

$$x_i: \hat{d}_i = \hat{w}^T x_i + \epsilon \quad [x_i \rightarrow i^{\text{th}} \text{ sample from training}_{xt}]$$

is the equation for linear regression where we know that

$$\epsilon \sim \text{Laplace}(\mu, \sigma)$$

\Rightarrow given input vectors x_n (training data) and labels y_n we have (for size n dataset)

~~$$\epsilon = \hat{d}_i - y_i$$~~

~~ϵ~~ $\ell(w) \rightarrow$ likelihood of w being the right weights

$$\ell(w) = \prod_{i=1}^n p(w_i)$$

$$= \prod_{i=1}^n \frac{1}{2\sigma} \exp\left(-\frac{|\epsilon_i|}{\sigma}\right)$$

$$= \prod_{i=1}^n \frac{1}{2\sigma} \exp\left(-\frac{|d_i - w^T x_i|}{\sigma}\right)$$

$$\log(\ell(w)) = \sum_{i=1}^N -\frac{1}{\sigma} |d_i - w^T x_i| + N \log\left(\frac{1}{2\sigma}\right)$$

$$= - \left[\sum_{i=1}^N \frac{1}{\sigma} |d_i - w^T x_i| + N \log(2\sigma) \right]$$

Ans -

The negative of this log likelihood is minimized, hence $-\log(\ell(w))$ is the desired objective function.