

TECHNICAL UNIVERSITY OF DENMARK

Written 3 hour exam, 7. December 2020

Course: Mathematics 2

01034/01035/01037

Allowed aids: All aids allowed by DTU.

Weights: Multiple-choice (electronic): 65%, Problem 1: 20%, Problem 2: 15%.

The weights are only guidelines. The exam is evaluated as a whole. In order to get full points in part B, you need to include intermediate calculations to a reasonable extent. Furthermore, all answers must be substantiated, if necessary with references to the text-book.

NB. The exam consists of two parts: **Part A**, electronic multiple-choice; **Part B**, see below.

- **To answer Part A:** Open the exam assignment “Mat2 Exam E20 Part A” and follow the instructions there.
- **Part B is included below.** Upload your answers to part B as a PDF file. (If your answers are hand-written, part B may be handed in on paper instead).

Part B

Problem 1

Consider the infinite series

$$f(t) = \sum_{n=1}^{\infty} \frac{\sin(nt)}{2^n}.$$

Explain your reasoning for each of your answers to the following questions:

1. Show that the series $f(t)$ converges for every $t \in \mathbb{R}$.
2. Is f an even function, an odd function, or neither?
3. Does the series converge uniformly on \mathbb{R} ?
4. Is f continuous?
5. Determine an $N \in \mathbb{N}$ such that for all $t \in \mathbb{R}$ the following inequality holds:

$$\left| f(t) - \sum_{n=1}^N \frac{\sin(nt)}{2^n} \right| \leq 10^{-3}.$$

6. Find the value of $\int_0^{2\pi} |f(t)|^2 dt$.

The exam set continues - Turn!

Problem 2

Consider the system of differential equations

$$\begin{aligned}x_1'(t) &= 5x_1(t) - x_2(t) + e^t, \\x_2'(t) &= 4x_1(t) + x_2(t).\end{aligned}\tag{1}$$

1. Show that the vector-valued function

$$\mathbf{x}(t) = \begin{pmatrix} te^{3t} \\ 2te^{3t} - e^{3t} \end{pmatrix}$$

is a solution to the corresponding *homogeneous* system.

2. Find the general real solution to the homogeneous system.
3. Find the general real solution to (1).

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End of Part B. Remember to answer Part A (Multiple Choice).