CSC_5RO11_TA

Bayesian Network

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Notation

To simplify the documentation, the following abbreviations will be used :

— **B** for Bronchitis
— **C** for Cancer
— **T** for Tuberculosis
— **S** for Stethoscope Test
— **X** for X-Ray Test
— + for Positive Result
— − for Negative Result

Question 1

Bayesian Network Diagram

The Bayesian network for this problem can be visualized as shown below :

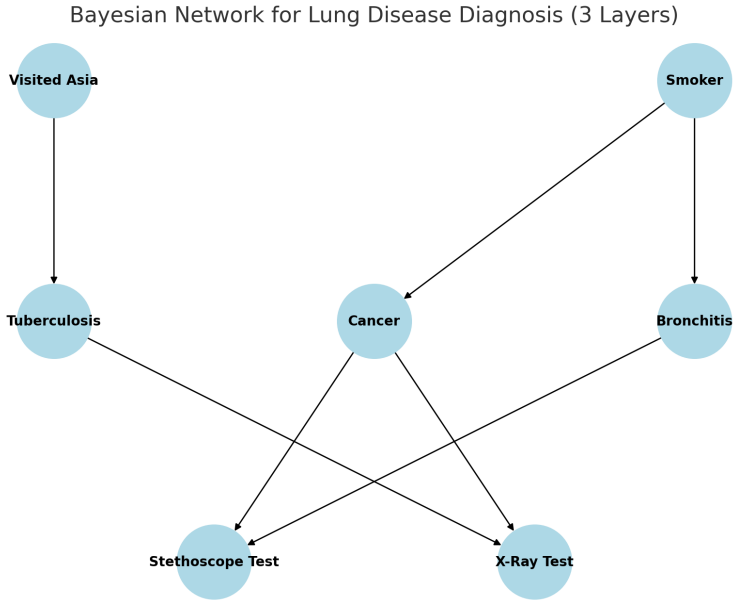

FIGURE 1 – Bayesian Network for Lung Disease Diagnosis

The network has three layers :

— **Layer 1 :** Questionnaires variables "Visited Asia" and "Smoker".
— **Layer 2 :** Disease variables "T", "C", and "B".
— **Layer 3 :** Test variables "S" and "X"

Conditional Probabilities

The conditional probabilities for each node in the network are as follows :

— **Prior Probabilities :**

$$P(\text{Visited Asia} = +) = 0.1,$$
$$P(\text{Smoker} = +) = 0.3.$$

— **T** : The probability of tuberculosis depends on whether the patient has recently visited Asia.

$$P(\text{T} = +|\text{Visited Asia} = +) = 0.1,$$
$$P(\text{T} = +|\text{Visited Asia} = -) = 0.01.$$

— **C** : The probability of cancer depends on whether the patient is a smoker.

$$P(\text{C} = +|\text{Smoker} = +) = 0.2,$$
$$P(\text{C} = +|\text{Smoker} = -) = 0.02.$$

— **B** : The probability of bronchitis also depends on whether the patient is a smoker.

$$P(\text{B} = +|\text{Smoker} = -) = 0.8,$$
$$P(\text{B} = +|\text{Smoker} = +) = 0.6.$$

— **S** : The result of the stethoscope test depends on whether the patient has bronchitis or cancer.

$$P(\text{S} = +|\text{B} = + \text{ or C} = +) = 0.6,$$
$$P(\text{S} = -|\text{B} = - \text{ and C} = -) = 0.99.$$

— **X** : The result of the X-ray test depends on whether the patient has tuberculosis or cancer.

$$P(\text{X} = +|\text{T} = + \text{ or C} = +) = 0.7,$$
$$P(\text{X} = -|\text{T} = - \text{ and C} = -) = 0.98.$$

Question 2

Given the conditions :

— The patient is **not a smoker**

— The patient has **not visited Asia**

Calculation of Probabilities for Each Disease :

1. **T** : Since the patient has not visited Asia, the probability of tuberculosis is given by :

$$P(\text{T} = +|\text{Visited Asia} = -) = 0.01.$$

2. **C** : Since the patient is not a smoker, the probability of cancer is :

$$P(\text{C} = +|\text{Smoker} = -) = 0.02.$$

3. **B** : For a non-smoker, the probability of bronchitis is :

$$P(\text{B} = +|\text{Smoker} = -) = 0.8.$$

Inference

Given these probabilities, we see that :

$$P(\text{B} = +|\text{Smoker} = -, \text{Visited Asia} = -) = 0.8$$

is significantly higher than the probabilities for tuberculosis (1%) and cancer (2%). Therefore, we can infer that the most likely diagnosis for this patient is **bronchitis**.

Question 3

Based on the inference in Question 2, bronchitis is the most likely diagnosis, with a probability of 80%. The doctor decides to use a stethoscope to confirm this diagnosis. The stethoscope test has a 60% probability of detecting bronchitis if it is present. A positive result would increase the confidence in the bronchitis diagnosis, while a negative result would indicate the need for a more refined assessment of the patient's condition.
Given that the stethoscope test result is negative, we need to update the probability of bronchitis based on this new information.
The patient is a **non-smoker** and has **not visited Asia**. Based on this information, the initial probabilities for each disease are as follows :

— **B** : $P(\text{B} = +) = 0.8$

— **C** : $P(\text{C} = +) = 0.02$

The stethoscope test result is **negative**, which impacts the probability of bronchitis and cancer, as the stethoscope test has a 60% probability of detecting these conditions if they are present. Given a negative test result :

— The probability of a negative result if the patient has bronchitis is $P(\text{S} = -|\text{B} = +) = 0.4$.

— The probability of a negative result if the patient has cancer is $P(\text{S} = -|\text{C} = +) = 0.4$.

— The probability of a negative result if the patient has both bronchitis and cancer (assuming independence) is $P(\text{S} = -|\text{B} = + \text{ and } \text{C} = +) = 0.4 \times 0.4 = 0.16$.

— The probability of a negative result if the patient has neither bronchitis nor cancer is $P(\text{S} = -|\text{B} = - \text{ and } \text{C} = -) = 0.99$.

To update the probability of bronchitis, we first calculate the total probability of observing a negative stethoscope test result, $P(\mathrm{S}=-)$, by considering all possible conditions :

$$\begin{aligned} P(\mathrm{S}=-) = {} & P(\mathrm{S}=-|\mathrm{B}=+ \text{ and } \mathrm{C}=-)\cdot P(\mathrm{B}=+ \text{ and } \mathrm{C}=-) \\ & + P(\mathrm{S}=-|\mathrm{B}=- \text{ and } \mathrm{C}=+)\cdot P(\mathrm{B}=- \text{ and } \mathrm{C}=+) \\ & + P(\mathrm{S}=-|\mathrm{B}=+ \text{ and } \mathrm{C}=+)\cdot P(\mathrm{B}=+ \text{ and } \mathrm{C}=+) \\ & + P(\mathrm{S}=-|\mathrm{B}=- \text{ and } \mathrm{C}=-)\cdot P(\mathrm{B}=- \text{ and } \mathrm{C}=-). \end{aligned}$$

Substituting the known values :

$$\begin{aligned} P(\mathrm{S}=-) &= (0.4\times 0.784) + (0.4\times 0.004) + (0.16\times 0.016) + (0.99\times 0.196) \\ &= 0.3136 + 0.0016 + 0.00256 + 0.19404 = 0.5118. \end{aligned}$$

Updated Probability of Bronchitis

$$P(\mathrm{B}=+|\mathrm{S}=-) = \frac{P(\mathrm{S}=-|\mathrm{B}=+)\cdot P(\mathrm{B}=+)}{P(\mathrm{S}=-)}.$$

Substituting the values :

$$P(\mathrm{B}=+|\mathrm{S}=-) = \frac{(0.4\times 0.784) + (0.16\times 0.016)}{0.5118} = \frac{0.31616}{0.5118} \approx 0.6177.$$

After observing a negative stethoscope test result, the probability of bronchitis decreases from 0.8 to approximately 0.6177. This reduction reflects the decreased likelihood of bronchitis given the negative test outcome.
We can also update the probability of cancer given the negative stethoscope test result :

$$P(\mathrm{C}=+|\mathrm{S}=-) = \frac{P(\mathrm{S}=-|\mathrm{C}=+)\cdot P(\mathrm{C}=+)}{P(\mathrm{S}=-)}$$

Substituting these values :

$$P(\mathrm{C}=+|\mathrm{S}=-) = \frac{0.4\times 0.02}{0.5118} = \frac{0.008}{0.5118} \approx 0.0156$$

Thus, the probability of cancer after the negative stethoscope test result is updated to approximately 0.0156.

Question 4

Given that the X-ray test result ($\mathbf{X}$) is positive, we will update the probabilities for each disease based on this new information. The X-ray test is effective at detecting $\mathbf{T}$ (Tuberculosis) and $\mathbf{C}$ (Cancer), with a 70% probability of a positive result if either of these diseases is present.
From the previous steps, we have the following updated probabilities after the negative stethoscope test result :

— **C** : $P(\text{C} = +) \approx 0.0156$

— **T** : $P(\text{T} = +) = 0.01$

To calculate the updated probabilities, we first find the total probability of a positive X-ray test, $P(\text{X} = +)$, considering all possible combinations of tuberculosis and cancer :

$$\begin{aligned} P(\text{X} = +) = P(\text{X} = +|P(\text{T} = + \text{ and C} = -) \cdot P(\text{T} = + \text{ and C} = -) \\ + P(\text{X} = +|P(\text{C} = + \text{ and T} = -) \cdot P(\text{C} = + \text{ and T} = -) \\ + P(\text{X} = +|\text{T} = + \text{ and C} = +) \cdot P(\text{T} = + \text{ and C} = +) \\ + P(\text{X} = +|\text{T} = - \text{ and C} = -) \cdot P(\text{T} = - \text{ and C} = -). \end{aligned}$$

We calculate the joint probabilities of the disease combinations :

$$P(\text{Tuberculosis only}) = P(T = +) \times (1 - P(C = +)) = 0.01 \times (1 - 0.0156) = 0.01 \times 0.9844 = 0.009844$$

$$P(\text{Cancer only}) = P(C = +) \times (1 - P(T = +)) = 0.0156 \times (1 - 0.01) = 0.0156 \times 0.99 = 0.015444$$

$$P(\text{Tuberculosis and Cancer}) = P(T = +) \times P(C = +) = 0.01 \times 0.0156 = 0.000156$$

$$P(\text{No Tuberculosis and No Cancer}) = 1 - 0.009844 - 0.015444 - 0.000156 = 0.974556$$

Substitute the joint probabilities into the total probability formula :

$$P(X = +) = (0.7 \times 0.009844) + (0.7 \times 0.015444) + (0.49 \times 0.000156) + (0.02 \times 0.974556) = 0.03726914$$

$$P(C = +|X = +) = \frac{P(X = +|C = +) \cdot P(C = +)}{P(X = +)}$$

Substituting the values :

$$P(C = +|X = +) = \frac{0.7 \times 0.0156}{0.03726914} = \frac{0.01092}{0.03726914} \approx 0.2937$$

Similarly, update the probability of tuberculosis :

$$P(T = +|X = +) = \frac{P(X = +|T = +) \cdot P(T = +)}{P(X = +)}$$

Substituting the values :

$$P(T = +|X = +) = \frac{0.7 \times 0.01}{0.03726914} = \frac{0.007}{0.03726914} \approx 0.1873$$

After the positive X-ray test result, the updated probabilities for each disease are :

— **Cancer (C)** : 0.2937

— **Tuberculosis (T)** : 0.1873

— **Bronchitis (B)** : 0.6177

Based on this updated information, cancer has become more probable than before, and tuberculosis has also increased in probability due to the positive X-ray result. However, bronchitis still has the highest probability, though the probabilities of both cancer and tuberculosis have risen significantly, making them potential diagnoses to consider alongside bronchitis.

Question5

After the stethoscope test was negative, bronchitis remained the most likely diagnosis, with a probability of approximately 61.77%. The probabilities for cancer and tuberculosis were relatively low, at 1.56% and 1%, respectively.
After the X-ray test was positive, the probabilities for cancer and tuberculosis increased significantly to 29.37% and 18.73%, respectively, indicating a notable increase in the likelihood of either cancer or tuberculosis.
Therefore, the X-ray test was indeed necessary, as it significantly raised the probability of both cancer and tuberculosis. This additional information led the doctor to reconsider diagnoses beyond bronchitis, especially since bronchitis would not affect the X-ray result, while the probabilities of cancer and tuberculosis increased significantly.