

~~Using~~ Using Bayes' rule, we see that the posterior can be obtained:

$$p(\theta|x) = \frac{\overset{\text{likelihood}}{p(x|\theta)} \overset{\text{prior}}{p(\theta)}}{\underset{\text{posterior}}{p(x)}}$$

Now since $p(x)$ is not available (calculating it as a marginal is intractable), we work with

$$\begin{aligned} p(\theta|x) &\propto p(x|\theta) p(\theta) \\ &= \theta^{\sum x_i} (1-\theta)^{b-\sum x_i} = \theta^{a-1} (1-\theta)^{b-1} \\ &= \theta^a (1-\theta)^{b+\sum x_i-1} \end{aligned}$$

Without the normalizing constant, we can still see that this is a Beta distribution.

If we normalize this, we ~~obtain~~ ^{can see} the parameters. a are $a+1, b+\sum x_i$

$p(\theta|x) \sim \text{Beta}(a+1, b+\sum x_i)$ Ans.

$$p(\theta|x) = \frac{\Gamma'(a+b+\sum x_i+1)}{\Gamma'(a+1)\Gamma'(b+\sum x_i)} \theta^a (1-\theta)^{b+\sum x_i-1}$$