SSY281 Model Predictive Control

Assignment 4

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Question 1: MPT and Polyhedral sets

a

The H- and V- representations of the given Polyhedron are plotted using MPT and is shown in figure 1.

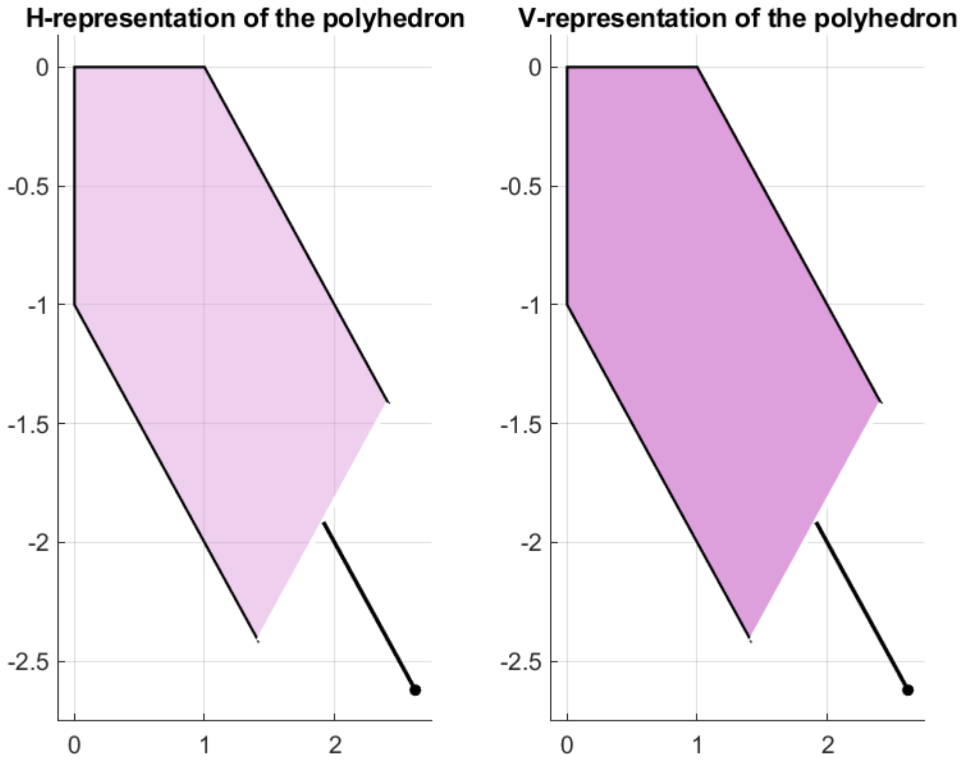

Figure 1: H- and V-representation of Polyhedron

As specified in the question, the V-representation was without considering the tail, i.e., without considering the unbounded part of the polyhedron. Hence the V-representation in the figure 1 is constructed with a ray R which shows the direction in which the polyhedron is unbounded. As expected, both H- and V-representations yield the same result.

b

The polyhedrons obtained with Minkowski sum and Pontryagin difference with the given polyhedrons P and Q are shown in figure 2. The polyhedron $(Q-P)+P$ is expected to be empty as $Q-P=\{\}$ and $\{\}\oplus P=\{\}$, but we get a polyhedron since MPT incorrectly returns P, which is verified in figure 2.

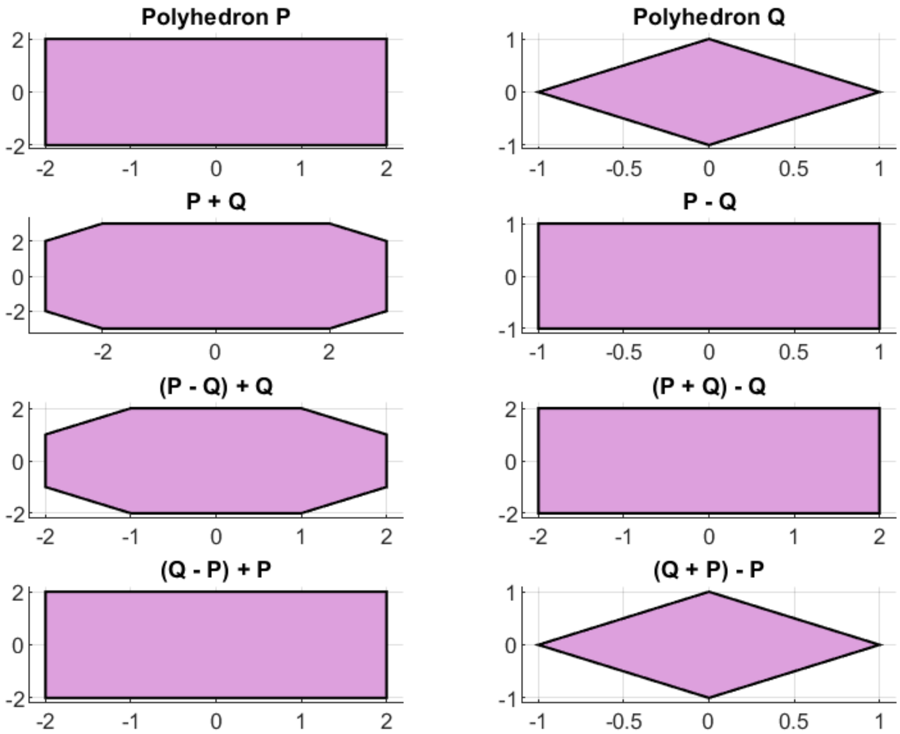

Figure 2: Minkowski sum and Pontryagin difference operations

Question 2: Forward and backward reachability

a

If the given set S is positively invariant, we can construct the reachable set $reach(S)$ of S with the given inequalities and definition as follows-

$$\forall x \in S \rightarrow x^+ \in S$$
$$x^+ = Ax$$
$$\implies A^{-1}x^+ = x$$
$$\text{and} \quad S := \{x : A_{in}x \leq b_{in}, x \in \mathbb{R}^2\}$$
$$\therefore reach(S) = \{x : A_{in}A^{-1}x \leq b_{in}, x \in \mathbb{R}^2\}$$

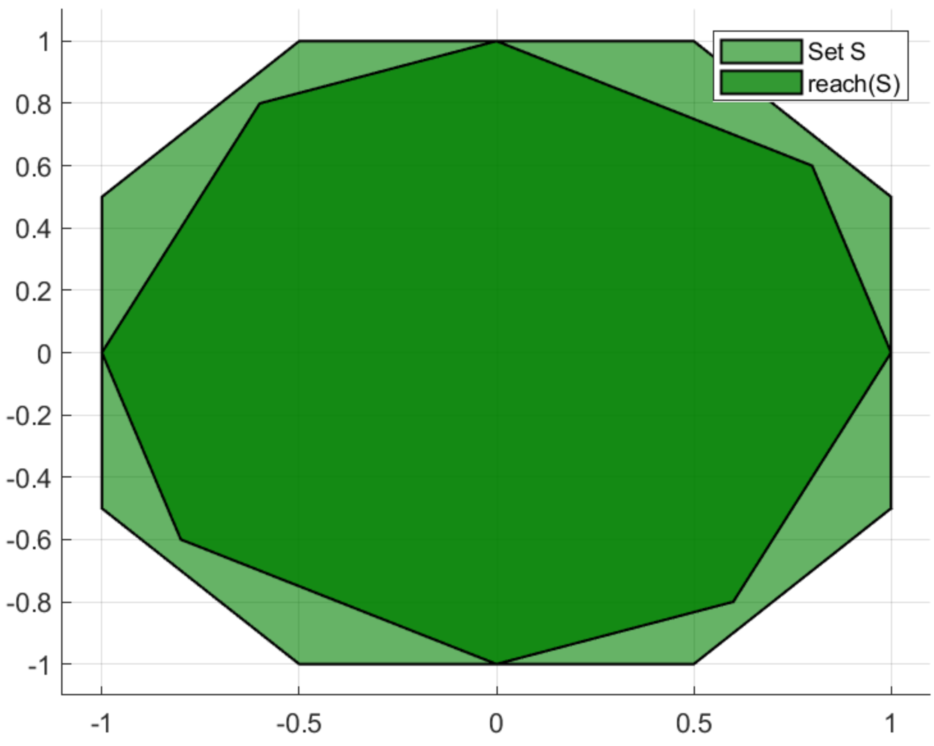

Figure 3: Set S and $reach(S)$

Since eigenvalues of $x^+ = Ax$ lie in the unit circle, it is stable and $reach(S)$ is positively invariant. Thus $reach(S) \subseteq S$, as shown in figure 3.

b

The given constraint can be rewritten as

$$U = \{u : A_u u \leq b_u, u \in \mathbb{R}\},$$
$$\text{where} \quad A_u = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, b_u = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

To obtain the one-step 3D reachable set,

$$x^+ = Ax + Bu$$
$$\implies x = A^{-1}x^+ - A^{-1}Bu$$
$$\implies A_{in}A^{-1}x^+ - A_{in}A^{-1}Bu \leq b_{in}$$
$$\implies \begin{bmatrix} A_{in}A^{-1} & -A_{in}A^{-1}B \\ 0 & A_u \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} \leq \begin{bmatrix} b_{in} \\ b_u \end{bmatrix}$$

The one-step reachable set, $reach(S)$ is within this 3D set and can be obtained by projecting the set onto x_1x_2 plane, computed using projection command in Matlab is shown in figure 4.

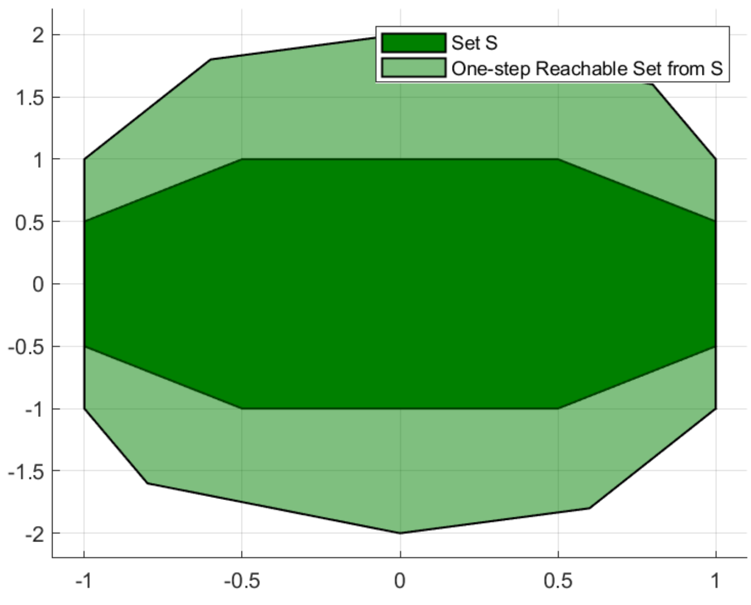

Figure 4: Set S and One-step Reachable set from S

c

To find the 3D one-step precursor set,

$$A_{in}(Ax + Bu) \leq b_{in}$$
$$\begin{bmatrix} A_{in}A & A_{in}B \\ 0 & A_u \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} \leq \begin{bmatrix} b_{in} \\ b_u \end{bmatrix}$$

The one-step precursor set, $pre(S)$ is within this 3D set and can be obtained by projecting the set onto $x_1 x_2$ plane, computed using projection command in Matlab is shown in figure 5.

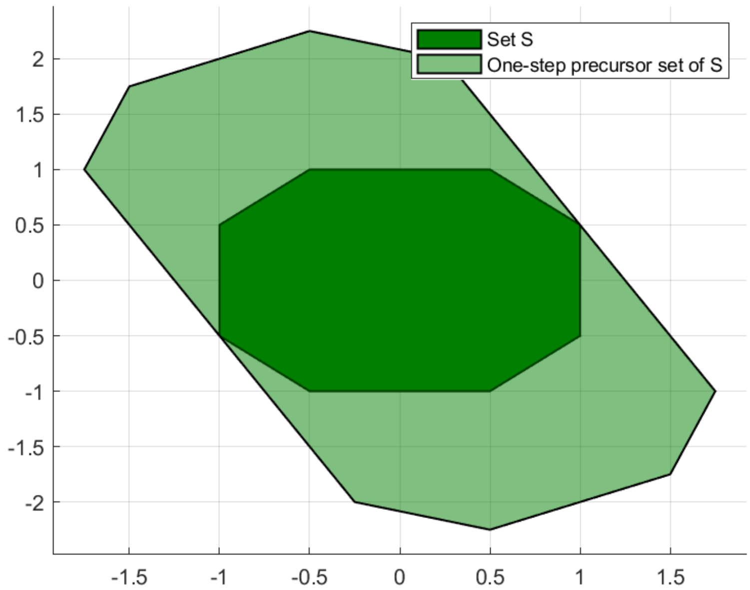

Figure 5: Set S and One-step precursor set of S

Question 3: Persistent feasibility

a

b

c