Problem Statement 1:

A company manufactures LED bulbs with a faulty rate of 30%. If I randomly select 6 chosen LEDs, what is the probability of having 2 faulty LEDs in my sample? Calculate the average value of this process. Also evaluate the standard deviation associated with it.

Solution 1: Let X be the random variable representing faulty bulbs in a sample

a) $X \sim Bin(6, 0.3)$, therefore the probability is given by

$$P(X = 2) = {n \choose x} * p^x * (1 - p)^{n - x}$$
$$= {6 \choose 2} * 0.3^2 * (1 - 0.3)^{6 - 2}$$
$$= 15 * 0.09 * 0.2401 = 0.324135$$

- b) The average value is given by E(X) = n p = 6 * 0.3 = 1.8
- c) The standard deviation associated with it is given by $s = \sqrt{Var(X)} = n \ p(1-p)$ $= \sqrt{6*0.3*(1-0.3)}$ $= \sqrt{1.8*0.7} = 1.122 \ (to 3dp)$

Problem Statement 2

Gaurav and Barakha are both preparing for entrance exams. Gaurav attempts to solve 8 questions per day with a correction rate of 75%, while Barakha averages around 12 questions per day with a correction rate of 45%. What is the probability that each of them will solve 5 questions correctly? What happens in cases of 4 and 6 correct solutions? What do you infer from it? What are the two main governing factors affecting their ability to solve questions correctly? Give a pictorial representation of the same to validate your answer.

Solution 2:

a) $G \sim Bin(8, 0.75)$, therefore the probability is given by

$$P(G will get 5 correct) = {8 \choose 5} * p^{x} * (1-p)^{n-x}$$
$$= {8 \choose 5} * 0.75^{5} * (1-0.3)^{8-5}$$
$$= 0.2076$$

a) $B \sim Bin(12, 0.45)$, therefore the probability is given by

$$P(B \text{ will get 5 correct}) = {12 \choose 5} * p^x * (1-p)^{n-x}$$
$$= {12 \choose 5} * 0.45^5 * (1-0.45)^{12-5}$$
$$= 0.2224$$

What happens in cases of 4 and 6 correct solutions?

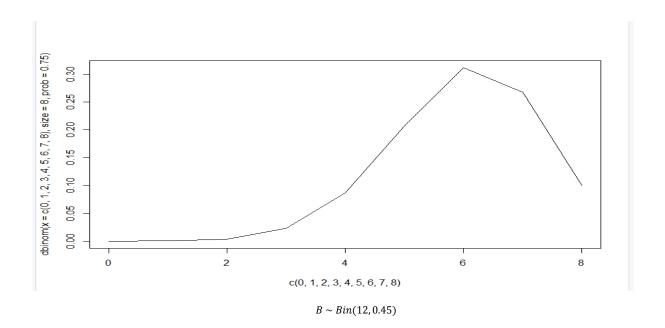
In cases of 4 and 6 probabilities of getting 6 correct is much higher than that of getting 4 correct

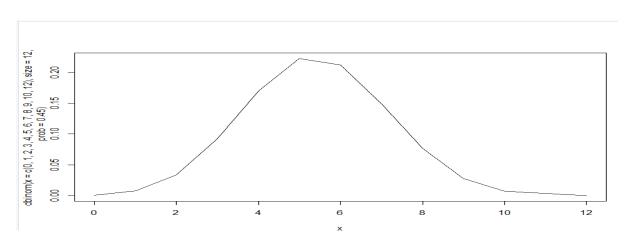
What are the two main governing factors affecting their ability to solve questions correctly?

- Number of trials
- Probability of correction

Pictorial View

 $G \sim Bin(8, 0.75)$





Problem Statement 3

Customers arrive at a rate of 72 per hour to my shop. What is the probability of k customers arriving in 4 minutes? a) 5 customers, b) not more than 3 customers, c) more than 3 customers.

Give a pictorial representation of the same to validate your answer.

Solution 3: Let X be the random variable that represents the number of customers who arrive at a shop. $\mu = \left(\frac{72}{60}\right) * 4 = 4.8$, thus $X \sim Po(4.8)$

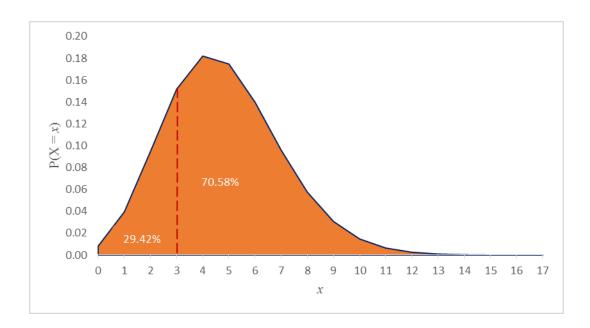
a)
$$P(X = 5) = \frac{e^{-\mu_* \mu^X}}{x!} = \frac{e^{-4.8_* \cdot 4.8^5}}{5!} = \mathbf{0.1747}$$

b)
$$P(X \le 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$$

= $\frac{e^{-4.8} \cdot 4.8^0}{0!} + \frac{e^{-4.8} \cdot 4.8^1}{1!} + \frac{e^{-4.8} \cdot 4.8^2}{2!} + \frac{e^{-4.8} \cdot 4.8^3}{3!} = \mathbf{0.2942}$

c)
$$P(X > 3) = 1 - P(X \le 3) = 1 - 0.2942 = 0.7058$$

d) The pictorial view of this is given by



Problem Statement 4

I work as a data analyst in Aeon Learning Pvt. Ltd. After analysing data, I make reports, where I have the efficiency of entering 77 words per minute with 6 errors per hour. What is the probability that I will commit 2 errors in a 455-word financial report? What happens when the no. of words increases (in case of 1000 words) or decreases (255 words)? How is the λ affected? How does it influence the PMF? Give a pictorial representation of the same to validate your answer.

Solution 4:

At 77 words per minute, it means in an hour the analyst will type 4260 words, in which he is susceptible to 6 errors. Let X be the randon variable that represents the number of errors in an hour at 77 words per minute (4260 per hour)

Thus $X \sim Po(6)$

a) For a 455 word document, the error rate becomes lower i.e. $\mu = \frac{455}{4260} * 6 = 0.591$, thus $X \sim Po(0.591)$

$$P(X = 2) = \frac{e^{-0.591} \cdot 0.591^2}{2!} = \mathbf{0.0964}$$

 $P(X=2) = \frac{e^{-0.591}*0.591^2}{2!} = \textbf{0.0964}$ b) For a 1000 word document, the error rate becomes $\mu = \frac{1000}{4260} * 6 = \frac{100}{77} = 1.299$, thus $X \sim Po(1.299)$

$$P(X = 2) = \frac{e^{-1.299} * 1.299^2}{2!} = 0.2301$$

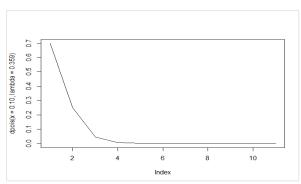
c) For a 255 word document, the error rate becomes $\mu = \frac{255}{4260} * 6 = 0.359$, thus $X \sim Po(0.359)$

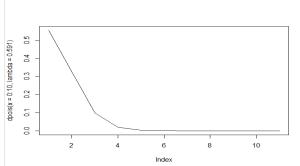
$$P(X=2) = \frac{e^{-0.359} * 0.359^2}{2!} = \mathbf{0.045}$$

The likelihood of making 2 errors increases as the number of words increases and decreases as the number of words in a document decreases.

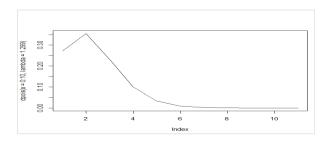
How does it influence the PMF?

255 words 455 words





1000 words



Problem Statement 5

The current measured in a copper wire is modelled by a continuous random variable (is in mA.) Assume that the range of X is [0, 20mA]. The probability density function is given by f(x) = 0.05 for $0 \le x \le 20$. What is the probability that a current measurement is less than 10 milliamperes? Draw the PDF and the CDF diagrams as well.

a)
$$P(X < 10) = \int_{0}^{10} 0.05 \, dx$$

$$= 0.05x \Big|_{0}^{10}$$

$$= 0.05(10) - 0.05(0)$$

$$= 0.5$$

b)

The PDF diagram is as follows

c) The CDF diagram is given below

