

ASSIGNMENT

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Course Name	Discrete Mathematics-2
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Department	Computer Science and Engineering
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Name of the Student	Shikhar singh
Reg. No	17ETCS002168
Semester/Year	5th/3rd
Course Leader/s	Ms. Pallavi R. Kumar and Mr. Narasimha Murthy K. R.

Declaration Sheet			
Student Name	Shikhar singh		
Reg. No	17ETCS002168		
Programme	B. Tech.	Semester/Year	5 th /3 rd
Course Code	CSC301A		
Course Title	Discrete Mathematics-2		
Course Date		to	
Course Leader	Ms. Pallavi		
<p>Declaration</p> <p>The assignment submitted herewith is a result of my own investigations and that I have conformed to the guidelines against plagiarism as laid out in the Student Handbook. All sections of the text and results, which have been obtained from other sources, are fully referenced. I understand that cheating and plagiarism constitute a breach of University regulations and will be dealt with accordingly.</p>			
Signature of the Student		Date	
Submission date stamp (by Examination & Assessment Section)			
Signature of the Course Leader and date		Signature of the Reviewer and date	

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Solution to question 1:

A. Specification of constraints

In the given problem there are total 200 students where each student must give 3 lab examination (three subjects: Computer Networks, Computer Simulation and Operating Systems). There are two laboratories (103C and 103D) allotted for conducting these examinations. And in each lab, there are total 30 computers and can accommodate three exam slots per day i.e. in a day, total 6 examinations can be conducted.

So, according to this given scenario, following constraints can be made:

1. Total number of batches = $\frac{\text{total number of students}}{\text{number of students per lab}} = \frac{200}{30} \approx 7$

Here, 6 batches will have 30 students per batch and 1 batch will remain with 20 students.

2. total number of labs = 2
3. no. of slots per lab = 3

B. Assumptions

Some of the assumptions are made while specifying the constraints in the solution to problem A. Apart from that, the assumptions to be made for generating the graph conditions are as follows:

Assumption 1: there will be only one examination for a batch in a day.

Assumption 2: 2. Assigning batch 1 to batch 3 students to lab 103C and batch 4 to 7 in lab 103D so as to minimize the shuffling of batches and to minimize the total number of papers to be printed (each lab will have a different set of paper.)

C. Application of Graph coloring method

Suppose B1, B2, B3 have lab exam in the same lab room(103C) where

1. B1, B2 and B3 have a lab exam on same day
2. B3, B1 and B2 have a lab exam on same day
3. B2, B3 and B1 have a lab exam on same day

Similarly, B4, B5, B6 and B7 have lab exam in the same lab room(103D) where

1. B4, B5 and B6 have a lab exam on same day
2. B7, B4 and B5 have a lab exam on same day
3. B6, B7 and B4 have a lab exam on same day

4. B5, B6 and B7 have a lab exam on same day

Now, generating a matrix for the above mentioned conditions and tabularizing it, we get:

Table 1.a: matrix of Batches as per the conditions

Batches	B1	B2	B3	B4	B5	B6	B7
B1	0	1	1	0	0	0	0
B2	1	0	1	0	0	0	0
B3	1	1	0	0	0	0	0
B4	0	0	0	0	1	1	0
B5	0	0	0	0	0	1	1
B6	0	0	0	1	0	0	1
B7	0	0	0	1	1	0	0

Creating a graph from the matrix in the table 1.a,

Graph colouring for examination held in lab room 103C:

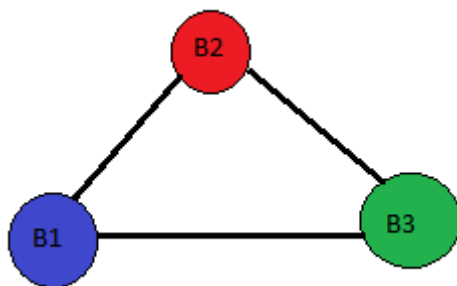


Figure 1.a: graph for lab103C

Here, B1 is assigned one colour. Say Blue, now B1 is adjacent to B2 and B3 hence no vertex can be assigned Blue now.

Therefore, B2 is assigned another colour Red. Now B2 is adjacent to B3 and B1 hence no vertex can be assigned Red now.

Assigning green to B3. As all the other nodes are coloured, no colour can be assigned furthermore.

- **Total number of colour used = 3**

Graph colouring for examination held in lab room 103D :

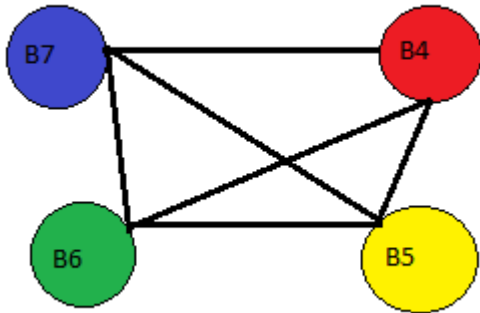


Figure 1.b: graph for lab 103d

Here, B7 is assigned one colour. Say Blue, now B7 is adjacent to B4, B6 and B5 hence no vertex can be assigned Blue now.

Therefore, B4 is assigned another colour Red. Now B4 is adjacent to B6, B7 and B5 hence no vertex can be assigned Red now.

Assigning yellow to B5. Now B5 is adjacent to B6, B7 and B4 hence no vertex can be assigned yellow now.

Assigning green to B6. As all the other nodes are coloured, no colour can be assigned furthermore.

- **Total number of colours used = 4**

Result: - from the figures 1.a and 1.b, it is evident that **103C** room has used total 3 different color so it means Batch 1, 2, and 3 had finished their exam in **3days**. Whereas for **103D** lab room total 4 different colors are used in drawing graph i.e. batch 4, 5, 6, and 7 had finished their lab exam in **4 days**.so, ultimately there are total 4 days in which all the batches will finish their examination optimally.

Tabulating these results, we get:-

For class 103C

Days	Computer networks	Computer simulation	Operating system
D1	B1	B2	B3
D2	B3	B1	B2
D3	B2	B3	B1

For class 103D

Days	Computer networks	Computer simulation	Operating system
D1	B4	B5	B6
D2	B7	B4	B5
D3	B6	B7	B4
D4	B5	B6	B7

D. Time Table Preparation

Now, tabulating the final time table for three laboratory examination of 7 batches (200 students):

Day	Computer networks	Computer simulation	Operating system	Computer networks	Computer simulation	Operating system
	103C			103D		
D1	B1	B2	B3	B4	B5	B6
D2	B3	B1	B2	B7	B4	B5
D3	B2	B3	B1	B6	B7	B4
D4				B5	B6	B7

Solution to question 2:

A. Specification of constraints: -

In the given problem there are total 4 teachers for external examination and their remuneration per day is different also there is availability of teachers i.e. A is available for 3 days, B for 1 days, C for 4 days, D for 2 days. But in the question, it was mention that no external examiner can be invited for more than three days so, teacher C availability will decrease to 3 from 4 days. With the help of that scenario we will generate equation 1, 2, 3 and 4 and with the help of question 1 equation 4 will generated.

$$x_1 \leq 3 \text{ -----eq 1}$$

$$x_2 \leq 1 \text{ -----eq 2}$$

$$x_3 \leq 3 \text{ -----eq 3}$$

$$x_4 \leq 2 \text{ -----eq 4}$$

$$x_1 + x_2 + x_3 + x_4 \geq 7 \text{ -----eq 5}$$

$$\text{and } x_1, x_2, x_3, x_4 \geq 0$$

Thus, total 5 constraints and 4 variables.

B. Assumption: -

Assumption: -

- Assume x_1, x_2, x_3 and x_4 as a number of days for teacher A, B, C and D.
- From the solution to problem 1, it can be assumed that summation of x_1, x_2, x_3 and x_4 is greater or equals to 7. Where 7 is the total number of batches.

C. Formulation and Method of Solution of LPP: -

In the given problem we have to minimize the total remuneration paid to the examiners so, the total remuneration is $P = 2000x_1 + 3400x_2 + 3000x_3 + 2500x_4$

Now,

$$\text{MIN } P = 2000x_1 + 3400x_2 + 3000x_3 + 2500x_4$$

subject to

$$x_1 \leq 3$$

$$x_2 \leq 1$$

$$x_3 \leq 3$$

$$x_4 \leq 2$$

$$x_1 + x_2 + x_3 + x_4 \geq 7$$

$$\text{and } x_1, x_2, x_3, x_4 \geq 0$$

now, in order to produce a maximization problem so that the Simplex Method may then be utilized.

This is the so-called dual problem for standard minimization using the Simplex Method.

So, we goanna transpose the constraint after making standardize minimum constraints by multiple - 1 both the side in equation 1, 2, 3 and 4

$$-x_1 \geq -3$$

$$-x_2 \geq -1$$

$$-x_3 \geq -3$$

$$-x_4 \geq -2$$

$$x_1 + x_2 + x_3 + x_4 \geq 7$$

$$\text{and } x_1, x_2, x_3, x_4 \geq 0$$

now,

transpose:

-1	0	0	0	-3
0	-1	0	1	-1
0	0	-1	0	-3
0	0	0	-1	-2
1	1	1	1	7
2000	3400	3000	2500	

So, after transpose we get,

$$Z = -3x_1 - x_2 - 3x_3 - 2x_4 + 7x_5$$

$$-x_1 + x_5 \leq 2000$$

$$-x_2 + x_5 \leq 3400$$

$$-x_3 + x_5 \leq 3000$$

$$-x_4 + x_5 \leq 2500$$

$$x_1 + x_2 + x_3 + x_4 \leq 7$$

$$\text{and } x_1, x_2, x_3, x_4 \geq 0$$

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropriate.

$$0P - X_1 + X_5 + S_1 + 0S_2 + 0S_3 + 0S_4 + 0S_5 \leq 2000$$

$$0P - X2 + X5 + S1 + 0S2 + 0S3 + 0S4 + 0S5 \leq 3400 \text{ -----i}$$

$$0P - X3 + X5 + S1 + 0S2 + 0S3 + 0S4 + 0S5 \leq 3000 \text{ -----ii}$$

$$0P - X4 + X5 + S1 + 0S2 + 0S3 + 0S4 + 0S5 \leq 2500 \text{ -----iii}$$

$$0P + X1 + X2 + X3 + X4 + 0S1 + 0S2 + 0S3 + 0S4 \leq -7 \text{ -----iv}$$

$$P - 3X1 - X2 - 3X4 - 2X5 + 7X5 + 0S1 + 0S2 + 0S3 + 0S4 + 0S5 = 0 \text{ ---v}$$

Now, solving the equations i, ii, iii, iv and v using simplex method and tabulating the results, we have:

Table 1:

x1	x2	x3	x4	x5	s1	s2	s3	s4	-z	RHS
-1	0	0	0	1	1	0	0	0	0	2000
0	-1	0	0	1	0	1	0	0	0	3400
0	0	-1	0	1	0	0	1	0	0	3000
0	0	0	-1	1	0	0	0	1	0	2500
3	1	3	2	-7	0	0	0	0	1	0

Table 2:

x1	x2	x3	x4	x5	s1	s2	s3	s4	-z	RHS
-1	0	0	0	1	1	0	0	0	0	2000
1	-1	0	0	0	-1	1	0	0	0	1400
1	0	-1	0	0	-1	0	1	0	0	1000
1	0	0	-1	0	-1	0	0	1	0	500
-4	1	3	2	0	7	0	0	0	1	14000

Table 3:

x1	x2	x3	x4	x5	s1	s2	s3	s4	-z	RHS
0	0	0	-1	1	0	0	0	1	0	2500
0	-1	0	1	0	0	1	0	-1	0	900
0	0	-1	1	0	0	0	1	-1	0	500
1	0	0	-1	0	-1	0	0	1	0	500
0	1	3	-2	0	3	0	0	4	1	16000

Table 4:

x1	x2	x3	x4	x5	s1	s2	s3	s4	-z	RHS
0	0	-1	0	1	0	0	1	0	0	3000
0	-1	1	0	0	0	1	-1	0	0	400
0	0	-1	1	0	0	0	1	-1	0	500
1	0	-1	0	0	-1	0	1	0	0	1000
0	1	1	0	0	3	0	2	2	1	17000

Optimal Solution: $z = -17000$ by using transpose

So, actual minimize for the total remuneration paid to the examiners is **Rs.17000**

Where $x_1=3$, $x_2 = 0$, $x_3= 2$, $x_4= 2$

$$P = 2000*3 + 3400*0 + 3000*2 + 2500*2 = ₹17,000$$

Hence, the minimum amount that must be paid in total to external examiners is Rs. 17,000 with the number of days each examiner has to be present being 3, 0, 2, 2 and 1 respectively.

D. Method used to solve the LPP and the reason to choose the method

Method: -

The method we are choosing to solve the above problem is **dual simplex** to produce a maximization problem so that the Simplex Method may then be utilized. This is the so-called dual problem for standard minimization using the Simplex Method.

Reason behind choosing dual simplex method: -

The dual simplex method starts with a super optimal (too good to be true) but infeasible solution and generates a sequence of progressively less infeasible (and less super optimal) ones until it arrives at a feasible solution (which will be optimal).

The main reason of using this method is avoiding the artificial and surplus variable introducing in the constraints, as the constraint is in the form of greater than or equal to ' \geq ' converted into less than or equal to ' \leq '. Also, economic interpretation of dual helps the management in making future decisions.

Bibliography

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