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Term Paper On

"Supply Chain Contracts"

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SUPPLY CHAIN CONTRACTS

Introduction:

Supply chain consists of multiple players, for example - retailer, supplier, manufacturer etc. All these players are concerned with optimizing their own objectives(*maximizing the profit/reducing the overall cos*t). Usually, this self-serving focus often leads to poor performance of the overall supply chain. However, if all the players agree on certain terms & conditions(*contracts*) such that their objectives align with the objective of the supply chain then an optimal performance of the supply chain can be achieved. We would like to introduce the basic idea of supply chain contracts. The term *contract* here refers to a *transfer payment* made by one player to another. The essence of the study of 'supply chain contracts' lies in deciding the size of these *transfer payments*.

Game Theory:

Supply chain contracts derive its idea from game theory. Let us introduce some terms of game theory and understand how supply chain chain contracts work. A *game* consists of two or more players. Each player may choose from a set of *strategies*, and a choice of strategies(one for each player) is called an *outcome*. For each outcome there is a *payoff* to each player. Given example shows two players A and B. Both of them have two strategies 1 and 2. Player A's and Player B's payoff are the first and second number in the pair respectively.

Payoffs for a sample game:		**Player B**	
		1	2
Player A	1	(1,1)	(-4,2)
	2	(2,-4)	(-2,-2)

Table 1

An outcome is called *Pareto optimal* if a player trying to further increase his payoffs leads to reduction in the other player's payoff. A *Nash Equilibrium* is an outcome in which the game ultimately moves to, if each player plays selfishly to increase his/her payoffs. In a game in which Nash equilibrium and Pareto optimal solution are different, players will have a worse solution if they play selfishly but can achieve better payoffs *(Pareto optimal*). Let us introduce a condition(*contract*) that towards the end of the game the players will split their profit & losses equally.

Payoffs after implementing the contract:		**Player B**	
		1	2
Player A	1	(1,1)	(-1,-1)
	2	(-1,-1)	(-2,-2)

Table 2

Now, both players even if they act selfishly will choose strategy 1(*Nash equilibrium*). But this outcome is also *Pareto optimal*. We observe that one simplistic contract can make the game move towards the best solution, even if the players act selfishly. Contract just modified the payoffs matrix so that the player chooses the outcome which is *Pareto optimal*. The aim of supply chain contracts is to convert the payoffs(*transfer payments)* in such a way that Nash equilibrium is the same as Pareto optimal solution.

Preliminary Analysis: **[For terminology see APPENDIX]**

We will start our analysis of supply chain contracts by considering the newsvendor(whom we will refer to as retailer) model that faces stochastic demand for a single period. The following set of events occur in this analysis:

1. Supplier chooses the parameters of the contract.
2. Retailer chooses the order quantity Q. We assume that the lead time is zero.
3. There is no backorder in our analysis. Any demand which is unmet will be considered as lost sales.
4. Costs are assessed, and payments are made between the supplier and the retailer.

We will start our analysis by formulating expected cost values for supplier and retailer as a function of Q. Let $S(Q)$ be the expected sales:

$$S(Q) = E[min\{Q, X\}] = \int_0^Q xf(x)dx + \int_Q^\infty Qf(x)dx = \int_0^\infty xf(x)dx - \int_Q^\infty xf(x)dx + Q\int_Q^\infty f(x)dx$$

$$= E[X] - \int_Q^\infty (x - Q)f(x)dx = \lambda - n(Q),$$

where $n(Q)$ denotes the loss function. Now, $S(Q) = \lambda - n(Q)$

$$\Rightarrow S'(Q) = \frac{d}{dQ}(\lambda - n(Q)) = -\frac{d}{dQ}\left\{\int_Q^\infty (x - Q)f(x)dx\right\} = \int_Q^\infty f(x)dx \quad \textit{(using Leibnitz's rule)}$$

$$\Rightarrow S'(Q) = F(\infty) - F(Q) = 1 - F(Q) = \overline{F}(Q)$$

Let $I(Q)$ be the expected inventory on hand at the end of the period:

$$I(Q) = E[max\{0, Q - X\}] = \int_0^Q (Q - x)f(x)dx = \int_0^\infty (Q - x)f(x)dx - \int_Q^\infty (Q - x)f(x)dx$$

$$= Q\int_0^\infty f(x)dx - E[X] + n(Q) = Q - \lambda + n(Q) = Q - S(Q)$$

Let $L(Q)$ be the expected lost sales:

$$L(Q) = E[max\{0, X - Q\}] = \int_Q^\infty (x - Q)f(x)dx = n(Q) = \lambda - S(Q) \qquad \{S(Q) = \lambda - n(Q)\}$$

Let T be the expected transfer payment. The size of T is decided by the type of contract. Now, the expected profit function for the retailer is: $\pi_r(Q) = rS(Q) + vI(Q) - b_rL(Q) - c_rQ - T$

$$= rS(Q) + v(Q - S(Q)) - b_r(\lambda - S(Q)) - c_rQ - T$$

$$= (r - v + b_r)S(Q) - (c_r - v)Q - b_r\lambda - T \qquad \text{.........(1)}$$

Similarly we can write the supplier's expected profit function as

$$\pi_s(Q) = -c_sQ - b_sL(Q) + T = -c_sQ - b_s(\lambda - S(Q)) + T = b_sS(Q) - c_sQ - b_s\lambda + T \quad \text{........(2)}$$

The supply chain's total expected profit function is sum of retailer's profit function $\{\pi_r(Q)\}$ and supplier's profit function $\{\pi_s(Q)\}$:

$$\Pi(Q) = \pi_r(Q) + \pi_s(Q) = (r - v + b_r)S(Q) - (c_r - v)Q - b_r\lambda - T + b_s S(Q) - c_s Q - b_s\lambda + T$$
$$= (r - v + b)S(Q) - (c - v)Q - b\lambda$$

Now we have to determine the order quantity Q say $Q°$ that maximizes the total supply chain profit. For this we differentiate total expected profit function $\Pi(Q)$. $\Pi'(Q°) = 0 \Rightarrow (r - v + b)S'(Q°) - (c - v) = 0$

$$\Rightarrow S'(Q°) = \overline{F}(Q°) = \frac{c-v}{r-v+b} \quad ....(3) \quad \Rightarrow F(Q°) = \frac{r+b-c}{r-v+b} \quad ....(4)$$

Now, $\Pi''(Q°) = (r - v + b)S''(Q°) = -(r - v + b)f(Q°) < 0$. Π is a concave function. Hence, $Q°$ maximes the supply chain's total expected profit function. We obtained the value of order quantity $Q = Q°$ that maximizes the total expected profit of the supply chain. The problem is whether the retailer will choose this value of $Q°$ as his order quantity or whether the supplier also prefers this $Q°$? If the retailer acts selfishly, he will prefer a Q(say Q_r^*) that maximizes (1) and if the supplier acts selfishly, he will prefer a $Q(say\ Q_s^*)$ that maximizes (2). This will lead to the supply chain ending in the *Nash equilibrium* which may/may not be *Pareto optimal*.. The supply chain will be said to be coordinated if $Q_r^* = Q_s^* = Q°$ and the solution is said to be *Pareto optimal*. A contract type is said to coordinate the supply chain if $Q_r^* = Q_s^* = Q°$ & both supplier as well as the retailer earn positive profit. Even if the order quantities coincides $i.e.$, $Q_r^* = Q_s^* = Q°$ but if one of the players earns negative profit then that player may or may enter into the contract which depends on some other factors which we will not discuss here. Now, we will analyze various types of contract and see whether the supply chain can be coordinated with some specific values of contract parameters.

1. The Wholesale Price Contract:

According to this contract, the retailer pays the supplier a given cost w per unit of the quantity ordered. The purchase cost w is the supplier's decision variable and the order quantity Q is the retailer's decision variable. The transfer payment (T_w) is given by: $T_w(Q, w) = wQ$

The retailer's and supplier's expected profit function for this contract can be obtained from Eq. (1) and Eq. (2) respectively. Here π_r and π_s are function of both Q and w.

Retailer's profit: $\pi_r(Q, w) = (r - v + b_r)S(Q) - (c_r - v)Q - b_r\lambda - wQ$

Supplier's profit: $\pi_s(Q, w) = b_s S(Q) - c_s Q - b_s\lambda + wQ$

Now the retailer and the supplier will choose those values of Q such that their own profit is maximized and will not think of others profit. Q and w are the decision variables so the supply chain is coordinated if there exist a value of w

such that $Q_r{}^* = Q_s{}^* = Q°$ where $Q_r{}^*, Q_s{}^*$ and $Q°$ are the order quantities that maximizes $\pi_r(Q,w)$, $\pi_s(Q,w)$, $\Pi(Q)$. Both $\pi_r(Q,w)$ & $\pi_s(Q,w)$ are concave functions. So Q_r^* and Q_s^* satisfy:

$$\frac{\partial \pi_r(Q,w)}{\partial Q}(Q = Q_r^*) = 0 \quad \Rightarrow S'(Q_r^*) = \frac{w+c_r-v}{r-v+b_r} \qquad \text{.......(5)}$$

$$\frac{\partial \pi_s(Q,w)}{\partial Q}(Q = Q_s^*) = 0 \quad \Rightarrow S'(Q_s^*) = \frac{c_s-w}{b_s} \qquad \text{.......(6)}$$

Let us try to obtain the value of w so that the supply chain is coordinated i.e. $Q_r^* = Q_s^* = Q°$. Using Eq. (3), (5) and (6) we get, $\Rightarrow \frac{w+c_r-v}{r-v+b_r} = \frac{c-v}{r-v+b}$ and $\frac{c_s-w}{b_s} = \frac{c-v}{r-v+b}$. Both these equations must give same value of w in order to make supply chain coordinated. The expression of w obtained is: $w = c_s - \{\frac{c-v}{r-v+b}\} b_s$. However, we have assumed that $v < c_r \le c < r$. This will lead to the coefficient of b_s being negative which will further lead to $w < c_s$. Therefore, for this value of Q, the supplier will earn a negative expected profit. Hence, though the supply chain is coordinated, the contract is not able to do the same. Let us consider the Eq. (), in which we got :

$$\pi_s'(Q,w) = b_s S'(Q_s^*) + (w - c_s) = b_s \overline{F}(Q_s^*) + (w - c_s) = b_s\{\overline{F}(Q_s^*) + \frac{(w-c_s)}{b_s}\} \; [\,\overline{F}(Q \text{ is decreasing in } Q]$$

Now, if $\{\frac{(w-c_s)}{b_s} < -1\}$ or $\{w < c_s - b_s\}$, then $\pi_s(Q,w)$ is strictly decreasing in Q. If $\{-1 < \frac{(w-c_s)}{b_s} < 0\}$, then $\pi_s(Q,w)$ is first increasing and then decreasing in Q. If $\{\frac{(w-c_s)}{b_s} > 0\}$ or $\{w > c_s\}$ then $\pi_s(Q,w)$ is strictly increasing in Q. Consider the following cases:

<u>Case 1:</u> $\frac{(w-c_s)}{b_s} > 0$, For this case, $Q_s^* = \infty$. In this case, the supplier earns a positive margin on the items sold to the retailer and the supplier does not share the risk of overage with the retailer.

<u>Case 2:</u> $-1 < \frac{(w-c_s)}{b_s} < 0$, For this case, Q_s^* = finite. In this case, supplier earns a negative margin ($w < c_s$) on the items sold to the retailer. But the supplier still agrees to this order quantity as small order quantities will lead to penalty cost. This will minimize the loss of the supplier.

<u>Case 3:</u> $\{\frac{(w-c_s)}{b_s} < -1\}$, For this case, $Q_s^* = 0$. In this case, the supplier will incur more loss by supplying the order quantity than it will incur from the stockout penalty. Hence, the supplier doesn't supply the order quantity.

Let us dive further into Case 1: $\{w > c_s\}$. One can show that if $\{w > c_s\}$ then $Q_r^* < Q°$. In this case the supplier earns positive expected profit , however the retailer will order less. This is because if the retailer orders $Q°(> Q_r^*)$, then he has to bear the entire risk of the overage because unsold items are not given back to the supplier. However, the risk of underage is shared by both retailer and supplier as supplier also pays for the stockout penalty (b_s).

Therefore, the retailer would order less than the supplier wants him to. Hence, as we will study further supply chain contracts we will see that the supplier shares the risk of overage with the retailer which provokes the retailer to buy more quantity from the supplier. Thus , one can observe that 'wholesale price contract' is a non-coordinating because no value of w that (a) leads to $Q_r^* = Q_s^* = Q°$ and (b) guarantees that both supplier and retailer earn positive expected profit.

2. The Buyback Contract:

According to this contract, the supplier costs the retailer w per unit purchased, but at the end of period, the supplier pays the retailer p for each unit of unsold inventory. In this contract, transfer payment $T_p(Q, w, p)$ becomes

$$T_p(Q, w, p) = wQ - pI(Q) = pS(Q) + (w - p)Q$$

We assume $0 \le p \le r - v + b_r$ ... (7) .Otherwise if $p + v - b_r > r$ then it will be better for the retailer to not to sell the product. Also $p \le w + c_r - v$ ...(8) .Otherwise retailer will not be in any risk of overstocking the product as he gets more by salvaging the product $(p + v)$ than what he paid for it $(w + c_r)$. Many suppliers agree to buy back the unsold inventory , otherwise the retailer might sell these goods at a discounted price and the supplier may not be happy with the fact that its products are getting sold at a lower price.

The retailer's and supplier's expected profit function for this contract can be obtained from Eq. (1) and Eq. (2) respectively. Here π_r and π_s are function of Q, w and p.

<u>Retailer's profit:</u> $\pi_r(Q, w, p) = (r - v + b_r)S(Q) - (c_r - v)Q - b_r\lambda - \{pS(Q) + (w - p)Q\}$

$$= (r - v + b_r - p)S(Q) - (c_r - v + w - p)Q - b_r\lambda \quad ......(9)$$

<u>Supplier's profit:</u> $\pi_s(Q, w, p) = b_sS(Q) - c_s(Q) - b_s\lambda + pS(Q) + (w - p)Q$

$$= (b_s + p)S(Q) + (w - p - c_s)Q - b_s\lambda \quad ......(10)$$

The Q that retailer will choose to maximize his profit say Q_r^* will vanish the derivative of retailer's profit function.

$$\frac{\partial \pi_r(Q,w,p)}{\partial Q}(Q = Q_r^*) = 0 \qquad \Rightarrow S'(Q_r^*) = \frac{w+c_r-v-p}{r-v+b_r-p}$$

Similarly we can get the expression for Q_s^* which maximizes the supplier's profit as $S'(Q_s^*) = \frac{c_s-w+p}{b_s+p}$

Since w and p are contract parameters it will be decided by the supplier. For given value of p subject to equation 7 and 8 supplier can choose the value of w that will coordinate the supply chain. This w can be obtained by applying the condition of coordination i.e. $Q_r^* = Q_s^* = Q°$. $\Rightarrow S'(Q_r^*) = S'(Q_s^*) = S'(Q°)$

Using equation 3,5 and 6 we get $\Rightarrow \frac{w+c_r-v-p}{r-v+b_r-p} = \frac{c-v}{r-v+b}$ and $\frac{c_s-w+p}{b_s+p} = \frac{c-v}{r-v+b}$

Both these equations must give the same value of w for given value of p to make the supply chain coordinated this indeed came out to be the same.

$$w(p) = p + c_s - (c - v)\frac{p+b_s}{r-v+b} \quad \ldots\ldots(11)$$

So the supply chain can be coordinated for the values of p satisfying $0 \leq p \leq r - v + b_r$ and $w(p)$ satisfying equation (11). Note that we have not used equation (8) for the permissible values of p because any p and $w(p)$ satisfying equation (7) and (11) will automatically satisfy equation (8).

Taking equation (11), $\quad w(p) = p + c_s - (c - v)\frac{p+b_s}{r-v+b}$

$$\Rightarrow w(p) + c_r - v = p + c_s - (c - v)\frac{p+b_s}{r-v+b} + c_r - v \quad \ldots\ldots(12)$$

Now from equation (7) we have :

$$p < r - v + b_r \quad \Rightarrow p + b_s < r - v + b_r + b_s = r - v + b$$

$$\Rightarrow \frac{p+b_s}{r-v+b} = \delta \text{ say, then } \delta < 1$$

Now putting this in equation (12) we get

$$w(p) + c_r - v = p + c - v - (c - v)\delta = p + (c - v)(1 - \delta) > p \text{ which is indeed Eq. (8).}$$

Now let us see whether the buyback contract coordinates the supply chain or not? Other than the conditions on p and $w(p)$ given by equation (7) and equation (11) we have to derive the condition which makes the buyback contract coordinating the supply chain means both supplier and retailer earns positive profit.

Let us define α as: $\alpha = \frac{r-v+b_r-p}{r-v+b} \quad \ldots\ldots(13)$

$= 1 - \frac{p+b_s}{r-v+b} = \frac{(c-v)-(c-v)\frac{p+b_s}{r-v+b}}{c-v}$ {multiplying $c - v$ in numerator and denominator}

$= \frac{p+c_s-(c-v)\frac{p+b_s}{r-v+b}-b+c_r-v}{c-v} = \frac{w(p)-p+c_r-v}{c-v} \quad \ldots\ldots(14)$

We see that since $b_r < b$ and $p \geq 0$ so $\alpha \leq 1$. Also $p \leq r - v + b_r$ by equation (7) so, from equation (9) and equation (10) we can rewrite the retailer's profit and supplier's profit with $w(p)$ given by equation (11) as:

$$\pi_r(Q_r^*, w, p) = (r - v + b_r - p)S(Q_r^*) - (w - p + c_r - v)Q_r^* - b_r\lambda$$

$$= \alpha(r - v + b)S(Q_r^*) - \alpha(c - v)Q_r^* - b_r\lambda = \alpha\Pi(Q^\circ) + \lambda(\alpha b - b_r)$$

where $\Pi(Q^\circ)$ is the total supply chain profit as derived before and $Q_r^* = Q_s^* = Q^\circ$ since equation (11) is used.

$$\pi_s(Q_s^*, w, p) = \Pi(Q^\circ) - \pi_r(Q^\circ, w, p) \qquad [\text{since } \Pi(Q) = \pi_r(Q, w, p) + \pi_s(Q, w, p)]$$

$$= \Pi(Q^\circ) - (\alpha\Pi(Q^\circ) + \lambda(\alpha b - b_r)) = (1 - \alpha)\Pi(Q^\circ) - \lambda(\alpha b - b_r)$$

Now, $\quad \pi_r(Q_r^*, w, p) > 0 \Rightarrow \alpha\Pi(Q^\circ) + \lambda(\alpha b - b_r) > 0 \quad \Rightarrow \alpha(\Pi(Q^\circ) + \lambda b) > \lambda b_r$

$$\Rightarrow \alpha > \frac{\lambda b_r}{\Pi(Q^\circ)+\lambda b} \quad \ldots\ldots(15)$$

Also, $\quad \pi_s(Q_s^*, w, p) > 0 \Rightarrow (1 - \alpha)\Pi(Q^\circ) - \lambda(\alpha b - b_r) > 0$

$$\Rightarrow \Pi(Q^\circ) + \lambda b_r > (\Pi(Q^\circ) + \lambda b)\alpha \Rightarrow \alpha < \frac{\Pi(Q^\circ)+\lambda b_r}{\Pi(Q^\circ)+\lambda b} \quad \ldots\ldots\ (16)$$

Using equation (13) be can make the condition on p for buyback to coordinate the supply chain.

$$\frac{\lambda b_r}{\Pi(Q°)+\lambda b} < \frac{r-v+b_r-p}{r-v+b} < \frac{\Pi(Q°)+\lambda b_r}{\Pi(Q°)+\lambda b}$$

$$\Rightarrow (r - v + b)\frac{\lambda b_r}{\Pi(Q°)+\lambda b} - (r - v + b_r) <- p < (r - v + b)\frac{\Pi(Q°)+\lambda b_r}{\Pi(Q°)+\lambda b} - (r - v + b_r)$$

$$\Rightarrow r - v + b_r - (r - v + b)\frac{\Pi(Q°)+\lambda b_r}{\Pi(Q°)+\lambda b} < p < r - v + b_r - (r - v + b)\frac{\lambda b_r}{\Pi(Q°)+\lambda b} \quad \text{......(17)}$$

Since $v < r$ this range of p satisfies equation (7) from RHS. To show that this range of p satisfies equation (7) from LHS we need to show $r - v + b_r - (r - v + b)\frac{\Pi(Q°)+\lambda b_r}{\Pi(Q°)+\lambda b} > 0$

$$\Leftrightarrow \frac{r-v+b_r}{r-v+b} > \frac{\Pi(Q°)+\lambda b_r}{\Pi(Q°)+\lambda b} \Leftrightarrow \frac{r-v+b_r}{r-v+b_r-(r-v+b)} < \frac{\Pi(Q°)+\lambda b_r}{\Pi(Q°)+\lambda b_r-(\Pi(Q°)+\lambda b)}$$

$$\Leftrightarrow (r - v + b_r)(b_r - b)\lambda < (\Pi(Q°) + \lambda b_r)(b_r - b) \Leftrightarrow (r - v)\lambda > \Pi(Q°) \quad \text{......(18)}$$

Now $\Pi(Q°)$ is the supply chain profit which takes maximum value when all the products ordered are sold.

$$\Pi(Q°) \leq \lambda(r - c_r - c_s) = \lambda(r - c) < \lambda(r - v) \qquad [\text{since } c > r]$$

So equation (18) is true and makes the argument true. So we get the range of p with $w = w(p)$ under which buyback contract coordinates the supply chain.

Now we will analyze if p does not fall in the range given by equation (17).

<u>**Case 1**</u>: $0 \leq p \leq r - v + b_r - (r - v + b)\frac{\Pi(Q°)+\lambda b_r}{\Pi(Q°)+\lambda b} = p_1$(say)

Now for this p corresponding α takes the range

$$\alpha \geq \frac{\Pi(Q°)+\lambda b_r}{\Pi(Q°)+\lambda b} \qquad [\text{from equation (16)}]$$

$$\Rightarrow \alpha(\Pi(Q°) + \lambda b) \geq \Pi(Q°) + \lambda b_r \Rightarrow \alpha\Pi(Q°) + \lambda(\alpha b - b_r) \geq \Pi(Q°) \Rightarrow \pi_r(Q_r^*, w, p) \geq \Pi(Q°)$$

So here retailer earns more than or equal to the supply chain expected profit and since sum of retailer's profit and supplier's profit has to be $\Pi(Q°)$ so supplier's earn zero or negative profit.

<u>**Case 2**</u>: $p_2 = r - v + b_r - (r - v + b)\frac{\lambda b_r}{\Pi(Q°)+\lambda b} \leq p \leq r - v + b_r$

Corresponding α takes the range

$$\alpha \leq \frac{\lambda b_r}{\Pi(Q°)+\lambda b} \qquad [\text{from equation (15)}]$$

$$\Rightarrow \alpha\Pi(Q°) + \lambda\alpha b \leq \lambda b_r \Rightarrow \alpha\Pi(Q°) + \lambda(\alpha b - b_r) \leq 0 \Rightarrow \pi_r(Q_r^*, w, p) \leq 0$$

So here the retailer earns zero or negative profit and the supplier earns more than or equal to the total expected profit of the supply chain.

3. <u>The Revenue Sharing Contract:</u>

According to this contract ,the retailer pays the supplier a wholesale price of w per unit, and the supplier receives a proportion of the retailer's revenue. The revenue is shared in both the cases: sales as well as salvage. Let us define ϕ as the fraction of revenue that the retailer keeps and $(1 - \phi)$ is the fraction he shares with the supplier. For this contract, the transfer payment $T_r(Q, w, \phi)$ is given by: $T_r(Q, w, \phi) = (w + (1 - \phi)v)Q + (1 - \phi)(r - v)S(Q)$

The retailer's and supplier's expected profit function for this contract can be obtained from Eq. (1) and Eq. (2) respectively. Here π_r and π_s are function of Q, w and ϕ.

Retailer's profit:

$$\pi_r(Q, w, \phi) = (r - v + b_r)S(Q) - (c_r - v)Q - b_r\lambda - wQ - (1 - \phi)rS(Q) - (1 - \phi)v(Q - S(Q))$$

$$= (r - v + b_r + (1 - \phi)(v - r))S(Q) - (c_r - v + w + (1 - \phi)v)Q - b_r\lambda$$

$$= (\phi(r - v) + b_r)S(Q) - (c_r + w - \phi v)Q - b_r\lambda$$

Supplier's profit:

$$\pi_s(Q, w, \phi) = b_sS(Q) - c_sQ - b_s\lambda + wQ + (1 - \phi)rS(Q) + (1 - \phi)v(Q - S(Q))$$

$$=(b_s + (1 - \phi)(r - v))S(Q) - (c_s - w - (1 - \phi)v)Q - b_s\lambda$$

Here also as w and ϕ are contract parameters it is to be decided by the supplier. Let us try to obtain the value of w for a given value of $\phi \in [0, 1]$ so that the supply chain is coordinated.

Retailer's maximum profit occur when $\frac{\partial \pi_r(Q,w,\phi)}{\partial Q}(Q = Q_r^*) = 0$ and $\frac{\partial^2 \pi_r(Q,w,\phi)}{\partial Q^2}(Q = Q_r^*) < 0$

$$\Rightarrow \{r - v + b_r + (1 - \phi)(v - r)\}S'(Q_r^*) - \{c_r - v + w + (1 - \phi)v\} = 0$$

$$\Rightarrow S'(Q_r^*) = \frac{c_r+w-\phi v}{\phi(r-v)+b_r}$$

$$\frac{\partial^2 \pi_r(Q,w,\phi)}{\partial Q^2}(Q = Q_r^*) = (b_s + (1 - \phi)(r - v))S''(Q) =- (b_s + (1 - \phi)(r - v))F'(Q) < 0$$

[since $v < r, S'(Q) = 1 - F(Q)$, $F(Q)$ is increasing function]

Similarly we can get the expression for Q_s^* which maximizes the supplier's profit as

$\frac{\partial \pi_s(Q,w,\phi)}{\partial Q}(Q = Q_s^*) = 0 \Rightarrow S'(Q_s^*) = \frac{c_s-w-(1-\phi)v}{(1-\phi)(r-v)+b_s}$. Now the supply chain is coordinated if $Q_r^* = Q_s^* = Q°$.

$\Rightarrow S'(Q_r^*) = S'(Q_s^*) = S'(Q°)$, on solving both these equations we get same value of $w(\phi)$ as

$$w(\phi) =- c_r + \phi v + (c - v)\frac{\phi(r-v)+b_r}{r-v+b} \quad \ldots(19) \qquad \left[S'(Q°) = \frac{c-v}{r-v+b}\right]$$

Similar to buyback contract we can define $\alpha = \frac{\phi(r-v)+b_r}{r-v+b} \leq 1$ [since $\phi \leq 1, b_r \leq b$]

Using equation (19),

$$\alpha = \frac{-\phi v+c_r+w}{c-v} \geq 0 \qquad [\text{since } v < c_r, \phi \leq 1, c_r \leq c]$$

Then $\pi_r(Q, w, \phi)$ becomes $\pi_r(Q, w, \phi) = \alpha\Pi(Q°) + \lambda(\alpha b - b_r)$ ……(20)

And $\pi_s(Q, w, \phi)$ becomes $\pi_s(Q, w, \phi) = \Pi(Q) - \pi_r(Q, w, \phi) = (1 - \alpha)\Pi(Q°) - \lambda(\alpha b - b_r)$ ……(21)

Now, similar to the buyback contract we make the condition that retailer's and supplier's profit must be positive for making this contract coordinating the supply chain. We obtain that both the person makes positive profit when

$$\frac{\lambda b_r}{\Pi(Q^\circ)+\lambda b} < \alpha < \frac{\Pi(Q^\circ)+\lambda b_r}{\Pi(Q^\circ)+\lambda b}$$

This is the same condition we saw in buyback contracts to make the contract coordinate the supply chain.

If $\alpha = \frac{\lambda b_r}{\Pi(Q^\circ)+\lambda b}$ then supplier earns the entire profit of supply chain i.e. $\pi_s(Q, w, \phi) = \Pi(Q)$. One can verify this by putting this α in equation (21). If $\alpha = \frac{\Pi(Q^\circ)+\lambda b_r}{\Pi(Q^\circ)+\lambda b}$ then retailer earns the entire profit of supply chain i.e. $\pi_r(Q, w, \phi) = \Pi(Q)$. One can verify this by putting this α in equation (20).

Equivalence between buyback contract and revenue sharing contract:

We saw that the buyback contract and revenue sharing contract are similar in the conditions in making the contract coordinates the supply chain. Considering the expression of transfer payment in both the contract, for sake of clarity in wholesale price w of buyback and revenue sharing we denote them by w_p and w_r.

Buyback Contract: $T_p(Q, w_p, p) = (w_p - p)Q + pS(Q)$

Revenue Sharing Contract: $T_r(Q, w_r, \phi) = (w_r + (1 - \phi)v)Q + (1 - \phi)(r - v)S(Q)$

If the expression of transfer payment will be the same then all further analysis will be the same i.e. supply chain profit, retailer's profit, supplier's profit and optimal order quantities. Transfer payment expression must be same for all Q. This can happen only when coefficient of Q and $S(Q)$ are the same in both the expressions.

$$w_p - p = w_r + (1 - \phi)v \quad \text{and} \quad p = (1 - \phi)(r - v)$$

Solving for w_p we get, $w_p - (1 - \phi)(r - v) = w_r + (1 - \phi)v$

$$\Rightarrow w_p = w_r + (1 - \phi)(v + r - v) \Rightarrow w_p = w_r + (1 - \phi)r$$

Hence the condition for the two contract to be equivalent is
$w_p = w_r + (1 - \phi)r$ and $p = (1 - \phi)(r - v)$. However, this is true only for the supply chain with one retailer. When more than one retailer is in the supply chain then the expression of transfer payment will be different and contracts will not be equivalent.

4. The Quantity Flexibility Contract:

According to this contract, the supplier charges a price of w per unit purchased to the retailer and the supplier pays the retailer for the goods which are unsold at the end of period. The difference between 'quantity flexibility' and 'buyback' contract lies in the fact that in the buyback contract the supplier pays *partly* to the retailer for *every* unsold item but in this contract, the supplier pays *fully* to the retailer for *a portion* of his unsold goods. The full price that the retailer bears for every unsold good is given by $\{w + c_r - v\}$. The supplier pays this much price for a portion (say δ) of unsold goods. Hence, the supplier pays the retailer $\{w + c_r - v\} * min\{I, \delta Q\}$, where I refers to the on-hand inventory and $\delta \in [0, 1]$. Please note that here δ is a decision variable. The expression for the transfer payment is:

$$T_q(Q,w,\delta) = wQ - (w + c_r - v)\left[\int_0^{(1-\delta)Q} \delta Q f(x)dx + \int_{(1-\delta)Q}^{Q} (Q - x)f(x)dx\right]$$

which on further solving gives, $T_q(Q,w,\delta) = wQ - (w + c_r - v) \int_{(1-\delta)Q}^{Q} F(x)dx$

$w(\delta) = \frac{(r-v+b_r)\overline{F}(Q°)}{\overline{F}(Q°)+(1-\delta)F((1-\delta)Q°)} - c_r + v$. The retailer's and supplier's expected profit function for this contract can be obtained from Eq. (1) and Eq. (2) respectively. Here π_r and π_s are function of Q, w and δ.

<u>Retailer's profit:</u> $\pi_r(Q,w,\delta) = (r - v + b_r)S(Q) - (c_r - v + w)Q + (w + c_r - v) \int_{(1-\delta)Q}^{Q} F(x)dx$

<u>Supplier's profit:</u> $\pi_s(Q,w,\delta) = b_s S(Q) - (c_s - w)Q - b_s\lambda - (w + c_r - v) \int_{(1-\delta)Q}^{Q} F(x)dx$

Let us try to find the value of $w(\delta)$ that coordinates the supply chain from the retailer's end.

<u>For retailer's end</u>: $\frac{\partial \pi_r(Q,w,\delta)}{\partial Q}(Q = Q_r^*) = 0 \quad \Rightarrow w(\delta) = \frac{(r-v+b_r)\overline{F}(Q_r^*)}{\overline{F}(Q_r^*)+(1-\delta)F((1-\delta)Q_r^*)} - c_r + v$

<u>For supplier's end:</u> $\frac{\partial \pi_s(Q,w,\delta)}{\partial Q}(Q = Q_s^*) = 0 \quad \Rightarrow w(\delta) = \frac{(r-v+b_r)\overline{F}(Q_s^*)}{\overline{F}(Q_s^*)+(1-\delta)F((1-\delta)Q_s^*)} - c_r + v$

Let us check if $\pi_r(Q,w,\delta)$ is a concave function or not. For this we will go for second derivative test:

$$\frac{\partial^2 \pi_r(Q,w,\delta)}{\partial Q^2} =- (r + b_r - w(\delta) - c_r)f(Q) - (w(\delta) - c_r - v)(1 - \delta)^2 f((1 - \delta)Q)$$

The above expression is negative if: $v - c_r \le w(\delta) \le r + b_r - c_r$, ……{valid $\forall\ \delta \in [0, 1]$}

$w(0) = \frac{(r-v+b_r)\overline{F}(Q°)}{\overline{F}(Q°)+F(Q°)} + v - c_r = (r - v + b_r)\ \overline{F}(Q°) + v - c_r \ge v - c_r$

$w(1) = r + b_r - c_r$

Let us calculate the second derivative of $\pi_s(Q,w,\delta)$ and check if Q_s^* maximizes $\pi_s(Q,w,\delta)$:

$\frac{\partial^2 \pi_s(Q,w,\delta)}{\partial Q^2} =- w(\delta)[f(Q) - (1 - \delta)^2 f((1 - \delta)Q] - b_s f(Q)$. This expression is not always negative. We observe that the 'quantity flexibility contract' coordinates the supply chain from the retailer's side but it does not always coordinate from the supplier's side also. The retailer would place an order of size $Q°(=Q_r^*)$ but the supplier would like to deliver an order of different size (say Q') which maximizes his profit. If $Q' > Q°$, then the retailer has a choice whether to accept a larger order size or not. If $Q' < Q°$,the supplier would want to supply an order less than the retailer's order size, but in this case there is a risk of goodwill loss, etc..

This brings us to the idea of a *compliance regime*. The regime is known as *voluntary compliance* when the retailer permits the supplier to deliver less than the order size. while if the retailer forces the supplier to deliver the entire order, the regime is called *forced compliance.*

Loss Aversion:

Loss aversion simply refers to a person who will try to do any such activity in which he/she instead of making profits ends up on a loss. Very easily one can see this loss can be avoided by actually not doing the activity. Let's consider the newsvendor model. Let assume one retailer orders Q amount and his demand be D. Lets see the case when $D \leq Q$.

Retailers Profit: $=-\ Qc + rD + v(Q - D) = Q(v - c) + D(r - v) = D(r - v) - Q(c - v)$

If we have $D < \frac{Q(c-v)}{(r-v)}$, we can see that Retailers Profit would be negative, so the retailers ends up on a loss in this case. A retailer would try to avoid any such case.

Loss-Averse Retailer's Ordering Policy under the Combined Contract:

Decision Step1 : Decision Maker - Distributor/Manufacturer**:** The distributor/manufacturer initially gives to the retailer an offer for a contract with the following details (p, β, w) where p is buyback cost, β is the threshold proportion of Q that retailer must order and w is the wholesale price.

Note: The offer for this contract from the distributor/manufacturer is made before the selling period starts.

Decision Step2 : Decision Maker - Retailer: Given the wholesale price w to the retailer he decides the order quantity Q for the product and informs the same to the distributor/manufacturer. Again this step occurs, before the actual selling season starts and Q comes from some prediction of the retailer. With the information Q, the distributor/manufacturer can start its preparation to be ready to provide the products in the selling season.

Decision Step3 : Decision Maker - Retailer **:**This decision step occurs when the retailer actually receives some information about the demand D. At this stage, the retailer can order within the range $[\beta Q, Q]$, where β was already specified by the distributor/manufacturer. The orders delivery then occurs immediately, which can be used to fulfill customer demands.

Decision Step4 **:** In this step no decision is made, it is just that all the unsold items are returned by the retailer to distributor/manufacturer with the price p.

Special case: If we have $\beta = 1$,combined contract reduces to the buyback contract.

Retailers Profit:

- If $0 \leq D < \beta Q \Rightarrow \Pi(Q, D, w, p, \beta) = (r - p)D - (w - p)\beta Q$
- If $\beta Q \leq D < Q \Rightarrow \Pi(Q, D, w, p, \beta) = (r - w)D$
- If $Q \leq D \Rightarrow \Pi(Q, D, w, p, \beta) = (r - w)Q$

Loss Averse behavior of the retailer: If we have $D < \frac{(w-b)}{(p-b)} \beta Q$, we can see that we have $\Pi(Q, D, w, b, \beta) < 0$, which basically means that the retailer suffers from a loss and thus would try to avoid that situation.

Gain/ Loss Sharing & Buyback:

The consequence of Gain-Loss Sharing Buyback contract for coordination in a supply chain system, with the help of a risk-neutral integrated firm's solution as the benchmark, had been studied. The order size is usually brought down by the retailer's loss aversion and this eventually brings down the total expected supply chain profit. Risk neutral suppliers have competitive pressure upon them to design contract type that shall be in the same faith with the retailer's decision-making behavior while aiming for supply chain profit maximization. According to the GLB contract (w, γ , β, p), the retailer shall pay a unit wholesale price of w to the supplier, the retail merchant shall also be entitled to b ∈ [v, w) from the supplier for every unit that remains at the end of the trading session. Next there is an option for the retailer:

1. To share a proportion (percentage) β ∈ [0, 1) of his profit with the supplier.
2. Or they are reimbursed a fraction γ ∈ [0, 1] by the supplier for the loss incurred by them.

In general practice the parameters r, c, v, and λ are considered exogenous (made available from the outside) and r, v, and λ are available and known to both the supplier and the retailer end. The flow and the series of events in a supply chain with a GLB contract is as follows:- Prior to the trade session, the supplier serves as the supply chain leader, providing the retail merchant a GLB contract (w, γ , β, p).

1. The retailer places an order with the supplier for Q units of the product at the unit wholesale price of w.
2. Before the trading session begins, production takes place and all finished products are given to the retailer.
3. Regular & consistent session demand is realized.
4. Just when the trading session starts, the retailer receives a price p from the supplier for each unit which shall remain unsold till the end of the season, the supplier receives salvage price v for each unit which remains unsold, and the retailer's gain or loss is tallied. The supplier either shares a portion of the merchant's profit or reimburses a portion of the merchant's loss. It has been defined that:

Retailer's breakeven selling quantity function: $q(Q, w, b) = (w - p)Q/(r - p)$

Retailer's expected loss function: $L(X, Q, w, b) = \int_{0}^{q(Q,w,b)} [px + b(Q - x) - wQ]f(x)dx$

Retailer's expected gain function: $\int_{q(Q,w,b)}^{Q} [px + b(Q - x) - wQ]f(x)\,dx + \int_{Q}^{\infty}(p - w)Qf(x)\,dx$

Retailer's expected utility:
$E[U(\pi_r(X, Q, w, \gamma, \beta, b))] = [\lambda(1 - \gamma) - (1 - \beta)]L(X, Q, w, b) + (1 - \beta)E[\pi_r(X, Q, w, b)]$

Retailer's expected profit: $E[\pi_r(X, Q, w, \gamma, \beta, b)] = (\beta - \gamma)L(X, Q, w, b) + (1 - \beta)E[\pi_r(X, Q, w, b)]$
Where $E[\pi_r(X, Q, w, b)] = L(X, Q, w, b) + G(X, Q, w, b)$ denotes the expected profit of the retailer under the

buyback contract. We can write the supplier's predicted profit function as follows, given the retailer's order quantity:

$$E[\pi_m(X, Q, w, \gamma, \beta, b)] = (w - c)Q + \gamma L(X, Q, w, b) + \beta G(X, Q, w, b) - \int_0^Q (b - v)(Q - x)f(x)dx$$

It can be deduced from the above mentioned equations that

$$E[\pi(X, Q)] = E[\pi_r(X, Q, w, \gamma, \beta, b)] + E[\pi_m(X, Q, w, \gamma, \beta, b)]$$

APPENDIX:

Terminology	Description
Demand per period, **X**	probability density function $f(\cdot)$ and distribution function $F(\cdot)$
λ	Mean demand $\boldsymbol{E}$**[X]**
$\boldsymbol{r}$	Selling price
$\boldsymbol{c_r}$	Retailer's cost per unit
$\boldsymbol{c_s}$	Supplier's production cost
$\boldsymbol{c}$	$= \boldsymbol{c_s}+\boldsymbol{c_r}$
$\boldsymbol{b_s}, \boldsymbol{b_r}$	Supplier's, retailer's stock out penalty
$\boldsymbol{b}$	$= \boldsymbol{b_s}+\boldsymbol{b_r}$
$\boldsymbol{v}$	Salvage value
$\boldsymbol{Q}$	Order quantity

Table 3

Here c_r is the cost that the retailer incurs on processing and marketing of product. Since this is one period model, any unmet demand is lost which incur a penalty cost of b_s, b_r to the supplier and the retailer respectively. Unsold product (if any) at the end of period will be salvaged for a salvage value of v per unit. We assume that $c_s + c_r < r$ because otherwise the cost incurred in ordering and supplying the product will be more than revenue obtained by selling and the supply chain system will make no profit. Also $v < c_r$ because otherwise if $v \geq c_r$ the retailer will choose to salvage the product instead of spending on doing marketing, shipping, and processing to sell the product.

REFERENCES:

1. Chapter 14 – "**Supply Chain Contracts**", **Fundamentals of Supply Chain Theory** by Lawerence V. Synder & Zuo-Jun Max Shen.

2. "**Channel coordination for a supply chain with a risk-neutral Manufacturer and a Loss averse Retailer**" – Charles X. Wang & Scott Webster

3. **"Supply Chain Coordination with a Loss averse Retailer and Combined Contract"** – Wei Lui, Shiji Song, Ying Qiao & Han Zhao