

Lecture: 1

Probability mass function (pmf) (discrete)

$$P_r\{Z=i\} \geq 0 \text{ for all } i$$

$$\sum_z P_r\{Z\} = 1$$

Probability density function (pdf) (continuous)

$$p(z) \geq 0 \text{ for all } z, \text{ and } \int p(z) dz = 1$$

Conditional distribution (product rule)

$$p(x, z) = p(z|x)p(x), \text{ if } p(x) \neq 0 \Rightarrow p(z|x) = \frac{p(x, z)}{p(x)}$$

Law of total probability (sum rule)

$$\bullet \text{ Discrete: } P_r\{Z\} = \sum_{x \in S_x} P_r\{x, Z\} = \sum_{x \in S_x} P_r\{Z|x\} P_r\{x\}$$

$$\bullet \text{ Continuous: } p(z) = \int_{x \in S_x} p(x, z) dx = \int_{x \in S_x} p(z|x) p(x) dx$$

Expected value (mean vector)

$$\mathbb{E}\{x\} = \int x p(x) dx$$

Covariance matrix

$$\text{Cov}\{x\} = \mathbb{E}\{[x - \mathbb{E}\{x\}][x - \mathbb{E}\{x\}]^T\}$$

Law of large numbers

$$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^n x_i = \mathbb{E}_{p(x)}\{x\}$$

Gaussian distribution

$$x \sim N(\mu, Q)$$

$$\Rightarrow \text{pdf: } p(x) = N(x; \mu, Q) = \frac{1}{\sqrt{|2\pi Q|}} \exp\left(-\frac{1}{2}(x-\mu)^T Q^{-1}(x-\mu)\right)$$