SSY345 - Sensor fusion

Lecture: 4

Linear and Gaussian model

$$x_k = A_{k-1} x_{k-1} + q_{k-1}, \quad q_{k-1} \sim N(\bar{q}_{k-1}, Q_{k-1})$$

$$y_k = H_k x_k + r_k, \quad r_k \sim N(\bar{r}_k, R_k)$$

and $x_0 \sim N(\bar{x}_0, P_{0|0})$

Kalman filter

- The kalman filter recursively computes

$$p(x_k | Y_{1:k-1}) = N(x_k; \hat{x}_{k|k-1}, P_{k|k-1}) \quad \text{(Prediction step)}$$

$$p(x_k | Y_{1:k}) = N(x_k; \hat{x}_{k|k}, P_{k|k}) \quad \text{(Update step)}$$

Prediction step

$$\hat{x}_{k|k-1} = A_{k-1} \hat{x}_{k-1|k-1}$$

$$P_{k|k-1} = A_{k-1} P_{k-1|k-1} A_{k-1}^T + Q_{k-1}$$

Update step

$$\left.\begin{aligned} \hat{x}_{k|k} &= \hat{x}_{k|k-1} + K_k v_k \\ P_{k|k} &= P_{k|k-1} - K_k S_k K_k^T \end{aligned}\right\} \text{where} \Rightarrow \left\{\begin{aligned} K_k &= P_{k|k-1} H_k^T S_k^{-1} \\ v_k &= y_k - H_k \hat{x}_{k|k-1} \\ S_k &= H_k P_{k|k-1} H_k^T + R_k \end{aligned}\right.$$

Decomposing joint expectations (product rule)

x and y are random variables

$$\mathbb{E}\{g(x,y)\} = \mathbb{E}\{\underbrace{\mathbb{E}\{g(x,y)|y\}}_{h(y)}\} = \mathbb{E}\{h(y)\}$$

well performing filter should satisfy

$$\mathbb{E}\{x_k - \hat{x}_{k|k}\} = 0$$

$$\underbrace{\mathbb{E}\{(x_k - \hat{x}_{k|k})(x_k - \hat{x}_{k|k})^T | y_{1:k}\}}_{P_{k|k}} = \underbrace{\mathbb{E}\{(x_k - \hat{x}_{k|k})(x_k - \hat{x}_{k|k})^T\}}_{MSE}$$

Innovation consistensy ($v_k = y_k - H_k \hat{x}_{k|k-1}$)

$$p(v_k | y_{1:k-1}) = N(v_k ; 0, S_k)$$

$$\mathrm{Cov}(v_k, v_{k-l}) = \begin{cases} \mathrm{Cov}\{v_k\} & \text{if } l = 0 \\ 0 & \text{otherwise} \end{cases}$$

whiteness

$$p(l) = \frac{\sum_{k=l+1}^{K} v_k^T v_{k-l}}{\sum_{\tau=l+1}^{K} v_\tau^T v_\tau}$$

LMMSE objective (static example)

- Find A and b such that $\hat{x} = Ay + b$ yields smallest MSE, $\mathbb{E}\{(x-\hat{x})^T(x-\hat{x})\}$