13/04/2023
Evening

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Memory Based
Answers & Solutions
for
JEE (Main)-2023 (Online) Phase-2

(Physics, Chemistry and Mathematics)

Time : 3 hrs. **M.M. : 300**

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. What should be the minimum size of antenna required for successful transmission of wave having wavelength λ?

(1) 2λ (2) $\frac{\lambda}{4}$

(3) $\frac{\lambda}{2}$ (4) λ

Answer (2)

Sol. Theoretical

2. A 10 μC charge is divided into two equal parts and kept at 1 cm distance. Find repulsion between charges.

(1) 225 N (2) 450 N

(3) 2250 N (4) 4500 N

Answer (3)

Sol. $F = \frac{9\times10^9\times(5\times10^{-6})^2}{(10^{-2})^2} = 9\times25\times10 = 2250\text{ N}$

3. Two identical trains cross each other moving on parallel tracks, opposite in direction. Speed of one of the train is 70 km/hr and second train has a speed of 110 km/hr. If it takes 8 seconds for two trains to cross each other then length of trains is equal to

(1) 100 m (2) 200 m

(3) 300 m (4) 400 m

Answer (2)

Sol. Total distance to cover relatively = 2ℓ

$\Rightarrow 8 = \frac{2\ell}{180\times\frac{5}{18}}$

$\Rightarrow \ell = 200$ m

4. A particle is performing S.H.M., whose distance from mean position varies as $x = A\sin(\omega t)$.

Find the position of particle from mean position where kinetic energy and potential energy is equal.

(1) $\left(\frac{A}{2}\right)$ (2) $\left(\frac{A}{\sqrt{2}}\right)$

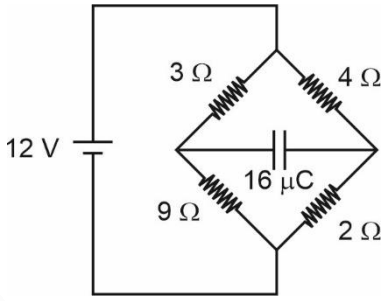
(3) $\left(\frac{A}{2\sqrt{2}}\right)$ (4) $\left(\frac{A}{4}\right)$

Answer (2)

Sol. $\frac{1}{2}m\omega^2\left(A^2-x^2\right) = \frac{1}{4}m\omega^2A^2$

$\Rightarrow A^2-x^2 = \frac{1}{2}A^2 \Rightarrow x^2 = \frac{A^2}{2} \Rightarrow x = \left(\frac{A}{\sqrt{2}}\right)$

5. Find energy stored in capacitor in given circuit.



(1) 0.2 mJ (2) 0.4 mJ

(3) 0.6 mJ (4) 0.8 mJ

Answer (1)

Sol. In steady state, capacitor will behave as open circuit

$R_{eq} = 4\ \Omega$ $\qquad i = \frac{12}{4} = 3$ A

$i_4 = \frac{12}{18}\times3 = 3$ A

$i_3 = 1$ A

So, P.D. across capacitor

$-3 - V + 8 = 0$

$V = 5$ volt

Energy stored = $\frac{1}{2}CV^2$

$= \frac{1}{2}\times16(\mu\text{C})\times25$

$= 200\ \mu$J

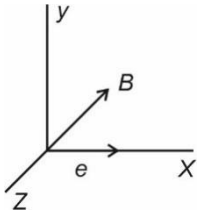
6. An electron is moving along positive x direction in x-y plane. Magnetic field points in negative z direction, then the force due to magnetic field on electron points in the direction

(1) $\hat{j}$ (2) $-\hat{j}$

(3) $\hat{k}$ (4) $-\hat{k}$

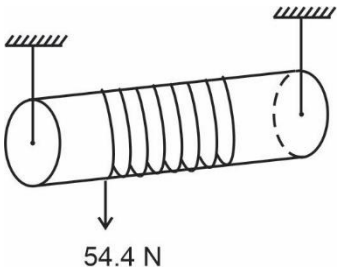
Answer (2)

Sol.



as electron is negative charge $\vec{F} = -e\,\vec{V}\times\vec{B}$

7. A force of 54.4 N is applied on free end of a string wrapped around a cylinder (solid) of mass 15 kg and radius 10 cm. Angular acceleration of the cylinder is equal to



(1) 94.10 rad/s^2 (2) 72.5 rad/s^2
(3) 14.50 rad/s^2 (4) 94.50 rad/s^2

Answer (2)

Sol. $0.1\times 54.4 = \frac{1}{2}\times 15\times (0.1)^2\times \alpha$

α = 72.53

8. Planet A has density and radius (ρ, R) while B has ($\rho/2$, $1.5R$). If g_{AS} and g_{BS} are the acceleration at the surface of planet A and B respectively, find $\frac{g_{BS}}{g_{AS}}$.

(1) $\frac{3}{2}$ (2) $\frac{3}{4}$
(3) $\frac{1}{4}$ (4) 3

Answer (2)

Sol. $\because \quad g = \frac{4\pi}{3}G\rho R$

$\therefore \quad \frac{g_{BS}}{g_{AS}} = \frac{\rho_B R_B}{\rho_A R_A} = \frac{\left(\frac{\rho}{2}\right)(1.5R)}{\rho R} = \frac{3}{4}$

9. **Assertion (A):** Binding energy per nucleon for nuclei (Atomic number 30 to 107) is independent of atomic number.

Reason (R): Nuclear force is short range force.

(1) (A) and (R) both are true and (R) explains (A) correctly
(2) (A) and (R) both are true but (R) does not explain A correctly
(3) (A) is true but (R) is false
(4) (A) and (R) both are false

Answer (1)

Sol. For nuclei with atomic number 30 to 107 there is a saturation in nuclear force per nucleon as nucleons being added at large distance do not make much impact on nuclear forces at a specific region.

10. If ratio of amplitudes is 2 : 1 in Young's double slit experiment, then find the ratio of maximum intensity to minimum intensity.

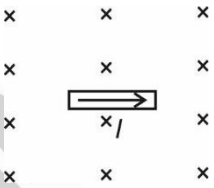
(1) 2 : 1
(2) 25 : 9
(3) 9 : 1
(4) 9 : 4

Answer (3)

Sol. Amplitudes are $2A$ and A.

$$\frac{I_{max}}{I_{min}} = \frac{(2A+A)^2}{(2A-A)^2} = \frac{9}{1}$$

11. A conducting rod carries current I hang freely in gravity space as shown in figure. If length of rod, current in rod and magnetic field strength are 0.5m, 2A and 0.4T respectively. Then find the mass of rod (Take g = 10 m/s^2)



(1) 20 gm
(2) 40 gm
(3) 60 gm
(4) 80 gm

Answer (2)

Sol. $ILB = mg \Rightarrow m = \frac{ILB}{g} = \frac{.2\times\frac{1}{2}\times 0.4}{10}$

$= \frac{4}{100}$ kg = 40 gm

12. A mixture of gases with adiabatic coefficient equal to $\frac{3}{2}$ is compressed from initial state (P_0, V_0) to one fourth volume adiabatically. Its final pressure will be equal to

(1) P_0 (2) $2P_0$
(3) $4P_0$ (4) $8P_0$

Answer (4)

Sol. PV^γ = Constant

$$\Rightarrow P_2 = P_1\times\left(\frac{V_1}{V_2}\right)^\gamma$$

$$= P_0\times(4)^{\frac{3}{2}}$$

$P_2 = 8P_0$

13. **Assertion:** If radius of ball (5 ± 0.1) mm, then error in terminal velocity is 4%.

Reason: Terminal velocity is directly proportional to radius (r).

(1) Both A and R are correct and R is the correct explanation of A.

(2) Both A and R are correct but R is not the correct explanation of A.

(3) A is true R is false

(4) A is false R is true

Answer (3)

Sol. $V_T = \frac{2}{9\eta}|\sigma - \rho| r^2 g \Rightarrow V_T \alpha r^2$

% error in $V_T = 2\times\left(\frac{0.1}{5}\times 100\%\right) = 4\%$

14. In series LCR circuit, value of resistance inductance and capacitance are 10 Ω, 0.1 H and 2 mF respectively. If angular frequency of AC source is 100 rad/s, then power factor of circuit is

(1) $\frac{1}{\sqrt{5}}$ (2) $\frac{2}{\sqrt{5}}$

(3) $\frac{3}{\sqrt{5}}$ (4) $\frac{1}{2\sqrt{5}}$

Answer (2)

Sol. $\cos\phi = \frac{R}{Z}$

$X_L = (0.1 \times 100) = 10\ \Omega$

$X_C = \frac{1}{\omega C} = \frac{1}{100\times 2\times 10^{-3}} = 5\ \Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2}$

$= \sqrt{10^2 + (10-5)^2} = \sqrt{10^2 + 5^2}$

$= \sqrt{125} = 5\sqrt{5}$

$\therefore \quad \cos\phi = \frac{10}{5\sqrt{5}} = \frac{2}{\sqrt{5}}$

15. Position of particle located on *x*-axis changes with time (*t*) as $x = 2.5t^2$. Speed of the particle at $t = 5$ seconds is equal to

(1) 5 m/s (2) 10 m/s

(3) 25 m/s (4) 50 m/s

Answer (3)

Sol. $x = 2.5t^2$

$v = \frac{dx}{dt}$

$= 5t$

at $t = 5$ s

$v = 25$ m/s

16. **Statement 1:** In purely inductive circuit average power consumed is very high.

Statement 2: In purely inductive circuit only, resonance can be achieved.

(1) (1) and (2) both are true

(2) (1) is false (2) is true

(3) (1) is true (2) is false

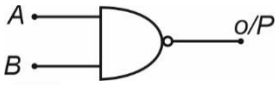
(4) (1) and (2) both are false

Answer (4)

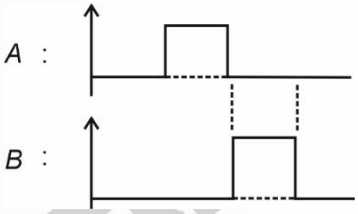
Sol. In purely inductive circuit $\cos\phi = 0$

$\Rightarrow$ Power consumed = 0

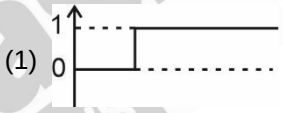
For resonance both *L* and *C* should be available in circuit.

17. 

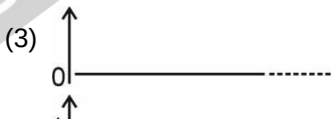
Input waveform at *A* and *B* are –



Output waveform will be?

(1) 

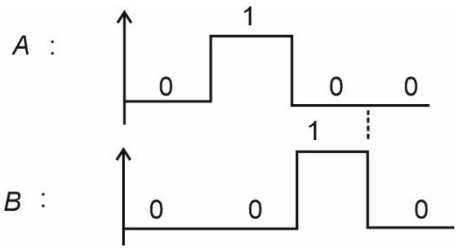
(2) 

(3) 

(4) 

Answer (2)

Sol.



$Y = \overline{A \cdot B}$

A	B	Y
0	0	1
1	0	1
0	1	1
0	0	1

18. Pressure (P), volume (V), temperature (T) are related to each other as per a relation

$$\left(P+\frac{a}{V^2}\right)(V-b)=RT$$

Dimension of $\frac{a}{b}$ are

(1) $[ML^{-2}]$

(2) $[M^2LT^{-2}]$

(3) $[ML^2T^{-2}]$

(4) $[ML^3T^{-1}]$

Answer (3)

Sol. $[a] = [P][V^2]$

$[b] = [V]$

$$\Rightarrow \frac{[a]}{[b]} = [P][V]$$

$= [ML^2T^{-2}]$

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. A bi-convex lens of focal length 10 cm is cut perpendicularly to principal axis. Find power of new lens.

Answer (5)

Sol. After cutting the bi-convex lens, focl length of individual lens is f = 20 cm = 0.2 m

$\therefore \quad P = 1/f = 5$ D

22. Body accelerates from rest to 4 m/s, energy is E. If it accelerates from rest to 24, then energy is nE. Find n.

Answer (4)

Sol. $E = \frac{1}{2}mu^2$

$$nE = \frac{1}{2}m(24)^2 = 4E \Rightarrow n = 4$$

23. If a substance absorbs 500 nm wavelength radiation and emits radiation of wavelength 600 nm, then the net change in energy is $x \times 10^{-2}$ eV

Find the value of x to the nearest integer.

Answer (41)

Sol. $\frac{hc}{\lambda} = E$

$$\Rightarrow \frac{1240}{500}\text{eV} = E_1 = 2.48 \text{ eV}$$

$$E_2 = \frac{1240}{600} = 2.07 \text{ eV}$$

$\Delta E = 2.48 - 2.07$ eV

$= 0.41$ eV $= 41 \times 10^{-2}$ eV

24. A car of mass 200 kg is revolving in a circular track of radius 70 m with angular velocity of 0.2 rad/sec, then find the centripetal force in newton.

Answer (560)

Sol. $F_C = m\omega^2 r$

$= 200 \times 0.04 \times 70$

$= 560$ N

25.

26.

27.

28.

29.

30.

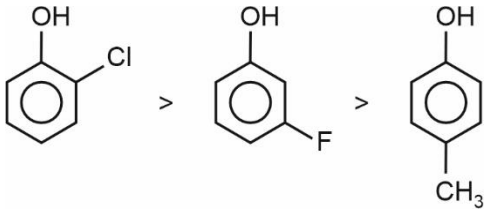
CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. **Assertion:** Acidic nature



Reason: F is better electron withdrawing group than Cl

(1) Assertion & Reason, both are correct and Reason is correct explanation of Assertion

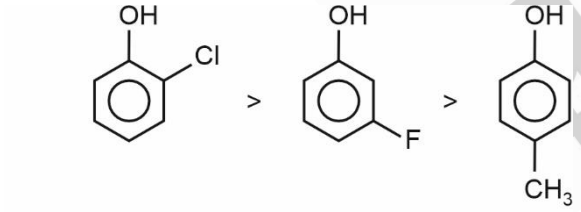
(2) Assertion and Reason, both are correct but Reason is not correct explanation of Assertion

(3) Assertion is correct, Reason is incorrect

(4) Assertion is incorrect, Reason is correct

Answer (2)

Sol.



pK_a	8.56	9.29	10.2

2. Which of the following the best method for preparation of BeF_2

(1) $Be + F_2 \rightarrow BeF_2$

(2) $BeH_2 + F_2 \rightarrow BeF_2$

(3) $BeH_2 + NaF \rightarrow$

(4) By $(NH_4)_2BeF_4$ (thermal decomposition)

Answer (4)

Sol. Best method for preparation of BeF_2 is by thermal decomposition of $(NH_4)_2\ BeF_4$

$$(NH_4)_2BeF_4 \xrightarrow{\Delta} NH_4F + BeF_2$$

Ref. NCERT (s-block)

3. The correct increasing order of the magnitude of standard enthalpies of formation for group-1 halides is

(1) NaI < NaF < NaBr < NaCl

(2) NaI < NaBr < NaCl < NaF

(3) NaF < NaCl < NaBr < NaI

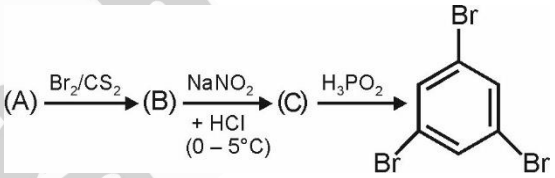
(4) NaCl < NaBr < NaF < NaI

Answer (2)

Sol.

Halide		ΔH_f° (kJ mol^{-1})
NaF	–	569
NaCl	–	400
NaBr	–	360
NaI	–	288

4. Consider the following reaction and identify the reactant (A)



(1) Aniline

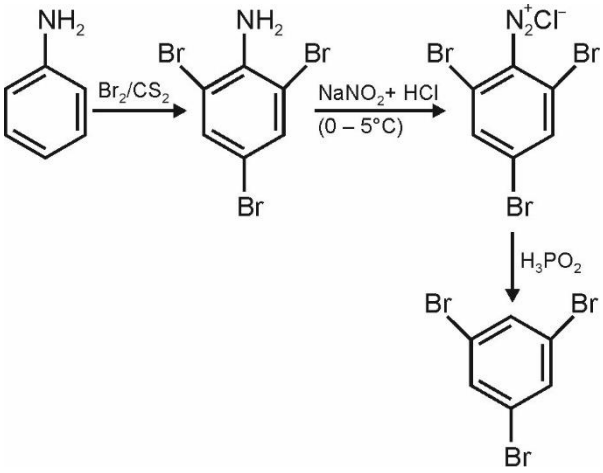
(2) Phenol

(3) Salicylic acid

(4) Acetanilide

Answer (1)

Sol. The reactant (A) is likely to be aniline because other options will undergo mono-bromination on reaction with Br_2 dissolved in CS_2.



5. **Assertion A :** Bond angle of SO_2 is less than H_2O

Reason R : Both form V-shaped structure.

(1) Assertion & Reason, both are correct and Reason is correct explanation of Assertion

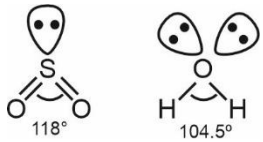
(2) Assertion and Reason, both are correct but Reason is not correct explanation of Assertion

(3) Assertion is correct, Reason is incorrect

(4) Assertion is incorrect, Reason is correct

Answer (4)

Sol.



6. Ba^{+2} cannot be precipitated as

(1) $BaCO_3$

(2) $Ba(OH)_2$

(3) $BaCrO_4$

(4) $BaSO_4$

Answer (2)

Sol. $Ba(OH)_2$ is soluble in water

$BaCO_3$ & $BaSO_4$ are white ppt

$BaCrO_4$ - Yellow ppt

7. Which of the following is oxidised by oxygen in acidic medium?

(1) Cl^-, Br^-

(2) Br^-, I^-

(3) Br^-

(4) I^-

Answer (2)

Sol. Reduction potential

$E^{o}_{I_2/I^-} = 0.54$ V

$E^{o}_{Br_2/Br^-} = 1.09$ V

$E^{o}_{O_2/H_2O} = 1.23$ V

$E^{o}_{Cl_2/Cl^-} = 1.36$ V

R. P. is in order $Cl_2 > Br_2 > I_2$

O.P. is revers in order

So, I^- and Br^- ion will get oxidised

8. A naturally occurring amino acid that contains only one basic functional group.

(1) Arginine

(2) Lysine

(3) Histidine

(4) Isoleucine

Answer (4)

Sol. Isoleucine has single nitrogenous base group.

9. Match the polymers given in column-I with their characteristics given in column-II

	Column-I		**Column-II**
(A)	Nylon 66	(P)	Thermosetting
(B)	Nylon 6	(Q)	Polyester
(C)	Phenol formaldehyde resin	(R)	Homopolymer
(D)	Dacron	(S)	Polyamide

(1) (A)-(P); (B)-(Q); (C)-(S); (D)-(R)

(2) (A)-(Q); (B)-(P); (C)-(R); (D)-(S)

(3) (A)-(P,Q); (B)-(R, S); (C)-(Q); (D)-(P)

(4) (A)-(S); (B)-(R, S); (C)-(P); (D)-(Q)

Answer (4)

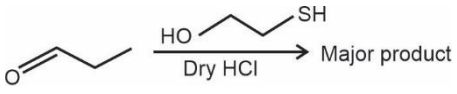
Sol. (A) Nylon 66 is a copolymer obtained by condensation polymerisation of hexamethylene diamine and adipic acid. It is a polyamide.

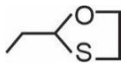
(B) Nylon 6 is a homopolymer of caprolactam. It is a polyamide.

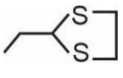
(C) Phenol formaldehyde resin is obtained by condensation polymerisation of phenol and formaldehyde. It is a thermosetting polymer.

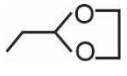
(D) Dacron is a copolymer obtained by condensation polymerisation of terephthalic acid and ethylene glycol. It is a polyester.

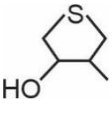
10. Identify the major product formed in the following reaction.



(1) 

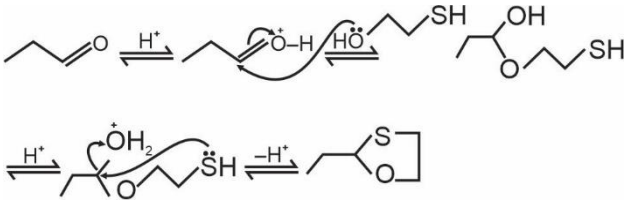
(2) 

(3) 

(4) 

Answer (1)

Sol.



11. Match reagent in Column-I with product in Column-II.

	Column-I Reagent 2-Bromopropane		Column-II Product
A	Alc.KOH	1	Nitrile
B	alc.KCN	2	Alkene
C	$AgNO_2$	3	Ester
D	CH_3COOAg	4	Nitro

(1) A–2; B–1; C–3; D–4

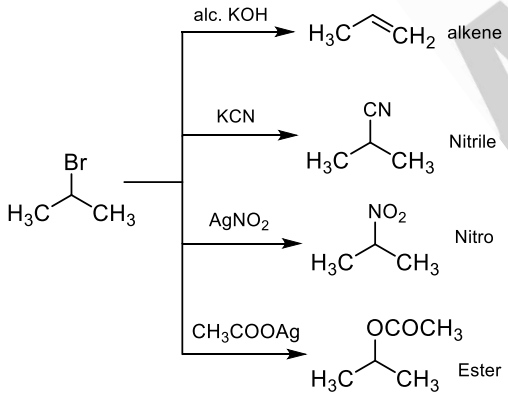
(2) A–2; B–1; C–4; D–3

(3) A–2; B–3; C–1; D–4

(4) A–1; B–2; C–4; D–3

Answer (2)

Sol.



12. S-I : Tropolone has 8π electron in total.

S-II : π-electrons of  are involved in aromaticity of tropolone.

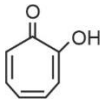
(1) Both S-I and S-II are true

(2) S-I is true, S-II is false

(3) S-I is false, S-II is true

(4) Both S-I and S-II are false

Answer (2)

Sol. Tropolone 

13.

14.

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. Consider the reaction

$$Cr_2O_7^{-2} + xH^{\oplus} + Fe^{+2} \longrightarrow yFe^{+3} + 2Cr^{+3} + zH_2O$$

Sum of x, y, z = ?

Answer (27)

Sol. $14H^{\oplus} + Cr_2O_7^{-2} + 6Fe^{+2} \longrightarrow 6Fe^{+3} + 2Cr^{+3} + 7H_2O$

x =14, y = 6, z = 7

x + y + z = 27

22. If the formula of Borax is

$Na_2B_yO_x(OH)_y \cdot zH_2O$, find the value of x + y + z?

Answer (17)

Sol. Formula is $Na_2B_4O_5(OH)_4.8H_2O$

$\therefore$ x = 5

y = 4

z = 8

x + y + z = 17

23. Given length of body diagonal of unit cell is 4 Å. Find the radius of Na atom forming bcc lattice (in Å).

Answer (1)

Sol. $4r = \sqrt{3}a$

$$r = \frac{\sqrt{3}\,a}{4}$$

$$r = \frac{4}{4} = 1\ \text{Å}$$

24. Find the orbital angular momentum of 3*s* orbital.

Answer (0)

Sol. Orbital angular momentum is given by $\sqrt{l(l+1)}$, l is the azimuthal quantum number.

For '*s*' orbital l = 0

$\therefore$ Orbital angular momentum = 0

25. Number of stereoisomers of $[Cr(OX)_2ClBr]^-$

Answer (03.00)

Sol. cis-2

Trans-1

26. Find out pH of resultant solution obtained when 20 mL of 0.1 M NaOH is mixed with 50 mL of 0.1 M CH_3COOH

pK_a of CH_3COOH = 4.74

log2 = 0.30; log3 = 0.47

Answer (04.57)

Sol.

$$CH_3COOH + NaOH \longrightarrow CH_3COONa + H_2O$$

5	2	–	–
↓	↓	↓	
3	0	2	

$$pH = pK_a + \log\frac{2}{3}$$

= 4.74 + 0.30 – 0.47

= 4.57

27. 23% NaCl and 19.5% $MgCl_2$ is present in salt water by weight. The degree of dissociation of both the salts is 100%. Find the normal boiling point of salt water (in ºC). (K_b = 0.52 K kg mol^{-1}) (Nearest integer)

Answer (113)

Sol. $\Delta T_b = iK_bm$

$$= \left(\frac{23\times2\times1000}{(58.5)\times57.5} + \frac{3\times19.5\times1000}{95\times57.5}\right)\times0.52$$

$$= \frac{(7.86+6.16)\times0.52}{57.5}\times100 \approx 12.66$$

$\therefore$ Boiling point $\approx 113^{o}C$

28. Consider a reaction

A(g) $\rightarrow$ 2B(g) + C(g)

Initial pressure (P_i) = 800 mm Hg.

At 10 minutes, total pressure is 1600 mm Hg, then find the total pressure at 30 minutes. (in mm Hg)

Answer (2200)

Sol. A(g) $\rightarrow$ 2B(g) + C(g)

800	—	—
800-p	2p	p

At 10 minutes, P_{total} = 800 + 2p = 1600

p = 400 mm Hg.

$\therefore$ 10 minutes means 1 half life

At t = 30 minutes, $p = \frac{7\times800}{8} = 700$ mm Hg

$\therefore$ P_{total} = (800 – 700) + 2 × 700 +700

= 800 + 1400

= 2200 mm Hg.

29.

30.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

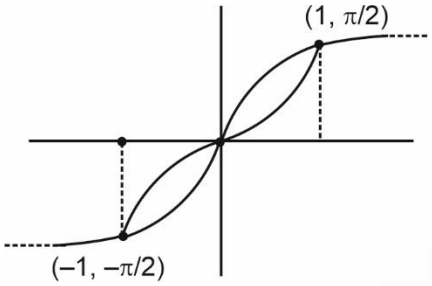
1. If $\sin^{-1}x = 2\tan^{-1}x$, then number of integral values of x is equal to

(1) 0 (2) 1

(3) 2 (4) More than 2

Answer (4)

Sol.



$$\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

$\Rightarrow$ clearly 1, 0, –1 are the real integral solution

2. If $x^2 - \sqrt{2}x + 2 = 0$ has roots α and β then $\alpha^{14} + \beta^{14}$ is

(1) –256 (2) –128

(3) $-128\sqrt{2}$ (4) $-256\sqrt{2}$

Answer (2)

Sol. $\alpha, \beta = \dfrac{\sqrt{2} \pm i\sqrt{6}}{2}$

$$= \frac{\sqrt{2}}{2}\left(1 + \sqrt{3}i\right)$$

$$= \sqrt{2}\left(\cos\frac{\pi}{3} \pm i\sin\frac{\pi}{3}\right)$$

$$\alpha^{14} + \beta^{14} = 2^7\left[\left(\cos\frac{14\pi}{3} + i\sin\frac{14\pi}{3}\right) + \left(\cos\frac{14\pi}{3} - i\sin\frac{14\pi}{3}\right)\right]$$

$$= 2^7 \cdot 2\cos\frac{2\pi}{3}$$

$$= -128$$

3. The range of $\dfrac{4 + (\sin x)^4}{1 + x^2}$ is

(1) [0, 1] (2) (0, 4]

(3) (0, 3] (4) None of these

Answer (2)

Sol. $f(x) = \dfrac{4 + (\sin x)^4}{1 + x^2}$

$\because$ $f(-x) = f(x)$

$\Rightarrow$ $f(x)$ is even

Now,

$$f'(x) = \frac{\left(1 + x^2\right)4\sin^3 x\cos x - 4 + (\sin x)^4 2x}{\left(1 + x^2\right)^2}$$

$\Rightarrow$ $f(x)$ is decreasing for $x \in (0, \infty)$

$f(x)$ is increasing for $x \in (-\infty, 0)$

$\therefore$ $f(x)_{max}$ will be for $x = 0$

$f(x) \in (0, 4]$

4. The coefficient of x^4 in $\left(2x^3 - \dfrac{1}{3x^8}\right)^5$ is

(1) $-\dfrac{80}{3}$ (2) $\dfrac{80}{3}$

(3) $\dfrac{40}{3}$ (4) $-\dfrac{40}{3}$

Answer (1)

Sol. $T_{r+1} = {}^5C_r (2x^3)^{5-r}\left(-\dfrac{1}{3x^8}\right)^r$

$$= {}^5C_r\, 2^{5-r}\left(-\frac{1}{3}\right)^r \cdot x^{15-11r}$$

For coefficient of x^4: $15 - 11r = 4$

$11r = 11$

$\boxed{r = 1}$

$\therefore$ Coefficient of $x^4 = {}^5C_1\, 2^4\left(-\dfrac{1}{3}\right)$

$$= -\frac{80}{3}$$

5. The number of six-digit number formed by using the digits {1, 2, 3, 4, 5, 6} which are divisible by 6 (Repetition is not allowed)

(1) 120 (2) 360

(3) 240 (4) 720

Answer (2)

Sol. 1 + 2 + 3 + 4 + 5 + 6 = 21 is divisible by 3, so all numbers are divisible by 3.

To be divisible by 6, last digit must be even.

Total numbers = 3.5!

= 360

6. $\int_0^{\frac{\pi}{4}} \frac{\tan^{50} x}{\tan^{51} x + \tan^{49} x} dx =$

(1) $\frac{1}{4}$ (2) $\frac{2}{3}$

(3) $\frac{3}{2}$ (4) $\frac{1}{2}$

Answer (1)

Sol. $\int_0^{\frac{\pi}{4}} \frac{dx}{\tan x + \cot x}$

$\Rightarrow \int_0^{\frac{\pi}{4}} \frac{2\sin x \cdot \cos x}{2} dx$

$\Rightarrow \frac{1}{2}\int_0^{\frac{\pi}{4}} \sin 2x dx$

$= -\frac{1}{2}\frac{\cos 2x}{2}\Big|_0^{\frac{\pi}{4}}$

$= -\frac{1}{4}(0-1) = \frac{1}{4}$

7. For matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ \alpha & 3 & 2 \\ 3 & 1 & 1 \end{bmatrix}$ and $|A| = 2$ then the value of $\left|\alpha \text{adj}\left(\alpha \text{ adj}(\alpha A)\right)\right|$ is

(1) 2^{25} (2) 2^{24}

(3) 2^{20} (4) 2^{16}

Answer (1)

Sol. $\because$ $|A| = 2$

$\Rightarrow 1(3-2) - 2(\alpha - 6) + 1(\alpha - 9) = 2$

$\Rightarrow \boxed{\alpha = 2}$

Now, $\left|\alpha \text{ adj}\left(\alpha \text{ adj}(\alpha A)\right)\right| = \left|2 \text{ adj}\left(2 \text{ adj}(2A)\right)\right|$

$= 2^3 \left|\text{adj } \left(2 \text{ adj}(2A)\right)\right|$

$= 2^3 \left|2 \text{ adj}(2A)\right|^2$

$= 2^3 \cdot 2^6 |2A|^4$

$= 2^9 \cdot 2^{12} \cdot 2^4$

$= 2^{25}$

8. In a given data set mean of 40 observations is 50 and standard deviation is 12. Two readings which were 20 and 25, were mistakenly taken as 40 and 45. Find correct variance of data set

(1) 169 (2) 150

(3) 178 (4) 180

Answer (3)

Sol. $\sum x_{i(\text{Correct})} = 40 \cdot 50 - 40$

$= 1960$

$\bar{x}_{i(\text{Correct})} = 49$

$\frac{\sum x^2_{i(\text{Wrong})}}{40} - (50)^2 = 144$

$\sum x^2_{i(\text{Wrong})} = 2644 \cdot 40$

$\sum x^2_{i(\text{Correct})} = 40 \cdot 2644 - (2600)$

Correct variance $= \frac{40 \cdot 2644 - 2600}{40} - (49)^2$

= 2644 – 65 – 2401

= 2644 – 2466

= 178

9. If for a complex number z, $\bar{z} = i\left(z^2 + \text{Re}(z)\right)$ then $|z|^2$ is sum of values of all

(1) 1 (2) 2

(3) 3 (4) 4

Answer (4)

Sol. Let $z = x + iy$

$x - iy = i(x^2 - y^2 + 2ixy + x)$

$x - iy = -2xy + i(x^2 - y^2 + x)$

$x = -2xy$ or $x^2 - y^2 + x + y = 0$

$x = 0$ for $x = 0$ $y = 0, 1$

or

$y = \frac{-1}{2}$ for $y = \frac{-1}{2}$

$$x^2 + x - \frac{1}{4} - \frac{1}{2} = 0$$

$$x^2 + x - \frac{3}{4} = 0$$

$$4x^2 + 4x - 3 = 0$$

$$4x^2 + 6x - 2x - 3 = 0$$

$$(2x - 1)(2x + 3) = 0$$

$z = 0 + 0i \rightarrow |z| = 0$

$z = 0 + i \rightarrow |z| = 1$

$z = \frac{1}{2} - \frac{1}{2}i \rightarrow |z| = \sqrt{\frac{1}{2}}$

$z = \frac{-3}{2} - \frac{1}{2}i \rightarrow |z| = \sqrt{\frac{10}{4}}$

Sum of $|z|^2 = 0 + 1 + \frac{1}{2} + \frac{10}{4}$

$$= \frac{16}{4} = 4$$

10. The area bounded by the curve

$x^2 \le y \le |x^2 - 4|$ and $y \ge 1$ is

(1) $4\sqrt{2} + 1$

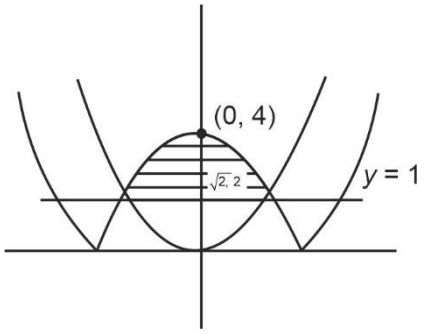
(2) $\frac{4}{3}\left(4\sqrt{2} - 1\right)$

(3) $\frac{4}{3}\left(4\sqrt{2} + 1\right)$

(4) $\frac{2}{3}\left(4\sqrt{2}\right)$

Answer (2)

Sol.



Required area $= 2\left[\int_1^2 \sqrt{y}dy + \int_2^4 \sqrt{4-y}dy\right]$

$$= 2\left[\left.\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right|_1^2 - \left.\frac{(4-y)^{\frac{3}{2}}}{\frac{3}{2}}\right|_2^4\right]$$

$$= \frac{4}{3}\left[\left(2\sqrt{2} - 1\right) - \left(0 - 2\sqrt{2}\right)\right]$$

$$= \frac{4}{3}\left(4\sqrt{2} - 1\right)$$

11. The geometric mean of 5th and 7th term is 2 and the product of 3rd and 6th term of the GP is $\frac{1}{3}$. If a_n is the nth term then $(a_3 + a_4).(a_5 + a_6)$ is

(1) $\frac{1 + (12)^{1/3}}{(12)^{1/3}}$ (2) $\frac{1 + (12)^{1/3}}{3}$

(3) $\left(\frac{1 + (12)^{1/3}}{3 \times (12)^{1/3}}\right)^2$ (4) $\frac{(1 + (12)^{1/3})^2}{3 \times (12)^{1/3}}$

Answer (4)

Sol. $(a_5 a_7)^{1/2} = 2$

$ar^4 ar^6 = 4$

$a^2 r^{10} = 4$ ...(1)

$a_3 a_6 = \frac{1}{3}$

$ar^2 ar^5 = \frac{1}{3}$

$a^2 r^7 = \frac{1}{3}$ ...(2)

$\frac{(1)}{(2)} = r^3 = 12$

$\Rightarrow r = (12)^{1/3}$

$a^2(12)^{7/3} = \frac{1}{3}$

$a^2 = \left(\frac{1}{3(12)^{7/3}}\right)$

$\because (a_3 + a_4) \cdot (a_5 + a_6)$

$= (ar^2 + ar^3)(ar^4 + ar^5)$

$= r^6a^2(1+r)^2 = \frac{1}{3(12)^{7/3}} \times 144\left(1+(12)^{1/3}\right)^2$

$= \frac{(1+(12)^{1/3})^2}{3\times(12)^{1/3}}$

12. The statement $((\sim p) \wedge q) \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q)$ is equivalent to

(1) Tautology (2) Fallacy

(3) $(p \vee q)$ (4) $\sim (p \wedge q)$

Answer (4)

Sol. $(\sim p \wedge q) \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q)$

$= \sim p \wedge (q \vee \sim q) \vee (p \wedge \sim q)$

$= \sim p \vee (p \wedge \sim q)$

$= (\sim p \vee p) \wedge (\sim p \vee \sim q)$

$= T \wedge \sim(p \wedge q)$

$= \sim(p \wedge q)$

13. Given $\frac{x+3}{-3} = \frac{y-2}{2} = \frac{z-5}{5}$ which of the following lines in options is coplanar with the given line?

(1) $\frac{x+1}{-1} = \frac{y-1}{1} = \frac{z-5}{5}$

(2) $\frac{x+}{1} = \frac{y+1}{-1} = \frac{z-5}{5}$

(3) $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-5}{5}$

(4) $\frac{x-1}{-1} = \frac{y+2}{-2} = \frac{z-5}{4}$

Answer (1)

Sol. For non-parallel lines to be coplanar

$(\vec{r}_1 - \vec{r}_2)\cdot(\vec{a}_1 \times \vec{a}_2) = 0$

For option A $\vec{r}_1 - \vec{r}_2 = -2\hat{i} + \hat{j}$

$\vec{a}_1 = -3\hat{j} + 2\hat{j} + 5\hat{k}$

$\vec{a}_2 = -\hat{i} + \hat{j} + 5\hat{k}$

$\begin{vmatrix} -2 & 1 & 0 \\ -3 & 2 & 5 \\ -1 & 1 & 5 \end{vmatrix} = 0$

$\therefore$ Option (A) is correct

Similarly, we can also check other option which comes out to be non-coplanar

14. For $\vec{a}, \vec{b}, \vec{c}, |\vec{a}| = 2, |\vec{b}| = 3$ and $\vec{a}\cdot\vec{b} = 4$ then $\left|(\vec{a}+2\vec{b})\times(2\vec{a}-3\vec{b})\right|^2$ is

(1) 280 (2) 980

(3) 480 (4) 1764

Answer (2)

Sol. $\left|(\vec{a}+2\vec{b})\times(2\vec{a}-3\vec{b})\right|^2$

$= \left|-3(\vec{a}\times\vec{b}) + 4(\vec{b}\times\vec{a})\right|^2$

$= \left|7(\vec{b}\times\vec{a})\right|^2 = 49\left(|\vec{a}|^2|\vec{b}|^2 - (\vec{a}\cdot\vec{b})^2\right)$

$= 49\,(36 - 16)$

$= 980$

15. A line is passing through $A(4, 5, 8)$ and $B(1, -7, 5)$ from point $C(1, 2, 5)$ a perpendicular is drawn on AB. If foot of perpendicular is N then distance of N from plane $2x - 2y + 2z - 3 = 0$ is

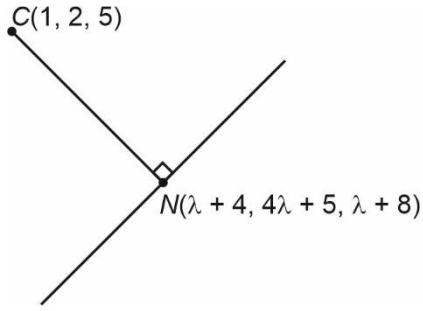
(1) $\frac{9}{2\sqrt{3}}$ (2) $\frac{15}{2\sqrt{3}}$

(3) $\frac{8}{3\sqrt{3}}$ (4) $\frac{7}{4\sqrt{3}}$

Answer (2)

Sol. $L: \frac{x-4}{3} = \frac{y-5}{12} = \frac{z-8}{3}$

$L: \frac{x-4}{1} = \frac{y-5}{4} = \frac{z-8}{1}$



Now, $NC \perp AB$

$< \lambda + 3, 4\lambda + 3, \lambda + 3 > \cdot < 1, 4, 1> = 0$

$\lambda + 3 + 16\lambda + 12 + \lambda + 3 = 0$

$18\lambda + 18 = 0$

$\lambda = -1$

$\therefore$ $N(3, 1, 7)$

$$\text{Distance} = \left|\frac{6-2+14-3}{\sqrt{2^2+2^2+2^2}}\right|$$

$$= \frac{15}{2\sqrt{3}} \text{ unit}$$

16. If centroid of triangle formed by the lines $2x + y = 10$, $x + 3y = 7$ and $3x - y = 5$ is (α, β). The quadratic equation whose roots are $\alpha + 2\beta$ and $2\alpha + \beta$ is

(1) $225x^2 + 3645x - 14690 = 0$

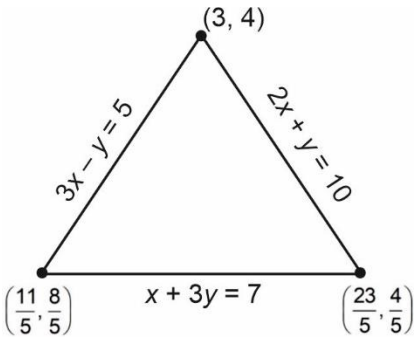
(2) $225x^2 - 3645x + 14690 = 0$

(3) $225x^2 - 3645x - 14690 = 0$

(4) $225x^2 + 3645x + 14690 = 0$

Answer (2)

Sol.



$$\therefore \text{ Centroid } = (\alpha, \beta) = \left(\frac{3+\frac{23}{5}+\frac{11}{5}}{3}, \frac{4+\frac{4}{5}+\frac{8}{5}}{3}\right)$$

$$\Rightarrow (\alpha, \beta) = \left(\frac{49}{15}, \frac{32}{15}\right)$$

$$\alpha + 2\beta = \frac{113}{15}$$

$$2\alpha + \beta = \frac{130}{15}$$

$\therefore$ required equation: $x^2 - \left(\frac{243}{15}\right)x + \frac{14690}{225} = 0$

$\Rightarrow$ $225x^2 - 3645x + 14690 = 0$

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. The remainder when 7^{103} is divided by 17 is

Answer (12)

Sol. $7 \equiv 7 \pmod{17}$

$7^2 \equiv -2 \pmod{17}$

$7^6 \equiv -8 \pmod{17}$

$7^8 \equiv -1 \pmod{17}$

$7^{96} \equiv 1 \pmod{17}$

$7^{103} \equiv 12 \pmod{17}$

$\therefore$ Remainder = 12

22. The value of

$\left[\sqrt{1}\right] + \left[\sqrt{2}\right] + \left[\sqrt{3}\right] + \ldots \left[\sqrt{120}\right]$ is equal to, where $[\cdot]$ denotes greatest integer function.

Answer (825)

Sol. $E = 1 + 1 + 1 + 2 + 2 + 2 + 2 + 2 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 4 + 4 + 4 + 4 + 4 + \ldots$

$E = 3 \times 1 + 5 \times 2 + 7 \times 3 + \ldots + 19 \times 9 + 10 \times 21$

$$= \sum_{r=1}^{10}(2r+1)r = 2\left[\frac{10\times11\times21}{6}\right] + \frac{10\times11}{2}$$

= 770 + 55 = 825

23. Rank of Monday in English dictionary if all alphabets are arranged in order?

Answer (327)

Sol.

3	5	4	2	1	6
M	O	N	D	A	Y
2	3	2	1	0	0
5!	4!	3!	2!	1!	0!

$\therefore$ Rank = (2 × 5! + 3 × 4! + 2 × 3! + 1 × 2!) + 1

= 240 + 72 + 12 + 2 + 1 = 327

24.

25.

26.

27.

28.

29.

30.

❑ ❑ ❑