In English | Log ud

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DTU INSIDE

CampusNet / 01034 Matematik 2 E19 / Opgaver

Mat2 Exam E19 Part A English

Side 1

There are 10 questions in total.

- [x] Vis rigtige svar
- [ ] Skjul rigtige svar

Spørgsmål 1

Consider the two infinite series:

$$R=\sum_{n=1}^{\infty}(-1)^n\frac{1+n^3}{4n^3+2n+1}, \qquad \text{and} \quad S=\sum_{n=1}^{\infty}(-1)^n\frac{2}{n+1}.$$

Which statement is true?

- [ ] Both R and S are convergent.
- [ ] Both R and S are divergent.
- [x] R is divergent and S is conditionally convergent.
- [ ] R is divergent and S is absolutely convergent.
- [ ] R is conditionally convergent and S is divergent.

Spørgsmål 2

The radius of convergence ρ for the power series

$$\sum_{n=1}^{\infty}\frac{n^2+\ln(n)}{3^n}x^n$$

is:

- [ ] $\rho=\frac{1}{3}$
- [x] $\rho=3$
- [ ] $\rho=1$
- [ ] $\rho=\infty$
- [ ] $\rho=\frac{1}{2}$
- [ ] $\rho=2$
- [ ] $\rho=0$

Spørgsmål 3

It is given that the characteristic polynomial for a 4th order homogeneous differential equation with constant coefficients is:

$$P(\lambda) = \lambda^2(\lambda^2 - 1).$$

The general real solution is:

- [ ] $y(t) = c_1e^t + c_2e^{-t} + c_3\cos(t) + c_4\sin(t), \quad c_1, c_2, c_3, c_4 \in \mathbb{R}.$
- [ ] $y(t) = c_1 + c_2t + c_3\cos(2t) + c_4\sin(2t), \quad c_1, c_2, c_3, c_4 \in \mathbb{R}.$
- [ ] $y(t) = c_1 + c_2e^t + c_3e^{-t}, \quad c_1, c_2, c_3 \in \mathbb{R}.$
- [ ] $y(t) = c_1 + c_2\cos(t) + c_3\sin(t), \quad c_1, c_2, c_3 \in \mathbb{R}.$
- [x] $y(t) = c_1 + c_2t + c_3e^t + c_4e^{-t}, \quad c_1, c_2, c_3, c_4 \in \mathbb{R}.$
- [ ] $y(t) = c_1 + c_2t + c_3e^t + c_4te^t$

Spørgsmål 4

Consider a pair of infinite series with positive terms,

$$R = \sum_{n=1}^{\infty} a_n$$

and

$$S = \sum_{n=1}^{\infty} b_n$$

where $a_n > 0,\, b_n > 0 \quad \text{for all } n \in \mathbb{N}$, and the sequence

$$c_n = \frac{1}{n},\ n \in \mathbb{N}.$$

It is given that $\sqrt{a_n} \leq b_n$ for all n, and that

$$\lim_{n\to\infty} \frac{b_n}{c_n} = 1.$$

Choose the statement that is correct:

- [ ] R and S are both divergent.
- [x] S is divergent and R is convergent.
- [ ] Both series are convergent.
- [ ] R is divergent and S is convergent.

Spørgsmål 5

Consider the differential equation

$y'(t) - t^3 y(t) = 0$

By setting:

$y(t) = \sum_{n=0}^{\infty} c_n t^n$

we can rewrite the differential equation as one of the following equations. Choose the correct one:

- [ ] $\sum_{n=0}^{\infty} (c_{n+1}(n+1) - c_{n+3})\, t^n = 0$
- [ ] $c_1 + 2c_2 t + 3c_3 t + \sum_{n=-3}^{\infty} (c_{n+1}(n+1) - c_{n+3})\, t^n = 0$
- [x] $c_1 + 2c_2 t + 3c_3 t^2 + \sum_{n=3}^{\infty} (c_{n+1}(n+1) - c_{n-3})\, t^n = 0$
- [ ] $c_1 + 2c_2 + 3c_3 + \sum_{n=3}^{\infty} (c_{n+1}(n+1) - c_{n-3})\, t^n = 0$
- [ ] $c_1 + 2c_2 t + 3c_3 t^2 + \sum_{n=3}^{\infty} (c_{n-1}(n-1) - c_{n+3})\, t^n = 0$
- [ ] $c_1 + 2c_2 t + 3c_3 t^2 + \sum_{n=3}^{\infty} (c_{n-1}(n-1) - c_{n-3})\, t^n = 0$

Side 2

Spørgsmål 6

Consider the series:

$$\sum_{n=0}^{\infty}(1-x)^n$$

On which of the following x-intervals is the series uniformly convergent?

- [ ] $-3/4 \leq x \leq -1/4$
- [ ] $-3 \leq x \leq 3$
- [ ] $-1 \leq x \leq 1$
- [ ] $-1/2 < x < 1/2$
- [x] $1/4 \leq x \leq 3/4$
- [ ] $0 < x < 1$

Spørgsmål 7

For the first order system of differential equations

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t)$$

it is given that the matrix

$$\mathbf{A} \in \mathbb{R}^{3\times 3}$$

has the eigenvalues:

$$\lambda_1 = -1 - a, \quad \lambda_2 = a$$

where the number $a \in \mathbb{R}$ is a real parameter.

For $a \neq -1/2$ it is also given that: λ_1 has algebraic multiplicity 1, while λ_2 has algebraic multiplicity 2 and geometric multiplicity 1.

For which values of the parameter a is the system stable?

- [ ] $-1 \leq a \leq 0$
- [x] $-1 \leq a < 0$
- [ ] $a < 0$
- [ ] $a > 1$
- [ ] $-\infty < a < \infty$
- [ ] There are no values of a for which the system is stable.

Spørgsmål 8

Let

$f : \mathbb{R} \to \mathbb{R}$

be an even and 2π-periodic function, given on the interval $[0, \pi]$ by the formula:

$$f(x) = \begin{cases} x & \text{for } x \in [0, 1[, \\ 3/2 & \text{for } x = 1, \\ 2 & \text{for } x \in]1, \pi]. \end{cases}$$

Which of the following statements about the Fourier series of f is valid?

- [x] The Fourier series converges pointwise, but not uniformly, to f.
- [ ] The Fourier series converges uniformly to f.
- [ ] The Fourier series converges pointwise, but not to f.
- [ ] The Fourier series converges uniformly, but not to f.
- [ ] There is at least one point x at which the Fourier series is divergent.

Spørgsmål 9

Consider the pair of infinite series R and S with non-constant terms given by:

$$R(x) = \sum_{n=1}^{\infty} \frac{\cos(nx)}{n}, \quad x \in \mathbb{R},$$ and $$S(x) = \sum_{n=1}^{\infty} \frac{\cos(n^2 x)}{n^2}, \quad x \in \mathbb{R}.$$

Select the statement that is valid:

- [ ] Both series have convergent majorant series.
- [ ] Both series have majorant series, but neither has a convergent majorant.
- [x] Both series have majorant series, but only S has a convergent majorant.
- [ ] Neither series has a majorant series.
- [ ] Both series have majorant series, but only R has a convergent majorant.

Spørgsmål 10

For a third order homogeneous differential equation with initial conditions:

$y(0) = 1, \quad y'(0) = 0, \quad y''(0) = 0,$

it is given that the power series method leads to the recursion formula:

$$c_{n+3} = \frac{c_n}{(n+3)(n+2)}, \quad n \geq 0,$$

for the power series solution

$$y(x) = \sum_{n=0}^{\infty} c_n x^n.$$

The 6th partial sum of y(x),

$$S_6(x) = \sum_{n=0}^{6} c_n x^n$$

is then given by:

- [x] $S_6(x) = 1 + \frac{1}{6}x^3 + \frac{1}{180}x^6$
- [ ] $S_6(x) = \frac{1}{6}x^3 + \frac{1}{180}x^6$
- [ ] $S_6(x) = x + \frac{1}{12}x^4$
- [ ] $S_6(x) = \frac{1}{2}x^2 + \frac{1}{40}x^5$
- [ ] $S_6(x) = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{12}x^4 + \frac{1}{40}x^5 + \frac{1}{180}x^6$

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Problem 1:
(1) $y(t) = c_1 e^{(1+i)t} + c_2 e^{(1-i)t}$, $c_1, c_2 \in \mathbb{C}$, $y(t) = c_1 e^t \cos t + c_2 e^t \sin t$, $c_1, c_2 \in \mathbb{R}$
(2) —
(3) $H(s) = \frac{1}{s^2-2s+2}$, $s \neq 1 \pm i$. $y(t) = \sum_{n=-N}^{N} \frac{c_n}{-n^2-2in+2} e^{int}$.
(4) $y(t) = c_1 e^{(1+i)t} + c_2 e^{(1-i)t} + \frac{1+t}{2} + \sum_{n=-N}^{N} \frac{c_n}{-n^2-2in+2} e^{int}$, $c_1, c_2 \in \mathbb{C}$.

Problem 2:
(i) Funktionen er ulige så alle $a_n = 0$. $b_2 = \frac{1}{2}$ og for alle andre n: $b_n = -\frac{4}{\pi} \frac{\sin\left(\frac{\pi n}{2}\right)}{n^2-4}$. Specielt, for alle $n = 2m \neq 2$: $b_{2m} = 0$.
(ii) $N \geq 255$.
(iii) Ja. F.eks. vha korollar 6.13.