph cuts the  $y$ -axis at  $-2$

$$\Rightarrow \text{at } \nu = 0, V_s = -2 \text{ V}$$

$$\Rightarrow \phi = 2 \text{ eV}$$

6. **Assertion (A):** Fan spins even after switch is in OFF.

**Reason (R):** Fan in rotation has rotational inertia.

- (1) A is correct and R is correct explanation of A  
 (2) A and R both are correct but R is not correct explanation of A.  
 (3) A is correct and R is incorrect  
 (4) Both A and R are incorrect

**Answer (1)**

7. Wire A and B have their Young's moduli in the ratio 1 : 3, area of cross-section in the ratio of 1 : 2 and lengths in ratio of 3 : 4. If same force is applied on the two wires to elongate then ratio of elongation is equal to

- (1) 8 : 1 (2) 1 : 12  
 (3) 1 : 8 (4) 9 : 2

**Answer (4)**

**Sol.**  $\frac{\Delta l_A}{\Delta l_B} = \frac{F l_A / A_A Y_A}{F l_B / A_B Y_B} = \left( \frac{Y_B}{Y_A} \right) \left( \frac{A_B}{A_A} \right) \left( \frac{l_A}{l_B} \right)$   
 $= \left( \frac{3}{1} \right) \left( \frac{2}{1} \right) \left( \frac{3}{4} \right) = \left( \frac{9}{2} \right)$

8. If half life for a radio-active decay reaction is  $T$ . Find the time after which  $\frac{7}{8}$ th of initial mass decays.

- (1)  $3T$  (2)  $2T$   
 (3)  $\frac{T}{2}$  (4)  $4T$

**Answer (1)**

**Sol.**  $\therefore t_{1/2} = T$

also,  $m_0 \xrightarrow{T} \frac{m_0}{2} \xrightarrow{T} \frac{m_0}{4} \xrightarrow{T} \frac{m_0}{8}$

total time taken is  $3T$ .

9. When electric field is applied to the electrons in a conductor it starts

- (1) Moving in straight line  
 (2) Drifting from higher potential to lower potential  
 (3) Drifting from lower potential to higher potential  
 (4) Moving with constant velocity

**Answer (3)**

**Sol.** Electron starts drifting from lower potential to higher potential.

10. A metallic slab of thickness  $\frac{2d}{3}$  and area of surface same as that of plates of capacitor of capacitance  $C_1$  is inserted parallel to plates of capacitor such that its new capacitance becomes equal to  $C_2$ . If  $d$  is space width between the two plates, then  $\frac{C_2}{C_1}$  is equal to

- (1) 1 (2) 2  
 (3) 3 (4) 4

**Answer (3)**

**Sol.**  $C_2 = \frac{C_1 d}{\left( d - \frac{2d}{3} \right)}$

$\frac{C_2}{C_1} = 3$

11. Consider two statements:

**Statement 1:** Magnetic susceptibility of diamagnetic substance is  $-1 \leq x < 0$ .

**Statement 2:** Diamagnetic substance moves from stronger to weaker magnetic field.

- (1) Both statements are correct  
 (2) Both are incorrect  
 (3) Statement 1 is correct and statement 2 is incorrect  
 (4) Statement 1 is incorrect and statement 2 is correct

**Answer (1)**

**Sol.** For diamagnetic,  $\mu_r < 1$

$\Rightarrow -1 \leq x < 0$

Also, diamagnetic material moves from stronger to weaker fields.

12. In communication of a message signal, if frequency of message signal and carrier signal are 3 kHz and 6 MHz respectively. Then, amplitude modulated signal will have bandwidth of

- (1) 12 MHz (2) 6 kHz  
 (3) 3 kHz (4) 6 MHz

**Answer (2)**

**Sol.** Bandwidth of AM signal =  $2 f_m$

$= 2 \times 3 \text{ kHz} = 6 \text{ kHz}$

13. In standard YDSE phase difference between two rays reaching at points  $P$  and  $Q$  is  $\frac{\pi}{3}$  and  $\frac{\pi}{2}$  respectively. Ratio of resultant intensity at  $P$  and  $Q$  is equal to

- (1)  $\frac{3}{2}$  (2)  $\frac{2}{3}$   
 (3)  $\frac{1}{4}$  (4)  $\frac{1}{2}$

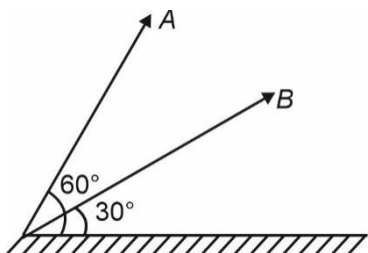
**Answer (1)**

**Sol.**  $I = I_0 \cos^2 \left( \frac{\Delta\phi}{2} \right)$

$$\Rightarrow \frac{I_P}{I_Q} = \frac{\cos^2 \left( \frac{\Delta\phi_P}{2} \right)}{\cos^2 \left( \frac{\Delta\phi_Q}{2} \right)}$$

$$= \frac{\frac{3}{4}}{\frac{1}{2}} = \frac{3}{2}$$

14. Two projectiles A and B are projected from the same point on ground with same speed of projection as shown. Find the ratio of maximum height attained by A to that of B.



- (1) 3 : 1                      (2) 1 : 3  
(3)  $\sqrt{3} : 1$                 (4)  $\sqrt{3} : 2$

**Answer (1)**

**Sol.**  $\frac{h_A}{h_B} = \frac{\frac{u^2 \sin^2 60^\circ}{2g}}{\frac{u^2 \sin^2 30^\circ}{2g}} = \frac{3}{1}$

15. Force acting on a particle at origin is  $\vec{F} = -c\hat{k}$ . The torque of this force about  $(-2, 3, 0)$  is  $-c(A\hat{i} + B\hat{j})$ .

If  $\frac{A}{B} = x$ , find  $x$ .

- (1)  $\frac{2}{3}$                       (2) 3  
(3) 1.5                    (4) 1

**Answer (3)**

**Sol.**  $\vec{\tau} = \vec{r} \times \vec{F}$   
 $= (2\hat{i} + 3\hat{j}) \times (-c\hat{k})$   
 $= 2c\hat{j} + 3c\hat{i}$   
 $= c(3\hat{i} + 2\hat{j})$   
 $\Rightarrow \frac{A}{B} = 1.5$

16. If time period for one revolution by satellite near the earth's surface is  $T$ . Then the time period of revolution of satellite at a height equal to radius of earth will be

- (1)  $\sqrt{8}T$   
(2)  $\sqrt{2}T$   
(3)  $\sqrt{4}T$   
(4)  $\sqrt{3}T$

**Answer (1)**

**Sol.**  $\therefore T^2 \propto r^3$

$$\left( \frac{T}{T'} \right)^2 = \left( \frac{R}{2R} \right)^3$$

$$T' = \sqrt{8}T$$

17. In a mixture 0.5 moles of  $O_2$  and 4 moles of Ne gas are taken at temperature  $T$ . The internal energy of the system is equal to

- (1)  $\left( \frac{13}{2} \right) RT$   
(2)  $\left( \frac{11}{4} \right) RT$   
(3)  $\left( \frac{29}{4} \right) RT$   
(4)  $\left( \frac{13}{4} \right) RT$

**Answer (3)**

**Sol.**  $U = U_{O_2} + U_{Ne}$   
 $= \frac{5}{2} n_{O_2} RT + \frac{3}{2} n_{Ne} RT$   
 $= \left( \frac{5}{4} + \frac{24}{4} \right) RT$

18. **Assertion (A):** Acceleration due to gravity is minimum at equator.

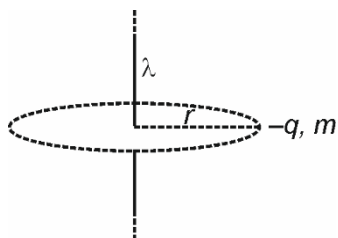
**Reason (R):** Rotation of earth influences acceleration.

- (1) A is correct, R is correct explanation of A  
(2) A is correct, R is incorrect explanation of A  
(3) A is correct and R is incorrect  
(4) Both A and R are incorrect

**Answer (1)**

**Sol.** Acceleration due to gravity depends on the rotation of earth and distance from the centre.

19. An infinite line charge of charge density  $\lambda$  has a charged particle  $(-q, m)$  revolving about it in a circle of radius  $r$ . Find the orbital speed.



- (1)  $\sqrt{\frac{\lambda a}{\pi \epsilon_0 m}}$   
 (2)  $\sqrt{\frac{\lambda q}{4\pi \epsilon_0 m}}$   
 (3)  $\sqrt{\frac{4\lambda a}{\pi \epsilon_0 m}}$   
 (4)  $\sqrt{\frac{\lambda q}{2\pi \epsilon_0 m}}$

**Answer (4)**

**Sol.**  $E = \frac{\lambda}{2\pi \epsilon_0 r}$

$$\Rightarrow \frac{\lambda q}{2\pi \epsilon_0 r} = \frac{mv^2}{r}$$

$$\Rightarrow v = \sqrt{\frac{\lambda q}{2\pi \epsilon_0 m}}$$

20. A gas is compressed adiabatically. Which of the following statements is not correct?

- (1) Internal energy is constant  
 (2) Temperature increases  
 (3)  $|\text{Work done}| = |\text{Change in internal energy}|$   
 (4) Heat is not supplied to the system

**Answer (1)**

**Sol.**  $\Delta Q = \Delta U + W$

For adiabatic,  $\Delta Q = 0 \quad \dots(1)$

Also, since compression  $\Rightarrow T$

$\Rightarrow U$  increases.

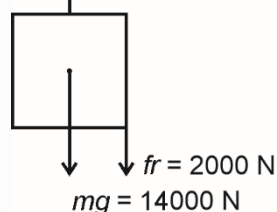
## SECTION - B

**Numerical Value Type Questions:** This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. Frictional force acts on the lift of mass 1400 kg is 2000 N. If lift moves with constant velocity of 3 m/s in upward direction, the power (in kW) of motor is (take  $g = 10 \text{ m/s}^2$ )

**Answer (48)**

**Sol.**  $\uparrow F = \text{Force by motor}$



$$P = F \cdot V$$

$$= 16000 \times 3$$

$$= 48000 \text{ watt} = 48 \text{ kW}$$

22. If the angular frequency of the given motion is  $k\omega$ , find the value of  $k$

$$y = \sin(\omega t) + \cos \omega t$$

**Answer (1)**

**Sol.**  $y = \sqrt{2} \left[ \sin(\omega t) \cdot \cos\left(\frac{\pi}{4}\right) + \cos(\omega t) \sin\left(\frac{\pi}{4}\right) \right]$

$$= \sqrt{2} \sin\left(\omega t + \frac{\pi}{4}\right)$$

Angular frequency =  $\omega$

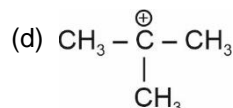
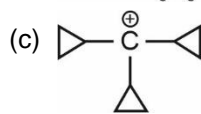
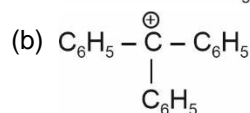
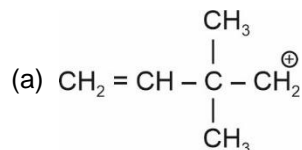
23.  
 24.  
 25.  
 26.  
 27.  
 28.  
 29.  
 30.



**Sol.**  $E = (2) \left( \frac{5R}{2} \right) (T) + (4) \left( \frac{3R}{2} \right) (T)$

$= 11 RT$

8. Which of the following is the correct hydride affinity order of carbocations



- (1) (c) < (b) < (d) < (a)    (2) (b) < (d) < (c) < (a)  
 (3) (a) < (d) < (b) < (c)    (4) (c) < (a) < (d) < (b)

**Answer (1)**

**Sol.** The correct hydride affinity order of carbocations will be decided by the stability of carbocation. Higher the stability of carbocation, lower will be hydride affinity.

$\therefore$  Correct hydride affinity order of carbocations is (c) < (b) < (d) < (a)

9. Water of crystallization in Soda ash and washing soda is respectively.

- (1) 0,10                                      (2) 10,0  
 (3) 0,0                                        (4) 0,1

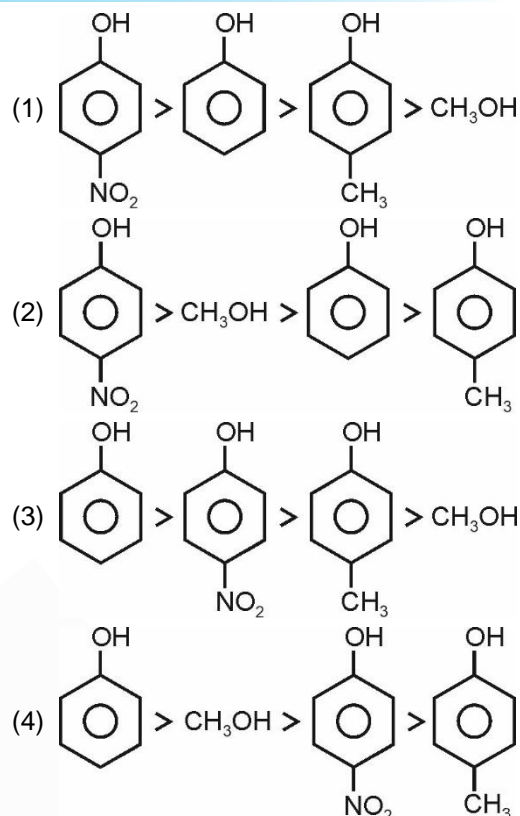
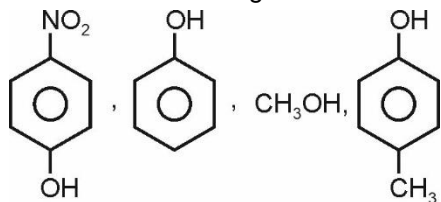
**Answer (1)**

**Sol.** Soda ash is  $\text{Na}_2\text{CO}_3$

Washing soda is  $\text{Na}_2\text{CO}_3 \cdot 10\text{H}_2\text{O}$ .

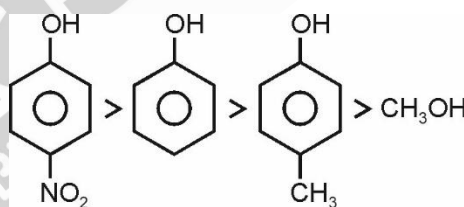
Therefore correct answer is 0,10.

10. Order of acidic strength of



**Answer (1)**

**Sol.** Correct order is

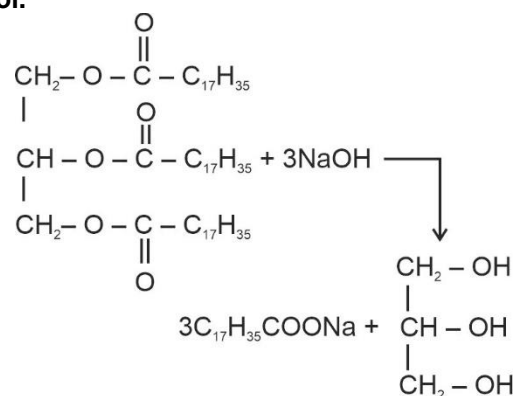


11. What process is used to make soap from fat?

- (1) Saponification  
 (2) Electrolysis  
 (3) Solvay process  
 (4) Haber process

**Answer (1)**

**Sol.**





12. **Assertion:** Higher energy is required for the conversion of  $\text{Mg}$  to  $\text{Mg}^{2-}$  than that for  $\text{Mg}$  to  $\text{Mg}^-$ .

**Reason:**  $\text{Mg}^{2-}$  has very small size and more charge.

- (1) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion  
(2) Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion  
(3) Assertion is correct but Reason is incorrect  
(4) Assertion is incorrect but Reason is correct

**Answer (3)**

**Sol.** Since  $\text{Mg}^{2-}$  has higher charge density than  $\text{Mg}^-$ , then interelectronic repulsion will be higher in case of  $\text{Mg}^{2-}$  as compared to  $\text{Mg}^-$ .

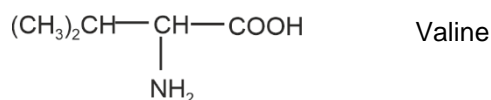
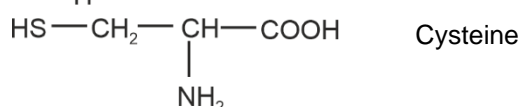
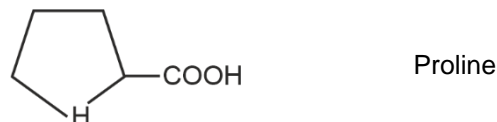
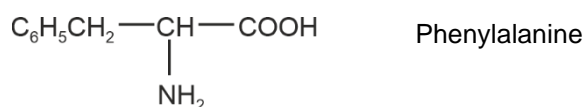
Hence, higher energy is required for the conversion of  $\text{Mg}$  to  $\text{Mg}^{2-}$  than that of  $\text{Mg}$  to  $\text{Mg}^-$ .

13. An unknown organic compound is heated with fuming  $\text{HNO}_3$ . The reaction mixture is treated with aq  $\text{BaCl}_2$  solution which gives white precipitate. Identify the unknown organic compound.

- (1) Phenylalanine (2) Proline  
(3) Cysteine (4) Valine

**Answer (3)**

**Sol.** The unknown organic compound contains S-atom which gets oxidised by fuming  $\text{HNO}_3$  to  $\text{SO}_4^{2-}$  ions. Addition of aq  $\text{BaCl}_2$  gives white precipitate of  $\text{BaSO}_4$ . Among the given compounds only cysteine has S-atom.



14. Following two columns are provided

	Column-I (Complex)		Column-II (CFSE)
a.	$[\text{Ti}(\text{H}_2\text{O})_6]^{2+}$	(i)	$-1.2 \Delta_0$
b.	$[\text{V}(\text{H}_2\text{O})_6]^{2+}$	(ii)	$-0.6 \Delta_0$
c.	$[\text{Mn}(\text{H}_2\text{O})_6]^{3+}$	(iii)	0
d.	$[\text{Fe}(\text{H}_2\text{O})_6]^{3+}$	(iv)	$-0.8 \Delta_0$

(1) a(iv); b(i); c(ii); d(iii) (2) a(i); b(ii); c(iv); d(iii)

(3) a(iv); b(iii); c(i); d(ii) (4) a(i); b(ii); c(iii); d(iv)

**Answer (1)**

**Sol.**  $\text{CFSE} = -\frac{2}{5} \Delta_0(t_{2g} \text{ electrons}) + \frac{3}{5} \Delta_0(e_g \text{ electrons})$

15.  
16.  
17.  
18.  
19.  
20.

## SECTION - B

**Numerical Value Type Questions:** This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. For a metal ion,  $\mu = 4.9$  B.M. Find out number of unpaired electrons

**Answer (04.00)**

**Sol.**  $\sqrt{n(n+2)} = 4.92$   
 $(n)(n+2) = 24$   
 $n = 4$

22. Find out difference in oxidation state of Xe in completely Hydrolysed form of  $\text{XeF}_4$  and  $\text{XeF}_6$

**Answer (00.00)**

**Sol.**  $\text{XeF}_6 + 3\text{H}_2\text{O} \xrightarrow[\text{Hydrolysis}]{\text{Complete}} \text{XeO}_3 + 6\text{HF}$   
(+6)

$\text{XeF}_4 + \text{H}_2\text{O} \xrightarrow[\text{Hydrolysis}]{\text{Complete}} \text{XeO}_3 + \text{Xe} + \text{O}_2 + \text{HF}$   
(+6)



23.  $\text{NH}_3$ ,  $\text{NO}$ ,  $\text{N}_2$ ,  $\text{F}_2$ ,  $\text{CO}$ ,  $\text{CO}_2$ ,  $\text{H}_2\text{O}$ , and  $\text{XeF}_4$

Fill the number of above molecules having only two lone pair of electrons.

**Answer (3)**

**Sol.** These are  $\text{N}_2$ ,  $\text{CO}$  and  $\text{H}_2\text{O}$ .

24. How many electrons are gained by  $\text{MnO}_4^-$  in strongly alkaline medium?

**Answer (1)**

**Sol.**  $\text{MnO}_4^-$  gains one electron to form  $\text{MnO}_4^{2-}$  in strongly alkaline medium.

25. Consider a reaction at equilibrium



If final pressure at equilibrium is 1 atm &  $k_p = \frac{1}{27}$ ,

then % dissociation of A will be (consider  $1 - \alpha \approx 1$ ) (nearest integer)

**Answer (21)**

**Sol.**  $\text{A(g)} \rightleftharpoons 2\text{B(g)} + \text{C(g)}$

P            ---            ---

$P(1-\alpha)$      $2P\alpha$              $P\alpha$

$\therefore P_{\text{total}} = P(1 + 2\alpha)$

$$k_p = \frac{(4P^2\alpha^2)P\alpha}{P(1-\alpha)} = \frac{4P^2\alpha^3}{1-\alpha}$$

$$k_p = \frac{4P_T^2\alpha^3}{(1-\alpha)(1+2\alpha)^2}$$

$$\frac{1}{27} = \frac{4P_T^2\alpha^3}{1}$$

$$P_T^2\alpha^3 = \frac{1}{108}$$

$$\alpha^3 = \frac{1}{108}$$

$$\alpha = \left(\frac{1}{108}\right)^{1/3} \times 100$$

$$= \frac{100}{4.762}$$

$$\alpha \approx 21$$

26. 0.02 M  $\text{CH}_3\text{COOH}$  has specific conductance,  $K = 5 \times 10^{-5} \text{ S cm}^{-1}$ . Also given limiting molar conductance of  $\text{CH}_3\text{COOH}$  is  $400 \text{ S cm}^2 \text{ mol}^{-1}$ .

Therefore,  $K_a$  for  $\text{CH}_3\text{COOH}$  is  $\text{---} \times 10^{-7} \text{ M}$

**Answer (8)**

**Sol.**  $\text{CH}_3\text{COOH} \rightleftharpoons \text{CH}_3\text{COO}^- + \text{H}^+$

$$\Lambda_m = \frac{K \times 1000}{M} = \frac{5 \times 10^{-5} \times 10^3}{2 \times 10^{-2}} = 2.5$$

$$\alpha = \frac{\Lambda_m}{\Lambda_m^\circ} = \frac{2.5}{400}$$

$$\therefore K = \frac{C\alpha^2}{1-\alpha}$$

$$= \frac{0.02 \times \left(\frac{2.5}{400}\right)^2}{1 - \frac{2.5}{400}}$$

$$= \frac{7.8125 \times 10^{-7}}{0.99375}$$

$$\approx 7.861 \times 10^{-7}$$

$$K_a \approx 8 \times 10^{-7} \text{ M}$$

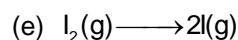
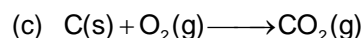
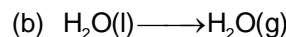
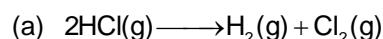
27. For a first-order reaction, if the value of  $t_{1/2}$  is T, then the value of  $t_{7/8}$  will be  $\text{---} T$ .

**Answer (3)**

**Sol.**  $t_{7/8}$  means 3 half lives.

$$\therefore t_{7/8} = 3T$$

28. Number of endothermic reactions among following



**Answer (04)**

**Sol.** Burning of carbon is exothermic, all other are endothermic.

29.

30.

**MATHEMATICS**

**SECTION - A**

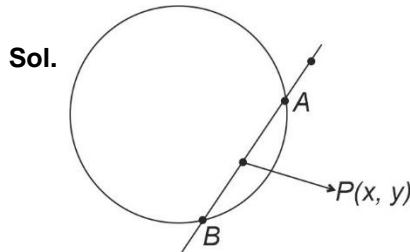
**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

**Choose the correct answer:**

1. Let a circle  $x^2 + y^2 = 16$  and line passing through (1, 2) cuts the curve at A and B then the locus of the mid-point of AB is

- (1)  $x^2 + y^2 + x + y = 0$  (2)  $x^2 + y^2 - x + 2y = 0$   
(3)  $x^2 + y^2 - x - 2y = 0$  (4)  $x^2 + y^2 + x + 2y = 0$

**Answer (3)**



Let  $P(x_1, y_1)$  be the mid-point of AB

Then  $T = S_1$

$$x_1^2 + y_1^2 - 16 = xx_1 + yy_1 - 16$$

$$\Rightarrow xx_1 + yy_1 = x_1^2 + y_1^2 \quad \dots(i)$$

$\therefore$  (i) passes through (1, 2)

$$\therefore x_1 + 2y_1 = x_1^2 + y_1^2$$

$\therefore$  required locus

$$x^2 + y^2 - x - 2y = 0$$

2. Consider  $f(x) = \sec^{-1}\left(\frac{2x}{5x+3}\right)$ , domain of  $f(x)$  is

$[\alpha, \beta) \cup (\gamma, \delta]$ , then the value of  $|3\alpha + 10\beta + 5\gamma + 21\delta|$  is

- (1) 22 (2) 23  
(3) 21 (4) 19

**Answer (3)**

**Sol.**  $\frac{2x}{5x+3} \geq 1$  OR  $\frac{2x}{5x+3} \leq -1$

$$\Rightarrow \frac{2x}{5x+3} - 1 \geq 0$$

$$\Rightarrow \frac{-3x-3}{5x+3} \geq 0$$

$$\Rightarrow \frac{x+1}{5x+3} \leq 0 \quad \Rightarrow x \in \left[-1, \frac{-3}{5}\right)$$

$$\frac{2x}{5x+3} + 1 \leq 0$$

$$\Rightarrow \frac{7x+3}{5x+3} \leq 0 \quad \Rightarrow x \in \left(\frac{-3}{5}, \frac{-3}{7}\right]$$

$$\therefore x \in \left[-1, \frac{-3}{5}\right) \cup \left(\frac{-3}{5}, \frac{-3}{7}\right]$$

$$\alpha = -1, \beta = \frac{-3}{5}, \gamma = \frac{-3}{5}, \delta = \frac{-3}{7}$$

$$|3\alpha + 10\beta + 5\gamma + 21\delta|$$

$$= |-3 - 6 - 3 - 9| = 21$$

3. 8 persons has to travel from A to B in 3 allotted cars. If a car can carry maximum 3 persons. Then find the number of ways they can travel.

- (1) 1880 (2) 1800  
(3) 1680 (4) 1600

**Answer (3)**

**Sol.**  $\frac{C_1}{3} \frac{C_2}{3} \frac{C_3}{2} \rightarrow$

Total  $\frac{8!}{3!3!2!2!}$  groups

So, they can travel in

$$\frac{8!}{3!3!2!2!} \times 31 \text{ ways}$$

$$= 1680$$

4. If  $\frac{z+i}{4z+2i}$  is purely real and  $z = x + iy$  ( $x, y \in R$ ) then one of the possibility is

- (1)  $x \neq 0, y \neq -1$  (2)  $x \neq 0, y = -1$   
(3)  $x = 0, 2y \neq -1$  (4)  $x = 1, 2y \neq -1$

**Answer (3)**

**Sol.**  $Im\left(\frac{x+i(y+1)}{4x+2i(2y+1)} \cdot \frac{4x-2i(2y+1)}{4x-2i(2y+1)}\right) = 0$

$$\Rightarrow \frac{4x(y+1) - 2x(2y+1)}{16x^2 + 4(2y+1)^2} = 0$$

$$\Rightarrow \frac{2x}{16x^2 + 4(2y+1)^2} = 0$$

$$\Rightarrow x = 0$$

But at  $x = 0, 2y \neq -1$

5. If  $\int \left( \left( \frac{x}{e} \right)^{2x} + \left( \frac{e}{x} \right)^{2x} \right) \ln x \, dx = \alpha \left( \frac{x}{e} \right)^{2x} + \beta \left( \frac{e}{x} \right)^{2x} + c$

where  $c$  is constant of integration, then

(1)  $\alpha + \beta = 0$  (2)  $\alpha + \beta = 1$

(3)  $\alpha\beta = \frac{1}{2}$  (4)  $\alpha\beta = \frac{1}{4}$

**Answer (1)**

**Sol.** Let  $\left( \frac{x}{e} \right)^{2x} = t$

$$2x(\ln x - 1) = \ln t$$

$$\left[ 2(\ln x - 1) + 2x \left( \frac{1}{x} \right) \right] dx = \frac{1}{t} dt$$

$$\ln x = \frac{1}{2t} dt$$

$$I = \int \left( t + \frac{1}{t} \right) \times \frac{1}{2t} dt = \frac{1}{2} \int \left( 1 + \frac{1}{t^2} \right) dt$$

$$= \frac{1}{2} \left( t - \frac{1}{t} \right) + c$$

$$= \frac{1}{2} \left( \left( \frac{x}{e} \right)^{2x} - \left( \frac{e}{x} \right)^{2x} \right) + c$$

$$\Rightarrow \alpha = \frac{1}{2}, \beta = \frac{-1}{2}$$

6. If a dice is thrown  $n$ -times and probability of getting 7 times odd is equal to 9 times even. Then

$P(2 \text{ even})$  is  $\frac{K}{2^{15}}$  then  $K$  is

- (1) 58 (2) 60  
(3) 48 (4) 65

**Answer (2)**

**Sol.**  $P(\text{getting odd 7 times}) = P(\text{getting even 9 times})$

$${}^nC_7 \left( \frac{1}{2} \right)^7 \left( \frac{1}{2} \right)^{n-7} = {}^nC_9 \left( \frac{1}{2} \right)^9 \left( \frac{1}{2} \right)^{n-9}$$

$${}^nC_7 = {}^nC_9$$

$$n = 9 + 7 = 16$$

$$P(2 \text{ times even}) = {}^{16}C_2 \left( \frac{1}{2} \right)^{14} \left( \frac{1}{2} \right)^2$$

$$= \frac{{}^{16}C_2}{2^{16}} = \frac{16!}{2! \times 14!} \times \frac{1}{2^{16}}$$

$$= \frac{15 \times 16}{2 \times 2^{16}}$$

$$= \frac{15 \times 4}{2^{15}} = \frac{K}{2^{15}}$$

$$\Rightarrow K = 60$$

7. If  $\int_0^{t^2} (f(x) + x^2) dx = \frac{4}{3} t^3$ , then  $f(x)$  is

- (1)  $x^2 - 2\sqrt{x}$  (2)  $x^2 + 2\sqrt{x}$   
(3)  $x^2$  (4)  $-x^2 + 2\sqrt{x}$

**Answer (4)**

**Sol.**  $(f(t^2) + t^4) 2t = 4t^2$

$$f(t^2) + t^4 = 2t$$

$$f(x^2) = -x^4 + 2x$$

Let  $x^2 = u$

$$f(u) = -u^2 + 2\sqrt{u}$$

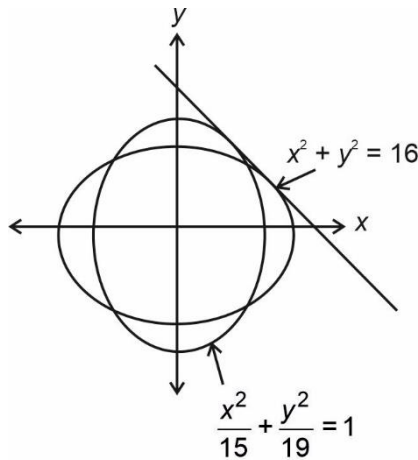
$$f(x) = -x^2 + 2\sqrt{x}$$

8. A tangent drawn to an ellipse  $19x^2 + 15y^2 = 285$  is also a tangent to a circle. This circle is concentric with the conic and its radius is 4 units. Then the angle made by the tangent with minor axis of ellipse is

- (1)  $\frac{\pi}{3}$  (2)  $\frac{\pi}{2}$   
(3)  $\frac{\pi}{6}$  (4)  $\frac{\pi}{4}$

**Answer (1)**

Sol.



$$19x^2 + 15y^2 = 285$$

$$\Rightarrow \frac{x^2}{15} + \frac{y^2}{19} = 1$$

Let equation of tangent of ellipse be

$$y = mx \pm \sqrt{15m^2 + 19} \quad \dots(i)$$

If it is tangent to  $x^2 + y^2 = 16$  then

$$\frac{\sqrt{15m^2 + 19}}{\sqrt{m^2 + 1}} = 4$$

$$15m^2 + 19 = 16m^2 + 16$$

$$m^2 = 3$$

$$m = \pm\sqrt{3}$$

$\therefore$  angle made by the tangent by minor axis  
(x-axis) will be  $= \frac{\pi}{3}$

9.  $y = f(x)$  is a quadratic function passing through  $(-1, 0)$  and tangent to it at  $(1, 1)$  is  $y = x$ . Find x intercept by normal at point  $(\alpha, \alpha + 1)$ ,  $(\alpha > 0)$

- (1) 11                      (2) -11  
(3) 5                        (4) -5

**Answer (1)**

**Sol.** Let  $f(x) = ax^2 + bx + (b - a)$

$$f(1) = 2a + b = 1 \text{ and } f(1) = 1 = 2b$$

$$b = \frac{1}{2}, a = \frac{1}{4}$$

$$\frac{-dx}{dy} = \frac{-1}{2ax + b} = \frac{-2}{x + 1}$$

$$\text{and } \alpha + 1 = \frac{1}{4}\alpha^2 + \frac{1}{2}\alpha + \frac{1}{4}$$

$$4(\alpha + 1) = (\alpha + 1)^2$$

$$\Rightarrow \alpha + 1 = 4 \text{ OR } \alpha = 3$$

$$\text{Slope of normal} = \frac{-2}{4} = \frac{-1}{2}$$

Equation of normal

$$y - 4 = \frac{-1}{2}(x - 3)$$

$$2y - 8 = -x + 3$$

$$2y + x = 11$$

10.

11.

12.

13.

14.

15.

16.

17.

18.

19.

20.

## SECTION - B

**Numerical Value Type Questions:** This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. The sum of all 4 digit number using the digits 1, 2, 2, 3 is

**Answer (26,664)**

**Sol.** Sum of all unit place numbers

$$= 3! \times 2 + \frac{3!}{2!} \times 3 + \frac{3!}{2!} \times 1$$

$$= 12 + 9 + 3 = 24$$

∴ Sum of all 4 digit no.

$$\begin{aligned}
 &= 1 \times 24 + 10 \times 24 + 100 \times 24 + 1000 \times 24 \\
 &= 24000 + 2400 + 240 + 24 \\
 &= 26,6764
 \end{aligned}$$

22. 4, 11, 21, 34 ..... then find the value of  $\frac{S_{29} - S_9}{60}$

**Answer (223)**

**Sol.**  $S = 4 + 11 + 21 + 34 + \dots + T_n$

$$\frac{S = 4 + 11 + 21 + \dots + T_{n-1} + T_n}{T_n = 4 + 7 + 10 + 13 + \dots + (T_n + T_{n-1})}$$

$n-1$  term

$$T_n = 4 + \frac{(n-1)}{2} [14 + (n-2)3]$$

$$= 4 + \frac{(n-1)}{2} [8 + 3n]$$

$$T_n = 4 + \frac{1}{2} (3n^2 + 5n - 8)$$

$$\begin{aligned}
 \sum T_n = S_n &= \frac{3}{2} \sum n^2 + \frac{5}{2} \sum n \\
 &= \frac{3}{2} \frac{n(n+1)(2n+1)}{6} + \frac{5}{2} \frac{n(n+1)}{2}
 \end{aligned}$$

$$\frac{S_{29} - S_9}{60} = 223$$

23. In  $\triangle ABC$ ,  $P$  is circumcentre,  $Q$  is orthocentre, then

$\vec{PA} + \vec{PB} + \vec{PC}$  is

(1)  $2\vec{PQ}$

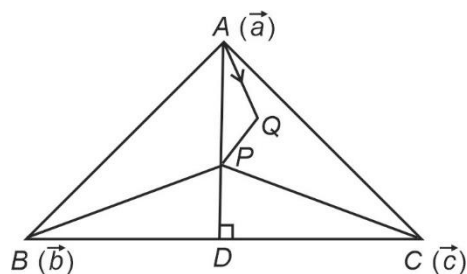
(2)  $\vec{PQ}$

(3)  $3\vec{PQ}$

(4)  $\frac{1}{2}\vec{PQ}$

**Answer (2)**

**Sol.**



Let  $P$  be origin then

$$\vec{PA} + \vec{PB} + \vec{PC} = \vec{a} + \vec{b} + \vec{c}$$

$$\vec{PD} = \frac{\vec{b} + \vec{c}}{2}$$

$$\vec{PQ} = 2\vec{PD}$$

$$= \vec{b} + \vec{c}$$

$$\vec{PQ} = \vec{a} + (\vec{b} + \vec{c})$$

$$= \vec{a} + \vec{b} + \vec{c}$$

24. Let  $S = \left\{ x \in \left[ -\frac{\pi}{2}, \frac{\pi}{2} \right] : 9^{1-\tan^2 x} + 9^{\tan^2 x} = 10 \right\}$  and

$$\beta = \sum_{x \in S} \left( \frac{x}{3} \right). \text{ Then } \frac{1}{7}(\beta - 14)^2 \text{ is}$$

**Answer (28)**

**Sol.** Let  $9^{\tan^2 x} = t$

$$\frac{9}{t} + t = 10$$

$$t^2 - 10t + 9 = 0$$

$$\Rightarrow t = 9 \text{ or } 1$$

$$9^{\tan^2 x} = 9$$

$$\Rightarrow \tan^2 x = 1$$

$$= \tan x = \pm 1$$

$$x = \pm \frac{\pi}{4}$$

or

$$\tan x = 0$$

$$\Rightarrow x = 0$$

$$\beta = \frac{0}{3} + \frac{\pi}{12} - \frac{\pi}{12} = 0$$

$$\therefore \frac{1}{7}(\beta - 14)^2 = \frac{14^2}{7} = 28$$

25. The coefficient of  $x$  and  $x^2$  in  $(1+x)^p (1-x)^q$  are 4 and  $-5$ , then  $2p+3q$  is

**Answer (63)**

**Sol.**  $(1+x)^p (1-x)^q = (1+px + \frac{p(p-1)}{2}x^2 + \dots)$

$$(1 - qx + q\frac{(q-1)}{2}x^2 + \dots)$$

$$\therefore \text{Coefficient of } x = p - q = 4 \quad \dots(i)$$

$$\begin{aligned} \text{Coefficient of } x^2 &= \frac{q(q-1)}{2} - pq + \frac{p(p-1)}{2} \\ &= -5 \end{aligned}$$

$$\Rightarrow (p-q)^2 - (p+q) = -10$$

$$\therefore p + q = 26 \quad \dots(ii)$$

$$\text{By (i) and (ii) } p = 15, q = 11$$

$$\text{So, } 2p + 3q = 30 + 33 = 63$$

26. Let  $\alpha$  be the remainder  $(22)^{2022} + (2022)^{22}$  is divided by 3 and  $\beta$  be the remainder when the same is divided by 7 then  $\alpha^2 + \beta^2$  is

**Answer (05)**

**Sol.**  $(22)^{2022} + (2022)^{22}$

**For  $\alpha$**

$$(21+1)^{2022} + \underbrace{(2022)^{22}}_{\text{divisible by 3}}$$

$$= (3k_1 + 1)$$

**For  $\beta$**

$$(21+1)^{2022} + (2023-1)^{22}$$

$$= (7\lambda + 1) + (7\mu + 1)$$

$$= 7k_2 + 2$$

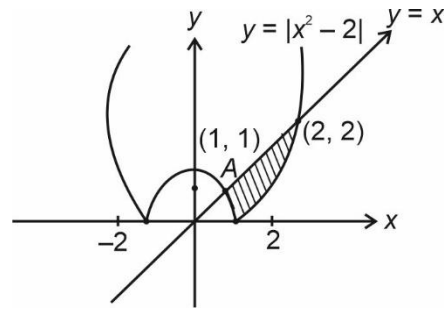
$$\text{So, } \alpha = 1, \beta = 2$$

$$\alpha^2 + \beta^2 = 5$$

27. If area bounded by region  $\{x, y \mid |x^2 - 2| \leq y \leq x\}$  is A, then  $6A + 16\sqrt{2}$  is

**Answer (27)**

**Sol.**



$\therefore$  Required area

$$= \int_1^{\sqrt{2}} x - \{-(x^2 - 2)\} dx + \int_{\sqrt{2}}^2 \{x - (x^2 - 2)\} dx$$

$$= \int_1^{\sqrt{2}} (x^2 + x - 2) dx + \int_{\sqrt{2}}^2 (-x^2 + x + 2) dx$$

$$= \left( \frac{x^3}{3} + \frac{x^2}{2} - 2x \right)_1^{\sqrt{2}} + \left( \frac{-x^3}{3} + \frac{x^2}{2} + 2x \right)_{\sqrt{2}}^2$$

$$= \left( \frac{2\sqrt{2}}{3} + 1 - 2\sqrt{2} \right) - \left( \frac{1}{3} + \frac{1}{2} - 2 \right)$$

$$+ \left( \frac{-8}{3} + 2 + 4 \right) - \left( \frac{-2\sqrt{2}}{3} + 1 + 2\sqrt{2} \right)$$

$$\therefore 6A + 16\sqrt{2} = 27 - 16\sqrt{2} + 16\sqrt{2} = 27$$

28.

29.

30.

