SSY281 - Model Predictive Control

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Lecture: 1

Limitations of linear control design:
- Buffer control
- Saturation on control and control rates
- Process output limitations

Important attributes of MPC
- MPC handles actuator limitations and process constraint
- MPC handles multivariable systems
- MPC is model based

The reciding horizon idea
1. At time k, predict response over finite predict horizon N. This depends on future control input over control horizon M
2. Pick control sequence with best performance depending on objective; cost or criterion
3. Apply first element and recalculate everything as done in Step 1.

Example: Reciding horizon control of integrator system
- System at time k, with output dynamics: $y(k+1)=y(k)+u(k)$
- Past inputs: $\{\dots, u(k-2), u(k-1)\}$
- $u(k+1|k)$: planned future control input at time $k+1$, given info at time k
- $\Delta u(k+1|k)=u(k+1|k)-u(k|k)$: Control increment
- $\hat{y}(k+1|k)$: predicted output at time $k+1$
- $r(k)$: a reference signal to be followed by the output

For a prediction horizon: $N=2$, the predicted outputs are written as

$$\hat{y}(k+1|k)=y(k)+u(k|k)=y(k)+u(k-1)+\Delta u(k|k)=y_f(k+1|k)+\Delta u(k|k)$$

$$\hat{y}(k+2|k)=y(k)+u(k|k)+u(k+1|k)=y(k)+2u(k|k)+\Delta u(k+1|k)$$

$$=y(k)+2u(k-1)+2\Delta u(k|k)+\Delta u(k+1|k)$$

$$=y_f(k+2|k)+2\Delta u(k|k)+\Delta u(k+1|k)$$

Case 1: Control horizon of $M=1$

Only one future control input to be choosen, assume control constant after that, i.e. $\Delta u(k+1|k)=0$

$$V_2=(\hat{y}(k+1|k)-r(k+1))^2+(\hat{y}(k+2|k)-r(k+2))^2$$
$$=(y_f(k+1|k)+\Delta u(k|k)-r(k+1))^2+(y_f(k+2|k)+2\Delta u(k|k)-r(k+2))^2$$

Solve by differentiate V_2 and set to 0.

$$\frac{\partial V_2}{\partial \Delta u(k|k)}=2\cdot(y_f(k+1|k)+\Delta u(k|k)-r(k+1))+2(y_f(k+2|k)+2\Delta u(k|k)-r(k+2))\cdot 2=0$$

This gives optimal (incremental) control

$$\Delta u(k|k)=\frac{1}{5}((r(k+1|k)-y_f(k+1|k))+2(r(k+2)-y_f(k+2|k)))$$

which gives a linear system to solve

$$\begin{cases}\hat{y}(k+1|k)=y_f(k+1|k)+\Delta u(k|k)=r(k+1)\\ \hat{y}(k+2|k)=y_f(k+2|k)+2\Delta u(k|k)=r(k+2)\end{cases}$$

Rewritten in vector notation

$$\underbrace{\begin{bmatrix}y_f(k+1|k)\\ y_f(k+2|k)\end{bmatrix}}_{Y_f}+\underbrace{\begin{bmatrix}1\\ 2\end{bmatrix}}_{\Theta}\Delta u(k|k)=\underbrace{\begin{bmatrix}r(k+1)\\ r(k+2)\end{bmatrix}}_{r}\iff Y_f+\Theta\Delta u(k|k)=r$$

Solution found by using least-squares on $\|\Theta\Delta u(k|k)-(r-Y_f)\|^2$. which is same as:

$$\Delta u(k|k)=(\Theta^T\Theta)^{-1}\Theta^T(r-Y_f)$$

Summary: The MPC example recipe

1. At time k, predict output N samples ahead
   $\hat{y}(k+1|k), \ldots, \hat{y}(k+N|k)$
2. Prediction depends on future control inputs
   $u(k|k), u(k+1|k), \ldots, u(k+M|k)$
3. Minimize a criterion
   $V(k) = V(\hat{y}(k+1:k+N|k), u(k:k+M|k))$ with
   respect to control sequence $u(k:k+M-1|k)$
4. Apply first control signal in sequence
   to the process: $u(k) = u(k|k)$
5. Increment time $k := k+1$ and go to step 1 again.

MPC Ingredients (focus: linear models, quadratic criteria)

- Internal model describing process and disturbance
- Estimator/predictor to determine evolution of state
- objective/criterion to express desired system behaviour
- online optimization algorithm to determine future control
- Receding horizon principle