Homework 1, Computational Complexity 2024

The deadline is 23:59 on Wednesday 9 October. Please submit your solutions on Moodle. Typing your solutions using LaTeX is strongly encouraged. The problems are meant to be worked on in groups of 2–3 students. Please submit only one writeup per team. You are strongly encouraged to solve these problems by yourself. If you must, you may use books or online resources to help solve homework problems, but you must credit all such sources in your writeup and you must never copy material verbatim.

1 A *decisive* nondeterministic Turing machine is one where each nondeterministic computation path outputs either *yes*, *no*, or *maybe*. We say such a machine decides a language L if the following holds: If $x \in L$, then all computations end up with *yes* or *maybe*, and at least one *yes*. If $x \notin L$, then all computations end up with *no* or *maybe*, and at least one *no*. Show that L is decided by a decisive polytime NTM if and only if $L \in \mathsf{NP} \cap \mathsf{coNP}$.

2 Show that the following *distinct-3SAT* problem is NP-complete:

$$\text{D3SAT} = \{\langle\varphi\rangle : \varphi \in \text{3SAT} \text{ and each clause of } \varphi \text{ involves three distinct variables}\}.$$

(For example, $(x \vee \overline{y} \vee \overline{z}) \wedge (\overline{x} \vee y \vee \overline{w}) \in \text{D3SAT}$, whereas $(x \vee \overline{y} \vee \overline{z}) \wedge (\overline{x} \vee y \vee \overline{y}) \notin \text{D3SAT}$ since the clause $(\overline{x} \vee y \vee \overline{y})$ involves only two distinct variables.)

3 Let $G = (V, E)$ be an undirected graph. We say that a vertex set $K \subseteq V$ is a *kernel* iff (i) for any two $v, u \in K$ we have $\{v, u\} \notin E$, and (ii) for every $u \in V \setminus K$ there is $v \in K$ such that $\{u, v\} \in E$. In other words, a kernel is a set that is both (i) independent and (ii) dominating*. Show that the following problem is NP-complete:

$$\textsc{Kernel} = \{\langle G, k\rangle : G \text{ has a kernel of size at most } k\}.$$

(You can use any NP*-complete problem discussed in class/exercises in your reductions.)*

*https://en.wikipedia.org/wiki/Dominating_set