15/04/2023
Morning

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Memory Based
Answers & Solutions
for
JEE (Main)-2023 (Online) Phase-2

(Physics, Chemistry and Mathematics)

Time : 3 hrs. **M.M. : 300**

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. *A* has half life of 5 years. Find the amount of *A* left after 15 years.

   (1) $\frac{1}{8}$ of initial value

   (2) $\frac{7}{8}$ of initial value

   (3) $\frac{1}{4}$ of initial value

   (4) $\frac{3}{4}$ of initial value

Answer (1)

Sol. $N = \frac{N_0}{(2)^3}$ as 15 years = 3 half life

$N = \left(\frac{N_0}{8}\right)$

2. A variable force $F = 5kx$ N acts on a body moving along x-axis. Find the work done by this force in displacing the body from $x = 2$ m to $x = 5$ m.

   (k is a constant)

   (1) $\left(\frac{205}{2}k\right)$J

   (2) $\left(\frac{105}{2}k\right)$J

   (3) (52k) J

   (4) (51k) J

Answer (2)

Sol. $W = \int F dx = \int_{x=2}^{x=5} 5kx \cdot dx$

$= 5k\frac{x^2}{2}\Big|_{x=2}^{x=5} = \frac{5}{2}k \times 21$

3. If de-Broglie wavelength is λ when energy is E, find wavelength at $\frac{E}{4}$ (kinetic energy).

   (1) 2λ (2) $\sqrt{2}\lambda$

   (3) λ (4) $\frac{\lambda}{\sqrt{2}}$

Answer (1)

Sol. $\because \quad \lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mk}}$

$\lambda' = \frac{h}{\sqrt{\frac{2mk}{4}}} = 2\lambda$

4. If position of particle is changing with time as $r = t^2 - 2t$ (m). Find the velocity at $t = 2$ second

   (1) 2 m/s (2) 3 m/s

   (3) 0 m/s (4) 4 m/s

Answer (1)

Sol. $\frac{dr}{dt} = (2t - 2)$

$\left.\frac{dr}{dt}\right|_{t=2} = 4 - 2 = 2$ m/s

5. Height of receiving and transmitting antenna in communication of a signal are 245 m and 180 m respectively. Find the maximum distance between the two antenna for proper communication

   (1) 104 km (2) 208 km

   (3) 52 km (4) 96 km

Answer (1)

Sol. Maximum distance $= \sqrt{2Rh_T} + \sqrt{2Rh_r}$

$= \sqrt{2 \times 6400 \times 10^3 \times 180} + \sqrt{2 \times 6400 \times 10^3 \times 245}$

= 1,04,000 m = 104 km

6. If position vector of a particle is given by $\vec{r}(t) = 8t\hat{i} + 5t^2\hat{j} + 6\hat{k}$, then the correct statement about the acceleration of the particle is

   (1) It is along positive y-axis

   (2) It is along positive x-axis

   (3) It is equally inclined to x and y-axes

   (4) It is along positive z-axis

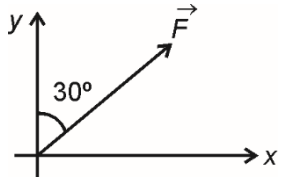
Answer (1)

Sol. $\vec{v} = 8\hat{i} + 10t\hat{j}$

$\vec{a} = 10\hat{j}$

$\therefore$ It is along positive y-axis

7. If *y*-component of a force acting in *x*-*y* plane is $2\sqrt{3}$ N. Then the *x*-component will be



(1) $2\sqrt{3}$ N (2) 2 N

(3) 3 N (4) $3\sqrt{2}$ N

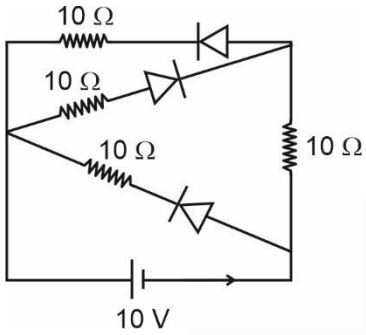
Answer (2)

Sol. $F = \cos 30 = 2\sqrt{3}$

F = 4 N

$\therefore F_n = F\sin 30^\circ = 2$ N

8. Find the value of current passing through battery.

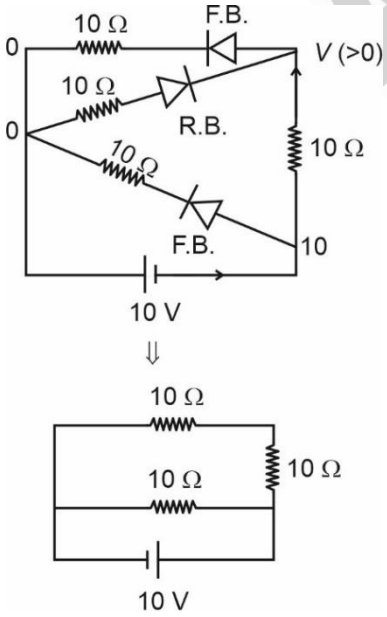


(1) 4 A (2) 1.5 A

(3) 0.5 A (4) 1 A

Answer (2)

Sol. The biasing of diode will be as shown in figure.



$$I_b = \frac{10(20+10)}{20\times 10} = \frac{3}{2}\text{ A}$$

9. A particle is released from a height equal to radius of earth. Find its velocity when it strikes the ground.

(1) $\sqrt{gR}$

(2) $\sqrt{\frac{gR}{2}}$

(3) $\sqrt{2gR}$

(4) $\sqrt{4gR}$

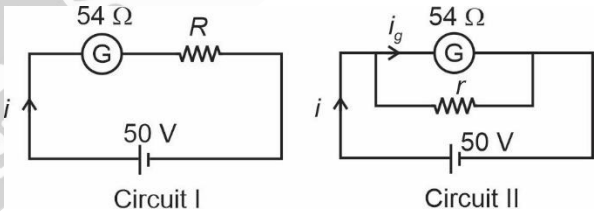
Answer (1)

Sol. From mechanical energy conservation-

$$-\frac{GMm}{2R} + 0 = -\frac{GMm}{R} + \frac{1}{2}mv^2$$

$$\Rightarrow v^2 = \frac{GM}{R} = gR \Rightarrow v = \sqrt{gR}$$

10. Circuit I is converted to voltmeter after adding resistance R and current in circuit I is 1 mA. Circuit II is converted to ammeter after adding resistance r as shown. If current through battery in circuit II is 10 mA and current through galvanometer is 1 mA, then find R and r if resistance of galvanometer is 54 Ω.



(1) R = 49946 Ω, r = 54 Ω

(2) R = 6 Ω, r = 49946 Ω

(3) R = 49946 Ω, r = 49946 Ω

(4) R = 49946 Ω, r = 6 Ω

Answer (4)

Sol. $1\times 10^{-3} = \frac{50}{54+R}$

$54 + R = 50000$

R = 49946 Ω

Now, i_g = 1 mA and current through r = 9 mA

So, 54 × 1 = r × 9

$r = \left(\frac{54}{9}\right) = 6\ \Omega$

11. Match the List-I with List-II and choose the correct option.

	List-I		List-II
A.	Micro-wave	(p)	400 nm – 1 nm
B.	Ultra violet	(q)	1 nm – 1 pm
C.	X-rays	(r)	2.5 μm – 750 nm
D.	Infrared	(s)	1 mm – 25 μm

(1) A(s), B(q), C(r), D(p)

(2) A(s), B(p), C(q), D(r)

(3) A(p), B(s), C(q), D(r)

(4) A(r), B(q), C(s), D(p)

Answer (2)

Sol. Theory based.

12. In a single slit diffraction experiment $\lambda = 600$ nm, if at $\theta = 30°$, first minima is formed then find the value of width of slit in μm.

(1) 1.2 (2) 1.5

(3) 1 (4) 1.8

Answer (1)

Sol. For first minima, $\sin\theta = \frac{\lambda}{a}$

$\Rightarrow a = 2\lambda = 1200$ nm $= 1.2$ μm

13. Two identical particles each of mass *m,* move in circular path due to their own mutual gravitational force. Find the velocity of the particle if the radius of circular path is a

(1) $\sqrt{\frac{4Gm}{a}}$

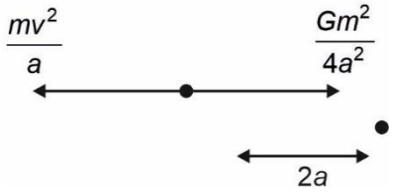
(2) $\sqrt{\frac{Gm}{2a}}$

(3) $\sqrt{\frac{2Gm}{a}}$

(4) $\sqrt{\frac{Gm}{4a}}$

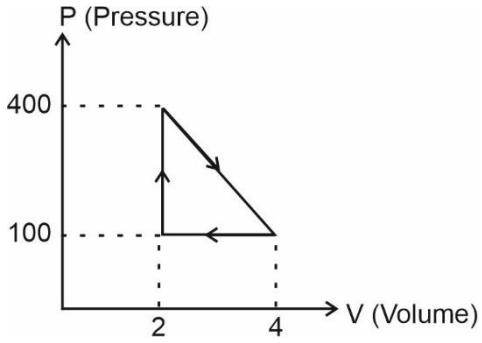
Answer (4)

Sol.



$$\frac{mv^2}{a} = \frac{Gm^2}{4a^2} \Rightarrow v = \sqrt{\frac{Gm}{4a}}$$

14. Calculate the work done by the cyclic process given in indicator diagram (Assume all values in S.I. unit)



(1) 300

(2) – 300

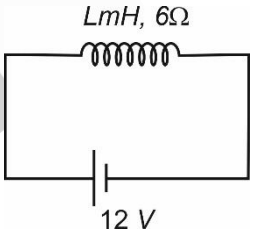
(3) 600

(4) – 600

Answer (1)

Sol. $W = \frac{1}{2} \times 2 \times 300 = 300$

15. In given L-R circuit connected with a D.C source of 12V, inductance is LmH and resistances is 6 Ω. If the emf induced in the inductor at t = 1mS is 10V, value of L is



(1) $\frac{3}{\ln(1.2)}$

(2) $\frac{6}{\ln(1.2)}$

(3) $\frac{3}{\ln(1.8)}$

(4) $\frac{6}{\ln(2.4)}$

Answer (2)

Sol. $i = i_0\,(1 - e^{-Rt/L})$

$$\therefore \text{Emf} = \left|\frac{Ldi}{dt}\right| = L\left(\frac{V}{R}\right)\left(\frac{R}{L}\right)e^{-Rt/L} = Ve^{-Rt/L}$$

$$10 = 12\left(e^{-6/L}\right) \Rightarrow L = \frac{6}{\ln(1.2)}$$

16. In a linear SHM,

A. acceleration is maximum at mean position,

B. velocity is maximum at extreme position,

C. acceleration is maximum at extreme position,

D. velocity is maximum at mean position.

(1) B, C and D are correct

(2) A and D are correct

(3) A and B are correct

(4) C and D are correct

Answer (4)

Sol. $|a| = \omega^2 x$ and $v = \omega\sqrt{A^2 - x^2}$

17. **Statement-I:** In a series combination of resistor, equivalent resistance is smaller than the individual resistance.

Statement-II: Resistivity of wire depends on the temperature.

(1) Statement-I is true, statement-II is false

(2) Statement-I is false, statement-II is true

(3) Both statement-I and statement-II are true

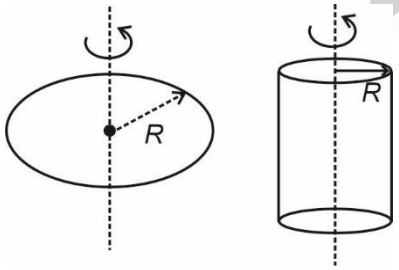
(4) Both statement-I and statement-II are false

Answer (2)

Sol. $R_{eq} = R_1 + R_2 \Rightarrow R_{eq} > (R_1, R_2, ...)$

$\rho = f(T)$

18. Find the ratio of radius of gyration of solid sphere and solid cylinder, both having same mass and radius.



(1) $\frac{2}{\sqrt{5}}$

(2) $\frac{\sqrt{5}}{2}$

(3) $\frac{\sqrt{2}}{\sqrt{5}}$

(4) $\frac{2}{\sqrt{3}}$

Answer (1)

Sol. For solid sphere

$$\frac{2}{5}mR^2 = mk^2$$

$$k = \sqrt{\frac{2}{5}}R$$

For solid cylinder

$$\frac{mR^2}{2} = mk'^2$$

$$\Rightarrow \quad k' = \frac{R}{\sqrt{2}}$$

$$\Rightarrow \quad \frac{k}{k'} = \frac{2}{\sqrt{5}}$$

19. In sonometer experiment, string of mass 18 g having linear mass density 20 g/m oscillates in fundamental mode of frequency 50 Hz. Find the velocity of transverse waves in the string.

(1) 70 m/s (2) 60 m/s

(3) 90 m/s (4) 110 m/s

Answer (3)

Sol. $f = \frac{1}{2l}\sqrt{\frac{T}{\mu}} = \frac{v}{2r}$

$$\therefore \quad 50 = \frac{1}{2\left(\frac{18}{20}\right)}(v)$$

$$\Rightarrow \quad v = 90 \text{ m/s}$$

20. Velocity of is defined as $v = \lambda^a g^b \rho^c$, where ρ is density of water, λ is wavelength and g is acceleration due to gravity.

Find the value of a, b and c in order (a, b, c)

(1) $\left(\frac{1}{2}, \frac{1}{2}, 0\right)$

(2) $\left(1, \frac{1}{2}, 0\right)$

(3) $\left(\frac{1}{2}, 1, 0\right)$

(4) (1, 1, 0)

Answer (1)

Sol. $M^0LT^{-1} = L^aL^bT^{-2b}\, M^cL^{-3c} = M^cL^{(a+b-3c)}\, T^{-2b}$

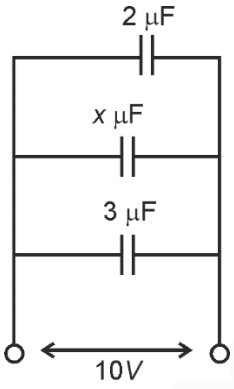
$$\Rightarrow \quad c = 0, -2b = -1 \Rightarrow b = \frac{1}{2}$$

$a + b - 3c = 1 \Rightarrow a + b = 1 \Rightarrow a = \frac{1}{2}$

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. If total charge stored in capacitors is equal to 50 μC, then x = ?



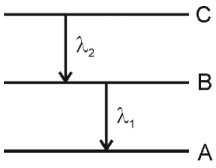
Answer (x = 0)

Sol. q_{Total} = 20 μC + 10x μC + 30 μC = 50 μC

x = 0

22. In the given transition states, A, B and C are first, second and third exited states respectively then

$\frac{\lambda_1}{\lambda_2} = \frac{7}{4n}$, find the value of n



Answer (5)

Sol. A → n = 2

B → n = 3

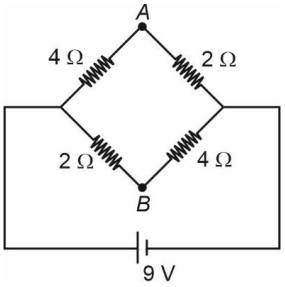
C → n = 4

$\lambda_1 = \frac{hc}{\Delta E_{AB}} \quad \Delta E_{AB} = Rz^2\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5Rz^2}{36}$

$\lambda_2 = \frac{hc}{(\Delta E)_{BC}} \quad (\Delta E)_{BC} = Rz^2\left(\frac{1}{9} - \frac{1}{16}\right) = \frac{7Rz^2}{144}$

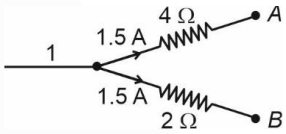
$\frac{\lambda_1}{\lambda_2} = \frac{\Delta E_{BC}}{\Delta E_{AB}} = \frac{7}{144} \times \frac{36}{5} = \left(\frac{7}{20}\right) = \frac{7}{4n} = n = 5$

23. Find the magnitude of potential difference in volt between A and B in given circuit.



Answer (3)

Sol. $i = \frac{9}{3} = 3A$



$\Rightarrow V_1 - 4 \times 1.5 = V_A = V_1 - 6$

$\Rightarrow V_1 - 1.5 \times 2 = V_B = V_1 - 3$

$V_A - V_B = (V_1 - 6) - (V_1 - 3)$

$= -3$

$|V_A - V_B| = 3$ V

24. The refractive index of equilateral prism is $\mu = \sqrt{2}$, then find its minimum angle of deviation in degree.

Answer (30)

Sol. For minimum deviation, $\delta_{min} = 2i - A$

Also, $\sin i = \sqrt{2}\sin\left(90^\circ - 60^\circ\right)$

$i = 45°$

$\delta_{min} = 90° - 60° = 30°$

25. Electric field due to a dipole at an equatorial point depends upon r^{-n}. Value of n is

Answer (3)

Sol. $\vec{E} = \frac{kP}{r^3} \Rightarrow E \propto \frac{1}{r^3}$

26.

27.

28.

29.

30.

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. In which of the following cities, photochemical smog formed is minimum?

(1) Kashmir (2) New Delhi

(3) Hyderabad (4) Kolkata

Answer (1)

Sol. Cold place will have minimum photochemical smog.

2. Number of P – O – P bonds in H_3PO_4, P_4O_{10} and $(HPO_3)_3$ are (respectively)

(1) 0, 6, 3 (2) 6, 3, 0

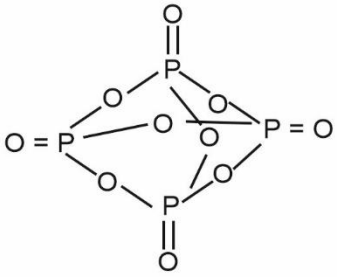
(3) 1, 4, 3 (4) 0, 5, 4

Answer (1)

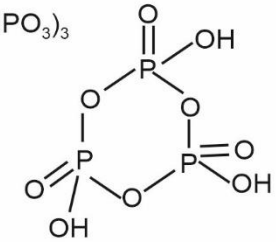
Sol. H_2PO_4

P – O – P bonds = 0

P_4O_{10} (P– O – P bonds = 6)



$(HPO_3)_3$



(3 P– O – P bonds)

3. **S-1:** According to Bohr's model, angular momentum is quantised for stationary orbits.

S-2: Bohr's Model doesn't follow Heisenberg's uncertainty principle.

(1) Both S-1 and S-2 are true

(2) S-1 is true and S-2 is false

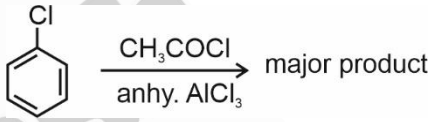
(3) S-1 is false and S-2 is true

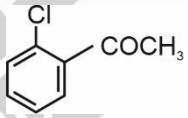
(4) Both S-1 and S-2 are false

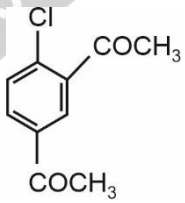
Answer (1)

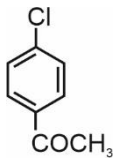
Sol. Both statements are true.

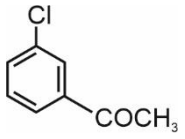
4. Consider the reaction :



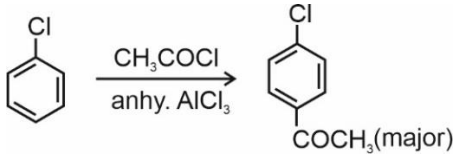
(1) 

(2) 

(3) 

(4) 

Answer (3)

Sol. 

5. Calculate ratio of radii of 2nd & 3rd Bohr's orbit of H-atom.

(1) 2 : 3

(2) 3 : 2

(3) 4 : 9

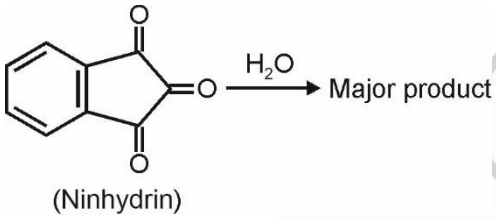
(4) 9 : 4

Answer (3)

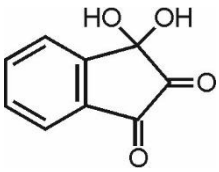
Sol. $r \propto n^2$

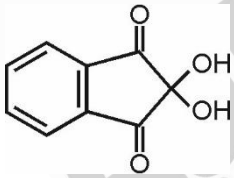
$$\frac{r_2}{r_3} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

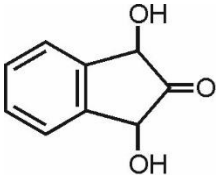
6. The major product formed in the following reaction is

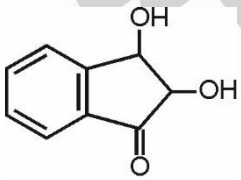


(Ninhydrin)

(1) 

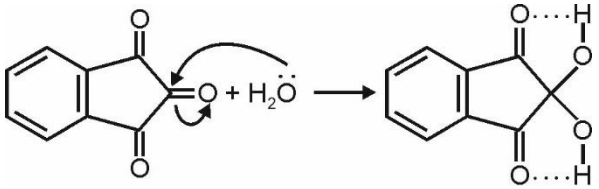
(2) 

(3) 

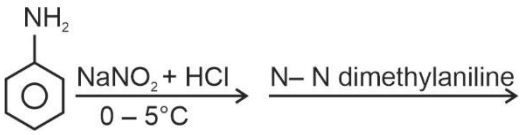
(4) 

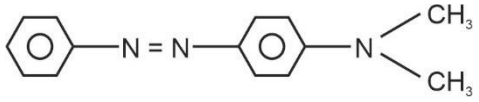
Answer (2)

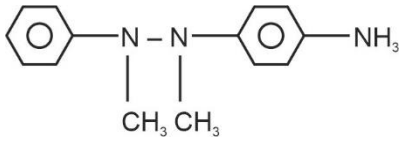
Sol. Ninhydrin has three carbonyl groups. Two of them are in conjugation with benzene ring. So water adds to the carbonyl group which relatively free forming gem diol. The gem diol does not undergo dehydration as it is stabilished by intramolecular H-bond.

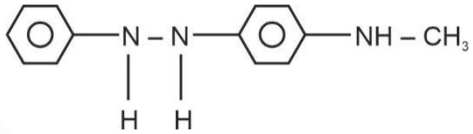


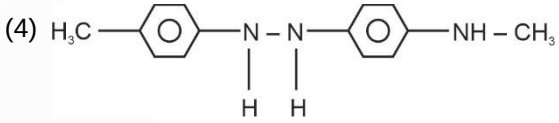
7. The product of the following reaction is:



(1) 

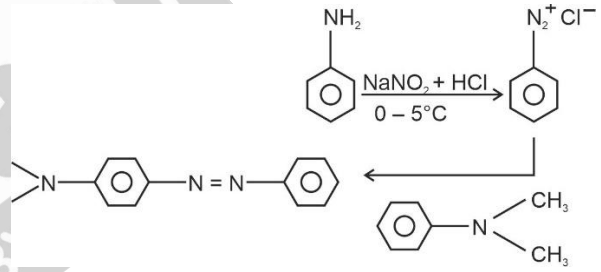
(2) 

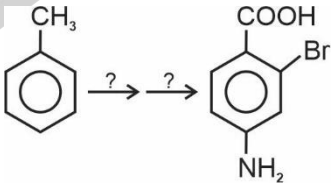
(3) 

(4) 

Answer (1)

Sol.



8. 

(1) (i) HNO_3/H_2SO_4
(ii) $Br_2/AlCl_3$
(iii) $KMnO_4/H^+$

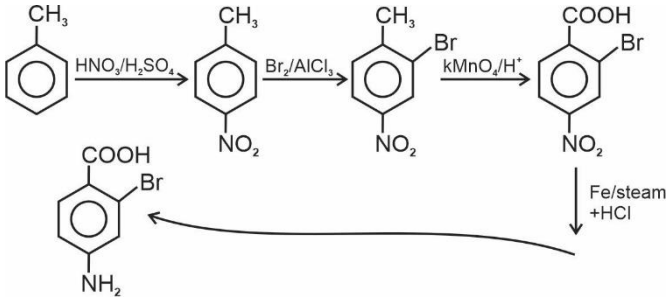
(2) (i) HNO_3/H_2SO_4
(ii) $Br_2/AlCl_3$
(iii) Fe/steam + HCl
(iv) $KMnO_4/H^+$

(3) (i) $Br_2/AlCl_3$
(ii) HNO_3/H_2SO_4
(iii) $KMnO_4/H^+$
(iv) Fe/steam + HCl

(4) (i) HNO_3/H_2SO_4
(ii) $Br_2/AlCl_3$
(iii) $KMnO_4/H^+$
(iv) Fe/steam + HCl

Answer (4)

Sol.



9. Out of the following which has maximum CFSE? (Consider with sign)

(1) $Fe(H_2O)_6]^{3+}$

(2) $[Ti(H_2O)_6]^{3+}$

(3) $[Mn(H_2O)_6]^{3+}$

(4) $[Cr(H_2O)_6]^{3+}$

Answer (1)

Sol. $[Fe(H_2O)_6]^{3+}$ has $t_{2g}^3\ e_g^2$ configuration and has zero value for CFSE.

10. In CsCl crystal, which of the following relations is true?

(1) ${}^{r}Cs^{\oplus} + {}^{r}Cl^{\Theta} = \frac{\sqrt{3}a}{2}$

(2) ${}^{r}Cs^{\oplus} + {}^{r}Cl^{\Theta} = \frac{a}{\sqrt{2}}$

(3) ${}^{r}Cs^{\oplus} + {}^{r}Cl^{\Theta} = \frac{a}{2}$

(4) ${}^{r}Cs^{\oplus} + {}^{r}Cl^{\Theta} = \frac{\sqrt{3}}{\sqrt{2}}\ a$

Answer (1)

Sol. As $Cs^{\oplus}$ occupies cubical voids, we have

$${}^{r}Cs^{\oplus} + {}^{r}Cl^{\Theta} = \frac{\sqrt{3}a}{2}$$

11.

	Column I	**Column-II**
	Monomer unit	**Polymer**
(a)	Acrylonitrile	(i) Orlon
(b)	Tetra-Fluoroethene	(ii) Natural Rubber
(c)	Caprolactam	(iii) Teflon
(d)	Isoprene	(iv) Nylon-6

(1) a → (i); b → (ii); c → (iii); d → (iv)

(2) a → (i); b → (iii); c → (iv); d → (ii)

(3) a → (ii); b → (iv); c → (iii); d → (i)

(4) a → (iii); b → (ii); c → (iv); d → (i)

Answer (2)

Sol.

Monomer unit		**Polymer**
Acrylonitrile	—	Orlon
Tetra-Fluoroethene	—	Teflon
Caprolactam	—	Nylon-6
Isoprene	—	Natural Rubber

12. In which of the following is not an example of calcination?

(1) $PbS + O_2 \rightarrow PbO + SO_2$

(2) $CaCO_3 \rightarrow CaO + CO_2$

(3) $MgCO_3 \xrightarrow{\Delta} MgO + CO_2$

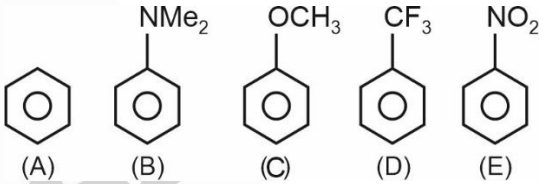
(4) $ZnCO_3 \rightarrow ZnO + CO_2$

Answer (1)

Sol. $PbS + O_2 \rightarrow PbO + SO_2$

is an example of roasting.

13. Rate of electrophilic aromatic substitution.



(1) B > C > A > D > E (2) A > B > C > D > E

(3) E > D > C > B > A (4) A > B > D > C > E

Answer (1)

Sol. B > C > A > D > E

Rate of electrophilic aromatic substitution ∝-electron density in benzene ring.

14. Identify the stationary phase (S) and mobile phase (M) in paper chromatography.

(1) S : Solvent
M : Chromatography paper

(2) S : Solvent
M : Water

(3) S : Water
M : Solvent

(4) S : Chromatography paper
M : Solvent

Answer (4)

Sol. In paper chromatography, a special quality paper called chromatography paper is used.

Chromatography paper contains water trapped in it which acts as a stationary phase. A strip of chromatography paper spotted at the base with the solution of a mixture is suspended in a suitable solvent which acts as a mobile phase.

15. $CH_3CH=CH_2 \xrightarrow[\substack{(2)\ PCC\\(3)\ CH_3MgBr\\(4)\ H_2O}]{\substack{(1)\ BH_3/THF\\H_2O_2/NaOH}} Product$

The product is

(1) $CH_3CH_2-\underset{\displaystyle OH}{\underset{|}{C}}H-CH_3$

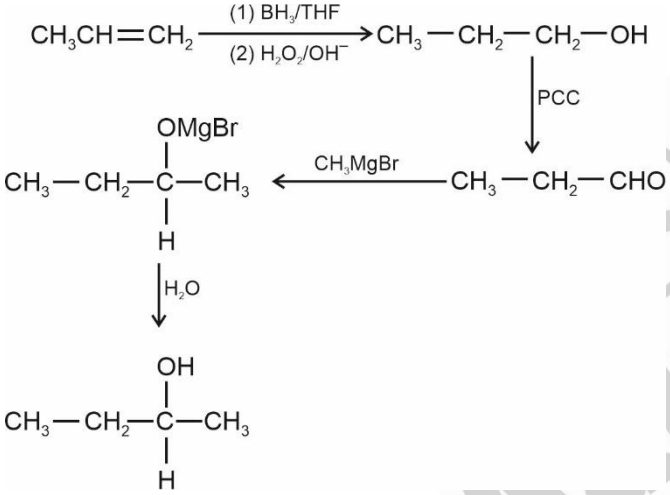
(2) CH_3CH_2COOH

(3) $CH_3CH_2-\underset{\displaystyle OH}{\underset{|}{\overset{\displaystyle CH_3}{\overset{|}{C}}}}H-CH_3$

(4) CH_3-CH_2CHO

Answer (1)

Sol.



16. The ratio of silica to alumina in cement is

(1) 5.5

(2) 2

(3) 3

(4) 1.5

Answer (3)

Sol. For good quality cement, the ratio of silica (SiO_2) to alumina (Al_2O_3) should be between 2.5 and 4.

17. **Statement I :** pH of 10^{-8} M HCl is 8 at 25°C

Statement II : Titration of weak acid & strong base at Half equivalence point gives pH = $\frac{pKa}{2}$.

(1) Statement I is correct and Statement II is correct

(2) Statement I and II both are incorrect

(3) Statement I is incorrect and Statement II incorrect

(4) Statement I is correct and Statement II is incorrect

Answer (2)

Sol. 10^{-8} M HCl is acidic solution hence pH will be less than 7 at 25°C and incomplete titration of weak acid by strong base upto half equivalence point results in buffer with pH = pKa of weak acid.

18. **Assertion A :** $MgCl_2$ and $BeCl_2$ gives flame test

Reason R : Ionization energy of Be and Mg is high

(1) A is incorrect but R is correct

(2) A is incorrect and R is also incorrect

(3) A is correct, R is correct and R is correct explanation of A

(4) A is correct, R is correct, R is not the correct explanation of A

Answer (1)

Sol. $MgCl_2$ and $BeCl_2$ do not give flame test as both have high ionization energy.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. How many of the following statements are correct.

(1) Conductivity (K) decreases with increase in dilution for both strong & weak electrolyte

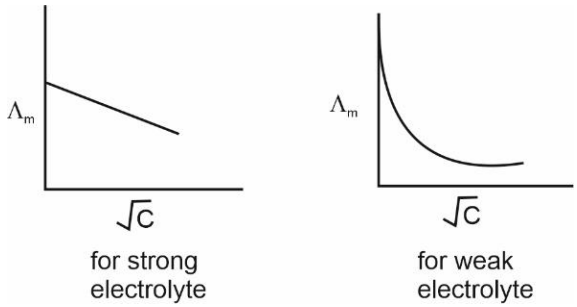
(2) Molar conductivity increases with increase in dilution for both strong and weak electrolyte

(3) Molar conductivity increases with increase in 'α' for weak electrolyte.

(4) Change in molar conductivity is same for both strong and weak electrolyte with increase in dilution.

Answer (3)

Sol. Except (4) all statements are correct.



for strong electrolyte

for weak electrolyte

22. Lowering of vapour pressure of 30% of aqueous solution of glucose. (in mm Hg)

$P_{H_2O} = 760$ mm of Hg

Answer (729.00)

Sol. $\frac{760 - x}{x} = \frac{\left(\frac{30}{180}\right)}{\left(\frac{70}{18}\right)} = \frac{3}{70}$

x = 728.7 mm Hg

≈ 729 mm Hg

23. What is the change in oxidation state of Mn in the reaction

$KMnO_4 + KI \xrightarrow{\text{acidic medium}}$

Answer (5)

Sol. $KMnO_4 + KI \xrightarrow{H^{\oplus}} I_2 + Mn^{2+}$

Change in oxidation state of Mn = 5

24. How many of the following have 10 electrons?

(i) O^{2-} (ii) O

(iii) Al^{3+} (iv) Al

(v) F (vi) F^-

(vii) Mg^{2+} (viii) Mg

(ix) N^{3-}

Answer (05)

Sol.

Species	Number of electrons
O^{2-}	10
O	8
Al^{3+}	10
Al	13
F	9
F^-	10
Mg^{2+}	10
Mg	12
N^{3-}	10

25. Oxidation state of Cr in chromyl chloride is

Answer (6)

Sol. : In CrO_2Cl_2; oxidation state of Cr is +6

26. For a radioactive decay $t_{1/2} = 15$ years. What will be the rate constant (yr^{-1})?

Answer (0.05)

Sol. $k = \frac{0.613}{t_{1/2}}$

= 0.0462 yr^{-1} □ 0.05 yr^{-1}

27.

28.

29.

30.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. There are 5 black and 3 white balls in a bag. A die is rolled, we need to pick the number of balls appearing on a die. The probability that all balls are white is

(1) $\frac{1}{12}$ (2) $\frac{1}{18}$

(3) $\frac{2}{9}$ (4) $\frac{1}{2}$

Answer (1)

Sol. $\frac{1}{6}\times\frac{{}^3C_1}{{}^8C_1}+\frac{1}{6}\times\frac{{}^3C_2}{{}^8C_2}+\frac{1}{6}\times\frac{{}^3C_3}{{}^8C_3}$

$=\frac{1}{6}\left(\frac{3}{8}+\frac{3}{28}+\frac{1}{56}\right)$

$=\frac{1}{6}\left(\frac{21+6+1}{56}\right)=\frac{1}{6}\times\frac{1}{2}=\frac{1}{12}$

2. The mean and variance of 15 observations is 20 and 64, respectively. If 55 is wrongly read as 40 as one of the observation, then the correct variance is _____ .

(1) $\frac{243}{3}$ (2) $\frac{167}{2}$

(3) $\frac{247}{3}$ (4) 96

Answer (3)

Sol. $64=\frac{\sum x_i^2}{15}-(20)^2$

$\Rightarrow \sum x_i^2=6950$

$\sigma^2=\frac{6950-40^2+50^2}{15}-(21)^2$

$=\frac{7850}{15}-441$

$=\frac{1235}{15}$

$=\frac{247}{3}$

3. Matrix A having order m has the value of its determinant as $(m)^{-n}$. The value of $\det(n\ \text{adj}(\text{adj}(mA)))$ is

(1) $n^m(m^{m-n})^{(m-1)^2}$ (2) $n^m(m^{m-n})^{(m-1)}$

(3) $m^n(m^{m-n})$ (4) $n^m(m^{n-m})^2$

Answer (1)

Sol. $\det(n\ adj(adj(mA)))$

$=n^m\ \det(adj(adj\ mA))$

$=n^m\cdot(\det(mA))^{(m-1)^2}$

$=n^m\cdot\left(m^m\ \det(A)\right)^{(m-1)^2}$

$=n^m\cdot m^{n(m-1)^2}\cdot(m^{-n})^{(m-1)^2}$

$=n^m\cdot(m^n)^{(n-1)^2}(m^{-n})^{(m-1)^2}$

$=n^m(m^{m-n})^{(m-1)^2}$

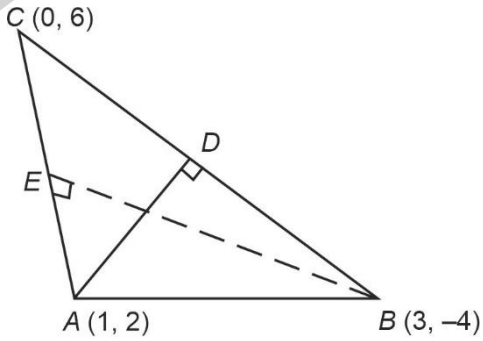
4. The orthocentre of a triangle having vertices as $A(1, 2)$, $B(3, -4)$, $C(0, 6)$ is

(1) (–129, –37) (2) (9, –1)

(3) (7, –3) (4) (28, –16)

Answer (1)

Sol.



$AD:(y-2)=\frac{3}{10}(x-1)$

$3x-10y+17=0$ ...(i)

$BE:(y+4)=\frac{1}{4}(x-3)$

$x-4y=19$ ...(ii)

Solving (i) and (ii)

(–129, –37) is orthocentre

5. The statement $p \wedge (q \wedge \sim (p \wedge q))$ is

(1) Tautology

(2) Fallacy

(3) Is equivalent to $p \wedge q$

(4) Is equivalent to $p \vee q$

Answer (2)

Sol. $p \wedge (q \wedge \sim (p \wedge q))$

$= p \wedge (q \wedge (\sim p \vee \sim q))$

$= p \wedge ((q \wedge \sim p) \vee (q \wedge \sim q))$

$= p \wedge (q \wedge \sim p)$

$= F$

6. If we have a ATM pin of 4 digit. The Sum of first two digits is equal to sum of last two digits and the greatest integer used is 7. Then the number of trials used to get the pin if all digits are different

(1) 194

(2) 192

(3) 200

(4) 220

Answer (2)

Sol. $\underline{a}\,\underline{b}\,\underline{c}\,\underline{d}$

According to condition $a + b = c + d$.

If sum is 3 $\rightarrow$ (0, 3) (1, 2)

If sum is 4 $\rightarrow$ (0, 4), (1, 3)

If sum is 5 $\rightarrow$ (0, 5), (1, 4), (2, 3)

If sum is 6 $\rightarrow$ (0, 6), (1, 5), (2, 4)

If sum is 7 $\rightarrow$ (0, 7), (1, 6), (2, 5), (3, 4)

If sum is 8 $\rightarrow$ (1, 7), (2, 6), (3, 5)

If sum is 9 $\rightarrow$ (2, 7), (3, 6), (4, 5)

If sum is 10 $\rightarrow$ (3, 7), (4, 6)

If sum is 11 $\rightarrow$ (4, 7), (5, 6)

Now total trials $= 4 \times 2! \times 2!\ 2! + 4 \times {}^3C_2 \times 2! \times 2! \times 2! + {}^4C_2 \times 2! \times 2! \times 2!$

$= 32 + 32 \times 3 + 64$

$= 32 + 96 + 64$

$= 192$

7. 3 points $A(1, 1, 1)$, $B(-2, 3, 2)$ and $C(0, 3, 0)$ lie on a plane. Line $\frac{x-1}{-2} = \frac{y+2}{-1} = \frac{z}{4}$ intersects the plane at P. The distance OP is (O is origin) _____.

(1) $\sqrt{349}$ (2) $\sqrt{231}$

(3) $\sqrt{341}$ (4) $\sqrt{168}$

Answer (3)

Sol. Equation of plane : $\begin{vmatrix} x & y-3 & z \\ 3 & -2 & -1 \\ 1 & -2 & 1 \end{vmatrix} = 0$

$\Rightarrow x(-2-2) - (y-3)(3+1) + z(-6+2) = 0$

$\Rightarrow -4x - (y-3)4 - 4z = 0$

$\Rightarrow x + y - 3 + z = 0$

$\Rightarrow x + y + z = 3$

Point on a line : $(-2k + 1, -k - 2, 4k)$

$(-2k + 1) + (-k - 2) + 4k = 3$

$\Rightarrow k = 4$

$\therefore P(-7, -6, 16)$

$OP = \sqrt{49 + 36 + 256}$

$= \sqrt{341}$

8. $A(5, -3)$, $C(7, 8)$ and $B(t, 0)$, $0 \le t \le 4$. The perimeter is maximum at $t = \alpha$ and minimum at $t = \beta$, then $\alpha^2 + \beta^2$ is ____

(1) 12 (2) 9

(3) 16 (4) 25

Answer (3)

Sol. perimeter = $AC + BC + AB$

$(\text{perimeter})^2 = 5\sqrt{5} + (t-7)^2 + 64 + (t-5)^2 + 9$

$= 73 + 5\sqrt{5} + 2t^2 - 24t + 74$

$\Rightarrow 2t^2 - 24t + 147 + 5\sqrt{5}$

$\Rightarrow 2(t-6)^2 + 75 + 5\sqrt{5}$

$(\text{perimeter})^2_{\max}$ at $t = 0 = \alpha$

$(\text{perimeter})^2_{\min}$ at $t = 4 = \beta$

$\therefore \alpha^2 + \beta^2 = 16$

9. Consider the circles $x^2 + y^2 - 13x - 15y + 13 = 0$ and $x^2 + y^2 - 6x - 6y - 7 = 0$, then number of common tangents is

(1) 2 (2) 0

(3) 1 (4) 4

Answer (1)

Sol : $c_1 \equiv \left(\frac{13}{2}, \frac{15}{2}\right)$ $c_2 \equiv (3, 3)$

$r_1 = \sqrt{\left(\frac{13}{2}\right)^2 + \left(\frac{15}{2}\right)^2 - 13}$; 9

$r_2 = \sqrt{9+9+7} = 5$

and $c_1c_2 = \sqrt{\left(\frac{7}{2}\right)^2 + \left(\frac{9}{2}\right)^2}$

So, $|c_1c_2| < r_1 + r_2$

$\therefore$ Total common tangents = 2

10. $f(x) = \int \frac{dx}{\sqrt{4-3x^2}\left(4x^2+3\right)}$, then $f(x)$ =

(1) $-\frac{1}{25}\left(\frac{\log\left(\frac{4}{x^2}-3\right)}{2} \frac{-\log\left(\frac{12}{x^2}+16\right)}{6}\right)+c$

(2) $\frac{1}{25}\left(\frac{\log\left(4-x^2\right)}{4} \frac{-\log\left(x^2-16\right)}{6}\right)$

(3) $-\frac{1}{25}\left[\log\left(4-3x^2\right)+\log\left(3x^2-16\right)\right]$

(4) $-\frac{1}{25}\left(\frac{\log\left(4-3x^2\right)}{2}+\frac{\log\left(12-16x^2\right)}{6}\right)$

Answer (1)

Sol. Let $x = \frac{1}{t}$

$dx = -\frac{1}{t^2}dt$

$\int \frac{\frac{-1}{t^2}dt}{\left(\sqrt{4-\frac{3}{t^2}}\right)\left(\frac{4}{t^2}+3\right)}$

$\Rightarrow -\int \frac{tdt}{\sqrt{4t^2-3}\left(4+3t^2\right)}$

$4t^2 - 3 = m^2$

$\Rightarrow 8t\,dt = 2\,mdm$

$= -\frac{1}{4}\int \frac{dm}{m\left(4+3\left(\frac{m^2+3}{4}\right)\right)}$

$= -\frac{1}{4}\int \frac{4dm}{m\left(3m^2+25\right)}$

$-\frac{1}{25}\int \frac{\left(3m^2+25\right)-m^2}{m\left(3m^2+25\right)}dm$

$= -\frac{1}{25}\int\left(\frac{1}{m}-\frac{m}{3m^2+25}\right)dm =$

$-\frac{1}{25}\left(\log m - \frac{\log\left(3m^2+25\right)}{6}\right)+c$

$\Rightarrow -\frac{1}{25}\left(\log\sqrt{4t^2-3} - \frac{\log\left(3\left(4t^2-3\right)+25\right)}{6}\right)+c$

$\Rightarrow -\frac{1}{25}\left(\log\sqrt{4t^2-3} - \frac{\log\left(12t^2+16\right)}{6}\right)+c$

$= -\frac{1}{25}\left[\frac{\log\left(\frac{4}{x^2}-3\right)}{2} \frac{-\log\left(\frac{12}{x^2}+16\right)}{6}\right]+c$

11.

12.

13.

14.

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. The number of solution of equation $x|x| + 5|x + 2| + 6 = 0$ is

Answer (01)

Sol. Case I

$x < -2$

$-x^2 - 5(x + 2) + 6 = 0$

$x^2 + 5x + 4 = 0$

$(x + 1)(x + 4) = 0$

$x = \underset{\text{(rejected)}}{-1}$ or -4

$\therefore x = -4$ is solution

Case II

$-2 < x < 0$

$-x^2 + 5(x + 2) + 6 = 0$

$x^2 - 5x - 16 = 0$

$x = \frac{5 \pm \sqrt{25 + 64}}{2} = \frac{5 \pm \sqrt{89}}{2}$

$\therefore$ No solution between $-2 < x < 0$

Case III

For $x > 0$

$x^2 + 5(x + 2) + 6 = 0$

$x^2 + 5x + 16 = 0$

$D < 0$

$\therefore$ No solution

$\Rightarrow$ Only one solution *i.e.*, $x = -4$

22. Let $f(x) = \log(4x^2 + 11x + 9) + \sin^{-1}(4x + 3) + \cos^{-1}\left(\frac{10x + 6}{3}\right)$ and if domain of $f(x)$ is $[\alpha, \beta]$, then $\left|10[\alpha - \beta]\right|$ is

Answer (04)

Sol. $4x^2 + 11x + 9 > 0$ $(\because 0 = 121 - 144 < 0)$

So, $-1 \le 4x + 3 \le 1$ and $-1 \le \frac{10x + 6}{3} \le 1$

$-4 \le 4x \le -2$ $\quad -9 \le 10x \le -3$

$-1 \le x \le -\frac{1}{2}$ $\quad -\frac{9}{10} \le x \le \frac{-3}{10}$

So, $D_f = \left[\frac{-9}{10}, \frac{-1}{2}\right]$

$\therefore \quad \alpha = \frac{-9}{10} \quad \beta = \frac{-1}{2}$

So, $\left|10\left(\frac{-9}{10} + \frac{1}{2}\right)\right| = 4$

23. 23. How many three-digit number can be formed which are divisible by 3 using the digits 1, 3, 5, 8 and repeatation is allowed

Answer (22)

Sol. I : All three digits are alike

1 1 1, 3 3 3, 5 5 5, 8 8 8 $\rightarrow 4$

II : 2 digits are alike

5 5 8 $\rightarrow \frac{3!}{2!} = 3$

8 8 5 $\rightarrow \frac{3!}{2!} = 3$

III : All three digits are different

1, 3, 5 $\rightarrow 6$

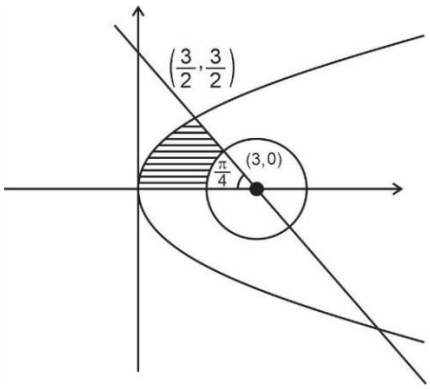
1, 3, 8 $\rightarrow 6$

$\therefore$ Total numbers = 22

24. Area bounded by the curve $2y^2 = 3x$ and the line $x + y = 3$ outside the circle $(x - 3)^2 + y^2 = 2$ and above x-axis is A. The value of $4(\pi + 4A)$ is

Answer (42)

Sol.



A = required area

$$= \int_0^{\frac{3}{2}} \left[(3-y) - \left(\frac{2y^2}{3} \right) \right] dy - \pi\left(\sqrt{2}\right)^2 \cdot \frac{1}{8}$$

$$\Rightarrow \left(3y - \frac{y^2}{2} - \frac{2}{9}y^3 \right)\Bigg|_0^{\frac{3}{2}} - \frac{\pi}{4}$$

$$\Rightarrow 3 \cdot \frac{3}{2} - \frac{9}{8} - \frac{2}{9} \cdot \frac{27}{8} - \frac{\pi}{4}$$

$$\Rightarrow \frac{36-9-6}{8} - \frac{\pi}{4} = \frac{21}{8} - \frac{\pi}{4}$$

$\Rightarrow 4(\pi + 4A)$

$$= 4\left(\frac{21}{2}\right) = 42$$

25. If $n \in [10, 100]$ and $n \in N$, then how many such n are possible where $3^n - 3$ is divisible by 7?

Answer (15)

Sol. $3^n - 3 = 7K, \quad K \in I$

$3^n = 7K + 3$

Now,

$3 \equiv 3 \pmod 7$

$3^2 \equiv 2 \pmod 7$

$3^3 \equiv -1 \pmod 7$

$3^6 \equiv 1 \pmod 7$

$3^7 \equiv 3 \pmod 7$

Since,

$3^{13} \equiv 3 \pmod 7$

$\vdots$

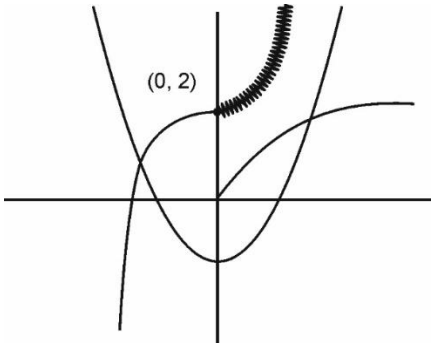
$\therefore$ n can be 13, 19, ……97

$\therefore$ Total $\boxed{n = 15}$

26. If $y = \max\left\{\sqrt{x}, x^2 - 4, x^3 + 2\right\}$, then number of solution(s) of $y = 1$ is/are ____.

Answer (0)

Sol.



As domain of y is $[0, \infty)$

$\therefore \quad y = \max\left(\sqrt{x}, x^2 - 4, x^3 + 2\right) = x^3 + 2$

$\forall x \in [0, \infty)$

$\therefore \quad x^3 + 2 = 1$

$\Rightarrow \quad x^3 = -1$

No solution in $[0, \infty)$

27. Let $A = \{1, 2, 3, 4\}$ if R on a set $A \times A$ such that $(a, b)\ R\ (c, d)$ iff $2a + 3b = 6c + 5d$, then number of elements in R is

Answer (04)

Sol. Maximum value of $2a + 3b = 20$ at $(4, 4)$

Minimum value of $6c + 5d = 11$ at $(1, 1)$

So, $6c + 5d$ can be 11, 16, 17

So, $2a + 3b = 11$

$(a, b) \equiv (4, 1), (1, 3)$

and

$2a + 3b = 16$ $\qquad$ $(6c + 5d = 16)$ $(1, 2)$

$(a, b) \equiv (2, 4)$

$2a + 3b = 17$

$(a, b) \equiv (4, 3)$

So, total elements = 4

28. If $f(x) = \max. \{1 + x + [x], x + 1, 1 - x + [x]\}$, $0 \le x \le 2$, then number of points where $f(x)$ is non-differentiable is

Answer (01)

Sol. $f(x) = \max. \{1 + x + [x], x + 1, 1 - x + [x]\}$

$$= \begin{cases} 1 + x, & 0 \ge x < 1 \\ 1 + x + [x] & 1 \le x \le 2 \end{cases}$$

$\therefore$ Number of points of non-differentiability = 1 (at $x = 1$)

29.

30.

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