T.I.M.E.
Triumphant Institute of Management Education Pvt. Ltd.

(KEY AND SOLUTIONS FOR AIMCAT0908)

Key

1.	3	8.	4	15.	2	22.	3	29.	1	36.	1	43.	1	50.	3	57.	2	64.	2	71.	3
2.	1	9.	2	16.	2	23.	2	30.	5	37.	2	44.	3	51.	3	58.	4	65.	5	72.	4
3.	2	10.	3	17.	3	24.	4	31.	1	38.	3	45.	1	52.	4	59.	5	66.	1	73.	2
4.	5	11.	4	18.	1	25.	4	32.	3	39.	3	46.	3	53.	1	60.	4	67.	3	74.	5
5.	4	12.	3	19.	4	26.	5	33.	4	40.	4	47.	2	54.	4	61.	4	68.	2	75.	5
6.	3	13.	4	20.	4	27.	4	34.	1	41.	2	48.	1	55.	2	62.	2	69.	1		
7.	2	14.	3	21.	2	28.	2	35.	3	42.	4	49.	5	56.	3	63.	5	70.	1		

Solutions

SECTION – I

Solutions for questions 1 to 5:

1. The returns are the maximum possible, when the company expected to give the highest returns (in rupees) gives double the expected returns, while the two companies which were expected to give the next highest returns (in rupees) gave as expected and the loss was for the company on which the investment was the minimum.

∴ Returns

$= 2 \times \frac{-20}{100} + 6 \times \frac{5}{100} + 10 \times \frac{60}{100} + 8 \times \frac{25}{100} + 4 \times \frac{40}{100}$

$= -0.4 + 0.3 + 6 + 2 + 1.6 =$ Rs.9.5 lakh

∴ Percentage returns $\frac{9.5}{30} \times 100 = 31\frac{2}{3}\%$

Choice (3)

2. We have seen that the maximum return he can make is Rs.9.5 lakh, i.e., $31\frac{2}{3}\%$. As the returns made in this case are $28\frac{2}{3}\%$, i.e., Rs.8.6 lakh, it is very close to the highest possible return. Let us check the higher returns possible if the return on company C is not double the expected return.

$= 2 \times \frac{-20}{100} + 6 \times \frac{5}{100} + 10 \times \frac{30}{100} + 8 \times \frac{50}{100} + 4 \times \frac{40}{100}$

$= -0.4 + 0.3 + 3 + 4 + 1.6 =$ Rs.8.5 lakh

∴ The maximum return is less than that obtained, which means that the return on company C was double that expected.

Now checking different combinations, we can see that a return of Rs.8.6 lakh is obtained when the return on company C is double that expected, and that on company D and E are exactly as expected, and that on company A was half as expected and the 20% loss in investment happened for investment on company B.

Choice (1)

The minimum possible return happens when he loses 20% on the company he invested the maximum amount in, half the expected returns are obtained on company D, and double the expected returns on the company which was expected to give the lowest return (in rupee terms). The returns

$= 2 \times \frac{40}{100} + 6 \times \frac{10}{100} + 10 \times \frac{-20}{100} + 8 \times \frac{12.5}{100} + 4 \times \frac{40}{100}$

$= 0.8 + 0.6 - 2 + 1 + 1.6 = 2$ lakh

Required percentage $\frac{2}{30} \times 100 = 6\frac{2}{3}$

Choice (2)

4. If the loss on one of the companies is exactly equal to the returns on another company, three cases are possible.

Case 1:

Company A gave double the expected return and company E's share price went down by 20%.

Case 2:

Company D's share price fell by 20 % and company E gave exactly the expected returns.

Case 3:

Company C's share price fell by 20% and company D gave exactly the expected returns.

∴ None of the given statements is necessarily true.

Choice (5)

5. For the returns earned from two companies to be equal, the only possibility is that company A gave double the expected returns and company E gave half the expected returns.

The maximum returns in this case would be

$2 \times \frac{40}{100} + 6 \times \frac{(-20)}{100} + 10 \times \frac{30}{100} + 8 \times \frac{25}{100} + 4 \times \frac{20}{100}$

$= 0.8 - 1.2 + 3 + 2 + 0.8 = 5.4$

∴ Required percentage = $\frac{5.4}{30} \times 100 = 18\%$

Choice (4)

Solutions for questions 6 to 10:

6. It is given that player C scored 25% of his total points scored in the tournament in match III and he scored 20% of the total points scored by the team in match III.

∴ As points scored by the players and points scored by C in each match are integers, the least value of the team's score in match III is 100 and the total points scored by C is 80. Now the points scored by him in different matches are

Match I – 12
Match II – 8
Match III – 20
Match IV – 12
Match V – 16
Match VI – 12

Now as he has scored at least 12 points in the first match, the total points scored by the team must be more than that and so the lowest possible value of the teams score in match I is 15 (the next possibility is 20, then 25 and so on).

If the team scored only 15 points in match I, then the team's score in different matches, the points scored by player C and the points scored by the other players would be as follows.

Match	Teams Score	Player C	Other Players
I	15	12	3
II	27	8	19
III	100	20	80
IV	54	12	42
V	36	16	20
VI	18	12	6
Total	250	80	170

As it is given that the share of A, B etc., are 25%, 15% etc., the total points scored by the other players cannot be 170, as it will lead to non-integer values.

Now we cannot increase the teams score in match III, as this will increase the points scored by player C in match III and subsequently other matches, which means that the team's scores in all the matches have to be increased. As seen earlier, the next possible set of points scored by the team in the matches are 20, 36, 72, 48 and 24, in which case we get the following table.

Match	Teams Score	Player C	Other Players
I	20	12	8
II	36	8	28
III	100	20	80
IV	72	12	60
V	48	16	32
VI	24	12	12
Total	300	80	220

In this case the points scored by different players can be integers and so the least points scored by the team in the six matches together is 300. Choice (3)

7. It can be seen that the lowest total points scored by others is 220 points, in which case the player who scored the highest points has to score $\frac{25}{100} \times 220 = 55$ points. But the least points scored by player C are 80 and so that would be the least possible score of the player who scored the highest number of points for the team. Choice (2)

8. Assume that the team scored 100 points in all the matches together, other than match III. Now as 8 % of the team's total points in the tournament were scored in match I, the points scored by the team in all the matches would be 125, i.e., 25 points were scored by the team in match III.

Points scored by player C = $\frac{20}{100} \times 25 \times \frac{100}{25} = 20$ points.

$\therefore$ Points scored by all others = 125 – 20 = 105

$\therefore$ Points scored by D = $\frac{20}{100} \times 105 = 21$

$\therefore$ Required ratio is 21 : 18 = 7 : 6 Choice (4)

9. Let the total points scored by the team in the tournament be 100 points.

$\therefore$ Player C scored 3 points in match IV or $3 \times \frac{100}{15} = 20$ points in the tournament.

$\therefore$ He scored $\frac{25}{100} \times 20 = 5$ points in match III.

$\therefore$ The points scored by the team in match III were $5 \times \frac{100}{20} = 25$

As the total points scored by the team in the tournament were 100, and 25 points were scored in match III, 75 points were scored in all the other matches together or 13.5 points were scored in match II, i.e., 13.5% of the total.

Note that we are getting non-integer values (13.5). For points scored as we have assumed total points in the tournament to be 100, which is not the case and minimum points scored in the tournament would be 400 in this case. Choice (2)

10. Let the total points scored by the team in match IV be 36. Player F scored 9 points and player C scored 6 points in match IV.

$\therefore$ Total points scored by player C in the tournament were 40 and the points scored by the team in match III were 50.

$\therefore$ Total points scored by the team in the tournament = 150 (matches I, II, IV, V and VI) + 50 (Match III).

As player C scored 40 points, points scored by all other players = 150 – 40 = 110

$\therefore$ Points scored by player F = $\frac{25}{100} \times 110 = 27.5$

$\therefore$ Required ratio = 27.5 : 40 = 11 : 16 Choice (3)

Solutions for questions 11 to 15:

11. It is given that the final rating of Chandu is 84.0

The final rating is the simple average of the skill ratings.

$\therefore$ If his skill rating in dialogue delivery is 'x'.

$\frac{86 + 83 + 83.5 + 83 + x}{5} = 84$

$\therefore x = 84.5$

$\therefore \frac{82 + y}{2} = 84.5 \quad \therefore y = 87$ Choice (4)

12. The boys who were rated at least 83 in at least one event in each of the skills tested are Arun and Chandu. Choice (3)

13. The contestants who were offered acting contracts with the channel were Arun, Bala, Chandu and Divya. Among these contestants, the male contestant who had the highest rating in the skill 'Action' was Chandu. Choice (4)

14. The contestants have to choose the event so as to maximise his/her final rating.

∴ If Arun had chosen Acting, his final score would have improved by $\frac{90-86}{5} = 0.8$ and if he had chosen 'Timing' his final rating would here improve by $\frac{9}{2\times5} = 0.9$.

∴ He has to choose 'Timing'.

Similarly Bala has to choose Acting, Chandu and Divya also, Acting, Eshwar should choose 'Body', Fana 'Western Dance' and so on. The improvements made to the final rating of each person would be

Aruna – 0.9
Bala – 1.9
Chandu – 1.4
Divya – 1.2
Eswar – 0.9

And so on, we need not consider the persons ranked lower as their final scores are low.

∴ Chandu would now have the highest final rating.

Choice (3)

15. The present skill rating in Action for the four contestants are Govind – 83.5, Hema – 83.5, Inder – 77 and Jacob – 77.5. Now, if each of them was rated 92, the increase in their final ratings would be 1.7, 1.7, 3.0 and 2.9 respectively.

∴ The final ratings of Govind, Hema, Inder and Jacob would be

Govind – 84.7
Hema – 84.0
Inder – 84.9
Jacob – 84.6

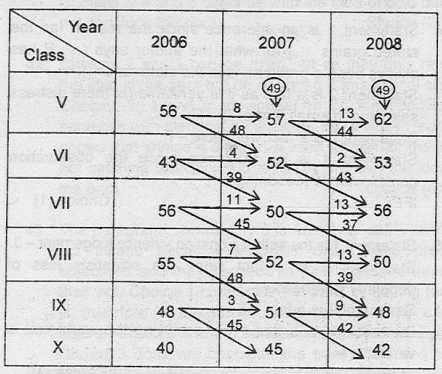
Choice (2)

Solutions for questions 16 to 20:

It is given that the school had a cent percent pass percentage in each of the given years. That is, 40 students passed out of class X in 2006 and 45 students passed out in 2007.

∴ All the students who are in class X in the year 2008 are those who were promoted from class IX.

∴ Of the 51 students in class IX in 2007, only 42 of them got promoted and 9 of them failed. So these 9 stay in class IX in the year 2008 also and so only 39 (48 – 9) students got promoted from class VIII and 52 – 39 = 13 of them had failed. The details of the students in each class in the different years would be as follows.



The table shows the number of students in each year who passed the annual exams and are promoted to the next class and students who failed and stay in the same class. For class V in the years 2007 and 2006, the number of new students who joined is also given.

16. The number of students who failed in the year 2006 is 8 + 4 + 11 + 7 + 3 = 33 Choice (2)

17. In class VII, 11 students out of a total of 56 students failed in the year 2006 and this was the lowest pass percentage. Choice (3)

18. The number of students who joined class V in both 2007 and 2008 is the same, i.e., 49. Choice (1)

19. Total students in all the classes = 307.
Number of students who passed
= 45 + 42 + 39 + 37 + 43 + 44 = 250

Pass percentage = $\frac{250}{307} \times 100 = 81.4\%$

Choice (4)

20. In each of the classes from V to IX, the number of students who failed in 2007 was more than that in 2006. Choice (4)

Solutions for questions 21 to 25:

The scores of the players in the different matches would be as follows (range of scores in case of players whose scores cannot be uniquely determined)

Player / Match	A	B	C	D	E
I	48	8 – 47	8 – 47	57	60
II	24 – 47	64	55	25 – 48	49
III	54	49	20 – 43	44	20 – 43

As no player scored the same number of points in any two matches, the range of scores for player A in match II is 24 – 47 and not 24 – 48.

21. The minimum number of points scored by D in the tournament is 57 + 25 + 44 = 126 Choice (2)

22. The R Index of A (48), B(49) and E(49) can be exactly calculated. Choice (3)

23. As the total number of points in all the three matches together is 670 (220 + 240 + 210) and as there are five players, at least one of the players would have a score less than 135. If one of the players, say C, is assumed to score the minimum points, then all other players would have more than 135 points. Choice (2)

24. The best value (lowest value) of K index for different players are

A – 54 – 47 = 7
B – 64 – 47 = 17
C – 55 – 43 = 12
D – 57 – 44 = 13
E – 60 – 43 = 17

∴ The best value can be for A, i.e., 7. Choice (4)

25. The highest numerical values of K index for A is 30 and that of E is 32 and it can be seen that depending on the scores they made in match I, either B or C would definitely have a numerical value of K index more than 30.
∴ The best possible case is when both B and C have a K index of 32 and they could match that of E.
∴ A would never have the worst value of K index for the tournament. Choice (4)

Difficulty level wise summary - Section I	
Level of Difficulty	Questions
Very Easy	12, 13
Easy	1, 3, 11, 15, 21, 22
Medium	4, 5, 14, 16, 17, 18, 19, 20, 23, 24
Difficult	2, 7, 8, 9, 10, 25
Very Difficult	6

SECTION – II

Solutions for questions 26 to 30:

Number of words and Explanatory notes for RC:

Passage : 933

26. The words in quote occur at the end of para 1. He does not believe his face reflects his true character or at least his true character as he sees it. Choice (5)

27. In the last para the author refers to Hitler and Churchill and says no one saw the streak of insanity in Hitler's face when he rose to power (which we now see). Similarly if we did not know of Churchill's greatness his face could be taken as an evidence of the ill effects of tobacco. In other words what we know of a person helps us to find evidence of it in his face.
Choice (4)

28. Statement 1 is true according to para 4.
Statement 2 is negated by the same para; while heredity determines our look to some extent, our experience and time draw the finer lines.
Statement 3 is true – para 6.
Statement 4 is true – para 3.
Statement 5 is true – last para. Choice (2)

29. Refer to para 2 end, which validates choice 1, that is, it is probably more true than what we take it to be.
Choice (1)

30. Choice 1 can be ruled out since the author does not develop the subject by argument and counter argument.
Choice 2 is ruled out because he does not trace the history in a linear fashion.
Choice 3 is ruled out as he does not treat the subject casually and he has a clear focus (not nebulous).
Choice 4 is ruled out as he is serious and not speaking in a lighter vein.
Choice 5 is true – the author has two or three related opinions regarding the face which he presents with examples. Choice (5)

Solutions for questions 31 to 35:

31. Statement 1 is an inference – sustained lobbying is a fact (may be through newspaper or other media), it has paid off is the author's conclusion – I.
Statement 2 is a fact – F (again media information).
Statement 3 is an inference the governments objective (aimed at boosting ....) is the author's inference based on his knowledge (fact) that jute is the mainstay of the WB economy – I.
Statement 4 is a fact since the focus of the sentence is on the jute industry coming under pressure. The rest of the sentence (seen as being) is an opinion but that is only a modifier – F.
IFIF Choice (1)

32. Statement 1 is an inference the 'if' clause and 'would' suggest that the author concludes what would happens under certain circumstances – I.
Statement 2 is a fact – the author is reporting what someone said which is verifiable and hence a fact – F.
Statement 3 is also a fact since the focus is on providing information although the first part has a bit of opinion (Were petrol pumps employing women? Are they doing so now?) – F.
Statement 4 appears to be an inference (the first part based on the second) but no part of it is a fact. A conclusion based on belief (opinion) hence a judgement – J.
IFFJ. Choice (3)

33. Statement 1 is clearly the author's opinion hence judgement – J.
Statement 2 is a fact that is verifiable (Have parents, teachers, attributed to Harry Potter children's interest in reading?) – F.
Statement 3 is judgement since they express the author's opinion ('not quits so straight forward a success story') – J.
Statement 4 is a fact since it is a statistics – F.
Statement 5 is an inference – the author's conclusion (no doubt....a publishing sensation) is followed by the facts on which it is based. (325 million copies world wide) – I.
JFJFI Choice (4)

34. Statement 1 is an inference since the reason (as the stored grains ....) for what the author says (.... B has always been a problem) is in the statement – I.
Statement 2 is a fact as it is verifiable (Is there distress sale among small farmer?) – F.
Statement 3 is a fact, verifiable – F.
Statement 4 is an inference since the observation comes with its reasoning – I.
IFFI Choice (1)

35. Statement 1 is the author's opinion – hence judgement – J.
Statement 2 is a fact since the scientists loss of popularly is verifiable – F.
Statement 3 is a fact – F.
Statement 4 is a fact – his achieving the world first is verifiable – I.
JFFF Choice (3)

Solutions for questions 36 to 40:

Number of words and Explanatory notes for RC:

Passage : 685

36. The words in quote occur at the beginning of para 2 and refer to what is stated in para 1. Choice 1 is the implied meaning. Choice 5 is not the author's opinion. Choices 2, 3 and 4 are not implied but partly stated.
Choice (1)

37. Refer to para 3 – we ask this question because we believe that everything should be based on reason. Even when we are led by an instinct we want to give it the appearance of rationality. Choice 5 is not the answer because the author is not sure that there is a logical basis for our love of reason. Choice 4 is not the answer because though it is the 'glint of the golden mean' that attracts us to the mean, we want to justify our choice as being reasonable (by giving a reason for it). Hence we are trying to make our instinctual response to appear to be based on reason (or cloaking it in reason).
Choice (2)

38. Option (1) is incorrect as the author hasn't given evidences to say that reason has been used by people of different times. The last para renders option (2) incorrect and option (3) correct, option (4) (5) are not the focus.
Choice (3)

39. Refer to para 2 which supports B and C.
Choice (3)

40. Refer to the last para which supports all the options and negates choice 4.
Choice (4)

Solutions for questions 41 to 45:

41. The paragraph begins by saying that global cooling was a real phenomenon (implying that we do not consider it to be so) and goes on to give an example that shows how it was a real phenomenon that changed world history. Choice 2 concludes the para by giving our present attitude (implied in the opening sentence). Choice 1 appears to be the concluding sentence but it merely reiterates the idea in the first sentence whereas choice 2 strengthens what is said in the first sentences. Choices 3, 4 and 5 continue with the idea of cold but do not conclude the para.
Choice (2)

42. Choices 2 and 3 can be ruled out as they only repeat the idea in the para but do not bring it to a logical close. Choices 1, 4 and 5 all appear to be the concluding sentence with identical beginnings. But a closer reading shows that choice 4 alone is a valid reason for the idea not catching on. Choice 5 is not in line with the idea in the para.
Choice (4)

43. The paragraph shows why it is foolish to call a terrorist act insane because terrorists often have precise objectives and they succeed in their objective through their act. Choice 1 concludes the para by saying that it is therefore a dangerous underestimation to call it insane. Choices 2 and 5 contradict the idea in the para. Choice 3 does not conclude the para. Choice 4 is in keeping with the idea and appears to conclude the para but the para is not about terrorist but our perception of terrorism and hence only choice 1 concludes the idea.
Choice (1)

44. The given para says that it is not difficult to attract butterflies to your garden since all they want is nectar, to feed and a place to lay eggs. Choice 3 concludes the para by saying that if you provide the two, you would succeed in attracting butterflies to your garden. Choices 1 and 2 relate to the idea in the para but do not conclude it choice 4 and 5 are very generalised and hence do not conclude.
Choice (3)

45. The paragraph begins with Indonesia's refusal to share the virus sample and goes on to give the reason for it. Choice 1 concludes the para by going back to the inequities in the system. Choice 5 can be ruled out on the basis of tense. Since the option says 'has devised' it means that the system is already in place which negates what precedes in the paragraph (there is a definite prospect ..... countries that had supplied ..... will have no right). Choice 2, 3 and 4 relate to Indonesia whereas the para has moved on from Indonesia to WHO.
Choice (1)

Solutions for questions 46 to 50:

Number of words and Explanatory notes for RC:

Passage : 901

46. According to the author, the society still patriarchal in nature, is willing at best to grant an 'honorary male' status to an empowered woman (para 3).
Choice (3)

47. Line 8 in para 4 talks of "..... taking the space which is now available ....." so the issue is not of non availability. Further the author states that 'lack of power' is not the malady. It is only the non-availment of powers. Once women avail themselves of the opportunities, power will automatically come.
Choice (2)

48. Refer to lines 6-8 of para 2 of the passage 'Treating empowerment as self-reliance ..... without any societal perspective of the problem'. Hence empowerment through self-reliance is only a egotistic perspective.
Choice (1)

49. The end of in para 2, 'It ignores the extent to which ..... position in society' show that (5) is one correct answer.
Choice (5)

50. Refer to lines 8-9 of para 4, 'Therefore the project of women's empowered ..... opportunity' points to choice 3.
Choice (3)

Difficulty level wise summary - Section II	
Level of Difficulty	Questions
Very Easy	–
Easy	–
Medium	26, 27, 28, 29, 38, 39, 40, 43,
	44, 47, 48, 49, 50
Difficult	30, 31, 32, 34, 35, 36, 41, 42,
	45, 46
Very Difficult	33, 37

SECTION – III

Solutions for questions 51 to 60:

51. $N_1^2 = N_2^2 + N_3^2 + \ldots + N_{100}^2$

If x is divisible by 3, x^2 is also divisible by 3. If x is not divisible by 3, x^2 leaves a remainder of 1, when divided by 3.

If none of the numbers in the RHS are divisible by 3, each would leave a remainder of 1 and the RHS would leave the same remainder as 99, i.e., 0, and thus N_1 would have to be divisible by 3.

[One possible solution for this is N_1 = say 12, and $N_2 = N_3 = \ldots N_{16} = 12$ and $N_{17} = N_{18} = \ldots\ldots = N_{100} = 1$]

$\therefore$ At least one number has to be divisible by 3.

Choice (3)

52. The data is tabulated below.

	Number of pages	Line spacing	$\frac{\text{Text width}}{\text{Page width}}$	Time taken to read	Effective length of text
Preethi's book	150	3	80%	3	(150) (3) (0.8) = 360
Savitha's book	250	4	75%	5	(250) (4) (0.75) = 750

P's reading speed = 120 units, S's is 150 units.

P takes more time to read S's (the longer) book.

After exchanging the books, Savitha will take 5 hours $\times \frac{360}{750}$ = 2 hours 24 minutes, i.e., she will finish the book by 10:24 p.m.

Preethi will take 3 hours $\times \frac{750}{360}$ = 6 hours 15 minutes, i.e., she will finish the book by 2:15 a.m.

Choice (4)

53. $2\log_5 (3^x - 65) = \log_5 (3^x - 17) + \log_5 4$

$\Rightarrow \log_5 (3^x - 65)^2 = \log_5 4(3^x - 17)$

$\Rightarrow (3^x - 65)^2 = 4(3^x - 17) \Rightarrow (3^x)^2 - 134\,(3^x) + 4293 = 0$

$\Rightarrow (3^x - 81)\,(3^x - 53) = 0$

$\Rightarrow 3^x = 81$ or 53

But $\log_5 (3^x - 17)$ and $\log_5 (3^x - 65)$ are defined only when $(3^x - 17)$, $(3^x - 65) > 0$, i.e., when $3^x > 65$.

$\therefore 3^x = 81$

$x = 4$, i.e., the square of an integer. Choice (1)

54. Let the present ages of Minna, Sony and Rohan be m years, s years and r years respectively.

$m + s + r = 90$ ------- (1)

Sony will be as old as Rohan is in $(r - s)$ years.

Rohan's age then = $(r + r - s)$ years = $2r - s$ years.

Minna's age then = $(m + r - s)$ years.

$$\frac{2r - s}{m + r - s - \frac{m}{4}} = \frac{2}{1}$$

$$\Rightarrow s = \frac{3m}{2} \text{ ---------- (2)}$$

Rohan was as old as Sony is, $(r - s)$ years ago.

Sony's age $(r - s)$ years ago = $(2s - r)$ years.

Minna's age then = $m - (r - s)$ years = $(m - r + s)$ years.

$$\frac{2s - r}{m - r + s} = \frac{2}{1}$$

$r = 2m$ ---------- (3)

Also, $m + r + s = 90$

From (1), (2) and (3), $\frac{2}{3}s + 2\left(\frac{2}{3}s\right) + s = 90$

$s = 30$ Choice (4)

55. Let M faces meet at a vertex. If we chop off this corner with a neat cut, we lose a vertex but gain M of them

$\Delta V = M - 1$.

We also introduce M new edges without losing an edge.

$\therefore \Delta E = M$

And we have introduced a new face.

$\therefore \Delta F = 1$

$\therefore \Delta V + \Delta F = \Delta E$. Choice (2)

Note: To obtain the relation between the number of faces, vertices and edges we consider a simple solid (a tetrahedron) F = 4, V = 4, E = 6.

$\therefore$ F + V = E + 2. This is called Euler's Theorem.

$\Rightarrow F_1 + V_1 = E_1 + 2$ (before cutting) and

$F_2 + V_2 = E_2 + 2$ (after cutting)

Hence $\Delta F + \Delta V = \Delta E$.

56. Let the initial speeds of X, Y and Z be x, y and z respectively.

$$x = \frac{y}{2} \text{ and } y = z - \frac{1}{3}z = \frac{2}{3}z$$

$$\therefore x = \frac{1}{3}z \text{ ------- (1)}$$

Total time of travel of x and z, usually = 2 hours.

$\Rightarrow$ Usually z runs for 30 minutes and x runs for 90 minutes. On the day that X runs 24 minutes behind schedule, total time of travel = 96 minutes. X increased its speed by $11\frac{1}{9}$ %. i.e., by $\frac{1}{9}$th of its original speed.

$\therefore$ New time travel of $x = \frac{9}{10}(90) = 81$ minutes. Z must have travelled for 96 – 81 = 15 minutes.

$\therefore z$ doubled its speed (since it usually takes 30 minutes).

$\therefore$ Required ratio = $(z)\,(2) : (x)\left(\frac{10}{9}\right)$

$= (3x)\,(2) : (x)\left(\frac{10}{9}\right) = 27 : 5$. Choice (3)

57. Let the number of apples, bananas and oranges bought be a, b and g.

Given that $a + b + g = 80$; $a \geq 25$, $b \geq 25$, $g \geq 25$

$\Rightarrow 25 \leq (a, b, g) \leq 30$

As the increase in cost per orange by Re.1 and the increase in cost per banana by Rs.4 increases the overall bill by Rs.136, g + 4b = 136

In order to satisfy, the above condition, g must be a multiple of 4. Hence, it has to be 28.

Hence, b is 27 and a is 25.

Hence, Arjun purchased 27 bananas. Choice (2)

58. Let $E = \dfrac{(x-1)(x-3)}{(x-2)(x-4)}$

E > 0	E < 0	E > 0	E < 0	E > 0
x	1	2	3	4

Consider the ranges $(-\alpha, 1)$, $(1, 2)$, $(2, 3)$, $(3, 4)$ and $(4, \alpha)$
The expression is > 0, < 0, > 0, < 0, and > 0 for the respective ranges.
We are looking for a range of values of x in which there is no value which satisfies E > 0.
The range of values we are looking for is a subset of E < 0 i.e. $(1, 2) \cup (3, 4)$

From the given options only $\left(\frac{5}{4}, \frac{5}{3}\right)$ satisfies E < 0.

Choice (4)

59. Given $a = 2b$ and $3c = 4d$
$\Rightarrow a : b = 2 : 1$ and $c : d = 4 : 3$
So, let $a = 2k$, $b = k$, $c = 4l$ and $d = 3l$, where k and l are non-zero.
$abc^2 + 2b^2d^2 = 32k^2l^2 + 18k^2l^2 = 50k^2l^2$
$a^2cd + 2bc^2d = 48k^2l^2 + 96kl^3 = 48kl^2\ (k + 2l)$
Since we do not know the ratio of k and l the ratio of $(abc^2 + 2b^2d^2)$ and $(a^2cd + 2bc^2d)$ cannot be uniquely determined. Choice (5)

60. The second worker (i.e., $i = 2$) joined the first worker after $2^{2-2}x = x$ days, i.e. 1 worker worked for x days.
The third worker joined the team after $2x$ days i.e., 2 workers worked for $2x$ days.
The fourth worker joined the team after $4x$ days, i.e. 3 workers worked for $4x$ days and so on.
The n^{th} worker joined $(2^{n-2})x$ days after the $(n-1)^{th}$ worker joined, i.e. $(n-1)$ workers worked for $2^{n-2}x$ days
The $(n+1)^{th}$ worker would have joined $(2^{n-1})x$ days after the n^{th} worker joined. But just before, the work was completed, and n workers worked for $2^{n-1}x$ days.

The first worker worked for
$x + 2x + 4x + \ldots + 2^{n-2}x + 2^{n-1}x = (2^n - 1)x$ days.....(i)

The entire work (W) is given by
$W = 1(x) + 2(2x) + 3(4x) + \ldots + n(2^{n-1})x$
$\Rightarrow 2W = (2x) + \ldots + (n-1)(2^{n-2})x + n(2^n)x$
$\therefore W = (n2^n - 1)x - (2x + \ldots + 2^{n-1}x)$
$= (n2^n - 1)x - (2^n - 2)x$ ..... (ii)
The first workers share = Rs.4094
The total wage = Rs.40962
$\therefore$ From (i) and (ii)

$$\frac{2^n - 1}{n2^n + 1 - 2^n} = \frac{2^n - 1}{(n-1)2^n + 1} = \frac{4094}{40962}$$

$$= \frac{2047}{20481} = \frac{2^{11} - 1}{10(2^{11}) + 1}$$

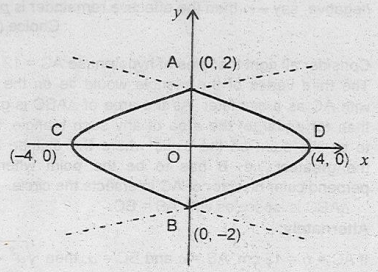
i.e., $\dfrac{2^n - 1}{(n-1)2^n + 1} = \dfrac{2^{11} - 1}{10(2^{11}) + 1}$

Comparing the two sides, we conclude that $n = 11$

Choice (4)

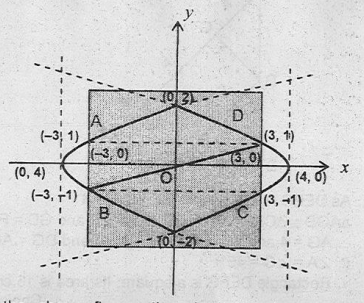
Solutions for questions 61 and 62:

61. The given curve $x = f(y)$ is formed by two parabolas which intersect the y-axis at (0, –2) and (0, 2), one facing to the right and the other facing to the left, as in the figure given below.



We need to find the maximum possible distance between any two points on the curve. $f(y)$ intersects x-axis at (–4, 0) and (4, 0). So clearly the maximum distance between any two points on the curve is CD, i.e., 8 cm. Choice (4)

62. When the original piece of cardboard is of less than 8 cm × 8 cm dimensions, some part of the curve lies out side the cardboard.



In the above figure, the shaded region shows the cardboard. Consider the rectangle ABCD of dimensions 2 cm × 6 cm inscribed in the cut out piece. BD, the diagonal, of length = $\sqrt{2^2 + 6^2} = \sqrt{40}$ cm

Choice (2)

Solutions for questions 63 to 75:

63. Each of the nine friends may either be invited or not invited. Hence 2 ways for each person. That is, a total of 2^9 ways of inviting but to invite at least two friends, we must subtract the following two cases
(i) inviting none – ${}^9C_0$ = one way
(ii) inviting one – ${}^9C_1$ = 9 ways
The number of ways of inviting at least two friends is
$\therefore 2^9 - 1 - 9 = 512 - 10 = 502$ ways. Choice (5)

64. If U is the sum of all the alternate digits of N starting from the units place and T is the sum of all the alternate digits of N starting from the tens place, the remainder when N is divided by 11 is same as the remainder when

U – T is divided by 11. For one group of ten consecutive digits of N, U – T = –5. For the 100 groups, it is –500, i.e. equivalent to a remainder of 6 when divided by 11. Therefore 5 is the least number that has to be added to N, so that the sum is divisible by 11.
[Note: If the remainder when N is divided by p is negative, say $-r$, then the effective remainder is $p - r$.
Choice (2)

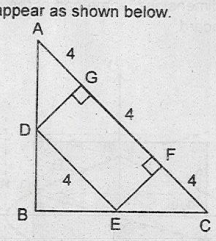
65. Consider all right triangles of hypotenuse AC = 12 cm. The third vertex of the triangle would lie on the circle with AC as a diameter. As the area of ΔABC is greater than (or equal to) the area of any such triangle, B has to be the point on the circle where the altitude BM is the greatest, i.e. B has to be the point where the perpendicular bisector of AC intersects the circle.
∴ ΔABC is isosceles with AB = BC.
Alternately:

If AC = b = 12 cm, AB = c and BC = a, then $\sqrt{a^2+c^2}$ = constant.

To maximise the area $\frac{1}{2} \times a \times c$, $a = c$, i.e., the triangle is isosceles.

Its area is $\frac{1}{2}(12)(6)$ cm^2 = 36 cm^2

As DE is parallel to BC and DE = 4 cm, the scaled figure would appear as shown below.



As DEFG is a rectangle GF = DE = 4
ΔAGD ≡ ΔCFE (∠A = ∠C, ∠G = ∠F and GD = FE)
∴ AG = 4 and FC = 4 and EF = FC and DG = AG
(∵ ∠A = ∠C = 45°)
∴ Rectangle DEFG is a square. Its area is 16 cm^2
Choice (5)

66. The number of players to be selected is 8 out of 6 + 4 + 3 = 13 with a minimum of 2 from each group. Consider the following table of possibilities:

S.no.	A	B	C	Total
1	4	2	2	8
2	3	3	2	8
3	3	2	3	8
4	2	4	2	8
5	2	3	3	8

The number of ways in which the first combination can be obtained is ${}^6C_4\ {}^4C_2\ {}^3C_2 = (15)(6)(3) = 270 \rightarrow (1)$
By similar argument the number of ways for the remaining 4 combinations will work out to 240, 120, 45, 60.
Hence, the total number of ways of selecting the players = 270 + 240 + 120 + 45 + 60 = 735
Choice (1)

67. If $y > 0$, $2^y < 4^y$ and $3^y < 4^y$
∴ $2^y + 2(3^y) < 3(4^y)$
If $y < 0$, $2^y > 4^y$ and $3^y > 4^y$
∴ $2^y + 2(3^y) > 3(4^y)$
If $y = 0$
$2^y + 2(3^y) = 3(4^y)$
Conversely, if $2^y + 2(3^y) > 3(4^y)$, it follows that $y < 0$.

$3x^2 + 2x - 2 < 0 \Rightarrow \frac{-\sqrt{7}-1}{3} < x < \frac{\sqrt{7}-1}{3}$

or (approximately), $\frac{-3.6}{3} < x < \frac{1.6}{3}$ or $-1.2 < x < 0.5$

∴ Among the options, –0.5 is the only possible value of x.
Choice (3)

68. The number of balls in the 9 layers are tabulated below

Layer	Number
1 (Top)	a
2	b
3	$a + b$
4	$2(a + b)$
5	$4(a + b)$
6	$8(a + b)$
7	$16(a + b)$
8	$32(a + b)$
9	$64(a + b)$

∴ The total number of balls = 128 $(a + b)$ = 768 (Given)
∴ $a + b = 6$
∴ Number of balls in 6th and 7th layers is $24(a + b) = 144$

Alternative solution:

Let the number of balls in the 6th layer be x.
i.e. The total number of balls in the top 5 layers is x.
The 7th layer has $2x$ balls.
The 8th layer has $4x$ balls.
The 9th layer has $8x$ balls.
∴ The total number of balls is $16x = 768$ (given)
∴ $x = 48$ The total number of balls in the 6th and 7th layers is $3x = 144$
Choice (2)

69. The average calculated by all the three approaches will be the same.
Hence, A = B = C.
∴ A + 2B = 3C.
Choice (1)

Note: The proof for this can be shown as given below.
Let us assume that there were n students in the class with height $h_1, h_2, h_3, h_4 \ldots h_n$.

$\therefore A = \frac{h_1 + h_2 + h_3 + \ldots\ldots\ldots h_n}{n}$ ------ (1)

Number of all possible pairs from n students will be ${}^nC_2$.
Out of ${}^nC_2$ groups of two students at a time, there will be $(n - 1)$ groups in which every individual student will be present, then

$B = \frac{\frac{h_1+h_2}{2} + \frac{h_2+h_3}{2} + \frac{h_3+h_4}{2} + \ldots\ldots\ldots {}^nC_2 \text{ times}}{{}^nC_2}$

$B = \frac{\frac{(n-1)(h_1+h_2+h_3+\ldots\ldots\ldots h_n)}{2}}{{}^nC_2}$

$$B = \frac{\left\{\frac{(n-1)}{1}\right\}}{\left\{\frac{n(n-1)}{2\times1}\right\}} \quad \frac{(h_1 + h_2 \ldots\ldots h_n)}{2}$$

$$B = \frac{h_1 + h_2 + \ldots\ldots h_n}{n} \quad \text{------- (2)}$$

For triplets, the total number of triplets will be ${}^nC_3$ and out of ${}^nC_3$ triplets on individual will be present in ${}^{n-1}C_2$ groups. Therefore

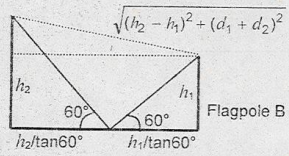
$$C = \frac{\frac{h_1 + h_2 + h_3}{3} + \frac{h_2 + h_3 + h_4}{3} + {}^nC_3 \text{ times}}{{}^nC_3}$$

$$C = \frac{\frac{{}^{n-1}C_2}{3}\{h_1 + h_2 + h_3 + \ldots\ldots h_n\}}{{}^nC_3}$$

$$C = \frac{h_1 + h_2 + \ldots\ldots h_n}{n}$$

$\therefore A = B = C$

70. If there is only point on the ground, from which each flagpole subtends 60°, then the flagpoles must be exactly at a distance equal to the sum of the distances from which they respectively subtend 60° (i.e. when the condition is that the poles are as far away from each other as possible. The other case that is possible is that the poles are at a distance equal to the difference of the distances from which they respectively subtend 60°).

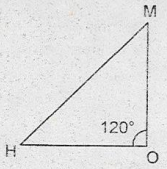


Hence $\frac{15\sqrt{3}}{\tan 60°} + \frac{30\sqrt{3}}{\tan 60°} = 45$ m

The distance between their tops

$= \sqrt{45^2 + (30\sqrt{3} - 15\sqrt{3})^2} = 30\sqrt{3}$ m Choice (1)

71. Let M and H represent the tips of the minute and hours hand.



Length of the hour hand $= \left(1 - \frac{1}{3}\right)(12) = 8$ cm

Angle between the hands of the clock at 8:00 = 120°

If the centre of the clock is O, HM^2

$= HO^2 + OM^2 - 2(HO)(OM)(\cos 120°)$

$= 12^2 + 8^2 - 2(12)(8)\left(-\frac{1}{2}\right) = 304$

$\therefore HM = 4\sqrt{19}$ cm Choice (3)

72. Let us assume that Mohan rented the truck for 8 hours or less. Total amount he should have paid as per the km charge = (9) (36) = Rs.324. But he paid Rs.330.

$\therefore$ Total amount paid by him was as per the hourly charge.

$\therefore$ Duration of his rent $= \frac{330}{60} = 5.5$ hours

Suppose Mohan rented the truck for more than 8 hours. Total amount he should have paid as per the km charge = (9 – 1) (36) = Rs.288. But he paid Rs.330.

$\therefore$ Total amount paid by him was as per the hourly charge.

$\therefore$ Duration of his rent $= \frac{330}{60-16} = 7.5$ hours, which is not possible.

From the above paragraphs, Mohan must have rented the truck for 5.5 hours.

Statement A is clearly true

Consider statements B and C.

If Sohan rented the truck for more than 8 hours, he should have paid.

max [(60 – 16) × (time of rent), (9 – 1) × (36)]

= max [44 × (> 8), 324]

= max [> 352, 288] which is > Rs.352.

If Sohan rented the truck for 8 hours or less, he should have paid max [(60) × (≤ 8), (9) × (36)] = max [≤ 480, 324] which is ≥ 324.

But the minimum possible rent time of Sohan cannot be determined.

$\therefore$ Statement B is true but statement C need not be true.

Consider statement D

It is known that Mohan rented the truck for 5.5 hours. Now, if Sohan rented the truck for less than 5.5 hours, then given the fact that both had driven for 36 km, Sohan would pay less than Mohan. Hence statement D is also necessarily true. Hence, statements A, B and D are necessarily true. Choice (4)

73. $A = 22\ldots\ldots2 = 2(11\ldots\ldots1) = \frac{2}{9}(99\ldots\ldots9) = \frac{2}{9}(10^{12} - 1)$

$B = 555555 = \frac{5}{9}(999999) = \frac{5}{9}(10^6 - 1)$

$\therefore C = \frac{8}{9}(A + B) + 1$

$= \frac{16}{9^2}(10^{12}) + \frac{40}{9^2}(10^6) - \frac{16}{9^2} - \frac{40}{9^2} + \frac{9^2}{9^2}$

$= \frac{16(10^{12}) + 40(10^6) + 25}{81} = \left[\frac{4(10^6) + 5}{9}\right]^2$

$\therefore \sqrt{C} = \frac{4(10^6) + 5}{9} = \frac{4000005}{9} = 444445$

The sum of the digits of $\sqrt{C} = 25$

Alternative solution:

Though the process is lengthy, this problem can also be done by directly calculating C and finding its square root using the long division approach.

Here, A + B = 222222777777 and $C = \frac{8}{9}(A + B) + 1$

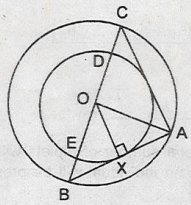
= 197531358025 and $\sqrt{C} = 444445$. Choice (2)

74. We get the highest power of p in $p!$ as p^5, when $p = 5k$, where k is a prime greater than 5, as k repeats for only 5 times and 5 repeats for more than 5 times.
$\therefore$ Here p has 12 possibilities (i.e., up to 5(47)).
If k is a composite number, then the highest power of p in $p!$ is always greater than p^5.
If $p = 2^a \times 3^b$, then for $a = 2$ and $b = 1$, the highest power of p in $p!$ is p^5.
$\therefore$ There are at least 13 possibilities.

Alternative solution:

Let k be the greatest prime factor of p. (Let $k > 5$). The highest power of p in $p!$ is given to be p^5. We need (at least) 5 'k' s among all the numbers upto p.
$\therefore p \geq 5k$. Also $p < 6k$ (if $p = 6k$ or more, k occurs at least 6 times in $p!$ and the other prime factors would occur more times and hence p^6 would also be a factor of $p!$).
But as p itself is a multiple of k, $p = 5k$.
There are 12 numbers of this kind which are less than or equal to 250, namely 5(7), 5(11), 5(13),....5(47)
$\therefore N \geq 12$ Choice (5)

75. $\angle BAC = 90°$ (angle in a semi circle)
$\angle OAB = 45°$ (OA is the bisector)
$\angle OBA = \angle OAB = 45°$
(OB and OA are the radii of the same circle)
If OX (radius) is drawn to the tangent AB at the point of tangency. $\angle OXB = 90°$. OX = 8 cm (radius)



Triangle OBX is an isosceles triangle.
OX = BX = AX = 8 cm

Area of triangle OAB = $\frac{1}{2} \times AB \times OX$

= $\frac{1}{2} \times 16 \times 8 = 64$ sq.cm. Choice (5)

Difficulty level wise summary - Section III	
Level of Difficulty	Questions
Very Easy	–
Easy	59, 63, 68, 69
Medium	52, 54, 55, 56, 57, 58, 64, 70,
	71, 75
Difficult	51, 53, 61, 62, 65, 66, 67, 72, 73
Very Difficult	60, 74