12/04/2023
Morning

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Memory Based
Answers & Solutions
for
JEE (Main)-2023 (Online) Phase-2

(Physics, Chemistry and Mathematics)

Time : 3 hrs. **M.M. : 300**

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. If a planet has mass equal to 16 times the mass of earth, and radius equal to 4 times that of earth. The ratio of escape speed of planet to that of earth is

(1) 2 : 1 (2) 1 : 2

(3) $\sqrt{2}:1$ (4) 4 : 1

Answer (1)

Sol. $\frac{V_P}{V_e}=\sqrt{\frac{2GM_P}{R_P}}\times\sqrt{\frac{R_e}{2GM_e}}$

$=\sqrt{\frac{16}{4}}=2$

2. Find ratio of de-Broglie wavelength of a proton and an α-particle, when accelerated through a potential difference of 2 V and 4 V respectively.

(1) 4 : 1 (2) 2 : 1

(3) 1 : 8 (4) 16 : 1

Answer (1)

Sol. $\frac{\lambda_p}{\lambda_\alpha}=\frac{\sqrt{2q_\alpha V_\alpha m_\alpha}}{\sqrt{2q_pV_pm_p}}=\sqrt{\frac{2\times4\times4}{1\times2\times1}}=\frac{4}{1}$

3. If a body of mass 5 kg is in equilibrium due to forces F_1, F_2 and F_3. F_2 and F_3 are perpendicular to each other. If F_1 is removed then find the acceleration of body. Given : F_2 = 6 N and F_3 = 8 N

(1) 2 m/s^2 (2) 3 m/s^2

(3) 4 m/s^2 (4) 5 m/s^2

Answer (1)

Sol. $F_{net}=\sqrt{6^2+8^2}=10$ N

$a=\frac{10}{5}=2$ m/s^2

4. If an object cools down from 80°C to 60°C in 5 minutes in a surrounding of temperature 20°C. The time taken to cool from 60°C to 40°C will be (assume Newton's law of cooling to be valid)

(1) $\frac{25}{3}$minutes (2) 5 minutes

(3) $\frac{25}{4}$minutes (4) 9 minutes

Answer (1)

Sol. $\frac{20}{5}=K(70-20)$ …(1)

also $\frac{20}{t}=K(50-20)$ …(2)

from (1) and (2)

$t=\frac{25}{3}$minutes

5. Ratio between rms speed of Ar to the most probable speed of O_2 at 27°C is

(1) $\sqrt{\frac{8}{\pi}}$ (2) $\sqrt{\frac{8}{3}}$

(3) $\sqrt{\frac{4}{\pi}}$ (4) $\sqrt{\frac{4}{3}}$

Answer (2)

Sol. $v_{\text{rms Ar}}=\sqrt{\frac{3RT}{M}}=\sqrt{\frac{3RT}{40}}$

$v_{\text{mp }O_2}=\sqrt{\frac{2RT}{M}}=\sqrt{\frac{2RT}{32}}$

$\frac{v_{\text{rms Ar}}}{v_{\text{mp }O_2}}=\sqrt{\frac{3}{40}\times\frac{32}{2}}=\sqrt{\frac{6}{5}}$

6. A dipole having dipole moment $\vec{M}$ is placed in two magnetic field of strength B_1 and B_2 respectively. If dipole oscillates 60 time in 20 seconds in B_1 magnetic field and 60 oscillations in 30 seconds in B_2 magnetic field. Then find the $\left(\frac{B_1}{B_2}\right)$.

(1) $\frac{3}{2}$ (2) $\frac{2}{3}$

(3) $\frac{4}{9}$ (4) $\frac{9}{4}$

Answer (4)

Sol. $\tau=\vec{M}\times\vec{B}$

$I\alpha=-MB\theta$

$\alpha=-\left(\frac{MB}{I}\right)\theta$

$T=2\pi\sqrt{\frac{I}{MB}}$

$\frac{T_1}{T_2}=\sqrt{\frac{B_2}{B_1}}$

$\Rightarrow\ \frac{20}{30}=\sqrt{\frac{B_2}{B_1}}$

$\Rightarrow \frac{B_1}{B_2} = \frac{9}{4}$

7. Mass of body = 500 kg, $\mu = 0.7$. Find work required to move a distance of 4 Km if the body moves with velocity 10 m/s.

(1) 3.5×10^6 J (2) 28×10^6 J

(3) 7×10^6 J (4) 14×10^6 J

Answer (4)

Sol. Since v = const. $\Rightarrow F = \mu mg = 0.7 \times 500 \times 10 = 3500$ N

$W = FS = 3.5 \times 10^3 \times 4 \times 10^3 = 14 \times 10^6$ J

8. Suppose a situation in which two planet orbits around the sun in same orbit. If the mass of planet 1 is twice the mass of planet 2, then what do they have same?

(1) Potential energy (2) Kinetic energy

(3) Total energy (4) Velocity

Answer (4)

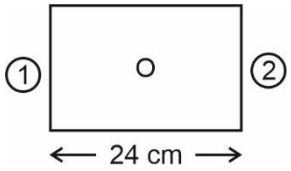
Sol. $v = \sqrt{\frac{GM}{r}}$; M = mass of sun

P.E. $= -\frac{GMm}{r}$ m, different so different P.E.

K.E. $= \frac{1}{2}mv^2$ m, different so different K.E.

T.E. will be different.

9. In a ice cube of thickness 24 cm, has bubble trapped in it as shown in figure. If apparent side are 12 cm and 4 cm from side ① and side ② respectively then refractive index of ice cube is



(1) $\frac{4}{3}$ (2) $\frac{3}{2}$

(3) 2 (4) 2.4

Answer (2)

Sol. $\frac{l}{\mu} = 12 + 4 = 16$ cm

$\frac{24}{16} = \mu$

$\Rightarrow \mu = \frac{3}{2}$

10. **Statement (1):** A truck and a car moving with equal kinetic energy are stopped by equal retarding force. Both will cover equal distance to stop.

Statement (2): A car moving towards east suddenly changes its direction towards north with same speed. Its acceleration is zero.

(1) Both (1) and (2) are true

(2) Both (1) and (2) are false

(3) (1) is true, (2) is false

(4) (1) is false, (2) is true

Answer (3)

Sol. For (1) $v \propto \frac{1}{\sqrt{m}}$, $a \propto \frac{1}{m}$

$\because$ $s = \frac{v^2}{2a} \rightarrow$ independent of mass

For (2) direction is changed, $\therefore a \neq 0$

11. Match the physical quantity in column-I with the respective dimension in column-II and choose the correct option

	Column-I		Column-II
I.	Spring constant	(P)	$[ML^2T^0]$
II.	Moment of inertia	(Q)	$[M^0L^0T^{-1}]$
III.	Angular momentum	(R)	$[ML^0T^{-2}]$
IV.	Angular speed	(S)	$[ML^2T^{-1}]$

(1) I(P), II(Q), III(R), IV(S)

(2) I(R), II(P), III(Q), IV(S)

(3) I(R), II(S), III(Q), IV(P)

(4) I(R), II(P), III(S), IV(Q)

Answer (4)

Sol. Theoretical

12. The length of a conductor having resistance 160 Ω, is compressed to 25% of its initial value. The new resistance will be

(1) 10 Ω

(2) 20 Ω

(3) 15 Ω

(4) 17 Ω

Answer (1)

Sol. At constant volume, $R \propto \ell^2$

$\therefore \quad \frac{160}{R'} = \frac{\ell^2}{\frac{\ell^2}{16}}$

$\boxed{R' = 10\ \Omega}$

13. Statement I : In LCR circuit, by increasing frequency current increases first then decreases

Statement II : Power factor of LCR circuit is one.

Choose the correct option

(1) Statement I is correct and statement II is incorrect
(2) Statement I is incorrect and statement I is correct
(3) Both Statement I and statement II are correct
(4) Both Statement I and statement II are incorrect

Answer (1)

Sol. $I = \frac{V}{Z}$

As ω increases, Z decreases first then increases

$\cos\phi = \left(\frac{R}{Z}\right)$

14. **Assertion (A):** An electrical dipole is enclosed in a closed gaussian surface. The total flux through the enclosed surface is zero.

Reason (R): Net charge inside the enclosed surface is zero.

(1) Both (A) and (R) are correct and (R) is correct explanation of (A)
(2) Both (A) and (R) are correct and (R) is not the correct explanation of (A)
(3) (A) is true, but (R) is false
(4) (A) and (R) both are false

Answer (1)

Sol. $\phi = \frac{q_{in}}{\epsilon_0}$ and $q_{in} = 0$ inside surface

15. A circular ring is placed in magnetic field of 0.4 T. Suddenly its radius starts shrinking at the rate of 1 mm/s. Find the induced emf in the ring at $r = 2$ cm.

(1) $16\pi\ \mu V$ (2) $8\pi\ \mu V$
(3) 16π mV (4) 8π mV

Answer (1)

Sol. $\phi = BA$

$$\varepsilon = \frac{d\phi}{dt} = \frac{BdA}{dt} = \frac{2\pi rBdr}{dt}$$

at $r = 2$ cm

$$\varepsilon_{induced} = \frac{2\pi\times 2}{100}\times 0.4\times\frac{.1}{1000}$$

$$= \frac{16\pi}{10^6} = 16\pi\times 10^{-6}\ V$$

16. A body is doing SHM with amplitude A. When it is at $x = +\frac{A}{2}$, find ratio of kinetic energy to potential energy

(1) 1 : 1 (2) 3 : 1
(3) 2 : 1 (4) 4 : 1

Answer (2)

Sol. $\frac{K}{U} = \frac{\frac{1}{2}m\omega^2(A^2 - x^2)}{\frac{1}{2}m\omega^2x^2}$

$$= \frac{A^2 - x^2}{x^2} = \frac{\frac{3A^2}{4}}{\frac{A^2}{4}} = \frac{3}{1}$$

17. Current flowing in a conductor at 0°C and 100°C is 2 A and 1.2 A respectively. The current at 80°C is

(1) 1.3 A (2) 1.5 A
(3) 1.6 A (4) 1.8 A

Answer (1)

Sol. $\because\ R \propto \frac{1}{i}$

Let $R = \frac{x}{i}$

also $\frac{\frac{x}{1.2} - \frac{x}{2}}{100 - 0} = \frac{\frac{x}{i} - \frac{x}{2}}{80 - 0}$

$i = \frac{30}{23} \approx 1.3$ A

18. Which of the following is more energetic between Infrared wave and microwave?

(1) IR wave
(2) Microwaves
(3) Both are same energetic
(4) Cannot predict

Answer (1)

Sol. $\because\ f_{IR} > f_{micro}$

$\therefore\ E_{IR} > E_{micro}$

IR waves are more energetic.

19. If carnot engines works between freezing point and boiling point of water then the efficiency of carnot engine is

(1) 35% (2) 27%
(3) 22% (4) 17%

Answer (2)

Sol. $\eta = 1 - \frac{T_L}{T_H} = 1 - \left(\frac{273}{373}\right) = \left(\frac{100}{373}\right) \approx 0.27$

20. In closed organ pipe, the resonance consecutive frequencies are in ratio 1 : 3 : 5… and 5th harmonic frequency is 405 Hz. Velocity of sound = 345 m/s. Find length of organ pipe.

(1) $\frac{108}{115}$ m (2) $\frac{81}{115}$ m

(3) $\frac{115}{108}$ m (4) $\frac{115}{81}$ m

Answer (3)

Sol. For 5th harmonic, $f = 5f_0 = 405$

or $5\frac{v}{\lambda} = 405 \Rightarrow 5\left(\frac{345}{4l}\right) = 405$

$\Rightarrow \quad l = \frac{5\times 345}{4\times 405}$

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. A particle is thrown vertically upward with initial velocity of 150 m/s. Find the ratio of its speed at $t = 3$ seconds and $t = 5$ seconds. (take $g = 10$ m/s^2)

Answer (01.20)

Sol. $\frac{v_3}{v_5} = \left(\frac{u - g\times 3}{u - g\times 5}\right) = \left(\frac{150-30}{150-50}\right) = \frac{120}{100} = 1.2$

22. 64 identical balls made of conducting material each having potential of 10 mV are joined to form a bigger ball. The potential of bigger ball is ____ V.

Answer (00.16)

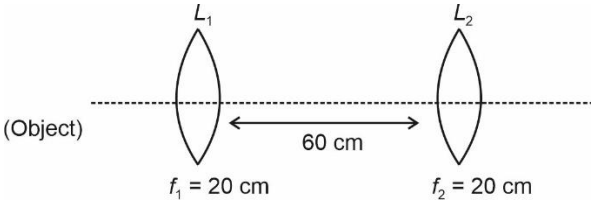
Sol. $64\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3} = \pi R^3 \Rightarrow R = 4r$

Also $Q' = 64Q$

$\because \quad \frac{KQ}{r} = 10 \text{ mV}$ then $V' = \frac{K(64Q)}{4r} = 16\times 10 \text{ mV}$

= 160 mV

23. An object placed at very large distance from lens L. The distance of final image formed from L_1 will be _____ m.



Answer (01.00)

Sol. I[st] image is formed at focus of L_1 which is at $2f_2$ from lens L_2.

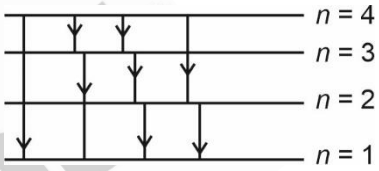
24. A photon of energy 12.75 eV falls a H-atom. Find out no. of spectral lines observed?

Answer (6)

Sol. $\because \Delta E = 13.6\left[1 - \frac{1}{n^2}\right]$ eV

For $n = 4$, $\Delta E = 12.75$ eV

In 4 energy level,



no. of spectral lines = $^4C_2 = 6$

25. A uniform solid sphere is rolling without slipping on a horizontal surface. The ratio of translational kinetic energy to the total kinetic energy is 5/x. Find the value of x.

Answer (7)

Sol. $\frac{\text{K.E}_{\text{Trans.}}}{\text{K.E}_{\text{Total}}} = \frac{\frac{1}{2}mR^2\omega^2}{\frac{1}{2}\left(\frac{2}{5}+1\right)mR^2\omega^2} = \frac{5}{7}$

26.

27.

28.

29.

30.

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. Consider the following reaction sequence:

$CaCl_2 + Na_2CO_3 \rightarrow \underset{Z}{\overset{\uparrow}{\underset{\uparrow}{X}}} + Y$

	X	Y	Z
(1)	$CaCO_3$	NaCl	HCl
(2)	CaO	NaCl + CO_2	KCl
(3)	CaO	NaCl + CO_2	NaCl
(4)	$CaCO_3$	NaCl	KCl

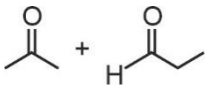
Answer (1)

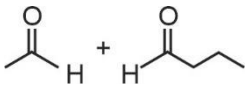
Sol. $CaCl_2 + Na_2CO_3 \rightarrow CaCO_3 + 2NaCl$

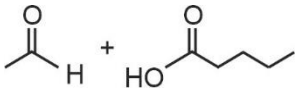
$CaCO_3 + 2HCl \rightarrow CaCl_2 + H_2O + CO_2$

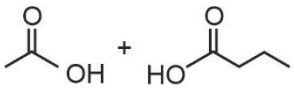
2. $\text{Hex} - 2 - \text{ene} \xrightarrow[H_2O_2]{O_3} A + B$

Product A and B are

(1) 

(2) 

(3) 

(4) 

Answer (4)

Sol. $CH_3 - CH = CH - CH_2 - CH_2CH_3 \xrightarrow[H_2O_2]{O_3} CH_3COOH + CH_3CH_2CH_2COOH$

3. Match the columns.

	Column-I (Type of hydride)		Column-II (Formula)
(A)	Electron deficient	(1)	MgH_2
(B)	Electron precise	(2)	HF
(C)	Electron rich	(3)	CH_4
(D)	Saline hydride	(4)	B_2H_6

(1) A → 4; B → 3; C → 1; D → 2

(2) A → 4; B → 1; C → 3; D → 2

(3) A → 4; B → 3; C → 2; D → 1

(4) A → 1; B → 2; C → 3; D → 4

Answer (3)

Sol. Electron deficient → B_2H_6

Electron precise → CH_4

Electron rich → HF

Saline hydride → MgH_2

4. Match the Columns

Column-I		Column-II	
A	Biodegradable	P	Polyacrylonitrile
B	Synthetic	Q	PHBV
C	Natural	R	Dacron
D	Polyester	S	Rubber

(1) A – Q; B – P; C – S; D – R

(2) A – Q; B – P; C – R; D – S

(3) A – P; B – Q; C – S; D – R

(4) A – Q; B – R; C – S; D – P

Answer (1)

Sol. PHBV is Biodegradable; Rubber is natural; Dacron is Polyester

5. **Assertion (A) :** 5*f* electrons can participate in bonding to a greater extent as compared to 4*f* electrons.

Reason (R) : Both resemble in their angular part of wave function, but 5*f* is not as buried as 4*f* orbitals.

(1) (A) is correct, (R) is correct and (R) is the correct explanation of (A)

(2) (A) is correct, (R) is correct and (R) is incorrect explanation of (A)

(3) (A) is correct, (R) is incorrect

(4) Both (A) and (R) are incorrect

Answer (1)

Sol. Both statements are correct and Reason is correct

[For explanation refer NCERT d & f block Elements]

6. Density of group-1 metal follows the order:

(1) Li > Na > K > Rb (2) Li > K > Na > Rb

(3) Rb > K > Na > Li (4) Rb > Na > K > Li

Answer (4)

Sol. $\left.\begin{matrix}\underline{Li = 0.53}\\ \underline{Na = 0.97}\\ \underline{K = 0.86}\\ \underline{Rb = 1.53}\end{matrix}\right\}$ all in gm/cc

7. Critical temperature of A, B, C & D are 5.3, 20.3, 128.5, 166.5. Then order of adsorption is

(1) A > B > C > D (2) D > C > B > A

(3) C > D > A > B (4) B > A > C > D

Answer (2)

Sol. Higher is critical temperature more readily gas is liquified, means more are forces of attraction. Thus, more readily gas will be adsorbed.

8. Molality of $MgCl_2$ is 1m (α = 80%)

Calculate vapour pressure of solution (in torr), if vapour pressure of pure solvent is 100 torr.

(1) 95.53 (2) 78.23

(3) 68.12 (4) 98.26

Answer (1)

Sol. $\frac{P^\circ - P_s}{P_s} = \frac{i.n_A}{n_B}$

$\frac{100 - P_s}{P_s} = (1+2\alpha) \times \frac{18}{1000}$

$= \frac{2.6 \times 18}{1000}$

$100 - P_s = 0.0468\ P_s$

$\Rightarrow P_s = 95.53$ torr

9. **Assertion A :** Boron is hardest element in group-13.

Reason R : High lattice enthalpy due to strong crystalline lattice.

(1) Both A and R are correct and R is correct explanation of Assertion

(2) Both A and R are correct but R is not correct explanation

(3) A is correct but R is wrong statement

(4) Both A and R are correct

Answer (1)

Sol. Due to very strong crystalline lattice Boron has unusually high M.P.

10. Bond order and magnetic property of acetylide is similar to

(1) NO^+ (2) NO^-

(3) O_2^+ (4) O_2^-

Answer (1)

Sol. $HC \equiv C^{\ominus}$ Bond order = 3

NO^+ also have bond order equal to 3.

11. Statement 1: In Ellingham diagram the change in slope for Mg to MgO reaction occurs at 1120°C.

Statement 2: Sudden change in entropy also occurs at 1120°C.

(1) Both statements are correct

(2) Both statements are incorrect

(3) Statement 1 is correct but statement 2 is incorrect

(4) Statement 1 is incorrect but statement 2 is correct

Answer (1)

Sol. Both statements are correct.

Reference: NCERT

12. A gas with MW = 42 AMU will have same RMS velocity (at 27ºC) as that of V_{mps} of which gas at 27ºC

(1) CO_2 (2) CO

(3) N_2O (4) NO_2

Answer (2)

Sol. $\sqrt{\frac{3}{42}} = \sqrt{\frac{2}{MW}}$

$\Rightarrow$ MW = 28 $\Rightarrow$ CO gas

13. A 12.5 eV electron beam is used to bombard gaseous hydrogen at room temperature. Calculate the total no. of spectral lines when electrons return to ground state

(1) 3 (2) 2

(3) 4 (4) 1

Answer (1)

Sol. $E_3 = \frac{-13.6}{9} = -1.5$ eV

$E_1 = -13.6$ eV

$E_3 - E_1 = 12.09$ eV

Hence, total spectral lines $= \frac{2(3)}{2} = 3$

14. Following reaction is taking place:

$2NO + Br_2 \rightarrow 2NOBr$

Step-01: $NO + Br_2 \rightarrow NOBr_2$ (fast)

Step-02: $NOBr_2 + NO \rightarrow 2NOBr$ (slow)

Find order of the given reaction:

(1) 01 (2) 02

(3) 03 (4) 04

Answer (3)

Sol. $r = k_3[NOBr_2][NO]$

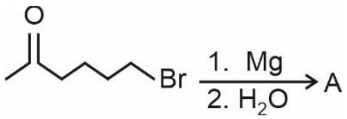
$[NOBr_2] = K_{eq.}[NO][Br_2]$

$r = k_3[NO][Br_2]\ K_{eq.}[NO]$

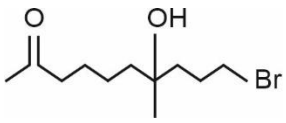
$r = k_3.\ K_{eq.}[NO]^2[Br_2]^1$

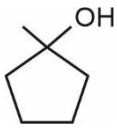
Order = 3

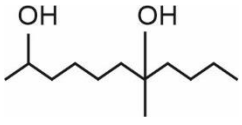
15. Consider the following reaction

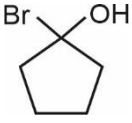


A is

(1) 

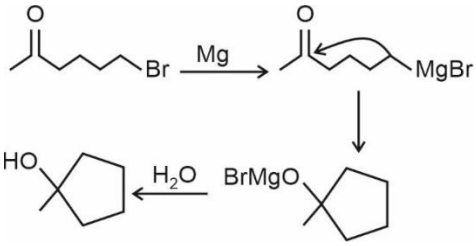
(2) 

(3) 

(4) 

Answer (2)

Sol.



16. Select correct statements about lead storage battery:

(1) $PbSO_4$ converts into PbO_2 at anode during discharging

(2) $PbSO_4$ converts into PbO_2 at cathode during discharge

(3) 38% H_2SO_4 solution is taken as the electrolyte

(4) H_2SO_4 is produced during discharging

Answer (3)

Sol.

Anode :→ $Pb \longrightarrow Pb^{+2} + 2e^-$

$Pb^{+2} + SO_4^{-2} \longrightarrow PbSO_4$

$Pb + SO_4^{-2} \longrightarrow PbSO_4 + 2e^{\ominus}$

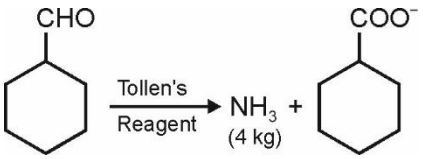
Cathode:→ $4H^{\oplus} + 2e^{\ominus} + PbO_2 \rightarrow Pb^{+2} + 2H_2O$

$Pb^{+2} + SO_4^{-2} \rightarrow PbSO_4$

$2e^{\ominus} + PbO_2 + 4H^{\oplus} + SO_4^{-2} \rightarrow PbSO_4 + 2H_2O$

Net reaction during discharging: -

$Pb + PbO_2 + 2H_2SO_4 \rightarrow 2PbSO_4 + 2H_2O$

17. 

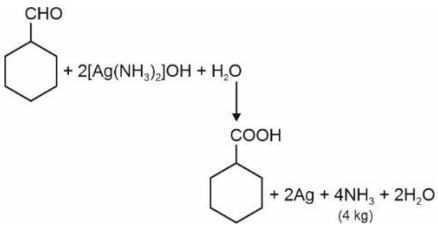
Calculate mass of Tollen's Reagent Required?

(1) 18.70 kg (2) 37.40 kg

(3) 9.35 kg (4) 55.10 kg

Answer (1)

Sol. The balanced equation is



No. of moles of NH_3 formed = $\frac{4\times10^3}{17}$

$\therefore$ No. of moles of tollen's reagent consumed $= \frac{2\times10^3}{17}$

So mass of tollen's reagent $= \frac{2\times10^3}{17}\times159$

= 18.70 kg

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. pH of 1 litre HCl solution is 1.0. How much water is added to make pH 2.

Answer (09.00)

Sol. $(10^{-1})(1) = (10^{-2})(V_2)$

$V_2 = 10$ L

Water added = (10 – 1)

= 9 L

22. Given P_i = 3 atm

$V_{initial}$ = 2 L

V_{final} = 3 L

T = 350 K

If isothermal reversible process is carried out, calculate ΔS for system (in Joules)

Answer (0.72)

Sol. $\Delta S = n \times R\, ln\left(\frac{V_2}{V_1}\right)$

$= n R \ln\frac{3}{2}$

$= \frac{P_i V_i}{T}\ln\frac{3}{2}$

$= \frac{6\times101.325\times2.303}{350} log\frac{3}{2}$

= 4 × (0.48 – 0.30)

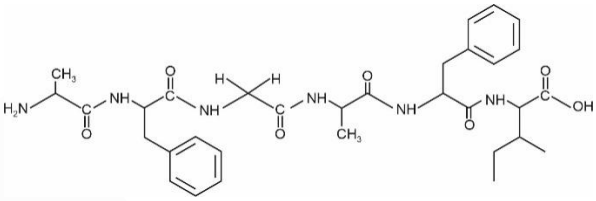
= 0.72 J/K

23. The number of sp^2 hybridized carbon atoms in the following peptide is

Ala – Phe – Gly – Ala – Phe - Ile

Answer (18)

Sol.



24. How many of the given metals will show photoelectric effect when light of 400 nm falls on below metal?

Metal	Li	Na	K	Mg	Cu	Ag
W(eV)	2.42	2.3	2.25	3.7	4.8	4.3

Answer (03.00)

Sol. $E_{photon} = \frac{12400}{4000} = 3.1$ eV

Metals Li; Na; K will show photoelectric effect.

25. A metal chloride contains 55% by mass of chlorine. 100 mL of vapours gives 0.57 gm of chlorine at STP. Calculate the molecular mass of metal chloride. (Nearest integer)

Answer (232.00)

Sol. $\frac{(1)\times MW}{(R)\times(273)} = \frac{.57}{(.1)}\times\frac{100}{55}$

MW = 232.28 (≈232)

26.

27.

28.

29.

30.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

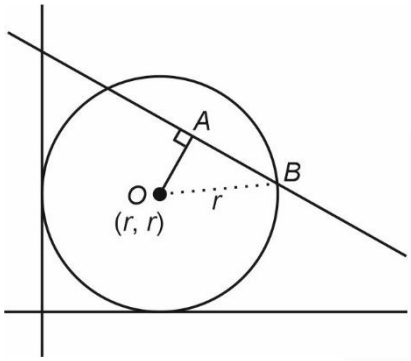
1. Two circles having radius r_1 and r_2 touch both the coordinate axes. Line $x + y = 2$ makes intercept as 2 on both the circles. The value of $r_1^2 + r_2^2 - r_1 \cdot r_2$ is

(1) $\frac{9}{2}$ (2) 6

(3) 7 (4) 8

Answer (3)

Sol.



$AB = 1$

$OA = \sqrt{r^2 - 1}$

$\Rightarrow \left|\frac{2r-2}{\sqrt{2}}\right| = \sqrt{r^2-1}$

$\Rightarrow \sqrt{2}(r-1) = \sqrt{r^2-1}$

$\Rightarrow 2(r-1)^2 = r^2 - 1$

$\Rightarrow 2r^2 - 4r + 2 = r^2 - 1$

$\Rightarrow r^2 - 4r + 3 = 0$

$\Rightarrow (r-1)(r-3) = 0$

$\Rightarrow r = 1, 3$

$\therefore r_1 = 1$ and $r_2 = 3$

$\therefore r_1^2 + r_2^2 - r_1 \cdot r_2 = 1 + 9 - 3 = 7$

2. Area of region enclosed by curve $y = x^3$ and its tangent at (–1, –1)

(1) 4 (2) 27

(3) $\frac{4}{27}$ (4) $\frac{27}{4}$

Answer (4)

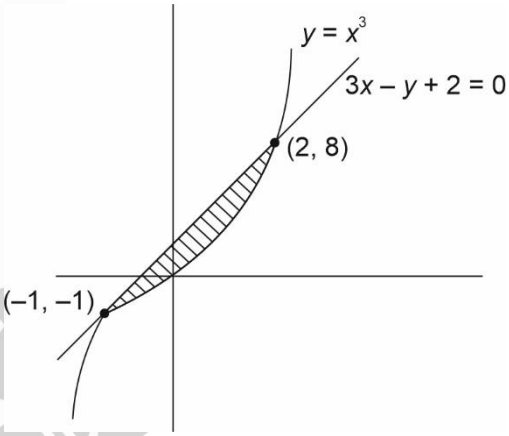
Sol. $y = x^3$

$y' = 3x^2$

$y'_{(-1,-1)} = 3$

$T : y + 1 = 3(x + 1)$

$T : 3x - y + 2 = 0$



$$\text{Area} = \int_{-1}^{2} (3x+2) - x^3 dx$$

$$= \frac{3x^2}{2} + 2x - \frac{x^4}{4}\Big]_{-1}^{2}$$

$$= \left|\frac{3}{2} \times 3 + 2 \times 3 - \frac{1}{4} \times 15\right|$$

$$= \frac{9}{2} + 6 - \frac{15}{4}$$

$$= \frac{27}{4} \text{ sq. units}$$

3. If $(1 + x^2)dy = y(y - x)dx$ and $y(1) = 1$. Then $y\left(2\sqrt{2}\right)$ is

(1) $\frac{4}{\sqrt{2}}$ (2) $\frac{3}{\sqrt{2}}$

(3) $\frac{1}{\sqrt{2}}$ (4) $\sqrt{2}$

Answer (3)

Sol. $\frac{dy}{dx}+\frac{x}{1+x^2}y=\frac{y^2}{1+x^2}$

$\frac{1}{y^2}\frac{dy}{dx}+\frac{x}{1+x^2}\times\frac{1}{y}=\frac{1}{1+x^2}$

Let $\frac{1}{y}=t$

$-\frac{1}{y^2}\frac{dy}{dx}=\frac{dt}{dx}$

$\frac{-dt}{dx}+\left(\frac{x}{1+x^2}\right)dt=\frac{1}{1+x^2}$

$\frac{dt}{dx}-\left(\frac{x}{1+x^2}\right)dt=-\frac{1}{1+x^2}$

IF $=e^{-\int\frac{x}{1+x^2}dx}=e^{-\frac{1}{2}\log|1+x^2|}=\frac{1}{\sqrt{1+x^2}}$

$\frac{t}{\sqrt{1+x^2}}=-\int\underbrace{\frac{1}{(1+x^2)\sqrt{1+x^2}}}_{I}dx$

Let $x=\tan\theta$

$dx=\sec^2\theta\,d\theta$

$I=\int\frac{\sec^2\theta}{\sec^2\theta\cdot\sec\theta}d\theta=\int\cos\theta=\sin\theta+C$

$\therefore\quad\frac{1}{y\sqrt{1+x^2}}=-\frac{x}{\sqrt{1+x^2}}+C$

$\because\quad y(1)=1$

$\Rightarrow\quad C=\sqrt{2}$

$\therefore\quad\frac{1}{y\sqrt{1+x^2}}+\frac{x}{\sqrt{1+x^2}}=\sqrt{2}$

$1+xy=\sqrt{2}y\sqrt{1+x^2}$

Now

$y\left(2\sqrt{2}\right)$

$1+2\sqrt{2}y=3\sqrt{2}y$

$\sqrt{2}y=1$

$y=\frac{1}{\sqrt{2}}$

4. For the expression $(1-x)^{100}$. Then sum of coefficient of first 50 terms is

(1) ${}^{99}C_{49}$ (2) $-\frac{{}^{100}C_{50}}{2}$

(3) $-{}^{99}C_{49}$ (4) $-{}^{101}C_{50}$

Answer (2)

Sol. Sum of coefficient of first 50 terms

$(t)={}^{100}C_0-{}^{100}C_1+\ldots+{}^{100}C_{49}$

Now

${}^{100}C_0-{}^{100}C_1+\ldots+{}^{100}C_{100}=0$

$2\left[{}^{100}C_0-{}^{100}C_1+\ldots\right]+{}^{100}C_{50}=0$

$\therefore\quad t=-\frac{1}{2}{}^{100}C_{50}$

5. Positive numbers a_1, a_2, …. a_5 are in geometric progression. Their mean and variance are $\frac{31}{10}$ and $\frac{m}{n}$ respectively. The mean of the reciprocals is $\frac{31}{40}$, then $m+n$ is

(1) 209 (2) 211

(3) 113 (4) 429

Answer (2)

Sol. $a\left(\frac{1}{r^2}+\frac{1}{r}+1+r+r^2\right)=\frac{31}{2}$

$\frac{1}{a}\left(\frac{1}{r^2}+\frac{1}{r}+1+r+r^2\right)=\frac{31}{8}$

$\Rightarrow\quad a^2=4$

$\Rightarrow\quad a=2$

$\Rightarrow\quad\frac{1}{r^2}+\frac{1}{r}+1+r+r^2=\frac{31}{4}$

$\Rightarrow\quad\left(r+\frac{1}{r}\right)^2+\left(r+\frac{1}{r}\right)=\frac{31}{4}+1=\frac{35}{4}$

$4t^2+4t-35=0$

$\Rightarrow\quad t=\frac{5}{2}$

$\Rightarrow\quad r=2$

$\therefore\quad$ numbers are $=\frac{1}{2},1,2,4,8$

$$\therefore \quad \sigma^2 = \frac{\frac{1}{4}+1+4+16+64}{5} - \left(\frac{31}{10}\right)^2$$

$$= \frac{341}{20} - \frac{961}{100}$$

$$= \frac{1705-961}{100}$$

$$= \frac{744}{100} = \frac{186}{25}$$

$\therefore \quad m + n = 186 + 25$

$= 211$

6. If $\Delta(k) = \begin{vmatrix} 1 & 2k-1 & 2k \\ n & n^2 & n(n+1) \\ \cos^2 n & \cos^2(n+1) & (n+2) \end{vmatrix}$, then

$\sum_{k=1}^{n} \Delta(k) =$

(1) n (2) 1

(3) $\frac{n^2}{2}$ (4) 0

Answer (4)

Sol. $\sum_{k=1}^{n} \Delta(k) = \begin{vmatrix} n & n^2 & n(n+1) \\ n & n^2 & n(n+1) \\ \cos^2 n & \cos^2(n+1) & (n+2) \end{vmatrix} = 0$

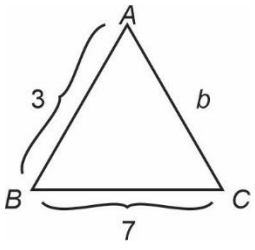
7. Given A, B, C represents angles of a ΔAB and $\cos A + 2\cos B + \cos C = 2$ and $AB = 3$ and $BC = 7$ then $\cos A - \cos C$ is

(1) $-\frac{10}{7}$ (2) $\frac{10}{7}$

(3) $\frac{5}{7}$ (4) $-\frac{5}{7}$

Answer (1)

Sol.



$\cos A + 2\cos B + \cos C = 2.$

$$\frac{9+b^2-49}{6b} + 2\left(\frac{49+9-b^2}{42}\right) + \left(\frac{49+b^2-9}{14b}\right) = 2$$

$$\frac{b^2-40}{6b} + \frac{58-b^2}{21} + \frac{40+b^2}{14b} = 2$$

$\Rightarrow$ $b = -4$ or 4 or 5

b cannot be -4 and 4

$\Rightarrow$ $b = 5$.

Now,

$\cos A - \cos C$

$$\frac{9+25-49}{2\times3\times5} - \frac{49+25-9}{2\times7\times5}$$

$$-\frac{1}{2} - \frac{13}{14} = \frac{-20}{14} = \frac{-10}{7}$$

8. Let $x^2 + \sqrt{6}x + 4 = 0$ be any quadratic equation and α, β are the roots of that equation then

$$\frac{\alpha^{34}\beta^{24} + \alpha^{32}\beta^{26} + 2\alpha^{33}\beta^{25}}{\alpha^{31}\beta^{20} + \alpha^{28}\beta^{23} + 3\alpha^{30}\beta^{21} + 3\alpha^{29}\beta^{22}}$$ is

(1) $\frac{-2^7}{3}\sqrt{6}$ (2) $\frac{2^7}{3}\sqrt{6}$

(3) $\frac{-2^8}{3}\sqrt{6}$ (4) $\frac{2^8}{3}\sqrt{6}$

Answer (1)

Sol. $x^2 + \sqrt{6}x + 4 = 0 \begin{matrix} \nearrow \alpha \\ \searrow \beta \end{matrix}$

$\therefore \quad \alpha + \beta = -\sqrt{6}, \quad \alpha\beta = 4$

Now $\frac{\alpha^{34}\beta^{24} + \alpha^{32}\beta^{26} + 2\alpha^{33}\beta^{25}}{\alpha^{31}\beta^{20} + \alpha^{28}\beta^{23} + 3\alpha^{30}\beta^{21} + 3\alpha^{29}\beta^{22}}$

$$= \frac{\alpha^{32}\beta^{24}\left[\alpha^2 + \beta^2 + 2\alpha\beta\right]}{\alpha^{28}\beta^{20}\left[\alpha^3 + \beta^3 + 3\alpha^2\beta + 3\alpha\beta^2\right]}$$

$$= (\alpha\beta)^4 \frac{\left[(\alpha+\beta)^2\right]}{(\alpha+\beta)^3} = \frac{4^4}{-\sqrt{6}} = \frac{-2^7}{3}\sqrt{6}$$

9. If a plane $4x - 3y + z = 2$ is rotated by an angle of $\frac{\pi}{2}$ at intersection point of another plane $3x + 11z - 4y = 12$, then $P(2, 3, 4)$ is at what distance from resultant plane?

(1) $\frac{250}{\sqrt{63245}}$ (2) $\frac{641}{\sqrt{66846}}$

(3) $\frac{925}{\sqrt{66215}}$ (4) $\frac{24}{\sqrt{11235}}$

Answer (2)

Sol : Equation of required plane:

$4x - 3y + z - 2 + \lambda(3x - 4y + 11z - 12) = 0$

If is perpendicular to $4x - 3y + z = 2$

$\therefore \quad (4 + 3\lambda)\cdot 4 + (-3 - 4\lambda)(-3) + (1 + 11\cdot\lambda)1 = 0$

$\Rightarrow \quad 16 + 12\lambda + 9 + 12\lambda + 1 + 11\lambda = 0$

$\Rightarrow \quad 35\lambda + 26 = 0$

$\Rightarrow \quad \lambda = -\frac{26}{35}$

$\therefore \quad x(4 + 3\lambda) - y(3 + 4\lambda) + z(1 + 11\lambda) - 2 - 12\lambda = 0$

$\Rightarrow \quad \frac{62x}{35} - y\left(\frac{1}{35}\right) + z\left(\frac{-251}{35}\right) + \left(\frac{242}{35}\right) = 0$

$\Rightarrow \quad 62x - y - 251z + 242 = 0$

Distance from (2, 3, 4)

$= \left|\frac{124-3-1004+242}{\sqrt{66846}}\right|$

$= \frac{641}{\sqrt{66846}}$

10. A circle with centre $z_0 = \frac{1}{2} + \frac{3i}{2}$ exists in an argand plane. A point $z_1 = 1 + i$ and z_2 lies outside the circle, such that $|z_0 - z_1|\,|z_0 - z_2| = 1$. Then the largest value of $|z_2|$ is

(1) $\sqrt{5} - \sqrt{2}$ (2) $\sqrt{\frac{5}{2}} - \sqrt{2}$

(3) $\sqrt{\frac{5}{2}}$ (4) $\sqrt{\frac{5}{2}} + \sqrt{2}$

Answer (4)

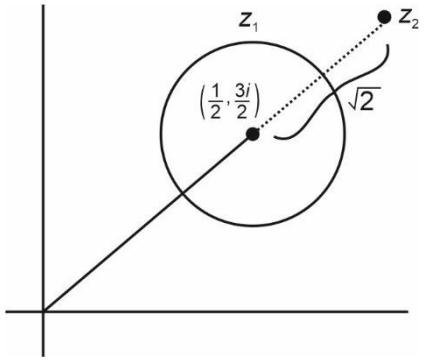
Sol. $\left|z - \frac{1}{2} - \frac{3i}{2}\right| = r \rightarrow$ Circle

Now,

$|z_0 - z_1|\,|z_0 - z_2| = 1$

$\frac{1}{\sqrt{2}}|z_0 - z_2| = 1$

$|z_0 - z_2| = \sqrt{2}$



Max $|z_2| = \sqrt{\frac{1}{4} + \frac{9}{4}} + \sqrt{2}$

$= \sqrt{\frac{10}{4}} + \sqrt{2}$

$= \left(\sqrt{\frac{5}{2}} + \sqrt{2}\right)$ unit

11. Let $\vec{a} = \lambda\hat{i} + \hat{j} - \hat{k}, \vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c}$ is a vector such that $(\vec{a} + \vec{b} + \vec{c}) \times \vec{c} = 0$ and $\vec{a}\cdot\vec{c} = -17, \vec{b}\cdot\vec{c} = -20$. Find $\left|\vec{c} \times (\lambda\hat{i} + \hat{j} + \hat{k})\right|^2$ given $(\lambda > 0)$

(1) 46

(2) 61

(3) 48

(4) 51

Answer (1)

Sol. $k(\vec{a} + \vec{b}) = \vec{c}$

$\vec{a}\cdot\vec{c} = -17, \vec{b}\cdot\vec{c} = -20$

$k(\lambda^2 + 3\lambda - 1) = -17, k(3\lambda + 11) = -20$

$\Rightarrow \quad \lambda = -\frac{69}{20}, 3$

$\lambda = 3, k = -1$

$\vec{c} = -1(\vec{a} + \vec{b})$

$= -((\lambda + 3)\hat{i} + \hat{k}) = -6\hat{i} - \hat{k}$

$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -6 & 0 & -1 \\ 3 & 1 & 0 \end{vmatrix} = \hat{i}(1) - \hat{j}(3) + \hat{k}(-6)$

$= \hat{i} - 3\hat{j} - 6\hat{k}$

$\left|\vec{c} \times (\lambda\hat{i} + \hat{j} + \hat{k})\right|^2 = 46$

12.

13.

14.

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. If $\frac{{}^nC_n}{n+1}+\frac{{}^nC_{n-1}}{n}+.....+\frac{1}{2}{}^nC_1+{}^nC_0=\frac{255}{8}$. Then value of n is

Answer (07)

Sol. $\int_0^1(1+x)^n=\int_0^1\left({}^nC_0+{}^nC_1x+{}^nC_2x^2+.....+{}^nC_nx^n\right)dx$

$$\frac{(1+x)^{n+1}}{n+1}\Big]_0^1={}^nC_0x+{}^nC_1\frac{x^2}{2}+{}^nC_2\frac{x^3}{3}+.....+\frac{{}^nC_nx^{n+1}}{n+1}\Big]_0^1$$

$$\frac{2^{n+1}}{n+1}-\frac{1}{n+1}={}^nC_0+\frac{{}^nC_1}{2}+\frac{{}^nC_2}{3}+.....+\frac{{}^nC_n}{n+1}$$

Now

$$\frac{2^{n+1}-1}{n+1}=\frac{255}{8}$$

$\Rightarrow \quad n = 7$

22. If the value of $\int_{-0.15}^{0.15}|100x^2-1|dx=\frac{k}{3000}$, then the value of k is _____.

Answer (575)

Sol. $I=2\int_0^{0.15}|100x^2-1|dx$

$$=2\left[\int_0^{0.1}-(100x^2-1)dx+\int_{0.1}^{0.15}(100x^2-1)dx\right]$$

$$=2\left[\left[x-\frac{100x^3}{3}\right]_0^{0.1}+\left[\frac{100x^3}{3}-x\right]_{0.1}^{0.15}\right]$$

$$=\frac{575}{3000}$$

$\Rightarrow \quad k = 575$

23. $N > 40000$, where N is divisible by 5. How many such 5 digits numbers using 0, 1, 3, 5, 7, 9?

Answer (120)

Sol. **Case I :** Number starts with 5

5 _ _ _ 0

↓ ↓ ↓

4 ways 3 ways 2 ways $= 4 \times 3 \times 2 = 24$

Case II : Number starts with 7

7 _ _ _ _

↓ ↓ ↓ ↓

4 3 2 2 $= 4 \times 3 \times 2 \times 2 = 48$

Case III : Number starts with 9

9 _ _ _ _

↓ ↓ ↓ ↓

4 3 2 2 $= 4 \times 3 \times 2 \times 2 = 48$

Total ways = 120

24. Three numbers a, b, c are in A.P. and they are used to make a 9-digits number using each digit thrice, such that at least 3 consecutive digits are in A.P. then number of such numbers is

Answer (1260)

Sol. | a | b | c | or | c | b | a |

So, total number $\frac{{}^7C_1\times 2\times 6!}{2!\,2!\,2!}=\frac{7!}{4}$

$= 7 \times 6 \times 5 \times 3 \times 2 = 1260$

$a\hat{i}+\hat{j}+k$

25. If $\hat{i}+b\hat{j}+k$ are co-planar,

$\hat{i}+\hat{j}+ck$

then the value of $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}$ is

Answer (1)

Sol. $\begin{vmatrix}a & 1 & 1\\ 1 & b & 1\\ 1 & 1 & c\end{vmatrix}=0$

$R_1 \to R_1 - R_2,\ R_2 \to R_2 - R_3$

$$\begin{vmatrix}a-1 & 1-b & 0\\ 0 & b-1 & 1-c\\ 1 & 1 & c\end{vmatrix}=0$$

$(a-1)[c(b-1)-(1-c)]+1[(1-b)(1-c)]=0$

$c(a-1)(b-1)-(a-1)(1-c)+(1-b)(1-c)=0$

Multiply and divide by $(1-a)(1-b)(1-c)$

$$-\frac{1-c-1}{1-c}+\frac{1}{1-b}+\frac{1}{1-a}=0$$

$$-1+\frac{1}{1-c}+\frac{1}{1-b}+\frac{1}{1-a}=0$$

$\therefore \quad \frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=1$

26. $f(x)=\lfloor[x]\rfloor+\sqrt{x-[x]}$. The number of points of discontinuity of $f(x)$ in [–2, 1] is.

Answer (2)

Sol. $f(x)=\lfloor[x]\rfloor+\sqrt{\{x\}}$

$x = -2$

$f(-2) = 2$

$f(-2^+) = 2 + 0 = 2$

$x = -1$

$f(-1) = 1 + 0 = 1$

$f(-1^-) = 2 + 1 = 3$

$\therefore$ discontinuous at $x = -1$

$x = 0$

$f(0) = 0$

$f(0^-) = 1 + 1 = 2$

$\therefore$ discontinuous at $x = 0$

$x = 1$

$f(1) = 1$

$f(1^-) = 0 + 1 = 1$

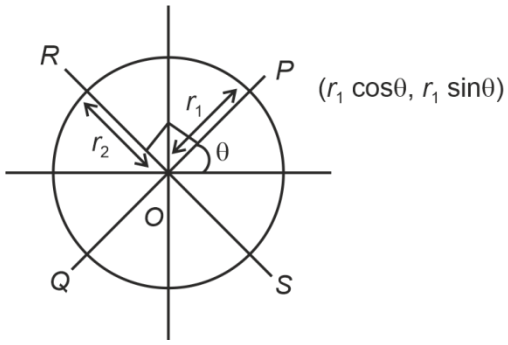
$\therefore$ discontinuous at $x = -1$ and at $x = 0$

$\therefore$ 2 points of discontinuity

27. Given $9x^2 + 4y^2 = 36$ and a point $P\left(\frac{2\sqrt{3}}{\sqrt{7}},\frac{6}{\sqrt{7}}\right)$ lie on ellipse. PQ is a diameter of ellipse and RS is a diameter which is perpendicular to PQ. If $\frac{1}{PQ^2}+\frac{1}{RS^2}=\frac{p}{m}$ in simplest form, then $p + m$ is

Answer (157)

Sol. $r_1=\sqrt{\frac{48}{7}}$



$\frac{r_1^2\cos^2\theta}{4}+\frac{r_1^2\sin^2\theta}{9}=1$

$\frac{\cos^2\theta}{4}+\frac{\sin^2\theta}{9}=\frac{7}{48}$ ...(i)

$\frac{r_2^2\sin^2\theta}{4}+\frac{r_2^2\cos^2\theta}{9}=1$

$\frac{\sin^2\theta}{4}+\frac{\cos^2\theta}{9}=\frac{1}{r_2^2}$

From (i), $\frac{1}{r_2^2}=\frac{1}{4}+\frac{1}{9}-\frac{7}{48}=\frac{31}{144}$

$\frac{1}{PQ^2}+\frac{1}{RS^2}=\frac{1}{4}\left(\frac{1}{r_1^2}+\frac{1}{r_2^2}\right)$

$=\frac{1}{4}\left(\frac{7}{48}+\frac{31}{144}\right)=\frac{13}{144}$

28.

29.

30.

❑ ❑ ❑